A Continuous Time Physical Graph based Formulation to Scheduled Service Network Design

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Abstract. Scheduled Service Network Design supports consolidation-based freight carriers in setting up a transportation network by selecting the transportation services to operate, with their schedules, and the itineraries of the commodities to move. We propose a new formulation to the problem that represents time in its continuous nature, directly over the physical graph, thus mitigating the drawbacks that a traditional formulation, relying on a time-space network, may have for large scale instances, due to the increase in its dimensions and the consequent intractability in solving the problem exactly. Preliminary numerical experiments comparing the new and traditional formulations on a set of randomly generated instances are performed. Results highlight that the proposed formulation is a valuable tool to solve large scale instances with a long schedule length.

Keywords: Scheduled Service Network Design, Freight Transportation, Continuous Time Representation, Integer Linear Programming

1 Introduction

Consolidation-based freight transportation carriers group and dispatch within the same vehicle or convoy (e.g., a truck, a container ship, a freight train) multiple shipments, each potentially associated with a different customer. Consolidation is crucial to the profitability of these carriers, since it increases resource utilization and reduces transportation costs and prices as economies of scale can be exploited. Postal and small-package transportation companies, less-than-truckload motor carriers, railroads, maritime liner navigation companies perform similar services for freight. Such carriers operate over a network composed of terminals connected by an infrastructure (e.g., highways or rail tracks) or conceptual links (e.g., maritime corridors). The terminals come in several designs and sizes, depending on the particular transportation modes. To achieve consolidation, the carriers operate *transportation services* over the network, according to a defined *route*, from an origin to a destination terminal, possibly with intermediate stops, and a *schedule*, giving timing information related to the time of departure from and arrival at each stop. Such transportation services are used to route shipments from their origin towards their destination terminals, through *itineraries* possibly visiting intermediate terminals where consolidation is fulfilled, and thus loading/unloading and service-to-service transfer operations are performed.

Jointly determining the scheduled services to operate and the commodity itineraries is a rather complex tactical-planning problem referred to as Scheduled Service Network Design (SSND). Its goal is to define the transportation plan (i.e., scheduled services and commodity itineraries) with the objective of minimizing the total transportation cost. The problem is normally addressed for a certain period of time, called *schedule period*, with respect to which some demand regularity is observed over a long period, called planning horizon. For example, a commodity requiring transportation every week over a period of six months gives rise to a weekly schedule period within a six month planning horizon. The obtained transportation plan is then cyclically repeated over the planning horizon (e.g., the weekly schedule is repeated over the six months).

Many variants of SSND have been studied in the literature [1]. Some papers focus on adapting the SSND to specific contexts and modes of transportation (see [2] for rail, truck and maritime transportation). Other papers, instead, address extensions of the classical SSND including additional management issues such as empty repositioning of resources (e.g., vehicles or containers), or resource management considerations [3,4,5,6]. Finally, a few contributions tackle the SSND through stochastic methodologies explicitly accounting for the uncertainty affecting, for instance, volume of demand [7,8] or travel time of services [9,10].

A traditional way to formulate the SSND is resorting to a *time-space network*, a particular graph structure that incorporates both temporal and spatial information related to the application to deal with. In case of SSND, a time-space network is constructed from the graph representing the physical network over which the carriers operate, where nodes represent terminals and arcs service legs. The schedule period is then partitioned into discrete intervals. The original nodes of the graph are replicated for each interval, and arcs are appropriately added to connect the timed-nodes. Although time-space networks provide a flexible modeling technique, the decision on the appropriate time discretization is crucial. A fine discretization usually gives good approximations to the continuous time formulation at the expense of large, and often intractable, models. A time-space network based on a one-minute time discretization gives a continuous time SSND [11]. As opposed, a coarse discretization, even though more computationally amenable, may yield to a poor quality solution due to a rough time approximation [12]. A recent branch of study for SSND is thus focusing on methods managing time according to its continuous nature, in order to mitigate such a loss of quality in solutions. Specifically, [11] proposes an iterative refinement algorithm, still based on time-space networks, to obtain continuous time SSND solutions, while [13] introduces a continuous time formulation based on the enumeration of consolidation paths of shipments, which may, however, be computationally ineffective when the number of shipments is large.

In this paper, we propose a new continuous time formulation using directly the physical graph of SSND (SSND-CPG, in the following), which does neither rely on a time-space network nor on the enumeration of consolidation paths of shipments. We also report some preliminary computational results on the comparison between the traditional formulation of SSND relying on the time-space network (SSND-TS, in the following) and SSND-CPG. The reported results highlight the good performance of SSND-CPG for instances characterized by a long schedule length. The paper is organized as follows. Section 2 describes the problem and the notation used, it contains a short recall of SSND-TS and the presentation of the proposed formulation SSND-CPG. Section 3 describes the computational study and the obtained results. Finally, Section 4 concludes the paper.

2 Problem description, notation and formulations

The physical network on which the carriers operate is represented by a directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$, with node set $\mathcal{N}$ representing terminals and arc set $\mathcal{A}$ modelling service legs. Let T be the chosen schedule length, and thus $[0, T]$ be the schedule period. The demand is represented by a set of commodities $\mathcal{K}$, each commodity $k \in \mathcal{K}$ requiring to transport a certain volume w_k from an origin terminal $o_k \in \mathcal{N}$ to a destination terminal $d_k \in \mathcal{N}$, according to its availability time at origin, $a_k \in [0, T]$, and due date at destination, b_k. We assume that each commodity k must follow a single path from the origin to the destination, and that its itinerary spans over T, i.e., $b_k \leq a_k + T$. The set of potential services the carriers may operate is specified by Σ. Each service $\sigma \in \Sigma$ follows a route in the physical network, represented by a directed path $\mathcal{P}_\sigma = (\mathcal{N}_\sigma, \mathcal{A}_\sigma)$, where $\mathcal{N}_\sigma$ describes the set of terminals visited by σ, from its origin o_σ to its destination d_σ, and $\mathcal{A}_\sigma$ models the service legs. Alongside with routing information, timing information is specified: $\hat{\phi}_{i,\sigma}$ denotes the departure time of service σ from node i, for any $i \in \mathcal{N}_\sigma \setminus \{d_\sigma\}$, while $\hat{\psi}_{i,\sigma}$ denotes the arrival time of service σ to node i, for any $i \in \mathcal{N}_\sigma \setminus \{o_\sigma\}$. As for the commodities, we assume that the route of each service σ spans over T, i.e., $\hat{\psi}_{d_\sigma,\sigma} \leq \hat{\phi}_{o_\sigma,\sigma} + T$. We also assume $\hat{\psi}_{d_\sigma,\sigma} \leq T$. Each service σ has a capacity u_σ. Finally, an activation cost f_σ is associated with each service σ, a unit commodity transportation cost $c^k_{(i,j)\sigma}$, differentiated for service, is associated with each commodity $k \in \mathcal{K}$, service $\sigma \in \Sigma$ and arc $(i, j) \in \mathcal{A}_\sigma$, and a cost c^k_i is associated with each commodity $k \in \mathcal{K}$ and each node $i \in \mathcal{N}$, modelling the unit cost of holding goods of type k at terminal i.

The goal is to select, among the potential services the carriers may activate, those services that allow to satisfy the transportation demand at the minimum cost. As previously indicated, the selected transportation plan is then cyclically repeated to cover a given planning horizon. Notice, in particular, that each commodity in $\mathcal{K}$ and each service in Σ occur once within the schedule period, while they occur within the planning horizon with periodicity T.

2.1 The time-space network formulation

For brevity, we report here a short presentation of the time-space network formulation, named SSND-TS, by referring to [1] for an in-depth description. Chosen a time discretization of the schedule period, the graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ is replicated according to it, generating the time-space network $\mathcal{G}_{TS} = (\mathcal{N}_{TS}, \mathcal{A}_{TS})$. Each node in $\mathcal{N}_{TS}$ represents a terminal at a specific time instant, while $\mathcal{A}_{TS}$ is composed of the set of *holding* arcs, i.e., arcs between representations of the same node in two consecutive periods, used to model idle time at terminals for freights or services, and the set of *moving arcs*, i.e., arcs between representations of two different nodes in two different periods, standing for potential service legs. Given the cyclic nature of the schedule, the time-space network wraps-around, allowing operations starting at the end of the schedule period to terminate at its beginning through appropriate arcs. SSND-TS then takes the form of a fixed-cost, capacitated, multi-commodity network design formulation over $\mathcal{G}_{TS}$, aiming at minimizing the transportation cost (service activation and commodity routing costs) under commodity flow-conservation and linking-capacity constraints. Notice that, depending on how time discretization is performed, SSND-TS could provide just an approximate formulation to the problem addressed.

2.2 The continuous time physical graph based formulation

The formulation we propose considers directly the physical graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$. Given the circularity of the schedule, any commodity in $\mathcal{K}$, besides been potentially transported by services travelling within the period $[0, T]$, may also be potentially transported by a service starting from its origin before time 0, or similarly, by a service arriving at its destination after time T. To account for this, we partition the service set Σ into two subsets, say Σ_1 and Σ_2. The subset Σ_1 contains those services leaving their origin before time 0 and arriving at their destination after time 0, while the services belonging to Σ_2 travel within [0, T], i.e., they leave their origin at a time greater than or equal to 0 and arrive at their destination within T. Notice that potential services leaving their origin within [0, T] and arriving at their destination after T are indeed translations in time of services belonging to Σ_1. In order to characterize those services which can be used to satisfy the demand in $\mathcal{K}$, we define an enlarged set $\tilde{\Sigma}$. It contains all the services in Σ, plus two copies of each service $\sigma_1 \in \Sigma_1$ and one copy of each service $\sigma_2 \in \Sigma_2$. Specifically, given $\sigma_1 \in \Sigma_1$, the two additional copies σ_1' and σ_1''

are defined in such a way that:

$$\begin{aligned}
&\mathcal{P}_{\sigma_1'} = \mathcal{P}_{\sigma_1} && \mathcal{P}_{\sigma_1''} = \mathcal{P}_{\sigma_1}\\
&\hat{\phi}_{i,\sigma_1'} = \hat{\phi}_{i,\sigma_1} + T \quad \forall\, i \in \mathcal{N}_{\sigma_1} \setminus \{d_{\sigma_1}\} && \hat{\phi}_{i,\sigma_1''} = \hat{\phi}_{i,\sigma_1} + 2T \quad \forall\, i \in \mathcal{N}_{\sigma_1} \setminus \{d_{\sigma_1}\}\\
&\hat{\psi}_{i,\sigma_1'} = \hat{\psi}_{i,\sigma_1} + T \quad \forall\, i \in \mathcal{N}_{\sigma_1} \setminus \{o_{\sigma_1}\} && \hat{\psi}_{i,\sigma_1''} = \hat{\psi}_{i,\sigma_1} + 2T \quad \forall\, i \in \mathcal{N}_{\sigma_1} \setminus \{o_{\sigma_1}\}\\
&c^k_{(i,j)\sigma_1'} = c^k_{(i,j)\sigma_1} \quad \forall\, k \in \mathcal{K},\ \forall\, (i,j) \in \mathcal{A}_{\sigma_1} && c^k_{(i,j)\sigma_1''} = c^k_{(i,j)\sigma_1} \quad \forall\, k \in \mathcal{K},\ \forall\, (i,j) \in \mathcal{A}_{\sigma_1}.
\end{aligned}$$

Similarly, given $\sigma_2 \in \Sigma_2$, the additional copy σ_2' is defined in such a way that:

$$\begin{aligned}
&\mathcal{P}_{\sigma_2'} = \mathcal{P}_{\sigma_2}\\
&\hat{\phi}_{i,\sigma_2'} = \hat{\phi}_{i,\sigma_2} + T \quad \forall\, i \in \mathcal{N}_{\sigma_2} \setminus \{d_{\sigma_2}\}\\
&\hat{\psi}_{i,\sigma_2'} = \hat{\psi}_{i,\sigma_2} + T \quad \forall\, i \in \mathcal{N}_{\sigma_2} \setminus \{o_{\sigma_2}\}\\
&c^k_{(i,j)\sigma_2'} = c^k_{(i,j)\sigma_2} \quad \forall\, k \in \mathcal{K},\ \forall\, (i,j) \in \mathcal{A}_{\sigma_2}.
\end{aligned}$$

Services σ_1' and σ_1'' thus represent translations in time of service $\sigma_1 \in \Sigma_1$, whose operations are shifted in the future after T and $2T$ periods, respectively, while service σ_2' represents the translation T periods ahead of service $\sigma_2 \in \Sigma_2$. We define such services as *twin-services*, and denote by Γ_σ the set of twin-services of service σ. By construction, $\tilde{\Sigma} = \Sigma \cup \bigcup_{\sigma \in \Sigma} \Gamma_\sigma$ is the set of those services that the commodities, whose availability date is in $[0,T]$, may potentially use to be transported towards their destination. Due to the definition of $\tilde{\Sigma}$, we schedule over an enlarged time period $[T', T'']$, where

$$T' = \min_{\sigma \in \tilde{\Sigma}} \ \hat{\phi}_{o_\sigma,\sigma}, \qquad T'' = \max_{\sigma \in \tilde{\Sigma}} \ \hat{\psi}_{d_\sigma,\sigma}.$$

The interval $[T', T'']$ will be called the *planning period*.

Figure 1 provides an example of twin-services and planning period. Specifically, in the considered schedule period $[0,T]$ there are three commodities to satisfy: commodity 1, whose availability and due dates are both within $[0,T]$, and commodities 2 and 3, whose availability dates fall in $[0,T]$ while the due dates are after T. They are represented by gray rectangles. Two services are available: $\sigma_1 \in \Sigma_1$ (black truck) and $\sigma_2 \in \Sigma_2$ (white truck). Therefore, $\tilde{\Sigma}$ contains five services: σ_1 and its two copies σ_1' and σ_1'', and σ_2 together with its copy σ_2', as shown in the figure. The planning period spans from $T' = \hat{\phi}_{o_{\sigma_1},\sigma_1}$ to $T'' = \hat{\psi}_{d_{\sigma_1''},\sigma_1''} = \hat{\psi}_{d_{\sigma_1},\sigma_1} + 2T$. Given the regularity in demand, the three commodities must be transported more than once within the planning period $[T', T'']$, with periodicity T, sometimes with an availability date outside the schedule period. Such a periodicity of demand with respect to T is represented in the figure by dashed gray rectangles. The five services in $\tilde{\Sigma}$ are those that may potentially be used by the three commodities to reach their destination. Consider for example service σ_1 and its twin-services σ_1' and σ_1''. As said, they all represent the same service, translated in time, over the planning period. Their operations are thus performed over the same set of physical links, shifted however

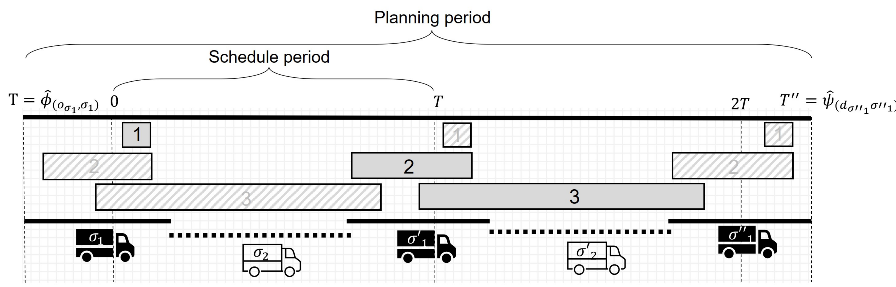

Fig. 1. Example of twin-services, schedule period and planning period.

ahead T times (considering σ_1') and $2T$ times (considering σ_1'') with respect to σ_1. Given the regularity in demand, in some time instants the three commodities could simultaneously be transported over some physical links of service σ_1, as the figure shows. In order to guarantee to service σ_1 enough capacity to transport commodities 1, 2 and 3 anytime, we therefore impose, through non-conventional capacity constraints, that the sum of the volumes of the commodities transported by σ_1 and by its two twin-services, i.e., σ_1' and σ_1'', does not exceed the capacity of σ_1. Similar constraints are imposed for σ_2.

2.3 Mathematical formulation SSND-CPG

We define four sets of variables:

- $y_\sigma \in \{0,1\}$, $\sigma \in \Sigma$, represents whether service σ is selected ($y_\sigma = 1$), or not ($y_\sigma = 0$);
- $x^k_{(i,j)\sigma} \in \{0,1\}$, $k \in \mathcal{K}$, $\sigma \in \tilde{\Sigma}$, $(i,j) \in \mathcal{A}_\sigma$, represents whether commodity k moves from i to j on board of service σ;
- $\varepsilon^k_i \geq 0$, $k \in \mathcal{K}$, $i \in \mathcal{N} \setminus \{d_k\}$, represents the time instant at which commodity k begins its movement from terminal i (it is 0 if k does not pass through i);
- $\eta^k_i \geq 0$, $k \in \mathcal{K}$, $i \in \mathcal{N} \setminus \{o_k\}$, represents the time instant at which commodity k ends its movement to terminal i (it is 0 if k does not pass through i).

The problem is formulated as follows:

$$\min \sum_{\sigma \in \Sigma} f_\sigma y_\sigma + \sum_{k \in \mathcal{K}} \sum_{\sigma \in \tilde{\Sigma}} \sum_{(i,j) \in \mathcal{A}_\sigma} c^k_{(i,j)\sigma} x^k_{(i,j)\sigma} + \sum_{k \in \mathcal{K}} \sum_{\substack{i \in \mathcal{N}: \\ i \neq o_k, d_k}} c^k_i \left(\varepsilon^k_i - \eta^k_i\right) + \sum_{k \in \mathcal{K}} c^k_{o_k} \left(\varepsilon^k_{o_k} - a_k\right) + \sum_{k \in \mathcal{K}} c^k_{d_k} \left(b_k - \eta^k_{d_k}\right) \quad (1)$$

$$\sum_{\substack{\sigma \in \tilde{\Sigma}: \\ i \in \mathcal{N}_\sigma \setminus \{d_\sigma\}}} \sum_{\substack{j \in \mathcal{N}: \\ (i,j) \in \mathcal{A}_\sigma}} x^k_{(i,j)\sigma} - \sum_{\substack{\sigma \in \tilde{\Sigma}: \\ i \in \mathcal{N}_\sigma \setminus \{o_\sigma\}}} \sum_{\substack{j \in \mathcal{N}: \\ (j,i) \in \mathcal{A}_\sigma}} x^k_{(j,i)\sigma} = \begin{cases} 1 & \text{if } i = o_k, \\ -1 & \text{if } i = d_k, \\ 0 & \text{otherwise,} \end{cases} \quad (2)$$

$$\forall\, k \in \mathcal{K},\ \forall\, i \in \mathcal{N},$$

$$\sum_{k \in \mathcal{K}} w^k \, x^k_{(i,j)\sigma} + \sum_{\rho \in \Gamma_\sigma} \sum_{k \in \mathcal{K}} w^k \, x^k_{(i,j)\rho} \leq u_\sigma \, y_\sigma \qquad \forall \, \sigma \in \Sigma, \; \forall \, (i,j) \in \mathcal{A}_\sigma, \quad (3)$$

$$\varepsilon^k_{o_k} \geq a_k \qquad \forall \, k \in \mathcal{K}, \quad (4)$$

$$\eta^k_{d_k} \leq b_k \qquad \forall \, k \in \mathcal{K}, \quad (5)$$

$$\varepsilon^k_i \geq \eta^k_i \qquad \forall \, k \in \mathcal{K}, \; \forall \, i \in \mathcal{N} \setminus \{o_k, d_k\}, \quad (6)$$

$$\varepsilon^k_i \geq \hat{\phi}_{i,\sigma} - (T'' - T') \left(1 - x^k_{(i,j)\sigma}\right) \qquad \forall \, k \in \mathcal{K}, \; \forall \, \sigma \in \tilde{\Sigma}, \; \forall \, (i,j) \in \mathcal{A}_\sigma, \quad (7)$$

$$\varepsilon^k_i \leq \hat{\phi}_{i,\sigma} + (T'' - T') \left(1 - x^k_{(i,j)\sigma}\right) \qquad \forall \, k \in \mathcal{K}, \; \forall \, \sigma \in \tilde{\Sigma}, \; \forall \, (i,j) \in \mathcal{A}_\sigma, \quad (8)$$

$$\eta^k_j \geq \hat{\psi}_{j,\sigma} - (T'' - T') \left(1 - x^k_{(i,j)\sigma}\right) \qquad \forall \, k \in \mathcal{K}, \; \forall \, \sigma \in \tilde{\Sigma}, \; \forall \, (i,j) \in \mathcal{A}_\sigma, \quad (9)$$

$$\eta^k_j \leq \hat{\psi}_{j,\sigma} + (T'' - T') \left(1 - x^k_{(i,j)\sigma}\right) \qquad \forall \, k \in \mathcal{K}, \; \forall \, \sigma \in \tilde{\Sigma}, \; \forall \, (i,j) \in \mathcal{A}_\sigma, \quad (10)$$

$$\varepsilon^k_i \leq b_k \sum_{\substack{\sigma \in \tilde{\Sigma}: \\ i \in \mathcal{N}_\sigma \setminus \{d_\sigma\}}} \sum_{\substack{j \in \mathcal{N}: \\ (i,j) \in \mathcal{A}_\sigma}} x^k_{(i,j)\sigma} \qquad \forall \, k \in \mathcal{K}, \; \forall \, i \in \mathcal{N} \setminus \{o_k, d_k\}. \quad (11)$$

The objective function (1) is the sum of the fixed costs associated with the selected transportation services (first term), transportation costs (second term) and holding costs at terminals (third, fourth and fifth term). Flow conservation constraints (2) ensure that each commodity is routed from its origin to its destination through a single path. Linking-capacity constraints (3) guarantee that commodities can use selected services only, and that the total commodity flow on any service, considering all of its copies, cannot exceed its capacity. Constraints (4) ensure that each commodity departs from its origin after it becomes available, while constraints (5) guarantee that each commodity arrives at destination before its due date. Relations (6) regulate the operations of commodities in terms of time by ensuring that the leaving time of a commodity from a terminal (except for its destination) be greater than or equal to the time at which the commodity arrived at that terminal. Constraints (7)–(8) ensure that the leaving time of a commodity from a terminal equals the scheduled departure time of the service on which it is loaded. Similarly, (9)–(10) ensure that the arrival time of a commodity to a terminal equals the scheduled arrival time of the service on which it is transported. Finally, constraints (11) force to 0 the leaving times of commodities from terminals when not passing through them (note that arrival times are also forced to 0 by constraints (6)).

3 Experimental plan and computational results

We compared SSND-CPG and SSND-TS computationally on 4 groups of randomly generated instances, each composed of 5 instances. The number of terminals is the same for all the instances and it is equal to 10. Origin and destination

Table 1. Features of instances.

Instance	Schedule Period (weeks)	Services	Commodities	SSND-CPG $\lvert\mathcal{N}\rvert$	$\lvert\mathcal{A}\rvert$	SSND-TS $\lvert\mathcal{N}_{TS}\rvert$	$\lvert\mathcal{A}_{TS}\rvert$
1	1	224	40	10	30	105000	105226
2	1	224	40	10	30	105000	105226
3	1	224	40	10	30	105000	105226
4	1	224	40	10	30	105000	105226
5	1	224	40	10	30	105000	105226
6	2	448	80	10	30	210000	210448
7	2	448	80	10	30	210000	210448
8	2	448	80	10	32	210000	210448
9	2	462	80	10	31	210000	210462
10	2	495	80	10	31	225000	225495
11	3	528	120	10	22	480000	480528
12	3	769	120	10	30	315000	315672
13	3	672	120	10	30	315000	315630
14	3	693	120	10	31	315000	315609
15	3	672	120	10	30	315000	315630
16	4	896	160	10	30	420000	419990
17	4	864	160	10	30	405000	405864
18	4	896	160	10	30	420000	419990
19	4	891	160	10	31	405000	405891
20	4	924	160	10	31	420000	420924

of each commodity as well as its volume have been generated starting from two different uniform distributions. Their availability and due dates are instead generated from normal distributions as in [11]. Origin and destination of each service have been generated using a uniform distribution as well. The 4 groups of instances differ in terms of schedule length, number of potential services to activate and number of commodities to transport.

The features of the instances are reported in Table 1, whose first 4 columns report the ID of the instance, the schedule period (in weeks), the number of potential services, and the number of commodities, respectively. The last 4 columns report the number of nodes and arcs of the physical network used in SSND-CPG, and the number of nodes and arcs of the time-space network used in SSND-TS, which is based on a one minute time discretization. This gives rise to a continuous-time service network design formulation, as discussed in [11].

We solved the instances with CPLEX 12.6 and a time limit of 3 hours. The experiments have been run on an Intel Xeon 5120 with 2.20 GHz and 32 GB of RAM. The obtained results are reported in Table 2. The table is divided in 4 panels, each referring to a subset of instances with the same schedule period, namely 1, 2, 3 and 4 weeks. In each panel, the time in seconds required by CPLEX to solve each instance of the corresponding group by SSND-CPG and SSND-TS are reported, as well as the percentage optimality gap at the end of the CPLEX execution (i.e., when an optimal solution is found or the time limit of 3 hours is reached). For the set of smallest instances, i.e., those with a schedule period of 1 week, SSND-TS shows its superiority in solving SSND, since it requires on

Table 2. Comparison between SSND-CPG and SSND-TS formulations.

1 week schedule period				
	SSND-CPG		SSND-TS	
Inst.	Time	Gap	Time	Gap
1	34.93	0%	27.53	0%
2	216.76	0%	107.30	0%
3	533.08	0%	146.37	0%
4	415.38	0%	119.15	0%
5	529.57	0%	142.18	0%
Avg.	345.94	0%	108.51	0%

2 weeks schedule period				
	SSND-CPG		SSND-TS	
Inst.	Time	Gap	Time	Gap
6	10800	10.67%	10800	9.92%
7	10800	12.27%	10800	10.64%
8	10800	10.79%	10800	7.25%
9	10800	8.10%	10800	7.07%
10	10800	8.74%	10800	7.60%
Avg.	10800	10.11%	10800	8.50%

3 weeks schedule period				
	SSND-CPG		SSND-TS	
Inst.	Time	Gap	Time	Gap
11	10800	9.43%	-	-
12	10800	9.17%	-	-
13	10800	11.73%	-	-
14	10800	10.89%	-	-
15	10800	9.89%	-	-
Avg.	10800	10.22%	-	-

4 weeks schedule period				
	SSND-CPG		SSND-TS	
Inst.	Time	Gap	Time	Gap
16	10800	12.84%	-	-
17	10800	12.70%	-	-
18	10800	12.48%	-	-
19	10800	13.05%	-	-
20	10800	15.46%	-	-
Avg.	10800	13.31%	-	-

average one third of the time needed by CPLEX to solve SSND-CPG. In both cases, however, optimal solutions are always found, although, in the considered data set, optimal solutions found by the alternative formulations do not always share the same set of activated services. When the schedule period is 2 weeks, CPLEX always reaches the time limit, no matter of the formulation considered. However, CPLEX still shows a better performance in terms of optimality gap when coupled with SSND-TS. On the other hand, when a schedule period of 3 or 4 weeks is considered, CPLEX is not able to find any solutions when SSND-TS is used, due to the huge dimension of the corresponding time-space networks. As opposed, when considering SSND-CPG solutions are always found within the 3 hours of time limit imposed, with an average optimality gap of about 10% and 13%, respectively. The proposed formulation SSND-CPG, modelling time in a continuous and compact way, appears thus to be a valuable tool for solving SSND when a long schedule length needs to be considered.

4 Conclusions

We have proposed a continuous time formulation for the Scheduled Service Network Design Problem, addressing tactical planning decisions for consolidation-based freight carriers. Our formulation represents time according to its continuous nature, directly over the physical graph, thus mitigating the drawbacks that a traditional formulation, relying on a time-space network, may have for large scale instances, due to the increase in its dimensions and the consequent intractability in solving the problem exactly. We have compared the new formulation and the traditional one computationally over a set of randomly generated instances. The preliminary experiments highlight that the proposed formulation is a valuable tool to solve large scale instances with a long schedule length.

Future research will investigate additional features of SSND, and the design of alternative resolution approaches for solving even larger SSND instances.

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