Equilibrium concepts in transportation networks: generalized Wardrop conditions and variational formulations*

M. Pappalardo† M. Passacantando‡

Abstract: We present two new Wardrop-type definitions of equilibrium for transportation networks and we investigate their relationships with variational formulations of Stampacchia or Minty-type.

Key words: Transportation network, equilibrium solutions, generalized Wardrop condition, Minty variational inequalities.

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1 Introduction

In a transportation network an equilibrium flow is classically defined by Wardrop's principle: each traveler tries to minimize his own travel time irrespective of the other travelers, this principle is known as the *user equilibrium principle*. A lot of analysis has been carried out in order to connect this principle with a variational formulation. In fact, variational inequalities are very useful tools for the study of an equilibrium flow. In this paper we propose new definitions of equilibrium flow, we propose some relationships with variational formulations and we show that this study permits us to select different equilibrium points. Moreover these new definitions aim to analyse the evolution of a traffic network when the flow is not an equilibrium flow.

Now we briefly recall a definition which will be useful in the sequel.

Definition 1.1. Given a closed and convex set $K \subset \mathbb{R}^n$ and a vector function $F : K \to \mathbb{R}^n$. The Variational Inequality we consider, say SVI(F,K), consists in determining a

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†Department of Applied Mathematics, via Bonanno 25/b, 56126 - Pisa, Italy

‡Department of Mathematics, via Buonarroti 2, 56127 - Pisa, Italy

vector $x^* \in K$, such that

$$\langle F(x^*), x - x^* \rangle \geq 0, \quad \forall\, x \in K,$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in $\mathbb{R}^n$; the corresponding Minty Variational Inequality, say MVI(F,K), consists in determining a vector $x^* \in K$, such that

$$\langle F(x), x - x^* \rangle \geq 0, \quad \forall\, x \in K.$$

The following known result establishes relationships between the solutions of SVI(F,K) and those of MVI(F,K).

Theorem 1.1. (Generalized Minty Lemma) The following statements hold:

1. if F is continuous on K, then each solution to MVI(F,K) is a solution of SVI(F,K);
2. if F is pseudomonotone on K, that is

   $$\langle F(y), x - y \rangle \geq 0 \Rightarrow \langle F(x), x - y \rangle \geq 0, \quad \forall\, x, y \in K,$$

   then each solution to SVI(F,K) is a solution to MVI(F,K).

2 Equilibrium model in a traffic network

Let (N, A, W) be a transportation network where $N = \{N_1, ..., N_p\}$ is the set of nodes, $A = \{A_1, ..., A_n\}$ the set of directed arcs and $W = \{W_1, ..., W_l\}$ the set of OD (origin-destination) pairs. We denote by $\mathcal{R}_j$ the set of those paths R_r, $r = 1, ..., r_j$, that connect the pair $W_j \in W$, and by F_r, $r = 1, ..., r_j$, the path flow on R_r. If we consider the set of all paths $\mathcal{R} = \bigcup_{j=1}^{l} \mathcal{R}_j$ and arrange the path flows into a vector $F \in \mathbb{R}^m$ where $m = r_1 + \cdots + r_l$, we obtain a column vector $F = (F_1, ..., F_m)$, whose components F_r represent the flow on the path R_r, $r = 1, ..., m$ suitably rearranged. A feasible flow has to satisfy demand requirements,

$$\sum_{r=1}^{r_j} F_r = \rho_j, \qquad j = 1, ..., l,$$

where $\rho \geq 0$ is given in R^l. Introducing the pair-path incidence matrix $\phi = (\phi_{j\,r})$, namely

$$\phi_{j\,r} = \begin{cases} 1 & \text{if } R_r \in \mathcal{R}_j, \\ 0 & \text{if } R_r \notin \mathcal{R}_j, \end{cases}$$

the demand requirements can be written as

$$\phi\, F = \rho.$$

Thus, the set of all feasible flows is given by

$$K = \{F \in \mathbb{R}^m : \ F \geq 0, \quad \phi\, F = \rho\}.$$

The flow on arc A_i is denoted by f_i and f denotes the column vector whose components are f_i, $i = 1, ..., n$. The travel cost on arc A_i is a given function of f which we denote by $c_i(f)$ and the column vector $c(f)$, whose components are $c_i(f)$, denotes the travel cost on all arcs. We denote by $\Delta = (\delta_{i\,r})$ the arc-path incidence matrix, namely

$$\delta_{i\,r} = \begin{cases} 1 \quad \text{if } A_i \in R_r \quad i = 1, ..., n \quad r = 1, ..., m, \\ 0 \quad \text{if } A_i \notin R_r \quad i = 1, ..., n \quad r = 1, ..., m, \end{cases}$$

hence it results

$$f_i = \sum_{r=1}^{m} \delta_{i\,r}\, F_r, \qquad i = 1, \ldots, n$$

that is $f = \Delta\, F$, and, denoting by $C_r(F)$ the travel cost on path R_r, we obtain

$$C_r(F) = \sum_{i=1}^{n} \delta_{i\,r}\, c_i(f), \qquad r = 1, \ldots, m,$$

that is

$$C(F) = \Delta^T c(\Delta\, F).$$

Therefore we have a cost function $C : K \to \mathbb{R}^m$ such that $C_r(F)$ gives the marginal cost of sending one additional unit of flow through path r, when the flow F is already present.

Now we may recall the classical Wardrop equilibrium condition.

Definition 2.1. A feasible flow $H \in K$ is called an **equilibrium flow** if and only if

$$\left[\begin{array}{c} \text{for each OD pair } W_j \in W \text{ and each } R_q,\ R_s \in \mathcal{R}_j, \\ H_s > 0 \Longrightarrow C_s(H) \leq C_q(H). \end{array} \right] \tag{1}$$

It is classically possible to characterize an equilibrium flow by means of a variational inequality.

Theorem 2.1. A feasible flow H is an equilibrium flow if and only if H is a solution to SVI(C,K).

Remark 2.1. If we consider a network equilibrium flow in the network standard model, namely a model where travel link cost functions are differentiable, positive, strictly increasing and separable in the sense that

$$c(f) = \sum_{i=1}^{n} c_i(f_i) \qquad i = 1, \ldots, n,$$

we have a more clear situation. In fact in this case the jacobian matrix of the cost function $C(F)$ is symmetric:

$$\frac{\partial C_q}{\partial F_s} = \frac{\partial C_s}{\partial F_q} \qquad \forall\ q, s = 1, \ldots, m.$$

and hence one has $C(F) = \nabla T(\Delta F)$ where

$$T(\Delta F) = T(f) = \sum_{i=1}^{n} \int_0^{f_i} c_i(s) ds.$$

Since functions c_i are strictly increasing, then the following functions

$$f_i \to \int_0^{f_i} c_i(s) ds \qquad i = 1, \ldots, n,$$

are strictly convex, thus $T(\Delta F)$ is also strictly convex and cost function C is strictly monotone.

Therefore in this special case the following are equivalent:

- Wardrop network equilibrium flow;
- minimization points of $T(\Delta F)$;
- solutions to SVI(C,K);
- solutions to MVI(C,K)

Since K is a compact set and $T(\Delta F)$ is a strictly convex function, then there exists a unique minimizer for $T(\Delta F)$.

Hence, in such symmetric network, there is a unique equilibrium flow H; moreover since C is strictly monotone, we obtain

$$\langle C(F), H - F \rangle < 0 \qquad \forall\ F \in K.$$

Wardrop equilibrium condition states that if, at a certain time, the network flow is an equilibrium flow, then no user wants to change his path and therefore the traffic flow will remain constant for all future times. Nevertheless, if the network flows are not equilibrium flows, then Wardrop condition is not useful to know the network traffic evolution.

Moreover we observe that to check whether or not a feasible flow H satisfies Wardrop condition, we need not to compare H to the other feasible flows; now we are going to introduce a new Wardrop-type condition for a flow $H \in K$ that involves all the network feasible flows. This is done in the following:

Definition 2.2. A feasible flow $H \in K$ is called a **strong equilibrium flow** if and only if

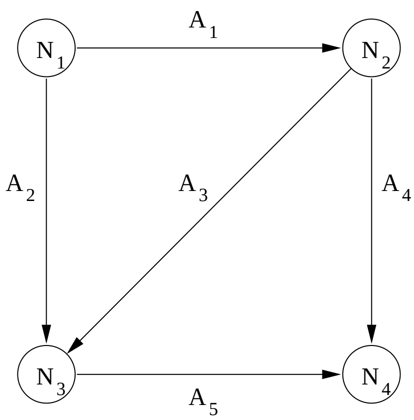
$$\left[\begin{array}{c} \text{for each OD pair } W_j \in W, \text{each } R_q,\ R_s \in \mathcal{R}_j, \\ \text{and each flow } F \in K, \\ C_q(F) < C_s(F) \Longrightarrow F_s > H_s \ \text{ or } \ F_s = H_s = 0. \end{array} \right] \qquad (2)$$

First we remark that a strong equilibrium flow is, in particular, an equilibrium flow.

To better understand the above definition suppose that for some OD pair $W_j \in W$, for some paths $R_q,\ R_s \in W_j$ and for some feasible flow $F \in K$, with $F_s > 0$, we have $C_q(F) < C_s(F)$, then network flow will go down on path R_s; now if a flow H satisfies condition (2), then flow on R_s approaches to H_s. In other words a strong equilibrium flow H takes into account the information $C_q(F) < C_s(F)$ over all the other flows F.

The following examples show a network in which there exists a strong equilibrium flow.

Example 2.1. Let us consider a network with four nodes $N_1,\ N_2,\ N_3,\ N_4$ and five arcs $A_1,\ A_2,\ A_3,\ A_4,\ A_5$:



We have only one OD pair (N_1, N_4) with travel demand $\rho = 10$ and three paths that connect this pair:

$$R_1 = A_1 \cup A_4, \qquad R_2 = A_2 \cup A_5, \qquad R_3 = A_1 \cup A_3 \cup A_5,$$

therefore the set of feasible flows is

$$K = \{F \in \mathbb{R}^3 :\ F \geq 0,\ F_1 + F_2 + F_3 = 10\}.$$

We assume that travel cost on all arcs is defined as follows:

$$\begin{cases} c_1(f_1) = f_1 \\ c_2 = 21 \\ c_3(f_3) = f_3 \\ c_4 = 21 \\ c_5(f_5) = f_5, \end{cases}$$

and hence the corresponding travel cost on paths is

$$\begin{cases} C_1(F) = F_1 + \ F_3 + 21 \\ C_2(F) = F_2 + \ F_3 + 21 \\ C_3(F) = F_1 + F_2 + 3\,F_3. \end{cases}$$

We remark that, for all feasible flows $F \in K$, one has:

$$C_1(F) > C_3(F), \quad \text{and} \quad C_2(F) > C_3(F),$$

therefore the feasible flow $H = (0, 0, 10)$ is the unique Wardrop equilibrium flow. Moreover, for an arbitrary flow $F \in K$, the inequality $C_q(F) < C_s(F)$ in the condition (2) is true only if $s = 1$ or $s = 2$, and $H_1 = H_2 = 0$, thus the condition

$$F_s > H_s \;\text{ or }\; F_s = H_s = 0$$

is equivalent to

$$F_s > 0 \;\text{ or }\; F_s = 0,$$

which is trivially true.

Example 2.2. Let us consider the same network as the example 2.1, in which travel demand is $\rho = 10$ and travel cost on arcs is defined as follows:

$$\begin{cases} c_1(f_1) = f_1 \\ c_2(f_3) = 10 + f_3 \\ c_3(f_3) = f_3 \\ c_4(f_3) = 10 + f_3 \\ c_5(f_5) = f_5, \end{cases}$$

and hence the corresponding travel cost on paths is

$$\begin{cases} C_1(F) = F_1 + 2\,F_3 + 21 \\ C_2(F) = F_2 + 2\,F_3 + 21 \\ C_3(F) = F_1 + F_2 + 3\,F_3. \end{cases}$$

We remark that for all feasible flow $F \in K$ one has:

$$C_1(F) \geq C_3(F), \quad \text{and} \quad C_2(F) \geq C_3(F),$$

therefore the feasible flow $H = (0, 0, 10)$ is the unique equilibrium flow. Moreover, for an arbitrary flow $F \in K$, the inequality $C_q(F) < C_s(F)$ in the condition (2) is true only if $s = 1$ or $s = 2$, and $H_1 = H_2 = 0$, thus the condition

$$F_s > H_s \;\text{ or }\; F_s = H_s = 0$$

is equivalent to

$$F_s > 0 \;\text{ or }\; F_s = 0,$$

which is trivially true.

Besides Wardrop equilibrium, also the condition (2) has connections with a variational formulation, as the following theorem states.

Theorem 2.2. If H is a strong equilibrium flow, then H is a solution to MVI(C,K).

Proof. Let $F \in K$ and $W_j \in W$ be arbitrary. We consider a path $R_q \in \mathcal{R}_j$ such that

$$C_q(F) = \min_{R_r \in \mathcal{R}_j} C_r(F).$$

Then from condition (2) one has

$$(C_r(F) - C_q(F))(F_r - H_r) \geq 0 \quad \forall\ R_r \in \mathcal{R}_j.$$

Thus

$$\sum_{r=1}^{r_j} C_r(F)(F_r - H_r) \geq C_q(F) \sum_{r=1}^{r_j} (F_r - H_r) = C_q(F)(\rho_j - \rho_j) = 0.$$

Hence

$$\langle C(F), F - H \rangle = \sum_{j=1}^{l} \sum_{r=1}^{r_j} C_r(F)(F_r - H_r) \geq 0,$$

hence H is a solution of MVI(C,K). □

The vice versa of theorem 2.2 is, in general, false as the following example shows.

Example 2.3. Let us consider the same network as in the example 2.1, where travel costs on paths are:

$$\begin{cases} C_1(F) = 3\,F_1 + 2\,F_3 \\ C_2(F) = 5\,F_2 + 2\,F_3 \\ C_3(F) = 2\,F_1 + F_2 + 3\,F_3. \end{cases}$$

We observe that $H = (5, 3, 2)$ is an equilibrium flow, because

$$C_1(H) = C_2(H) = C_3(H) = 19,$$

and H is solution of $MVI(C, K)$, in fact for all $F \in K$ we can write $F_3 = 10 - F_1 - F_2$, and thus we have

$$\langle C(F), F - H \rangle = 2F_1^2 + 5F_2^2 - F_1F_2 - 17F_1 - 25F_2 + 80 = \phi(F_1, F_2).$$

Since

$$\nabla^2 \phi(F_1, F_2) = \begin{pmatrix} 4 & -1 \\ -1 & 10 \end{pmatrix}$$

has positive eigenvalues $\lambda_{1,2} = 7 \pm \sqrt{10}$, ϕ is a strictly convex function, his minimum point is $(5, 3)$ and $\phi(5, 3) = 0$, hence $\phi(F_1, F_2) \geq 0$ for all $(F_1, F_2) \in \mathbb{R}^2$, that is H is solution of $MVI(C, K)$.

However H is not a global equilibrium flow, because, if we consider $F = (6, 3, 1) \in K$, one has

$$17 = C_2(F) < C_3(F) = 18 \qquad \text{and} \qquad 1 = \mathrm{F}_3 < \mathrm{H}_3 = 2.$$

For a feasible flow $H \in K$ we now introduce another Wardrop-type condition giving the "stability" of H with respect to some perturbations of flow H on a arbitrary couple of paths connecting an OD-pair.

Definition 2.3. A feasible flow $H \in K$ is called a **"stable" equilibrium flow** if and only if

$$\left[\begin{array}{c} \text{for each OD pair } W_j \in W, \text{ each } R_q,\ R_s \in \mathcal{R}_j, \\ \text{and each flow } F \in K \text{ such that } F_i = H_i, \forall\, i \neq q, s \text{ and } F_s < H_s \\ H_s > 0 \Longrightarrow C_s(F) \leq C_q(F) \end{array}\right] \tag{3}$$

To better understand the above definition, suppose that a feasible flow H be a "stable" equilibrium flow, R_q, R_s be two paths connecting a certain OD pair and that the flow H_s be positive; if we perturb H only on the paths R_q, R_s pushing flow from R_s to R_q, then the path R_q does not cost less than the path R_s.

In the following theorem we check the relationships between stable equilibrium flow and solutions of SVI(C,K) and MVI(C,K).

Theorem 2.3.

1. If a flow $H \in K$ solves MVI(C,K), then H is a stable equilibrium flow;

2. if $H \in K$ is a stable equilibrium flow and the cost function C is continuous, then H is an equilibrium flow.

Proof.

1. We consider an arbitrary OD pair W_j and two paths $R_q, R_s \in \mathcal{R}_j$ fixed. Since $H \in K$ is a solution to MVI(C,K) then for all feasible flow $F \in K$ one has

$$\langle C(F), F - H \rangle = \sum_{j=1}^{l} \sum_{r=1}^{r_j} C_r(F)(F_r - H_r) \geq 0;$$

   in particular, if we choose a feasible flow F such that $F_i = H_i, \forall\, i \neq q, s$ and $F_s < H_s$, then

$$(C_s(F) - C_q(F))(F_s - H_s) \geq 0,$$

   and hence $C_s(F) \leq C_q(F)$.

2. For each OD pair W_j and for each $R_q, R_s \in \mathcal{R}_j$, with $H_s > 0$, we have $C_s(H) \leq C_q(H)$ by continuity of the cost function C.

□

From the above theorem it follows that the stable equilibrium flows are selected network equilibrium flows, with the following property: if a feasible flow H is a network equilibrium flow and R_q, R_s are two paths connecting a certain OD pair W_j, with $H_s > 0$, then we know that $C_s(H) \leq C_q(H)$; moreover, if H is a stable equilibrium flow then $C_s(F) \leq C_q(F)$ for all perturbation flows F which shift flow from R_s to R_q.

We remark that a stable equilibrium flow does not coincide with all the network equilibrium flows as the following example shows.

Example 2.4. Let us consider the same network as the example 2.1, in which travel cost on arcs is:

$$\begin{cases} c_1(f_1) = f_1 \\ c_2(f_2, f_3) = f_2 + 4\, f_3 \\ c_3(f_2, f_3) = f_2 + 2\, f_3 \\ c_4(f_3, f_4) = 2\, f_3 + 2\, f_4 \\ c_5(f_5) = f_5, \end{cases}$$

and therefore the travel cost on paths is

$$\begin{cases} C_1(F) = 3\, F_1 + 3\, F_3 \\ C_2(F) = 2\, F_2 + 5\, F_3 \\ C_3(F) = F_1 + 2\, F_2 + 4\, F_3. \end{cases}$$

We assume that travel demand is $\rho = 10$. An equilibrium flow is $H = (4, 2, 4)$, in fact

$$C_1(H) = C_2(H) = C_3(H) = 24,$$

but H is not a stable equilibrium flow because if we perturb H on paths R_2 and R_3 so that $F_1 = 4,\ F_2 = 2 + x$ and $F_3 = 4 - x$, with $0 < x < 4$, one has

$$C_3(F) = 24 - 2\, x > C_2(F) = 24 - 3\, x.$$

Moreover we can remark, by the following example, that all the stable equilibrium flows are not solution to MVI(C,K).

Example 2.5. We consider the same network as the example 2.1, with the following travel cost on paths:

$$\begin{cases} C_1(F) = 5\, F_1 + 4\, F_3 \\ C_2(F) = 5\, F_2 + 4\, F_3 \\ C_3(F) = 3\, F_1 + 3\, F_2 + 3\, F_3 \end{cases}$$

and travel demand is $\rho = 9$.

An equilibrium flow is $H = (3, 3, 3)$, in fact

$$C_1(H) = C_2(H) = C_3(H) = 27,$$

moreover H is a stable equilibrium flow. In fact if we choose a feasible flow F such that $F_1 = 3,\ F_2 = 3 - x$ and $F_3 = 3 + x$, with $0 < x < 3$, one has

$$C_2(F) = 27 - x < 27 = C_3(F),$$

if $F_2 = 3,\ F_1 = 3 - x$ and $F_3 = 3 + x$ then

$$C_1(F) = 27 - x < 27 = C_3(F),$$

if $F_3 = 3,\ F_1 = 3 - x$ and $F_2 = 3 + x$ then

$$C_1(F) = 27 - 5\,x < 27 + 5\,x = C_2(F).$$

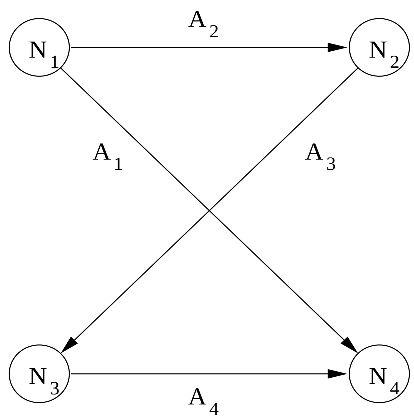
If we choose the perturbation $-3 < x < 0$ we will obtain analogous inequalities.

However H is not solution to MVI(C,K) because for $F = (0, 0, 9)$ we have

$$\langle C(F), F - H\rangle = \langle (36, 36, 27), (-3, -3, 6)\rangle = -54.$$

Finally we show an example that gives a global picture:

Example 2.6. Let us consider the following network with four nodes $N_1,\ N_2,\ N_3,\ N_4$ and four arcs $A_1,\ A_2,\ A_3,\ A_4$:



We have only one OD pair (N_1, N_4) with travel demand $\rho = 10$ and only two paths connecting this pair:

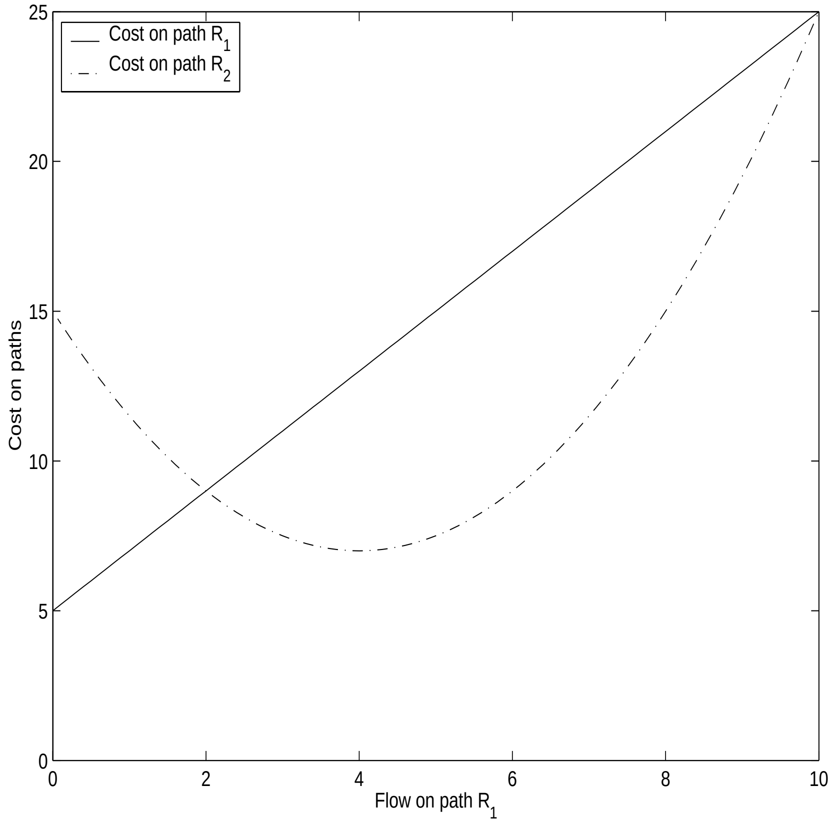
$$R_1 = A_1, \qquad R_2 = A_2 \cup A_3 \cup A_4,$$

therefore the set of feasible flows is

$$K = \{F \in \mathbb{R}^2 :\ F \geq 0,\ F_1 + F_2 = 10\}.$$

We assume that travel cost on paths is defined as follows:

$$\begin{cases} C_1(F) = 2\,F_1 + 5 \\ C_2(F) = \frac{1}{2}\,F_1^2 - 4\,F_1 + 15 \end{cases}$$

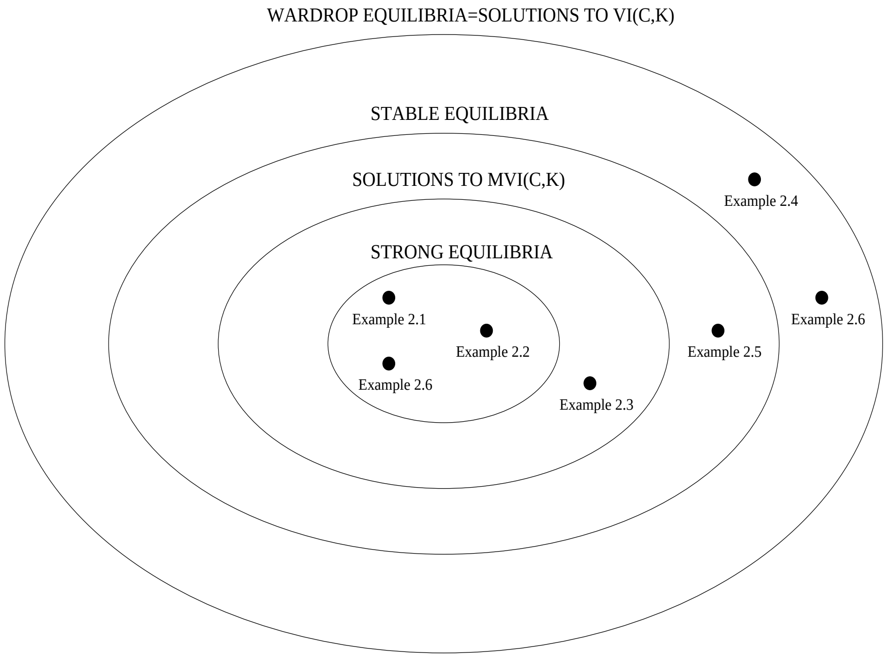


From the figure it is easy to argue that there are two equilibrium flows: $H' = (2, 8)$ and $H'' = (10, 0)$.

We remark that H' is a strong equilibrium flow because: if $C_2(F) > C_1(F)$ then $F_1 < H'_1$, that is $F_2 > H'_2$, and if $C_1(F) > C_2(F)$, then $F_1 > H'_1$.

Moreover H'' is an equilibrium flow but it is not a stable equilibrium: indeed $H''_1 > 0$ but $C_1(F) > C_2(F)$ for each feasible flow F such that $2 < F_1 < 10$.

Finally, we sum up in the following figure the relationships between Wardrop-type equilibria and solutions to SVI(C,K) and MVI(C,K) when the cost function C is assumed continuous on K.



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