Trans-dimensional Monte Carlo sampling applied to the magnetotelluric inverse problem

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Trans-dimensional Monte Carlo sampling applied to the magnetotelluric inverse problem

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Abstract. The data required to build geological models of the subsurface are often unavailable from direct measurements or well logs. In order to image the subsurface geological structures several geophysical methods have been developed. The magnetotelluric (MT) method uses natural, time-varying electromagnetic (EM) fields as its source to measure the EM impedance of the subsurface. The interpretation of these data is routinely undertaken by solving inverse problems to produce 1D, 2D or 3D electrical conductivity models of the subsurface. In classical MT inverse problems the investigated models are parametrized using a fixed number of unknowns (i.e. fixed number of layers in a 1D model, or a fixed number of cells in a 2D model), and the non-uniqueness of the solution is handled by a regularization term added to the objective function. This study presents a different approach to the 1D MT inverse problem, by using a trans-dimensional Monte Carlo sampling algorithm, where trans-dimensionality implies that the number of unknown parameters is a parameter itself. This construction has been shown to have a built-in Occam razor, so that the regularization term is not required to produce a simple model. The influences of subjective choices in the interpretation process can therefore be sensibly reduced. The inverse problem is solved within a Bayesian framework, where posterior probability distribution of the investigated parameters are sought, rather than a single best-fit model, and uncertainties on the model parameters, and their correlation, can be easily measured.

1. Introduction

MT method is an electromagnetic sounding tomographic technique used to infer the resistivity distribution of the subsurface by measurements of orthogonal components of horizontal EM fields on the Earth surface. The MT inverse problem is known to be both unstable and highly non-linear. Despite these irregular traits, linearized inversion of MT data are routinely undertaken and lead to affordable and robust results [1]. The theoretical problem with this approach is in the inverse problem set up. The linearized version of the problem, even if solved in a 1D layered domain, requires in fact several key choices to be made, for example the number of parameters used to describe the subsurface, the presence of a regularization term in the objective function, the trade-off balance between a regular model and a model that fit the data better and so on. In the past, several authors used Bayesian inversion to interpreter MT data (cf. e.g. [2]). In this paper we present a trans-dimensional procedure in which the number of parameters to be estimated is an unknown itself. From a Bayesian perspective the inverse problem is not solved by a single model that fits the dataset, rather the solution of the inverse problem is given in a statistical form, returning the posterior probability distribution (PPD) of models that fit the

Table 1. Synthetic model used in the algorithm test. Layers are numbered top to bottom.

Layer number	Layer thickness (km)	Resistivity ($\Omega \cdot m$)
1	1.0	1000
2	1.0	100
3	∞	1000

dataset. The PPD is given by Bayes's rule as

$$p(\vec{m}|\vec{d}, \mathcal{I}) = \frac{p(\vec{m}|\mathcal{I})p(\vec{d}|\vec{m}, \mathcal{I})}{p(\vec{d}|\mathcal{I})} \quad (1)$$

in which p is the probability distribution, $\star|\star$ denotes the conditional dependence, $\vec{d}$ is the array of measured data and $\vec{m}$ is the array that stores the model parameters. Details about the quantities used in the Equation 1 can be found in Piana Agostinetti [3] and references therein. The reverse jump Markov chain Monte Carlo (rjMCMC) algorithm used here is formally identical to the one described by Piana Agostinetti [3].

As the MT method in 1D is mainly sensitive to conductance rather than conductivity (cf. e.g. [1]), the implicit parsimony in the model complexity that characterizes rjMCMC solutions helps the interpretation of data that cannot be adequately described by a simple model and that requires regularization terms or complex model to be fitted.

2. Numerical Experiment

The algorithm has been validated by the simulation of an inversion using data from a synthetic model, summarized in Table 1. The synthetic model is built as a stack of three layers: a resistive basement, a conductive layer and a resistive topmost layer. This model presents two the classic challenges for a MT inverse problem: retrieving the bottom of a conductor and fit the data with a model that is not overparametrized.

First we tested the prior probability distribution sampler – setting manually the likelihood to 1– to check that the candidate models are effectively drawn from the prior probability distribution. Let us pretend we miss a good *a priori* information about the parameters distribution, in this case we purpose as prior distribution for the resistivity ρ an uniform distribution in the interval $-0.5 < \log_{10}(\rho) < 3.5$. Frequency distribution of the sampled values for electrical resistivity, shown in Figure 1, demonstrate that the sampler draws this parameter from a uniform distribution, in the correct limits. Even the number of interfaces, and their location in depth, plotted respectively in Figure 3 and 5 result uniformly distributed.

We thus simulated data from the synthetic model, we added 3% random noise and test the algorithm, using 191 Markov chains with 500.000 models per chain.

3. Results and discussion

Overall the algorithm retrieved the (supposed unknown) synthetic model, with the resistivity distribution shown in Figure 2. The top interface of the conductor is well resolved, as expected, while the depth of the deeper resistor results blurred. This is due to the lack of sensitivity of the method on resistive region beneath conductors. Figure 4 proves the implicit parsimony of the algorithm showing that frequently the data can be fitted using three interfaces (i.e., two layers), one less than the true value. Even the distribution of the interfaces in depth (Figure 6 results overall correct, with the distribution peaks centered at 1 km and 2 km depth.

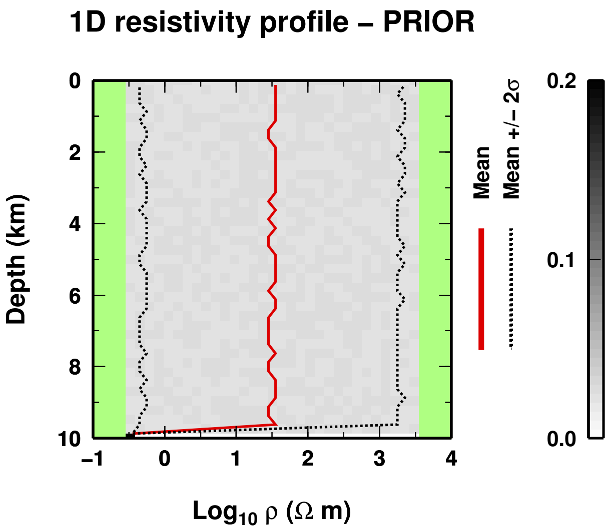

Figure 1. Prior sampling of the resistivity.

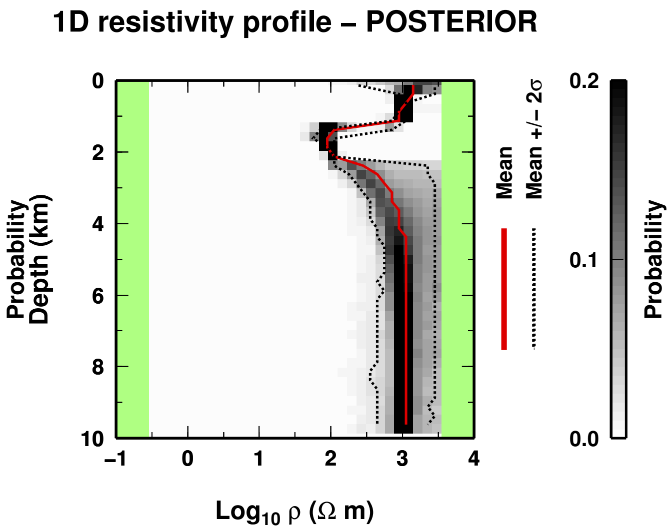

Figure 2. Posterior sampling of the resistivity.

Finally, Figure 7 and 8 show the data fit obtained using respectively the prior and posterior resistivity distributions. Without any affordable *a priori* information, it s natural that the prior

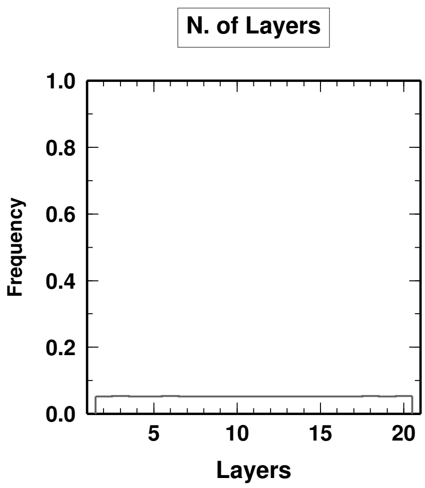

Figure 3. Prior sampling of the number of interfaces.

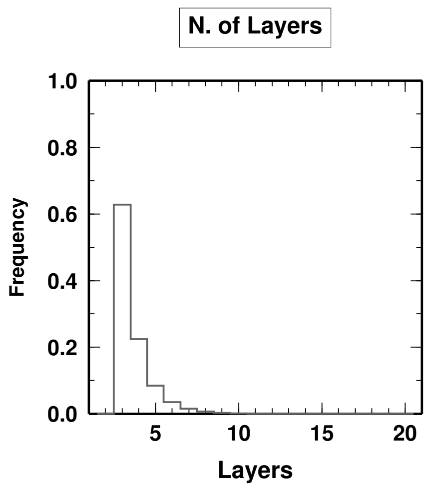

Figure 4. Posterior sampling of the number of interfaces.

distribution does not fit the data (red crosses). On the contrary the posterior distribution fits the dataset with accuracy, with the exception due to the MT method. The high frequency data are not fitted with accuracy as the topmost part of the model can be constrained only by higher frequency signals, in fact in Figure 2 is clearly shown the ambiguity in the corresponding region of the distribution.

In general the algorithm behaves as expected, retrieving the correct distribution and fitting the data using the simplest possible model. The trans-dimensional nature of the method allows to avoid the artificial use of en explicit regularization term and the superimposed selection of the appropriate number of parameters that is driven only by the data. Nevertheless, in order to validate the purposed method, other tests with both more complex synthetic models and real data are needed.

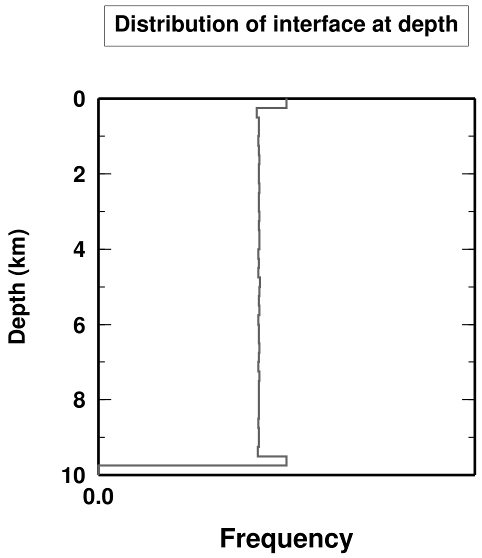

Figure 5. Prior sampling of the depth of the interfaces.

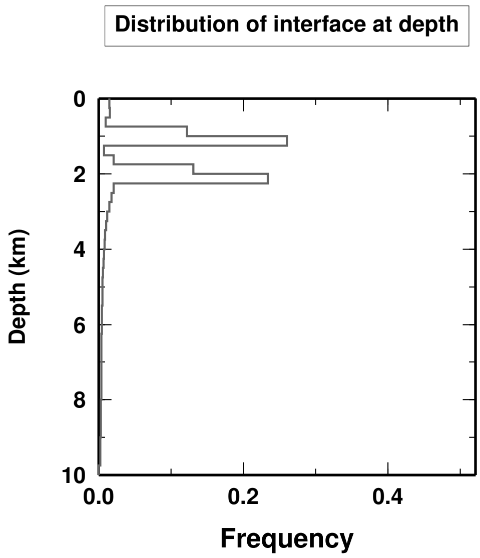

Figure 6. Posterior sampling of the depth of the interfaces.

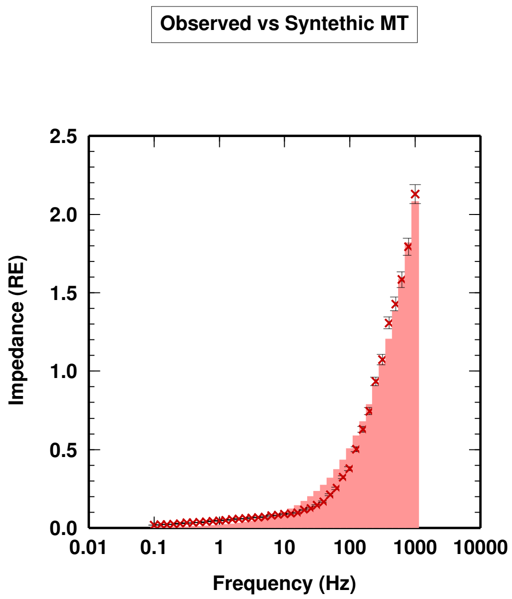

Figure 7. Prior sampling of the resistivity.

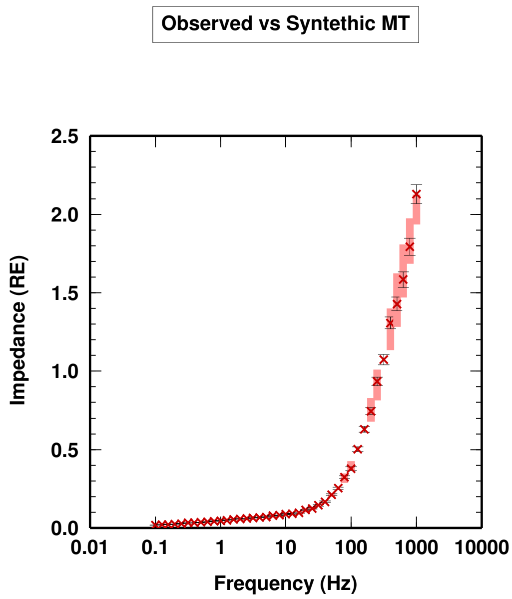

Figure 8. Posterior sampling of the resistivity.

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[2] Grandis H, Menvielle M and Roussignol M 1999 *Geophysical Journal International* **138** 757–768
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