The Impact of Risk Aversion on the Rigidity of Insurance Premiums

Vanda Tulli[1] and Gerd Weinrich[2]

[1]Università di Milano-Bicocca

[2]Università Cattolica di Milano

AMASES 2023

Introduction

Two concepts of major relevance in the theory of insurance:

Risk aversion and **Premium rigidity**

Why risk aversion?

- A risk-averse insurance company (IC) management acts in the best interest of those shareholders whose wealth is heavily concentrated in that single company.
- IC management could be pursuing its own objectives: owners are not perfectly diversified when it comes to their own assets.
- Usually, their most important component of total wealth is their human capital: know-how largely specific to the IC.
- Risk of insolvency with its consequences for thousands of policyholders. A management responsible for such a large-scale failure would see its future prospects for employment and earnings damaged.

Why premium rigidity?

- Empirical research suggests considerable premium rigidity.
- Institutional aspects of the insurance market, e.g. authorization of vigilance organs.
- Long-term contracts (sometimes, but contracts are typically one year).
- **Here: risk aversion.**

Scenario:

- IC insures a client's enterprise, premium P if quota $q = 1$ (full insurance). For $q < 1$ the client pays qP.
- At the renewal of the contract: how would the client react to a change in the premium?
- *Status quo*: (q_0, P_0), **no uncertainty.**
- For $P \neq P_0$ IC is uncertain regarding q, has a conjecture expressed by a (subjective) probability distribution.
- **Uncertainty increases with distance from status quo.**

We show:

- IC may keep the premium fixed even though an otherwise identical IC would change it.
- Due to *risk aversion of order one* (Segal & Spivak, 1990).
- If no fixed premiums exist: the size of the premium adjustment decreases with increasing risk aversion.
- In case of adjustment costs: increasing risk aversion diminishes the minimum cost sufficient to keep the premium fixed.

The Model

Consider IC and client with enterprise having possible loss X with $EX =: \overline{x}$.

IC's subjective probability distribution over the r.v. $Q(P|P_0)$ (the client's reaction) given by $F(q, P|P_0) = \text{Prob}\{Q(P|P_0) \leq q\}$ with $EQ(P|P_0) =: \overline{q}(P \mid P_0)$. Let

$$\delta(q \mid q_0) = \begin{cases} 1, & \text{if } q \geq q_0 \\ 0, & \text{if } q < q_0 \end{cases} \qquad \begin{matrix}\text{degenerate} \\ \text{distribution}\end{matrix} \quad .$$

(A1) For $P \neq P_0$ $F(q, P \mid P_0)$ is given by means of a density function $f(q, P \mid P_0)$ continuous in q and differentiable in P, while $F(q, P_0 \mid P_0) = \delta(q \mid q_0)$ for all q.

(A2) $\overline{q}(P \mid P_0)$ is decreasing and continuously differentiable in P for all $P > 0$ and $P_0 > 0$. Moreover, if $P \to P_0$, then $\int g(q)\, dF(q, P \mid P_0) \to \int g(q)\, d\delta(q \mid q_0) = g(q_0)$ for any continuous function $g : [0, 1] \to \mathbb{R}$.

Simplify notation: $\overline{q}(P \mid P_0) = \overline{q}(P)$, $F(q, P \mid P_0) = F(q, P)$ etc.

Let $u(\cdot)$ be IC's (Bernoullian) utility function.

IC's expected utility function:

$$U(P) := \int u(\underbrace{q \times (P - \overline{x})}_{profit})dF(q, P) .$$

Consider also

$$V(P) := u(\overline{\pi}(P))$$

with $\overline{\pi}(P) := \overline{q}(P) \times (P - \overline{x})$ = IC's expected profit. Then:

- $U(P) \leq V(P)\ \forall P$ if IC is risk averse, i.e. $u(\cdot)$ is concave.
- $U(P) = V(P)\ \forall P$ if IC is risk neutral, i.e. $u(\cdot)$ is linear.
- $U(P_0) = u(q_0 \times (P_0 - \overline{x})) = V(P_0)$ since at P_0 there is no uncertainty.
- $\lim_{P \to P_0^-} U'(P) \geq V'(P_0) \geq \lim_{P \to P_0^+} U'(P)$

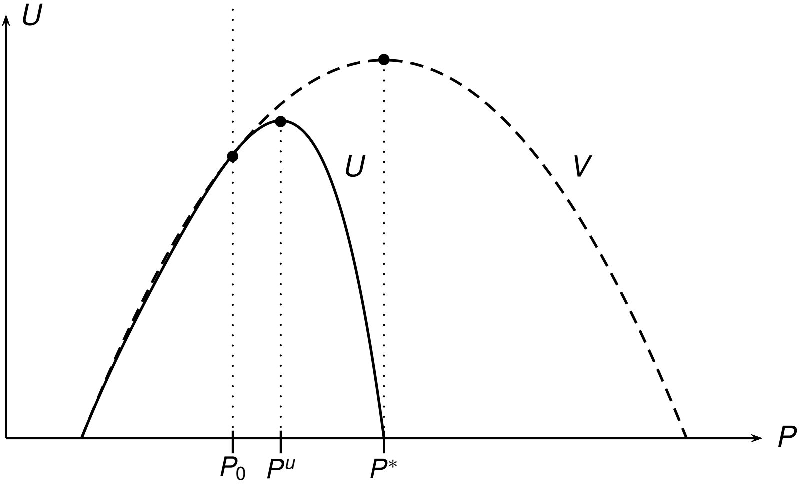
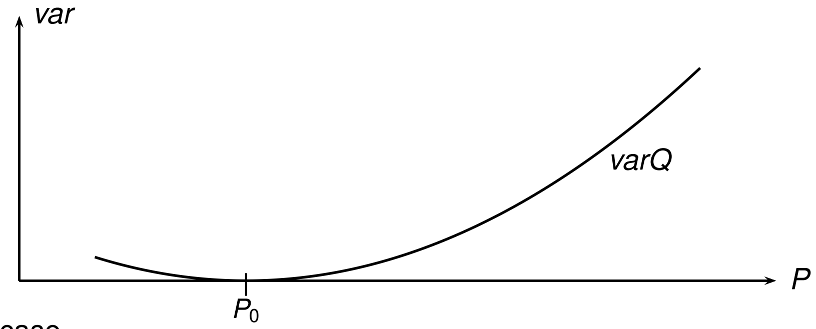


Figure : Expected utility functions in case of risk aversion (U, solid) and risk neutrality (V, dashed). Note: $P^u \neq P_0$.

Fixed Premiums

Next consider the function $P \mapsto varQ(P)$.



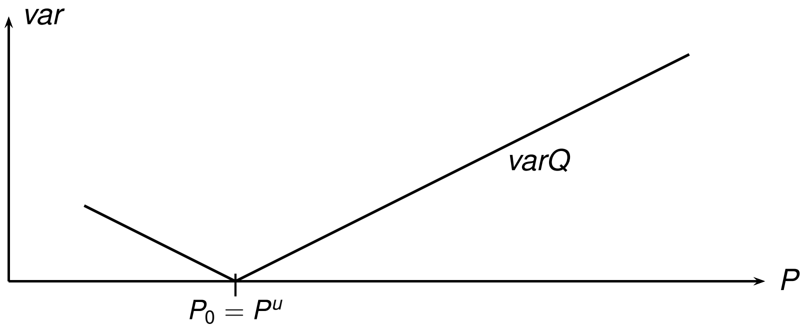
In this case

$$\lim_{P \to P_0^-} \frac{dvarQ\,(P)}{dP} = 0 = \lim_{P \to P_0^+} \frac{dvarQ\,(P)}{dP}$$

The variance function is smooth.

Then consider the case

$$\lim_{P \to P_0^-} \frac{dvarQ\,(P)}{dP} < 0 \quad \text{and} \quad \lim_{P \to P_0^+} \frac{dvarQ\,(P)}{dP} > 0$$



The variance function is kinked at P_0.

Proposition 1 *Assume* $(A1)$ *and* $(A2)$. *Then the IC's expected utility function* $U(P)$ *has a kink at* $P = P_0$ *if, and only if, it is risk averse and the function* $varQ(P)$ *is kinked at* $P = P_0$.

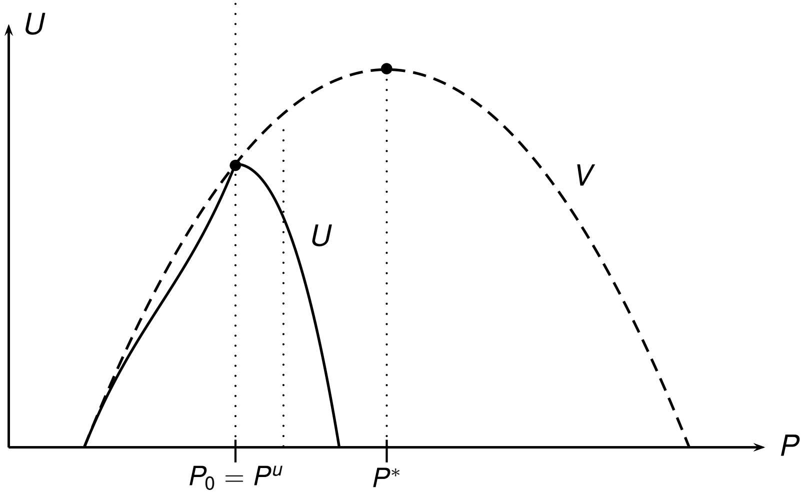


Figure : Expected utility in case of a kink of $varQ(P)$ at $P = P_0 = P^u$.

Note: In the previous scenario P_0 is a fixed premium. The risk averse IC does not change it although the risk neutral IC would do so.

The kink in the variance function is equivalent to *Risk aversion of order one* according to Segal & Spivak (1990).

Risk aversion of order two: the variance function is convex but there is no kink.

Intuition of why the kink in the variance function is transferred to the expected utility function: consider the special case

$$U(P) = T(\overline{\pi}(P), var\Pi(P)),$$

with $\Pi(P) = Q(P)(P - \overline{x})$ = r.v. profit, $var\Pi(P) = varQ(P)(P - \overline{x})^2$.
$U(P)$ has a kink $\Leftrightarrow$ $var\Pi(P)$ has a kink $\Leftrightarrow$ $varQ(P)$ has a kink since $\overline{\pi}(P)$ is by assumption differentiable.

Example 1 Assume

- $q(P) = \max\{1 - cP, 0\}$, $c \in (0, 1/\overline{x})$, client's true demand function, unknown to IC.
- IC has observed the status quo (q_0, P_0); it satisfies $q_0 = 1 - cP_0$.
- IC conjectures that the change in q for $P \neq P_0$, $z := q - q_0$, is normally distributed with parameters $\nu(\rho)$ and $\sigma(\rho)$, where $\rho := P - P_0$.
- Setting

$$g(z; \nu, \sigma) := \left(\sigma\sqrt{2\pi}\right)^{-1} e^{-\frac{1}{2}(z-\nu)^2/\sigma^2}$$

  $Q(P)$ is distributed, for $P \neq P_0$, according to the density

$$f(q_0 + z, P_0 + \rho) = g(z; \nu(\rho), \sigma(\rho))$$

  while $Q(P_0) = q_0$ with probability one.
- $\nu(\rho) := -c\rho,\ \sigma(\rho) := \beta|\rho|^{\gamma},\ \beta, \gamma > 0$

Then:

- $\overline{q}(P_0+\rho) = q_0 - c\rho = q_0 - c(P - P_0) = 1 - cP$
  $\Rightarrow$ IC has correct beliefs regarding the expected value of q.
- $varQ(P_0+\rho) = \beta^2 |\rho|^{2\gamma}$; uncertainty increases with ρ.
- $|dvarQ(P_0+\rho)/d\rho| = 2\beta^2\gamma|\rho|^{2\gamma-1}$;
  tends to zero for $P \to P_0$ (i.e. $\rho \to 0$) $\Leftrightarrow \gamma > 1/2$.
- Thus for $\gamma \leq 1/2$ the IC's variance function and hence its expected utility function has a kink (or a cusp) at the status quo, and there exist fixed premiums. (For the case $\gamma = 1/2$ see previous figure p. 10.)
- Note: a continuous infinitesimal variation of a single parameter (γ) in the specification of the company's beliefs changes its behaviour from flexible to rigid.

Premium inertia when no Fixed Premiums exist

New assumptions:

(B1) For $P \neq P_0$ $Q(P)$ is normally distributed with density
$$f(q_0 + z, P_0 + \rho) = g(z; -c\rho, |\rho|) = \left(|\rho|\sqrt{2\pi}\right)^{-1} e^{-\frac{1}{2}(z+c\rho)^2/|\rho|^2},$$
where $z = q - q_0$, $\rho = P - P_0$ and $c \in (0, 1/\overline{x}]$;
$Q(P_0) = q_0$ with probability one.

(B2) The IC's utility function is $u_\alpha(\pi) = -e^{-\alpha\pi}$ when $\alpha > 0$, whereas $u_0(\pi) = \pi$.

In (B1): As in Example 1, but with $\gamma = 1 \Rightarrow$ there do not exist fixed premiums.

In (B2): IC has constant absolute risk aversion $\alpha > 0$; for $\alpha \to 0$ it tends to become risk neutral.

We show: IC's expected utility function becomes

$$U(\rho,\alpha) = -e^{-\alpha\tau(\rho,\alpha)} \text{ if } \alpha > 0,$$

with

$$\tau(\rho,\alpha) = \overline{\pi}(P_0+\rho) - \frac{\alpha}{2} var\Pi(P_0+\rho).$$

To maximize $U(\rho,\alpha)$ is equivalent to maximizing $\tau(\rho,\alpha)$.

This can be extended to $\alpha = 0$ in which case $\tau(\rho,0) = \overline{\pi}(P_0+\rho)$.

Denote

$$\rho^*(\alpha) := \arg\max_{\rho} U(\rho,\alpha)$$

Proposition 2 *Assume* $(B1)$, $(B2)$ *and* $P_0 \neq P^*$. *Then (a) the absolute value of the optimal deviation from the status-quo premium is a decreasing function of the IC's absolute risk aversion, i.e.* $d|\rho^*(\alpha)|/d\alpha < 0$, *and (b)* $\lim_{\alpha\to\infty} \rho^*(\alpha) = 0$.

Example 2

Assume $c = 1/4$, $\overline{x} = 2$, (q_0, P_0) = $(3/8, 5/2)$ status quo
$\Rightarrow \pi_0 = 3/16$.

What would the risk-neutral IC do?

$\max_\rho \tau(\rho, 0) \Rightarrow \rho^*(0) = 1/2 \Rightarrow (\overline{q}^*, P^*) = (1/4, 3) \Rightarrow \overline{\pi}^* = 1/4$

The risk neutral IC would increase the premium by $1/2$ and, although this would decrease $\overline{q}$ by $\overline{q}^* - q_0 = -1/8$, it would increase its expected profit by $\overline{\pi}^* - \pi_0 = 1/16$.

What would the risk-averse IC do?

$\max_\rho \tau(\rho, \alpha) \Rightarrow 8\alpha\rho^3 + 6\alpha\rho^2 + (2+\alpha)\rho - 1 \overset{!}{=} 0$

solution: $$\rho^*(\alpha) = \sqrt[3]{\frac{3}{32\alpha} + \left(\left(\frac{3}{32\alpha}\right)^2 + \left(\frac{4-\alpha}{48\alpha}\right)^3\right)^{1/2}} + \sqrt[3]{\frac{3}{32\alpha} - \left(\left(\frac{3}{32\alpha}\right)^2 + \left(\frac{4-\alpha}{48\alpha}\right)^3\right)^{1/2}} - \frac{1}{4}$$

Proposition 2 *Assume* $(B1)$, $(B2)$ *and* $P_0 \neq P^*$. *Then (a) the absolute value of the optimal deviation from the status-quo premium is a decreasing function of the IC's absolute risk aversion, i.e.* $d\left|\rho^*(\alpha)\right|/d\alpha < 0$, *and (b)* $\lim_{\alpha\to\infty} \rho^*(\alpha) = 0$.

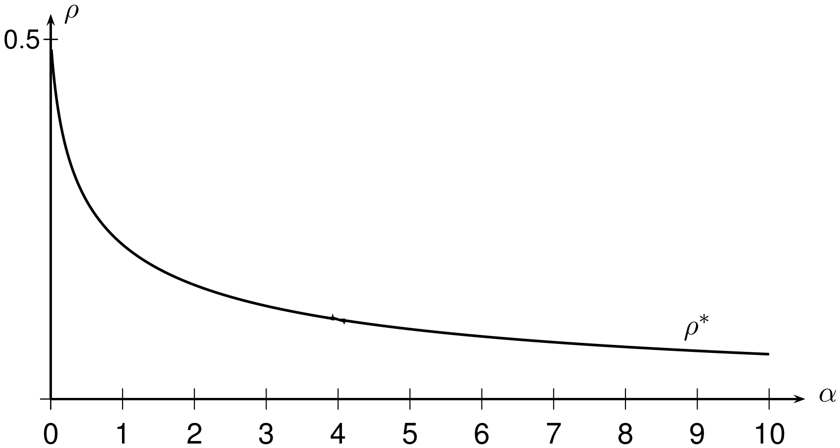


Figure : The optimal premium adjustment ρ^* as depending on the degree of risk aversion α.

Risk Aversion and Premium Adjustment Costs

Consider now small premium adjustment costs (e.g. administrative and/or government supervision expenses).

When risk aversion increases, the size of optimal premium adjustment decreases and the gain in expected utility decreases. Premium adjustment costs may then render it not convenient to adjust the premium. The minimum cost sufficient to block premium adjustment will decrease with increasing risk aversion.

Formally, assume that, for any given q, expected profit is

$$\pi = q\,(P - \overline{x}) - \lambda\ ,$$

where $\lambda \geq 0$ is a premium-adjustment cost parameter when $P \neq P_0$ and $\pi = q_0\,(P_0 - \overline{x}) = \pi_0$ otherwise.

The corresponding expected utility then is

$$U\,(\rho, \alpha, \lambda) = -e^{-\alpha[\tau(\rho,\alpha)-\lambda]}$$

The IC adjusts its premium iff $U\,(\rho, \alpha, \lambda) > u_\alpha\,(\pi_0)$.

Proposition 3 *Assume (B1), (B2) and $P_0 \neq P^*$. Then, for any degree of risk aversion $\alpha \geq 0$ there exists $\overline{\lambda}(\alpha) > 0$ such that the IC adjusts its premium iff its adjustment cost is smaller than $\overline{\lambda}(\alpha)$. Moreover, $\overline{\lambda}(\alpha)$ is decreasing with $\lim_{\alpha\to\infty} \overline{\lambda}(\alpha) = 0$.*

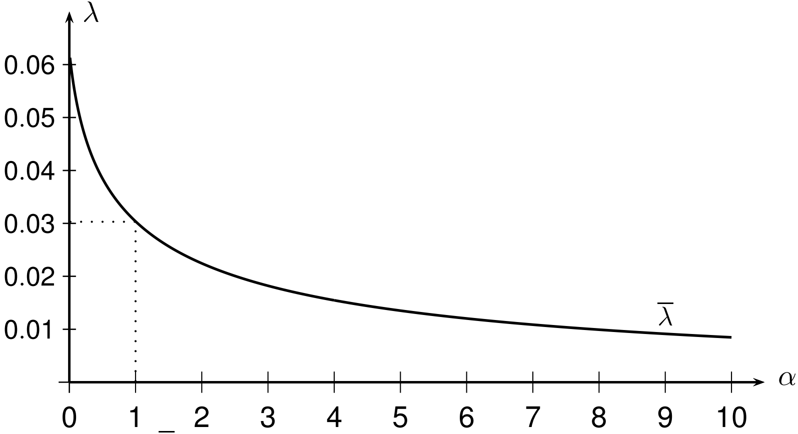


Figure : Minimum cost $\overline{\lambda}$ sufficient to block premium adjustment at the status-quo premium P_0 as a function of the degree of risk aversion α.

Note: the previous figure is derived from the numbers in Example 2. The formula for $\overline{\lambda}(\alpha)$ is

$$\overline{\lambda}(\alpha) = \left(\frac{3}{8} - \frac{1}{4}\rho^*(\alpha)\right)\left(\frac{1}{2} + \rho^*(\alpha)\right) - \frac{\alpha}{2}(\rho^*(\alpha))^2\left(\frac{1}{2} + \rho^*(\alpha)\right)^2 - \frac{3}{16}.$$

From this one can calculate for the function $\alpha \mapsto \rho^* \mapsto \overline{\lambda}$ the values

α	0	1	2	10
ρ^*	0.500	0.215	0.159	0.062
$\overline{\lambda}$	0.062	0.030	0.022	0.008

Note: the change from risk neutrality (i.e. $\alpha = 0$) to the modest degree of risk aversion $\alpha = 1$ already reduces the blocking cost by more than 50%, i.e. from 0.062 to 0.030.

Concluding remarks

- Risk aversion may be a crucial element in explaining premium rigidity.
- A risk-averse IC may keep its premium fixed where a risk-neutral would change it if the IC's variance function displays a kink at the status quo.
- Examples show that this could easily happen by means of a continuous change in a single parameter of the IC's beliefs.
- In the absence of fixed premiums risk aversion still makes a crucial difference with respect to risk neutrality as it significantly reduces the size of premium adjustment as well as the adjustment costs sufficient to block adjustment.

- Regarding the kink in the variance function, from a psychological point of view its occurrence seems quite plausible if one takes into account the following fact:

  Zero probability, that is certainty, is often perceived in a qualitatively different way in relation to other probabilities.

  The switch from a situation of certainty of no loss to, say, a 5% probability of a loss is quite likely more worrying than a change in the probability of loss from 5% to 10%. To be sure of something is one thing, not to be sure another. This "discontinuity" in perception at probability zero is formally reflected in our mathematical model by the discontinuity of the variance function's slope at the status quo.

- Arrow, K.J. (1965), The Theory of Risk Aversion, Chapter 2 of *Aspects of the Theory of Risk Bearing*. Helsinki: Yrjö Jahnsonin Saatio.
- Dionne, G. and S.E. Harrington (2017), Insurance and Insurance Markets, *Handbook of the Economics of Risk and Uncertainty*, 1st Edition, W.K. Viscusi and M. Machina (Eds.), North Holland, Amsterdam, 203-261, 2014. Available at http://dx.doi.org/10.2139/ssrn.2943685.
- Paluszynski, R. and Pei-Cheng Yu (2022), Commitment versus Flexibility and Sticky Prices: Evidence from Life Insurance, forthcoming in *Review of Economic Dynamics*, https://doi.org/10.1016/j.red.2022.07.003.
- Segal, U. and A. Spivak (1990), First Order versus Second Order Risk Aversion, *Journal of Economic Theory* 51, 111-125.
- Zweifel, P., Eisen, R. and D.L. Eckles (2021), *Insurance Economics* (2nd ed.), New York, NY Springer.

Thank you for your attention