LONG-RANGE CORRELATIONS AND UNIVERSALITY IN PLASMA EDGE TURBULENCE

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1. Introduction

Recently, it has been conjectured that turbulent transport in fusion plasmas may be of a self-similar nature. This conclusion arises naturally in the context of the, still speculative, Self-Organized Criticality (SOC) models [1]. These models predict transport by avalanches, which would generate self-similar behaviour in space and time of the turbulent data. The attractiveness of the hypothesis lies in several predictions that seem to be in accord with observations of fusion plasmas: (1) the existence of a critical gradient that is robust to a certain extent (the well-known "profile consistency" of fusion plasmas that causes the profiles to be rather insensitive to the power deposition profile), (2) long-range interactions (the observed radial cold and heat pulses that travel faster than diffusively), and (3) Bohm scaling of the diffusivities.

To test the SOC hypothesis, first one needs to demonstrate the self-similar nature of the turbulence. This can in principle be tested by accurately determining the shape of the autocorrelation function (ACF) of turbulent signals. The ability to discern between an algebraic or exponential decay of the ACF at large lags would provide an indication whether SOC models could be appropriate descriptions of the turbulence. Unfortunately, the critical information is present in the tail of the distribution where statistics are generally poor, making this approach inviable for experimental data analysis.

There exist several other equivalent measures of self-similarity. One is the variance of the m-point averaged signal, which decays with increasing m as $m^{-\beta}$, with decay exponent β, which is the same as the decay exponent of the ACF. For uncorrelated (random) signals, $\beta = 1$. Another alternative, known as the Rescaled-Range analysis technique or Hurst analysis, is more promising [2]. This type of analysis is far more robust against random noise perturbations than the direct determination of the ACF and gives more reliable results for finite-length data series. A closely related and equally robust technique is the Scaled Windowed Variance method [3].

We present the results of applying this technique to data obtained with Langmuir probes in fusion plasmas in a wide variety of devices, and find significant and similar values of the Hurst exponent in all devices, pointing towards possible universality.

2. Description of the method

For a time series of length n, $X \equiv \{X_t : t = 1, 2, ..., n\}$, with mean $\bar{X}(n)$ and variance $S^2(n)$ the *R*/*S* ratio (or Rescaled Range) is defined as [4, 5]

$$\frac{R(n)}{S(n)} = \frac{\max(0, W_1, W_2, ..., W_n) - \min(0, W_1, W_2, ..., W_n)}{\sqrt{S^2(n)}}$$

where $W_k = X_1 + X_2 + \cdots + X_k - k\bar{X}(n)$. For a sequence with only short-range dependencies, the expected value of this ratio scales as $E[R(n)/S(n)] \xrightarrow[n\to\infty]{} \lambda\, n^{0.5}$. However, for phenomena characterized by long-range dependencies, it scales as $E[R(n)/S(n)] \xrightarrow[n\to\infty]{} \lambda\, n^H$ with $H > 0.5$. The parameter H is referred to as the Hurst parameter, and is related to the variance decay exponent through $H = 1 - \beta/2$. When the dependence of $R(n)/S(n)$ on n is well-fitted by the function λn^H over a long range of values of n (spanning several decades), then one has a strong indication of self-similarity of the signal over this range[6].

The robustness of this method to random noise perturbations is shown in Fig. 1. It shows the R/S graph for data obtained with a Langmuir probe in the edge region of the TJ-I tokamak upon adding various levels of noise to the raw data. The slope of the line is affected very little even at noise levels around 60%.

For purposes of testing and calibrating the method, we have analyzed computer-generated data from sandpile models, yielding high values of H (≈0.8) for this prototypical SOC model, which could be reduced to $H = 0.5$ by adding a decorrelating shear flow. Further, we have analyzed data from Langmuir probes in non-fusion devices (e.g. Thorello, a toroidal device without rotational transform [7]) yielding values of H close to 0.5. This indicates that these devices do not exhibit sufficient confinement to generate a self-organized state.

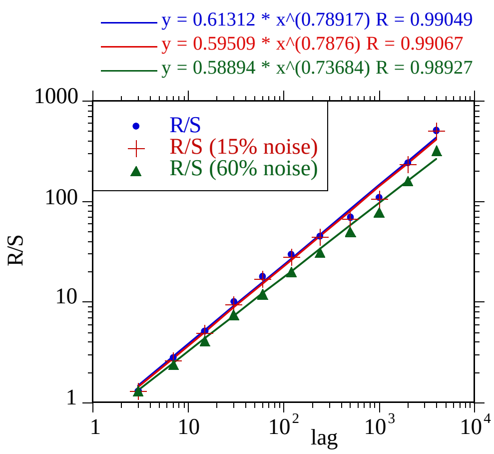

Fig. 1. R/S graph for experimental data taken with a Langmuir probe in TJ-I with various levels of noise added.

3. Analysis of turbulence measured with Langmuir probes in fusion plasmas

The analysis of data from Langmuir probes (the Ion Saturation Current, I_{sat}) taken at the plasma edge in a wide variety of fusion devices (tokamaks, stellarators, RFPs) reveals the existence of self-similar behaviour or long-range correlations in all devices studied (see Table 1 and Fig. 2). The observed variation of the Hurst exponent in the plasma edge, $0.62 < H < 0.75$, is small in spite of the variety of devices. On the other hand, the variation of H in the scrape-off layer (SOL) is much larger. In Wendelstein VII-AS, a slight decrease in H at the sheared flow layer was observed (Fig. 3), possibly corresponding to a local decorrelation effect. In Fig. 3, the error bars are calculated from a power fit to R/S taking into account the statistical error in the individual estimates of R/S. Although the error bars are comparable to the radial variation, the radial profile is reproducible.

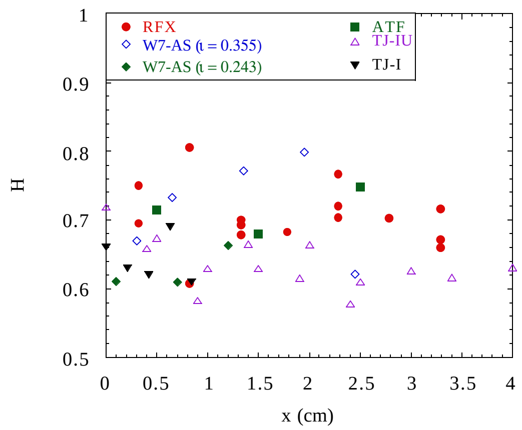

Fig. 2. Observed values of H within the last flux surface (x = distance from LCFS) in 5 different fusion devices

The significance of the obtained results was checked by re-analyzing the data after shuffling (i.e., re-ordering the temporal sequence in a random manner). After shuffling, H was always reduced close to its random value (0.5), increasing confidence in the obtained values.

The repeated occurrence of values of H differing significantly from the value corresponding to random noise ($H = 0.5$) in all fusion devices points to a universal aspect of the underlying turbulence. Further, the degree of self-similarity detected implies the existence of long-range correlations (with respect to the correlation time).

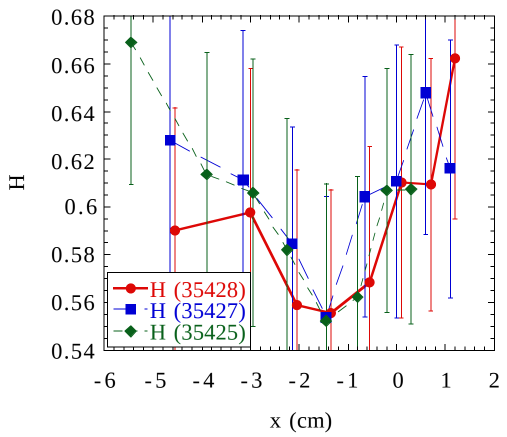

Fig. 3. Radial profile of the Hurst parameter H for three different plasma discharges with the same global parameters in W7-AS and for the configuration with $\iota_a = 0.234$.

4. Conclusions and discussion

We have analyzed data from Langmuir probes taken at the plasma edge in a wide variety of fusion devices with the Rescaled-Range technique in order to clarify the possible existence of self-similar behaviour or long-range correlations. The observed variation of the Hurst exponent in the plasma edge, $0.62 < H < 0.75$, is small in spite of the variety of devices. On the other hand, the variation of *H* in the scrape-off layer is much larger. In Wendelstein VII-AS, a slight decrease in *H* at the sheared flow layer was observed, possibly corresponding to a local decorrelation effect.

Device	Nº Series	$\langle H \rangle_{edge}$	$\langle H \rangle_{SOL}$
TJ-I	9	0.75 ± 0.03	0.75 ± 0.04
JET Limiter	4	–	0.52 ± 0.04
JET Divertor	4	–	0.63 ± 0.03
TJ-IU Torsatron	21	0.64 ± 0.03	0.67 ± 0.01
W7-AS, $\iota_a = 0.243$	24	0.62 ± 0.01	0.60 ± 0.04
W7-AS, $\iota_a = 0.355$	29	0.72 ± 0.07	0.66 ± 0.06
ATF	20	0.71 ± 0.03	0.92 ± 0.07
RFX (RFP)	29	0.69 ± 0.04	–

Table 1. Comparison of obtained values of the Hurst exponent in different devices and at different radial positions.

The repeated occurrence of values of *H* differing significantly from the value corresponding to random noise (*H* = 0.5) in all machines points to a universal aspect of the underlying turbulence. Further, the degree of self-similarity detected implies the existence of long-range correlations (several orders of magnitude longer than the experimental turbulence decorrelation time). This may either be due to a response of the system to perturbations that is much slower than the turbulence decorrelation time, or to long-range correlations generated by some mechanism, for which the avalanches of SOC models are a good candidate. In other words, although the evidence for self-similarity of the studied data is compelling, this by itself is no proof of SOC behaviour. The detection of SOC behaviour would require the analysis of both time- and space-resolved measurements to detect avalanches. Analysis techniques similar to the Rescaled Range technique that can handle such twodimensional data sets are under development.

References

[1] P. Bak, C. Tang, and K. Weisenfeld: *Self-Organized Criticality: An Explanation of 1/f Noise*, Phys. Rev. Lett. **59** (1987) 381

[2] B.A. Carreras, B. Ph. van Milligen, M.A. Pedrosa, et al.: *Long-Range Correlations in Plasma Edge Turbulence*, Phys. Rev. Lett. **80**, (1998) 4438

[3] M.J. Cannon, D.B. Percival, D.C. Caccia, et al.: Physica **A** 241 (1997) 606

[4] B. B. Mandelbrot and J. R. Wallis: Water Resources Research **4**, 909-918 (1969)

[5] H. E. Hurst: Trans. Am. Soc. Civil Eng. **116**, 770 (1951)

[6] B.A. Carreras, B. Ph. van Milligen, M.A. Pedrosa, et al.: *Self-similarity of the Plasma Edge Fluctuations*, Submitted to Phys. Plasmas (1998)

[7] C. Riccardi, D. Xuantong, M. Salierno, L. Gamberale, M. Fontanesi: Phys. Plasmas **4** (11) 3749 (1997).