Decomposing changes in CO_2 emission inequality over time: the roles of re-ranking and changes in per capita CO_2 emission disparities

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Abstract

This paper analyzes the effects of changes in country ranking and per capita CO_2 emissions on the change in CO_2 emission inequality over time. For this purpose, we introduce a three-term decomposition of the change occurring in the Gini index of per capita CO_2 emissions when moving from an initial to a final per capita CO_2 emission distribution. The decomposition explains the link between the inequality trend and the changes in country ranking, population size, and per capita CO_2 emission disparities. We show that all components of inequality change can be further decomposed by subgroup. This provides analysts with a decomposition technique detecting the within-group and between-group contributions to each component of inequality change. The decomposition is used to analyze the change in per capita CO_2 emission inequality in Europe over the 1991-2011 period.

Keywords: CO_2 emission, Decomposition, Gini index, Inequality, Re-ranking

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1. Introduction

The impact of CO_2 emissions on the environment and climate change highlights the need to analyze CO_2 emission distribution across countries [1, 2, 3]. Increasing attention is being paid to the analysis of international inequality in energy consumption and per capita CO_2 emissions [4, 5, 6], in order to provide policy-makers with information useful for deciding specific mitigation policy objectives [7]. This also requires the definition of measurement tools for exploring the inequality dynamic over time. Recently, Duro [8] has shown that the change in per capita CO_2 emission inequality between two points in time can be decomposed in two terms measuring the contributions of changes in population shares and per capita CO_2 emissions. The two components are clearly of interest since analysts can detect the effects of changes in population shares and per capita CO_2 emissions on the change in inequality in the per capita CO_2 emission distribution. However, some aspects of the distributional change between the initial and the final time are not evident observing the component attributable to the change in per capita CO_2 emissions. Indeed, when moving from the initial to the final time, the ranking of countries in the per capita CO_2 emission distribution can change [9]. In addition, analysts may be interested in assessing whether the relative emission disparities between the countries toward the bottom of the initial distribution, i.e. low per capita emitters, and those toward the top, i.e. high per capita emitters, increase or decrease over the analyzed time period. In this article, we propose a decomposition explaining the link among inequality change, re-ranking, population variations, and changes in relative emission disparities between low and high per capita emitters.

When an inequality index of per capita CO_2 emissions is calculated, the per capita CO_2 emissions are the units and the population shares are the unit weights [10]. Duro [8] pointed out that the inequality change over time depends on both the change in population shares and the change in per capita CO_2 emissions; he then decomposed the inequality change into a component attributable to population change and another component attributable to per capita CO_2

emission change. As shown in [8], this two-term decomposition can be applied to various inequality indices, including the Gini index. Starting from the Duro decomposition of the change in the Gini index, a matrix decomposition of the change in inequality is introduced. This decomposition clearly separates the roles of population change and per capita CO_2 emission change. To derive the matrix decomposition, we follow the matrix approach proposed by Mussini for decomposing the Gini index by subgroup and source [11]. This requires the generalization of the Mussini matrix approach to take into account the population shares representing the weighting system to be used when calculating the inequality index.

We then extend the matrix decomposition of the inequality change to identify two further components: one caused by the re-ranking of per capita emitters from the initial to the final time; the other resulting from the change in relative emission disparities between initially low and high per capita emitters from the initial to the final time. More specifically, these two components are obtained by splitting the matrix expression originally attributed to the per capita CO_2 emission changes in the Duro decomposition. For instance, suppose that the process of economic growth in poorer countries with low per capita CO_2 emissions leads to an increase in their per capita CO_2 emissions from the initial to the final time. Besides, suppose that, in the same period, richer countries with high per capita CO_2 emissions improve energy efficiency, reducing demand for fossil fuel energy and consequently their per capita CO_2 emissions. Let us assume that countries exchange their positions within the per capita CO_2 emission distribution from the initial to the final time. In such a scenario, one of the two new components explains the actual reduction in relative emission disparities among countries by keeping their positions in the initial per capita CO_2 emission distribution fixed. The second component measures the re-ranking effect partially offsetting the equalizing effect caused by economic growth in countries initially at the bottom of the per capita CO_2 emission distribution and by energy efficiency improvements in countries initially at the top. Using the new three-term decomposition, analysts can explain the various contributions of population change, re-ranking and changes in relative emission disparities among per capita emitters to the overall change in inequality over time.

The paper also shows that the three components of the change in inequality can be decomposed by subgroup, separating the within-group contribution from the between-group contribution for each component of inequality change. Hence, using the matrix decomposition approach, we combine the decomposition by subgroup with the decomposition of the change in inequality over time.

The new decomposition is applied to decompose the change in per capita CO_2 emission inequality among European countries over the 1991-2011 period. The entire period is split into four 5-year sub-periods to perform a more detailed analysis of the inequality change over time. Our findings indicate that re-ranking has a non-negligible effect over the 1991-2011 period but the change in relative emission disparities between low and high per capita emitters plays the major role in determining the changes in inequality. Results show that a small change in inequality over a sub-period is the outcome of the offsetting interaction between the re-ranking of emitters and the change in relative emission disparities between low and high per capita emitters; that is, a small change in inequality is not the result of few changes in the per capita CO_2 emission distribution over the sub-period.

The rest of the article is organized as follows. Section 2 introduces notation and the matrix expression of the Gini index used to measure the inequality in per capita CO_2 emissions. Section 3 sets out the new decomposition of the change in inequality. In Section 4, the decomposition is applied to the change in inequality between European countries over the 1991-2011 period. Section 5 concludes.

2. Preliminaries and notation

Consider k countries and assume that, for every country i, the population n_i and the per capita CO_2 emission are known, with $i = 1, \ldots, k$ and $n = \sum_{i=1}^{k} n_i$. Let e_i be the per capita CO_2 emission of country i and $p_i = n_i/n$ be its population share. Duro [8] suggested using the Gini index to measure the inequality in per capita CO_2 emissions. Using the previous symbols, the Gini index formulation is:

$$G = \frac{1}{2\bar{e}\left(p\right)} \sum_{i=1}^{k} \sum_{j=1}^{k} p_i p_j \left|e_i - e_j\right|, \tag{1}$$

where $\bar{e}\left(p\right)$ is the weighted average of per capita CO_2 emissions where the weights are the population shares.

The Gini index in equation 1 can be expressed in matrix form by following the approach proposed by Mussini [11] for the measurement of income inequality. For this purpose, the following notation is used. Let $\mathbf{e} = \left(e_1, \ldots, e_k\right)^T$ be the $k \times 1$ vector of per capita CO_2 emissions sorted in decreasing order and $\mathbf{p} = \left(p_1, \ldots, p_k\right)^T$ be the $k \times 1$ vector of the corresponding population shares. $\mathbf{1}_k$ being the $k \times 1$ vector with each element equal to 1, the $k \times k$ skew-symmetric matrix $\mathbf{E}$ is defined as follows:

$$\mathbf{E} = \frac{1}{\bar{e}\left(p\right)} \left(\mathbf{1}_k \mathbf{e}^T - \mathbf{e}\mathbf{1}_k^T\right) = \begin{bmatrix} \frac{e_1 - e_1}{\bar{e}(p)} & \cdots & \frac{e_k - e_1}{\bar{e}(p)} \\ \vdots & \ddots & \vdots \\ \frac{e_1 - e_k}{\bar{e}(p)} & \cdots & \frac{e_k - e_k}{\bar{e}(p)} \end{bmatrix}. \tag{2}$$

The generic (i, j)-th element of $\mathbf{E}$ is equal to the difference between e_j and e_i divided by the weighted average of per capita CO_2 emissions. Hence, $\mathbf{E}$ contains the k^2 relative pairwise differences between the per capita CO_2 emissions as ordered in $\mathbf{e}$. The Gini index can be expressed as follows:[1]

$$G\left(\mathbf{p}, \mathbf{e}\right) = \frac{1}{2} tr\left(\mathbf{GPE}^T\mathbf{P}\right), \tag{3}$$

where $\mathbf{P} = diag\left\{\mathbf{p}\right\}$ is the $k \times k$ diagonal matrix with the diagonal elements equal to the population shares in $\mathbf{p}$, and $\mathbf{G}$ is a $k \times k$ G-matrix (a skew-symmetric matrix with upper diagonal elements equal to -1, lower diagonal elements equal to 1 and diagonal elements equal to 0) (see Silber in [12]). Using the circular

[1] The proof is given in Appendix A.

property of the trace, expression in equation 3 can be written as

$$\begin{aligned} G(\mathbf{p}, \mathbf{e}) &= \frac{1}{2} tr \left(\mathbf{PGPE}^T \right) \\ &= \frac{1}{2} tr \left(\tilde{\mathbf{G}} \mathbf{E}^T \right). \end{aligned} \tag{4}$$

The matrix $\tilde{\mathbf{G}} = \mathbf{PGP}$ in equation 4 is a generalization of the Silber G-matrix introduced in order to take into account the weighting vector $\mathbf{p}$, or any other weighting vector, when calculating the Gini index. $\tilde{g}_{ij}$ being the (i, j)-th element of $\tilde{\mathbf{G}}$, when $i > j$ $\tilde{g}_{ij}$ is equal to $p_i p_j$ whereas $\tilde{g}_{ji}$ is equal to $-p_i p_j$. Since $\tilde{\mathbf{G}}$ contains the weights of the pairwise differences in $\mathbf{E}$, we refer to $\tilde{\mathbf{G}}$ as the weighting G-matrix throughout the article.

The next section shows that the above matrix approach is useful to detect the components of inequality change between two points in time.

3. A multi-dimensional decomposition of the change in inequality

The analysis of the change in CO_2 emission inequality between two points in time, say t and $t+1$, is performed by identifying the components of inequality change over time [8]. Duro [8] pointed out that the change in inequality depends on the change in population shares and the change in per capita CO_2 emissions moving from t to $t+1$. Duro suggested a two-term decomposition of the change in the Gini inequality index, in which one term measures the contribution of the change in disparities between per capita CO_2 emissions and a second term gauges the contribution of the change in population shares.

In this section, we first propose a matrix approach for calculating the two terms of the Duro decomposition (Section 3.1). One appealing feature of this matrix approach is the evident separation between the population effect and the per capita CO_2 emission effect in determining the inequality change; moreover, the matrix formulation is suitable for further developing the decomposition of the inequality change. Section 3.2 shows that the Duro decomposition can be extended, isolating two new components: one gauging for re-ranking of emitters in the per capita CO_2 emission distribution, the other measuring the change in relative emission disparities between low and high per capita emitters in the t distribution when passing to the $t+1$ distribution. Hence, using the new decomposition, the contributions of population change, re-ranking and change in relative emission disparities between per capita emitters can be detected. In Section 3.3, the components of the three-term decomposition are decomposed by subgroup, providing a multi-dimensional decomposition of the change in inequality enabling analysts to detect the within-group and between-group contributions to the effects of changes in population, ranking of countries and relative emission disparities.

3.1. A matrix approach to the decomposition of the inequality change

Assume that per capita CO_2 emissions and population shares are observed in t and $t+1$. The Duro decomposition can be expressed in a matrix form which clearly identifies the role of the change in population shares and that of the change in per capita CO_2 emissions. Let $\mathbf{e}_t$ be the $k \times 1$ vector of the t per capita emissions sorted in decreasing order and $\mathbf{p}_t$ be the $k \times 1$ vector of the corresponding population shares. Let $\mathbf{e}_{t+1}$ be the $k \times 1$ vector of the $t+1$ per capita emissions sorted in decreasing order and $\mathbf{p}_{t+1}$ be the $k \times 1$ vector of the corresponding population shares. The change in inequality between t and $t+1$ is given by the Gini index in $t+1$ minus the Gini index in t:

$$\Delta G = G\left(\mathbf{p}_{t+1}, \mathbf{e}_{t+1}\right) - G\left(\mathbf{p}_t, \mathbf{e}_t\right) = \frac{1}{2} tr\left(\tilde{\mathbf{G}}_{t+1}\mathbf{E}_{t+1}^T\right) - \frac{1}{2} tr\left(\tilde{\mathbf{G}}_t\mathbf{E}_t^T\right). \quad (5)$$

Let $\mathbf{p}_{t|t+1}$ stand for the $k \times 1$ vector of the t population shares arranged by the decreasing order of the corresponding $t+1$ per capita emissions. Let $\bar{e}_{t+1}\left(p_t\right)$ denote the weighted average of the $t+1$ per capita emissions calculated by using the population shares in t, and $\lambda = \bar{e}_{t+1}\left(p_{t+1}\right)/\bar{e}_{t+1}\left(p_t\right)$ be the ratio of the actual $t+1$ average per capita emission to the fictitious $t+1$ average per capita emission calculated by using the population shares in t. Since $\mathbf{P}_{t|t+1} = diag\left\{\mathbf{p}_{t|t+1}\right\}$, the Gini index of $t+1$ per capita emissions calculated by using the t population shares is

$$
\begin{aligned}
G\left(\mathbf{p}_{t|t+1}, \mathbf{e}_{t+1}\right) &= \frac{1}{2} tr\left(\mathbf{G}\mathbf{P}_{t|t+1} \lambda \mathbf{E}_{t+1}^{T} \mathbf{P}_{t|t+1}\right) \\
&= \frac{1}{2} tr\left(\mathbf{P}_{t|t+1} \mathbf{G}\mathbf{P}_{t|t+1} \lambda \mathbf{E}_{t+1}^{T}\right) \\
&= \frac{1}{2} tr\left(\tilde{\mathbf{G}}_{t|t+1} \lambda \mathbf{E}_{t+1}^{T}\right)
\end{aligned}
\tag{6}
$$

where $\tilde{\mathbf{G}}_{t|t+1} = \mathbf{P}_{t|t+1} \mathbf{G}\mathbf{P}_{t|t+1}$ is the weighting G-matrix obtained by using the population shares in t instead of those in $t+1$. In equation 6, multiplying the matrix $\mathbf{E}_{t+1}^{T}$ by λ ensures that the pairwise differences between the $t+1$ per capita emissions are divided by $\bar{e}_{t+1}(p_t)$ instead of $\bar{e}_{t+1}(p_{t+1})$. By adding and subtracting $G\left(\mathbf{p}_{t|t+1}, \mathbf{e}_{t+1}\right)$ in equation 5, Duro [8] separated the contribution to ΔG attributable to the change in population shares from that attributable to the change in disparities between per capita CO_2 emissions:

$$
\begin{aligned}
\Delta G &= \left[G\left(\mathbf{p}_{t|t+1}, \mathbf{e}_{t+1}\right) - G\left(\mathbf{p}_{t}, \mathbf{e}_{t}\right)\right] + \left[G\left(\mathbf{p}_{t+1}, \mathbf{e}_{t+1}\right) - G\left(\mathbf{p}_{t|t+1}, \mathbf{e}_{t+1}\right)\right] \\
&= E + S.
\end{aligned}
\tag{7}
$$

The component E measures the contribution to ΔG coming from the change in disparities between per capita CO_2 emissions, whereas the component S measures the contribution of the change in population shares to ΔG. Focusing on S, its matrix expression can be re-arranged as follows:

$$
\begin{aligned}
S &= \frac{1}{2} tr\left(\tilde{\mathbf{G}}_{t+1} \mathbf{E}_{t+1}^{T}\right) - \frac{1}{2} tr\left(\tilde{\mathbf{G}}_{t|t+1} \lambda \mathbf{E}_{t+1}^{T}\right) \\
&= \frac{1}{2} tr\left[\left(\tilde{\mathbf{G}}_{t+1} - \lambda \tilde{\mathbf{G}}_{t|t+1}\right) \mathbf{E}_{t+1}^{T}\right] \\
&= \frac{1}{2} tr\left(\mathbf{S}\mathbf{E}_{t+1}^{T}\right),
\end{aligned}
\tag{8}
$$

where $\mathbf{S} = \tilde{\mathbf{G}}_{t+1} - \lambda \tilde{\mathbf{G}}_{t|t+1}$ captures the change in inequality caused by the changes in population shares from t to $t+1$. If country populations have equi-proportionate changes, then $S = 0$ since $\tilde{\mathbf{G}}_{t+1} = \tilde{\mathbf{G}}_{t|t+1}$ and $\lambda = 1$; that is, the population changes do not affect the change in inequality. The matrix expression in equation 8 clearly shows that the population component is solely due to the change in population shares, irrespective of the change in the disparities between per capita CO_2 emissions.

3.2. A three-term decomposition of the inequality change

The contribution to the inequality change measured by E is clearly of interest; however, it does not provide information on the mobility of countries within the per capita CO_2 emission distribution over time. In addition, analysts may be interested in assessing whether the relative emission disparities between the low per capita emitters in t and the high per capita emitters in t increase or decrease when passing from t to $t+1$. This point can be investigated by keeping countries sorted by their per capita CO_2 emissions in t and comparing their t and $t+1$ per capita emissions, in order to measure the actual change in the relative disparities between the per capita emissions of the countries toward the bottom of the t distribution and those of the countries toward the top of the t distribution when moving from t to $t+1$. Similar distributional aspects were explored by Jenkins and Van Kerm in their decomposition of the change in income inequality over time [13]. Mussini expressed the Jenkins and Van Kerm decomposition in a matrix form which is subgroup decomposable [14]. Based on the approach in [14], we show that E can be decomposed into two components measuring the effects on ΔG attributable to re-ranking and change in relative emission disparities between per capita emitters by keeping their ranking in t fixed.

Let $\mathbf{e}_{t+1|t}$ be the $k \times 1$ vector of the $t+1$ per capita emissions sorted by the decreasing order of the respective t per capita emissions. The concentration index of the $t+1$ per capita emissions sorted by the t per capita emissions, calculated by using the t population shares, is defined as follows:

$$C\left(\mathbf{p}_t, \mathbf{e}_{t+1|t}\right) = \frac{1}{2} tr\left(\tilde{\mathbf{G}}_t \mathbf{E}^T_{t+1|t}\right), \tag{9}$$

where $\mathbf{E}_{t+1|t} = \left[1/\bar{e}_{t+1}\left(p_t\right)\right]\left(\mathbf{1}_k \mathbf{e}^T_{t+1|t} - \mathbf{e}_{t+1|t}\mathbf{1}^T_k\right)$ is the $k \times k$ skew-symmetric matrix having its (i,j)-th element equal to the difference between $e_{j,t+1|t}$ and $e_{i,t+1|t}$ divided by $\bar{e}_{t+1}\left(p_t\right)$; that is, $\mathbf{E}_{t+1|t}$ contains the k^2 relative pairwise differences between the per capita CO_2 emissions as arranged in $\mathbf{e}_{t+1|t}$. The concentration index $C\left(\mathbf{p}_t, \mathbf{e}_{t+1|t}\right)$ can be re-written as a function of $\mathbf{E}_{t+1}$ instead of $\mathbf{E}_{t+1|t}$. Let $\mathbf{B}$ stand for the $k \times k$ permutation matrix re-arranging the elements of $\mathbf{e}_{t+1}$ to obtain $\mathbf{e}_{t+1|t}$, that is $\mathbf{e}_{t+1|t} = \mathbf{B}\mathbf{e}_{t+1}$. After some algebraic manipulations, we obtain $\mathbf{E}_{t+1|t} = \mathbf{B}\lambda\mathbf{E}_{t+1}\mathbf{B}^T$. By replacing $\mathbf{E}_{t+1|t}$ with $\mathbf{B}\lambda\mathbf{E}_{t+1}\mathbf{B}^T$ in equation 9, $C\left(\mathbf{p}_t, \mathbf{e}_{t+1|t}\right)$ can be re-written as:

$$
\begin{aligned}
C\left(\mathbf{p}_t, \mathbf{e}_{t+1|t}\right) &= \frac{1}{2} tr\left(\tilde{\mathbf{G}}_t \mathbf{B}\lambda\mathbf{E}_{t+1}^T \mathbf{B}^T\right) \\
&= \frac{1}{2} tr\left(\mathbf{B}^T \tilde{\mathbf{G}}_t \mathbf{B}\lambda\mathbf{E}_{t+1}^T\right).
\end{aligned}
\tag{10}
$$

Given equations 9 and 10, the term E can be decomposed as follows:

$$
\begin{aligned}
E &= \left[G\left(\mathbf{p}_{t|t+1}, \mathbf{e}_{t+1}\right) - C\left(\mathbf{p}_t, \mathbf{e}_{t+1|t}\right)\right] - \left[G\left(\mathbf{p}_t, \mathbf{e}_t\right) - C\left(\mathbf{p}_t, \mathbf{e}_{t+1|t}\right)\right] \\
&= \left[\frac{1}{2} tr\left(\tilde{\mathbf{G}}_{t|t+1} \lambda\mathbf{E}_{t+1}^T\right) - \frac{1}{2} tr\left(\mathbf{B}^T \tilde{\mathbf{G}}_t \mathbf{B}\lambda\mathbf{E}_{t+1}^T\right)\right] \\
&- \left[\frac{1}{2} tr\left(\tilde{\mathbf{G}}_t \mathbf{E}_t^T\right) - \frac{1}{2} tr\left(\tilde{\mathbf{G}}_t \mathbf{E}_{t+1|t}^T\right)\right] \\
&= \frac{1}{2} tr\left[\left(\tilde{\mathbf{G}}_{t|t+1} - \mathbf{B}^T \tilde{\mathbf{G}}_t \mathbf{B}\right)\lambda\mathbf{E}_{t+1}^T\right] - \frac{1}{2} tr\left[\tilde{\mathbf{G}}_t \left(\mathbf{E}_t^T - \mathbf{E}_{t+1|t}^T\right)\right] \\
&= \frac{1}{2} tr\left(\mathbf{R}\lambda\mathbf{E}_{t+1}^T\right) - \frac{1}{2} tr\left(\tilde{\mathbf{G}}_t \mathbf{D}^T\right) \\
&= R - D,
\end{aligned}
\tag{11}
$$

where $\mathbf{R} = \tilde{\mathbf{G}}_{t|t+1} - \mathbf{B}^T \tilde{\mathbf{G}}_t \mathbf{B}$ and $\mathbf{D} = \mathbf{E}_t - \mathbf{E}_{t+1|t}$.

In equation 11, R is the component measuring the re-ranking among countries moving from t to $t+1$. It always yields a non-negative contribution to the change in inequality since $0 \leq R \leq 2G\left(\mathbf{p}_{t|t+1}, \mathbf{e}_{t+1}\right)$.[2] From the matrix expression of R, we can see that the movements of countries are tracked by using the matrix $\mathbf{R}$ which detects the pairs of countries re-ranking when passing from the per capita CO_2 emission distribution in t to that in $t+1$. Since r_{ij} is the generic (i,j)-th element of $\mathbf{R}$, r_{ij} is equal to $2p_{i,t|t+1}\, p_{j,t|t+1}$ $\left(-2p_{i,t|t+1}\, p_{j,t|t+1}\right)$ if $i > j$ $(i < j)$ and the (i,j)-th entry of $\mathbf{E}_{t+1}$ is filled by the relative difference between the per capita CO_2 emissions belonging to two re-ranking countries;

[2] R is a mobility measure which coincides with the Atkinson-Plotnick re-ranking coefficient used to measure re-ranking between income receivers in income distribution [13]. The Atkinson-Plotnick re-ranking coefficient equals 0 if the ranking of income receivers is unchanged from t to $t+1$, whereas it is equal to two times the Gini index in $t+1$ if the ranking in $t+1$ is completely reverse compared to the ranking in t.

otherwise r_{ij} is equal to 0.[3] If the ranking of countries is unchanged from t to $t+1$, then $\mathbf{B} = \mathbf{I}_k$ and $\tilde{\mathbf{G}}_{t|t+1} = \tilde{\mathbf{G}}_t$ implying $R = 0$. The matrix expression of R clearly shows that the re-ranking effect is solely due to changes in position of countries within the per capita CO_2 emission distribution.

D is the component which measures the change in relative emission disparities between per capita emitters by keeping their ranking in t fixed. This component can reduce or increase the inequality between t and $t+1$. The generic (i, j)-th element of $\mathbf{D}$, denoted by d_{ij}, compares the relative difference between the t per capita emissions of the countries in positions j and i in $\mathbf{e}_t$ with the relative difference between the $t+1$ per capita emissions of the same two countries in $\mathbf{e}_{t+1|t}$, since both $\mathbf{E}_t$ and $\mathbf{E}_{t+1|t}$ are obtained by keeping countries sorted by the decreasing order of their per capita CO_2 emissions in t. If $d_{ij} > 0$ with $i > j$ (and consequently $d_{ji} < 0$ since $\mathbf{D}$ is skew-symmetric), the relative disparity between the t per capita emissions of two countries is greater than the relative disparity between the $t+1$ per capita emissions of the same countries; hence, an equalizing effect is attributable to the change in per capita CO_2 emissions from t to $t+1$. If $d_{ij} < 0$ with $i > j$ (and consequently $d_{ji} > 0$), the relative disparity between the t per capita emissions of two countries is less than the relative disparity between the $t+1$ per capita emissions of the same countries; hence, a disequalizing effect is attributable to the change in per capita CO_2

[3] For instance, suppose that the vector $\mathbf{e}_t = (e_{1,t} = 7, e_{2,t} = 5, e_{3,t} = 3, e_{4,t} = 1)^T$ contains the per capita emissions of four countries in t sorted in decreasing order and that the vector $\mathbf{p}_t = (p_{1,t} = 0.2, p_{2,t} = 0.4, p_{3,t} = 0.1, p_{4,t} = 0.3)^T$ includes the corresponding population shares in t. Let $\mathbf{e}_{t+1|t} = \left(e_{1,t+1|t} = 6, e_{2,t+1|t} = 8, e_{3,t+1|t} = 4.5, e_{4,t+1|t} = 0.5\right)^T$ be the per capita emissions in $t+1$ sorted by the decreasing order of the per capita emissions in t. Therefore the vector of the $t+1$ per capita emissions sorted in decreasing order is $\mathbf{e}_{t+1} = (e_{1,t+1} = 8, e_{2,t+1} = 6, e_{3,t+1} = 4.5, e_{4,t+1} = 0.5)^T$, in which re-ranking has occurred between the two highest per capita emitters in t when moving to $t+1$. The 4×4 matrix R detects the re-ranking by showing $r_{21} = 2p_{2,t|t+1}p_{1,t|t+1}$ and $r_{12} = -2p_{2,t|t+1}p_{1,t|t+1}$, whereas its remaining elements are equal to zero. Since $\mathbf{p}_{t|t+1} = \left(p_{1,t|t+1} = 0.4, p_{2,t|t+1} = 0.2, p_{3,t|t+1} = 0.1, p_{4,t|t+1} = 0.3\right)^T$, we obtain $r_{21} = 0.16$ and $r_{12} = -0.16$.

emissions from t to $t+1$.[4] When $D > 0$, the relative disparities in per capita CO_2 emissions globally decrease moving from t to $t+1$; in other words, the relative disparities between low and high per capita emitters in t diminish when passing to $t+1$, producing an inequality reduction effect caused by the changes in per capita CO_2 emissions. When $D < 0$, the relative disparities in per capita CO_2 emissions globally increase from t and $t+1$; that is, the relative disparities between low and high per capita emitters in t increase when moving to $t+1$, producing an inequality growth effect caused by the change in per capita CO_2 emissions. If all per capita CO_2 emissions change in the same proportion from t to $t+1$, $D = 0$ since the relative disparity in per capita CO_2 emissions is unchanged for every pair of countries.

Now, given equation 11, a three-term decomposition of the change in inequality is obtained:

$$\begin{aligned} \Delta G &= R - D + S \\ &= \frac{1}{2} tr \left(\mathbf{R} \lambda \mathbf{E}_{t+1}^T \right) - \frac{1}{2} tr \left(\tilde{\mathbf{G}}_t \mathbf{D}^T \right) + \frac{1}{2} tr \left(\mathbf{S} \mathbf{E}_{t+1}^T \right). \end{aligned} \tag{12}$$

The decomposition in equation 12, provides additional information compared to the Duro decomposition, since the three-term decomposition identifies two further components of inequality change. Another attractive characteristic of this matrix decomposition approach is the evident separation between the roles of the various components: each component is identified by a specific matrix containing all the information necessary to determine its effect on the inequality change. For instance, component D is fully determined by matrix $\mathbf{D}$ measuring the change in relative emission disparities between per capita emitters, since both the per capita emitter ranking and the population shares are fixed in t.

In the next section, we show that the terms of the decomposition in equation 12 can be decomposed by subgroup, enabling analysts to further develop the

[4]For instance, referring to the numerical example shown in footnote 3, the 4×4 matrix D has $d_{41} = 0.4$ and $d_{14} = -0.4$. This indicates that the relative emission disparity between the lowest and highest per capita emitters in t has decreased from t to $t+1$.

analysis of the inequality change over time.

3.3. A subgroup decomposition of the components of inequality change

The Gini index can be decomposed by subgroup into within-group and between-group components. The decomposition of the Gini index into a component measuring income disparities within subgroups and a component measuring income disparities between subgroups was introduced by Dagum [15]. More recently, Mussini [16] proposed a decomposition detecting the within-group and between-group contributions to the components of income inequality and poverty changes over time. We extend the decomposition in equation 12 by isolating the within-group and between-group contributions to each component.

Suppose that countries are split into r subgroups according to a given criterion (e.g., region, GDP level). Let $\mathbf{w}_{h,t}$ be the $k \times 1$ vector with nonzero elements equal to 1 in the corresponding positions filled by the t per capita emissions of countries belonging to subgroup h (with $h = 1, \ldots, r$) in $\mathbf{e}_t$. The $k \times k$ matrix $\mathbf{W}_{h,t} = \mathbf{w}_{h,t}\mathbf{w}_{h,t}^T$ has its (i, j)-th entry equal to 1 if and only if the (i, j)-th entry of $\mathbf{E}_t$ is filled by the relative difference between the per capita emissions of two countries belonging to subgroup h, otherwise the (i, j)-th entry of $\mathbf{W}_{h,t}$ is 0. Using the Hadamard product,[5] the relative pairwise differences between the per capita emissions of the countries within subgroup h can be selected from $\mathbf{E}_t$:

$$\mathbf{E}_{h,t} = \mathbf{W}_{h,t} \odot \mathbf{E}_t. \tag{13}$$

The relative pairwise differences between the $t + 1$ per capita emissions of subgroup h in $\mathbf{E}_{t+1|t}$ fill the same entries in which the relative pairwise differences between the t per capita emissions of subgroup h are arranged in $\mathbf{E}_t$. Therefore, $\mathbf{W}_{h,t}$ can also be used to select the relative pairwise differences of subgroup h from $\mathbf{E}_{t+1|t}$:

$$\mathbf{E}_{h,t+1|t} = \mathbf{W}_{h,t} \odot \mathbf{E}_{t+1|t}\,. \tag{14}$$

[5]Let $\mathbf{X}$ and $\mathbf{Y}$ be $n \times n$ matrices. The Hadamard product $\mathbf{X} \odot \mathbf{Y}$ is defined as the $n \times n$ matrix with the (i, j)-th element equal to $x_{ij}y_{ij}$. The Hadamard product is the element-by-element matrix product [17].

Since $\mathbf{w}_{h,t+1}$ is the $k \times 1$ vector with nonzero elements equal to 1 in the corresponding positions filled by the $t+1$ per capita emissions of countries within subgroup h in $\mathbf{e}_{t+1}$, the $k \times k$ matrix $\mathbf{W}_{h,t+1} = \mathbf{w}_{h,t+1}\mathbf{w}_{h,t+1}^T$ selects the relative pairwise differences between the $t+1$ per capita emissions of the countries within subgroup h from $\mathbf{E}_{t+1}$:

$$\mathbf{E}_{h,t+1} = \mathbf{W}_{h,t+1} \odot \mathbf{E}_{t+1}. \tag{15}$$

Since $\mathbf{D} = \mathbf{E}_t - \mathbf{E}_{t+1|t}$, the Hadamard product between $\mathbf{W}_{h,t}$ and $\mathbf{D}$ gives the matrix with nonzero elements equal to the elements of $\mathbf{D}$ involving the countries of subgroup h:

$$\mathbf{D}_h = \mathbf{E}_{h,t} - \mathbf{E}_{h,t+1|t} = \mathbf{W}_{h,t} \odot \left(\mathbf{E}_t - \mathbf{E}_{t+1|t}\right) = \mathbf{W}_{h,t} \odot \mathbf{D}. \tag{16}$$

By replacing $\mathbf{E}_{t+1}$ and $\mathbf{D}$ in equation 12 with $\mathbf{E}_{h,t+1}$ and $\mathbf{D}_h$, respectively, we obtain the decomposition of the change in the inequality contribution of subgroup h:

$$\begin{aligned}\Delta G_h &= \frac{1}{2} tr\left(\mathbf{R}\lambda\mathbf{E}_{h,t+1}^T\right) - \frac{1}{2} tr\left(\tilde{\mathbf{G}}_t\mathbf{D}_h^T\right) + \frac{1}{2} tr\left(\mathbf{S}\mathbf{E}_{h,t+1}^T\right) \\ &= R_h - D_h + S_h.\end{aligned} \tag{17}$$

The $k \times k$ matrix $\mathbf{W}_{gh,t} = \mathbf{w}_{g,t}\mathbf{w}_{h,t}^T + \mathbf{w}_{h,t}\mathbf{w}_{g,t}^T$ has nonzero elements equal to 1 in the entries corresponding to the relative pairwise differences between the per capita emissions of subgroup g and those of subgroup h in both $\mathbf{E}_t$ and $\mathbf{E}_{t+1|t}$; hence, $\mathbf{W}_{gh,t}$ selects the between-group pairwise differences from both the matrices:

$$\mathbf{E}_{gh,t} = \mathbf{W}_{gh,t} \odot \mathbf{E}_t \tag{18}$$

and

$$\mathbf{E}_{gh,t+1|t} = \mathbf{W}_{gh,t} \odot \mathbf{E}_{t+1|t}. \tag{19}$$

The Hadamard product between $\mathbf{W}_{gh,t}$ and $\mathbf{D}$ selects the elements of $\mathbf{D}$ measuring the change in the relative disparities between the per capita emissions of subgroup g and the per capita emissions of subgroup h:

$$\mathbf{D}_{gh} = \mathbf{E}_{gh,t} - \mathbf{E}_{gh,t+1|t} = \mathbf{W}_{gh,t} \odot \left(\mathbf{E}_t - \mathbf{E}_{t+1|t}\right) = \mathbf{W}_{gh,t} \odot \mathbf{D}. \tag{20}$$

The $k \times k$ matrix $\mathbf{W}_{gh,t+1} = \mathbf{w}_{g,t+1}\mathbf{w}_{h,t+1}^T + \mathbf{w}_{h,t+1}\mathbf{w}_{g,t+1}^T$ has nonzero elements equal to 1 in the entries corresponding to the relative pairwise differences between the per capita emissions of subgroup g and the per capita emissions of subgroup h in $\mathbf{E}_{t+1}$. Hence, the matrix

$$\mathbf{E}_{gh,t+1} = \mathbf{W}_{gh,t+1} \odot \mathbf{E}_{t+1} \tag{21}$$

contains the relative pairwise differences between the $t+1$ per capita emissions of subgroup g and those of subgroup h. By replacing $\mathbf{E}_{t+1}$ and $\mathbf{D}$ in equation 12 with $\mathbf{E}_{gh,t+1}$ and $\mathbf{D}_{gh}$, respectively, we obtain the decomposition of the change in the contribution of inequality between subgroups g and h:

$$\begin{aligned}\Delta G_{gh} &= \frac{1}{2}tr\left(\mathbf{R}\lambda\mathbf{E}_{gh,t+1}^T\right) - \frac{1}{2}tr\left(\tilde{\mathbf{G}}_t\mathbf{D}_{gh}^T\right) + \frac{1}{2}tr\left(\mathbf{S}\mathbf{E}_{gh,t+1}^T\right) \\ &= R_{gh} - D_{gh} + S_{gh},\end{aligned} \tag{22}$$

where $\Delta G_{gh} = \Delta G_{hg}$ by construction.

Given equations 17 and 22, the subgroup decomposition of the components of inequality change is:

$$\begin{aligned}\Delta G &= \sum_{h=1}^{r}(R_h - D_h + S_h) + \sum_{h=2}^{r}\sum_{g=1}^{h-1}(R_{gh} - D_{gh} + S_{gh}) \\ &= \Delta G^{WG} + \Delta G^{BG},\end{aligned} \tag{23}$$

where ΔG^{WG} and ΔG^{BG} are respectively the within-group and between-group components of the inequality change. In equation 23, the three-term decomposition obtained in Section 3.2 is further developed by linking the subgroup decomposition dimension with the time decomposition dimension expressed by the decomposition of the inequality change over time. This provides a multi-dimensional decomposition which detects the inequality contributions of the combinations between the subgroup components and the components of inequality change over time.

4. Application

We apply the multi-dimensional decomposition introduced in Section 3.3 to analyze the inequality change in per capita CO_2 emissions from fuel combustion in Europe over the 1991-2011 period. Data are from the International Energy Agency (IEA) [18]. European countries are divided into two subgroups: one subgroup comprising OECD countries, the other including non-OECD countries. The non-OECD subgroup comprises several countries of the former USSR and Yugoslavia;[6] for these countries, disaggregated data on per capita CO_2 emissions were not available before 1990. To the best of our knowledge, this is the first study analyzing inequality in CO_2 emissions in Europe including the new European countries of the former USSR and Yugoslavia.[7] In addition, our analysis can provide EU policy-makers with a preliminary overview of inequality in CO_2 emissions in case of enlargement of the EU through the accession of new members, since candidate countries are usually EU neighboring countries.[8] Using multi-dimensional decomposition, we can detect the effects of re-ranking, population changes and changes in relative emission disparities between per capita emitters on the inequality changes within and between the two subgroups over the 1991-2011 period. Since the 1991-2011 period covers two decades, the entire period is divided into four 5-year sub-periods, for detailed investigation of the components of inequality change.

We henceforth refer to OECD and non-OECD countries as subgroups 1 and 2, respectively. Table 1 shows the multi-dimensional decomposition of the change in inequality over the 1991-2011 period. Table 1 shows that inequality decreases over the 1991-1996, 1996-2001 and 2001-2006 sub-periods but increases from 2006 to 2011. We explore the effects of the components of inequality change over the various sub-periods in detail below.

[6] The list of countries in the two subgroups is shown in Appendix B.

[7] In a recent paper by Duro [8], inequality in CO_2 emissions is analyzed worldwide, with European countries forming two subgroups: one including OECD countries, the second comprising non-OECD countries; however, countries of the former USSR and those of the former Yugoslavia are not considered separately. Padilla and Duro [7] analyze inequality in CO_2 emissions among EU-27 member countries, however European countries which are not members of the EU are not considered.

[8] Albania, Iceland, Macedonia, Montenegro, Serbia and Turkey are formal candidates to join the EU in the next few years.

Inequality diminishes over the 1991-1996, 1996-2001 and 2001-2006 sub-periods; however, the within-group and between-group components of R and D play different roles in the various sub-periods. The relative emission disparities increase within subgroup 1 (-0.00476) from 1991 to 1996; this reinforces the disequalizing effects of re-ranking (0.00136) and change in population shares (0.00240) within subgroup 1.[9] The between-group re-ranking has a remarkable disequalizing effect (0.03269) between 1991 and 1996. This disequalizing effect is more than offset by the concomitant equalizing effect of the change in relative emission disparities between subgroups (0.06388) which determines the decrease in inequality between OECD and non-OECD countries (-0.03100) over the 1991-1996 sub-period.[10]

Within subgroup 1 (i.e., among OECD countries), the relative emission disparities between low and high per capita emitters in 1996 decrease when moving from 1996 to 2001 (0.01030), contributing to the overall equalizing effect of D (0.01393) which exceeds the disequalizing effects of re-ranking (0.00534) and population shares (0.00221). It is worth mentioning that the change in population shares has a disequalizing effect over the 1996-2001 sub-period which is larger than the population share effects shown in the other sub-periods.

The inequality over the 2001-2006 sub-period is almost unchanged (-0.00262), but this is the outcome of the interaction between R, D and S instead of very small changes in the per capita CO_2 emission distribution. Indeed, the combined effect of R and S nearly offsets the equalizing effect of D, producing a small reduction in inequality. More specifically, the between-group re-ranking contribution (0.00552) overcomes the contribution to D from the changes in the between-group relative disparities between the low and high per capita emitters in the 2001 distribution (0.00353).

[9] Since D enters negatively in the decomposition (see equations 12 and 23), a negative value of D provides a contribution increasing inequality from the initial to the final time.

[10] Appendix C shows the Lorenz and concentration curves of CO_2 emissions for the 1991-1996 period.

The increase in relative emission disparities between low and high per capita emitters (-0.02296) plays the most important role in determining the rise in inequality between 2006 and 2011. More specifically, the disequalizing effect of D is mainly due to the increase in relative emission disparities between subgroups (-0.02001). This disequalizing effect between subgroups is reinforced by the reranking effect (0.00270) and the population share effect (0.00094), determining an increase in between-group inequality (0.02365) which is the majority of the overall inequality growth between 2006 and 2011. The change in relative emission disparities between low and high per capita emitters shows a disequalizing effect within the subgroup of non-OECD countries (-0.00819) whereas it has an equalizing effect among OECD countries (0.00525). All three components of inequality change contribute to increasing inequality from 2006 to 2011.

Table 1: Multi-dimensional decomposition of the inequality change over the 1991-2011 period.

Period	Subgroup component	G_t	G_{t+5}	ΔG	R	D	S
91-96	1	0.06550	0.07402	0.00852	0.00136	-0.00476	0.00240
	2	0.03812	0.03409	-0.00402	0.00189	0.00496	-0.00096
	12	0.13535	0.10434	-0.03100	0.03269	0.06388	0.00019
	Total	0.23896	0.21246	-0.02650	0.03595	0.06408	0.00162
96-01	1	0.07402	0.06850	-0.00552	0.00200	0.01030	0.00277
	2	0.03409	0.03459	0.00050	0.00034	-0.00122	-0.00106
	12	0.10434	0.10299	-0.00135	0.00300	0.00485	0.00050
	Total	0.21246	0.20608	-0.00638	0.00534	0.01393	0.00221
01-06	1	0.06850	0.06321	-0.00529	0.00021	0.00803	0.00253
	2	0.03459	0.03491	0.00032	0.00048	-0.00110	-0.00126
	12	0.10299	0.10532	0.00233	0.00552	0.00353	0.00034
	Total	0.20608	0.20344	-0.00264	0.00621	0.01047	0.00162
06-11	1	0.06321	0.06073	-0.00248	0.00165	0.00525	0.00111
	2	0.03491	0.04302	0.00811	0.00019	-0.00819	-0.00028
	12	0.10532	0.12897	0.02365	0.00270	-0.02001	0.00094
	Total	0.20344	0.23271	0.02927	0.00454	-0.02296	0.00178

Source: Own elaborations on IEA data.

To sum up, inequality decreases between 1991 and 2006 due to the reduction in relative emission disparities between low and high per capita emitters; this reduction is more evident for pairwise comparisons between OECD and non-OECD countries. The disequalizing effect of re-ranking is noticeable over the 1991-1996 sub-period, especially between OECD and non-OECD countries. Our findings indicate that the tendency to inequality reduction between OECD and non-OECD countries from 1991 to 2006 does not hold over the more recent sub-period (2006-2011). The growth in relative emission disparities between low and high per capita emitters plays the major role in increasing inequality from 2006 to 2011. It is worth mentioning that the change in population shares has a disequalizing effect over the entire 1991-2011 period, however the role of S is less important than those of R and D in determining the changes in inequality.

Overall, multi-dimensional decomposition reveals that the re-ranking effect is non-negligible since countries move within the per capita CO_2 emission distribution over time; furthermore, the analysis shows that the change in relative emission disparities between initially low and high per capita emitters cannot be detected by observing the change in the inequality index since country positions change when moving from the initial to the final time.

5. Conclusions

The contribution of the article is twofold. First, we have developed a matrix decomposition which extends the Duro decomposition of the change in per capita CO_2 emission inequality, since the former detects two further components of inequality change: the re-ranking of per capita emitters and the change in relative emission disparities between initially low and high per capita emitters when moving from the initial to the final time. This provides analysts and policy-makers with information on the distributional changes in the per capita CO_2 emission distribution which cannot be detected by observing the change in per capita CO_2 emission inequality. For instance, a small change in per capita CO_2 emission inequality may suggest that the per capita CO_2 emission distribution is nearly stable in terms of inequality; however, that small change in inequality may result from a dynamic distribution in which countries have exchanged their positions and the countries with low per capita CO_2 emissions have reduced the relative gap with those having high per capita CO_2 emissions.

Second, we have shown that the matrix expressions of the components of inequality change can be further decomposed by subgroup, identifying the within-group and between-group effects on inequality caused by population changes, re-ranking, and changes in relative emission disparities between per capita emitters. For each component of inequality change, the within-group contribution is separated from the between-group contribution by distinguishing the pairwise comparisons between members of the same subgroup from those between members of different subgroups; that is, the two components are obtained by simply splitting the inequality contributions arising within subgroups from those arising between subgroups. This links the subgroup decomposition approach with the decomposition of the inequality change over time, providing a multi-dimensional matrix decomposition technique. Since subgroups can be formed according to any criterion (GDP levels, geographical areas, membership in international organizations), multi-dimensional decomposition can be broadly applied to studies on the CO_2 emission inequality dynamic. In such studies, monitoring the changes in within-group and between-group inequalities is relevant to policy-makers since increasing between-group disparities may generate subgroups with conflicting interests and hence less consensus among countries on emission mitigation objectives [4]. Multi-dimensional decomposition enables analysts and policy-makers to detect the combined effects of subgroup components and components of inequality change over time.

The decomposition of the change in inequality among European Countries over the 1991-2011 period shows three main points. First, inequality diminishes in the initial years of the period (from 1991 to 1996) and increases in the last years (from 2006 to 2011), but is almost unchanged over the decade from 1996 to 2006. Second, the small changes in inequality from 1996 to 2006 are not the result of a nearly stable per capita CO_2 emission distribution, since both re-ranking and changes in relative emission disparities between countries occur over that sub-period; however, these distributional changes generate effects which nearly offset each other, determining small changes in inequality. Third, the distributional changes between OECD and non-OECD countries play the major role in determining the changes in inequality over the 1991-2011 period.

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Appendix A. Proof of equation 3

This appendix proves that

$$\frac{1}{2}tr\left(\mathbf{GPE}^T\mathbf{P}\right) = \frac{1}{2\bar{e}\left(p\right)}\sum_{i=1}^{k}\sum_{j=1}^{k} p_i p_j \left|e_i - e_j\right|. \tag{A.1}$$

Let $\mathbf{u}_i$ stand for the $k \times 1$ unit vector with the i-th element equal to 1 and the other elements equal to 0. The matrix expression on the left-hand side of equation A.1 can be re-arranged as follows:

$$\begin{aligned}\frac{1}{2}tr\left(\mathbf{GPE}^T\mathbf{P}\right) &= \frac{1}{2}\sum_{i=1}^{k}\left(\mathbf{GPE}^T\mathbf{P}\right)_{ii} \\ &= \frac{1}{2}\sum_{i=1}^{k}\mathbf{u}_i^T\mathbf{GPE}^T\mathbf{P}\mathbf{u}_i \\ &= \frac{1}{2}\sum_{i=1}^{k}p_i\mathbf{u}_i^T\mathbf{GPE}^T\mathbf{u}_i \\ &= \frac{1}{2}\sum_{i=1}^{k}\sum_{j=1}^{k}p_i p_j\left(\mathbf{G}\right)_{ij}\left(\mathbf{E}\right)_{ij}.\end{aligned} \tag{A.2}$$

In equation A.2, the product between $\left(\mathbf{G}\right)_{ij}$ and $\left(\mathbf{E}\right)_{ij}$ is equal to the absolute value of the difference between e_j and e_i divided by $\bar{e}\left(p\right)$. Since the element-by-element product between the elements of $\mathbf{G}$ and those of $\mathbf{E}$ gives the absolute values of the k^2 relative pairwise differences between per capita emissions, we obtain

$$\frac{1}{2\bar{e}(p)}\sum_{i=1}^{k}\sum_{j=1}^{k}p_i p_j\left|e_i-e_j\right| = \frac{1}{2}\sum_{i=1}^{k}\sum_{j=1}^{k}p_i p_j\left(\mathbf{G}\right)_{ij}\left(\mathbf{E}\right)_{ij} = \frac{1}{2}tr\left(\mathbf{GPE}^T\mathbf{P}\right). \tag{A.3}$$

Appendix B. Countries included in the analysis

OECD Europe: Austria, Belgium, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Luxemburg, Netherlands, Norway, Poland, Portugal, Slovak Republic, Slovenia, Spain, Sweden, Switzerland, Turkey, United Kingdom.

non-OECD Europe: Albania, Armenia, Azerbaijan, Belarus, Bosnia and Herzegovina, Bulgaria, Croatia, Cyprus, Georgia, Gibraltar, Kazakhstan, Kyrgyzstan, Latvia, Lithuania, FYR of Macedonia, Malta, Republic of Moldova, Romania, Russian Federation, Serbia, Tajikistan, Turkmenistan, Ukraine, Uzbekistan.

Appendix C. Lorenz and concentration curves of CO_2 emissions

This appendix shows that components R and D can be also explained in terms of the Lorenz and concentration curves underlying the Gini and concentration indices of CO_2 emissions defined in Section 3. Conventionally, the Lorenz curve shows the share of total income held by the poorest P share of population, with P varying from 0 to 1. The Lorenz curve can be used to represent inequality in CO_2 emissions [19], where incomes and income receivers are respectively replaced by per capita CO_2 emissions and emitters. Since every per capita CO_2 emission is weighted by its population share, the Lorenz curve shows the cumulative shares of CO_2 emissions on the vertical axis and the cumulative shares of population on the horizontal axis. The relationship between the Lorenz curve and the Gini index is immediate since the Gini index is equal to twice the area between the line of equality (i.e., the 45-degree line running from the bottom-left corner to the top-right) and the Lorenz curve.

When the Lorenz curve of CO_2 emissions is plotted, emitters are sorted in increasing order by per capita CO_2 emission; this ensures that the slope of the curve is monotonically increasing.[11] If emitters are sorted by a different variable, the concentration curve of CO_2 emissions is plotted. For instance, the concentration curve of CO_2 emissions in time $t+1$ can be plotted keeping emitters sorted in increasing order by per capita CO_2 emission in time t. In this case, the slope of the concentration curve may be non-monotonically increasing as emitters are not necessarily sorted in increasing order by per capita emission in time $t+1$. The concentration index is equal to twice the area between the line of equality and the concentration curve.

The Lorenz and concentration curves of CO_2 emissions for the 1991-1996 period are plotted in Figure C.1: $Lc\left(\mathbf{p}_{91|96}, \mathbf{e}_{96}\right)$ denotes the Lorenz curve showing the cumulative shares of CO_2 emissions in 1996 plotted against the cumulative shares of population in 1991; $Lc\left(\mathbf{p}_{91}, \mathbf{e}_{91}\right)$ denotes the Lorenz curve showing the cumulative shares of CO_2 emissions in 1991 plotted against the cumulative shares of population in 1991; $Cc\left(\mathbf{p}_{91}, \mathbf{e}_{96|91}\right)$ denotes the concentration curve showing the cumulative shares of CO_2 emissions in 1996 plotted against the cumulative shares of population in 1991, the emitters having been sorted in increasing order by per capita CO_2 emission in 1991. In Figure C.1, $Cc\left(\mathbf{p}_{91}, \mathbf{e}_{96|91}\right)$ is above $Lc\left(\mathbf{p}_{91}, \mathbf{e}_{91}\right)$ since the relative emission disparities between low and high per capita emitters in 1991 decrease moving to 1996. The concentration index $C\left(\mathbf{p}_{91}, \mathbf{e}_{96|91}\right)$ is equal to twice the area between the line of equality and $Cc\left(\mathbf{p}_{91}, \mathbf{e}_{96|91}\right)$ while the Gini index $G\left(\mathbf{p}_{91}, \mathbf{e}_{91}\right)$ is equal to twice

[11] Let $e_{(i)}$ denote the per capita CO_2 emission with rank i in the increasing parade of country per capita CO_2 emissions and $P\left(e_{(i)}\right)$ denote the cumulative share of population with per capita CO_2 emission lower than or equal to $e_{(i)}$. As shown in Groot [19], the slope of the Lorenz curve at any $P\left(e_{(i)}\right)$ is equal to the ratio of $e_{(i)}$ to the average per capita CO_2 emission; thus, the slope of the curve is monotonically increasing as emitters are sorted in increasing order by per capita CO_2 emission when plotting the Lorenz curve.

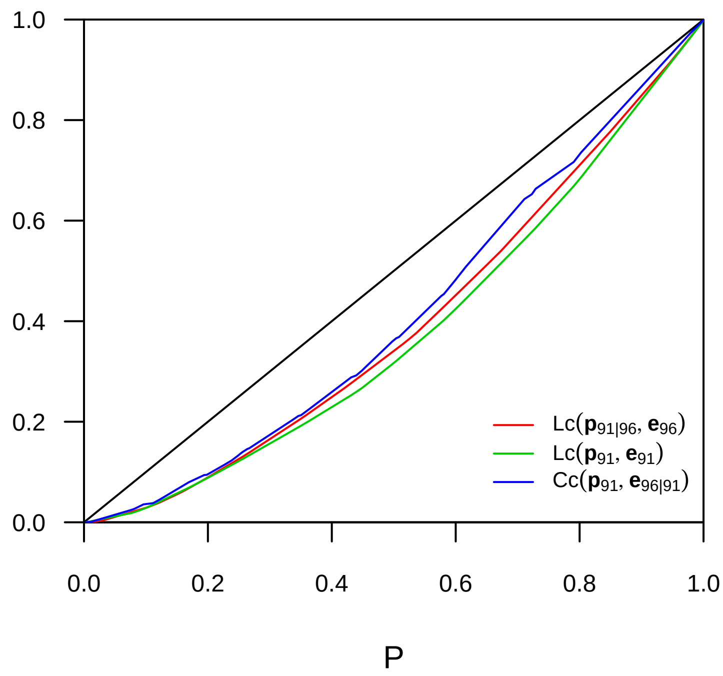

Figure C.1: Lorenz and concentration curves in 1991-1996.

the area between the line of equality and $Lc\left(\mathbf{p}_{91}, \mathbf{e}_{91}\right)$; thus, the component D is twice the area between $Cc\left(\mathbf{p}_{91}, \mathbf{e}_{96|91}\right)$ and $Lc\left(\mathbf{p}_{91}, \mathbf{e}_{91}\right)$. $Cc\left(\mathbf{p}_{91}, \mathbf{e}_{96|91}\right)$ is above $Lc\left(\mathbf{p}_{91|96}, \mathbf{e}_{96}\right)$ since re-ranking occurs from 1991 to 1996. The Gini index $G\left(\mathbf{p}_{91|96}, \mathbf{e}_{96}\right)$ is equal to twice the area between the line of equality and $Lc\left(\mathbf{p}_{91|96}, \mathbf{e}_{96}\right)$, so that the component R is equal to twice the area between $Cc\left(\mathbf{p}_{91}, \mathbf{e}_{96|91}\right)$ and $Lc\left(\mathbf{p}_{91|96}, \mathbf{e}_{96}\right)$.

The figures showing the Lorenz and concentration curves in the other sub-periods are available upon request to the authors.

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