An Act of Mathematisation: Familiarisation with Fractions

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The discussion in our group concerned activities that have been carried out in some third and fourth grade classes in primary schools aimed at familiarizing children with the concept of fractions. Our proposal seeks to give an effective answer to the long-standing problem of unsatisfactory results in teaching and learning fractions. It starts with a process of mathematization that identifies the comparison of two homogeneous quantities by a pair of natural numbers. The concept of fractions is then introduced as the comparison-measure: A fraction is the measure of the quantity Q with respect to the whole W; the second quantity in the comparison, W, is always referred to as the "whole." Studies have shown that children can identify the relationship between fractions and division themselves. Therefore, the next step is a mutual interaction between the teacher and the children that allows them to arrive at Euclidean division. Thanks to this approach, Euclidean division is experienced by the children not as a formula to be memorized but as the icon of their active process of learning. Euclidean division is covered in fourth grade: All the *subconstructs of the construct of rational numbers* have their roots in Euclidean division and are related to it. In this way a new universe of fractions is structured with Euclidean division as core; the didactic process, while exploring the various contexts, keeps Euclidean division in mind and comes back to it constantly.

The discussion within our group covered three main points.

Linguistic splitting. The measure is the comparison between a quantity and the "special" quantity called "the whole" (formalized as n/n). The term *unit* (formalised as $1/n$) is reserved to indicate the common unit. So the whole differs from *the unit*. However: Two Names → Two Substantives → Two "Substances." This splitting results in the unusual classroom activity of the construction of the whole.

Exercise books as tools for noticing. In our class activities, the children's work in finding answers was relatively light in that they were able to come up with

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adequate results fairly easily. However, the teachers found many difficulties, e.g. the usage of common manipulatives. In our discussion group, the following question was addressed: Were teachers called on to change their paradigms for a revolution? The discussion has outlined the key role of children's exercise books as objects of noticing: they allow notice of the features that are fundamental in scaffolding the universe of fractions.

Mathematisation is first. Our approach is based on a process of mathematisation and it aims to construct a *new universe* of fractions. Which is the *true universe* of fractions? What is its relation with the *technical universe* of fractions? We have proposed that the universe of fractions is a plurality of *tuned universes*. What does *tuned* mean in this context? The discussion about these questions remains open. It certainly requires further analyses and reflections.