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|---|---|---|
| | Received | 17 December 2014 |
| Schedule | Revised | |
| | Accepted | 12 May 2015 |



Abstract

We study complex phenomena arising from a simple optimal choice consumer model, starting from the classical framework in Benhabib and Day (Rev Econ Stud 48(3):459–471, 1981). We introduce elements of increasing complexity (dynamic adjustment processes, nonlinear social interdependence, agents heterogeneity, agents local interaction) and we investigate their effects on the resulting social behaviors. The dynamics introduce the dependence of current preferences on past consumers actions. A non-monotone updating preference function allows us to obtain a threshold effect, according to which the agents adopt a bandwagon/snob behavior if the preferences are below/above a certain saturation level. We consider homogeneous and heterogeneous agents and we introduce local/global interaction when they are spatially distributed. We show through simulations several phenomena which are absent in the classical model and which reproduce significant social behaviors, such as path dependence, coexistence of different periodic and chaotic attractors, emergence of spatial patterns.

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|---|---|
| Keywords (separated by '-') | Consumer model - Dynamic adjustment process - Nonlinear threshold effect - Agents heterogeneity - Local interaction - Fashion cycle |
| Footnote Information | |

Mind Soc
DOI 10.1007/s11299-015-0169-9





# Emergence of complex social behaviors from the canonical consumption model

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Received: 17 December 2014 / Accepted: 12 May 2015


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**Keywords** Consumer model · Dynamic adjustment process · Nonlinear threshold effect · Agents heterogeneity · Local interaction · Fashion cycle

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Journal : Small-ext 11299 | Dispatch : 15-5-2015 | Pages : 11
Article No. : 169 | □ LE | □ TYPESET
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## 1 Introduction

The aim of our contribution is to show how elements of increasing complexity in a simple optimal choice consumer model allow describing significant social behaviors, such as the dynamics of endogenous preferences, and in particular of locally or globally interdependent preferences, which can not be encompassed in a classical model. To this end, we study a simple optimal choice consumer model, starting from the classical framework proposed in Benhabib and Day (1981), and we progressively introduce and study the effects of a dynamic adjustment process, non-linear social interdependence, agents heterogeneity and spatial distribution of locally interacting agents.

The presence of a *dynamic adjustment* mechanism, by means of which it is taken into account the dependence of current preferences on the past consumers behavior, together with the modeling of the social interaction among agents by means of nonlinear functions, allow describing the *path dependence* of the evolution of the consumption level (see Arthur 1994). Indeed, also thanks to the occurrence of *multistability* (i.e., coexistence of different attractors) when introducing *hetero-geneity* among agents, we can investigate the effects of past choices on the possible evolutions and equilibria that can be achieved. Moreover, the existence of *periodic* or *chaotic* attractors describes the possible erratic or fluctuating behaviors of some social phenomena, like fashion cycles (see Simmel 1904).

More precisely, starting from the modeling of bandwagon and snob effects considered in Naimzada and Tramontana (2009) and in Di Giovinazzo and Naimzada (2014), we examine a kind of dependence on past choices of the next period preferences which is strongly *nonlinear*, as both bandwagon and snob effects are simultaneously present. In particular, we introduce a threshold effect (see Granovetter 1978), so that the agents follow a bandwagon behavior if the average consumption is below a certain saturation level, above which their behavior is snob. Moreover, varying the shape of the preference functions, we can study the influence of agents heterogeneity. Finally, we show a simple way to model complex spatial social interactions and to study the effects of agents distribution on the possible diffusion, synchronization and pattern formation of social behaviors.

The remainder of the paper is organized as follows. In Sect. 2 we present our model. In Sect. 3 we add a strong nonlinearity, producing a threshold effect. In Sect. 4 we investigate both the introduction of heterogeneity and local interaction among agents. In Sect. 5 we summarize our results and we outline future research developments.



## 2 The model

As mentioned in the Introduction, our starting point is given by the consumer problem, which is a classical model in economic theory, where an agent has to choose the amount of the various available consumption goods in order to maximize his objective function subject to a budget constraint, in which the commodity prices



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are involved. In particular it is assumed that the agent has a high degree of rationality, which is reflected in his informational and computational skills: indeed the agent is supposed to be able to maximize his utility function, that describes his preferences for the various goods, taking into account the limitations imposed by the budget constraint. Rather than considering a single agent, it may be assumed that there are $N$ agents, indexed by $i \in \{1, \ldots, N\}$ and endowed with income $m_i$, that have to choose between two goods using Cobb–Douglas utility functions $u_i(x_i, y_i) = x_i^{\alpha_i} y_i^{1-\alpha_i}$, where $x_i$ and $y_i$ denote the amounts of two goods consumed within a given period and $\alpha_i \in (0,1)$ is the utility weight assigned to commodity $x$. As it is well known, when solving the consumer problem for the generic agent $i$, i.e., when maximizing $u_i(x_i, y_i)$ subject to the usual budget constraint $px_i + qy_i = m_i$, where $p$, $q$ are positive quantities representing the prices for the two goods, we find $x_i = \alpha_i m_i / p$ and $y_i = (1-\alpha_i) m_i / q$ as demand functions.

In Benhabib and Day (1981), in order to endogenize the preferences in the above model, time is introduced and it is assumed that the utility weight parameters $\alpha_i$ are no more constant, but that they rather depend on (immediate) past agent's choices, i.e., that, for all $t$,

$$\alpha_{i,t+1} = g_i(x_t, y_t), \quad \text{where} \quad g_i : (0, m_i/p) \times (0, m_i/q) \to (0,1)$$

are suitable maps. Hence, the demand functions for agent $i$ at time $t+1$ become $x_{i,t+1} = \alpha_{i,t+1}\, m_i/p$ and $y_{i,t+1} = (1-\alpha_{i,t+1}) m_i/q$. The limit of such approach is represented by the absence of interdependence among agents, so that the social nature of consumption is missing, as agents' choices only depend on their own past actions.

This issue may be overcome supposing that there are $N \geq 2$ (possibly) different agents that, in choosing between two goods, have personalized preferences and incomes, and their current choices depend on the immediate past choices of all (or some) agents, including themselves. In symbols, setting $\mathbf{x}_t = (x_{1,t}, \ldots, x_{N,t})$ and $\mathbf{y}_t = (y_{1,t}, \ldots, y_{N,t})$, where, for $i \in \{1, \ldots, N\}$, $x_{i,t}$ and $y_{i,t}$ denote the amounts of the two goods consumed within period $t$ by agent $i$, utility functions become $u_i(x_{i,t+1}, y_{i,t+1}) = x_{i,t+1}^{\alpha_{i,t+1}} y_{i,t+1}^{1-\alpha_{i,t+1}}$, where

$$\alpha_{i,t+1} = g_i(\mathbf{x}_t, \mathbf{y}_t), \quad \text{with} \quad g_i : \prod_{i=1}^{N} (0, m_i/p) \times \prod_{i=1}^{N} (0, m_i/q) \to (0,1)$$

a suitable updating preference function. Agent $i \in \{1, \ldots, N\}$ maximizes with respect to $(x_{i,t+1}, y_{i,t+1})$ his utility function $u_i(x_{i,t+1}, y_{i,t+1})$ subject to the budget constraint $px_{i,t+1} + qy_{i,t+1} = m_i$, where $p$, $q$ and $m_i$ are positive quantities representing respectively the prices for the two goods and the income of agent $i$.

A particular case of such setting has been considered in Naimzada and Tramontana (2009) and in Di Giovinazzo and Naimzada (2014), where consumption is the result of continuous interaction between two types of agents, i.e., the "snobs" and the "bandwagoners". In those papers it is assumed that there are two commodities, $x$ and $y$, the former being the social interaction good, i.e., that through which the social interaction occurs and on which authors focus their attention. The





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bandwagoners (snobs) are then those agents for which preference for the consumption of the social interaction good increases (decreases) with the rise of the average collective consumption of that good in the previous period. Bandwagoners and snob agents can be respectively modeled through increasing and decreasing functions $g$, depending on commodity $x$ only, and in particular on its average consumption level in the past period. Since we will analyze a similar framework, along the paper we will not deal with commodity $y$, nor with its price $q$, anymore, and for simplicity we will set $p = 1$ in all our simulations. In Di Giovinazzo and Naimzada (2014) it is shown that, if all agents are snob, there may be convergent dynamics, diverging dynamics or a period-two cycle; if instead all agents are bandwagoner, only converging dynamics are possible. This is caused by the fact that the monotonic nonlinearities of the updating preference functions are not essential. Hence, in order to obtain interesting dynamics, those authors had to take into account scenarios with both snob and bandwagoner agents simultaneously.

To show the effects of increasing the model complexity, in the next sections we will consider some possible generalizations of the previous framework. In particular, in Sect. 3 we will investigate the consequences of a non-monotonic dependence of the preferences on the average consumption of the social interaction good $x$, in order to take into account a threshold effect, still considering homogeneous agents. Then, in Sect. 4 we will introduce further elements of complexity to the homogeneous model, considering heterogeneous agents (Sect. 4.1) and spatially distributed agents (Sect. 4.2).

In symbols, the general shape of the dynamical system we are going to deal with in Sects. 3 and 4.1 is given by

$$\bar{x}_{t+1} = \frac{\sum_{i=1}^{N} f_i(\bar{x}_t) m_i}{Np}, \tag{1}$$

where

$$\bar{x}_t = \frac{\sum_{i=1}^{N} x_{i,t}}{N} \tag{2}$$

is the average consumption for the social interaction good at time $t$, and, for $i \in \{1, \ldots, N\}, f_i : (0, +\infty) \to (0, 1)$ is a continuous non-monotone map describing the updating preference function of agent $i$, i.e., $\alpha_{i,t+1} = g_i(\mathbf{x}_t, \mathbf{y}_t) = f_i(\bar{x}_t)$, with $f_i$ as in (3). Notice that (1) is obtained by solving the individual consumer maximization problems.

In Sect. 4.2 we will assume that agents, in addition to the global average consumption level, in forming their preferences consider also the individual consumption choices of their neighbors. In such case, it is not possible to aggregate all consumption choices anymore and thus we will deal with an equation for each agent [see (6)].





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## 3 Introducing a significant nonlinearity

In order to simultaneously encompass both bandwagon and snob effects, in the updating preference functions we will introduce a threshold effect (see Granovetter 1978), so that agents are bandwagoner if the average consumption level for the social interaction good in the previous period is below a certain saturation level, above which they become snob. The threshold represents the switching consumption level between the two different behaviors and affects the shape of the updating preference functions $f_i$. Below such threshold, the preferences increase as the average consumption level increases, hence functions $f_i$ are increasing; conversely, above the switching level, an increase of the average consumption level induces a preference reduction, which is modeled by decreasing functions $f_i$. Consequently, the updating preference functions are unimodal, with the maximum preference level in correspondence to the threshold point.

In symbols, we will then focus on the case in which, for each agent $i \in \{1, \ldots, N\}$, $\alpha_{i,t+1} = f_i(\bar{x}_t)$, with

$$f_i : (0, +\infty) \to (0, 1), \quad f_i(\bar{x}_t) = a_i e^{-\sigma_i (\bar{x}_t - \mu_i)^2}, \tag{3}$$

where $a_i \in (0, 1)$ represents the maximum level of preference for good $x$ of agent $i$, while $\sigma_i$ and $\mu_i$ are nonnegative parameters describing the reactivity and the saturation level of agent $i$, respectively. Notice that when $\sigma_i = 0$ the preferences of agent $i$ are fixed and thus there is no interaction for such agent with the rest of the population: hence, $\sigma_i$ may be also interpreted as the social interaction degree for agent $i$. See Patidar (2006) for an analysis of the dynamics generated by the Gaussian map related to (3).

In the present section, we will show that rich dynamics, and also chaotic behaviors, may arise even under the assumption that all agents are homogeneous. Hence, let us consider the case in which the incomes, as well as the updating preferences functions, coincide for all agents, i.e., $m_i = m$, $a_i = a$, $\mu_i = \mu$ and $\sigma_i = \sigma$, for all $i \in \{1, \ldots, N\}$, so that the resulting discrete dynamical system

$$\bar{x}_{t+1} = \frac{m}{p} \alpha_{t+1} = \frac{m}{p} f(\bar{x}_t) = \frac{m}{p} a e^{-\sigma(\bar{x}_t - \mu)^2} \tag{4}$$

will admit one or more steady states. More precisely, we find that there may be at most three steady states, of which at most one larger than $\mu$.

A possible framework, showing how, even in a homogeneous setting, the introduction of the threshold effect allows for describing more complex situations than those generated by either bandwagon or snob agents, is that in Fig. 1, where we consider $a = 0.98, m = 0.8$ and $\sigma = 12$. As the saturation level $\mu$ is varied, we can focus on different consumer behaviors characterized by the preponderance/balancing of either bandwagon or snob effects. In fact, as already noticed, in this case we can have up to three equilibria, the number and position of which determine the possible resulting dynamics.

For a given income $m$, if the saturation level $\mu$ is sufficiently large (Fig. 1a, $\mu = 0.9$), (4) has a unique equilibrium average consumption $\bar{x}_1^* < \mu$ (represented by

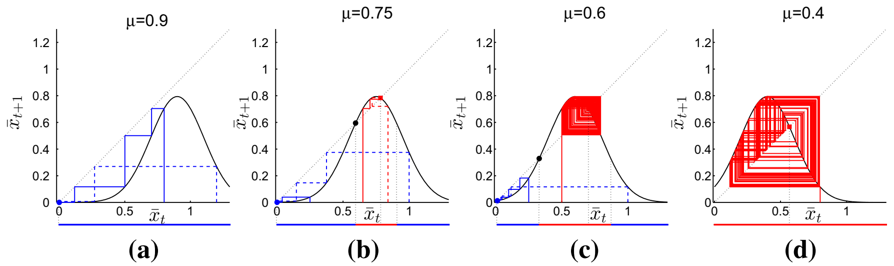


**Fig. 1** In each plot, we report the shape of the RHS of (4), the possible steady states and the cobweb diagrams representing possible evolutions of some initial data and, below each plot, a *line* showing the basins of attraction of each attractor, depicted with the same color used for the corresponding attractor. **a** One stable (*blue dot*) steady state in the bandwagon interval. **b** Two stable steady states, one in the bandwagon interval (*blue dot*), the other in the snob interval (*red square*), separated by a repelling steady state (*black dot*). **c** A stable steady state in the bandwagon interval (*blue dot*) and an unstable one in the snob interval (*red square*), which gives rise to a chaotic attractor. **d** An unstable steady state in the snob interval (*red square*), surrounded by a chaotic attractor

the blue dot), which is in general quite close to the origin due to the negative feedback effect of the small relative income-price $m/p$. The steady state lies in the bandwagon region $(0, \mu)$ of the preference function, so that, in this case, the presence of the threshold $\mu$ has no significant effects and the resulting dynamics (solid and dashed blue lines) are very similar to those obtained in Di Giovinazzo and Naimzada (2014) for the case of bandwagon agents. However, for smaller saturation levels, the resulting dynamics can be totally different. In fact, for $\mu \approx 0.818$ a so-called fold bifurcation occurs, at which the graph of the map associated to (4) is tangent to the line $x_{t+1} = x_t$, and a couple of stable ($\bar{x}_2^*$)/unstable ($\bar{x}_U^*$) new equilibria arises (represented respectively by a red square and by a black dot in Fig. 1b). This results in a situation in which both bandwagon and snob behaviors may be present. In particular, if $\bar{x}_2^* > \mu$, it holds that $\bar{x}_1^*$ lies in the bandwagon interval, while $\bar{x}_2^*$ lies in the snob interval $(\mu, +\infty)$ of the updating preference function. Let us examine Fig. 1b. If, at any time $t$, the average consumption level is too small ($\bar{x}_t < \bar{x}_U^*$), then its evolution (solid blue line) is similar to that described in Fig. 1a. However, if $\bar{x}_t$ is larger but still in the bandwagon interval ($\bar{x}_U^* < \bar{x}_t < \mu$), the relative income-price is sufficiently large to activate the bandwagon effect, and the average consumption level increases (solid red line). Conversely, if $\mu < \bar{x}_t$ we are in the snob interval and oscillating trajectories arise (dashed red line). The snob behavior tends to lower the average consumption, which can either generate a bandwagon behavior again (dashed red line) or give rise to damping oscillations, internal to the snob interval (final iterations of solid red line, not visible in the picture), converging toward the steady state $\bar{x}_2^*$. Finally, if $\bar{x}_t$ is sufficiently large, the snob effect can be so strong that the next period average consumption decreases to $\bar{x}_{t+1} < \bar{x}_U^*$ (dashed blue line), and then it tends toward $\bar{x}_1^*$.

In the previous framework we have two coexisting stable steady states, characterized by disconnected basins of attractions, which reflect the coexistence of both bandwagon and snob effects. However, the snob behavior can give rise to





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more complicated stable attractors, as shown in Fig. 1c, where the chaotic interaction between the snob and bandwagon effects generates oscillating complex dynamics.

The just described phenomena are two different possible examples of multistability induced by the strong nonlinearity of the updating preference function.

Finally, if the saturation level is sufficiently small with respect to the relative income-price, equilibrium $\bar{x}_1^*$ disappears as a second fold bifurcation occurs for $\mu \approx 0.5155$, after which $\bar{x}_2^*$ is the only attractor. Since it lies in the snob interval $(\mu, +\infty)$, it can give rise to oscillating trajectories which may be chaotic (as in Fig. 1d), periodic (cycling between two or more values) or which can damp down on equilibrium $\bar{x}_2^*$.

In Fig. 2a, b, keeping $a$, $m$ and $\sigma$ like in Fig. 1, for various initial conditions we report some average consumption level evolutions for the cases with $\mu = 0.75$ and $\mu = 0.6$, to highlight the path dependence of the average consumption levels in the cases of coexisting snob and bandwagon behaviors considered in Fig. 1b, c. The oscillating dynamics of the average consumption qualitatively reproduce the well-known fashion cycles, which, as explained in Simmel (1904), are a consequence of the coexistence of both bandwagon and snob effects.

In Fig. 2c we show the bifurcation diagram of $\bar{x}$, obtained for decreasing values of $\mu \in (0, 1)$ and setting $\bar{x}_0 = 0.7$ as initial condition, from which we can see that the loss of stability of $\bar{x}_2^*$ occurs through a flip bifurcation (for $\mu \approx 0.712$), by means of which the stable equilibrium $\bar{x}_2^*$ is replaced by a period-2 cycle, that evolves toward chaos through a cascade of period doublings. Then, through a sequence of period halvings, the dynamics qualitatively stabilize on a period-2 cycle again (for $\mu \approx 0.168$).

## 4 Increasing complexity

In the previous section we showed that, even in a homogeneous agents setting, in the presence of a non-monotone function $f$, complex dynamical phenomena (such as



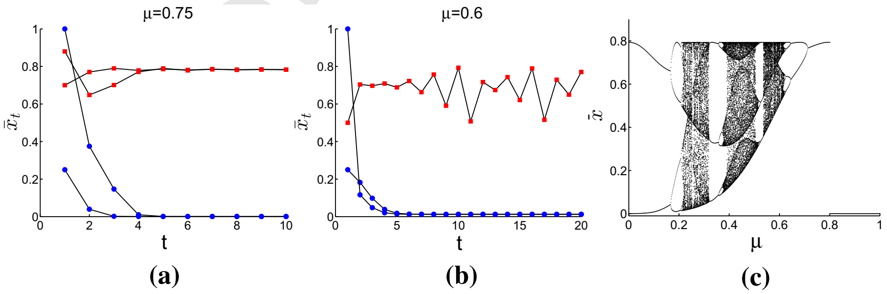


**Fig. 2** Time series. Case of coexisting stable equilibria in **a** and of a stable equilibrium coexisting with a chaotic attractor in **b**, for some different initial data, showing path dependence and disconnectedness of the basins of attraction. **c** Bifurcation diagram for values of $\mu$ decreasing from 1 to 0



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multistability and path dependence) may arise. Here, starting from the framework in Sect. 3, we investigate the possible effects on the dynamics of the addition of further levels of complexity, represented by the heterogeneity among agents and possible local interactions.

## 4.1 Heterogeneity

If we assume that agents are characterized by different updating preference functions and incomes, the resulting nonlinearity characterizing the dynamics can become much more complicated than those observed so far. In fact, in this situation, the aggregate updating preference function, obtained by taking the average of the updating preference functions of all agents, is no more necessarily unimodal, and thus characterized by the presence of a unique maximum point, because we can now have several intervals in which preferences either increase or decrease with respect to the average consumption level. In fact, we will now deal with the following dynamical system

$$\bar{x}_{t+1} = \frac{\sum_{i=1}^{N} m_i a_i e^{-\sigma_i(\bar{x}_t-\mu_i)^2}}{Np}, \tag{5}$$

where all terms have been defined in the previous section. When $N \geq 2$, the map may become multimodal, which is a necessary condition for the coexistence of at least two complex (periodic or chaotic) attractors, while when the map is unimodal we can find at most the coexistence between a locally stable fixed point and a complex attractor. In Fig. 3 we report a scenario with four agents and $a_1 = 0.98, \sigma_1 = 1, \mu_1 = 1, m_1 = 5,$ $a_2 = 0.98, \sigma_2 = 2, \mu_2 = 3, m_2 = 14,$ $a_3 = 0.98, \sigma_3 = 0.5, \mu_3 = 5, m_3 = 21$ and $a_4 = 0.98, \sigma_4 = 3, \mu_4 = 9, m_4 = 37$. In this case, we have two stable equilibria (blue and green dots) coexisting with a period-2 cycle (around the red dot) and a chaotic attractor (around the light blue dot).

## 4.2 Local interaction

In the previous models, the agents formed their next time preferences on the basis of the average consumption, computed considering the consumption levels of all



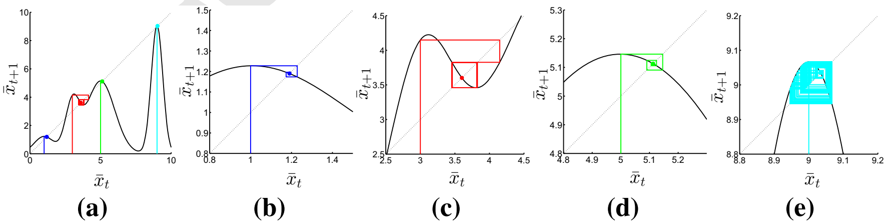


**Fig. 3** The case with $N = 4$ heterogeneous agents. **a** Shape of the RHS of (5); **b–e** magnifications near steady states. We find coexistence among stable equilibria (**b**, **d**), a periodic attractor (**c**) and a chaotic attractor (**e**)



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agents. This resulted in a dynamic adjustment of the global average consumption level. But what happens if the agents are spatially distributed and each agent affects to a greater extent his neighboring agents? In addition to Naimzada and Tramontana (2009), similar issues have been considered in Bell (2002) and Knasnička (2012) for frameworks with only bandwagoner agents. In the present framework, agents' spatial distribution is crucial, as the resulting evolution of the consumption levels also depends on the local interactions of each agent. To focus on our problem, we assume that $N$ agents are distributed on a ring, so that each agent $i \in \{1, \ldots, N\}$ has the two neighbors $i-1$ and $i+1$, where we identify 0 with $N$ and $N+1$ with 1.

Let us now suppose that each agent, in forming his next period's preferences, takes into account both the global average consumption of all agents in (2), as well as the local average consumption by himself $x_{i,t}$ and of his two neighboring agents $x_{i-1,t}, x_{i+1,t}$. We then introduce the constant $\omega \in [0, 1]$, which represents the degree of local interaction and weights the importance given by agents to the local influence in comparison with the global average consumption. When $\omega = 0$, we recover (4) and we just have the global interaction, while for $\omega = 1$ the interaction is completely local. Moreover, we suppose that each agent, in computing his local average, can apply a different weight to his consumption level with respect to those of his neighbors. To such end, we introduce the degree of local imitation $v \in [0, 1]$, so that the local average reads as

$$(x_{i+1,t} + x_{i-1,t})v/3 + (1 - 2v/3)x_{i,t}.$$

When $v = 1$, the local average is the arithmetic mean of $x_{i-1,t}, x_{i,t}$ and $x_{i+1,t}$; as $v$ decreases, each agent gives more and more weight to his consumption level. The most extreme scenario of stubbornness corresponds to the choice of $\omega = 1$ and $v = 0$, so that each agent evolves independently from the others, just taking into account his own past consumption level.

The resulting consumption adjustment for the $i$th agent is then given by

$$x_{i,t+1} = \frac{m}{p} f\left( (1-\omega)\bar{x}_{i,t} + \omega \left( \frac{x_{i+1,t} + x_{i-1,t}}{3} v + \left(1 - \frac{2v}{3}\right) x_{i,t} \right) \right), \tag{6}$$

with $f$ as in (4). Notice that the model is now represented by an $N$-dimensional system, as it is no more possible to aggregate the individual equations.

In order to show the possible effects of switching between global and local interaction, we focus on a framework in which there are $N = 80$ agents and we set $a = 0.98, \sigma = 8, \mu = 0.7, m = 0.81$. The graph of the one-dimensional map $\varphi$, identical across agents and corresponding to the RHS of (6) when $\omega = 1$ and $v = 0$, is shown in Fig. 4a and has two stable equilibria at $x_A^* \approx 0.02$ and $x_B^* \approx 0.765$. For the $N = 80$ agents, we consider an initial random distribution as represented by the red line in Fig. 4b, c. In the first simulation, we consider $v = 1$. If $\omega = 0$, all the agents immediately synchronize on the global average consumption level. Since, for the particular choice of the parameters and of the initial datum, such level lies in the basin of attraction of $x_A^*$, all agents simultaneously converge to 0.02. Increasing $\omega$, if the agents give sufficient weight to the local average with respect to the global one, the same starting condition can give rise to trajectories that converge for each agent





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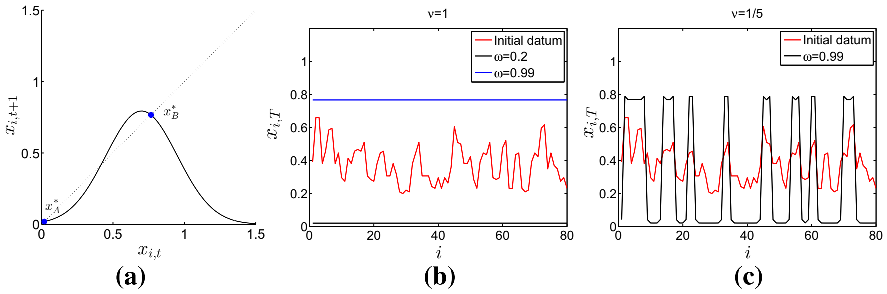


**Fig. 4** Spatially distributed agents. The shape of the RHS of (6) when $\omega = 1$ and $v = 0$ is plotted in **a**. When $v = 1$ and the interaction degree $\omega$ is small ($\omega = 0.2$), all the agents choices tend toward $x_A^*$, while when the interaction degree $\omega$ is large ($\omega = 0.99$), all the agents progressively choose $x_B^*$ (**b**). If we only consider local interaction, for suitable local weights we can have the formation of spatial patters, as in **c** for $\omega = 0.99$ and $v = 1/5$

either to $x_A^*$ (as for example when $\omega = 0.2$) or to $x_B^*$ (for $\omega = 0.99$). The resulting dynamics mainly depend on the belonging of the global/local average consumptions to the basin of attraction of a particular steady state, together with the value of the degree of local interaction. We remark that the degree of local interaction also affects the speed of convergence toward the equilibrium.

Conversely, if we consider different weights in the local mean (i.e., $v \neq 1$), the final steady state may be not spatially homogeneous, and the consumption levels may arrange into a spatial pattern, as shown in Fig. 4c for a local interaction ($\omega = 0.99$) with $v = 1/5$.

## 5 Concluding remarks

Starting from a simple optimal choice consumer model, in the present paper we illustrated how elements of increasing complexity allow describing significant social behaviors, such as the dynamics of locally or globally interdependent preferences. More precisely, we showed that the presence of a dynamic adjustment mechanism, by means of which we could take into account the dependence of current preferences on the past consumers behavior, together with the modeling of the social interaction among agents by means of nonlinear functions, allow describing the path dependence of the evolution of the consumption level. In particular, varying the shape of the preference functions, we could also study the influence of agents heterogeneity. We notice that the found periodic and chaotic attractors describe the possible erratic and fluctuating behaviors of some social phenomena, like fashion cycles. Finally, we showed a simple way to model complex spatial social interactions and to study the effects of agents distribution on the possible diffusion, synchronization and pattern formation of social behaviors.

We stress that the proposed framework of local interaction is very simple, and further complexity can be taken into account as well. For example we could consider heterogeneous agents, also with respect to the degrees of local interaction





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and imitation, study more complicated spatial distributions or local interactions, or even endogenize $\omega$. A development of such lines of research will be performed in some forthcoming papers.

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