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Endogenous evolution of heterogeneous consumers preferences: Multistability and coexistence between groups



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HIGHLIGHTS

- We propose an evolutive model in an exchange economy setting.
- Agents are heterogeneous in the structure of the preferences.
- The share updating mechanism is non-monotone in the calorie intake.
- We find multistability phenomena involving equilibria with heterogeneous agents.

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ABSTRACT

We propose an exchange economy evolutionary model with agents heterogeneous in the structure of preferences. Assuming that the share updating mechanism is non-monotone in the calorie intake, we find multistability phenomena involving equilibria characterized by the coexistence of heterogeneous agents.

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1. Introduction

Chang and Stauber (2009) propose a model on the evolution of the population shares in an exchange economy setting, in which it is assumed that there are two groups of agents characterized by a different structure of the preferences. Indeed, the weights assigned to the two consumption goods in the Cobb–Douglas utility functions do not coincide across groups. The mechanism according to which shares are updated in Chang and Stauber (2009) is monotone in the calorie intake: the larger such intake by a group,

the more that group share increases. The results in that paper concern the existence and local stability of trivial and nontrivial market stationary equilibria, where we call equilibria trivial if they are not characterized by the coexistence between the two groups of agents, as one of the two groups totally prevails on the other and remains alone. Chang and Stauber (2009) find at most one nontrivial market stationary equilibrium, which when exists is stable, and two trivial equilibria.

According to Chang and Stauber (2009), a monotone population growth rate is suitable to represent the long-run centuries-old trend, as the diet of a population group affects its long-term survival. On the other hand, in that paper it is remarked that it would be interesting to consider more general biological payoff functions. In fact, supported by the empirical literature, we believe that a biological payoff function monotonically increasing in the calorie intake is not well suited to describe

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the framework of contemporary developed countries. Namely, according to [Ponthiere \(2011\)](#), monotone survival functions do not fit aggregate data (cf. Fig. 1 therein); moreover, a broad epidemiological literature (see e.g. [Adams et al., 2006](#); [Bender et al., 1998](#); [Fontaine et al., 2003](#) and [Solomon and Manson, 1997](#)) has shown the negative effects of overconsumption on health and survival, emphasizing a non-monotone relationship between the corpulence, measured by BMI (Body-Mass Index, i.e., the weight in kilograms divided by the square of the height in meters), and mortality risks. Such non-monotone relationship is clearly represented in the (BMI, mortality)-plane by [Waalder \(1984\)](#)'s U-shaped curves, which have been commented by [Fogel \(1994\)](#) from an intertemporal viewpoint. The relevance of those findings gave rise to the so-called “economics of obesity” (see the survey by [Philipson and Posner, 2008](#)).

In light of the above observations, we then aim to reconsider the model in [Chang and Stauber \(2009\)](#), replacing the monotone population growth rate assumed therein with a bell-shaped map, increasing with the calorie intake up to a certain threshold value, above which it becomes decreasing. Even with such a crucial change, we still obtain a one-dimensional continuous-time dynamical system, for which, in addition to the three equilibria in [Chang and Stauber \(2009\)](#), we find (up to) two additional nontrivial market stationary equilibria. Moreover, the (possibly existing) nontrivial equilibrium found in [Chang and Stauber \(2009\)](#) may become unstable in our context, and also the two trivial equilibria may have different dynamic behaviors with respect to [Chang and Stauber \(2009\)](#). We perform a qualitative bifurcation analysis on varying the parameter describing the threshold value at which the growth rate becomes decreasing. In particular, differently from the framework in [Chang and Stauber \(2009\)](#), our setting displays multistability phenomena, characterized by the presence of multiple, trivial and nontrivial, locally stable market stationary equilibria.

We remark that multistability may be considered as a source of richness for the framework under analysis because, other parameters being equal, i.e., under the same institutional, cultural and social conditions, it allows to explain different trajectories and evolutionary paths. The initial conditions, leading to the various attractors, represent indeed a summary of the past history, which in the presence of multistability phenomena does matter in determining the evolution of the system. Such property, in the literature on complex systems, is also called “path-dependence” (see [Arthur, 1994](#)).

Moreover, in the specific context we deal with, the presence of multiple equilibria well represents the variety of historical experiences across different countries in relation to the approach they adopt towards food, diet and consequently towards obesity (consider e.g., according to [Philipson and Posner, 2008](#), the different scenarios in the US and in the Mediterranean countries).

Finally, we stress that in our model we also analyzed the setting in which the two groups of agents may differ in the threshold level, above which an increase in the calorie intake becomes harmful rather than beneficial. Such scenario can describe for instance the case in which the two groups of agents differ in the amount of sports they play: inactive people need a lower calorie intake than athletes. Since the possible dynamic scenarios we found were similar to those obtained in the setting in which the two threshold levels coincide, we preferred to confine ourselves to the simpler framework, which also allows a deeper analytical treatment.

2. The model

We start our discussion recalling the framework in [Chang and Stauber \(2009\)](#), where the authors consider a continuous-time model describing an exchange economy with a continuum

of agents, which may be of type α or of type β . There are two consumption goods, x and y , and agent preferences are described by Cobb–Douglas utility functions, i.e., $U_i(x, y) = x^i y^{1-i}$, for $i \in \{\alpha, \beta\}$, with $0 < \beta < \alpha < 1$. Both kinds of agents have the same endowments of the two goods, denoted respectively by w_x and w_y . The analysis is performed in terms of the relative price $p(t) = p_y(t)/p_x(t)$, where $p_x(t)$ and $p_y(t)$ are the prices at time t for goods x and y , respectively. The size of the population of kind α (β) at time t is denoted by $A(t)$ ($B(t)$). The calorie intake $K_i(t)$ of an agent of type $i \in \{\alpha, \beta\}$ at time t is given by a linear combination of the units $x_i(t)$ and $y_i(t)$ of goods x and y he consumes, weighted respectively with the calories that each agent derives from the consumption of a unit of good x and of good y , i.e., $K_i(t) = c_x x_i(t) + c_y y_i(t)$. In [Chang and Stauber \(2009\)](#), denoting by \bar{K} the calorie subsistence level, the growth rate of the population of type i is then assumed to be

$$K_i(t) - \bar{K}, \tag{2.1}$$

so that the evolution of the two groups of consumers is described by the following system

$$\begin{cases} \frac{dA(t)}{dt} = (K_\alpha(t) - \bar{K})A(t) \\ \frac{dB(t)}{dt} = (K_\beta(t) - \bar{K})B(t). \end{cases} \tag{2.2}$$

Introducing the normalized variables $a(t) = (A(t))/(A(t) + B(t))$ and $b(t) = (B(t))/(A(t) + B(t))$, describing the fractions of the population composed by the agents of type α and β , respectively, and noticing that $b(t) = 1 - a(t)$, System (2.2) becomes equivalent to

$$\frac{da(t)}{dt} = (K_\alpha(t) - K_\beta(t))a(t)(1 - a(t)). \tag{2.3}$$

The (normalized) price at which agents exchange their endowments is determined by solving the consumer maximization problem and using a market clearing condition. According to [Chang and Stauber \(2009\)](#), the market equilibrium price, i.e., the price that clears the market, is then given by

$$p^*(t) = \frac{[1 - (a(t)\alpha + (1 - a(t))\beta)]w_x}{(a(t)\alpha + (1 - a(t))\beta)w_y} \tag{2.4}$$

and the consumer equilibrium quantities of the two goods for agent of type $i \in \{\alpha, \beta\}$, compatible with the market equilibrium, are

$$\begin{aligned} x_i^*(t) &= i(w_x + p^*(t)w_y) = \frac{iw_x}{a(t)\alpha + (1 - a(t))\beta}, \\ y_i^*(t) &= (1 - i) \left(\frac{w_x}{p^*(t)} + w_y \right) = \frac{(1 - i)w_y}{1 - (a(t)\alpha + (1 - a(t))\beta)}. \end{aligned} \tag{2.5}$$

Hence, (2.3) can be rewritten as

$$\begin{aligned} \frac{da(t)}{dt} &= (\alpha - \beta)a(t)(1 - a(t)) \\ &\times \left(\frac{c_x w_x}{a(t)\alpha + (1 - a(t))\beta} - \frac{c_y w_y}{1 - a(t)\alpha - (1 - a(t))\beta} \right). \end{aligned} \tag{2.6}$$

The market stationary equilibria, at which for every t the population shares, and thus also the market equilibrium price and the consumer equilibrium quantities, are constant, will be called trivial if they are not characterized by the coexistence between the two groups of agents, and nontrivial otherwise. In addition to the trivial market stationary equilibria $a = 0$ and $a = 1$, a nontrivial market stationary equilibrium is given by $a = a^*$, with

$$a^* = \frac{(1 - \beta)c_x w_x - \beta c_y w_y}{(\alpha - \beta)(c_x w_x + c_y w_y)}, \tag{2.7}$$

as long as $a^* \in (0, 1)$, i.e., for $c_x w_x \in ((\beta c_y w_y)/(1 - \beta), (\alpha c_y w_y)/(1 - \alpha))$. Such market stationary equilibrium, when it exists, is always stable for the model considered in Chang and Stauber (2009). In that paper no comments are made on the local stability of the dynamical system at $a = 0$ and $a = 1$. However, a simple continuity argument shows that, when $a^* \in (0, 1)$, then $a = 0$ and $a = 1$ are always unstable. When instead $a^* \notin (0, 1)$, $a = 0$ may be unstable and $a = 1$ stable, or vice versa.

The framework we are going to analyze differs from the one recalled above in a crucial aspect. Indeed, instead of dealing with the monotone growth rate in (2.1), we assume, in agreement with the quoted empirical literature, the existence for the growth rate of a threshold value, above which an increasing calorie intake becomes harmful, rather than beneficial. In symbols, as growth rate we consider

$$\frac{1}{1 + \sigma(K_i(t) - \widehat{K})^2}, \tag{2.8}$$

where σ is a positive parameter describing the intensity of the decrease in the growth rate due to an increase in the distance between the calorie intake $K_i(t)$ and the threshold value \widehat{K} . In this manner \widehat{K} is no more interpretable as the calorie subsistence level \bar{K} in (2.1), but as the desirable calorie intake, which allows maximizing the growth rate. With such modification, the evolution of the two groups of consumers gets described by the following system

$$\begin{cases} \frac{dA(t)}{dt} = \frac{A(t)}{1 + \sigma(K_\alpha(t) - \widehat{K})^2} \\ \frac{dB(t)}{dt} = \frac{B(t)}{1 + \sigma(K_\beta(t) - \widehat{K})^2} \end{cases} \tag{2.9}$$

which, introducing the population fractions $a(t)$ and $b(t) = 1 - a(t)$, is equivalent to

$$\begin{aligned} \frac{da(t)}{dt} &= a(t)(1 - a(t)) \\ &\times \left(\frac{1}{1 + \sigma(K_\alpha(t) - \widehat{K})^2} - \frac{1}{1 + \sigma(K_\beta(t) - \widehat{K})^2} \right) \\ &= a(t)(1 - a(t))(K_\alpha(t) - K_\beta(t)) \\ &\times \left(\frac{\sigma(2\widehat{K} - K_\alpha(t) - K_\beta(t))}{(1 + \sigma(K_\alpha(t) - \widehat{K})^2)(1 + \sigma(K_\beta(t) - \widehat{K})^2)} \right). \end{aligned} \tag{2.10}$$

Recalling the expressions for the market equilibrium values in (2.4) and (2.5), then (2.10) can be rewritten as Eq. (2.11) (in Box 1).

Such equation admits, in addition to the trivial market stationary equilibria $a = 0$ and $a = 1$, up to three nontrivial equilibria. As we shall see in Proposition 3.1, one of them is again given by $a = a^*$, with a^* as in (2.7), for $c_x w_x \in ((\beta c_y w_y)/(1 - \beta), (\alpha c_y w_y)/(1 - \alpha))$, while the other two are $a = a_{1,2}^*$, with

$$a_{1,2}^* = \frac{2\widehat{K}(1 - 2\beta) + c_x w_x(\alpha + \beta) - c_y w_y(2 - \alpha - \beta) \pm \sqrt{\Delta}}{4\widehat{K}(\alpha - \beta)}, \tag{2.12}$$

where

$$\begin{aligned} \Delta &= 4\widehat{K}^2 + (c_x w_x(\alpha + \beta) - c_y w_y(2 - \alpha - \beta))^2 \\ &\quad - 4\widehat{K}(c_x w_x(\alpha + \beta) + c_y w_y(2 - \alpha - \beta)), \end{aligned}$$

as long as they are real and belong to $(0, 1)$. Due to the heavy expressions of $a_{1,2}^*$ and in order to compare our results to those in Chang and Stauber (2009), in Proposition 3.2 we will analytically investigate the local stability of (2.11) just at $a = 0$, $a = a^*$ and $a = 1$.

In view of the subsequent analysis, it is expedient to introduce the one-dimensional maps $f, g : [0, 1] \rightarrow \mathbb{R}$ related to (2.6) and (2.11), respectively, and defined as Eqs. (2.13) and (2.14) (in Box II).

3. Stability, bifurcation analysis, and possible scenarios

As a first step in the study of our dynamical system, in the next result we derive the expressions of the market stationary equilibria for (2.11).

Proposition 3.1. Eq. (2.11) admits $a = 0$, $a = 1$, $a = a^*$ in (2.7), and $a = a_{1,2}^*$ in (2.12) as market stationary equilibria, as long as they are real and belong to $(0, 1)$.

Proof. The conclusion immediately follows by observing that $a = 0$, $a = 1$, $a = a^*$ in (2.7), and $a = a_{1,2}^*$ in (2.12) are all the solutions to the equation $g(a) = 0$, with g as in (2.14). \square

In the following proposition we investigate the stability conditions for the market stationary equilibria $a = 0$, $a = a^*$, $a = 1$, we have in common with Chang and Stauber (2009).

Proposition 3.2. Eq. (2.11) is locally asymptotically stable:

- at 0 if $((1 - \beta)c_x w_x - \beta c_y w_y) \left(2\widehat{K} - \frac{(\alpha + \beta)c_x w_x}{\beta} - \frac{(2 - \alpha - \beta)c_y w_y}{1 - \beta} \right) < 0$;
- at a^* in (2.7) if $\widehat{K} > c_x w_x + c_y w_y$;
- at 1 if $((1 - \alpha)c_x w_x - \alpha c_y w_y) \left(2\widehat{K} - \frac{(\alpha + \beta)c_x w_x}{\alpha} - \frac{(2 - \alpha - \beta)c_y w_y}{1 - \alpha} \right) > 0$.

Proof. The stability conditions follow by direct computations, imposing respectively $g'(0) < 0$, $g'(a^*) < 0$ and $g'(1) < 0$, with g as in (2.14). \square

Hence, the local stability of $a = 0$, $a = a^*$ and $a = 1$, being influenced in our framework also by \widehat{K} , is independent from the stability of the same market stationary equilibria in Chang and Stauber (2009). Indeed, if we wish to have the same dynamic behavior at $a = 0$, $a = a^*$ and $a = 1$ as in Chang and Stauber (2009), it suffices to take \widehat{K} large enough, while, in order to modify it, we just need to take \widehat{K} sufficiently small.²

As already recalled in Section 2, in the framework by Chang and Stauber (2009), when $a^* \in (0, 1)$, then $a = 0$ and $a = 1$ are always unstable (see the graph of f in red in Fig. 1(A)–(D)); when instead $a^* \notin (0, 1)$, $a = 0$ may be unstable and $a = 1$ stable, or vice versa. Hence, in the monotone growth rate setting no multistability phenomena, characterized by the presence of multiple locally stable market stationary equilibria, may arise. On the other hand, in the non-monotone growth rate framework, in addition to reproducing all the scenarios arising from the setting in Chang and Stauber (2009), we also find multistability phenomena, involving both trivial and nontrivial equilibria (see the graph of g in blue in Fig. 1(B)–(D)).

We shall now better analyze the mutual relationship between the stability of the equilibria in the monotone and non-monotone growth rate frameworks, performing a qualitative bifurcation analysis, i.e., investigating the emergence/disappearance and stability gain/loss of equilibria on varying $\bar{K} = \widehat{K}$. Indeed, although as seen in Section 2 the parameters \bar{K} and \widehat{K} have a different interpretation, an increase in either of the two produces an analogous effect, i.e., a reduction of the population growth rate, which in Chang and Stauber (2009) may also become negative. Due

² Namely, it is easy to see that (2.6) is locally asymptotically stable at 0 if $(1 - \beta)c_x w_x < \beta c_y w_y$, at 1 if $(1 - \alpha)c_x w_x > \alpha c_y w_y$, and always at a^* .

$$\frac{da(t)}{dt} = (\alpha - \beta)a(t)(1 - a(t)) \left(\frac{c_x w_x}{a(t)\alpha + (1 - a(t))\beta} - \frac{c_y w_y}{1 - a(t)\alpha - (1 - a(t))\beta} \right) \cdot \frac{\sigma \left(2\widehat{K} - \frac{c_x w_x}{a(t)\alpha + (1 - a(t))\beta} (\alpha + \beta) - \frac{c_y w_y}{1 - a(t)\alpha - (1 - a(t))\beta} (2 - \alpha - \beta) \right)}{\left(1 + \sigma \left(\frac{c_x w_x \alpha}{a(t)\alpha + (1 - a(t))\beta} + \frac{c_y w_y (1 - \alpha)}{1 - a(t)\alpha - (1 - a(t))\beta} - \widehat{K} \right)^2 \right) \left(1 + \sigma \left(\frac{c_x w_x \beta}{a(t)\alpha + (1 - a(t))\beta} + \frac{c_y w_y (1 - \beta)}{1 - a(t)\alpha - (1 - a(t))\beta} - \widehat{K} \right)^2 \right)}. \quad (2.11)$$

Box I.

$$f(a) = (\alpha - \beta)a(1 - a) \left(\frac{c_x w_x}{a\alpha + (1 - a)\beta} - \frac{c_y w_y}{1 - a\alpha - (1 - a)\beta} \right), \quad (2.13)$$

$$g(a) = (\alpha - \beta)a(1 - a) \left(\frac{c_x w_x}{a\alpha + (1 - a)\beta} - \frac{c_y w_y}{1 - a\alpha - (1 - a)\beta} \right) \cdot \frac{\sigma \left(2\widehat{K} - \frac{c_x w_x}{a\alpha + (1 - a)\beta} (\alpha + \beta) - \frac{c_y w_y}{1 - a\alpha - (1 - a)\beta} (2 - \alpha - \beta) \right)}{\left(1 + \sigma \left(\frac{c_x w_x \alpha}{a\alpha + (1 - a)\beta} + \frac{c_y w_y (1 - \alpha)}{1 - a\alpha - (1 - a)\beta} - \widehat{K} \right)^2 \right) \left(1 + \sigma \left(\frac{c_x w_x \beta}{a\alpha + (1 - a)\beta} + \frac{c_y w_y (1 - \beta)}{1 - a\alpha - (1 - a)\beta} - \widehat{K} \right)^2 \right)}. \quad (2.14)$$

Box II.

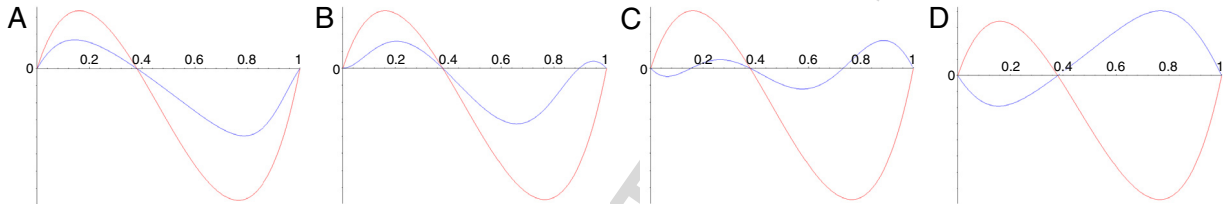


Fig. 1. The graphs of f in red and of g in blue with $K = 4.5$ in (A), $K = 3.8$ in (B), $K = 3.5$ in (C), $K = 3$ in (D). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

to such similarity, in what follows we will investigate the dynamics obtained in the monotone and non-monotone settings on varying $K = \bar{K} = \widehat{K}$.

In Fig. 1 we plot the graphs of the first iterate of f in (2.13) in red and of g in (2.14) in blue for different values of K , while the other parameters are fixed as follows: $c_x = 1.5$, $w_x = 1$, $c_y = 1.3$, $w_y = 1.4$, $\alpha = 0.7$, $\beta = 0.3$, $\sigma = 0.9$.

Fig. 1 shows that for large values of K maps f and g are analogous from a stability viewpoint (see (A) for $K = 4.5$), and thus we find for our model the same dynamic behavior as in Chang and Stauber (2009), while for small values of K maps f and g display an opposite behavior (see (D) for $K = 3$); for intermediate decreasing values of K , the behavior of f and g becomes less and less similar (see (B) for $K = 3.8$ and (C) for $K = 3.5$). We stress that, just by suitably modifying the values of w_y and K , it is possible to obtain a framework symmetric to that in (B), in which there exists a unique zero for g which is not a zero for f , and that is close to 0, rather than to 1. For instance, such a framework can be achieved for $w_y = 0.8$ and $K = 3$. In particular, still on varying K , it is possible to replicate the sequence of scenarios from (A) to (D), but passing through the framework symmetric to (B).

As concerns multistability, we notice that the locally stable equilibria are $a = a^*$, $a = 1$ in (B); $a = 0$, $a = a^*$, $a = 1$ in (C); $a = 0$, $a = 1$ in (D). Hence, at most one of the multiple locally stable equilibria is characterized by the coexistence between the two groups of agents, while in the other one(s) a group completely prevails and remains alone. The unstable equilibria in (B)–(D) play the role of separating the basins of attraction of the locally stable equilibria: trajectories will be attracted by one or the other of the various locally stable equilibria according to the chosen initial condition.

In regard to (A), since there the monotone and non-monotone growth rate frameworks are dynamically equivalent and since in Chang and Stauber (2009) no multistability phenomena may arise, the unique stable equilibrium we find is given by $a = a^*$.

We finally investigate what happens to the calorie intakes K_α and K_β in the stable equilibria. At first, we observe that, in general, at $a = a^*$ it holds that $K_\alpha = K_\beta \neq K$ and that $K_\alpha = K_\beta = K$ if and only if $K = c_x w_x + c_y w_y$, i.e., if K coincides with the stability threshold level found in Proposition 3.2 for (2.11) at $a = a^*$. For the sake of generality, we focus on the case in which $K_\alpha = K_\beta \neq K$ at $a = a^*$, considering for instance the parameter setting in Fig. 1(C), where, with respect to Chang and Stauber (2009), in addition to $a = a^*$, also $a = 0$ and $a = 1$ are locally stable. In such framework at $a = a^*$ we have $K_\alpha = K_\beta = 3.32 < K = 3.5$, at $a = 0$ we have $K_\alpha = 4.28$, $K_\beta = 3.32$ and at $a = 1$ we have $K_\alpha = 3.32$, $K_\beta = 4.89$. Hence, at $a = 0$ and at $a = 1$ the calorie intake of the only surviving group coincides with the calorie intake at $a = a^*$ and thus it is below the desirable calorie intake K , which allows maximizing the growth rate, while both at $a = 0$ and at $a = 1$ for the extinguishing group we observe an excess calorie intake. Actually, this is true not only at $a = 0$ and at $a = 1$, but also along the trajectories tending towards them.

4. Conclusion

We believe the setting we proposed can be a starting point for other research works.

For instance, from a modeling viewpoint, it could be modified to represent the fashion cycle. In such case, we would still deal with a bell-shaped map, describing, rather than the relationship between calorie intake and population growth rate, the link between

consumption and imitative behavior, below the saturation level, and between consumption and snob behavior, above such level. In order to interpret the fashion cycle, and in particular its multistability phenomena, we need to identify (at least) two lifestyles, described by different preference structures; for each lifestyle we shall introduce an attractivity degree, which depends in a nonlinear bell-shaped manner on the consumption of the representative agent belonging to the population share who adopts that particular preference structure. Then, the two attractivities jointly determine the population switching mechanism between the different lifestyles.

From a mathematical viewpoint, it would instead be interesting to study the model we presented considering time as discrete, rather than continuous, in order to investigate how the dynamics change and which new phenomena arise.

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