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FOREWORD

Welcome to the Twenty-Third IEEE Conference on Decision and Control, the annual meeting of the IEEE Control Systems Society. The CDC is held in cooperation with the Society for Industrial and Applied Mathematics and, for the first time, with the Operations Research Society of America.

This is a CDC with many distinctions: It is the first to be held in Las Vegas; it is the largest in terms of technical sessions and contributed and invited papers; and it is the IEEE Centennial year CDC. The operating and program committees have made every effort to make this an interesting conference. It is thanks to you, the authors and attendees, that the promise of the Program will hopefully be fullfilled.

The Program features three plenary sessions and a luncheon. The two traditional plenary talks highlight two unofficial themes of the Conference. The first talk is by Professor Thomas M. Cover. His topic is related to control under uncertainty, which has been receiving increased emphasis recently, in view of the many papers on robust, adaptive, and stochastic systems. The second is by Arne P. Rasmussen, whose manufacturing systems topic provides the setting for two invited sessions on production and industrial processes. It is of high interest in both industry and academia in all engineering disciplines. The third plenary session, with Dr. Stephen Kahne, who will discuss the history of controls and the Control Systems Society, commemorates the IEEE Centennial. The luncheon features President J. Boyd Pearson's address and Professor Harold W. Sorenson as master of ceremonies. Awards will be presented, including Outstanding Transactions Papers, Distinguished Members, IEEE Fellows, IEEE Centennial Medals, and culminating in the presentation of the IEEE Field Award in Control Systems Science and Engineering.

The Program Chairman Dr. Michael P. Polis and his committee had a very difficult task in selecting papers and sessions in view of the record number of invited sessions proposals and contributed papers submissions. Even the sixty-one technical sessions could not accommodate all the good papers or invited sessions which were submitted. Mike and his committee organized an excellent program, and I take this opportunity to thank them for a superior job.

The CDC is preceded by a tutorial workshop on Microprocessor-Based Digital Control System Design, organized by Professor Gene Hostetter. Dr. Kenneth M. Sobel complemented this theme by organizing several industrial exhibits involving mini- and micro-computers in controls, as well as textbooks and software packages related to the systems field. We thank Ken and the exhibitors for their contributions to narrowing the gaps between theory and practice in this area.

The publication of the Advance Program and the CDC three-volume Proceedings were the responsibility of Professor Frank L. Lewis, who produced as complete and error-free a Proceedings as possible. Many thanks to Frank and to Finance Chairman Professor Kenneth A. Loparo and Registration Chairman Professor Hassan K. Khalil, who watched out for the many painstaking details that are essential for the success of the Conference.

The success of our Publicity Chairman Dr. Sol W. Gully is attested to by the record number of submissions, and is hopefully to be followed by a record number of attendees. Last, but not least, the efforts of the Local Arrangements Chairman James H. Beggs will be apparent as the conference unfolds. His efforts range from providing smoothly run sessions and interesting social events to guided attractions of the Las Vegas area and its surroundings (including a trip to Death Valley).

Finally, special thanks are due to Ms. Elizabeth R. Ducot, who maintained an excellent and accurate CDC mailing list, and to Mrs. Annie Rogers for her support to the program committee.

In closing, the operating and program committees join me in welcoming you to this exciting CDC and hope that you, the contributors and attendees, will make it a successful conference.

Abraham H. Haddad
General Chairman

PROGRAM SCHEDULE

		1	2	3	4	5	6	7	8	9	10	11
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FRIDAY	8:30-9:30	BALLROOM C: Plenary II—Arne P. Rasmussen										
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WP = Wednesday afternoon
TA = Thursday morning
TP = Thursday afternoon
FA = Friday morning
FP = Friday afternoon
(I) = Invited paper
(SIAM) = Paper or session contributed by the Society for Industrial and Applied Mathematics
(ORSA) = Paper or session contributed by the Operations Research Society of America

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SESSION WA8: Conference Rooms 13–14
CONTROL SYSTEMS GOVERNED BY NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS

SESSION WA9: Conference Rooms 7–8
GENERALIZED STATE-SPACE SYSTEMS

SESSION WA10: Conference Rooms 9–10
FREQUENCY-DOMAIN DESIGN OF MULTIVARIABLE CONTROL SYSTEMS

WEDNESDAY AFTERNOON
DECEMBER 12, 1984

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IDENTIFICATION I

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Co-Chairman: J. Tugnait, *Exxon Production Research*

SESSION WP2: Ballroom E
ADAPTIVE CONTROL: IMPLEMENTATION ISSUES AND APPLICATIONS

Chairman: T. Jordanides, *California State, Long Beach*
Co-Chairman: H. Rauch, *Lockheed*

SESSION WP3: Ballroom F
ANALYSIS AND SYNTHESIS OF NONLINEAR INPUT-OUTPUT RESPONSES

Organizers: D. Normand-Cyrot, *C.N.R.S., E.S.E., France*, and S. Monaco, *University of Rome, Italy*
Chairman: S. Monaco, *University of Rome, Italy*
Co-Chairman: D. Normand-Cyrot, *C.N.R.S., E.S.E., France*

SESSION WP11: Ballroom C
FAILURE DETECTION AND FAULT-TOLERANT SYSTEMS

Chairman: B. Friedland, Singer Co.
Co-Chairman: R.-W. Liu, *University of Notre Dame*

THURSDAY MORNING DECEMBER 13,1984

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LARGE SCALE COMPUTING—TUTORIALS

Organizers and Chairpersons: Jane Cullum, *IBM T.J. Watson Research Center*, and A. Laub, *University of California, Santa Barbara*

SESSION TA2: Ballroom E
ADAPTIVE CONTROL—THEORY

Chairman: A. Chakravarty, *Boeing Aerospace Corp.*
Co-Chairman: R.D. Bell, *Macquarie University, Australia*

PARTICIPANT INDEX

WA8 - 11:15

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THE ROLE OF DUALITY IN SYSTEM IDENTIFICATION

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Abstract

The duality rule is applied to the identification of a spatially varying parameter which appears in an elliptic equation. Three kinds of error functionals are considered and preliminary computational results are given for two of them.

1. Introduction

The distinction between control and observation [1] as well as the one between optimal control [2] and identification [3] is to some extent formal rather than substantial . Although a control or "direct" problem is more easily stated, identification or "inverse" problems can be related to the former by applying the duality rule, which has its weakest setting in cathegory theory, as it could be shown by starting e.g. from Goguen and Varela's approach [4]. Of greater interest to control theory and to state-space systems in particular is however the analytical formulation of this rule, which consists of introducing an auxiliary state vector , the solution of the adjoint system. Given a state equation, which may be either linear or non-linear, ordinary or partial differential, equipped with some adequate initial and/or boundary conditions, Green's formula allows the adjoint state vector to be defined. Regardless of the non-linearity affecting the primal system, the adjoint state equation is always linear, as a consequence of the variational procedure followed.

To be more specific we shall restrict ourselves to a parameter identification problem for an elliptic partial differential equation subject to non - homogeneous boundary conditions of Dirichlet type.

2. The primal system

We denote by $p(.,.)$ the solution of :

$$\operatorname{div}(a(x,y) \operatorname{grad} p(x,y)) = f(x,y) \quad ;(x,y) \text{ in } D \subset\subset R^2 \tag{2.1}$$

$$p\big|_S = g(x,y) \qquad (x,y) \text{ in } \partial D := S \tag{2.2}$$

where D is a bounded domain of R^2, the boundary of which is S; $a(.,.)$ is the parameter function ; $f(.,.)$ and $g(.,.)$ are given functions.

(2.1),(2.2) model a number of physical systems, among which the steady state flow of a monophasic non-compressible fluid (e.g. water) in a confined isotropic porous medium. In this case $a(.,.)$ stands for transmissivity, $f(.,.)$ for the discharge rate, $p(.,.)$ for aquifer pressure.

The standard theory of elliptic equations[5] shows that the search for $p(.,.)$ in $H^1(D)$ is a well-posed problem if C is smooth enough and if :

$$f(.,.) \text{ in } H^{-1}(D) \tag{2.3}$$

$$g(.,.) \text{ in } H^{1/2}(S) \tag{2.4}$$

$$a(.,.) \text{ in } A_{ad} := \left\{ a \;\middle|\; a \text{ in } L^{\infty}(D),\ a(x,y)>0 ,\ \forall\ (x,y) \text{ in } D \right\} , \tag{2.5}$$

where $H^s(.)$ are Sobolev spaces of order s.
We stress that a strict positivity constraint applies to $a(.,.)$.

3. Some error functionals and the related adjoint systems

Let us now assume some information on $p(.,.)$, i.e. a function thereof, is available and that $a(.,.)$ is unknown. Of course (2.1) and (2.2) are unsuitable as such to yield $a(.,.)$,given $p(.,.)$, $f(.,.)$ etc., because $a(.,.)$ is acted upon by a first order differential operator and because the boundary condition (2.2) affects $p(.,.)$. If (2.1) were taken for a transport equation for $a(.,.)$, then boundary conditions for it ought to be given on a subset of S, say S_1, such that characteristic lines originated at points on S_1 could cover the whole of D. As this kind of condition is not physically realisable, we must keep the original one and work at the $p(.,.)$ system, which contains a non-linear relationship between $a(.,.)$ and $p(.,.)$. The $a(.,.)$ we look for shall then be a solution in the least squares sense as we will show presently.

Information on $p(.,.)$ results from observations carried out on the system. We are interested in comparing three types of "distributed" observation laws, denoted by C_1,C_2,C_3 respectively, which yield some functions of $p(.,.)$ having the whole of D as a support. Actually none of these laws has a physical counterpart, because practical measurements are always carried out across some subdomains say $D_i \subset D$, $1 \le i \le I$, centered at points (x_i,y_i) in D and having areas much smaller than D.

We let :

$$C_1 := \mathbb{1}_D, \text{ i.e. } C_1 p(x,y) = p(x,y),\ \forall (x,y) \text{ in } D \qquad (3.1)$$

$$C_2 : H^1(D) \longrightarrow (H^\circ(D))^3$$
$$p(x,y) \longmapsto C_2 p(x,y) := \text{grad } p(x,y) \qquad (3.2)$$

$$C_3 : H^1(D) \longrightarrow H^{-1}(D)$$
$$p(x,y) \longmapsto C_3 p(x,y) := \text{div}(b(x,y) \text{grad } p(x,y)$$
$$b(.,.) \text{ in } A_{ad} \qquad (3.3)$$

Every identification problem consists of comparing some measured quantities with the computed ones: physical insight suggests that the former belong to the range of an observation map, such as one of the above. By assuming that both measured and computed quantities belong to the same regularity class , in addition to being physically homogeneous, it makes sense to introduce the following mean squared absolute error functionals, which are linked to maps C_1, C_2, C_3 respectively .

$$J_{e1} := \int_D (p - z)^2 \, dD \ , \qquad (3.4)$$

$$J_{e2} := \int_D |\text{grad } p - \text{grad } z|^2 \, dD \ , \qquad (3.5)$$

$$J_{e3} := \int_D (\text{div}(b \text{ grad } z) - f)^2 \, dD \ , \qquad (3.6)$$

where $z(.,.)$ is the measured quantity, i.e. a part of the data set together with $f(.,.)$, $g(.,.)$ and the domain geometry.

J_{e1} is the well-known "squared output error" functional; J_{e2} depends on computed and measured gradients, whereas J_{e3} is the so called squared "equation error" functional or "residual" ([6],p.11). All functionals depend explicitly on the data and implicitly or explicitly on the unknown, $a(.,.)$.

We now define the functionals :

$$J_i := J_{ei} + c \, J_r \quad ; \ c > 0 \ ; \ i = 1,2 \text{ or } 3 \ , \qquad (3.7)$$

where J_r is the regularising term. In all of the three cases we shall deal with :

$$J_r := X_{Aad} \, \| b \|^2_{Aad} \ , \qquad (3.8)$$

where X_{Aad} is the index function of A_{ad} :

$$X_{Aad} = \begin{cases} 0 \text{ if } b \text{ in } A_{ad} \\ + \infty \text{ elsewhere} \end{cases} \qquad . \qquad (3.9)$$

typos fixed

Then solving the identification problem in the least squares sense amounts to finding $a(.,.)$ in $A_{ad}(D)$ which minimises $J_i(.)$:

$$J_i(a) = \inf_{b \text{ in } A_{ad}} J_i(b) \quad . \qquad (3.10)$$

(3.10) is equivalent to the following variational inequality :

$$\langle J_i'(a), b - a \rangle_{L^1, L^\infty} \geq 0 \ , \ \forall \ b \text{ in } A_{ad} \ , \qquad (3.11)$$

where the bra $\langle J_i'(a),.$ is the gradient of $J_i(.)$ with respect to $b(.,.)$ evaluated at $a(.,.)$, usually an element of $L^1(D)$, given the duality between $L^\infty(.)$ and $L^1(.)$.

Now the duality rule enables us to write these gradients as functions of $p(.,.)$ and, where needed, of the solutions $q_i(.,.)$ of the adjoint systems we shall define presently.

As the standard variational procedure (see e.g. Sect. 4 of [7]) shows, minimisation of J_1 is equivalent to :

- solving (2.1),(2.2) ,
- solving the q_1 - adjoint system :

$$\begin{cases} \text{div}(a \text{ grad } q_1) = -2(p - z) \\ q_1 |_S = 0 \end{cases} \qquad (3.12)$$

- satisfying the variational inequality :

$$\langle \text{grad } p \ . \ \text{grad } q_1 \ , \ b - a \rangle \geq 0, \ \forall \ b \text{ in } A_{ad} \ . \qquad (3.13)$$

Minimisation of J_2 can be also restated in terms of the solution of the $q_2(.,.)$ adjoint system and of a variational inequality. The former is needed because J_2, as well as J_1, does not depend explicitly on $b(.,.)$. If we consider $b(.,.)$ and grad $p(.,.)$ as independent variables in the functional differentiation procedure, we get the adjoint state equation :

$$b \text{ grad } q_2 = 2(\text{grad } p - \text{grad } z) \quad \text{in } D \ . \qquad (3.14)$$

By the help of some vector algebra we can prove that solving (3.14) is a well-posed problem if a condition at a single point, $(\bar{x},\bar{y})$, on the boundary S is given :

$$q_2(\bar{x},\bar{y}) = 0 \ , \quad (\bar{x},\bar{y}) \text{ in } S \ , \qquad (3.15)$$

which makes sense at least if q_2 in $C^\circ(\bar{D})$. The variational inequality characterising the minimum of J_2 reads :

$$\int_S q_2 \partial_n p(a).(b - a) \, dS - \int_D \text{grad } p(a).\text{grad } q_2(a)$$
$$(b - a) \, dD \geq 0 \ , \ \forall \ b \text{ in } A_{ad} \ . \qquad (3.16)$$

It differs from (3.13) because of the boundary integral term.

Finally the equation error functional J_3: since it contains $b(.,.)$ explicitly, the duality rule helps in proving that no adjoint system is needed to write $J_3'(.)$, the gradient with respect to $b(.,.)$. After some straightforward computations the latter is shown to read :

$$J_3'(.) = \int_S 2(w - f)\Big|_S \partial_n z(.)\, dS -$$

$$-2\int_D \mathrm{grad}(w - f).\mathrm{grad}\, z(.)\ \ dD \ , \qquad (3.17)$$

where $w := \mathrm{div}(b(.,.)\mathrm{grad}\ z(.,.))$

4. Functional minimisation : the infinite dimensional case

Let us briefly consider existence and uniqueness of minimising elements. For J_1 and J_2, existence is insured by addition of the J_r term. Uniqueness cannot be proved because both functionals are not convex. On the contrary, J_3 is convex,+) as it can be easily shown. This property makes J_3 an interesting functional to work with (see below, Sect. 6).

In order to construct a minimising element, we use the functional gradients introduced above. A constrained minimisation algorithm based on the steepest descent plus projection may then be considered, which updates the estimate a^k at the k-th iteration by the following rule :

$$a^{k+1} = P(a^k - s^k J_i'(a^k)) \ , \qquad (4.1)$$

where s^k is an adequate updating step and $P(.)$ is the projector on A_{ad}. In general (4.1) is however meaningless (see e.g. Sect. 4 of [7]). If we required more regularity for all quantities appearing e.g. in (3.13), by setting :

$$\mathrm{grad}\ p(.,.),\ \mathrm{grad}\ q_1(.,.)\ \text{in}\ L^\infty(D) \qquad (4.2)$$

and similar constraints for the operands in (3.16) or (3.17), we could read e.g. (3.13) as an inequality for an inner product in $L^2(D)$ made up of elements belonging to $L^\infty(D)$. Since however $L^\infty(.)$ is not closed in $L^2(.)$, we lack a basic hypothesis for the convergence of the sequence $\{a^k\}$ defined by (4.1) to be proved (see e.g. Ch. 1 of [8]). This matter therefore needs some further investigation.

Another item which deserves our attention is the way minimising elements depend on data, on $z(.,.)$ in particular. This property is also named "stability" or "continuity". Some results on the continuous dependence of $a(.,.)$ on $z(.,.)$, when the latter is slightly perturbed, are available in the realm of Tichonov's regularisation theory (Ch. 2 of [6]): see e.g. [9]. The effects of larger variations in $z(.,.)$ could be evaluated by applying Sokolovsky's method [10], which has originally been developed for direct problems.

+) on the set K_a where $a(.) \dashrightarrow J_3(a)$ is injective

footnote added

5. Discretisation : general remarks

The inconclusive result about convergence of the infinite dimensional gradient method is however of little relevance when the identification problems listed in Sect. 3 are restated in finite dimensional spaces. Since the duality rule also works in the finite dimensional spaces which approximate the ones introduced in Sect. 2 (see e.g. Sect. 5 of [7]), the exact gradients of J_{d1}, J_{d2}, J_{d3}, the discretised counterparts of the functionals defined by (3.7), are easily written. In this setting an updating law such as (4.1), which we rewrite :

$$a_d^{k+1} = P_d(a_d^k - s_d^k J_{di}'(a_d^k)) \qquad (5.1)$$

makes sense and convergence of the $\{a_d^k\}$ can be proved. Here a_d^k is the discretised a^k.

The flow charts of the algorithms which minimise functionals J_{d1} to J_{d3} can be easily generated. For J_{d1} the Reader may refer e.g. to [7,11] and to the literature quoted therein. Similar procedures apply to J_{d2} and J_{d3}. All of them need an initial estimate of the unknown, $a_d^0(.,.)$, i.e. a set of as many elements as there are nodes in the discretised domain D_d.

If the functional is not convex the $\{a_d^k\}$ sequence converges towards an element which depends on the initial estimate. In principle at least J_{d3} should not suffer from this drawback, although in practice an initial estimate too "far away" from the exact value (in the $A_{ad}(D_d)$ norm) often gives rise to a slow convergence rate.

An essential check to be performed during minimisation is the comparison between the attained value of J_{di}^k at the k-th step and a reference value, say v (> 0). As soon as :

$$J_{di}^k \le v \qquad (5.2)$$

computation is discontinued. The literature (see e.g. Ch. 1 and 2 of [6]) gives some hints on how to evaluate v.

The main contributions to v come from :

- data noise, i.e. measurement errors,
- discretisation errors, owed to approximations of the partial differential operators appearing in the primal and adjoint state equations,
- discretisation errors of functions, e.g. gradients, defined on or near the boundary.

When computer experiments are performed on test examples, the exact solutions of which are known, the first contribution is neglegible and the next two can be thoroughly investigated.

6. Discretisation: computational tests

The idea behind computational tests is something we call "meta-duality".

On one hand we have meta-duality between :

- the infinite dimensional and
- the finite dimensional

settings of a given identification problem. The former provides a high level symbolic manipulation language , by which quantities appearing in the latter are dealt with and computer codes are developed.

On the other hand there is a meta-duality relationship between :

- the finite dimensional problem and
- the computer implementing the algorithm.

The latter is the physical system of which the former claims to be the model.

In both of these meta-duality relationships correspondence is far frm being complete: an example is given by rules (4.1) vs. (5.1). Nonetheless it is worthwhile investigating how computer codes based on the theory briefly presented above perform in some cases.

At present we are interested in comparing minimisation of J_{d1} vs. J_{d3}.

We choose a rectangular domain and a set of functions $p(.,.)$,$f(.,.)$,$a(.,.)$ of polynomial or of exponential type, which satisfy (2.1). We let the boundary condition (2.2) hold and assume noiseless data, i.e. $z(x,y)=p(x,y)$ everywhere in D. We discretise the domain with a rectangular grid having 42 to 234 nodes in the extreme cases, hence we have as many unknown values of $a_d(x,y)$ to identify. We already know [11,12] that the algorithm based on J_{d1} identifies $a_d(.,.)$ in a satisfactory way after 30 to 100 iterations if initial values $a_d^o(x,y)$ are the same $\forall$ (x,y) in D_d and range approximately between one tenth and ten times the average of the exact solution over D_d. Results for J_{d3} will be available soon.

One feature which appears with both functionals will help us in evaluating discretisation errors and computing v of (5.2). If we let $a_d^o(.,.)$ consist of samples of the exact solution, we find that both J_{d1}^1 and J_{d3}^1 are, within the same order of magnitude, as large as the squared errors made when the algorithm starts with the average value of the exact solution. Moreover, if the number of grid nodes in D_d increases about 5 times, both J_{d1}^1 and J_{d3}^1 decrease by one to two orders of magnitude. This is due to the finite difference scheme which approximately solves (2.1),(2.2) and (3.12), when J_{d1} is worked at and to the low order Newton formula (see e.g. Ch. XV of [13]) used to estimate all derivatives which appear in (3.17). As the number of iterations increases both J_{d1}^k and J_{d3}^k decrease (typically two to three orders of magnitude when k=30). This means the gradient algorithm incorporates some self correcting or noise filtering features : actually discretisation errors contribute to overall noise as well as data errors do.

The finite difference scheme just mentioned is solved by Gauss-Seidel's iterative procedure, which stops when :

$$\max_{(x,y)\text{ in } D_d} \left| p_d^n(x,y) - p_d^{n-1}(x,y) \right| \le u , \qquad (6.1)$$

where n is the iteration number (not to be confused with the main loop index k) and u is a given positive quantity, the "iteration error". In some cases the absolute "estimation error" :

$$\max_{(x,y)\text{ in } D_d} \left| p_d^n(x,y) - p_d^{exact}(x,y) \right| := t \qquad (6.2)$$

for n such that (6.1) holds, may be unrelated to u. $u = 10^{-8}$ may yield $t = 10^{-2}$, which then affects the value of J_{d1}^k and the algorithm convergence rate.

This remark has been made to stress the need for accurate <u>direct</u> problem solvers when inverse ones are dealt with.

Conclusion

A number of open problems appear to exist in the field of parameter identification, although this is by no means a newly born subject.

Duality, as we have seen, is a valuable tool, which must however be used in connection with other methods if some understanding of inverse problems is to be gained.

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