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Sectoral shocks and banking crises in a Schumpeterian model of endogenous firm dynamics.

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Introduction

The dissertation is an essay on Schumpeterian firms' dynamic. We investigate the role of endogenous firms entry and exit in productivity shock propagation and we exploit a heterogeneous firms' set-up in order to understand which is the contribute of idiosyncratic productivity in driving business cycle fluctuations. The thesis is divided into two chapters. The two chapters share a non-trivial technology diffusion mechanism based on the work of Piersanti and Tirelli (2018 [26]). In this set-up, firms draw their technology level from a Pareto distribution and choose to pay a fixed cost and produce or to exit the market. This market structure, allows us to identify an entry threshold, depending on the price level and costs of production factor, that makes the entry and exit decision purely endogenous. Additionally, we are able to model an innovation spreading from younger to older firms, recognizing New Entrants as technology innovators. The main advantage of this formulation is that we are able to model endogenous exit flows without the need of keeping track of the idiosyncratic evolution of each incumbent efficiency. This way of modelling makes our basic model able to reproduce the responses of the economy to exogenous changes peculiar in more complex DSGE models.

In the first chapter, we build a two-sector (capital and final goods) model with endogenous firm dynamics to study the effects of sectoral productivity shock. Firms are characterized by idiosyncratic productivity levels and decreasing returns to scale. Shocks are modelled as a sudden improvement of the technology frontier accessed by new entrants, which then gradually spreads to incumbent firms. The shock drives less efficient firms out of the market and unambiguously raises productivity and output in the long run. By contrast, creative destruction is strongly limited by the initial fall in the relative price of capital goods. This latter result is driven by the wealth effect of the shock on consumption dynamics and by the ensuing reduction in savings and in demand for capital goods. The smaller scale of production of this sector is associated with increased efficiency and to a reduced relative price of capital goods. As a result, production costs in the final goods sector, fall and fewer incumbents exit the market. Relative to what would happen in a standard one sector model, we obtain a dramatic contraction in the initial employment fall associated with the shock.

The second chapter enriches the model with a financial sector, where financial intermediaries have the possibility to roll-over debt conditions. Thanks to this possibility, we allow the existence of non-performing firms (the so-called *Zombie firms*), which are firms that are not productive enough to produce in absence of a banking sector but are kept alive by the financial intermediaries, that finds convenient to renegotiate their debt condition instead of repossessing and reallocating the landed capital. We found that allowing financial intermediaries to renegotiate debt condition to unproductive incumbents, the economy recovers hardly after a crisis. Secondly, technology innovation brought by New Entrants spreads slowly to the whole production sector in presence of Non-performing firms. Additionally, we show that the presence of Non-performing firms' has a negative effect on TFP in crisis periods.

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Chapter 1

Schumpeterian firm dynamics, sectoral innovation and the business cycle.

Bianca Barbaro and Patrizio Tirelli

Abstract We build a two-sector (capital and final goods) model with endogenous firm dynamics to study the effects of permanent productivity shocks in the final goods sector. Firms are characterised by idiosyncratic productivity levels and decreasing returns to scale. Shocks are modelled as a sudden improvement of the technology frontier accessed by new entrants, which then gradually spreads to incumbent firms. The shock drives less efficient firms out of the market and unambiguously raises productivity and output in the long run. By contrast, creative destruction is strongly limited by the initial fall in the relative price of capital goods. This latter result is driven by the wealth effect of the shock on consumption dynamics and by the ensuing reduction in savings and in demand for capital goods. The smaller scale of production of this sector is associated with increased efficiency and to a reduced relative price of capital goods. As a result, production costs in the final goods sector fall and fewer incumbents exit the market. Relative to what would happen in a standard one sector model, we obtain a contraction in the initial employment fall associated with the shock.

1.1 Introduction

We study the effects of permanent productivity improvement, in a business cycle model characterised by distinct final and capital goods sectors (C-sector and K-sector) and by endogenous firm dynamics.

The study of technology shocks is an evergreen topic in macroeconomic literature and we can consider as a stylized fact that exogenous improvements in firms' productivity generate on impact a positive co-movement of output, consumption and worked hours. However, improvements in measuring total factor productivity (TFP) and the slow recovery from the recent financial crises have raised concerns on the adequacy of standard macroeconomic models in providing a convincing representation of the business cycle fluctuations induced by technology improvements. Indeed, the renewed interest in this topic goes beyond the role played by productivity shocks in driving business cycle fluctuations and is driven by the purpose of obtaining consistency between the predictions of macroeconomic models and the estimated responses of macro variables to technological improvements. (Sims, 2011[30]). Typically, standard RBC models are not able to capture the short-therm countercyclicality in the response of worked hours and capital to a TFP shock (Galí, 1999 [18]; Basu et al., 2006 [7]; Sims, 2011 [30]).

Fernald (2014 [16]) and Liu et al. (2012 [24]) find that the economy's response differs depending on the sectoral nature of the shock, where technology improvements in the consumption goods sector elicit a procyclical response, whereas the opposite holds true for shocks arising in the capital goods production sector.

Recent contributions that recall the Schumpeterian theory of *creative destruction* suggest that new firm drive the pattern of technological change, and technological change therefore implies exit and a consequential reallocation of inputs from incumbents to new entrants (Caballero and Hammour, 1996[11]; Campbell, 1998[13], Foster et al, 2001[17]).

Within this framework, our contribution explores the role of sectoral interdependence and reallocation of resources. To this aim we model a two-sector business cycle model characterised by endogenous firms dynamics where innovation brought by new entrants drives productivity improvement through an endogenous diffusion process.

Our modelling strategy closely follows Piersanti and Tirelli, 2018[26]. Producers are characterised by a decreasing return to scale production function and by idiosyncratic efficiency. In each period, firms draw their technology level from a Pareto distribution and choose to pay a fixed cost and produce or to exit the market. New entrants are by construction more productive than the incumbents since they benefit from an exogenous advantage in the technology frontier. With a lag, incumbents learn the technology adopted by new entrants. This market structure, allows us to identify an entry threshold, depending on the price level and costs of production factor, that makes both the entry and exit decisions endogenous. The main advantage of this formulation is that we are able to model endogenous exit flows without the need of keeping track of the idiosyncratic evolution of each incumbents' efficiency. Further, the process of technology diffusion is endogenous to the sectoral relative price, which co-determines entry-exit thresholds. One appealing feature of the technology diffusion adopted here is that we can allow firms to use both labour and capital in the production process, and for this reason we can focus on sectoral reallocation in consequence of technological change. This is indeed a critical innovation of the paper relative to contributions that keep track of individual firms' evolution over time and, for this reason, are constrained to assume a labour-only technology. This is because, using capital as an input, we would have to deal with a state variable that depends on firms' idiosyncratic productivities. In our model, firms produce only if they meet a technology requirement, expressed as a function of aggregate variables only. This set-up gives us the key advantages to study all the variables, including, capital, at their aggregate level without keeping track of the evolution of idiosyncratic productivity for each firm.

When a positive productivity shock hits a sector, the entry of more productive firms rises the productivity threshold consistent with non-negative profits, pushing less productive incumbents out of the market. The updating process of incumbents' productivity distribution mimics a direct technology spillover for which, after a shock, incumbents innovate in response to new entrants technology level (Schumpeter, 1942[28]; Aghion and Howitt, 1990[3]; Piersanti and Tirelli, 2018[26]). We share with Piersanti and Tirelli 2018 the strategy adopted for modelling the technology diffusion process, but our main concern here is for the effects of a permanent productivity shock in the final goods sector.

Our results in a nutshell. The shock drives less efficient firms out of the market and unambiguously raises productivity and output in the long run. By contrast, creative destruction is strongly limited by the initial fall in the relative price of capital goods. This latter result is driven by the wealth effect of the shock on consumption dynamics and by the ensuing reduction in savings and in demand for capital goods. The smaller scale of production of this sector is associated to increased efficiency and to a reduced relative price of capital goods. As a result, production costs in the final goods sector fall and fewer incumbents exit the market. Relative to what would happen in a standard one sector model, we obtain a contraction in the initial employment fall associated to the shock. We also detect a strikingly similarity between what happens in out model and the response of a standard RBC model to "news" shocks. Thus our results could be seen as a micro-foundation of such shocks in RBC models.

Our paper contributes to a growing strand of literature that includes endogenous firms dynamics in RBC and DSGE models. The use of a model with endogenous entry have the capabilities to generates an important propagation mechanism for business cycle model due to a typical sluggish response of the number of producers. The seminal work of Bilbiie, Ghironi and Melitz (2012 [10]) studies the role of endogenous entry in propagating business cycle fluctuations focusing on extensive margins, other studies include different levels of competition (Jaimovich and Floetotto, 2008[23]; Colciago and Etro, 2008[15]). All of these studies agree that firm entry and exit amplify and propagate the effects of aggregate shocks.

We do not focus our study on imperfect competition, and investigate the role of creative destruction in the context of competitive markets. In this regard, the closest contributions to our paper are Clementi and Palazzo (2016 [14]) and Hamano and Zanetti (2017 [21]). The distinctive features of our work our twofold. First, in our model dynamics of aggregate TFP are endogenous whereas in these models they are exogenous. Second, we are able to rationalize the effects of sectoral shocks. To the best of our knowledge, we are the first to use this feature to analyse the interaction between sectoral productivity level. The rest of the paper is organized as follows. Section 2 describes the model, section 3 presents calibration and impulse response function analysis. Section 4 concludes. Technical details and the de-trended version of the model are left to the Appendix.

1.1.1 Technology diffusion

Production sectors are modelled as in Piersanti and Tirelli (2018). Producers (both C-sector and K-sector) are characterised by a decreasing return to scale production function and by idiosyncratic efficiency. In each period, firms draw their technology level from a Pareto distribution and choose to pay a fixed cost and produce or to exit the market. New entrants are by construction more productive than the incumbents since they benefit from an exogenous advantage in the technology frontier. Incumbents learn the technology adopted by new entrants when they will join the surviving incumbents (in $t+1$). This market structure, allows us to identify an entry threshold, depending on the price level and costs of production factor, that makes the entry and exit decision purely endogenous. The main advantage of this formulation is that we are able to model endogenous exit flows without the need of keeping track of the idiosyncratic evolution of each incumbent efficiency. The Pareto distribution, help us to reproduce "creative destruction". When a positive productivity shock hits a sector, the entry of more productive firms rises the productivity entry requirement, pushing less productive incumbent out of the market and increasing the quality of producing firms. We are able to observe the path of productivity through the aggregate productivity level and through the entry and exit thresholds, identified by efficiency level that meets the zero-profit condition for which production is worthy, that will indicate the minimum productivity level required to participate the market. The updating process of incumbents' distribution, reproduce a direct technology spillover for which, after a shock, incumbents are forced to innovate in order to face new entrants technology level (Schumpeter, 1942[28] ; Aghion and Howitt, 1990[3]; Piersanti and Tirelli, 2018 [26]). The shocks we analyse are permanent, following suggestion on shock structure proposed by Sims (2011) and Basu et al. (2006).

1.2 The model

Our Two-sector Business Cycle model is characterised by the presence of three agents: households, consumption good firms (C-firms) and capital producing firms (K-firms). In each period, households purchase capital and consumption goods from the respective producers, supply labour, borrow capital to C-firms and sell their savings, in terms of consumption goods to K-firms. In each period, C-sector new entrants and incumbents, will know their idiosyncratic productivity level and choose if enter or exit the market. If their productivity level is high enough to produce, they will demand capital and labour to households and sell them the final produced good. Similarly, K-firms new entrants and incumbents decide to produce if their technology level is sufficiently high. K-sector production inputs consists households' saving that they will transform in investment goods (or capital goods). In each sector, entry and exit are purely endogenous.

1.2.1 Households

Standard household preferences are defined over consumption C_t, and labour effort L_t. The utility is characterised by the following preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t (lnC_t - \psi \frac{L_t^{1+\varphi}}{1+\varphi}) \tag{1.1}$$

the flow budget constraint is:

$$C_t + Q_t I_t = w_t L_t + r_t^k K_{t-1} + \Pi_t^{K,F} \tag{1.2}$$

Where w_t is the real wage, Q_t is the relative price of investment goods in terms of consumption goods, r_t^k is the real rental rate of capital and $\Pi_t^{K,L}$ are firms profits. Households supply labour L_t, and choose the optimal level of consumption C_t. Households savings S_t are transferred to K-producers to purchase investment goods I_t.

Capital depreciates at the rate δ and evolves accordingly to the following law of motion:

$$K_t = I_t + (1-\delta)K_{t-1} \tag{1.3}$$

The households' first order conditions are:

$$L_t = \left(\frac{w_t}{\psi} \cdot \frac{1}{C_t} \right)^{\frac{1}{\varphi}} \tag{1.4}$$

$$\frac{E_t\{C_{t+1}\}}{C_t} = \beta \left[E_t \frac{\{r_{t+1}^k\}}{Q_t} + (1-\delta) \frac{Q_{t+1}}{Q_t} \right] \tag{1.5}$$

1.2.2 Consumption goods producers (C-firms).

In the final goods sector, the fully competitive market is characterised by new entrants (NE^C) and incumbents (I^C) firms which are heterogeneous in their productivity level. The production function of a generic firm j is:

$$y_{j,t}^C = A_{j,t}^C \left[(k_{j,t}^C)^\alpha (l_{j,t}^C)^{(1-\alpha)} \right]^\gamma,$$

where $C = NE, I$, $A_{j,t}^C$ defines the firm-specific level of productivity, and $\gamma < 1$ is the degree of C-firms decreasing return to scale. In each period C-firms maximize their profits:

$$\pi_{j,t}^C = y_{j,t}^C - r_t^k k_{j,t}^C - w_t l_{j,t}^C - \phi_t^C \tag{1.6}$$

Where w_t is the real wage, r_t^k is the rental rate of capital and ϕ_t^C is a fixed production cost[1]. In the perfectly competitive market each firm will produce until the marginal cost of production is equal to the market price and the optimal demand for labour and capital of a generic j firms are:

$$k_{j,t}^C = \alpha\gamma \frac{y_{j,t}^C}{r_t^k} \tag{1.7}$$

[1]The fixed cost can be intended as an entry cost for NEs and as an operation cost for INCs.

$$l_{j,t}^C = (1-\alpha)\gamma \frac{y_{j,t}^C}{w_t} \tag{1.8}$$

Production decision.

Both new entrants and incumbents, will produce only if they can achieve a non negative profit. Using the first order conditions (1.7) and (1.8) and the profit function (1.6), we can derive the C-sector technology thresholds $\hat{A}_t^C$ for new entrants and incumbents associated to the zero profit condition,

$$y_{j,t}^C - r_t^k k_{j,t}^C - w_t l_{j,t}^C - \phi_t^C \geq 0 \rightarrow$$

$$\hat{A}_t^C = \left[\frac{\phi_t^C}{(1-\gamma)}\right]^{1-\gamma} \cdot \frac{\left[\left[\frac{r_t^k}{\alpha}\right]^{\alpha}\left[\frac{w_t}{(1-\alpha)}\right]^{(1-\alpha)}\right]^{\gamma}}{\gamma^{\gamma}} \tag{1.9}$$

The zero profit condition, shows that the technology requirement $\hat{A}_{j,t}^C$ is increasing in the cost of production.

New entrants.

Firms' productivity distribution and firms' entry and exit are modelled as in Piersanti and Tirelli (2018). $NE^C s$ draw their productivity level $A_{j,t}^{NE}$ from the Pareto distribution,

$$f_t(A_t^{NE}) = \int_{\underline{z}_t}^{+\infty} \frac{\xi \underline{z}_t^{\xi}}{(A_t^{NE})^{\xi+1}} d(A_t^{NE}) = 1 \qquad with\ A_t^{NE} \geq \underline{z}_t \tag{1.10}$$

Where ξ is the shape of the Pareto distribution ($\xi > 1$) and $\underline{z}_t$ defines the technology frontier. The evolution of $\underline{z}_t$ is described by the following equations:

$$\underline{z}_t = \underline{z}_{t-1} g_{z,t} \tag{1.11}$$

$$ln(g_{z,t}) = (1-\rho_z)ln(g_z) + \rho_z ln(g_{z,t-1}) + \sigma^z \epsilon_t^z \tag{1.12}$$

They enter the economy at time t if they meet the zero-profit condition. The mass of new entrants NE_t^C will be:

$$NE_t^C = \int_{\hat{A}_t^{NE}}^{+\infty} \frac{\xi \underline{z}_t^{\xi}}{(A_t^{NE})^{\xi+1}} d(A_t^{NE}) = (\frac{\underline{z}_t}{\hat{A}_t^{NE}})^{\xi} =$$

$$= \underline{z}_t^{\xi} \cdot \left\{\left[\frac{(1-\gamma)}{\phi_t^{NE}}\right]^{1-\gamma} \cdot \frac{\gamma^{\gamma}}{\left\{\left[\frac{r_t^k}{\alpha}\right]^{\alpha}\left[\frac{w_t}{(1-\alpha)}\right]^{(1-\alpha)}\right\}^{\gamma}}\right\}^{\xi} \tag{1.13}$$

The mass of new entrants is increasing in the technology frontier $\underline{z}_t$ and decreasing in the threshold $\hat{A}_t^{NE}$ and so, in production costs r_t^k, w_t and ϕ_t^{NE}.

Incumbents.

At the end of period $t-1$, the number of active firms will be:

$$\eta_{t-1}^{C} = NE_{t-1} + INC_{t-1} \tag{1.14}$$

At the beginning of each period t the η_{t-1}^{C} active firms draw their idiosyncratic productivity level $A_{j,t}^{I}$ from a Pareto distribution identified by the technology frontier $\hat{A}_{t-1}^{NE}$,

$$f_t(A_t^I) = \int_{\hat{A}_{t-1}^{NE}}^{+\infty} \frac{\xi(\hat{A}_{t-1}^{NE})^{\xi}}{(A_t^I)^{\xi+1}} d(A_t^I) \tag{1.15}$$

Then only firms that draw a productivity level $A_t^I \succeq \hat{A}_t^I$ will produce. This formulation allows us to model endogenous exit flows without the need of keeping track of the idiosyncratic evolution of each incumbent efficiency. The number of incumbents that will produce in t will be defined by:

$$INC_t^C = \eta_{t-1}^C \int_{\hat{A}_t^I}^{+\infty} \frac{\xi(\hat{A}_{t-1}^{NE})^{\xi}}{(A_t^I)^{\xi+1}} d(A_t^I) =$$

$$= \eta_{t-1}(\frac{\hat{A}_{t-1}^{NE}}{\hat{A}_t^I})^{\xi} = \eta_{t-1}\left[\left(\frac{\phi_{t-1}^{NE}}{\phi_t^I}\right)^{1-\gamma} \cdot \left(\left(\frac{r_{t-1}^k}{r_t^k}\right)^{\alpha}\left(\frac{w_{t-1}}{w_t}\right)^{1-\alpha}\right)^{\gamma}\right]^{\xi} \tag{1.16}$$

Just like the mass of new entrants, the mass of incumbents (1.16) will depend on costs of production. However, the number of active incumbents is a function of costs evolution: i.e. an increase in rental rate and wages will force less productive firms to exit the market. Equation (1.16) captures the slower innovation that characterizes incumbents. Since they update their technology learning from NE_{t-1}^C, their productivity will be ad hoc for the previous period. However, their technology level, has to be high enough to face costs in t. Notice that the cost of entry in $t-1$ will positively affect the number of producing incumbent in t. The entry cost has indeed two principal effects: if, on one hand, higher costs of entry are related to higher technology requirement an lower entry, on the other hand, an increase in the entry thresholds A_{t-1}^{NE} will shift the future productivity distribution of incumbents, insuring a decrease in exit and an increase in the average incumbents productivity. The sum of new entrants and incumbent in t, will define the number of active firms η_t^C, that will evolve accordingly to the following equation:

$$\eta_t^C = NE_t^C + INC_t^C =$$

$$= \underline{z}_t^{\xi} \cdot \left\{\left[\frac{(1-\gamma)}{\phi_t^I}\right]^{1-\gamma} \cdot \frac{\gamma^{\gamma}}{\left\{\left[\frac{r_t^k}{\alpha}\right]^{\alpha}\left[\frac{w_t}{(1-\alpha)}\right]^{(1-\alpha)}\right\}^{\gamma}}\right\}^{\xi} + \eta_{t-1}^C\left\{\left(\frac{\phi_{t-1}^{NE}}{\phi_t^I}\right)^{(1-\gamma)} \cdot \left[\left(\frac{r_{t-1}^k}{r_t^k}\right)^{\alpha}\left(\frac{w_{t-1}}{w_t}\right)^{1-\alpha}\right]^{\gamma}\right\}^{\xi} \tag{1.17}$$

The mass of exiting firms will be endogenously determined by the evolution of production costs, and it is described by the following equation:

$$E_t^C = \eta_{t-1}^C \left\{ 1 - \left[\left(\frac{\phi_{t-1}^{NE}}{\phi_t^I} \right)^{(1-\gamma)} \left(\frac{r_{t-1}^k}{r_t^k} \right)^{\alpha\gamma} \left(\frac{w_{t-1}}{w_t} \right)^{(1-\alpha)\gamma} \right]^{\xi} \right\}$$

C-sector aggregation.

The aggregate output will be the sum of the output of new entrants and of the output of incumbents:

$$Y_t = Y_t^{NE} + Y_t^I \tag{1.18}$$

Starting from the production function, we can obtain the aggregate output of new entrants, aggregating for the idiosyncratic productivity. Since the entry threshold in t is defined as $\hat{A}_t^{NE}$, the aggregate output of new entrants is:

$$Y_t^{NE} = \int_{\hat{A}_t^{NE}}^{+\infty} A_t^{NE} \left[\left(K_t^{NE}\right)^{\alpha} \left(L_t^{NE}\right)^{1-\alpha} \right]^{\gamma} dF(A_t^{NE}) =$$

$$= \frac{\xi \underline{z}_t^{\xi} \gamma^{\gamma\xi}}{\xi(1-\gamma)-1} \left[\frac{1-\gamma}{\phi_t^{NE}} \right]^{(1-\gamma)\xi} \left[\left(\frac{\alpha}{r_t^k} \right)^{\alpha} \left(\frac{1-\alpha}{w_t} \right)^{1-\alpha} \right]^{\gamma\xi} \tag{1.19}$$

Similarly, the aggregate output of incumbents' will depend on the threshold $\hat{A}_t^I$:

$$Y_t^I = \int_{\hat{A}_t^I}^{+\infty} A_t^I \left[\left(K_t^I\right)^{\alpha} \left(L_t^I\right)^{1-\alpha} \right]^{\gamma} dF(A_t^I) =$$

$$= \eta_{t-1} \frac{\xi \phi_t^I}{\xi(1-\gamma)-1} \left[\frac{\phi_{t-1}^{NE}}{\phi_t^I} \right]^{\xi-1} \left[\left(\frac{r_{t-1}^k}{r_t^k} \right)^{\alpha} \left(\frac{w_{t-1}}{w_t} \right)^{1-\alpha} \right]^{\gamma\xi} \tag{1.20}$$

Equation (1.19) and (1.20) can be simplified according to (1.13) and (1.16). We can rewrite the aggregate output (1.18) as:

$$Y_t = \frac{\xi}{\xi(1-\gamma)-1} \left(NE_t \phi_t^{NE} + INC_t \phi_t^I \right) \tag{1.21}$$

The aggregate output in t will be increasing in the number of active firms and in the fixed costs of production, that can be intended as a proxy of the firms' productivity.

To conclude the C-sector description, the aggregate productivity of active C-firms will be:

$$\bar{A}_t^C = \frac{\xi}{1-\xi} \left(NE_t^C \hat{A}_t^{NE} + INC_t^C \hat{A}_t^I \right) \tag{1.22}$$

1.2.3 Capital goods producers (K-Firms).

The K-sector structure follows the set up of Piersanti and Tirelli (2018). At the end of each period K-firms transform the households savings, in form of final goods, in investment goods which are then sold back to households. K-firms are endowed with decreasing return to scale (indexed with χ) and with idiosyncratic productivity levels. A generic K-firm k is characterised by the following production function:

$$I_{k,t} = A^K_{k,t}(S^K_{k,t})^{\chi} \tag{1.23}$$

where K= NE, INC and $\chi < 1$. Firms' profit is defined by:

$$\Pi^K_{k,t} = Q_t I^K_{k,t} - S^K_{k,t} - f^K_t \tag{1.24}$$

Solving the maximization problem, we can derive the optimal demand of savings:

$$S^K_{k,t} = \left(Q_t \chi A^K_{k,t}\right)^{\frac{1}{1-\chi}} \tag{1.25}$$

Production decision.

Entry and exit in the K-sector follows the same dynamic pattern of the C-sector. We can derive the productivity entry threshold from the zero profit condition. Imposing $\Pi^K_{k,t} = 0$ and substituting for the optimal demand of savings (1.25), we get

$$Q_t I^K_{k,t} - S^K_{k,t} - f^K_t \geq 0 \rightarrow \hat{A}^K_t = \left(\frac{f^K_t}{1-\chi}\right)^{1-\chi} \frac{1}{Q_t \chi^{\chi}} \tag{1.26}$$

New entrants.

At the beginning of each period, the potential K-NEs draw their productivity $A^{K,NE}_t$ from a new and more efficient Pareto distribution.

$$f_t(A^{K,NE}_t) = \int_{\underline{e}_t}^{+\infty} \frac{\omega \underline{e}^{\omega}_t}{(A^{K,NE}_t)^{\omega+1}} d(A^{K,NE}_t) = 1 \tag{1.27}$$

where ω is the tail index describing the distribution skewness and $\underline{e}_t = \underline{e}_{t-1} g^e_t$ represent the technology frontier identifying the K-sector productivity shock dynamics, where

$$ln(g^e_t) = (1-\rho_e) ln(g^e) + \rho_e ln(g^e_{t-1}) + \sigma^e \varepsilon^e_t \tag{1.28}$$

Cutting the pdf at the entry threshold, we obtain the mass of effective new entrants NE^K_t

$$NE^K_t = \left[Q_t \chi^{\chi} \underline{e}_t \left(\frac{1-\chi}{f^{NE}_t}\right)^{1-\chi}\right]^{\omega} \tag{1.29}$$

The mass of new entrants in the K-sector will depend negatively on the fixed cost and positively on the relative price of capital goods Q_t.

Incumbents.

We postulate a technology diffusion mechanism which is identical to the one discussed for the C-firms. The η^K_{t-1} firms draw their idiosyncratic productivity level $A^{K,I}_{j,t}$ from the Pareto distribution.

$$f_t(A_t^{K,I}) = \int_{\hat{A}_{t-1}^{K,NE}}^{+\infty} \frac{\omega(\hat{A}_{t-1}^{K,NE})^{\omega}}{(A_t^{K,I})^{\omega+1}} d(A_t^{K,I}) \tag{1.30}$$

K-sector incumbents will produce only if $A_t^{K,I} > \hat{A}_t^K$. The the mass of active INC^K in t will be:

$$INC_t^K = \eta_{t-1}^K \left[\left(\frac{f_{t-1}^{NE}}{f_t^I} \right)^{1-\chi} \frac{Q_t}{Q_{t-1}} \right]^{\omega} \tag{1.31}$$

As for the C-sector incumbents, the mass of INC_t^K will depend both on present and on past values because of the technology updating process. We can write the mass of active capital good producing firms in t as the sum ok NE_t^K and INC_t^K

$$\eta_t^K = NE_t^K + INC_t^K = \left[Q_t \chi^{\chi} \underline{e}_t \left(\frac{1-\chi}{f_t^{NE}} \right)^{1-\chi} \right]^{\omega} + \eta_{t-1}^K \left[\left(\frac{f_{t-1}^{NE}}{f_t^I} \right)^{1-\chi} \frac{Q_t}{Q_{t-1}} \right]^{\omega} \tag{1.32}$$

The mass of exiting firms in K-sector will be:

$$E_t^K = \eta_{t-1}^K \left\{ 1 - \left[\left(\frac{f_{t-1}^{NE}}{f_t^I} \right)^{1-\chi} \frac{Q_t}{Q_{t-1}} \right]^{\omega} \right\} \tag{1.33}$$

K-sector aggregation.

The supply functions of $NE^K s$ and $INC^K s$ are obtained aggregating 1.88 for the idiosyncratic productivity levels. Since the productivity threshold for NE^Ks is $\hat{A}_t^{NE}$, the aggregate investment good supply of K-sector new entrants will be:

$$I_t^{K,NE} = \int_{\hat{A}_t^{K,NE}}^{+\infty} A_t^{K,NE} \left(S_t^{K,NE} \right)^{\chi} dF(A_t^{K,NE}) =$$

$$= \frac{\omega \underline{e}_t^{\omega} \chi^{\chi\omega}}{\omega(1-\chi)-1} \left(\frac{1-\chi}{f_t^{NE}} \right)^{(1-\chi)\omega-1} \left(\frac{1}{Q_t} \right)^{1-\omega} \tag{1.34}$$

Similarly, we can derive the supply of investment good for K-sector incumbents,

$$I_t^{K,I} = \int_{\hat{A}_t^{K,I}}^{+\infty} A_t^{K,I} \left(S_t^{K,I} \right)^{\chi} dF(A_t^{K,I}) =$$

$$= \frac{\eta_{t-1}^K}{Q_t} \frac{\omega f_t^I}{\omega(1-\chi)-1} \left(\frac{f_{t-1}^{NE}}{f_t^I} \right)^{(1-\chi)\omega} \left(\frac{Q_{t-1}}{Q_t} \right)^{\omega} \tag{1.35}$$

The sum of the aggregate outputs (1.34) and (1.35) will determine the aggregate investment goods supply.

$$I_t = I_t^{K,NE} + I_t^{K,I} = \frac{1}{Q_t} \frac{\omega}{\omega(1-\chi)} \left(NE_t^K f_t^{NE} + INC_t^K f_t^I \right) \tag{1.36}$$

Equation (1.36) shows that the supply of investment goods will be increasing in the number of producing firms and in the fixed costs (that can proxy firms' productivity). Additionally, a decrease in the relative cost of investment goods will encourage their demand from households and, as an increase fixed costs, raise the technology requirements and the K-sector aggregate productivity. As we did for the K-good supply, we can also derive the demand aggregate K-firms demand of savings.

$$S_t = \frac{\omega}{\omega(1-\chi)}\left(NE_t^K f_t^{NE} + INC_t^K f_t^I\right) \tag{1.37}$$

And the aggregate K-sector productivity will be:

$$\bar{A}_t^K = \frac{\omega}{\omega - 1}\left(NE_t^K f_t^{NE} + INC_t^K f_t^I\right) \tag{1.38}$$

1.2.4 Market clearing

Finally, market clears if the following condition is respected.

$$C_t = Y_t - S_t - INC_t\phi_t^I - NE_t\phi_t^{NE} - NE_t^K f_t^{NE} - INC_t^K f_t^I \tag{1.39}$$

1.2.5 Calibration

We calibrate the initial condition of NEs fixed cost ϕ^{NE} to be equal to the 5% of the total output ex post (BGM 2012; Etro and Colciago 2010; Colciago and Rossi 2012). Additionally, in both sectors, we calibrate the Incumbents' fixed cost as a function of the entry cost to set the share of New Entrants in the economy at $H = 10\%$[2] (Piersanti and Tirelli, 2018[26]; Colciago and Etro, 2010[15]). We calibrated the technology frontiers' initial values $\underline{z}_{ss}$ and $\underline{e}_{ss}$ to obtain an unitary mass of firms. Finally, the steady state value of the relative price of investment $Q_{ss} = 1$ and $L_{ss} = 0.33$ are pinned down respectively by the initial value of K-sector entry cost f^{NE} and by preference parameter ψ. The Pareto distribution parameters are calibrated following the work by Piersanti and Tirelli (2018 [26]) and Asturias et al. (2017[4]). In particular, we calibrate the shape of both the productivity distribution as $\xi = \omega = 6.1$. The values of elasticity of labour supply and the labour disutility parameters are pinned in function of $L_{ss} = 0.33$. The other parameters are standard in RBC and DSGE literature and are summarized in Table (1.1). Calibration is quarterly.

1.3 Impulse response analysis

In this section we analyse the response of the economy to a C-sector productivity shock and to an improvement in K-sector firms' productivity. The results refers to two specification of the model: As first, we reduce the K-sector to a standard set-up, preserving the C-sector firm dynamic; Secondly, we restore entry and exit in the K-sector to study the effect of sectoral technology improvement in a two sector economy with firms dynamic. Table (1.2), shows the long term response of the main variables to a C-sector and to an K-sector productivity permanent shock. Finally, we compare our results with different kinds of shocks. Particularly, we will compare the response of our model (with firms dynamic in the C-sector) with the response of a benchmark RBC to a news shock, finding that our set-up is able to replicate a news shock (Barsky and Sims, 2011[6]). A robustness check of results' consistency with different Pareto distribution parameters is provided in Figure (1.7). We changed the concentration of firms around the technology frontiers and our results hold.

[2] Since we calibrate on a quarterly base $H = 2.5\%$

Table 1.1: Parameters calibration

Households		
ϕ	0.25	Inverse Frisch elasticity of labour supply
β	0.99	Discount rate
δ	0.025	Depreciation rate
ψ	2.6	labour disutility parameter
Intermediate Producers		
γ	0.8	Decreasing return index
α	0.33	Capital share
ξ	6.1	Pareto distribution shape parameter
$\underline{z}_{ss}$	0.68	Technology frontier (C-sector) initial condition
H	0.025	Share of NEs over total C-firms
Capital Producers		
χ	0.8	Decreasing return index
ω	6.1	Pareto distribution shape parameter
$\underline{e}_{ss}$	0.38	Technology frontier (K-sector) initial condition
H	0.025	Share of K-NEs over total K-firms

Table 1.2: Variables transition to a 5% permanent shock to z_t and e_t

Variable	*% Permanent deviation from the steady state*		
	One-sector BC	*Two-sector BC*	
	C-tech improvement	*C-tech improvement*	*K-tech improvement*
C	6.64	6.73	1.78
Y	6.64	6.73	1.78
K	5.31	5.62	6.48
Q	1.33	1.10	-4.70
W	6.64	6.73	1.78
r^k	1.32	1.10	-4.70
η	6.64	6.73	1.78

1.3.1 C-sector productivity shock

Shocking the NEs efficiency draws, we induce a sudden shift to the right of NE's productivity distribution. From condition (1.13) can be shown that the principal effect of the shock is the sudden entry of new and more productive firms. The inflow of young and productive firms strengths the competition among the final good sector and the higher level of technology will allow firms to pay higher wages and will increase the cost of production for C-firms. Consequentially, the incumbents (INC) that are not able to gain non-negative profits will be forced exit the market, and their number drops. Incumbents' behaviour in the following periods, reproduce the following technology spillover: surviving firms adopt the new entrants' technology and the number of incumbent recovers after 6 periods to reach a new and higher equilibrium level. At the end of the initial period, surviving incumbents will be more productive, the production cost will increase and the raise in NEs cut-off, will gradually arrest the boom in entries. The total number of active firms

η starts immediately to benefit from the new entries, after a negligible drop of (-0.04%) and in the long term the whole economy benefits from the productivity, as can be observed by the raise in Output (Y) and Consumption (C). According to standard RBC literature, a technological improvement should increase the labour supply through higher wages. In our model, the expectation on higher future wages, initially decrease labour supply, and encourage households' preference for consumption. Simultaneously, the increase in wages makes the technology requirement more demanding, lowering the labour demand of unproductive incumbent, that cannot afford to pay new wages. Input demand behaviour is in line with DSGE literature according to which, technological improvement creates a temporary contraction in input demand in the short term, and thanks to our set-up we are able to obtain this result without the need of price stickiness Households are compensated in the fall in hour worked by the higher wages level. A preference for consumption will initially lower the investment demand and so the price of investment goods, but after the initial drop the investment level will grow an will reach an higher steady state level. The reaction of the main variables are similar the two specification of the model. However, in the model with dynamic K-sector Q and r^k increase slightly less than in the model with no entry in both sectors, amplifying the positive effect of the shock on Consumption, Output and on the number of active firms. Focusing on the additional variables, the long term increase in investment demand, caused by a TFP improvement, stimulates the entry in the K-sector. At the same time, the higher investment goods price will lower the entry threshold $\hat{A}_t^{K,NE}$, reducing the exit and increasing the number of surviving incumbents. Despite the lower entry requirement, the aggregate productivity of the K-sector increases, as shown by the reaction of $\bar{A}^K$, highlighting a positive spillover from one sector to the other. However, the role of K-sector in shock propagation is relevant and highlighted by the comparison in Figure 1.6. The adjustment of the relative price of investment goods Q_t magnifies the impact of the technology shock in C-sector efficiency. With no K-sector, the technology improvement creates a contraction in output (creative destruction), while the endogenous determination of investment good prices mitigates the contractionary effect of the technology improvement and keeps the output above the steady state level. This effect is improved with entry and exit in the K-sector.

1.3.2 K-sector productivity shock

The third column of Table 1.2 shows how a permanent Investment technology shock, cause a permanent increase in capital K (the same happens fo investment I), and a consequent fall in the price of investment goods Q and in the rental rate of capital r^k. The new equilibrium will be characterised by higher consumption and output, despite the increased preference in savings. Focusing on the impulse response functions, the investment technology shock implies an inflow of more productive new entrants (NE^K) and the fall in Q_t will force less productive incumbents (INC^K) to exit the market, lowering the number of active firms and the initial fall in investment supply (I_t^K) below the steady-state level. After the initial reaction, the incumbents will update their technology and the K-sector will recover to increase permanently the level of investment and the number of active firms. The investment supply shift will keep on lowering the relative price Q_t, and households will reduce consumption in favour of investment. The fall in K-incumbents in the first periods is more severe than the one observed in the C-sector after a sectoral technology improvement and the impulse response of investment and K-incumbents underlines the "creative destruction" process. The slow increase in consumption and the fall in investment demand lowers the demand of final good and, as a consequence, production and employment will face an initial recession. Moving the attention to the C-sector productivity, the entry level will react following the output level. Initially, the lower final good demand and the higher entry requirement will discourage new entry and reduce the number of surviving incumbent. However, $\hat{A}_t$ will fall because of the reduction in costs of production, increasing the mass of active firms in the C-sector. In the long term, the aggregate TFP will increase, suggesting that the number of highly productive new entrant firms compensates the lower entry requirement.

1.3.3 Creative destruction

The impulse response functions point out a substantial difference between the responses to the two shocks. After an improvement in C-sector productivity, the economy starts immediately to grow and output never goes below the steady state level. On the other hand, an increase in K-sector productivity brings to a "creative destruction" process that lowers the level of investment and output on impact. The presence of a K-sector, and the adjustment in the relative price of investment goods, neutralizes the "destruction" effect of the entry of more productive firms. Figure 1.6 shows a comparison with a model without K-sector. In absence of K-sector, in response to a productivity shock, the return on capital drops on impact decreasing the investment level. Households, increase consumption more then in the model with K-sector and the drop in labour supply is more severe. As a consequence, the level of output falls on impact, highlighting "creative destruction". In our model, the adjustment of the relative price of investment goods strongly mitigates this contractionary effect. Households save more and consumption increases less than in the case without a K-sector. In this scenario, exit is reduced and output always above the steady state level.

1.3.4 Comparison with TFP and news shock

1.3.5 Permanent vs temporary shock

In order to provide a deeper understanding of our results, we evaluated the model response to shocks at different levels of persistence (Figure 1.4). We found that our model reacts to a non persistent temporary shock to NEs distribution replicating the effect of a persistent AR(1) shock to TFP in a standard Real Business Cycle. The temporary increase in the number of new entrants, reproduces the typical response of a Real Business Cycle to a standard TFP shock (Figure 1.4). The sudden improvement in A_t, driven by the entry of productive new entrants, increases the remuneration on investment and wages, increasing labour supply and capital goods demand. Although the shock we evaluate is not persistent, the technology spillover from new entrants to incumbents, adds persistence to the technological improvement ($\rho^A = 0.975$). Firms, that are more productive, are able to face the increase in production costs and this leads to the immediate increase in input demand that we cannot find in our model. Impulse response functions in Figure 1.4, highlight that with different levels of shocks persistence, we have different short therm responses of hour worked. Increasing the persistence of the shock to new entrants productivity, the short term reduction in production inputs become more severe.

Permanent shock and news shock

As shown in Figure (1.5), with a permanent shock to the New Entrants distribution, our model leads to a permanent increase in average productivity. The technology improvement starts from new entrants and spreads to Incumbent with a lag, so we can interpret it as a news shock. We compared our model response to a permanent increase in NE's productivity distribution with the response of a RBC to news shock modelled as in Barsky and Sims (2011[6]). The comparison of the variables responses shown in Figure (1.5) confirms our intuition: a permanent technology improvement that spreads from new entrants to incumbents has the same effect of a news shock and our model can provide a micro-foundation of the latter.

1.4 Conclusions

We built an Business Cycle model able to inspect the effect of neutral and investment technology shocks in an economy characterised by endogenous entry and exit. We find that our set-up captures the reduction of the input demand due to technological improvements from the simpler specification. This result is driven by both C-sector and K-sector technology shocks. From second order moments comparison, we can see that C and K technology improvement are respectively responsible for the 43% and of the 57% of capital and labour variance. We also confirmed the "creative destruction"effect of productivity shocks suggested by the Schumpeterian literature, clearly observable in the response of the K-sector to an investment stochastic technology shock. On the other hand, a C-sector productivity shock has an expansionary effect on output and consumption, for each specification from the first quarter of the simulations. This is because the entry of new productive firms compensates immediately the exit of unproductive incumbents and because the relative price of investment goods adjusts mitigating the creative destruction effect. The model is able to capture shock propagation from one sector to another, an improvement in C-sector productivity reduces the entry efficiency level of K-firms, but, after a short-term drop, raises the aggregate K-sector productivity in the long term. Our suggestion is that the lighter entry requirements encourage the entry of innovative new entrants, raising the aggregate productivity of the whole sector. A specular effect can be observed in the response of the C-sector to a K-sector innovation. Finally, we modelled a dynamic able to provide a micro foundation for TFP and news shocks. Our work can be extended to study the reallocation of workers across two different sectors. In our specification, the K-sector firms do not use labour as an input, but the initial decrease in aggregate K-productivity suggests that the introduction of workers mobility may change the impact of the shocks on hours worked, reproducing a workers migration from the innovative sector to the other. Secondly, our entry and exit dynamic can be useful to study the effect of credit policies. Since the entry threshold of C-firms depends on the costs of production factor, the introduction of a financial sector can be an interesting framework to study the effect of interest rate dynamics on aggregate productivity.

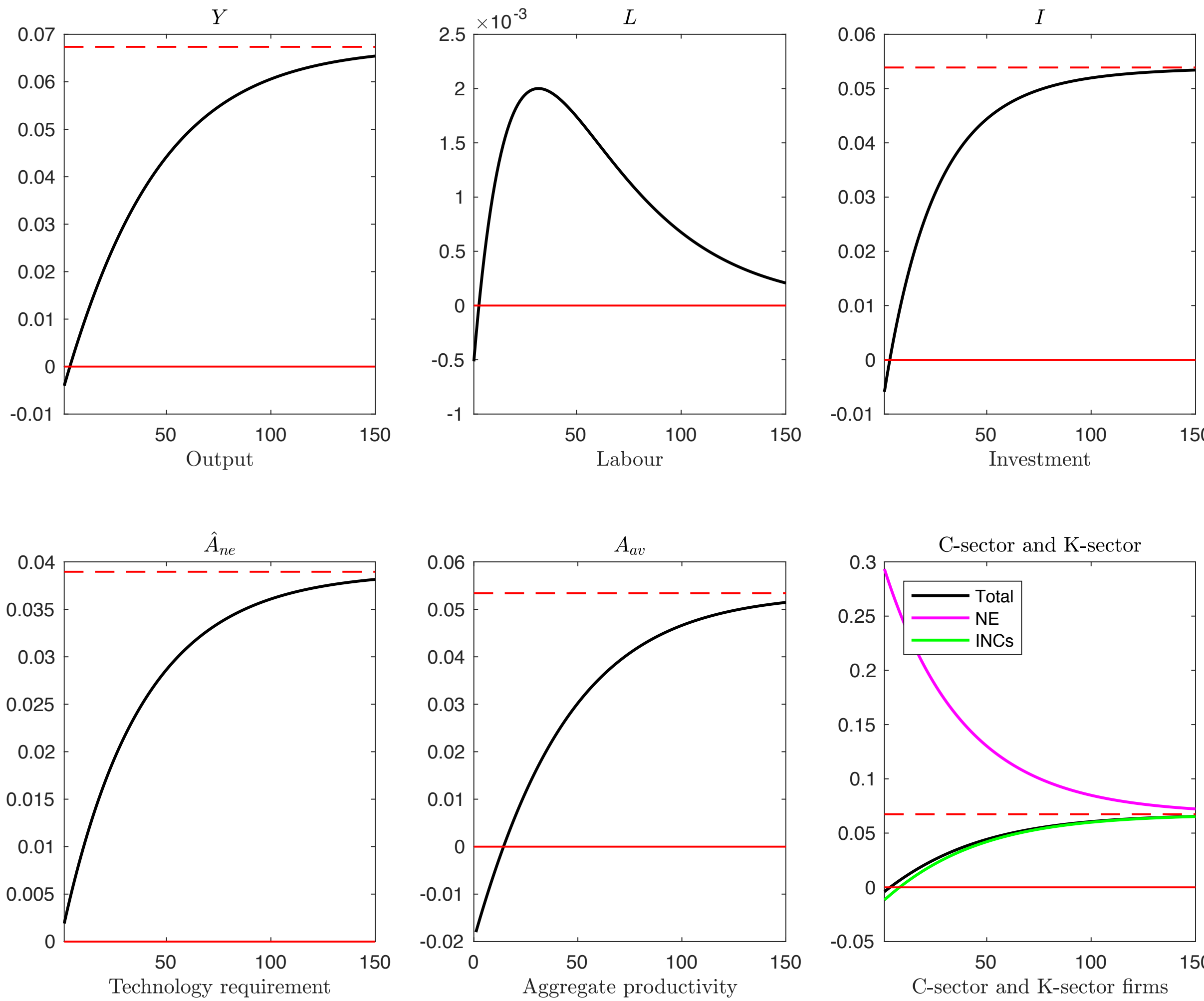

Figure 1.1: IRF of the model with firms dynamic in C to a C-sector productivity shock with $\rho^g = 0$ $\sigma^g = 0.05$.

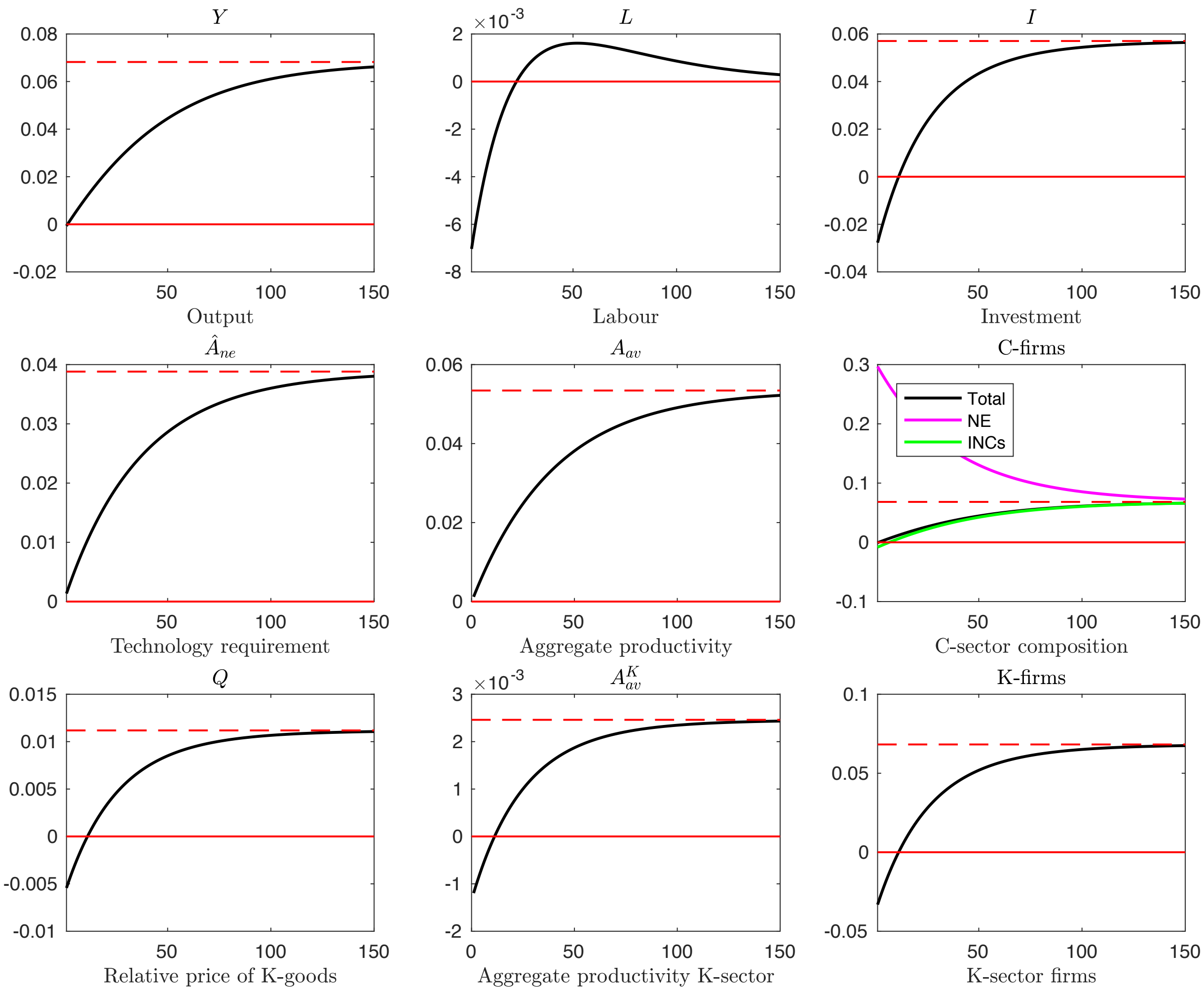

Figure 1.2: IRF of the model with firms dynamic in C and K-sector to a C-sector productivity shock with $\rho^g = 0$ $\sigma^g = 0.05$

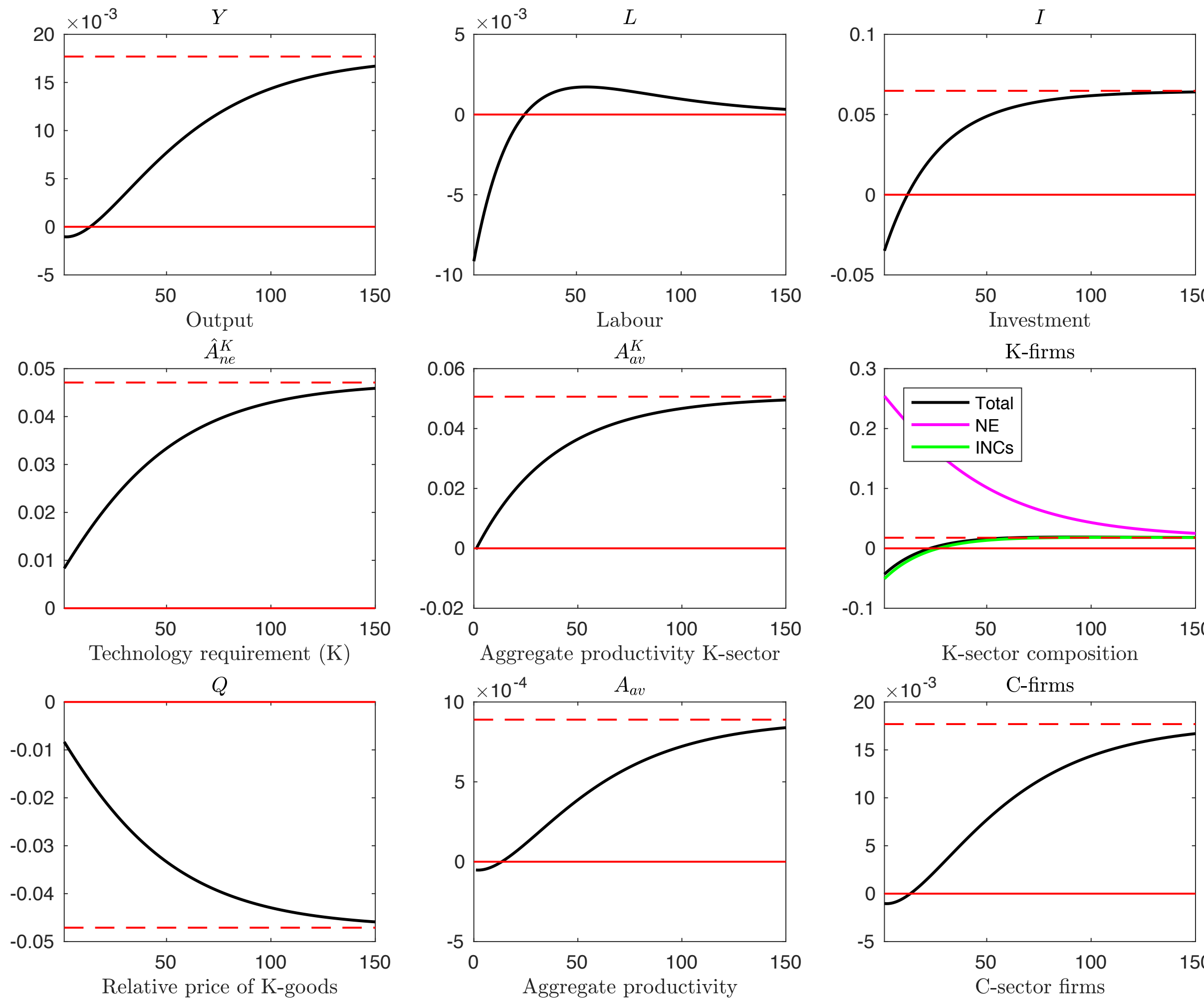

Figure 1.3: IRF of the model with firms dynamic in C and K-sector to a shock in K-sector $\rho^e = 0$ $\sigma^e = 0.05$ (blue line).

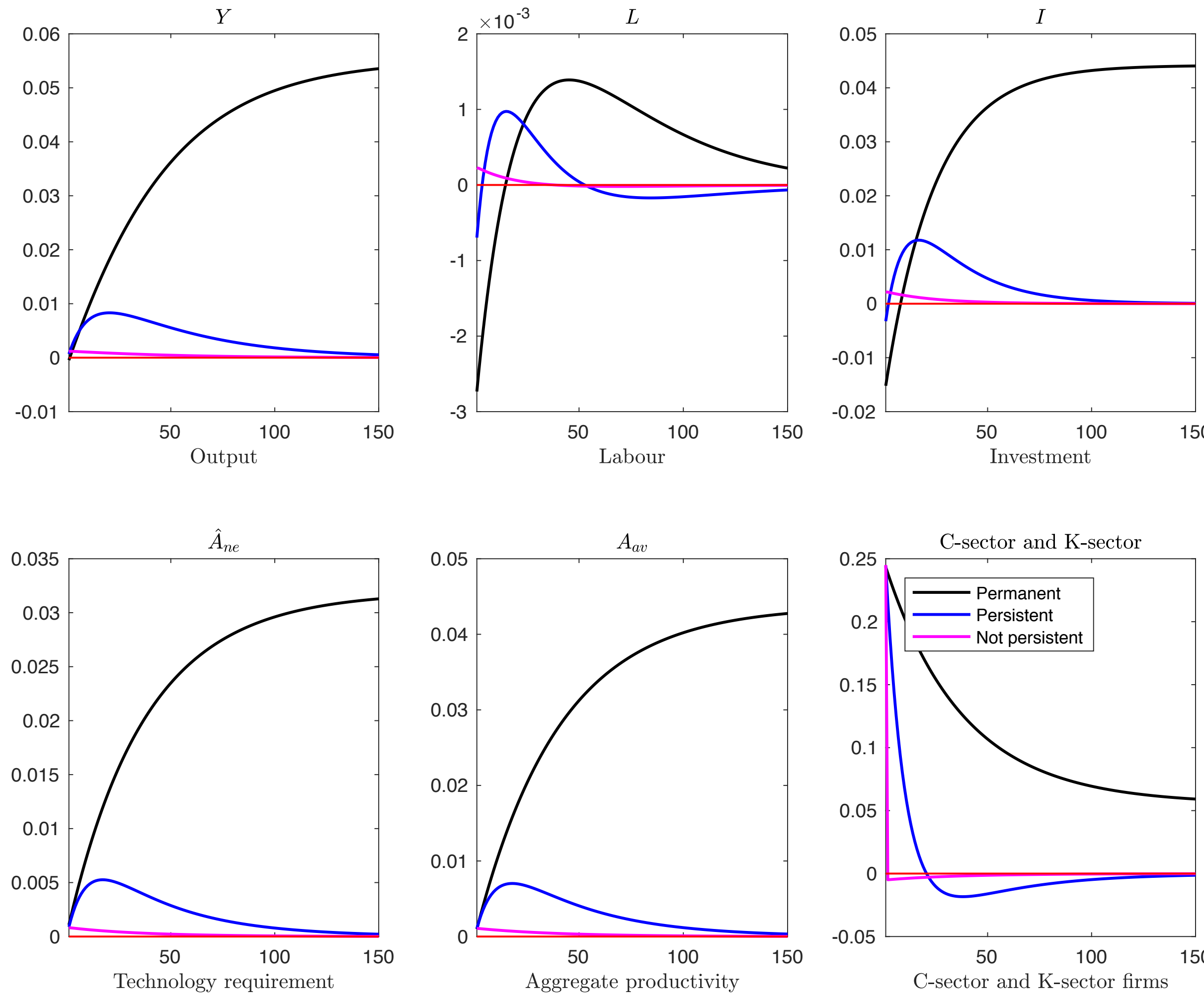

Figure 1.4: IRF of the model with firms dynamic in C-sector to a permanent (in black), a temporary shock with persistence $\rho^g = 0.98$ (green) and to a non persistent temporary shock (blue). $\sigma^g = 0.05$.

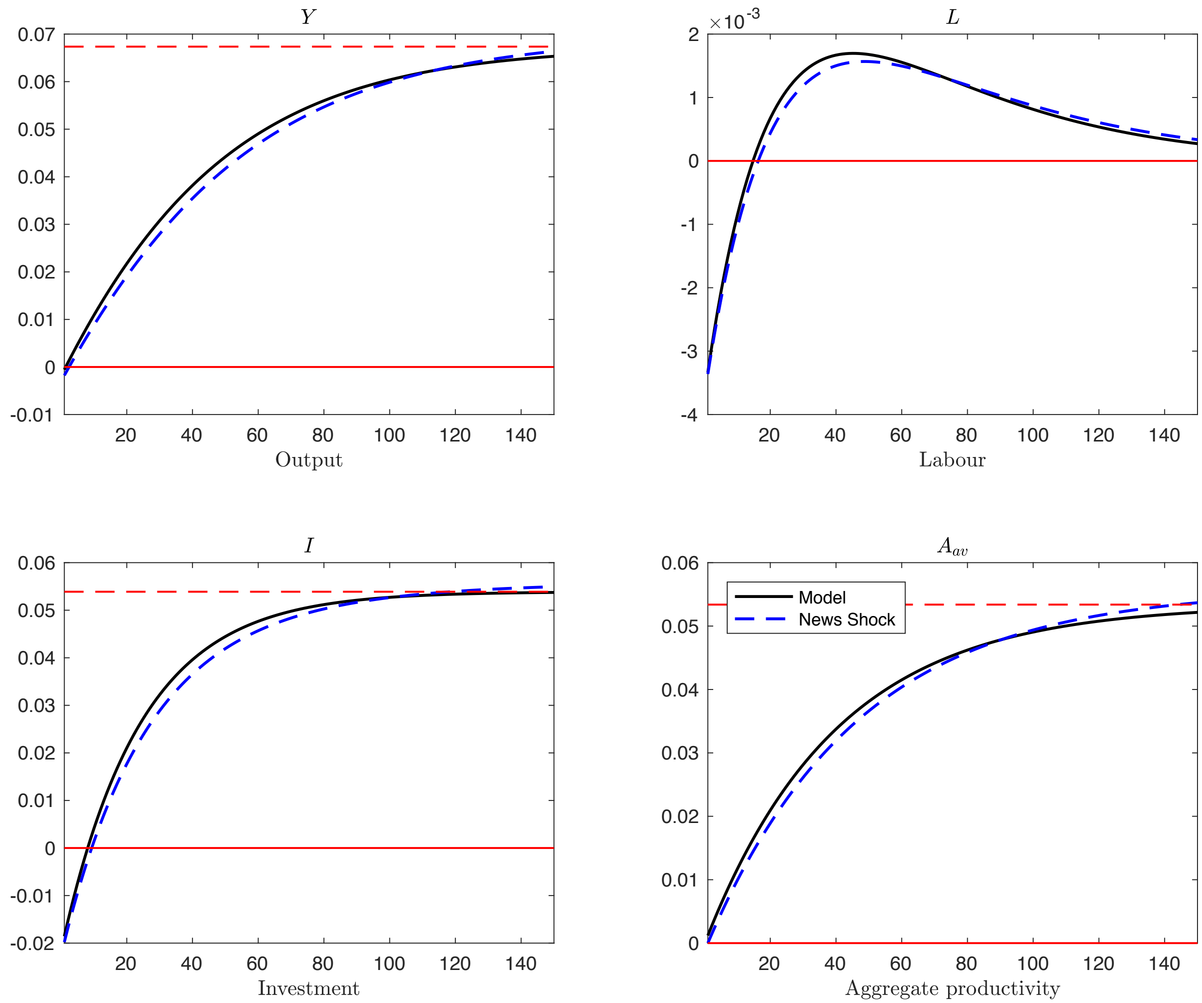

Figure 1.5: IRF of the model with firms dynamic in C-sector to a $\sigma^g = 0.05$ technology shock in C-sector (in black) , compared with a $\sigma^n = 0.001$ news shock in the benchmark RBC (in blue).

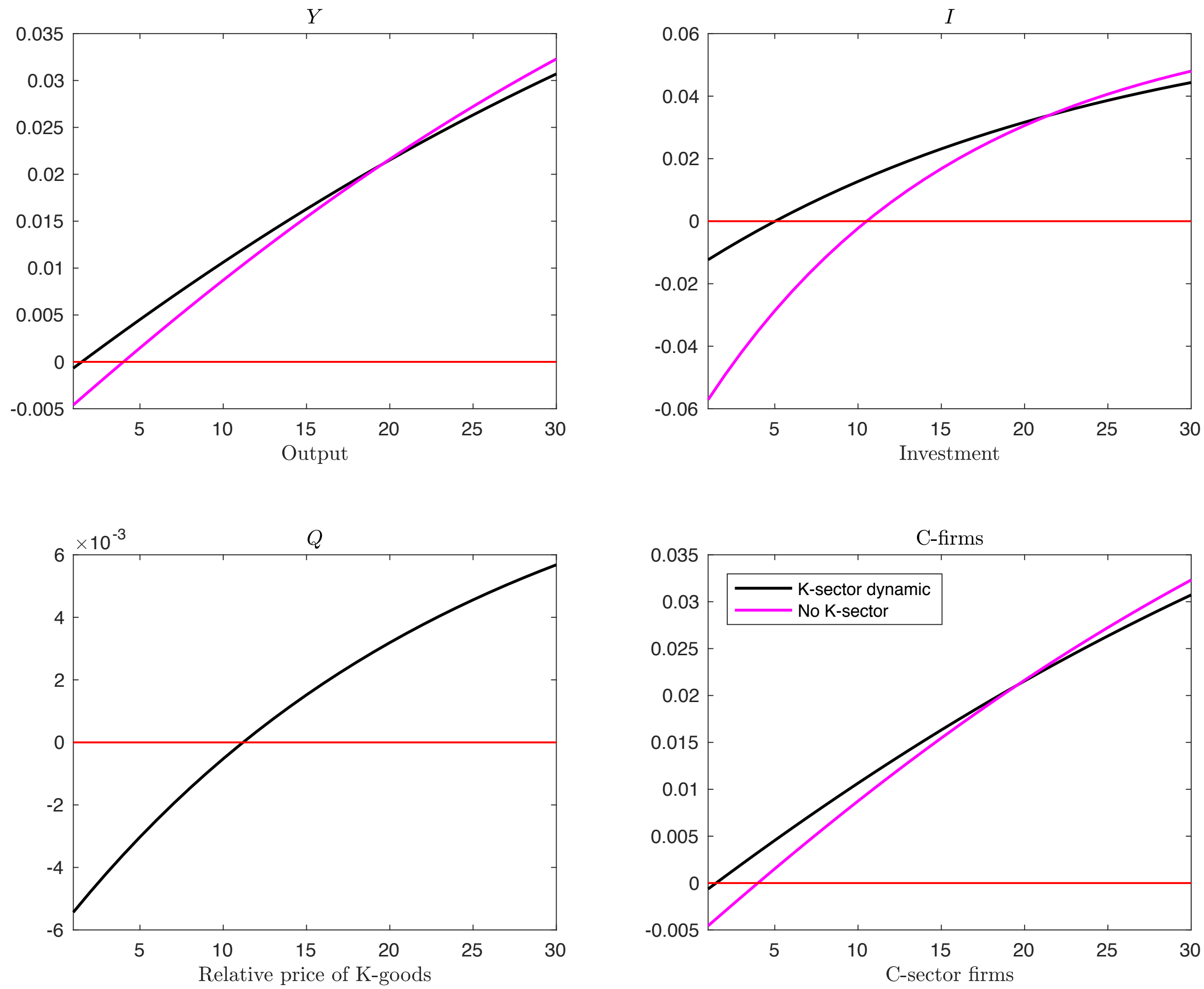

Figure 1.6: IRF of the model with firms dynamic in C-sector (magenta line) and of a no K-sector RBC (dashed line) to a shock in C-sector $\rho^e = 0$ $\sigma^e = 0.05$.

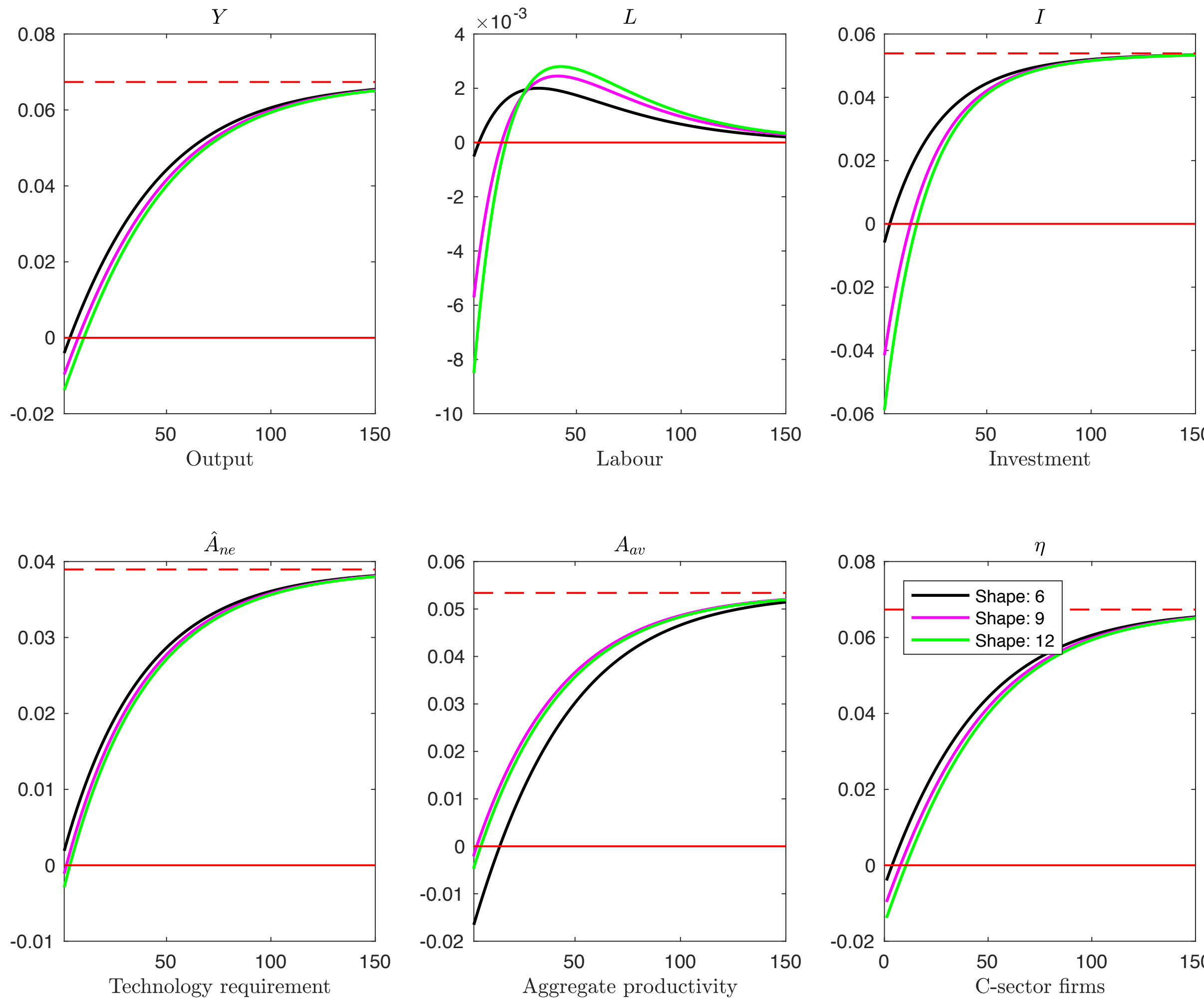

Figure 1.7: IRF of the model with firms dynamic in C-sector to a C-sector productivity shock with $\rho^g = 0$ $\sigma^g = 0.05$, with different shape parameters of Pareto distribution.

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1.5 Appendix to Chapter 1.

Model Dynamic Equations:

Exogenous processes:[3]

Evolution of C-sector technology frontier:

$$\underline{z}_t = g_t^z \, \underline{z}_{t-1} \tag{1.40}$$

$$ln\left(g_t^z\right) = ln\left(g\right) + \varepsilon_t^z \tag{1.41}$$

Evolution of K-sector technology frontier:

$$\underline{e}_t = g_t^e \underline{e}_{t-1} \tag{1.42}$$

$$ln\left(g_t^e\right) = ln\left(g\right) + \varepsilon_t^e \tag{1.43}$$

C-sector fixed costs:[4]

$$\phi_t^{NE} = g_{*C}^t \phi^{NE} \tag{1.44}$$

$$\phi_t^I = g_{*C}^t \phi^I \tag{1.45}$$

K-sector fixed costs:

$$f_t^{NE} = g_{*K}^t f^{NE} \tag{1.46}$$

$$f_t^I = g_{*K}^t f^I \tag{1.47}$$

Households:

$$\lambda_t = \frac{1}{C_t} \tag{1.48}$$

$$W_t = \frac{\psi \, {L_t}^{\phi}}{\lambda_t} \tag{1.49}$$

$$\lambda_t = \beta \, \lambda_{t+1} \left(\frac{r_{t+1}^k}{Q_t} + \frac{Q_{t+1}}{Q_t} \, (1-\delta) \right) \tag{1.50}$$

C-sector:

Productivity thresholds:

$$\hat{A}_t^{NE} = \left(\frac{\phi_t^{NE}}{1-\gamma} \right)^{(1-\gamma)} \frac{\left(\left(\frac{r_t^k}{\alpha} \right)^{\alpha} \left(\frac{W_t}{1-\alpha} \right)^{1-\alpha} \right)^{\gamma}}{\gamma^{\gamma}} \tag{1.51}$$

$$\hat{A}_t^{I} = \left(\frac{\phi_t^{I}}{1-\gamma} \right)^{(1-\gamma)} \frac{\left(\left(\frac{r_t^k}{\alpha} \right)^{\alpha} \left(\frac{W_t}{1-\alpha} \right)^{1-\alpha} \right)^{\gamma}}{\gamma^{\gamma}} \tag{1.52}$$

[3]Where g_{*y_C} and g_{*y_K} ($\neq g$) respect Balance Growth Path conditions.
[4]Notice that $\phi^{NE,I}$ and $f^{NE,I}$ are initial conditions.

Mass of firms:

$$NE_t = \left(\frac{z_t}{\hat{A}_t^{NE}}\right)^{\xi} \tag{1.53}$$

$$INC_t = \eta_{t-1}\left(\frac{\hat{A}_{t-1}^{NE}}{\hat{A}_t^{I}}\right)^{\xi} \tag{1.54}$$

$$\eta_t = NE_t + INC_t \tag{1.55}$$

Output:[5]

$$Y_t^{NE} = NE_t \phi_t^{NE}\, \frac{\xi}{\xi(1-\gamma)-1} \tag{1.56}$$

$$Y_t^{I} = INC_t \phi_t^{I}\, \frac{\xi}{\xi(1-\gamma)-1} \tag{1.57}$$

$$Y_t = Y_t^{NE} + Y_t^{I} \tag{1.58}$$

Production input demand:

$$K_{t-1} = \frac{\alpha\,\gamma\,Y_t}{r_t^k} = \left(\frac{\alpha}{r_t^k}\right)\frac{\gamma\xi}{\xi(1-\gamma)-1}\cdot\left(NE_t\phi_t^{NE} + INC_t\phi_t^{I}\right) \tag{1.59}$$

$$L_t = \frac{(1-\alpha)\,\gamma Y_t}{W_t} = \left(\frac{1-\alpha}{W_t}\right)\frac{\gamma\xi}{\xi(1-\gamma)-1}\cdot\left(NE_t\phi_t^{NE} + INC_t\phi_t^{I}\right) \tag{1.60}$$

K-sector:

Productivity thresholds:

$$\hat{A}_t^{K,NE} = \left(\frac{f_t^{NE}}{1-\chi}\right)^{1-\chi}\frac{1}{Q_t\chi^{\chi}} \tag{1.61}$$

$$\hat{A}_t^{K,I} = \left(\frac{f_t^{I}}{1-\chi}\right)^{1-\chi}\frac{1}{Q_t\chi^{\chi}} \tag{1.62}$$

Mass of firms:

$$NE_t^K = \left(\frac{e_t}{\hat{A}_t^{K,NE}}\right)^{\omega} \tag{1.63}$$

$$INC_t^K = \eta_{t-1}^K\left(\frac{\hat{A}_{t-1}^{K,NE}}{\hat{A}_t^{K,I}}\right)^{\omega} \tag{1.64}$$

$$\eta_t^K = NE_t^K + INC_t^K \tag{1.65}$$

Investment supply:

$$I_t^{K,NE} = NE_t^K\frac{\omega(1-\chi)}{\omega(1-\chi)-1}(\hat{A}_t{}^{K,NE})^{\frac{1}{1-\chi}}\,(Q_t\chi)^{\frac{\chi}{\chi-1}} \tag{1.66}$$

[5] Aggregation in 2.136

$$I_t^{K,I} = INC_t^K \frac{\omega(1-\chi)}{\omega(1-\chi)-1} (\hat{A}_t^{K,I})^{\frac{1}{1-\chi}} (Q_t \chi)^{\frac{\chi}{\chi-1}} \tag{1.67}$$

$$I_t^K = I_t^{K,I} + I_t^{K,NE} \tag{1.68}$$

Savings demand:

$$S_t^{K,I} = \frac{\omega\chi}{\omega(1-\chi)-1} INC_t^K f_t^I \tag{1.69}$$

$$S_t^{K,NE} = \frac{\omega\chi}{\omega(1-\chi)-1} NE_t^K f_t^{NE} \tag{1.70}$$

$$S_t^K = S_t^{K,I} + S_t^{K,NE} \tag{1.71}$$

Market clearing conditions:

$$C_t = Y_t - S_t - INC_t \phi_t^I - NE_t \phi_t^{NE} - f_t^I INC_t^K - f_t^{NE} NE_t^K \tag{1.72}$$

$$K_t = (1-\delta)\ K_{t-1} + I_t^K \tag{1.73}$$

Appendix A. : Model derivation.

A.a. Aggregation

A.a.1. C-sector aggregate demand of inputs:

To obtain the aggregate demand of capital and labour, we start from consumption good producers' first order conditions:

$$k_{j,t}^C = \alpha\gamma \frac{y_{j,t}^C}{r_t^k} \tag{1.74}$$

$$l_{j,t}^C = (1-\alpha)\gamma \frac{y_{j,t}^C}{W_t} \tag{1.75}$$

$$\frac{l_{j,t}^C}{k_{j,t}^C} = \frac{1-\alpha}{\alpha} \frac{r_t^k}{W_t} \tag{1.76}$$

Given the production function, $y_{j,t} = A_{j,t}\left(k_{j,t}^{\alpha} l_{j,t}^{(1-\alpha)}\right)^{\gamma}$, we can rewrite (1.74) as

$$k_{j,t} = \left(\frac{r_t^k}{\alpha\gamma}\right)^{\frac{1}{\alpha\gamma-1}} \cdot l_{j,t}^{\frac{(1-\alpha)\gamma}{1-\alpha\gamma}} \cdot A_{j,t}^{\frac{1}{1-\alpha\gamma}} \rightarrow$$

$$k_{j,t} = \left(\frac{1-\alpha}{\alpha}\frac{r_t^k}{W_t} k_{j,t}\right)^{\frac{(1-\alpha)\gamma}{1-\alpha\gamma}} \cdot \left(\frac{r_t^k}{\alpha\gamma}\right)^{\frac{1}{\alpha\gamma-1}} \cdot A_{j,t}^{\frac{1}{1-\alpha\gamma}} \rightarrow$$

$$k_{j,t} = \left(\frac{1-\alpha}{\alpha}\frac{r_t^k}{W_t}\right)^{\frac{(1-\alpha)\gamma}{1-\gamma}} \cdot \left(\frac{\alpha\gamma}{r_t^k}\right)^{\frac{1}{1-\gamma}} \cdot A_{j,t}^{\frac{1}{1-\gamma}} \tag{1.77}$$

Now we can aggregate the demand of capital fo NEs and INCs, integrating for the idiosyncratic productivity levels. NEs draw their individual productivity every period from the Pareto distribution:

$$f_t(A_t^{NE}) = \int_{\underline{z}_t}^{+\infty} \frac{\xi \underline{z}_t^{\xi}}{(A_t^{NE})^{\xi+1}} d(A_t^{NE}) = 1 \qquad with\ A_t^{NE} \geq \underline{z}_t \tag{1.78}$$

We can write the aggregate demand of capital from NEs:

$$K_t^{NE} = \left(\frac{1-\alpha}{\alpha}\frac{r_t^k}{W_t}\right)^{\frac{(1-\alpha)\gamma}{1-\gamma}} \cdot \left(\frac{\alpha\gamma}{r_t^k}\right)^{\frac{1}{1-\gamma}} \cdot \int_{\hat{A}_t^{NE}}^{+\infty} (A_t^{NE})^{\frac{1}{1-\gamma}} dF(A_{j,t}^{NE}) \rightarrow$$

$$K_t^{NE} = \left(\frac{1-\alpha}{\alpha}\frac{r_t^k}{W_t}\right)^{\frac{(1-\alpha)\gamma}{1-\gamma}} \cdot \left(\frac{\alpha\gamma}{r_t^k}\right)^{\frac{1}{1-\gamma}} \cdot \int_{\hat{A}_t^{NE}}^{+\infty} (A_t^{NE})^{\frac{1}{1-\gamma}} f(A_{j,t}^{NE}) d(A_{j,t}^{NE}) \rightarrow$$

$$K_t^{NE} = \left(\frac{1-\alpha}{\alpha}\frac{r_t^k}{W_t}\right)^{\frac{(1-\alpha)\gamma}{1-\gamma}} \cdot \left(\frac{\alpha\gamma}{r_t^k}\right)^{\frac{1}{1-\gamma}} \cdot \xi \underline{z}_t^{\xi} \int_{\hat{A}_t^{NE}}^{+\infty} (A_t^{NE})^{\frac{1}{1-\gamma}-\xi-1} d(A_{j,t}^{NE}) \rightarrow$$

$$K_t^{NE} = \left(\frac{1-\alpha}{\alpha}\frac{r_t^k}{W_t}\right)^{\frac{(1-\alpha)\gamma}{1-\gamma}} \cdot \left(\frac{\alpha\gamma}{r_t^k}\right)^{\frac{1}{1-\gamma}} \cdot \frac{\xi(1-\gamma)}{\xi(1-\gamma)-1} \left(\frac{\underline{z}_t}{\hat{A}_t^{NE}}\right)^{\xi} \left(\hat{A}_t^{NE}\right)^{\frac{1}{1-\gamma}} \rightarrow$$

$$K_t^{NE} = \left(\frac{1-\alpha}{\alpha}\frac{r_t^k}{W_t}\right)^{\frac{(1-\alpha)\gamma}{1-\gamma}} \cdot \left(\frac{\alpha\gamma}{r_t^k}\right)^{\frac{1}{1-\gamma}} \cdot \frac{\xi}{\xi(1-\gamma)-1} \cdot \left(\frac{\left[\left[\frac{r_t^k}{\alpha}\right]^{\alpha}\left[\frac{W_t}{(1-\alpha)}\right]^{(1-\alpha)}\right]^{\gamma}}{\gamma^{\gamma}}\right)^{\frac{1}{1-\gamma}} NE_t\phi_t^{NE} \rightarrow$$

$$K_t^{NE} = \left(\frac{\alpha}{r_t^k}\right) \frac{\gamma\xi}{\xi(1-\gamma)-1} \cdot NE_t\phi_t^{NE} \tag{1.79}$$

Similarly, INCs draw their individual productivity every period from the Pareto distribution:

$$f_t(A_t^I) = \int_{\hat{A}_{t-1}^{NE}}^{+\infty} \frac{\xi(\hat{A}_{t-1}^{NE})^{\xi}}{(A_t^I)^{\xi+1}} d(A_t^I) \tag{1.80}$$

We can write the aggregate demand of capital from INCs:

$$K_t^I = \left(\frac{1-\alpha}{\alpha}\frac{r_t^k}{W_t}\right)^{\frac{(1-\alpha)\gamma}{1-\gamma}} \cdot \left(\frac{\alpha\gamma}{r_t^k}\right)^{\frac{1}{1-\gamma}} \cdot \int_{\hat{A}_t^I}^{+\infty} (A_t^I)^{\frac{1}{1-\gamma}} dF(A_{j,t}^I) \rightarrow$$

$$K_t^I = \left(\frac{1-\alpha}{\alpha}\frac{r_t^k}{W_t}\right)^{\frac{(1-\alpha)\gamma}{1-\gamma}} \cdot \left(\frac{\alpha\gamma}{r_t^k}\right)^{\frac{1}{1-\gamma}} \cdot \int_{\hat{A}_t^I}^{+\infty} (A_t^I)^{\frac{1}{1-\gamma}} f(A_{j,t}^I) d(A_{j,t}^I) \rightarrow$$

$$K_t^I = \left(\frac{1-\alpha}{\alpha}\frac{r_t^k}{W_t}\right)^{\frac{(1-\alpha)\gamma}{1-\gamma}} \cdot \left(\frac{\alpha\gamma}{r_t^k}\right)^{\frac{1}{1-\gamma}} \cdot \xi\eta_{t-1}(\hat{A}_{t-1}^I)^{\xi}\int_{\hat{A}_t^I}^{+\infty}(A_t^I)^{\frac{1}{1-\gamma}-\xi-1}d(A_{j,t}^I) \rightarrow$$

$$K_t^I = \left(\frac{1-\alpha}{\alpha}\frac{r_t^k}{W_t}\right)^{\frac{(1-\alpha)\gamma}{1-\gamma}} \cdot \left(\frac{\alpha\gamma}{r_t^k}\right)^{\frac{1}{1-\gamma}} \cdot \frac{\xi}{\xi(1-\gamma)-1} \cdot \left(\frac{\left[\left[\frac{r_t^k}{\alpha}\right]^{\alpha}\left[\frac{W_t}{(1-\alpha)}\right]^{(1-\alpha)}\right]^{\gamma}}{\gamma^{\gamma}}\right)^{\frac{1}{1-\gamma}} INC_t\phi_t^I \rightarrow$$

$$K_t^I = \left(\frac{\alpha}{r_t^k}\right)\frac{\gamma\xi}{\xi(1-\gamma)-1} \cdot INC_t\phi_t^I \tag{1.81}$$

The sum of K_{t-1}^{NE} and K_{t-1}^I will give the aggregate demand of capital in $t-1$.

$$K_t = \left(\frac{\alpha}{r_t^k}\right)\frac{\gamma\xi}{\xi(1-\gamma)-1} \cdot \left(NE_t\phi_t^{NE} + INC_t\phi_t^I\right) \tag{1.82}$$

Using (1.82) and (1.76) we can derive the aggregate demand of labour in t.

$$L_t = \left(\frac{1-\alpha}{W_t}\right)\frac{\gamma\xi}{\xi(1-\gamma)-1} \cdot \left(NE_t\phi_t^{NE} + INC_t\phi_t^I\right) \tag{1.83}$$

Since the aggregate demand of capital and labour depend by the costs of production, the technology frontier $\underline{z}_t$ and by the entry thresholds $\hat{A}_t^{NE}$, $\hat{A}_t^I$, we can avoid of keeping track of the evolution of firms' idiosyncratic technology levels.

A.a.2. C-sector aggregate output.

The aggregate output will be the sum of the output of New Entrants and of the output of Incumbents:

$$Y_t = Y_t^{NE} + Y_t^I \tag{1.84}$$

Starting from the production function, we can obtain the aggregate output of New Entrants as:

$$Y_t^{NE} = \int_{\hat{A}_t^{NE}}^{+\infty} A_t^{NE}(Z_t^{NE})^{\gamma}dF(A_{j,t}^{NE}) \rightarrow$$

$$Y_t^{NE} = \int_{\hat{A}_t^{NE}}^{+\infty} A_t^{NE}\left(\frac{A_t^{NE}\gamma}{\left[\frac{r_t^k}{\alpha}\right]^{\alpha}\left[\frac{W_t}{(1-\alpha)}\right]^{(1-\alpha)}}\right)^{\frac{\gamma}{1-\gamma}} dF(A_{j,t}^{NE}) \rightarrow$$

$$Y_t^{NE} = \left(\frac{\gamma}{\left[\frac{r_t^k}{\alpha}\right]^{\alpha}\left[\frac{W_t}{(1-\alpha)}\right]^{(1-\alpha)}}\right)^{\frac{\gamma}{1-\gamma}} \int_{\hat{A}_t^{NE}}^{+\infty}(A_t^{NE})^{\frac{1}{1-\gamma}}f(A_{j,t}^{NE})d(A_{j,t}^{NE}) \rightarrow$$

$$Y_t^{NE} = \xi \underline{z}_t^{\xi} \left(\frac{\gamma}{\left[\frac{r_t^k}{\alpha}\right]^{\alpha} \left[\frac{W_t}{(1-\alpha)}\right]^{(1-\alpha)}} \right)^{\frac{\gamma}{1-\gamma}} \int_{\hat{A}_t^{NE}}^{+\infty} (A_t^{NE})^{\frac{1}{1-\gamma}-\xi-1} d(A_{j,t}^{NE}) \rightarrow$$

$$Y_t^{NE} = NE_t^C \frac{\xi(1-\gamma)}{\xi(1-\gamma)-1} (\hat{A}_t^{NE})^{\frac{1}{1-\gamma}} \left(\frac{\gamma}{\left[\frac{r_t^k}{\alpha}\right]^{\alpha} \left[\frac{W_t}{(1-\alpha)}\right]^{(1-\alpha)}} \right)^{\frac{\gamma}{1-\gamma}}$$

$$Y_t^{NE} = NE_t^C \phi_t^{NE} \frac{\xi}{\xi(1-\gamma)-1} \tag{1.85}$$

Where NE_t^C are defined as $\left(\frac{z_t}{\hat{A}_t^{NE}}\right)^{\xi}$. Similarly, the aggregate output of Incumbents will be:

$$Y_t^I = \int_{\hat{A}_t^I}^{+\infty} A_t^I (Z_t^I)^{\gamma} dF(A_t^I) =$$

$$Y_t^I = \int_{\hat{A}_t^I}^{+\infty} A_t^I \left(\frac{A_t^I \gamma}{\left[\frac{r_t^k}{\alpha}\right]^{\alpha} \left[\frac{W_t}{(1-\alpha)}\right]^{(1-\alpha)}} \right)^{\frac{\gamma}{1-\gamma}} dF(A_{j,t}^I) \rightarrow$$

$$Y_t^I = \left(\frac{\gamma}{\left[\frac{r_t^k}{\alpha}\right]^{\alpha} \left[\frac{W_t}{(1-\alpha)}\right]^{(1-\alpha)}} \right)^{\frac{\gamma}{1-\gamma}} \int_{\hat{A}_t^I}^{+\infty} (A_t^I)^{\frac{1}{1-\gamma}} f(A_{j,t}^I) d(A_{j,t}^I) \rightarrow$$

$$Y_t^I = \xi \eta_{t-1} (\hat{A}_{t-1}^{NE})^{\xi} \left(\frac{\gamma}{\left[\frac{r_t^k}{\alpha}\right]^{\alpha} \left[\frac{W_t}{(1-\alpha)}\right]^{(1-\alpha)}} \right)^{\frac{\gamma}{1-\gamma}} \int_{\hat{A}_t^I}^{+\infty} (A_t^I)^{\frac{1}{1-\gamma}-\xi-1} d(A_{j,t}^I) \rightarrow$$

$$Y_t^I = \xi \eta_{t-1} \left(\frac{\hat{A}_{t-1}^{NE}}{\hat{A}_t^I} \right)^{\xi} \left(\frac{\gamma}{\left[\frac{r_t^k}{\alpha}\right]^{\alpha} \left[\frac{W_t}{(1-\alpha)}\right]^{(1-\alpha)}} \right)^{\frac{\gamma}{1-\gamma}} \int_{\hat{A}_t^I}^{+\infty} (A_t^I)^{\frac{1}{1-\gamma}-\xi-1} d(A_{j,t}^I) \rightarrow$$

$$Y_t^I = INC_t^C \frac{\xi(1-\gamma)}{\xi(1-\gamma)-1} (\hat{A}_t^I)^{\frac{1}{1-\gamma}} \left(\frac{\gamma}{\left[\frac{r_t^k}{\alpha}\right]^{\alpha} \left[\frac{W_t}{(1-\alpha)}\right]^{(1-\alpha)}} \right)^{\frac{\gamma}{1-\gamma}}$$

$$Y_t^I = INC_t^C \phi_t^I \frac{\xi}{\xi(1-\gamma)-1} \tag{1.86}$$

Where INC_t^C are defined as $\eta_{t-1} \left(\frac{\hat{A}_{t-1}^{NE}}{\hat{A}_t^I} \right)^{\xi}$. We can easily write the aggregate output of consumption goods in t,

$$Y_t = \frac{\xi}{\xi(1-\gamma)-1} \left(NE_t^C \phi_t^{NE} + INC_t^C \phi_t^I \right) \tag{1.87}$$

A.a.3. K-sector demand of savings and supply of investment goods.

A generic K-firm k is characterised by the following production function:

$$I_{k,t} = A^K_{k,t}(S^K_{k,t})^{\chi} \tag{1.88}$$

Where the optimal demand of savings $S^K_{k,t}$ is given by

$$S^K_{k,t} = \left(Q_t \chi A^K_{k,t}\right)^{\frac{1}{1-\chi}} \tag{1.89}$$

At the beginning of each period, the potential K-NEs draw their productivity $A^{K,NE}_t$ from the Pareto distribution:

$$f_t(A^{K,NE}_t) = \int_{e_t}^{+\infty} \frac{\omega e^{\omega}_t}{(A^{K,NE}_t)^{\omega+1}} d(A^{K,NE}_t) = 1 \tag{1.90}$$

At the same time, K-INC draw their productivity from a second Pareto distribution:

$$f_t(A^{K,I}_t) = \int_{\hat{A}^{K,NE}_{t-1}}^{+\infty} \frac{\omega(\hat{A}^{K,NE}_{t-1})^{\omega}}{(A^{K,I}_t)^{\omega+1}} d(A^{K,I}_t) = 1 \tag{1.91}$$

The aggregate demand of saving in t for K-NEs will be:

$$S^{K,NE}_t = (Q_t\chi)^{\frac{1}{1-\chi}} \int_{\hat{A}^{K,NE}_t}^{+\infty} (A^{K,NE}_t)^{\frac{1}{1-\chi}} dF(A^{K,NE}_t) \rightarrow$$

$$S^{K,NE}_t = (Q_t\chi)^{\frac{1}{1-\chi}} \omega \underline{e}^{\omega}_t \int_{\hat{A}^{K,NE}_t}^{+\infty} (A^{K,NE}_t)^{\frac{1}{1-\chi}-\omega-1} d(A^{K,NE}_t) \rightarrow$$

$$S^{K,NE}_t = (Q_t\chi)^{\frac{1}{1-\chi}} \frac{\omega(1-\chi)}{\omega(1-\chi)-1} NE^K_t (\hat{A}^{K,NE}_t)^{\frac{1}{1-\chi}}$$

$$S^{K,NE}_t = \frac{\omega\chi}{\omega(1-\chi)-1} NE^K_t f^{NE}_t \tag{1.92}$$

Where NE^K_t are defined as $\left(\frac{e_t}{\hat{A}^{k,NE}_t}\right)^{\omega}$ and $\hat{A}^{K,NE}_t$ is the productivity threshold for the Incumbents. Similarly, for the K-INCs,

$$S^{K,I}_t = (Q_t\chi)^{\frac{1}{1-\chi}} \int_{\hat{A}^{K,I}_t}^{+\infty} (A^{K,I}_t)^{\frac{1}{1-\chi}} dF(A^{K,I}_t) \rightarrow$$

$$S^{K,I}_t = (Q_t\chi)^{\frac{1}{1-\chi}} \omega(\hat{A}^{K,NE}_{t-1})^{\omega} \int_{\hat{A}^{K,I}_t}^{+\infty} (A^{K,I}_t)^{\frac{1}{1-\chi}-\omega-1} d(A^{K,I}_t) \rightarrow$$

$$S^{K,I}_t = (Q_t\chi)^{\frac{1}{1-\chi}} \frac{\omega(1-\chi)}{\omega(1-\chi)-1} INC^K_t (\hat{A}^{K,I}_t)^{\frac{1}{1-\chi}}$$

$$S^{K,I}_t = \frac{\omega\chi}{\omega(1-\chi)-1} INC^K_t f^I_t \tag{1.93}$$

Where INC_t^K are defined as $\eta_{t-1}^K \left(\frac{\hat{A}_t^{k,I}}{\hat{A}_{t-1}^{k,NE}}\right)^\omega$ and $\hat{A}_t^{K,I}$ is the productivity threshold for the Incumbents. The aggregate demand of savings in t will be the sum of (1.92) and (1.93),

$$S_t^K = \frac{\omega\chi}{\omega(1-\chi)-1}\left(NE_t^K f_t^{NE} + INC_t^K f_t^I\right) \tag{1.94}$$

With the same iter, we can derive the aggregate supply of Investment goods,

$$I_t^K = \frac{1}{Q_t}\frac{\omega}{\omega(1-\chi)-1}\left(NE_t^K f_t^{NE} + INC_t^K f_t^I\right) \tag{1.95}$$

A.b. Market Clearing conditions.

The market clearing conditions is given by the following equation.

$$C_t = Y_t - S_t - INC_t\phi_t^I - NE_t\phi_t^{NE} - NE_t^K f_t^{NE} - INC_t^K f_t^I \tag{1.96}$$

In equilibrium, the demand of Investment good has to be satisfied by the production of the K-sector (1.95).

$$I_t^K = K_t - (1-\delta)K_{t-1} \tag{1.97}$$

Where I_t^K is the optimal Households' demand for investment goods.

Appendix B. : Deterministic Steady State.

The modelled economy follows a Balanced Growth Path (BGP), the stationary variables in steady state are L_{ss}, Q_{ss} and r^k_{ss} and the number of firms in both sectors. The other variables grow at the endogenous rate g_{*y}. Further, fixed costs of production in C-sector and K-sector grows respectively at the rates $g^t_{*\phi}$ and g^t_{*f} and the technology frontiers $\underline{z}_t$ and $\underline{e}_t$ grows respectively at the exogenous rates g^t_z and g^t_e. In order to compute the deterministic steady state, we have to identify the relation that binds the different growth rates. To clarify notation, a generic variable $x_{ss,t}$ is identified by the deterministic process $x_{ss}g^t_x$, where x_{ss} is the initial condition that we calibrate.

B.1. Households.

We can start our computation from the Households first order conditions. Since we know that C grows at the same rate of Y, we can show that the Lagrangian multiplier s.s. follows this path:

$$\lambda_{ss,t} = \frac{1}{C_{ss}g^t_{*y}} \tag{1.98}$$

From the Households Euler conditions, we can find the steady state return on capital and the wage:

$$\lambda_{ss,t} = \beta\,\lambda_{ss,t+1}\left(\frac{r^k_{ss}}{Q_{ss}} + (1-\delta)\right) \rightarrow$$

$$r^k_{ss} = \left(\frac{g_{*y}}{\beta} - 1 + \delta\right) Q_{ss} \tag{1.99}$$

$$W_{ss,t} = g^t_{*y}\frac{\psi\, {L_{ss}}^{\phi}}{\lambda_{ss}} \tag{1.100}$$

B.2. C-sector

Once that we have define the costs of production we can compute the C-sector s.s. productivity thresholds:

$$\hat{A}^{NE}_{ss,t} = \left(\frac{\phi^{NE}g^t_{*\phi}}{1-\gamma}\right)^{(1-\gamma)} \frac{\left(\left(\frac{r^k_{ss}}{\alpha}\right)^{\alpha}\left(\frac{W_{ss}g^t_{*y}}{1-\alpha}\right)^{1-\alpha}\right)^{\gamma}}{\gamma^{\gamma}} =$$

$$= (g^t_{*\phi})^{(1-\gamma)}(g^t_{*y})^{(1-\alpha)\gamma}\left(\frac{\phi^{NE}}{1-\gamma}\right)^{(1-\gamma)} \frac{\left(\left(\frac{r^k_{ss}}{\alpha}\right)^{\alpha}\left(\frac{W_{ss}}{1-\alpha}\right)^{1-\alpha}\right)^{\gamma}}{\gamma^{\gamma}} \tag{1.101}$$

$$\hat{A}^{I}_{ss,t} = \left(\frac{\phi^{I}g^t_{*\phi}}{1-\gamma}\right)^{(1-\gamma)} \frac{\left(\left(\frac{r^k_{ss}}{\alpha}\right)^{\alpha}\left(\frac{W_{ss}g^t_{*y}}{1-\alpha}\right)^{1-\alpha}\right)^{\gamma}}{\gamma^{\gamma}} =$$

$$= (g^t_{*\phi})^{(1-\gamma)}(g^t_{*y})^{(1-\alpha)\gamma}\left(\frac{\phi^{I}}{1-\gamma}\right)^{(1-\gamma)} \frac{\left(\left(\frac{r^k_{ss}}{\alpha}\right)^{\alpha}\left(\frac{W_{ss}}{1-\alpha}\right)^{1-\alpha}\right)^{\gamma}}{\gamma^{\gamma}} \tag{1.102}$$

Substituting (2.141) and (2.140) in the NEs and INCs equations:

$$NE_{ss} = \left(\frac{z_{ss}g_z^t}{\hat{A}_{ss,t}^{NE}}\right)^{\xi} = \left(\frac{z_{ss}g_z^t}{(g_{*\phi}^t)^{(1-\gamma)}(g_{*y}^t)^{(1-\alpha)\gamma}\hat{A}_{ss}^{NE}}\right)^{\xi} \tag{1.103}$$

$$INC_{ss} = \eta_{ss}\left(\frac{\hat{A}_{ss,t-1}^{NE}}{(\hat{A}_{ss}^{I})^t}\right)^{\xi} = \eta_{ss}\left(\frac{\hat{A}_{ss}^{NE}(g_{*\phi}^{t-1})^{(1-\gamma)}(g_{*y}^{t-1})^{(1-\alpha)\gamma}}{\hat{A}_{ss}^{I}(g_{*\phi}^t)^{(1-\gamma)}(g_{*y}^t)^{(1-\alpha)\gamma}}\right)^{\xi} = \eta_{ss}\left(\frac{\hat{A}_{ss}^{NE}}{\hat{A}_{ss}^{I}(g_{*\phi})^{(1-\gamma)}(g_{*y})^{(1-\alpha)\gamma}}\right)^{\xi} \tag{1.104}$$

Both Incumbents and New Entrants are stationary. Their sum will give us the stationary s.s. number of firms η_{ss}:

$$\eta_{ss} = NE_{ss} + INC_{ss} \tag{1.105}$$

$$\eta_{ss} = \left(\frac{z_{ss}g_z^t}{(g_{*\phi}^t)^{(1-\gamma)}(g_{*y}^t)^{(1-\alpha)\gamma}\hat{A}_{ss}^{NE}}\right)^{\xi} + \eta_{ss}\left(\frac{\hat{A}_{ss}^{NE}(g_{*\phi}^{t-1})^{(1-\gamma)}(g_{*y}^{t-1})^{(1-\alpha)\gamma}}{\hat{A}_{ss}^{I}(g_{*\phi}^t)^{(1-\gamma)}(g_{*y}^t)^{(1-\alpha)\gamma}}\right)^{\xi} \rightarrow$$

$$\eta_{ss} = \frac{\left(\frac{z_{ss}g_z^t}{(g_{*\phi}^t)^{(1-\gamma)}(g_{*y}^t)^{(1-\alpha)\gamma}\hat{A}_{ss}^{NE}}\right)^{\xi}}{\left[1 - \left(\frac{\hat{A}_{ss}^{NE}}{\hat{A}_{ss}^{I}(g_{*\phi})^{(1-\gamma)}(g_{*y})^{(1-\alpha)\gamma}}\right)^{\xi}\right]} \tag{1.106}$$

Where we calibrate $\left[1 - \left(\frac{\hat{A}_{ss}^{NE}}{\hat{A}_{ss}^{I}(g_{*\phi})^{(1-\gamma)}(g_{*y})^{(1-\alpha)\gamma}}\right)^{\xi}\right] > 1$. Further, for the number of firms to be constant, it must hold that:

$$g_{*y}^t = \left[\frac{g_z^t}{(g_{*\phi}^t)^{(1-\gamma)}}\right]^{\frac{1}{(1-\alpha)\gamma}} \tag{1.107}$$

We can now use the conditions above and aggregate output to show that the fixed costs grow at the same rate of aggregate output:

$$Y_{ss,t} = \frac{\xi(1-\gamma)}{\xi(1-\gamma)-1}\left(NE_{ss}\phi^{NE}g_{*\phi}^t + INC_{ss}\phi^{I}g_{*\phi}^t\right) \rightarrow$$

$$Y_{ss}g_{*y}^t = \frac{\xi(1-\gamma)}{\xi(1-\gamma)-1}g_{*\phi}^t\left(NE_{ss}\phi^{NE} + INC_{ss}\phi^{I}\right) \tag{1.108}$$

For (1.108) to hold, requires that we calibrate $g_{*\phi}$ such that:

$$g_{*y}^t = g_{*\phi}^t \tag{1.109}$$

From the aggregate demand of capital and labour, we can derive the last two conditions for the C-sector[6].

$$K_{ss,t} = (g_z^t)^{\frac{1}{(1-\alpha)\gamma+(1-\gamma)}}\left(\frac{\alpha}{r_{ss}^k}\right)\frac{\gamma\xi}{\xi(1-\gamma)-1}\left(NE_{ss}\phi^{NE} + INC_{ss}\phi^{I}\right) \tag{1.110}$$

[6]From condition (2.150) we replaced $g_{*\phi}^t$ with $(g_z^t)^{\frac{1}{(1-\alpha)\gamma+(1-\gamma)}}$.

$$L_{ss} = \left(\frac{1-\alpha}{W_{ss}}\right)\frac{\gamma\xi}{\xi(1-\gamma)-1}\left(NE_{ss}\phi_{ss}^{NE} + INC_{ss}\phi_{ss}^{I}\right) \tag{1.111}$$

Where the steady demand for labour (1.111) is stationary.

B.3. K-sector:

From the capital law of motion we know that investment demand grows at the same rate of capital g_{*y}:

$$K_{ss,t} = I_{ss,t}^{K} + (1-\delta)K_{ss,t-1} \rightarrow$$

$$K_{ss}g_{*y}^{t} = I_{ss,t}^{K} + (1-\delta)K_{ss}\frac{g_{*y}^{t-1}}{g_{*y}^{t}}g_{*y}^{t} \rightarrow$$

$$I_{ss,t}^{K} = (g_z^t)^{\frac{1}{(1-\alpha)\gamma+(1-\gamma)}}\left(1 - \frac{(1-\delta)}{(g_z)^{\frac{1}{(1-\alpha)\gamma+(1-\gamma)}}}\right)K_{ss} \tag{1.112}$$

Additionally, from the investment supply, we can show that the K-sector fixed cost grows at the BGP rate g_{*y}:

$$I_{ss,t}^{K} = \frac{g_{*f}^{t}}{Q_{ss}}\frac{\omega}{\omega(1-\chi)-1}\left(NE_{ss}^{K}f^{NE} + INC_{ss}^{K}f^{I}\right) \rightarrow \tag{1.113}$$

To insure that (1.113) holds we calibrate g_{*f} such that:

$$g_{*y}^{t} = g_{*f}^{t} \tag{1.114}$$

In K-sector the entry thresholds are defined as:

$$\hat{A}_{ss,t}^{K,NE} = \left(\frac{f^{NE}g_{*f}^{t}}{1-\chi}\right)^{1-\chi}\frac{1}{Q_{ss}\chi^{\chi}} = ((g_z^t)^{\frac{1}{(1-\alpha)\gamma+(1-\gamma)}})^{1-\chi}\left(\frac{f^{NE}}{1-\chi}\right)^{1-\chi}\frac{1}{Q_{ss}\chi^{\chi}} \tag{1.115}$$

$$\hat{A}_{ss,t}^{K,I} = \left(\frac{f^{I}g_{*f}^{t}}{1-\chi}\right)^{1-\chi}\frac{1}{Q_{ss}\chi^{\chi}} = ((g_z^t)^{\frac{1}{(1-\alpha)\gamma+(1-\gamma)}})^{1-\chi}\left(\frac{f^{I}}{1-\chi}\right)^{1-\chi}\frac{1}{Q_{ss}\chi^{\chi}} \tag{1.116}$$

Substituting (1.115) and (1.116) in K-sector NEs and INCs equations:

$$NE_{ss}^{K} = \left(\frac{e_{ss}g_e^t}{\hat{A}_{ss,t}^{K,NE}}\right)^{\omega} = \left(\frac{e_{ss}g_e^t}{\hat{A}_{ss}^{K,NE}(g_z^t)^{\frac{1-\chi}{(1-\alpha)\gamma+(1-\gamma)}}}\right)^{\omega} \tag{1.117}$$

$$INC_{ss}^{K} = \eta_{ss}^{K}\left(\frac{\hat{A}_{ss,t-1}^{K,NE}}{\hat{A}_{ss,t}^{K,NE}}\right)^{\omega} = \eta_{ss}^{K}\left(\frac{\hat{A}_{ss}^{K,NE}}{\hat{A}_{ss}^{K,I}g_z^{\frac{1-\chi}{(1-\alpha)\gamma+(1-\gamma)}}}\right)^{\omega} \tag{1.118}$$

Both Incumbents and New Entrants are stationary. Their sum will give us the stationary s.s. number of firms η_{ss}^{K}:

$$\eta_{ss} = NE_{ss} + INC_{ss} \tag{1.119}$$

$$\eta_{ss}^{K} = \left(\frac{\underline{e}_{ss} g_e^t}{(g_z^t)^{\frac{1-\chi}{(1-\alpha)\gamma+(1-\gamma)}} \hat{A}_{ss}^{K,NE}} \right)^{\omega} + \eta_{ss}^{K} \left(\frac{\hat{A}_{ss}^{K,NE}}{\hat{A}_{ss}^{K,I} g_z^{\frac{1-\chi}{(1-\alpha)\gamma+(1-\gamma)}}} \right)^{\omega} \rightarrow$$

$$\eta_{ss}^{K} = \frac{\left(\frac{\underline{e}_{ss} g_e^t}{(g_z^t)^{\frac{1-\chi}{(1-\alpha)\gamma+(1-\gamma)}} \hat{A}_{ss}^{K,NE}} \right)^{\omega}}{\left[1 - \left(\frac{\hat{A}_{ss}^{K,NE}}{\hat{A}_{ss}^{K,I} (g_z)^{\frac{1-\chi}{(1-\alpha)\gamma+(1-\gamma)}}} \right)^{\omega} \right]}$$

Where, accordingly to calibration, $\left[1 - \left(\frac{\hat{A}_{ss}^{K,NE}}{\hat{A}_{ss}^{K,I} g_{*f}^{1-\chi}} \right)^{\omega} \right] > 1$ and does not depend on time. Since the number of firms is constant, we calibrate:

$$g_e^t = (g_z^t)^{\frac{(1-\chi)}{(1-\alpha)\gamma+(1-\gamma)}} \tag{1.120}$$

Savings demand will follow the same trend of investment,

$$S_{ss,t}^{K} = (g_z^t)^{\frac{1}{(1-\alpha)\gamma+(1-\gamma)}} \frac{\omega\chi}{\omega(1-\chi)-1} \left(NE_{ss}^{K} f^{NE} + INC_{ss}^{K} f^{I} \right) \tag{1.121}$$

B.4. Market clearing:

We can conclude the deterministic steady state computation showing through the market clearing that output, consumption, savings and fixed costs must share the same trend consistently with our computation.

$$C_{ss} g_{*y}^t = Y_{ss} g_{*y}^t - S_{ss} g_{*y}^t - INC_{ss} \phi^I g_{*y}^t - NE_{ss} \phi^{NE} g_{*y}^t - INC_{ss}^{K} f^I g_{*y}^t - NE_{ss}^{K} f^{NE} g_{*y}^t \tag{1.122}$$

The stationary relative price of investment to insure that the investment demand (2.185) equals the investment supply (1.113) is:

$$Q_{ss} = \frac{\omega}{\omega(1-\chi)-1} \frac{\left(NE_{ss}^{K} f^{NE} + INC_{ss}^{K} f^{I} \right)}{\left(1 - \frac{(1-\delta)}{(g_z)^{\frac{1}{(1-\alpha)\gamma+(1-\gamma)}}} \right) K_{ss}} \tag{1.123}$$

Appendix C: Steady State Initial Conditions

We calibrate the initial condition of NEs fixed cost ϕ^{NE} such that it is equal to the 5% of the total output ex post (BGM 2012; Etro and Colciago 2010; Colciago and Rossi 2012). Additionally, in both sectors, we calibrate the Incumbents' fixed cost as a function of the entry cost to set the share of New Entrants in the economy at $H = 10\%$[7](Etro and Colciago, 2010). We calibrated the technology frontiers' initial values $\underline{z}_{ss}$ and $\underline{e}_{ss}$ to obtain an unitary mass of firms. Finally, the steady state value of the relative price of investment $Q_{ss} = 1$ and $L_{ss} = 0.33$ are pinned down respectively by the initial value of K-sector entry cost f^{NE} and by preference parameter ψ.

The market composition of the C-sector is described as:

$$\eta_{ss} = NE_{ss} + INC_{ss} \tag{1.124}$$

$$INC_{ss} = \left(\frac{\hat{A}^{NE}_{ss}}{\hat{A}^{I}_{ss} g_z}\right)^{\xi} \eta_{ss} = (1-H)\eta_{ss} \tag{1.125}$$

$$NE_{ss} = \left(\frac{\underline{z}_{ss}}{\hat{A}^{NE}_{ss}}\right)^{\xi} = H\eta_{ss} \tag{1.126}$$

where the steady state values of the technology thresholds are defined as:

$$\hat{A}^{NE,I}_{ss,t} = g_z^t \left(\frac{\phi^{NE,I}}{1-\gamma}\right)^{(1-\gamma)} \frac{\left(\left(\frac{r^k_{ss}}{\alpha}\right)^{\alpha}\left(\frac{W_{ss}}{1-\alpha}\right)^{1-\alpha}\right)^{\gamma}}{\gamma^{\gamma}} \tag{1.127}$$

To respect condition (1.124)-(1.126), we have to calibrate di initial value of ϕ^I according to the following condition:

$$\frac{INC_{ss}}{\eta_{ss}} = (1-H) \rightarrow \left[\left(\frac{\phi^{NE}}{\phi^{I}}\right)^{(1-\gamma)} \frac{1}{g_z}\right]^{\xi} = 1 - H \rightarrow \frac{\phi^{I}}{\phi^{NE}} = \left[g_z (1-H)^{\frac{1}{\xi}}\right]^{\frac{1}{(\gamma-1)}} \tag{1.128}$$

To respect the calibration of $\eta_{ss} = 1$, we pinned down the value of the technology frontier that allows us to respect the following condition:

$$\eta_{ss} = \underline{z}^{\xi}_{ss} \frac{\left[\left(\frac{\phi^{NE}}{1-\gamma}\right)^{(1-\gamma)} \frac{\left(\left(\frac{r^k_{ss}}{\alpha}\right)^{\alpha}\left(\frac{W_{ss}}{1-\alpha}\right)^{1-\alpha}\right)^{\gamma}}{\gamma^{\gamma}}\right]^{-\xi}}{\left[1-\left[\left(\frac{\phi^{NE}}{\phi^{I}}\right)^{(1-\gamma)} \frac{1}{g_z}\right]^{\xi}\right]} = 1 \rightarrow$$

$$\underline{z}_{ss} = H^{\frac{1}{\xi}} \left[\left(\frac{\phi^{NE}}{1-\gamma}\right)^{(1-\gamma)} \frac{\left(\left(\frac{r^k_{ss}}{\alpha}\right)^{\alpha}\left(\frac{W_{ss}}{1-\alpha}\right)^{1-\alpha}\right)^{\gamma}}{\gamma^{\gamma}}\right]$$

Using the steady state values of wage and rental rate of capital,

$$W_{ss} = \left(\frac{1-\alpha}{L_{ss}}\right) \frac{\gamma\xi}{\xi(1-\gamma)-1} \cdot \left(NE_{ss}\phi^{NE} + INC_{ss}\phi^{I}\right) \tag{1.129}$$

[7] Since we calibrate on a quarterly base $H = 2.5\%$

$$r^k_{ss} = \left[\frac{g_{*y}}{\beta} + (1-\delta)\right] Q_{ss} \tag{1.130}$$

we can compute the initial value of the C-sector technology frontier:

$$\underline{z}_{ss} = \frac{(\phi^{NE})^{(1-\gamma)+(1-\alpha)\gamma}}{(1-\gamma)^{(1-\gamma)}\gamma^{\gamma}} H^{\frac{1}{\xi}} \left(\frac{\frac{\gamma\xi}{\xi(1-\gamma)-1} \cdot \left(H + (1-H)\left[g_z(1-H)^{\frac{1}{\xi}}\right]^{\frac{1}{(\gamma-1)}} \right)}{\left(\frac{\alpha}{r^k_{ss}}\right)^{\frac{\alpha}{1-\alpha}} L_{ss}} \right)^{(1-\alpha)\gamma}$$

The market composition of the K-sector is described as:

$$\eta^K_{ss} = NE^K_{ss} + INC^K_{ss} \tag{1.131}$$

$$INC^K_{ss} = \left(\frac{\hat{A}^{KNE}_{ss}}{\hat{A}^I_{ss} g_e}\right)^{\omega} \eta^K_{ss} = (1-H)\eta_{ss} \tag{1.132}$$

$$NE^K_{ss} = \left(\frac{\underline{e}_{ss}}{\hat{A}^{K,NE}_{ss}}\right)^{\omega} = H\eta^K_{ss} \tag{1.133}$$

where the steady state values of the technology thresholds are defined as:

$$\hat{A}^{K,NE\ K,I}_{ss,t} = g^t_e \left(\frac{f^{NE,I}}{1-\chi}\right)^{1-\chi} \frac{1}{Q_{ss}\chi^{\chi}} \tag{1.134}$$

To respect the share of NEs in the K-sector (0.0025 as in the C-sector) we calibrate the K-sector INCs fixed cost as we did for the C-sector.

$$\frac{INC^K_{ss}}{\eta^K_{ss}} = \left(\frac{\hat{A}^{K,NE}_{ss}}{\hat{A}^{K,I}_{ss} g_e}\right)^{\omega} \rightarrow \left[\left(\frac{f^{NE}_{ss}}{f^I_{ss}}\right)^{(1-\chi)} \frac{1}{g_e}\right]^{\omega} = 1 - H \rightarrow \frac{f^I_{ss}}{f^{NE}_{ss}} = \left[\frac{1}{g_e * (1-H)^{\frac{1}{\omega}}}\right]^{\frac{1}{1-\chi}} \tag{1.135}$$

As for the C-sector, the steady state number of firms in the K-sector is $\eta^K_{ss} = 1$.

$$\eta^K_{ss} = \frac{\underline{e}^{\omega}_{ss}\left[\left(\frac{f^{NE}}{1-\chi}\right)^{1-\chi} \frac{1}{Q_{ss}\chi^{\chi}}\right]^{-\omega}}{1 - \left[\frac{g_e}{(1-H)^{\frac{1}{\omega}}}\right]^{\frac{1}{1-\chi}}} = 1 \rightarrow$$

$$\underline{e}_{ss} = \left(\frac{f^{NE}}{1-\chi}\right)^{1-\chi} \frac{H^{\frac{1}{\omega}}}{Q_{ss}\chi^{\chi}}$$

In equilibrium, the demand of investment (1.112) has to be satisfied by the investment good supply (1.113). We pin down the initial value of the entry costs that allow us to respect this condition at the calibrated investment goods' relative price $Q_{ss} = 1$,

$$\left(1 - \frac{(1-\delta)}{g_{*y}}\right) K_{ss} = \frac{1}{Q_{ss}} \frac{\omega}{\omega(1-\chi)-1} \left(NE^K_{ss} f^{NE} + INC^K_{ss} f^I\right) \rightarrow$$

$$f^{NE} = \frac{\left(1 - \frac{(1-\delta)}{g_{*y}}\right) K_{ss}}{\frac{\omega}{\omega(1-\chi)-1}\left(1 + \frac{(1-H)}{H}\left[\frac{1}{g_e(1-H)^{\frac{1}{\omega}}}\right]^{\frac{1}{1-\chi}}\right)}$$

We are now able to compute the initial values of the remaining variables as output, savings and consumption,

$$Y_{ss} = \frac{\xi}{\xi(1-\gamma)-1}\left(NE_{ss}\phi^{NE} + INC_{ss}\phi^{I}\right) \tag{1.136}$$

$$S^K_{ss} = \frac{\omega\chi}{\omega(1-\chi)-1}\left(NE^K_{ss}f^{NE} + INC^K_{ss}f^{I}\right) \tag{1.137}$$

$$C_{ss} = Y_{ss} - S_{ss} - INC_{ss}\phi^I - NE_{ss}\phi^{NE} - INC^K_{ss}f^I - NE^K_{ss}f^{NE} \tag{1.138}$$

To conclude we calibrate the preference parameter ψ to respect the calibration $L_{ss} = 0.33$

$$\psi = \frac{W_{ss}}{L^{\phi}_{ss}C_{ss}} \tag{1.139}$$

Below, the complete set of the initial conditions.

Steady steate initial conditions:

Households:

$$\lambda_{ss} = \frac{1}{C_{ss}} \tag{1.140}$$

$$W_{ss} = \frac{\psi\, {L_{ss}}^{\phi}}{\lambda_{ss}} \tag{1.141}$$

$$r^k_{ss} = \left[\frac{g_{*y}}{\beta} + (1-\delta)\right] Q_{ss} \tag{1.142}$$

C-sector:

Productivity thresholds:

$$\hat{A}^{NE}_{ss} = \left(\frac{\phi^{NE}}{1-\gamma}\right)^{(1-\gamma)} \frac{\left(\left(\frac{r^k_{ss}}{\alpha}\right)^{\alpha}\left(\frac{W_{ss}}{1-\alpha}\right)^{1-\alpha}\right)^{\gamma}}{\gamma^{\gamma}} \tag{1.143}$$

$$\hat{A}^{I}_{ss} = \left(\frac{\phi^{I}}{1-\gamma}\right)^{(1-\gamma)} \frac{\left(\left(\frac{r^k_{ss}}{\alpha}\right)^{\alpha}\left(\frac{W_{ss}}{1-\alpha}\right)^{1-\alpha}\right)^{\gamma}}{\gamma^{\gamma}} \tag{1.144}$$

Mass of firms:

$$NE_{ss} = \left(\frac{z_{ss}}{\hat{A}_{ss}^{NE}}\right)^{\xi} \tag{1.145}$$

$$INC_{ss} = \eta_{ss} \left(\frac{\hat{A}_{ss}^{NE}}{\hat{A}_{ss}^{I} g_{*y}}\right)^{\xi} \tag{1.146}$$

$$\eta_{ss} = NE_{ss} + INC_{ss} \tag{1.147}$$

Output:

$$Y_{ss}^{NE} = NE_{ss}\phi^{NE} \frac{\xi}{\xi(1-\gamma)-1} \tag{1.148}$$

$$Y_{ss}^{I} = INC_{ss}\phi^{I} \frac{\xi}{\xi(1-\gamma)-1} \tag{1.149}$$

$$Y_{ss} = Y_{ss}^{NE} + Y_{ss}^{I} \tag{1.150}$$

Production input demand:

$$K_{ss} = \frac{\alpha\,\gamma\,Y_{ss}}{r_{ss}^{k}} = \left(\frac{\alpha}{r_{ss}^{k}}\right) \frac{\gamma\xi}{\xi(1-\gamma)-1} \cdot \left(NE_{ss}\phi^{NE} + INC_{ss}\phi^{I}\right) \tag{1.151}$$

$$L_{ss} = \frac{(1-\alpha)\,\gamma Y_{ss}}{W_{ss}} = \left(\frac{1-\alpha}{W_{ss}}\right) \frac{\gamma\xi}{\xi(1-\gamma)-1} \cdot \left(NE_{ss}\phi^{NE} + INC_{ss}\phi^{I}\right) \tag{1.152}$$

K-sector:

Productivity thresholds:

$$\hat{A}_{ss}^{K,NE} = \left(\frac{f^{NE}}{1-\chi}\right)^{1-\chi} \frac{1}{Q_{ss}\chi^{\chi}} \tag{1.153}$$

$$\hat{A}_{ss}^{K,I} = \left(\frac{f^{I}}{1-\chi}\right)^{1-\chi} \frac{1}{Q_{ss}\chi^{\chi}} \tag{1.154}$$

Mass of firms:

$$NE_{ss}^{K} = \left(\frac{e_{ss}}{\hat{A}_{ss}^{K,NE}}\right)^{\omega} \tag{1.155}$$

$$INC_{ss}^{K} = \eta_{ss}^{K} \left(\frac{\hat{A}_{ss}^{K,NE}}{\hat{A}_{ss}^{K,I} g_{*y}}\right)^{\omega} \tag{1.156}$$

$$\eta_{ss}^{K} = NE_{ss}^{K} + INC_{ss}^{K} \tag{1.157}$$

Investment supply:

$$I_{ss}^{K,NE} = NE_{ss}^{K}\frac{\omega(1-\chi)}{\omega(1-\chi)-1}(\hat{A}_{ss}^{K,NE})^{\frac{1}{1-\chi}}(Q_{ss}\chi)^{\frac{\chi}{\chi-1}} \tag{1.158}$$

$$I_{ss}^{K,I} = INC_{ss}^{K}\frac{\omega(1-\chi)}{\omega(1-\chi)-1}(\hat{A}_{ss}^{K,I})^{\frac{1}{1-\chi}}(Q_{ss}\chi)^{\frac{\chi}{\chi-1}} \tag{1.159}$$

$$I_{ss}^{K} = I_{ss}^{K,I} + I_{ss}^{K,NE} \tag{1.160}$$

Savings demand:

$$S_{ss}^{K,I} = \frac{\omega\chi}{\omega(1-\chi)-1}INC_{ss}^{K}f^{I} \tag{1.161}$$

$$S_{ss}^{K,NE} = \frac{\omega\chi}{\omega(1-\chi)-1}NE_{ss}^{K}f^{NE} \tag{1.162}$$

$$S_{ss}^{K} = S_{ss}^{K,I} + S_{ss}^{K,NE} \tag{1.163}$$

Market clearing conditions:

$$C_{ss} = Y_{ss} - S_{ss} - INC_{ss}\phi^{I} - NE_{ss}\phi^{NE} - INC_{ss}^{K}f^{I} - NE_{ss}^{K}f^{NE} \tag{1.164}$$

$$I_{ss}^{K} = \left(1 - \frac{(1-\delta)}{g_{*y}}\right)K_{ss} \tag{1.165}$$

Chapter 2

Schumpeterian firms' dynamic and financial constraint.

Bianca Barbaro and Patrizio Tirelli

Abstract We build a business cycle model characterized by endogenous firms dynamic, based on idiosyncratic productivity levels and by a non-trivial financial sector. We extend a financial sector à la Gerlter and Karadi (2011 [20]), we extend the financial sector including firms' default and the possibility to roll-over borrowing condition to some unproductive firms. This category includes firms that are not productive enough to repay their debt but are kept alive by the financial intermediaries, that find it convenient to roll over debt instead of repossessing and reallocating the loan. Existence of non-negligible repossession costs is crucial for a debt roll over strategy to be viable. We find that a technology improvement discourages debt roll-over, reducing the share of Non-performing loans (NPL) and unproductive incumbent through the entry of new and more productive firms. New entrants, raise market competition and increase interest rates, financial intermediaries incentive to renegotiated debt condition decrease and the same happens to the share of NPLs. Furthermore, an adverse shock to financial intermediaries capital triggers an ever-greening mechanism that increases the share of NPLs in bankers balance sheets and persistently reduces aggregate productivity.

2.1 Introduction

We study the effect of productivity improvement and of a reduction of bankers net worth in a business cycle model with financial friction and debt roll-over. We allow for the existence of three type of firms: new entrants, incumbents and non-performing firms. This category includes all the otherwise insolvent firms that are not productive enough to gain profits and repay their debt but are kept alive by the financial intermediaries, that finds convenient to renegotiate their debt condition instead of repossessing and reallocating the landed capital.

The survival of firms characterized by non-performing loans (NPL) can be linked with the so-called "Zombie firms"(Caballero et al., 2008 [12]), which are considered responsible for prolonging the Japanese macroeconomic stagnation in the 1990s. The Japanese banks supplied credit to weak firms that were de facto candidate for insolvency in 1990s and in the early 2000s. This ever-greening tendency brought to an important market distortion: firms that were not productive enough to survive did not exit the market, creating a congestion that kept productive firms out of the credit market (Caballero et al, 2008[12]; Peek and Rosengren, 2005 [25]).

The productivity slowdown over the past decade has increased scholars' interest around potential barriers to productivity growth and innovation, bringing new interest to the adverse effects of "Zombie firms". Is Europe following the same path of Japan? Acharya et al. (2018, [1]) investigate the reaction of European banks to the ECB Outright Monetary Transactions (OMT [1]) program that brought stability to the banking sector and increased its liquidity. The study shows that implementation of the OMT plan was not sufficient to stimulate an adequate recovery but was in fat associated to an inefficient allocation of loans that favoured survival of less productive firms.

Credit misallocation could generate adverse effects on aggregate productivity, not only because inefficient firms are "artificially" kept alive, but also because their very survival may crowd out investment in more

productive firms. To confirm this hypothesis, Adalet (2018, [2]) finds that the share of unproductive firms across non-financial companies has increased significantly in the wake of the Great Financial Crisis and that the creative-destruction process that leads an efficient economic growth is slowing down. Siemer (2014 [29]) analyses annual Business Dynamics Statistics (BDS) data from 1978 to 2011 and finds that the financial crisis reduced the entry in the U.S. up to the creation a proper missing generation of new firms.

We link our theoretical research to this empirical strand of literature. The model is characterised as follows. The production sector á la Hopenhayn (1992 [22]) is modelled as in Piersanti and Tirelli (2018). At the beginning of each period all firms discover their idiosyncratic productivity level and, due to the existence of fixed production costs, decide whether they want to operate or not. One key difference exists between incumbents and potential new entrants, because the former are by assumption burdened by inherited debt, whereas new entrants are not. This assumption, combined with loans repossession costs is sufficient to induce banks to roll over debt of a fraction of incumbents instead of reallocating loans to more productive firms.

[1] "Once activated towards a specific country, the OMT program allows the ECB to buy a theoretically unlimited amount of the country's government bonds in secondary markets". (Acharya, 2018 [1])

One key feature of the model is that the share of Non performing loans is inversely rated to variations in the market rate of return on loans. The lower the market interest rate the smaller the banks incentive to repossess and to reallocate credit. As a result, a technology improvement discourages debt roll-over, reducing the share of Non-performing loans (NPL) and unproductive incumbent through the entry of new and more productive firms. New entrants, indeed, raise market competition and increase interest rates, financial intermediaries incentive to renegotiated debt condition decrease and the same happens to the share of NPLs. Furthermore, an adverse shock to financial intermediaries capital triggers an ever-greening mechanism that increases the share of NPLs in bankers balance sheets and persistently reduces aggregate productivity.

We contribute to two strands of macroeconomic literature. As first, we contribute to the growing field of studies on endogenous firms dynamic. The seminal work of Bilbiie, Ghironi and Melitz (2012[10]) studies the role of endogenous entry in propagating business cycle fluctuations focusing on extensive margins, other studies include different levels of competition (Jaimovich and Floetotto, 2008[23]; Colciago and Etro, 2008[15]). All of these studies agree that firm entry an exit amplify and propagate the effects of aggregate shocks, however their results may be amplified by monopolistic competition and increasing return to scale. As Clementi and Palazzo (2016 [14]) our economy is perfectly competitive and the principal difference between our model and the majority of previous studies is that technology improvement is endogenously driven by entry. Hamano and Zanetti ([21] 2017) find that a reduction in operational firms costs allows less productive firms to survive and stay in the market, creating a share of "zombie" firms in the economy. In our model, the share of non performing firms, is endogenously determined by the financial intermediaries problem.

Our second contribution to this field is technical. Models with endogenous firms dynamic and heterogeneous firms are usually characterised by labour as unique production input. This is because, using capital as an input, we would have to deal with a state variable that depends on firms' idiosyncratic productivities. In our model, firms produce only if they meet a technology requirement, expressed as a function of aggregate variables only. This set-up gives us the key advantages to study all the variables at their aggregate level without keeping track of the evolution of idiosyncratic productivity for each firm. As we explain in the paper, the specific features of the debt contract between the bank and the incumbent firm are carefully crafted to preserve this innovative feature of our model.

We also place our work in macroeconomic and financial friction strand of literature. The seminal works on financial friction analyse the role of imperfect financial markets with a constant number of firms and agree on the fact that financial friction magnifies the impact of shocks on the economy response (Gertler and Karadi, 2011[20], Bernanke et al., 1994 [9], Gerali et al., 2010 [19]). Some recent papers, however, introduce endogenous firms dynamic in models with financial markets shedding a light on the interaction between these two features as the moderating effect of the adjustment in firm numbers on the impact of financial shocks on aggregate output (Bergin et al., 2018 [8]). Rossi (2019 [27]) extended the firms dynamic contribute to financial friction literature, considering the role of endogenous entry and exit, finding that, a positive technology shock increases firms creation and reduces firms destruction on impact.

The role of the financial sector mitigates the shock effect on firms creation and destruction, because of higher production costs faced by firms generated by the monopolistic banking sector. In our model, perfect competition in financial sector does not generate mark-up in interest rate but, the unproductive firms' presence, will increase the interest rate and lead to higher costs of production for productive firms. Siemer (2014 [29]) uses a heterogeneous firms model with endogenous entry and financial constraint to show that a large financial shock reduces firm entry, creating a "missing generation" of firms entry and a consequent slow recovery. Our model, through the debt roll-over mechanism generates an explanation to the missing generation of new firms: the credit crush that follows the financial crises is worsened by the credit misallocation to zombie firms and this generates the drop in firms' entry. The paper is organized as follows: section 2 describes the model, section 3 analyses the response of the economy to productivity and financial intermediaries' net worth shock, section 4 concludes.

2.2 The model

2.2.1 Households

The Households' problem follows Gertler and Karadi (2011 [20]). As in GK, households can be workers or financial intermediary owners. Workers supply labor and return their wage to the household, bankers manage a financial intermediary which earnings are transferred to their household. The utility is characterized by the following preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(ln(C_t) - \psi \frac{L_t^{1+\varphi}}{1+\varphi} \right) \tag{2.1}$$

the flow budget constraint is:

$$C_t + I_t + D_t = w_t L_t + r_{t-1}^d D_{t-1} + \Pi_t^{B,F} \tag{2.2}$$

Where w_t is the real wage, r_t^d is the risk-free remuneration on deposits D_t landed to financial intermediaries and $\Pi_t^{B,F}$ are the financial intermediaries (B) profit and firms' profits (F) that households gain in behalf of their ownership. Finally, households supply labor L_t, and choose the optimal level of consumption C_t.

The first order conditions are the following:

$$\lambda_t = \frac{1}{C_t} \tag{2.3}$$

$$L_t = \left(\frac{\lambda_t w_t}{\psi} \right)^{\frac{1}{\varphi}} \tag{2.4}$$

$$\lambda_t = \beta r_t^d E_t\{\lambda_{t+1}\} \tag{2.5}$$

2.2.2 Firms

The production sector follows Piersanti and Tirelli (2018 [26]). The market is characterized by firms of different age (New entrants and Incumbent) which are heterogeneous in their productivity level and produce in a perfectly competitive environment according to a decreasing return to scale production function. We assume that, at the end of each period, firms, aware of the aggregate economic conditions, demand loans to financial intermediaries to produce in the next period. At the beginning of the following period, preexistent firms draw their technology level from a Pareto distribution and, depending on their expected profits, choose to pay a fixed cost and produce or to exit the market. If the idiosyncratic productivity of the firm is not high enough to let her produce, financial intermediaries can let her exit the market and repossess the loan net of a monitoring cot μ or, if it is more convenient, renegotiate the debt contract, save the firm and allow her to produce. Notice that, the uncertainty at the end of $t-1$ is uniquely around the idiosyncratic productivity level and firms are able to correctly forecast interest rates, wages and so, the marginal cost of production. We assume that New entrants are by construction averagely more productive than the incumbents since they benefit from an exogenous advantage in the technology frontier. Incumbents, learn the technology adopted by New entrants when they will join the surviving incumbents (in $t+1$).

Incumbents (INC).

At the end of period $t-1$, surviving incumbents obtain funds $b_{j,t-1}$ from a financial intermediary at the contractual rate r_t^k. Loan's demand is based on the firms' expected technology level. In period t, the expected profit function of a j firm is:

$$E_{t-1}\{\pi_{j,t}^I\} = E_{t-1}\{y_{j,t}^I\} - r_t^k E_{t-1}\{k_{j,t}^I\} + (1-\delta)E_{t-1}\{k_{j,t}^I\} - w_t E_{t-1}\{l_{j,t}^I\} - \phi_t^I \tag{2.6}$$

where:

$$E_{t-1}\{y_{j,t}^I\} = E_{t-1}\left\{A_{j,t}^I[(k_{j,t}^I)^\alpha (l_{j,t}^I)^{1-\alpha}]^\gamma\right\},$$

$$b_{j,t-1}^I = k_{j,t}^I$$

w_t is the real wage, and ϕ_t^I is a fixed cost. Since we do not have a capital good sector, we assume that the depreciation of capital impacts on firms' constraint. Incumbents base their demand of loan at the end of $t-1$ on $E_{t-1}\{A_{j,t}^I\}$ which is identical across firms. Given this assumption, the expected optimal bundle of production factor will be identical across firms too. Depending on their expected productivity, Incumbents will demand their demand of loans in order to maximize her expected profit:

$$E_{t-1}\{\pi_t^I\} = E_{t-1}\{y_t^I\} - E_{t-1}\{k_t\}(r_t^k - 1 + \delta) + E_{t-1}\{l_t^I\}w_t - \phi_t^I \tag{2.7}$$

We can derive the demand loan of potential incumbents in t, which is equal to the demand of capital based on technology expectations.

$$\bar{b}^I_{t-1} = \alpha\gamma \frac{E_{t-1}\{y^I_t\}}{(r^k_t - 1 + \delta)} = \frac{\alpha}{(r^k_t - 1 + \delta)} \left[\frac{\gamma E_{t-1}\{A^I_t\}}{\left[\left(\frac{(r^k_t - 1 + \delta)}{\alpha} \right)^{\alpha} \left(\frac{w_t}{(1-\alpha)} \right)^{(1-\alpha)} \right]^{\gamma}} \right]^{\frac{1}{1-\gamma}} \tag{2.8}$$

After purchasing the capital stock, incumbent firms will learn their idiosyncratic productivity $A^I_{j,t}$ which is a random draw from $f_t(A^I_t) = \int_{\hat{A}^{NE}_{t-1}}^{+\infty} \frac{\xi(\hat{A}^{NE}_{t-1})^{\xi}}{(A^I_t)^{\xi+1}} d(A^I_t)$. Depending on their idiosyncratic productivity, firms can freely integrate their demand of capital (if $\bar{b}^I_{t-1}$ is less than the optimal level) asking additional credit to the bank at the interest rate r^k_t or can lend their capital in excess at the same rate. So, they will maximize their final profit $\pi_{j,t}$, choosing the optimal production input demand based on their actual productivity level,

$$\pi^I_{j,t} = y^I_{j,t} - (r^k_t - 1 + \delta)k^I_{j,t} - w_t l^I_{j,t} - \phi^I_t \tag{2.9}$$

$$k^I_{j,t} = \alpha\gamma \frac{y_{j,t}}{(r^k_t - 1 + \delta)} \tag{2.10}$$

$$l^I_{j,t} = (1-\alpha)\gamma \frac{y_{j,t}}{w_t} \tag{2.11}$$

If the optimal demand of production factors and their idiosyncratic technology level allows firms to gain non-negative profits, they will decide pay the fixed cost ϕ^I_t and produce. Otherwise, firms will exit the market without adjusting their capital or purchasing labour and the financial intermediaries will repossess the landed capital. At the given contractual rate, we have a profitability threshold that defines the exit decision defined by the zero-profit condition:

$$\pi^I_{j,t} = A^I_{j,t} \left[(k^I_{j,t})^{\alpha} (l^I_{j,t})^{(1-\alpha)} \right]^{\gamma} - (r^k_t - 1 + \delta)k^I_{j,t} - w_t l^I_{j,t} - \phi^I_t \geq 0 \rightarrow$$

$$\hat{A}^I_t = \left[\frac{\phi^I_t}{(1-\gamma)} \right]^{1-\gamma} \cdot \frac{\left[\left[\frac{(r^k_t - 1 + \delta)}{\alpha} \right]^{\alpha} \left[\frac{w_t}{(1-\alpha)} \right]^{(1-\alpha)} \right]^{\gamma}}{\gamma^{\gamma}} \tag{2.12}$$

The higher is the cost of production (fixed and variable), the higher will be the entry requirement.

New entrants (NE).

Unlike the incumbents, New entrants are allowed to demand capital when they learn their idiosyncratic productivity at the beginning of t. As the Incumbent, they will get capital through a loan $b^{NE}_{j,t}$ that firms will payback at a contractual rate r^k_t. However, their loan demand will be based on their actual productivity level. The profit function of a j, NE firm is :

$$\pi^{NE}_{j,t} = y^{NE}_{j,t} - k^{NE}_{j,t}(r^k_t - 1 + \delta) - w_t l^{NE}_{j,t} - \phi^{NE}_t \tag{2.13}$$

$$b^{NE}_{j,t} = k^{NE}_{j,t}$$

The optimal demand of inputs from New entrants will be:

$$k_{j,t}^{NE} = \alpha\gamma \frac{y_{j,t}^{NE}}{(r_t^k - 1 + \delta)} \tag{2.14}$$

$$l_{j,t}^{NE} = (1-\alpha)\gamma \frac{y_{j,t}^{NE}}{w_t} \tag{2.15}$$

As for the Incumbents, the NE threshold will be defined by their zero profit condition, and will be:

$$\pi_{j,t}^{NE} = y_{j,t}^{NE} - k_{j,t}^{NE}(r_t^k - 1 + \delta) - w_t l_{j,t}^{NE} - \phi_t^{NE} \geq 0 \rightarrow$$

$$\hat{A}_t^{NE} = \left[\frac{\phi_t^{NE}}{(1-\gamma)}\right]^{1-\gamma} \cdot \frac{\left[\left[\frac{(r_t^k - 1+\delta)}{\alpha}\right]^{\alpha}\left[\frac{w_t}{(1-\alpha)}\right]^{(1-\alpha)}\right]^{\gamma}}{\gamma^{\gamma}} \tag{2.16}$$

Debt Rollover and Non-performing firms (NP).

When an Incumbent does not meet the entry requirement, the financial intermediary repossesses the landed funds, net of a transaction cost μ, and reallocates them. However, we introduced the possibility of not repossessing a fraction of unproductive incumbent loan and "save"a share of defaulting firms, accepting firms' revenues as loan repayment. The decision process of financial intermediaries follows this principle: if the financial intermediary repossesses and lends to other firms she gets $r_t^k\,(1-\mu)\,\bar{b}_{t-1}^I$, if she will save the unproductive firm and pay the fixed cost for her, will her profit (net of wages and fixed cost) plus the revenues from the reallocation of excess capital $r_t^k(\bar{b}_{t-1}^I - k_{j,t}^{NP})$ on which the repossession cost μ does not have to be paid in behalf of intermediaries' control on production. The total profit the intermediaries get from saving the Non performing firms is described by the following equation:

$$\Pi_{j,t}^{NP} = y_{j,t}^{NP} - w_t l_{j,t}^{NP} + r_t^k(\bar{b}_{t-1}^I - k_{j,t}^{NP}) + (1-\delta)k_{j,t}^{NP} - \phi_t^I \tag{2.17}$$

The production input combination is obtained from the maximization of (2.20),

$$k_{j,t}^{NP} = \alpha\gamma \frac{y_{j,t}^{NP}}{(r_t^k - 1 + \delta)} \tag{2.18}$$

$$l_{j,t}^{NP} = (1-\alpha)\gamma \frac{y_{j,t}^{NP}}{w_t} \tag{2.19}$$

In t the intermediary has the incentive to roll over debt, if the expected value of saving the firm is higher than the repossession value:

$$A_{j,t}^I (k_{j,t}^{NP})^{\alpha\gamma}(l_{j,t}^{NP})^{(1-\alpha)\gamma} - w_t l_{j,t}^{NP} + r_t^k(\bar{b}_{t-1}^I - k_{j,t}^{NP}) - \phi_t^I + (1-\delta)k_{j,t}^{NP} \succeq r_t^k\,(1-\mu)\,\bar{b}_{t-1}^I \tag{2.20}$$

From (2.20) we can derive the minimum productivity requirement for debt roll-over:

$$\hat{A}_t^{NP} = \left[\frac{\phi_t^I - \mu r_t^k \bar{b}_t^I}{(1-\gamma)}\right]^{1-\gamma} \cdot \frac{\left[\left[\frac{(r_t^k - 1 + \delta)}{\alpha}\right]^{\alpha} \left[\frac{w_t}{(1-\alpha)}\right]^{(1-\alpha)}\right]^{\gamma}}{\gamma^{\gamma}} \tag{2.21}$$

The technology requirement for debt-rollover mainly depend on the spread between the fixed cost of production and the reallocation value $\mu r_t^k \bar{b}_{t-1}^I$. This means that the lower is the interest rate at which banks rent the capital, the bigger will be the distance between the entry threshold for the Incumbents (2.12) and the Non-performing firms minimum technology level. A large distance between the two technology thresholds will increase the share of Non-performing firms' saved. Defining Non-performing firms as (NP_t), the aggregate profits, transferred to the financial sector will be:

$$\Pi_t^{NP} = Y_t^{NP} - w_t L_t^{NP} - NP_t^{NP} \phi_t^I \tag{2.22}$$

Mass of firms.

New entrants: As in Piersanti and Tirelli (2018 [26]), NEs draw their productivity level $A_{j,t}^{NE}$ from the following Pareto distribution:

$$f_t(A_t^{NE}) = \int_{\underline{z}_t}^{+\infty} \frac{\xi \underline{z}_t^{\xi}}{(A_t^{NE})^{\xi+1}} d(A_t^{NE}) = 1 \qquad with\ \hat{A}_t^{NE} \geq \underline{z}_t \tag{2.23}$$

Where ξ is the shape of the Pareto distribution ($\xi > 1$) and $\underline{z}_t$ defines the technology frontier. The evolution of $\underline{z}_t$ is described by the following equations:

$$\underline{z}_t = \underline{z}_{t-1} g_{z,t} \tag{2.24}$$

$$ln(g_{z,t}) = (1-\rho_z) ln(g_z) + \rho_z ln(g_{z,t-1}) + \sigma^z \epsilon_t^z, \quad \epsilon_t^z \sim \mu(0,1) \tag{2.25}$$

They enter the economy at time t if they meet the zero-profit condition. The mass of New entrants NE_t will be:

$$NE_t = \int_{\hat{A}_t^{NE}}^{+\infty} \frac{\xi \underline{z}_t^{\xi}}{(A_t^{NE})^{\xi+1}} d(A_t^{NE}) = \left(\frac{\underline{z}_t}{\hat{A}_t^{NE}}\right)^{\xi} =$$

$$= \underline{z}_t^{\xi} \cdot \left\{ \left[\frac{(1-\gamma)}{\phi_t^{NE}}\right]^{1-\gamma} \cdot \frac{\gamma^{\gamma}}{\left\{\left[\frac{(r_t^k - 1 + \delta)}{\alpha}\right]^{\alpha} \left[\frac{w_t}{(1-\alpha)}\right]^{(1-\alpha)}\right\}^{\gamma}} \right\}^{\xi} \tag{2.26}$$

Equation (2.26) shows how the mass of New entrants is increasing in the technology frontier $\underline{z}_t$ and decreasing in the threshold $\hat{A}_t^{NE}$ and in production costs r_t^k, w_t and ϕ_t^{NE}.

[1] Details in the following subsection

Incumbents: At the end of period $t-1$, the number of active firms will be:

$$\eta_{t-1} = NE_{t-1} + INC_{t-1} \tag{2.27}$$

At the beginning of each period t the η_{t-1} active firms draw their idiosyncratic productivity level $A^I_{j,t}$ from a Pareto distribution identified by the technology frontier $\hat{A}^{NE}_{t-1}$,

$$f_t(A^I_t) = \int_{\hat{A}^{NE}_{t-1}}^{+\infty} \frac{\xi(\hat{A}^{NE}_{t-1})^\xi}{(A^I_t)^{\xi+1}} d(A^I_t) \tag{2.28}$$

Then only firms that draw a productivity level $A^I_t \succeq \hat{A}_t$ will produce. This formulation allows us to model endogenous exit flows without the need of keeping track of the idiosyncratic evolution of each incumbent efficiency. The number of incumbents that will produce in t will be defined by:

$$INC_t = \eta_{t-1} \int_{\hat{A}^I_t}^{+\infty} \frac{\xi(\hat{A}^{NE}_{t-1})^\xi}{(A^I_t)^{\xi+1}} d(A^I_t) =$$

$$= \eta_{t-1}(\frac{\hat{A}^{NE}_{t-1}}{\hat{A}^I_t})^\xi = \eta_{t-1}\left[\left(\frac{\phi^{NE}_{t-1}}{\phi^I_t}\right)^{1-\gamma} \cdot \left(\left(\frac{(r^k_{t-1}-1+\delta)}{(r^k_t-1+\delta)}\right)^\alpha \left(\frac{w_{t-1}}{w_t}\right)^{1-\alpha}\right)^\gamma\right]^\xi \tag{2.29}$$

Just like the mass of New entrants, the mass of Incumbents (2.29) will depend on the costs of production and on their past value of these variables: i.e. an increase in rental rate and wages will decrease the the number of the incumbent. Equation (2.29) captures the slower innovation that characterizes incumbents. Incumbents, that learn their technology from NE_{t-1} will be characterized by a technology level ad hoc for the previous period. However, their technology level, has to be high enough to face costs in t. For this reason, the number of INC_t will be defined mostly by the costs evolution.

Non-performing Incumbents: The mass of Non-performing incumbent, is defined as the mass of firms below the Incumbents' technology threshold $\hat{A}^I_t$, but with productivity higher than $\hat{A}^{NP}_t$.

$$NP_t = \eta_{t-1} \int_{\hat{A}^{NP}_t}^{\hat{A}^I_t} \frac{\xi(\hat{A}^{NE}_{t-1})^\xi}{(A^{NP}_t)^{\xi+1}} d(A^{NP}_t) = \eta_{t-1}\left[(\frac{\hat{A}^{NE}_{t-1}}{\hat{A}^{NP}_t})^\xi - (\frac{\hat{A}^{NE}_{t-1}}{\hat{A}^I_t})^\xi\right] \tag{2.30}$$

The mass of Non-performing incumbents will depend positively on the spread between the entry requirement of NPs and NEs, and negatively on the spread between the entry requirement of INCs and NEs. The latter depends on costs evolution, while the difference between NEs and NPs thresholds depend on both the changes in the cost of production and on the cost of loans reallocation. From 2.127 we learn that the cost of reallocation has a principal role in determining the share of Non-performing incumbents. The bigger the reallocation cost will be, the higher will be the incentive of financial intermediaries to save firms and roll-over the debt to Non-performing incumbents. The second determinant of NPs mass can be found in costs evolutions, which role is to reproduce the delay in technology acquisition that characterizes Incumbents: if on one hand an increase in costs of production reduces the number of saved firms, on the other it will reduce generally the number of less productive, encouraging unproductive firms destruction and increasing financial intermediaries appetite for new investments. The term $\frac{\hat{A}^{NE}_{t-1}}{\hat{A}^{NP}_t}$ amplifies in absolute therm the share of Non-performing firms, while the ratio $\frac{\hat{A}^{NE}_{t-1}}{\hat{A}^{I}_t}$ contributes to a ceteris paribus decrease in Non-performing and productive incumbent, improving the economy efficiency in general. The sum of New Entrants, Incumbents and Non-performing[2] firms in t, will define the number of active firms η_t:

$$\eta_t = NE_t + INC_t + NP_t$$

Finally, the mass of defaulting firms, is defined by:

$$E_t = \eta_{t-1}\left[1 - \left(\frac{\hat{A}^{NE}_{t-1}}{\hat{A}^{NP}_t}\right)^{\xi}\right] \tag{2.31}$$

Comparing (2.127) and (2.31), we see that the higher is the difference between the New entrants' and Non-performing incumbents' thresholds, the lower will be the exit and the bigger will be the number of saved firms. Since a higher interest rate reduces this spread, we can suggest that lower interest rate policies may impact on efficient credit reallocation.

Aggregation

The aggregate demand of loans for each type of firm, is defined by the following set of equations:

$$B^I_t = \left(\frac{\alpha\gamma}{(r^k_t - 1 + \delta)}\right)\frac{\xi}{\xi(1-\gamma)-1}\cdot INC_t\phi^I_t \tag{2.32}$$

$$B^{NE}_t = \left(\frac{\alpha\gamma}{(r^k_t - 1 + \delta)}\right)\frac{\xi}{\xi(1-\gamma)-1}\cdot NE_t\phi^{NE}_t \tag{2.33}$$

$$B^{NP}_t = \left(\frac{\alpha\gamma}{(r^k_t - 1 + \delta)}\right)\frac{\xi}{\xi(1-\gamma)-1}\cdot NP_t\mu r^k_t\bar{b}^I_{t-1} \tag{2.34}$$

Where (2.128) defines the capital which is used by financial intermediaries tosave Non performing firms and produce.

Since $\bar{k}_{j,t}$ is demanded ex-ante and is based on idiosyncratic productivity expectations, we define $E_{t-1}\{A^I_{j,t}\}$ as the average productivity level of the Incumbents Pareto distribution. Given the distribution $f_t(A^I_t) = \int_{\hat{A}^{NE}_{t-1}}^{+\infty}\frac{\xi(\hat{A}^{NE}_{t-1})^{\xi}}{(A^I_t)^{\xi+1}}d(A^I_t)$, we can rewrite the ex-ante demand of loans 2.102 as:

$$\bar{b}^I_{t-1} = \frac{\alpha}{(r^k_t - 1 + \delta)}\left[\frac{\gamma\frac{\xi}{\xi-1}\hat{A}^{NE}_{t-1}}{\left[\left(\frac{(r^k_t-1+\delta)}{\alpha}\right)^{\alpha}\left(\frac{w_t}{(1-\alpha)}\right)^{(1-\alpha)}\right]^{\gamma}}\right]^{\frac{1}{1-\gamma}} \tag{2.35}$$

Total production is defined by:

$$Y_t = Y^{NE}_t + Y^{INC}_t + Y^{NP}_t \tag{2.36}$$

[2]Extended NPs equation: $NP_t = \eta_{t-1}\left\{\left[\left(\frac{\phi^{NE}_{t-1}}{\phi^I_t - \mu r^k_t\bar{b}^I_t}\right)^{(1-\gamma)\xi} - \left(\frac{\phi^{NE}_{t-1}}{\phi^I_t}\right)^{(1-\gamma)\xi}\right]\cdot\left(\left(\frac{(r^k_{t-1}-1+\delta)}{(r^k_t-1+\delta)}\right)^{\alpha}\left(\frac{w_{t-1}}{w_t}\right)^{1-\alpha}\right)^{\gamma\xi}\right\}$

where,

$$Y_t^{NE} = NE_t \frac{\xi(1-\gamma)}{\xi(1-\gamma)-1} (\hat{A}_t^{NE})^{\frac{1}{1-\gamma}} \left(\frac{\gamma}{\left[\frac{(r_t^k - 1 + \delta)}{\alpha}\right]^{\alpha} \left[\frac{w_t}{(1-\alpha)}\right]^{(1-\alpha)}} \right)^{\frac{\gamma}{1-\gamma}} \tag{2.37}$$

$$Y_t^{I} = INC_t \frac{\xi(1-\gamma)}{\xi(1-\gamma)-1} (\hat{A}_t^{I})^{\frac{1}{1-\gamma}} \left(\frac{\gamma}{\left[\frac{(r_t^k - 1 + \delta)}{\alpha}\right]^{\alpha} \left[\frac{w_t}{(1-\alpha)}\right]^{(1-\alpha)}} \right)^{\frac{\gamma}{1-\gamma}} \tag{2.38}$$

$$Y_t^{NP} = NP_t \frac{\xi(1-\gamma)}{\xi(1-\gamma)-1} \left[(\hat{A}_t^{I})^{\frac{1}{1-\gamma}} - (\hat{A}_t^{NP})^{\frac{1}{1-\gamma}} \right] \left(\frac{\gamma}{\left[\frac{(r_t^k - 1 + \delta)}{\alpha}\right]^{\alpha} \left[\frac{w_t}{(1-\alpha)}\right]^{(1-\alpha)}} \right)^{\frac{\gamma}{1-\gamma}} \tag{2.39}$$

Finally, the aggregate productivity is given by:

$$\bar{A}_t = \frac{\xi}{\xi - 1} \left(INC_t \phi_t^I + NE_t \phi_t^{NE} + NP_t \mu r_t^k \bar{b}_{t-1}^I \right) \tag{2.40}$$

2.2.3 Financial intermediaries

The banking sector extends the one in Gertler and Karadi (2011 [20]). Defining $NW_{h,t}$ as the amount of net worth that a banker h has at the end of period t; $D_{h,t}$ the deposit the intermediary gets from households; $B_{h,t}$ the quantity of financial claims on consumption-goods producers that the intermediary holds. The banker's balance sheet is:

$$B_{h,t} = NW_{h,t} + D_{h,t} \tag{2.41}$$

$$s.t.\ B_{h,t} = B_{h,t}^I + B_{h,t}^{NP} + B_{h,t}^{NE}$$

In each period, bankers gain r_t^k from lending to productive firms (NE_t and INC_t) and the profit of Non-performing firms. Additionally, since the bank will repossess the capital anticipated to defaulted firms to efficiently reallocate it, we have to consider a repossession cost equal to $\mu r_t^k \bar{b}_{t-1} E_t$, where E_t is the number of firms that exit the market. The average revenue on loans is defined as r_t^b :

$$r_t^b = \frac{r_t^k B_t^I + r_t^k B_t^{NE} + \Pi_t^{NP} - \mu r_t^k \bar{b}_{t-1} E_t}{B_t} \tag{2.42}$$

Net worth of bankers evolves accordingly to the following law of motion, governed by the spread between the average return on assets and the interest payments on households deposits:

$$NW_{h,t+1} = r_{t+1}^b B_{h,t} - r_t^d (B_{h,t} - NW_{h,t}) = (r_{t+1}^b - r_t^d) B_{h,t} + r_t^d NW_{h,t} \tag{2.43}$$

The intermediary gains from lending, if the average return on loans is higher than cost of borrowing. This means that the following participation constraint must be respected:

$$E_t \beta^i \frac{\lambda_{t+1+i}}{\lambda_t} (r^b_{t+1+i} - r^d_{t+i}) \geq 0, \quad i \geq 0 \tag{2.44}$$

If with perfect capital markets the risk premium is zero, with imperfect capital markets, however, condition (2.44) may not bind. Therefore, the intermediary will keep on expanding her assets until she can gain a non negative premium exiting the market. The intermediary objective function is:

$$V_{h,t} = E_t(1-\theta_b) \sum_{i=0}^{+\infty} \theta_b^i \beta^i \frac{\lambda_{t+1+i}}{\lambda_{t+i}} NW_{h,t+1+i} =$$

$$= E_t(1-\theta_b) \sum_{i=0}^{+\infty} \theta_b^i \beta^i \frac{\lambda_{t+1+i}}{\lambda_t} \left[\left(r^b_{t+1+i} - r^d_{t+i} \right) \varphi^b_{t+i} B_{h,t+1} + r^d_{t+i} NW_{h,t+i} \right] \tag{2.45}$$

Where θ_b is the survival rate. At each period the bank can divert a fraction λ_b of funds and exit the market, so the incentive compatibility constraint for lenders to be willing to supply funds to the banker, must be:

$$V_{h,t} \geq \lambda_b B_{h,t} \tag{2.46}$$

where the left hand side can be expressed as,

$$V_{h,t} = \mu^b_t + \nu^b_t NW_{h,t} \tag{2.47}$$

and μ^b_t is the expected discounted marginal benefit of expanding assets by a unit and ν^b_t is the expected discounted value of an additional unit of net worth.

$$\mu^b_t = E_t \left[(1-\theta_b)\beta \frac{\lambda_{t+1}}{\lambda_t} \left(r^b_{t+1} - r^d_t \right) + \theta_b \beta \frac{\lambda_{t+1}}{\lambda_t} m_{t+1} \mu^b_{t+1} \right]$$

$$\nu^b_t = E_t \left[(1-\theta_b)\beta \frac{\lambda_{t+1}}{\lambda_t} r^d_t + \theta_b \beta \frac{\lambda_{t+1}}{\lambda_t} n_{t+1} \nu^b_{t+1} \right]$$

Defining the gross growth rate in lending as:

$$m_t = \frac{B_{h,t}}{B_{h,t-1}}$$

and the gross growth rate in net worth as:

$$n_t = \frac{NW_{h,t}}{NW_{h,t-1}}$$

we can rewrite (2.46) and (2.47) as:

$$B_{h,t} = \frac{\nu^b_t}{\lambda_b - \mu^b_t} NW_{h,t} = \phi^b_t NW_{h,t} \tag{2.48}$$

where Φ^b_t is the leverage ratio in t.

It follows that, net worth evolution can be defined as:

$$NW_t = \left[(r_t^b - r_{t-1}^d)\Phi_t^b + r_{t-1}^d\right] NW_{t-1} \tag{2.49}$$

and,

$$n_t = \frac{NW_{h,t}}{NW_{h,t-1}} = (r_t^b - r_{t-1}^d)\Phi_t^b + r_{t-1}^d \tag{2.50}$$

$$m_t = \frac{\Phi_t^b}{\Phi_{t-1}^b} n_t \tag{2.51}$$

Finally, as in GK, households finance new banks (start-up funds) in each period. Households transfer a small fraction of the value of assets that exiting bankers had intermediated in their final operating period. Given the exit probability $(1-\theta_b)$, if the households will transfer a fraction equal to $\frac{\omega}{(1-\theta_b)}$

$$NW_{n,t} = \omega \frac{K_{t-1}}{\Phi_t^b} \tag{2.52}$$

The pre-existing bankers net worth is equal to:

$$NW_{e,t} = \theta_b \left[(r_t^b - r_{t-1}^d)\Phi_t^b + r_{t-1}^d\right] NW_{t-1} \tag{2.53}$$

The total net worth in t is defined by:

$$NW_t = NW_{e,t}\zeta_t + NW_{n,t} \tag{2.54}$$

where ζ_t indicates the quality of pre-existent net worth that evolves accordingly to an AR(1) process,

$$ln(\zeta_t) = \rho_\zeta ln(\zeta_{t-1}) + \sigma^\zeta \epsilon_t^\zeta, \quad \epsilon_t^\zeta \sim \mu(0,1) \tag{2.55}$$

2.2.4 Market clearing

Finally, capital evolves following a standard law of motion,

$$K_t = I_t + (1-\delta)K_{t-1} \tag{2.56}$$

and the market clears if the aggregate resources constraint is satisfied:

$$Y_t = C_t + I_t + INC_t\phi_t^I + NE_t\phi_t^{NE} + NP_t\mu r_t^k \bar{b}_{t-1} \tag{2.57}$$

2.2.5 Calibration

Our model follows a quarterly calibration. We set the decreasing return to scale parameter $\gamma = \chi = 0.8$ and $\xi = \omega = 6.1$ (Asturias et al., 2017[4]). We calibrate the initial condition of NEs fixed cost ϕ^{NE} such that the total cost of entry is the 5% of the total output ex post (Etro and Colciago, 2010 [15]; Rossi, 2019 [27]). The Incumbents' fixed cost initial condition is calibrated as a function of the entry cost to set the share of

New Entrants in the economy at $H = 10\%$[3] (Etro and Colciago, 2010 [15]). Additionally, we calibrate the monitoring cost μ to respect the calibration of the share of Non Performing firms at 9%, following pre-crisis data[4] elaboration from Banerjee et al. (2018 [5]). We calibrated the technology frontiers' initial values $\underline{z}_{ss}$ to obtain an unitary mass of firms. Following Gertler and Karadi (2011), we calibrate the spread between the return on loans r^b_{ss} and interest rate on deposit r^d_{ss} at $r^{diff}_{ss} = 0.0025$ and the leverage at $\phi^b_{ss} = \frac{0.1}{r^{diff}_{ss}}$. Finally, the steady state value of labour $L_{ss} = 0.33$ is pinned down by the preference parameter ψ.

The initial condition for NEs technology shifter is set to$\underline{z}_{ss} = 0.68$. Finally, NEs are set to be the 10% of the total firms (Piersanti and Tirelli, 2018[26]; Etro and Colciago, 2010[15]) and the share of Non-performing firms is set to be the 12% of the total firms (Banerjiee, 2018 [5]). The values of elasticity of labor supply and the labor disutility parameters are pinned in function of $L_{ss} = 0.33$. To calibrate the financial sector parameters, we started from the GK steady state spread between interest rate on loans $r^{diff}_t = r^k_{ss} - r^d_{ss} = 0.0025$. From this calibration, we pinned steady state leverage equal to $\phi_t = 4$, while the other parameters, presented in Table 2.1 are calibrated to match GK financial sector equilibrium conditions. The other parameters are standard in RBC and DSGE literature and are summarized in the table.

Table 2.1: Parameters calibration

	Households	
ϕ^{NE}	0.1	Initial entry cost
ϕ	0.25	Inverse Frisch elasticity of labor supply
β	0.99	Discount rate
δ	0.025	Depreciation rate
ψ	2.6	labor disutility parameter
h	0.815	Habit parameter
	Production Sector	
γ	0.8	Decreasing return index
α	0.33	Capital share
ξ	6.1	Pareto distribution shape parameter
$\underline{z}_{ss}$	0.734	Technology frontier initial condition
H	0.025	Share of NEs over total C-firms
H_{np}	0.0225	Share of NPs over total C-firms
	Financial Intermediaries	
λ_b	0.338	Fraction of capital that can be diverted
θ_b	0.9725	Survival probability of banks
ω	0.002	Proportional transfer to the entering bankers

2.3 Impulse response analysis

In this section, we analyse the response of the economy to a productivity positive shock and to a negative shock to financial intermediaries' net worth. Each subsection will compare the response of our model to a

[3] Since we calibrate on a quarterly base $H = 2.5\%$

[4] Data from Datastream Worldscope

benchmark in which we did not allow the financial sector to roll-over the debt contract (No NPLs).

2.3.1 IRF to permanent technology improvement.

The economy reaction to a permanent shift in NE's technology distribution is shown in Figure (2.1). Shocking the NEs efficiency draws, we induce a sudden shift to the right of The inflow of young and productive firms strengths the competition among the final good sector and the higher level of technology will allow firms to pay higher wages and will increase the cost of production for C-firms. Consequentially, the incumbents (INC) that are not able to gain non-negative profits will be forced to exit the market, and their number drops. The slow growth effect, typical of Schumpeterian frameworks, let the output increase slowly. Incumbents' behavior in the following periods, reproduce the following technology spillover: surviving firms adopt the New entrants' technology and the number of incumbents eventually recover to reach a new and higher equilibrium level. At the end of the initial period, surviving incumbents will be more productive, the production cost will increase and the rise in NEs cut-off will gradually arrest the boom in entries. Moving the attention to the households, a preference for consumption will initially lower the investment demand and this unanticipated decline will produce a deterioration in intermediary balance sheets, pushing up the premium.

The role of Non-performing firms.

The responses of the model economy to an increase in NEs technology frontier qualitatively follows the benchmark model response. This means that, after a technological improvement, the Non-performing firms' friction does not affect the sign of the economy response. The higher productivity requirements jointed to a higher risk premium for bankers disincentives debt roll-over. As shown in the last panel, the share of Non-performing loans will drop persistently in the long run. However, the presence of firms with productivities below the threshold makes the spillover process from New entrants to incumbent slightly slower. We can explain this difference through the evolution of interest rate on loans. Financial intermediaries have to compensate the losses in revenues due to Non-performing loans, increasing the contract interest rate to safe firms. The innovation in technology, discourage debt roll-over and the financial intermediaries will be able to provide loans to productive at a lower contractual rate. This will slightly lower the competition with respect to the benchmark model and slow down the innovation spreading.

2.3.2 IRF to a temporary shock in financial intermediaries' net worth.

A negative shock to the intermediaries' net worth aims to reproduce a sudden deterioration of financial institutions' assets. As stressed in Gertler and Karadi (2011 [20]), this kind of shock has to be intended as a rare, but persistent event. The initiating shock is a 5% drop in pre-existent net worth, with persistence 0.66 (quarterly). As shown in Figure 2.2, a negative shock to intermediaries' net worth decreases on the impact the risk premium. This fall will result in an immediate decline of the loan supply and in a consequent drastic drop in the number of firms and in output. In absence of favorable exogenous changes, the economy will eventually recover when the net worth returns to its steady-state level.

The role of Non-performing loans.

If we allow for banks to roll-over debt to Non-performing firms, after a negative net worth shock, intermediaries will prefer to roll-over pre-existent debt instead of reallocating. The presence of Non-performing loans in the banks' balance sheet, leads to a short term drop on the impact of the spread between expected returns on loans and the risk-free deposit rate. This condition, will incentive intermediaries to roll-over debt to less productive firms, adding persistence to the recession. Even when the net worth returns to its steady-state level, with NP firms, the output cannot easily return to the steady state level. This happens because a higher share of unproductive firms, will lower the competition and so the productivity requirement for firms. After 25 periods, the entry threshold $\hat{A}_t^{NE}$ will fall below the steady state, letting less productive firms in the market and slowing down the recovery. Figure 2.2 shows the impact of a net worth shock on aggregate productivities. In our model, the incentive to roll-over debt to unproductive firms leads to a drastic and persistent decrease in firms' average productivity. This is imputable to two main factors. As first, the presence of Non-performing firms reduces aggregate productivity. Secondly, the shock has a positive effect on entry requirements. Although the increase in the productivity thresholds should increase the average productivity of the whole production sector, it will also enlarge the share of Non-performing incumbents, leading to a persistent decrease in total productivity in the long run.

2.4 Conclusions

We found that a technology improvement discourages debt roll-over, reducing the share of Non-performing loans and unproductive incumbent. Furthermore, an adverse shock to financial intermediaries capital triggers an ever-greening mechanism that persistently reduces aggregate productivity. We confirm the empirical literature thoughts on Zombie firms' dynamic: in an expansionary scenario, banks prefer to finance productive firms and the entire economy benefits from the innovation, while, in a crisis scenario (in our case the banking crises simulation), the opportunistic behavior of keeping zombie firms' alive arises and the recovery from the recession is slower. In our model, the monitoring role of households is determinant in limiting the leverage level of financial intermediaries. To assume that the monitoring conditions do not change with loans ever-greening is a strong assumption that may not represent the financial scenario of the last decades. We are willing to enrich the model, analyzing the role of external supervision. Apart from that, our future steps are the following: we will enrich our model with price stickiness and we will extend the model with a capital goods sector. We suggest that our results, obtained with an almost frictionless business cycle model, can be magnified by frictions in price settings. The presence of a Capital sector, as suggested by the work presented in the first chapter, has an important role in containing the creative destruction process and we want to inspect how the adjustment in the relative price of investment good can interact with the banks' rollover decision.

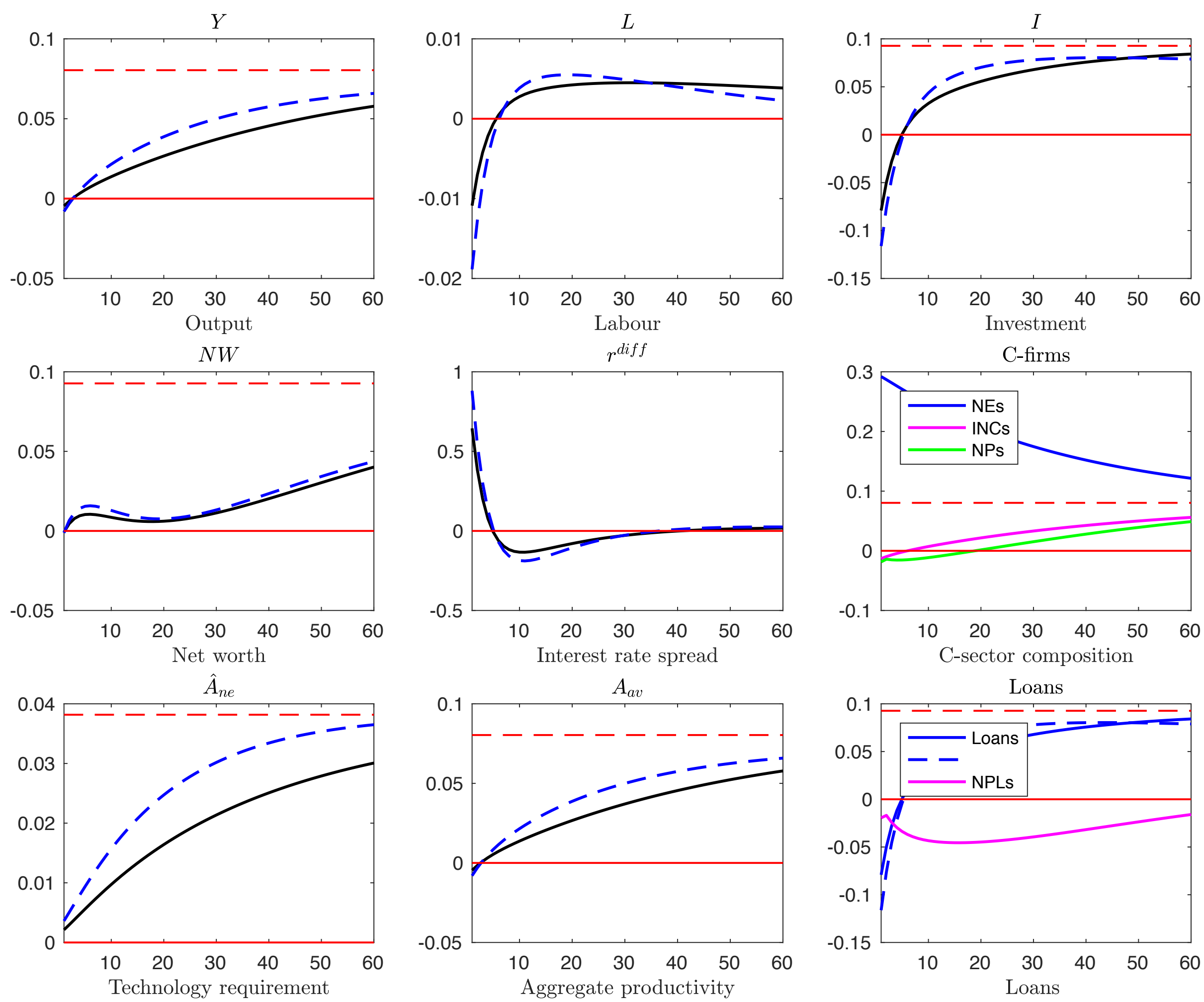

Figure 2.1: IRFs to a permanent technology improvement, $\rho^z = 0$ $\sigma^z = 0.05$. Model responses in black, benchmark responses in green dashed line.

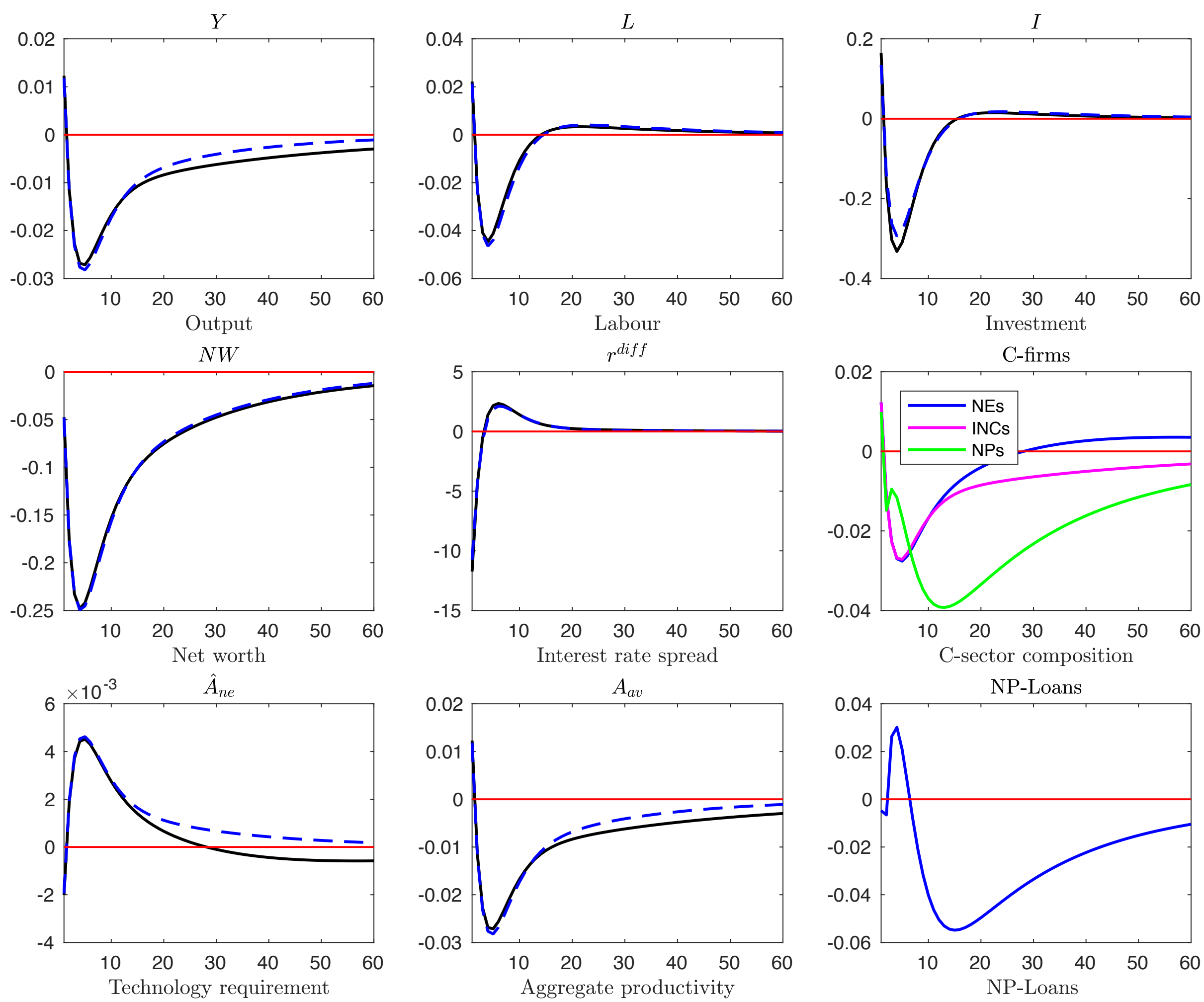

Figure 2.2: IRFs to a negative shock to intermediaries net worth, $\rho^{\zeta} = 0.66$ $\sigma^{\zeta} = 0.05$. Model responses in black, benchmark responses in green dashed line.

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2.5 Appendix to Chapter 2.

Model Dynamic Equations:

Exogenous processes:

Evolution of production sector technology frontier:

$$\underline{z}_t = g_t \, \underline{z}_{t-1} \tag{2.58}$$

$$ln\,(g_t^z) = ln\,(g^z) + \varepsilon_t^z \tag{2.59}$$

$$ln\,(\nu_t) = \rho^\nu \, ln\,(\nu_{t-1}) + (1-\rho^\nu)\, ln\,(\nu) + \varepsilon_t^\nu \tag{2.60}$$

Fixed costs of production:

$$\phi_t^{NE} = g_{*C}^t \phi^{NE} \tag{2.61}$$

$$\phi_t^I = g_{*C}^t \phi^I \tag{2.62}$$

Households:

$$\lambda_t = \frac{1}{C_t} \tag{2.63}$$

$$W_t = \frac{\psi \, {L_t}^\phi}{\lambda_t} \tag{2.64}$$

$$r_t^d = \frac{\lambda_t}{\beta \lambda_{t+1}} \tag{2.65}$$

$$\Lambda_t = \frac{\lambda_t}{\lambda_{t-1}} \tag{2.66}$$

Production sector:

Productivity thresholds:

$$\hat{A}_t^{NE} = \left(\frac{\phi_t^{NE}}{1-\gamma}\right)^{1-\gamma} \left[\frac{\left(\frac{(r_t^k - 1 + \delta)}{\alpha}\right)^\alpha \left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}}{\gamma}\right]^\gamma \tag{2.67}$$

$$\hat{A}_t^{I} = \left(\frac{\phi_t^{I}}{1-\gamma}\right)^{1-\gamma} \left[\frac{\left(\frac{(r_t^k - 1 + \delta)}{\alpha}\right)^\alpha \left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}}{\gamma}\right]^\gamma \tag{2.68}$$

$$\hat{A}_t^{NP} = \left(\frac{\phi_t^{I} - r_t^k \, \mu \, \bar{b}_t}{1-\gamma}\right)^{1-\gamma} \left[\frac{\left(\frac{(r_t^k - 1 + \delta)}{\alpha}\right)^\alpha \left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}}{\gamma}\right]^\gamma \tag{2.69}$$

Mass of firms:

$$NE_t = \left(\frac{z_t}{\hat{A}_t^{NE}}\right)^{\xi} \tag{2.70}$$

$$INC_t = \eta_{t-1} \left(\frac{\hat{A}_{t-1}^{NE}}{\hat{A}_t^{I}}\right)^{\xi} \tag{2.71}$$

$$NP_t = \eta_{t-1} \left[\left(\frac{\hat{A}_{t-1}^{NE}}{\hat{A}_t^{NP}}\right)^{\xi} - \left(\frac{\hat{A}_{t-1}^{NE}}{\hat{A}_t^{I}}\right)^{\xi}\right] \tag{2.72}$$

$$\eta_t = NP_t + NE_t + INC_t \tag{2.73}$$

Exit

$$E_t = \eta_{t-1} \left[1 - \left(\frac{\hat{A}_{t-1}^{NE}}{\hat{A}_t^{I}}\right)^{\xi}\right] \tag{2.74}$$

Output

$$Y_t^{NE} = \frac{\xi}{(\xi(1-\gamma)-1)}\, NE_t \phi_t^{NE} \tag{2.75}$$

$$Y_t^{I} = \frac{\xi}{(\xi(1-\gamma)-1)}\, INC_t \phi_t^{I} \tag{2.76}$$

$$Y_t^{NP} = \frac{\xi}{(\xi(1-\gamma)-1)}\, NP_t r_t^k \,\mu\, \bar{b}_t \tag{2.77}$$

$$Y_t = Y_t^{NP} + Y_t^{NE} + Y_t^{I} \tag{2.78}$$

Production input demand:

$$K_{t-1} = B_t = \frac{\alpha\,\gamma}{(r_t^k - 1 + \delta)} \frac{\xi}{(\xi(1-\gamma)-1)} \left(NE_t \phi_t^{NE} + INC_t \phi_t^{I} + NP_t r_t^k \,\mu\, \bar{b}_t\right) \tag{2.79}$$

$$L_t = \frac{(1-\alpha)\,\gamma}{W_t} \frac{\xi}{(\xi(1-\gamma)-1)} \left(NE_t \phi_t^{NE} + INC_t \phi_t^{I} + NP_t r_t^k \,\mu\, \bar{b}_t\right) \tag{2.80}$$

Financial sector:

Ex ante demand of loans:

$$\bar{b}_t = \frac{\alpha}{(r_t^k - 1 + \delta)} \left[\frac{\gamma \frac{\xi}{\xi-1} \hat{A}_{t-1}^{NE}}{\left[\left(\frac{(r_t^k - 1 + \delta)}{\alpha}\right)^{\alpha} \left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}\right]^{\gamma}}\right]^{\frac{1}{1-\gamma}} \tag{2.81}$$

Demands of loans:

$$B_t^{NE} = \frac{\alpha\,\gamma}{(r_t^k - 1 + \delta)} \frac{\xi}{(\xi(1-\gamma)-1)} NE_t \,\phi_t^{NE} \tag{2.82}$$

$$B_t^I = \frac{\alpha\,\gamma}{(r_t^k - 1 + \delta)}\,\frac{\xi}{(\xi(1-\gamma)-1)}\,INC_t\,\phi_t^I \tag{2.83}$$

$$B_t^{NP} = \frac{\alpha\,\gamma}{(r_t^k - 1 + \delta)}\,\frac{\xi}{(\xi(1-\gamma)-1)} NP_t\,\bar{b}_t\,r_t^k\,\mu \tag{2.84}$$

$$B_t = B_t^{NE} + B_t^I + B_t^{NP} \tag{2.85}$$

Non-performing firms transfer:

$$\Pi_t^{NP} = Y_t^{NP} - W_t L_t^{NP} - NP_t^{NP}\phi_t^I \tag{2.86}$$

Return on loans:

$$r_t^b = \frac{(r_t^k B_t^I + r_t^k B_t^{NE} + \Pi_t^{NP} - \mu r_t^k \bar{b}_t E_t)}{B_t} \tag{2.87}$$

Optimality conditions for financial intermediaries:

$$\Phi_t^b = \frac{\nu_t^b}{\lambda^b - \mu_t^b} \tag{2.88}$$

$$n_t = r_{t-1}^d + \left(r_t^b - r_{t-1}^d\right)\,\Phi_{t-1}^b \tag{2.89}$$

$$m_t = n_t\,\frac{\Phi_t^b}{\Phi_{t-1}^b} \tag{2.90}$$

$$\mu_t^b = \beta\,(1-\theta_b)\,\Lambda_{t+1}\,\left(r_{t+1}^b - r_t^d\right) + \Lambda_{t+1}\,\beta\,\theta_b\,m_{t+1}\,\mu_{t+1}^b \tag{2.91}$$

$$\nu_t^b = r_t^d\,\beta\,(1-\theta_b)\,\Lambda_{t+1} + \Lambda_{t+1}\,\beta\,\theta_b\,n_{t+1}\,\nu_{t+1}^b \tag{2.92}$$

$$NW_t = n_t\,\theta_b\,NW_{t-1}\nu_t + K_{t-1}\,\omega \tag{2.93}$$

$$r_t^{diff} = r_{t+1}^b - r_t^d \tag{2.94}$$

Credit policy:

$$K_t = \Phi_t^b\,NW_t \tag{2.95}$$

Market clearing conditions:

$$K_t = I_t + K_{t-1}\,(1-\delta) \tag{2.96}$$

$$C_t = Y_t - I_t - INC_t\,\phi_t^I - NE_t\,\phi_t^{NE} - NP_t \mu r_t^k \bar{b}_{t-1}^I \tag{2.97}$$

Appendix A. : Aggregation.

A.1. Firms' demand of loans.

A.1.1. The ex-ante demand of loans: At the end of $t-1$, Incumbents forecast their productivity and maximize expected profits[5] according to:

$$E_{t-1}\{\pi_t^I\} = E_{t-1}\{A_t^I\}\left[(\bar{k}_{t-1}^I)^{\alpha}(E_{t-1}\{l_t^I\})^{(1-\alpha)}\right]^{\gamma} -$$

$$- \left[(\bar{k}_{t-1}^I)^{\alpha}(E_{t-1}\{l_t^I\})^{(1-\alpha)}\right] \cdot \left[\frac{(r_t^k - 1 + \delta)}{\alpha}\right]^{\alpha}\left[\frac{w_t}{(1-\alpha)}\right]^{(1-\alpha)} - \phi_t^I \tag{2.98}$$

Where, the solutions to the ex-ante maximization are:

$$\bar{k}_{t-1}^I = \alpha\gamma\frac{E_{t-1}\{y_t^I\}}{(r_t^k - 1 + \delta)} \tag{2.99}$$

$$l_t^I = (1-\alpha)\gamma\frac{E_{t-1}\{y_t^I\}}{W_t} \tag{2.100}$$

$$\frac{l_t^I}{\bar{k}_{t-1}^I} = \frac{1-\alpha}{\alpha}\frac{(r_t^k - 1 + \delta)}{W_t} \tag{2.101}$$

If we assume that the Incumbents' expected productivity corresponds with the average of the Pareto distribution[6], the demand of loans based on technology expectations $\bar{k}_{t-1}^I = \bar{b}_{t-1}^I$ will be:

$$\bar{b}_{t-1}^I = \frac{\alpha}{(r_t^k - 1 + \delta)}\left[\frac{\gamma E_{t-1}\{A_t^I\}}{\left[\left(\frac{(r_t^k - 1+\delta)}{\alpha}\right)^{\alpha}\left(\frac{w_t}{(1-\alpha)}\right)^{(1-\alpha)}\right]^{\gamma}}\right]^{\frac{1}{1-\gamma}} = \frac{\alpha}{(r_t^k - 1 + \delta)}\left[\frac{\frac{\gamma\xi}{\xi-1}\hat{A}_{t-1}^{NE}}{\left[\left(\frac{(r_t^k - 1+\delta)}{\alpha}\right)^{\alpha}\left(\frac{w_t}{(1-\alpha)}\right)^{(1-\alpha)}\right]^{\gamma}}\right]^{\frac{1}{1-\gamma}} \tag{2.102}$$

A.1.2. Aggregate Incumbents' demand of loans: After purchasing the capital stock, incumbent firms will learn their idiosyncratic productivity $A_{j,t}^I$ and will freely adjust their demand of capital. So, they will maximize their final profit $\pi_{j,t}$, choosing the optimal production input demand,

$$\pi_{j,t}^I = y_{j,t}^I - (r_t^k - 1 + \delta)k_{j,t}^I - w_t l_{j,t}^I - \phi_t^I \tag{2.103}$$

$$s.t. \quad y_t = A_{j,t}^I\left[(k_j^I)^{\alpha}(l_{j,t}^I)^{(1-\alpha)}\right]^{\gamma},$$

$$b_t^I = k_{j,t}^I \tag{2.104}$$

[5]Expectation are only on idiosyncratic productivity, firms are aware of the aggregate state of the economy and, consequentially, of the production costs.

[6]Incumbents draw their productivity form a Pareto distribution with shape ξ and support $A_{j,t}^I = [\hat{A}_{t-1}^{NE}, \infty)$

And we can derive the following first order conditions:

$$k_{j,t} = \alpha\gamma \frac{y_{j,t}}{(r_t^k - 1 + \delta)} \tag{2.105}$$

$$l_{j,t} = (1-\alpha)\gamma \frac{y_{j,t}}{w_t} \tag{2.106}$$

$$\frac{l^I_{j,t}}{k^I_{j,t}} = \frac{1-\alpha}{\alpha} \frac{(r_t^k - 1 + \delta)}{W_t} \tag{2.107}$$

We can aggregate (2.105), integrating for the INCs' idiosyncratic productivity levels.

Given the constraints (2.104) and the demand of capital (2.105), we can derive the firm j^{NP} demand of loans in t,

$$b^I_{j,t} = k^I_{j,t} = \left(\frac{(r_t^k - 1 + \delta)}{\alpha\gamma}\right)^{\frac{1}{\alpha\gamma - 1}} \cdot (l^I_{j,t})^{\frac{(1-\alpha)\gamma}{1-\alpha\gamma}} \cdot (A^I_{j,t})^{\frac{1}{1-\alpha\gamma}} \rightarrow$$

$$b^I_{j,t} = \left(\frac{1-\alpha}{\alpha} \frac{(r_t^k - 1 + \delta)}{W_t} b_{j,t}\right)^{\frac{(1-\alpha)\gamma}{1-\alpha\gamma}} \cdot \left(\frac{(r_t^k - 1 + \delta)}{\alpha\gamma}\right)^{\frac{1}{\alpha\gamma - 1}} \cdot (A^I_{j,t})^{\frac{1}{1-\alpha\gamma}} \rightarrow$$

$$b^I_{j,t} = \left(\frac{1-\alpha}{\alpha} \frac{(r_t^k - 1 + \delta)}{W_t}\right)^{\frac{(1-\alpha)\gamma}{1-\gamma}} \cdot \left(\frac{\alpha\gamma}{(r_t^k - 1 + \delta)}\right)^{\frac{1}{1-\gamma}} \cdot (A^I_{j,t})^{\frac{1}{1-\gamma}} \tag{2.108}$$

Given the PDF,

$$f_t(A^I_t) = \int_{\hat{A}^{NE}_{t-1}}^{+\infty} \frac{\xi(\hat{A}^{NE}_{t-1})^{\xi}}{(A^I_t)^{\xi+1}} d(A^I_t) = 1 \tag{2.109}$$

and the mass of surviving Incumbents in t, with productivity $A^I_t \geq \hat{A}^I_t$:

$$INC_t = \eta_{t-1} f_t(\hat{A}^I_t) = \eta_{t-1} \int_{\hat{A}^I_t}^{+\infty} \frac{\xi(\hat{A}^{NE}_{t-1})^{\xi}}{(A^I_t)^{\xi+1}} d(A^I_t) = \eta_{t-1} \left(\frac{\hat{A}^{NE}_{t-1}}{\hat{A}^I_t}\right)^{\xi} \tag{2.110}$$

We can write the aggregate demand of capital from INCs:

$$B^I_t = \left(\frac{1-\alpha}{\alpha} \frac{(r_t^k - 1 + \delta)}{W_t}\right)^{\frac{(1-\alpha)\gamma}{1-\gamma}} \cdot \left(\frac{\alpha\gamma}{(r_t^k - 1 + \delta)}\right)^{\frac{1}{1-\gamma}} \cdot \int_{\hat{A}^I_t}^{+\infty} (A^I_t)^{\frac{1}{1-\gamma}} dF(A^I_{j,t}) \rightarrow$$

$$B^I_t = \left(\frac{1-\alpha}{\alpha} \frac{(r_t^k - 1 + \delta)}{W_t}\right)^{\frac{(1-\alpha)\gamma}{1-\gamma}} \cdot \left(\frac{\alpha\gamma}{(r_t^k - 1 + \delta)}\right)^{\frac{1}{1-\gamma}} \cdot \int_{\hat{A}^I_t}^{+\infty} (A^I_t)^{\frac{1}{1-\gamma}} f(A^I_{j,t}) d(A^I_{j,t}) \rightarrow$$

$$B^I_t = \left(\frac{1-\alpha}{\alpha} \frac{(r_t^k - 1 + \delta)}{W_t}\right)^{\frac{(1-\alpha)\gamma}{1-\gamma}} \cdot \left(\frac{\alpha\gamma}{(r_t^k - 1 + \delta)}\right)^{\frac{1}{1-\gamma}} \cdot \xi\eta_{t-1}(\hat{A}^{NE}_{t-1})^{\xi} \int_{\hat{A}^I_t}^{+\infty} (A^I_t)^{\frac{1}{1-\gamma} - \xi - 1} d(A^I_{j,t}) \rightarrow$$

$$B^I_t = \left(\frac{1-\alpha}{\alpha} \frac{r_t^k}{W_t}\right)^{\frac{(1-\alpha)\gamma}{1-\gamma}} \cdot \left(\frac{\alpha\gamma}{(r_t^k - 1 + \delta)}\right)^{\frac{1}{1-\gamma}} \cdot \frac{\xi}{\xi(1-\gamma) - 1} \cdot \left(\frac{\left[\left[\frac{(r_t^k - 1 + \delta)}{\alpha}\right]^{\alpha} \left[\frac{W_t}{(1-\alpha)}\right]^{(1-\alpha)}\right]^{\gamma}}{\gamma^{\gamma}}\right)^{\frac{1}{1-\gamma}} INC_t \phi^I_t \rightarrow$$

$$B_t^I = \left(\frac{\alpha}{(r_t^k - 1 + \delta)}\right) \frac{\gamma\xi}{\xi(1-\gamma) - 1} \cdot INC_t \phi_t^I \tag{2.111}$$

A.1.3. Aggregate New Entrants' demand of loans: New entrants demand loans ti financial intermediaries when they learn their idiosyncratic productivity at the beginning of t. The profit function of a j^{NE} firm is :

$$\pi_{j,t}^{NE} = y_{j,t}^{NE} - (r_t^k - 1 + \delta)k_{j,t-1}^{NE} - w_t l_{j,t}^{NE} - \phi_t^{NE} \tag{2.112}$$

$$s.t. \quad y_t = A_{j,t}^{NE}\left[(k_{j,t}^{NE})^{\alpha}(l_{j,t}^{NE})^{(1-\alpha)}\right]^{\gamma},$$

$$b_t^{NE} = k_{j,t}^{NE} \tag{2.113}$$

We can derive the following first order conditions:

$$k_{j,t}^{NE} = \alpha\gamma \frac{y_{j,t}^{NE}}{(r_t^k - 1 + \delta)} \tag{2.114}$$

$$l_{j,t}^{NE} = (1-\alpha)\gamma \frac{y_{j,t}^{NE}}{w_t} \tag{2.115}$$

$$\frac{l_{j,t}^{NE}}{k_{j,t}^{NE}} = \frac{1-\alpha}{\alpha}\frac{(r_t^k - 1 + \delta)}{W_t} \tag{2.116}$$

Given the constraints (2.113) and the demand of capital (2.114), we can derive the firm j^{NE} demand of loans in t, integrating for the NEs' idiosyncratic productivity levels.

$$b_{j,t}^{NE} = k_{j,t-1}^{NE} = \left(\frac{(r_t^k - 1 + \delta)}{\alpha\gamma}\right)^{\frac{1}{\alpha\gamma - 1}} \cdot (l_{j,t}^{NE})^{\frac{(1-\alpha)\gamma}{1-\alpha\gamma}} \cdot (A_{j,t}^{NE})^{\frac{1}{1-\alpha\gamma}} \rightarrow$$

$$b_{j,t}^{NE} = \left(\frac{1-\alpha}{\alpha}\frac{(r_t^k - 1 + \delta)}{W_t}k_{j,t-1}\right)^{\frac{(1-\alpha)\gamma}{1-\alpha\gamma}} \cdot \left(\frac{(r_t^k - 1 + \delta)}{\alpha\gamma}\right)^{\frac{1}{\alpha\gamma - 1}} \cdot (A_{j,t}^{NE})^{\frac{1}{1-\alpha\gamma}} \rightarrow$$

$$b_{j,t}^{NE} = \left(\frac{1-\alpha}{\alpha}\frac{(r_t^k - 1 + \delta)}{W_t}\right)^{\frac{(1-\alpha)\gamma}{1-\gamma}} \cdot \left(\frac{\alpha\gamma}{(r_t^k - 1 + \delta)}\right)^{\frac{1}{1-\gamma}} \cdot (A_{j,t}^{NE})^{\frac{1}{1-\gamma}} \tag{2.117}$$

Given the PDF,

$$f_t(A_t^{NE}) = \int_{\underline{z}_t}^{+\infty} \frac{\xi \underline{z}_t^{\xi}}{(A_t^{NE})^{\xi+1}} d(A_t^{NE}) = 1 \tag{2.118}$$

and the mass of entrants in t, with productivity $A_t^{NE} \geq \hat{A}_t^{NE}$:

$$NE_t = \int_{\hat{A}_t^{NE}}^{+\infty} \frac{\xi \underline{z}_t^{\xi}}{(A_t^I)^{\xi+1}} d(A_t^I) = \left(\frac{\underline{z}_t}{\hat{A}_t^{NE}}\right)^{\xi} \tag{2.119}$$

We can write the aggregate demand of loans from NEs:

$$B_t^{NE} = \left(\frac{1-\alpha}{\alpha}\frac{(r_t^k - 1 + \delta)}{W_t}\right)^{\frac{(1-\alpha)\gamma}{1-\gamma}} \cdot \left(\frac{\alpha\gamma}{(r_t^k - 1 + \delta)}\right)^{\frac{1}{1-\gamma}} \cdot \int_{\hat{A}_t^{NE}}^{+\infty} (A_t^{NE})^{\frac{1}{1-\gamma}} dF(A_{j,t}^{NE}) \rightarrow$$

$$B_t^{NE} = \left(\frac{1-\alpha}{\alpha}\frac{(r_t^k - 1 + \delta)}{W_t}\right)^{\frac{(1-\alpha)\gamma}{1-\gamma}} \cdot \left(\frac{\alpha\gamma}{(r_t^k - 1 + \delta)}\right)^{\frac{1}{1-\gamma}} \cdot \int_{\hat{A}_t^{NE}}^{+\infty} (A_t^{NE})^{\frac{1}{1-\gamma}} f(A_{j,t}^{NE}) d(A_{j,t}^{NE}) \rightarrow$$

$$B_t^{NE} = \left(\frac{1-\alpha}{\alpha}\frac{(r_t^k - 1 + \delta)}{W_t}\right)^{\frac{(1-\alpha)\gamma}{1-\gamma}} \cdot \left(\frac{\alpha\gamma}{(r_t^k - 1 + \delta)}\right)^{\frac{1}{1-\gamma}} \cdot \xi \underline{z}_t^{\xi} \int_{\hat{A}_t^{NE}}^{+\infty} (A_t^{NE})^{\frac{1}{1-\gamma}-\xi-1} d(A_{j,t}^{NE}) \rightarrow$$

$$B_t^{NE} = \left(\frac{1-\alpha}{\alpha}\frac{(r_t^k - 1 + \delta)}{W_t}\right)^{\frac{(1-\alpha)\gamma}{1-\gamma}} \cdot \left(\frac{\alpha\gamma}{(r_t^k - 1 + \delta)}\right)^{\frac{1}{1-\gamma}} \cdot \frac{\xi(1-\gamma)}{\xi(1-\gamma)-1} \left(\frac{\underline{z}_t}{\hat{A}_t^{NE}}\right)^{\xi} \left(\hat{A}_t^{NE}\right)^{\frac{1}{1-\gamma}} \rightarrow$$

$$B_t^{NE} = \left(\frac{\alpha}{(r_t^k - 1 + \delta)}\right) \frac{\gamma\xi}{\xi(1-\gamma)-1} \cdot NE_t \phi_t^{NE} \tag{2.120}$$

A.1.4. Aggregate renegotiated loan: In t the intermediary has the incentive to roll over debt of otherwise exiting firms (Non Performing firms) if the expected value of saving is higher than the repossession value of the defaulted loan:

$$A_{j,t}^I (k_{j,t}^{NP})^{\alpha\gamma} (l_{j,t}^{NP})^{(1-\alpha)\gamma} - w_t l_{j,t}^{NP} + r_t^k (\bar{b}_{t-1}^I - k_{j,t}^{NP}) - \phi_t^I + (1-\delta) k_{j,t}^{NP} \succeq r_t^k (1-\mu) \bar{b}_{t-1}^I \tag{2.121}$$

The optimal amount of renegotiated loan and the labour supplied by financial intermediaries on behalf of the saved firm j^{NP}, will be:

$$b_{j,t}^{NP} = \alpha\gamma \frac{y_{j,t}}{(r_t^k - 1 + \delta)} \tag{2.122}$$

$$l_{j,t}^{NP} = (1-\alpha)\gamma \frac{y_{j,t}^{NP}}{w_t} \tag{2.123}$$

$$\frac{l_{j,t}^{NP}}{b_{j,t}^{NP}} = \frac{1-\alpha}{\alpha}\frac{(r_t^k - 1 + \delta)}{W_t} \tag{2.124}$$

From (2.122) we can rewrite the quantity of capital addressed to save a generic firm j^{NP} in terms of debt roll-over:

$$b_{j,t}^{NP} = \left(\frac{(r_t^k - 1 + \delta)}{\alpha\gamma}\right)^{\frac{1}{\alpha\gamma-1}} \cdot (l_{j,t}^{NP})^{\frac{(1-\alpha)\gamma}{1-\alpha\gamma}} \cdot (A_{j,t}^I)^{\frac{1}{1-\alpha\gamma}} \rightarrow$$

$$b_{j,t}^{NP} = \left(\frac{1-\alpha}{\alpha}\frac{(r_t^k - 1 + \delta)}{W_t} k_{j,t-1}\right)^{\frac{(1-\alpha)\gamma}{1-\alpha\gamma}} \cdot \left(\frac{(r_t^k - 1 + \delta)}{\alpha\gamma}\right)^{\frac{1}{\alpha\gamma-1}} \cdot (A_{j,t}^I)^{\frac{1}{1-\alpha\gamma}} \rightarrow$$

$$b_{j,t}^{NP} = \left(\frac{1-\alpha}{\alpha}\frac{(r_t^k - 1 + \delta)}{W_t}\right)^{\frac{(1-\alpha)\gamma}{1-\gamma}} \cdot \left(\frac{\alpha\gamma}{(r_t^k - 1 + \delta)}\right)^{\frac{1}{1-\gamma}} \cdot (A_{j,t}^I)^{\frac{1}{1-\gamma}} \tag{2.125}$$

Given the PDF,

$$f_t(A_t^I) = \int_{\hat{A}_{t-1}^{NE}}^{+\infty} \frac{\xi(\hat{A}_{t-1}^{NE})^{\xi}}{(A_t^I)^{\xi+1}} d(A_t^I) = 1 \tag{2.126}$$

and the mass of saved firms in t, with productivity $\hat{A}_t^I \geq A_t^I \geq \hat{A}_t^{NP}$:

$$NP_t = \eta_{t-1} f_t(\hat{A}_t^{NP}) = \eta_{t-1} \int_{\hat{A}_t^{NP}}^{\hat{A}_t^I} \frac{\xi(\hat{A}_{t-1}^{NE})^\xi}{(A_t^{NP})^{\xi+1}} d(A_t^{NP}) = \eta_{t-1} \left[\left(\frac{\hat{A}_{t-1}^{NE}}{\hat{A}_t^{NP}} \right)^\xi - \left(\frac{\hat{A}_{t-1}^{NE}}{\hat{A}_t^I} \right)^\xi \right] \tag{2.127}$$

We can write the aggregate amount of renegotiated loans:

$$B_t^{NP} = \left(\frac{1-\alpha}{\alpha} \frac{(r_t^k - 1 + \delta)}{W_t} \right)^{\frac{(1-\alpha)\gamma}{1-\gamma}} \cdot \left(\frac{\alpha\gamma}{(r_t^k - 1 + \delta)} \right)^{\frac{1}{1-\gamma}} \cdot \int_{\hat{A}_t^{NP}}^{\hat{A}_t^I} (A_t^I)^{\frac{1}{1-\gamma}} dF(A_{j,t}^I) \rightarrow$$

$$B_t^{NP} = \left(\frac{1-\alpha}{\alpha} \frac{(r_t^k - 1 + \delta)}{W_t} \right)^{\frac{(1-\alpha)\gamma}{1-\gamma}} \cdot \left(\frac{\alpha\gamma}{(r_t^k - 1 + \delta)} \right)^{\frac{1}{1-\gamma}} \cdot \int_{\hat{A}_t^{NP}}^{\hat{A}_t^I} (A_t^I)^{\frac{1}{1-\gamma}} f(A_{j,t}^I) d(A_{j,t}^I) \rightarrow$$

$$B_t^{NP} = \left(\frac{1-\alpha}{\alpha} \frac{(r_t^k - 1 + \delta)}{W_t} \right)^{\frac{(1-\alpha)\gamma}{1-\gamma}} \cdot \left(\frac{\alpha\gamma}{(r_t^k - 1 + \delta)} \right)^{\frac{1}{1-\gamma}} \cdot \xi \eta_{t-1} (\hat{A}_{t-1}^{NE})^\xi \int_{\hat{A}_t^{NP}}^{\hat{A}_t^I} (A_t^I)^{\frac{1}{1-\gamma} - \xi - 1} d(A_{j,t}^I) \rightarrow$$

$$B_t^{NP} = \left(\frac{1-\alpha}{\alpha} \frac{r_t^k}{W_t} \right)^{\frac{(1-\alpha)\gamma}{1-\gamma}} \cdot \left(\frac{\alpha\gamma}{(r_t^k - 1 + \delta)} \right)^{\frac{1}{1-\gamma}} \cdot \frac{\xi(1-\gamma)}{\xi(1-\alpha)\gamma - 1} \eta_{t-1} (\hat{A}_{t-1}^{NE})^\xi \left[(\hat{A}_t^{NP})^{\frac{1}{1-\gamma} - \xi} - (\hat{A}_t^I)^{\frac{1}{1-\gamma} - \xi} \right] \rightarrow$$

$$B_t^{NP} = \left(\frac{\alpha\gamma}{(r_t^k - 1 + \delta)} \right) \frac{\xi}{\xi(1-\gamma) - 1} \cdot NP_t \mu r_t^k \bar{b}_{t-1}^I \tag{2.128}$$

A.2. Firms' demand of labour.

From conditions (2.107), (2.116) and (2.124), we can easily derive the demand of labour from New Entrants, Incumbents and financial intermediaries in behalf of saved firms.

$$L_t^I = \left(\frac{(1-\alpha)\gamma}{W_t} \right) \frac{\gamma\xi}{\xi(1-\gamma) - 1} \cdot INC_t \phi_t^I \tag{2.129}$$

$$L_t^{NE} = \left(\frac{(1-\alpha)\gamma}{W_t} \right) \frac{\gamma\xi}{\xi(1-\gamma) - 1} \cdot NE_t \phi_t^{NE} \tag{2.130}$$

$$L_t^{NP} = \left(\frac{(1-\alpha)\gamma}{W_t} \right) \frac{\xi}{\xi(1-\gamma) - 1} \cdot NP_t \mu r_t^k \bar{b}_{t-1}^I \tag{2.131}$$

A.3. Aggregate output.

The aggregate output will be the sum of the output of New Entrants, Incumbents and Non Performing Firms':

$$Y_t = Y_t^I + Y_t^{NE} + Y_t^{NP} \tag{2.132}$$

Starting from the production function, we can obtain the aggregate output of Incumbents as:

$$Y_t^I = \int_{\hat{A}_t^I}^{+\infty} A_t^I (Z_t^I)^\gamma dF(A_t^I) \rightarrow$$

$$Y_t^I = \int_{\hat{A}_t^I}^{+\infty} A_t^I \left(\frac{A_t^I \gamma}{\left[\frac{(r_t^k - 1 + \delta)}{\alpha}\right]^\alpha \left[\frac{W_t}{(1-\alpha)}\right]^{(1-\alpha)}} \right)^{\frac{\gamma}{1-\gamma}} dF(A_{j,t}^I) \rightarrow$$

$$Y_t^I = \left(\frac{\gamma}{\left[\frac{(r_t^k - 1 + \delta)}{\alpha}\right]^\alpha \left[\frac{W_t}{(1-\alpha)}\right]^{(1-\alpha)}} \right)^{\frac{\gamma}{1-\gamma}} \int_{\hat{A}_t^I}^{+\infty} (A_t^I)^{\frac{1}{1-\gamma}} f(A_{j,t}^I) d(A_{j,t}^I) \rightarrow$$

$$Y_t^I = \left(\frac{\gamma}{\left[\frac{(r_t^k - 1 + \delta)}{\alpha}\right]^\alpha \left[\frac{W_t}{(1-\alpha)}\right]^{(1-\alpha)}} \right)^{\frac{\gamma}{1-\gamma}} \xi \eta_{t-1} (\hat{A}_{t-1}^{NE})^\xi \int_{\hat{A}_t^I}^{+\infty} (A_t^I)^{\frac{1}{1-\gamma} - \xi - 1} d(A_{j,t}^I) \rightarrow$$

$$Y_t^I = \left(\frac{\gamma}{\left[\frac{(r_t^k - 1 + \delta)}{\alpha}\right]^\alpha \left[\frac{W_t}{(1-\alpha)}\right]^{(1-\alpha)}} \right)^{\frac{\gamma}{1-\gamma}} \xi \eta_{t-1} \left(\frac{\hat{A}_{t-1}^{NE}}{\hat{A}_t^I} \right)^\xi \int_{\hat{A}_t^I}^{+\infty} (A_t^I)^{\frac{1}{1-\gamma} - \xi - 1} d(A_{j,t}^I) \rightarrow$$

$$Y_t^I = \left(\frac{\gamma}{\left[\frac{(r_t^k - 1 + \delta)}{\alpha}\right]^\alpha \left[\frac{W_t}{(1-\alpha)}\right]^{(1-\alpha)}} \right)^{\frac{\gamma}{1-\gamma}} \frac{\xi(1-\gamma)}{\xi(1-\gamma) - 1} INC_t (\hat{A}_t^I)^{\frac{1}{1-\gamma}}$$

$$Y_t^I = \frac{\xi}{\xi(1-\gamma) - 1} INC_t \phi_t^I \tag{2.133}$$

Similarly, the aggregate output of New Entrants will be:

$$Y_t^{NE} = \int_{\hat{A}_t^{NE}}^{+\infty} A_t^{NE} (K_t^{NE} L_t^{NE})^\gamma dF(A_{j,t}^{NE}) \rightarrow$$

$$Y_t^{NE} = \int_{\hat{A}_t^{NE}}^{+\infty} A_t^{NE} \left(\frac{A_t^{NE} \gamma}{\left[\frac{(r_t^k - 1 + \delta)}{\alpha}\right]^\alpha \left[\frac{W_t}{(1-\alpha)}\right]^{(1-\alpha)}} \right)^{\frac{\gamma}{1-\gamma}} dF(A_{j,t}^{NE}) \rightarrow$$

$$Y_t^{NE} = \left(\frac{\gamma}{\left[\frac{(r_t^k - 1 + \delta)}{\alpha}\right]^\alpha \left[\frac{W_t}{(1-\alpha)}\right]^{(1-\alpha)}} \right)^{\frac{\gamma}{1-\gamma}} \int_{\hat{A}_t^{NE}}^{+\infty} (A_t^{NE})^{\frac{1}{1-\gamma}} f(A_{j,t}^{NE}) d(A_{j,t}^{NE}) \rightarrow$$

$$Y_t^{NE} = \left(\frac{\gamma}{\left[\frac{(r_t^k - 1 + \delta)}{\alpha}\right]^\alpha \left[\frac{W_t}{(1-\alpha)}\right]^{(1-\alpha)}} \right)^{\frac{\gamma}{1-\gamma}} \xi \underline{z}_t^\xi \int_{\hat{A}_t^{NE}}^{+\infty} (A_t^{NE})^{\frac{1}{1-\gamma} - \xi - 1} d(A_{j,t}^{NE}) \rightarrow$$

$$Y_t^{NE} = \left(\frac{\gamma}{\left[\frac{(r_t^k - 1 + \delta)}{\alpha}\right]^\alpha \left[\frac{W_t}{(1-\alpha)}\right]^{(1-\alpha)}} \right)^{\frac{\gamma}{1-\gamma}} \frac{\xi(1-\gamma)}{\xi(1-\gamma) - 1} NE_t (\hat{A}_t^{NE})^{\frac{1}{1-\gamma}}$$

$$Y_t^{NE} = \frac{\xi}{\xi(1-\gamma) - 1} NE_t \phi_t^{NE} \tag{2.134}$$

To conclude, the Non performing firms' output is given by:

$$Y_t^{NP} = \int_{\hat{A}_t^{NP}}^{\hat{A}_t^I} A_t^I (K_t^{NP} L_t^{NP})^\gamma dF(A_{j,t}^{NE}) \rightarrow$$

$$Y_t^{NP} = \int_{\hat{A}_t^{NP}}^{\hat{A}_t^I} A_t^{NE} \left(\frac{A_t^{NE}\gamma}{\left[\frac{(r_t^k - 1 + \delta)}{\alpha}\right]^\alpha \left[\frac{W_t}{(1-\alpha)}\right]^{(1-\alpha)}} \right)^{\frac{\gamma}{1-\gamma}} dF(A_{j,t}^{NE}) \rightarrow$$

$$Y_t^{NP} = \left(\frac{\gamma}{\left[\frac{(r_t^k - 1 + \delta)}{\alpha}\right]^\alpha \left[\frac{W_t}{(1-\alpha)}\right]^{(1-\alpha)}} \right)^{\frac{\gamma}{1-\gamma}} \int_{\hat{A}_t^{NP}}^{\hat{A}_t^I} (A_t^I)^{\frac{1}{1-\gamma}} f(A_{j,t}^I) d(A_{j,t}^I) \rightarrow$$

$$Y_t^{NP} = \left(\frac{\gamma}{\left[\frac{(r_t^k - 1 + \delta)}{\alpha}\right]^\alpha \left[\frac{W_t}{(1-\alpha)}\right]^{(1-\alpha)}} \right)^{\frac{\gamma}{1-\gamma}} \xi (\hat{A}_{t-1}^{NE})^\xi \int_{\hat{A}_t^{NP}}^{\hat{A}_t^I} (A_t^I)^{\frac{1}{1-\gamma} - \xi - 1} d(A_{j,t}^I) \rightarrow$$

$$Y_t^{NP} = \left(\frac{\gamma}{\left[\frac{(r_t^k - 1 + \delta)}{\alpha}\right]^\alpha \left[\frac{W_t}{(1-\alpha)}\right]^{(1-\alpha)}} \right)^{\frac{\gamma}{1-\gamma}} \frac{\xi(1-\gamma)}{\xi(1-\gamma)-1} \eta_{t-1} (\hat{A}_{t-1}^{NE})^\xi \left[(\hat{A}_t^{NP})^{\frac{1}{1-\gamma} - \xi} - (\hat{A}_t^I)^{\frac{1}{1-\gamma} - \xi} \right]$$

$$Y_t^{NP} = \frac{\xi}{\xi(1-\gamma)-1} NP_t \mu r_t^k \bar{b}_{t-1}^I \tag{2.135}$$

We can easily write the aggregate output of consumption goods in t,

$$Y_t = \frac{\xi}{\xi(1-\gamma)-1} \left(INC_t \phi_t^I + NE_t \phi_t^{NE} + NP_t \mu r_t^k \bar{b}_{t-1}^I \right) \tag{2.136}$$

Appendix B: Deterministic Steady State.

The modelled economy follows a Balanced Growth Path (BGP), the stationary variables in steady state are L_{ss}, r^d_{ss}, r^k_{ss}, r^b_{ss} and the number of firms. The other variables grow at the endogenous rate g_{*y}. Further, fixed costs of production grows at the rate $g^t_{*\phi}$ and the technology frontier $\underline{z}_t$ grows at the exogenous rates g^t_z. In order to compute the deterministic steady state, we have to identify the relation that binds the different growth rates. To clarify notation, a generic variable $x_{ss,t}$ is identified by the deterministic process $x_{ss} g^t_x$, where x_{ss} is the initial condition that we calibrate.

B.1. Households:

We can start our computation from the Households first order conditions. Since we know, form the first order condition on consumption, that C grows at the same rate of Y, we can show that the Lagrangian multiplier s.s. follows this path:

$$\lambda_{ss,t} = (C_{ss,t} - hC_{ss,t-1})^{-1} - \beta h (C_{ss,t+1} - hC_{ss,t})^{-1} \rightarrow$$

$$\lambda_{ss,t} = (C_{ss} g^t_{*y} - hC_{ss} g^{t-1}_{*y})^{-1} - \beta h (C_{ss} g^{t+1}_{*y} - hC_{ss} g^t_{*y})^{-1} \rightarrow$$

$$\lambda_{ss,t} = \frac{1}{g^t_{*y}} \left[\left(C_{ss} - \frac{hC_{ss}}{g_{*y}} \right)^{-1} - \beta h \left(C_{ss} g_{*y} - hC_{ss} \right)^{-1} \right] \tag{2.137}$$

From the Households Euler conditions, we can find the steady state return on deposits and the wage:

$$\lambda_{ss,t} = \beta \, \lambda_{ss,t+1} r^d_{ss} \rightarrow$$

$$r^d_{ss} = \frac{g_{*y}}{\beta} \tag{2.138}$$

$$W_{ss,t} = g^t_{*y} \frac{\psi \, {L_{ss}}^{\phi}}{\lambda_{ss}} \tag{2.139}$$

B.2. Production sector:

Going through the production sector, let us start by stating that the MPK of capital r^k_{ss}, that corresponds with the rental rate of loans is stationary and show it later. Once that we have defined that the wage grows at the BGP rate g_{*y} and the fixed cost grows at the rate $g_{*\phi}$, we can compute the C-sector s.s. productivity thresholds.

$$\hat{A}^{NE}_{ss,t} = \left(\frac{\phi^{NE} g^t_{*\phi}}{1-\gamma} \right)^{(1-\gamma)} \frac{\left(\left(\frac{r^k_{ss} - 1 + \delta}{\alpha} \right)^{\alpha} \left(\frac{W_{ss} g^t_{*y}}{1-\alpha} \right)^{1-\alpha} \right)^{\gamma}}{\gamma^{\gamma}} =$$

$$= (g^t_{*\phi})^{(1-\gamma)} (g^t_{*y})^{(1-\alpha)\gamma} \left(\frac{\phi^{NE}}{1-\gamma} \right)^{(1-\gamma)} \frac{\left(\left(\frac{r^k_{ss} - 1 + \delta}{\alpha} \right)^{\alpha} \left(\frac{W_{ss}}{1-\alpha} \right)^{1-\alpha} \right)^{\gamma}}{\gamma^{\gamma}} \tag{2.140}$$

$$\hat{A}^{I}_{ss,t} = \left(\frac{\phi^I g^t_{*\phi}}{1-\gamma}\right)^{(1-\gamma)} \frac{\left(\left(\frac{r^k_{ss}-1+\delta}{\alpha}\right)^{\alpha}\left(\frac{W_{ss}g^t_{*y}}{1-\alpha}\right)^{1-\alpha}\right)^{\gamma}}{\gamma^{\gamma}} =$$

$$= (g^t_{*\phi})^{(1-\gamma)}(g^t_{*y})^{(1-\alpha)\gamma}\left(\frac{\phi^I}{1-\gamma}\right)^{(1-\gamma)} \frac{\left(\left(\frac{r^k_{ss}-1+\delta}{\alpha}\right)^{\alpha}\left(\frac{W_{ss}}{1-\alpha}\right)^{1-\alpha}\right)^{\gamma}}{\gamma^{\gamma}} \tag{2.141}$$

The technology thresholds of NPs is affected by the trend of the ex-demand of loans $\bar{b}_{t-1}$.

$$\bar{b}^{I}_{ss,t-1} = \frac{\alpha}{(r^k_{ss}-1+\delta)}\left[\frac{\frac{\gamma\xi}{\xi-1}\hat{A}^{NE}_{ss}(g^{t-1}_{*\phi})^{(1-\gamma)}(g^{t-1}_{*y})^{(1-\alpha)\gamma}}{\left[\left(\frac{(r^k_{ss}-1+\delta)}{\alpha}\right)^{\alpha}\left(\frac{w_{ss}g^{t-1}_{*y}}{(1-\alpha)}\right)^{(1-\alpha)}\right]^{\gamma}}\right]^{\frac{1}{1-\gamma}} =$$

$$= g^{t-1}_{*\phi}\frac{\alpha}{(r^k_{ss}-1+\delta)}\left[\frac{\phi^{NE}}{(1-\gamma)}\right]\left(\frac{\gamma\xi}{\xi-1}\right)^{\frac{1}{1-\gamma}} \tag{2.142}$$

Given (2.142),

$$\hat{A}^{NP}_{ss,t} = \left(\frac{\phi^I g^t_{*\phi} - \mu r^k_{ss}\bar{b}^I_{ss}g^{t-1}_{*\phi}}{1-\gamma}\right)^{(1-\gamma)} \frac{\left(\left(\frac{r^k_{ss}-1+\delta}{\alpha}\right)^{\alpha}\left(\frac{W_{ss}g^t_{*y}}{1-\alpha}\right)^{1-\alpha}\right)^{\gamma}}{\gamma^{\gamma}} =$$

$$= (g^t_{*\phi})^{(1-\gamma)}(g^t_{*y})^{(1-\alpha)\gamma}\left(\frac{\phi^I - \frac{\mu r^k_{ss}\bar{b}^I_{ss}}{g_{*\phi}}}{1-\gamma}\right)^{(1-\gamma)} \frac{\left(\left(\frac{r^k_{ss}-1+\delta}{\alpha}\right)^{\alpha}\left(\frac{W_{ss}}{1-\alpha}\right)^{1-\alpha}\right)^{\gamma}}{\gamma^{\gamma}} \tag{2.143}$$

As stated before, the number of firms is stationary. Substituting (2.141), (2.140) and (2.143) in the INCs, NEs and NPs equations,

$$NE_{ss} = \left(\frac{z_{ss}g^t_z}{\hat{A}^{NE}_{ss,t}}\right)^{\xi} = \left(\frac{z_{ss}g^t_z}{(g^t_{*\phi})^{(1-\gamma)}(g^t_{*y})^{(1-\alpha)\gamma}\hat{A}^{NE}_{ss}}\right)^{\xi} \tag{2.144}$$

$$INC_{ss} = \eta_{ss}\left(\frac{\hat{A}^{NE}_{ss,t-1}}{(\hat{A}^{I}_{ss})^t}\right)^{\xi} = \eta_{ss}\left(\frac{\hat{A}^{NE}_{ss}(g^{t-1}_{*\phi})^{(1-\gamma)}(g^{t-1}_{*y})^{(1-\alpha)\gamma}}{\hat{A}^{I}_{ss}(g^{t}_{*\phi})^{(1-\gamma)}(g^{t}_{*y})^{(1-\alpha)\gamma}}\right)^{\xi} = \frac{\eta_{ss}}{\left[(g_{*\phi})^{(1-\gamma)}(g_{*y})^{(1-\alpha)\gamma}\right]^{\xi}}\left(\frac{\hat{A}^{NE}_{ss}}{\hat{A}^{I}_{ss}}\right)^{\xi} \tag{2.145}$$

$$NP_{ss} = \eta_{ss}\left[\left(\frac{\hat{A}^{NE}_{ss,t-1}}{\hat{A}^{NP}_{ss,t}}\right)^{\xi} - \left(\frac{\hat{A}^{NE}_{ss,t-1}}{\hat{A}^{I}_{ss,t}}\right)^{\xi}\right] = \frac{\eta_{ss}}{\left[(g_{*\phi})^{(1-\gamma)}(g_{*y})^{(1-\alpha)\gamma}\right]^{\xi}}\left[\left(\frac{\hat{A}^{NE}_{ss}}{\hat{A}^{NP}_{ss}}\right)^{\xi} - \left(\frac{\hat{A}^{NE}_{ss}}{\hat{A}^{I}_{ss}}\right)^{\xi}\right] \tag{2.146}$$

Both Incumbents and New Entrants are stationary. Their sum will give us the stationary s.s. number of firms η_{ss}:

$$\eta_{ss} = NE_{ss} + INC_{ss} + NP_{ss} \rightarrow$$

$$\eta_{ss} = \frac{\left(\frac{z_{ss} g_z^t}{(g_{*\phi}^t)^{(1-\gamma)} (g_{*y}^t)^{(1-\alpha)\gamma} \hat{A}_{ss}^{NE}}\right)^{\xi}}{\left[1 - \left(\frac{\hat{A}_{ss}^{NE}}{\hat{A}_{ss}^{NP} (g_{*\phi})^{(1-\gamma)} (g_{*y})^{(1-\alpha)\gamma}}\right)^{\xi}\right]} \tag{2.147}$$

Where we calibrate $\left[1 - \left(\frac{\hat{A}_{ss}^{NE}}{\hat{A}_{ss}^{NP} (g_{*\phi})^{(1-\gamma)} (g_{*y})^{(1-\alpha)\gamma}}\right)^{\xi}\right] > 1$. Further, for the number of firms to be constant, it must hold that:

$$g_{*y}^t = \left[\frac{g_z^t}{(g_{*\phi}^t)^{(1-\gamma)}}\right]^{\frac{1}{(1-\alpha)\gamma}} \tag{2.148}$$

We can now use the conditions above and aggregate output to show that the fixed costs grow at the same rate of aggregate output:

$$Y_{ss,t} = \frac{\xi(1-\gamma)}{\xi(1-\gamma)-1} \left(NE_{ss}\phi^{NE} g_{*\phi}^t + INC_{ss}\phi^I g_{*\phi}^t + NP_{ss}\mu r_{ss}^k \bar{b}_{ss}^I g_{*\phi}^t\right) \rightarrow$$

$$Y_{ss} g_{*y}^t = \frac{\xi(1-\gamma)}{\xi(1-\gamma)-1} g_{*\phi}^t \left(NE_{ss}\phi^{NE} + INC_{ss}\phi^I + NP_{ss}\mu r_{ss}^k \bar{b}_{ss}^I\right) \rightarrow \tag{2.149}$$

For (2.147) to hold, requires that we calibrate $g_{*\phi}$ such that:

$$g_{*y}^t = g_{*\phi}^t \tag{2.150}$$

and,

$$g_{*y}^t = (g_z^z)^{\frac{1}{(1-\alpha)\gamma + (1-\gamma)}} \tag{2.151}$$

From the aggregate demand of capital (or loans) and labour, we can derive the last two conditions for the C-sector.

$$K_{ss,t} = B_{ss,t} = g_{*y}^t \left(\frac{\alpha}{(r_{ss}^k - 1 + \delta)}\right) \frac{\gamma\xi}{\xi(1-\gamma)-1} \left(NE_{ss}\phi^{NE} + INC_{ss}\phi^I + NP_{ss}\mu r_{ss}^k \bar{b}_{ss}^I\right) \tag{2.152}$$

$$L_{ss} = \left(\frac{1-\alpha}{W_{ss}}\right) \frac{\gamma\xi}{\xi(1-\gamma)-1} \left(NE_{ss}\phi^{NE} + INC_{ss}\phi^I + NP_{ss}\mu r_{ss}^k \bar{b}_{ss}^I\right) \tag{2.153}$$

Where the steady demand for labour (2.153) is stationary. From (2.152) and (2.149), we can finally show that r_{ss}^k is stationary,

$$r_{ss}^k = \frac{\alpha\gamma Y_{ss} g_{*y}}{K_{ss} g_{*y}} = \frac{\alpha\gamma Y_{ss}}{K_{ss}} \tag{2.154}$$

B.3. Financial sector:

Financial intermediaries return on loans is stationary, as shown in the following equation:

$$r^b_{ss} = \frac{g^t_{*y}(r^k_{ss}B^I_{ss} + r^k_{ss}B^{NE}_{ss} + \Pi^{NP}_{ss} - \mu r^k_{ss}\frac{\bar{b}_{ss,t}}{g_{*y}}E_{ss})}{g^t_{*y}B_{ss}} \tag{2.155}$$

Where, $\Pi^{NP}_{ss,t}$ is the net transfer from NPs to financial intermediaries and grows at the BGP rate g_{*y},

$$\Pi^{NP}_{ss,t} = g^t_{*y}(Y^{NP}_{ss} - w_{ss}L^{NP}_{ss} - NP^{NP}_{ss}\phi^I) = g^t_{*y}[Y^{NP}_{ss}\left[1-(1-\alpha)\gamma\right] - NP^{NP}_{ss}\phi^I] \tag{2.156}$$

and, in $\mu r^k_t \bar{b}_{t-1}E_{ss}$, the only non stationary component is given by the ex-ante demand of loans (2.142) of defaulting firms E_{ss}.

The financial firms' net worth, grows according to the following law of motion.

$$NW_{ss,t+1} = (r^b_{ss} - r^d_{ss})B_{ss,t} + r^d_t NW_{ss,t} \tag{2.157}$$

Where, we know that the loan demand grows at the rate g^t_{*y}. It follows that we can rewrite (2.157) as:

$$NW_{ss,t}\frac{g^{t+1}_{*y}}{g^t_{*y}}g^t_{*y} = (r^b_{ss} - r^d_{ss})B_{ss,t}g^t_{*y} + r^d_{ss}NW_{ss,t}g^t_{*y}$$

$$NW_{ss,t}g_{*y} = (r^b_{ss} - r^d_{ss})B_{ss,t}g^t_{*y} + r^d_{ss}NW_{ss,t}g^t_{*y}$$

$$NW_{ss,t} = g^t_{*y}\frac{(r^b_{ss} - r^d_{ss})B_{ss,t}g^t_{*y}}{(g_* - r^d_{ss})}$$

The gross growth rate in lending and the growth rate in net worth are stationary,

$$m_{ss} = \frac{B_{ss,t}}{B_{ss,t-1}} = g_{*y}$$

$$n_{ss} = \frac{NW_{ss,t}}{NW_{ss,t-1}} = g_{*y}$$

Additionally, the relation between loans and net worth is defined by the following equation:

$$B_{ss,t} = \phi^b_{ss}NW_{ss,t} \tag{2.158}$$

Where ϕ^b_{ss} is the private leverage and is given by:

$$\Phi^b_{ss} = \frac{\nu^b_{ss}}{\lambda^b - \mu^b_{ss}} \tag{2.159}$$

For (2.158) to hold, we can claim that no component of ϕ^b_{ss} is governed by any growth rate. In details, the s.s. discounted marginal benefit of expanding assets by a unit μ^b_{ss} and the s.s. expected discounted value of an additional unit of net worth ν^b_{ss} are stationary and given by:

$$\mu^b_{ss} = (1-\theta_b)\beta\frac{\lambda_{ss,t+1}}{\lambda_{ss,t}}\left(r^b_{ss} - r^d_{ss}\right) + \theta_b\beta\frac{\lambda_{ss,t+1}}{\lambda_{ss,t}}m_{ss}\mu^b_{ss} =$$

$$= \frac{(1-\theta_b)\beta\frac{1}{g_{*y}}\left(r^b_{ss}-r^d_{ss}\right)}{1-\theta_b\beta\frac{1}{g_{*y}}m_{ss}} = \frac{1}{g_{*y}}\frac{(1-\theta_b)\beta\left(r^b_{ss}-r^d_{ss}\right)}{1-\theta_b\beta} \tag{2.160}$$

$$\nu^b_{ss} = (1-\theta_b)\beta\frac{\lambda_{ss,t+1}}{\lambda_{ss,t}}r^d_{ss} + \theta_b\beta\frac{\lambda_{ss,t+1}}{\lambda_{ss,t}}n_{ss}\nu^b_{ss} =$$

$$= \frac{(1-\theta_b)\beta\frac{1}{g_{*y}}r^d_{ss}}{1-\theta_b\beta\frac{1}{g_{*y}}n_{ss}} = \frac{1}{g_{*y}}\frac{(1-\theta_b)\beta r^d_{ss}}{1-\theta_b\beta} \tag{2.161}$$

B.4. Market clearing:

From the capital law of motion we know that investment demand grows at the same rate of capital g_{*y}:

$$K_{ss,t} = I^K_{ss,t} + (1-\delta)K_{ss,t-1} \rightarrow$$

$$K_{ss}g^t_{*y} = I^K_{ss,t} + (1-\delta)K_{ss}\frac{g^{t-1}_{*y}}{g^t_{*y}}g^t_{*y} \rightarrow$$

$$I^K_{ss,t} = g^t_{*y}\left(1-\frac{(1-\delta)}{g_{*y}}\right)K_{ss} \tag{2.162}$$

We can conclude the deterministic steady state computation showing through the market clearing condition that output, consumption, investments and fixed costs must share the same trend consistently with our computation.

$$C_{ss}g^t_{*y} = Y_{ss}g^t_{*y} - I_{ss}g^t_{*y} - INC_{ss}\phi^I g^t_{*y} - NE_{ss}\phi^{NE}g^t_{*y} - NP\mu r^k_{ss}\bar{b}_{ss}g^t_{*y} \tag{2.163}$$

Appendix C: Steady State Initial Conditions

We calibrate the initial condition of NEs fixed cost ϕ^{NE} such that the total cost of entry is the 5% of the total output ex post (BGM 2012; Etro and Colciago 2010; Colciago and Rossi 2012). We calibrate the Incumbents' fixed cost as a function of the entry cost to set the share of New Entrants in the economy at $H = 10\%$[7](Etro and Colciago, 2010). Additionally, we calibrate the monitoring cost μ to respect the calibration of the share of Non Performing firms at 9%, following pre-crisis data from Datastream Worldscope. We calibrated the technology frontiers' initial values $\underline{z}_{ss}$ to obtain an unitary mass of firms. Following Gertler and Karadi (2011), we calibrate the spread between the return on loans r^b_{ss} and interest rate on deposit r^d_{ss} at $r^{diff}_{ss} = 0.0025$ and the leverage at $\phi^b_{ss} = \frac{0.1}{r^{diff}_{ss}}$. Finally, the steady state value of labour $L_{ss} = 0.33$ is pinned down by the preference parameter ψ.

We start our calibration from the first order condition and from GK calibration on interest rates,

$$r^d_{ss} = \frac{g_*}{\beta} \tag{2.164}$$

$$r^{diff}_{ss} = 0.0025 \tag{2.165}$$

$$r^b_{ss} = r^{diff}_{ss} + r^d_{ss} \tag{2.166}$$

To respect (2.166), we have to calibrate the interest rate on loans r^k_{ss} according to:

$$r^b_{ss} = \frac{(r^k_{ss} B^I_{ss} + r^k_{ss} B^{NE}_{ss} + \Pi^{NP}_{ss} - \mu r^k_{ss} \frac{\bar{b}_{ss,t}}{g_{*y}} E_{ss})}{B_{ss}} \tag{2.167}$$

Where,

$$B^{NE}_{ss} = \frac{\alpha}{r^k_{ss} - 1 + \delta} \frac{\gamma\xi}{\xi(1-\gamma) - 1} NE_{ss}\phi^{NE} \tag{2.168}$$

$$B^I_{ss} = \frac{\alpha}{r^k_{ss} - 1 + \delta} \frac{\gamma\xi}{\xi(1-\gamma) - 1} INC_{ss}\phi^I \tag{2.169}$$

$$B^{NP}_{ss} = \frac{\alpha}{r^k_{ss} - 1 + \delta} \frac{\gamma\xi}{\xi(1-\gamma) - 1} NP_{ss}\mu r^k_{ss} \frac{\bar{b}_{ss,t}}{g_{*y}} \tag{2.170}$$

$$B_{ss} = B^{NE}_{ss} + B^I_{ss} + B^{NP}_{ss} \tag{2.171}$$

$$\Pi^{NP}_{ss} = Y^{NP}_{ss} \left[1 - (1-\alpha)\gamma\right] - NP^{NP}_{ss}\phi^I \tag{2.172}$$

$$Y^{NP}_{ss} = \frac{\xi}{\xi(1-\gamma) - 1} NP_{ss}\mu r^k_{ss} \frac{\bar{b}_{ss,t}}{g_{*y}} \tag{2.173}$$

In order to solve the system of equations (2.167)-(2.173), we need to make some clarification about the composition of the production sector, which is described by the following set of equations:

$$\eta_{ss} = NE_{ss} + INC_{ss} + NP_{ss} \tag{2.174}$$

[7] Since we calibrate on a quarterly base $H = 2.5\%$

$$NE_{ss} = \left(\frac{z_{ss}}{\hat{A}^{NE}_{ss}}\right)^{\xi} = H^{NE}\eta_{ss} \tag{2.175}$$

$$NP_{ss} = \eta_{ss}\left[\left(\frac{\hat{A}^{NE}_{ss}}{\hat{A}^{NP}_{ss}}\right)^{\xi} - \left(\frac{\hat{A}^{NE}_{ss}}{\hat{A}^{I}_{ss}}\right)^{\xi}\right] = H^{NP}\eta_{ss} \tag{2.176}$$

$$INC_{ss} = \eta_{ss}\left(\frac{\hat{A}^{NE}_{ss}}{\hat{A}^{I}_{ss} g_z}\right)^{\xi} = (1 - H^{NE} - H^{NP})\eta_{ss} \tag{2.177}$$

where the steady state values of the technology thresholds are defined as:

$$\hat{A}^{NE,I}_{ss,t} = g_z^t \left(\frac{\phi^{NE,I}}{1-\gamma}\right)^{(1-\gamma)} \frac{\left(\left(\frac{r^k_{ss}-1+\delta}{\alpha}\right)^{\alpha}\left(\frac{W_{ss}}{1-\alpha}\right)^{1-\alpha}\right)^{\gamma}}{\gamma^{\gamma}} \tag{2.178}$$

$$\hat{A}^{NP}_{ss,t} = g_z^t \left(\frac{\phi^I - \frac{\mu r^k_{ss}\bar{b}^I_{ss}}{g_{*y}}}{1-\gamma}\right)^{(1-\gamma)} \frac{\left(\left(\frac{r^k_{ss}-1+\delta}{\alpha}\right)^{\alpha}\left(\frac{W_{ss}}{1-\alpha}\right)^{1-\alpha}\right)^{\gamma}}{\gamma^{\gamma}} \tag{2.179}$$

To respect condition (2.174)-(2.176), we calibrate di initial value of ϕ^I and the monitoring cost μ according to the following conditions:

$$\frac{INC_{ss}}{\eta_{ss}} = (1 - H^{NE} - H^{NP}) \rightarrow \left[\frac{1}{g_z}\left(\frac{\phi^{NE}}{\phi^I}\right)^{(1-\gamma)}\right]^{\xi} = (1 - H^{NE} - H^{NP}) \rightarrow$$

$$\frac{\phi^I}{\phi^{NE}} = \left[\frac{1}{g_z(1 - H^{NE} - H^{NP})^{\frac{1}{\xi}}}\right]^{\frac{1}{(1-\gamma)}} \tag{2.180}$$

$$\frac{NP_{ss}}{\eta_{ss}} = H^{NP} \rightarrow \left[\frac{1}{g_z}\left(\frac{\phi^{NE}}{\phi^I - \frac{\mu r^k_{ss}\bar{b}_{ss}}{g_{*y}}}\right)^{(1-\gamma)}\right]^{\xi} - \left[\frac{1}{g_z}\left(\frac{\phi^{NE}}{\phi^I}\right)^{(1-\gamma)}\right]^{\xi} = H^{NP} \rightarrow$$

$$\left[\frac{1}{g_z}\left(\frac{\phi^{NE}}{\phi^I - \frac{\mu r^k_{ss}\bar{b}_{ss}}{g_{*y}}}\right)^{(1-\gamma)}\right]^{\xi} - (1 - H^{NE} - H^{NP}) = H^{NP} \rightarrow$$

$$\left(\frac{\phi^{NE}}{\phi^I - \frac{\mu r^k_{ss}\bar{b}_{ss}}{g_{*y}}}\right) = \left[(1 - H^{NE})^{\frac{1}{\xi}} g_z\right]^{\frac{1}{1-\gamma}} \rightarrow$$

$$\left(\frac{\phi^{NE}}{\left[(1 - H^{NE})^{\frac{1}{\xi}} g_z\right]^{\frac{1}{1-\gamma}}}\right) = \phi^I - \frac{\mu r^k_{ss}\bar{b}_{ss}}{g_{*y}} \rightarrow$$

$$\mu = \left\{\phi^I - \left[\frac{\phi^{NE}}{\left[(1 - H^{NE})^{\frac{1}{\xi}} g_z\right]^{\frac{1}{1-\gamma}}}\right]\right\} \frac{g_{*y}}{r^k_{ss}\bar{k}_{ss}} \rightarrow$$

$$\mu = \left\{ \left[\frac{1}{g_z(1-H^{NE}-H^{NP})^{\frac{1}{\xi}}} \right]^{\frac{1}{(1-\gamma)}} - \left[\frac{1}{\left[(1-H^{NE})^{\frac{1}{\xi}} g_z\right]^{\frac{1}{1-\gamma}}} \right] \right\} \frac{\phi^{NE} g_{*y}}{r^k_{ss}\bar{k}_{ss}} \tag{2.181}$$

To respect the calibration of $\eta_{ss} = 1$, we pinned down the value of the technology frontier that allows us to respect the following condition:

$$\eta_{ss} = \underline{z}^{\xi}_{ss} \frac{\left[\left(\frac{\phi^{NE}}{1-\gamma}\right)^{(1-\gamma)} \frac{\left(\left(\frac{r^k_{ss}}{\alpha}\right)^{\alpha} \left(\frac{W_{ss}}{1-\alpha}\right)^{1-\alpha}\right)^{\gamma}}{\gamma^{\gamma}} \right]^{-\xi}}{\left[1 - \left[\left(\frac{\phi^{NE}}{\phi^I - \frac{\mu r^k_{ss} \bar{b}^I_{ss}}{g_{*\phi}}} \right)^{(1-\gamma)} \frac{1}{g_z} \right]^{\xi} \right]} = 1 \rightarrow$$

$$\frac{\underline{z}^{\xi}_{ss}}{H^{NE}} \left[\left(\frac{\phi^{NE}}{1-\gamma}\right)^{(1-\gamma)} \frac{\left(\left(\frac{r^k_{ss}}{\alpha}\right)^{\alpha} \left(\frac{W_{ss}}{1-\alpha}\right)^{1-\alpha}\right)^{\gamma}}{\gamma^{\gamma}} \right]^{-\xi} = 1 \tag{2.182}$$

Substituting the steady state values of NEs, INCs and NPs in (2.167), we can easily solve the system of equations (2.167)-(2.173) and calibrate r^k_{ss}. We can also compute the steady state values of W_{ss}, which is given by:

$$W_{ss} = \left(\frac{1-\alpha}{L_{ss}}\right) \frac{\gamma\xi}{\xi(1-\gamma)-1} \cdot \left(NE_{ss}\phi^{NE} + INC_{ss}\phi^I + NP_{ss}\mu r^k_{ss} \frac{\bar{b}_{ss,t}}{g_{*y}} \right) \tag{2.183}$$

Now we can finally compute the initial value of the C-sector technology frontier:

$$\underline{z}_{ss} = \frac{(\phi^{NE})^{(1-\gamma)+(1-\alpha)\gamma}}{(1-\gamma)^{(1-\gamma)}\gamma^{\gamma}} H^{NE} \cdot \left[\frac{\frac{\gamma\xi}{\xi(1-\gamma)-1}}{\left(\frac{\alpha}{r^k_{ss}}\right)^{\frac{\alpha}{1-\alpha}} L_{ss}} \right]^{(1-\alpha)\gamma} \cdot$$

$$\cdot \left(H^{NE} + (1-H^{NE}) \left[\frac{1}{g_z(1-H^{NE}-H^{NP})^{\frac{1}{\xi}}} \right]^{\frac{1}{(1-\gamma)}} - H^{NP} \left[\frac{1}{\left[(1-H^{NE})^{\frac{1}{\xi}} g_z\right]^{\frac{1}{1-\gamma}}} \right] \right)^{(1-\alpha)\gamma} \tag{2.184}$$

Since $K_{ss} = B_{ss}$, using (2.171), we can obtain now the steady state level of investment,

$$I^K_{ss} = \left(1 - \frac{(1-\delta)}{g_{*y}}\right) K_{ss} \tag{2.185}$$

and the s.s. of output and consumption,

$$Y_{ss} = \frac{\xi(1-\gamma)}{\xi(1-\gamma)-1} \left(NE_{ss}\phi^{NE} + INC_{ss}\phi^I + NP_{ss}\mu r^k_{ss}\bar{b}^I_{ss} \right) \tag{2.186}$$

$$C_{ss} = Y_{ss} - I_{ss} - INC_{ss}\phi^I - NE_{ss}\phi^{NE} - NP_{ss}\phi^I \tag{2.187}$$

To conclude we calibrate the preference parameter ψ to respect the calibration $L_{ss} = 0.33$

$$\psi = \frac{W_{ss}}{L_{ss}^{\phi} C_{ss}} \tag{2.188}$$

Steady State initial values:

Households:

$$\lambda_{ss} = \left[\left(C_{ss} - \frac{hC_{ss}}{g_{*y}} \right)^{-1} - \beta h \left(C_{ss} g_{*y} - hC_{ss} \right)^{-1} \right] \tag{2.189}$$

$$W_{ss} = \frac{\psi \, {L_{ss}}^{\phi}}{\lambda_{ss}} \tag{2.190}$$

$$r^d_{ss} = \frac{g_{*y}}{\beta} \tag{2.191}$$

Production sector:

Productivity thresholds:

$$\hat{A}^{NE}_{ss} = \left(\frac{\phi^{NE}}{1-\gamma} \right)^{1-\gamma} \left[\frac{\left(\frac{(r^k_{ss} - 1 + \delta)}{\alpha} \right)^{\alpha} \left(\frac{W_{ss}}{1-\alpha} \right)^{1-\alpha}}{\gamma} \right]^{\gamma} \tag{2.192}$$

$$\hat{A}^{I}_{ss} = \left(\frac{\phi^{I}}{1-\gamma} \right)^{1-\gamma} \left[\frac{\left(\frac{(r^k_{ss} - 1 + \delta)}{\alpha} \right)^{\alpha} \left(\frac{W_{ss}}{1-\alpha} \right)^{1-\alpha}}{\gamma} \right]^{\gamma} \tag{2.193}$$

$$\hat{A}^{NP}_{ss} = \left(\frac{\phi^{I} - r^k_{ss} \, \mu \, \bar{b}_{ss}}{1-\gamma} \right)^{1-\gamma} \left[\frac{\left(\frac{(r^k_{ss} - 1 + \delta)}{\alpha} \right)^{\alpha} \left(\frac{W_{ss}}{1-\alpha} \right)^{1-\alpha}}{\gamma} \right]^{\gamma} \tag{2.194}$$

Mass of firms:

$$NE_{ss} = \left(\frac{z_{ss}}{\hat{A}^{NE}_{ss}} \right)^{\xi} \tag{2.195}$$

$$INC_{ss} = \eta_{t-1} \left(\frac{\hat{A}^{NE}_{ss}}{g_z \hat{A}^{I}_{ss}} \right)^{\xi} \tag{2.196}$$

$$NP_{ss} = \eta_{ss} \left[\left(\frac{\hat{A}^{NE}_{t-1}}{g_z \hat{A}^{NP}_{ss}} \right)^{\xi} - \left(\frac{\hat{A}^{NE}_{t-1}}{g_z \hat{A}^{I}_{ss}} \right)^{\xi} \right] \tag{2.197}$$

$$\eta_{ss} = NP_{ss} + NE_{ss} + INC_{ss} \tag{2.198}$$

Exit:

$$E_{ss} = NE_{ss} \tag{2.199}$$

Output

$$Y^{NE}_{ss} = \frac{\xi}{(\xi(1-\gamma) - 1)} \, NE_{ss} \phi^{NE}_{ss} \tag{2.200}$$

$$Y^{I}_{ss} = \frac{\xi}{(\xi(1-\gamma) - 1)} \, INC_{ss} \phi^{I}_{ss} \tag{2.201}$$

$$Y_{ss}^{NP} = \frac{\xi}{(\xi(1-\gamma)-1)} NP_{ss} r_{ss}^k \, \mu \, \bar{b}_{ss} \tag{2.202}$$

$$Y_{ss} = Y_{ss}^{NP} + Y_{ss}^{NE} + Y_{ss}^{I} \tag{2.203}$$

Production input demand:

$$K_{ss} = B_{ss} = \frac{\alpha \, \gamma}{(r_{ss}^k - 1 + \delta)} \frac{\xi}{(\xi(1-\gamma)-1)} \left(NE_{ss}\phi_{ss}^{NE} + INC_{ss}\phi_{ss}^{I} + NP_{ss} r_{ss}^k \, \mu \, \bar{b}_{ss} \right) \tag{2.204}$$

$$L_{ss} = \frac{(1-\alpha)\, \gamma}{W_{ss}} \frac{\xi}{(\xi(1-\gamma)-1)} \left(NE_{ss}\phi_{ss}^{NE} + INC_{ss}\phi_{ss}^{I} + NP_{ss} r_{ss}^k \, \mu \, \bar{b}_{ss} \right) \tag{2.205}$$

Financial sector:

Ex ante demand of loans:

$$\bar{b}_{ss} = \frac{\alpha}{g_{*y}(r_{ss}^k - 1 + \delta)} \left[\frac{\gamma \frac{\xi}{\xi-1} \hat{A}_{ss}^{NE}}{\left[\left(\frac{(r_{ss}^k - 1 + \delta)}{\alpha} \right)^{\alpha} \left(\frac{W_{ss}}{1-\alpha} \right)^{1-\alpha} \right]^{\gamma}} \right]^{\frac{1}{1-\gamma}} \tag{2.206}$$

Demands of loans:

$$B_{ss}^{NE} = \frac{\alpha \, \gamma}{(r_{ss}^k - 1 + \delta)} \frac{\xi}{(\xi(1-\gamma)-1)} NE_{ss} \, \phi_{ss}^{NE} \tag{2.207}$$

$$B_{ss}^{I} = \frac{\alpha \, \gamma}{(r_{ss}^k - 1 + \delta)} \frac{\xi}{(\xi(1-\gamma)-1)} INC_{ss} \, \phi_{ss}^{I} \tag{2.208}$$

$$B_{ss}^{NP} = \frac{\alpha \, \gamma}{(r_{ss}^k - 1 + \delta)} \frac{\xi}{(\xi(1-\gamma)-1)} NP_{ss} \, \bar{b}_{ss} \, r_{ss}^k \, \mu \tag{2.209}$$

$$B_{ss} = B_{ss}^{NE} + B_{ss}^{I} + B_{ss}^{NP} \tag{2.210}$$

Non-performing firms transfer:

$$\Pi_{ss}^{NP} = Y_{ss}^{NP} - W_{ss} L_{ss}^{NP} - NP_{ss}^{NP} \phi_{ss}^{I} \tag{2.211}$$

Return on loans:

$$r_{ss}^b = \frac{(r_{ss}^k B_{ss}^I + r_{ss}^k B_{ss}^{NE} + \Pi_{ss}^{NP} - \mu r_{ss}^k \bar{b}_{ss} E_{ss})}{B_{ss}} \tag{2.212}$$

Optimality conditions for financial intermediaries:

$$\Phi_{ss}^b = \frac{(1-\omega) g_{*y} - \theta_b r_{ss}^d}{(r_{ss}^b - r_{ss}^d)\theta_b} \tag{2.213}$$

$$n_{ss} = g_{*y} \left[(r_{ss}^b - r_{ss}^d)\Phi_{ss}^b + r_{ss}^d \right] \tag{2.214}$$

$$m_{ss} = n_{ss} \tag{2.215}$$

$$\mu^b_{ss} = \frac{(1-\theta_b)\beta\left(r^b_{ss} - r^d_{ss}\right)}{1 - \theta_b \beta g_{*y}\left[(r^b_{ss} - r^d_{ss})\Phi^b_{ss} + r^d_{ss}\right]} \tag{2.216}$$

$$\nu^b_{ss} = \frac{(1-\theta_b)\beta r^d_{ss}}{1 - \theta_b \beta g_{*y}\left[(r^b_{ss} - r^d_{ss})\Phi^b_{ss} + r^d_{ss}\right]} \tag{2.217}$$

$$r^{diff}_{ss} = r^b_{ss} - r^d_{ss} \tag{2.218}$$

Credit policy:

$$K_{ss} = \Phi^b_{ss}\, NW_{ss} \tag{2.219}$$

Market clearing conditions:

$$I^K_{ss} = \left(1 - \frac{(1-\delta)}{g_{*y}}\right) K_{ss} \tag{2.220}$$

$$C_{ss} = Y_{ss} - I_{ss} - INC_{ss}\, \phi^I - NE_{ss}\, \phi^{NE} - NP_{ss} \mu r^k_{ss} \bar{b}_{ss} \tag{2.221}$$