

SIS - CLADAG



CLADAG 2015

10° Scientific Meeting of the Classification and Data Analysis Group of the Italian Statistical Society

Flamingo Resort, Santa Margherita di Pula, October 8-10, 2015

BOOK OF ABSTRACTS

Editors:

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ISBN: 978 88 8467 749 9



Univeristà degli Studi di Cagliari



Fondazione Banco di Sardegna

CUEC EDITRICE

by Sardegna Novamedia Soc. Coop.

Via Basilicata n. 57/59 09127 Cagliari, ITALY Tel. & Fax +39 070 271573 www.cuec.eu info@cuec.eu

ISBN: 978-88-8467-749-9 First Edition CUEC © 2015

PREFACE

CLADAG 2015, the 10th Scientific Meeting of the Classification and Data Analysis Group of the Italian Statistical Society (SIS), will be held in Santa Margherita di Pula, Cagliari, Italy, from October 8th to October 10th 2015. The local organizer is the Department of Business and Economics of the University of Cagliari.

CLADAG 2015 will take place under the auspices of the International Federation of Classification Societies (IFCS) and of the Italian Statistical Society (SIS). It promotes advanced methodological research in multivariate statistics with a special vocation in Data Analysis and Classification. CLADAG supports the interchange of ideas in these fields of research, including the dissemination of concepts, numerical methods, algorithms, computational and applied results. It will also benefit of the support of Fondazione Banco di Sardegna.

CLADAG is a member of the International Federation of Classification Societies (IFCS). Among its activities, CLADAG organizes a biennial scientific meeting, schools related to classification and data analysis, publishes a newsletter, and cooperates with other member societies of the IFCS to the organization of their conferences.

The scientific program comprises three Keynote Lectures, an Invited Session, 10 Specialized Sessions, 15 Solicited Sessions and 15 Contributed Sessions. All the Specialized and Solicited Sessions have been promoted by the members of the Scientific Program Committee. The organizers wish to thank them for their cooperation in contributing to the success of CLADAG 2015.

The Book of Abstracts contains short papers of all the presentations scheduled in the conference program. It is organized according to type of session/lecture: Keynote Lectures, Specialized Sessions, Solicited Sessions and Contributed Sessions.

The editors would like to express their gratitude to the Rector of the University of Cagliari, the Director of the Department of Business and Economics and to all the statisticians working in the Department of Business and Economics for their enthusiasm in supporting the organization of this event from the very beginning, as well as to all people who worked hard to make it a success. Special thanks go to Dr. Massimo Cannas, Dr. Luca Frigau and Dr. Farideh Tavazoee for their editorial support

Last but not least, we thank all authors and participants, without whom the conference would not have been possible.

Cagliari, October 8 2015.

Francesco Mola, Claudio Conversano

Conference Themes

The 10th Meeting is orientated towards all topics related to data analysis, classification, multivariate and computational statistics. Submission of papers addressing these topics in both methodological and practical perspective has been encouraged by the members of the Scientific Program Committee.

The list of topics includes, but is not limited to, the following:

A Classification Theory

Bayesian Classification Biplots Clustering models Consensus of Classifications Correspondence Analysis Discrimination and Classification Factor Analysis and Dimension Reduction Methods Fuzzy Methods Genetic Algorithms Hierarchical Classification Multidimensional Scaling Multiway Scaling Multiway Methods Neural Networks for Classification Non Hierarchical Classification Similarities and Dissimilarities Software algorithms for classification Unfolding and Related Scaling Methods

B Data Analysis

Bayesian data Analysis Big data analysis- Categorical Data Analysis Covariance Structure Analysis Data Mining Data Science Data Visualization Decision Trees Functional data analysis Mixture and Latent Class Models Multilevel data Analysis Non Linear Data Analysis Nonparametric and Semiparametric Regression Partial Least Squares Pattern recognition Robustness and Data Diagnostics Social networks- Software algorithms for multivariate analysis Spatial Data Analysis Symbolic Data Analysis.

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Table of Contents

Keynote Lectures	
Mining key networks	21
Variable selection for model-based clustering of categorical data Brendan Murphy	22
EIGENVALUES IN MIXTURE MODELING: GEOMETRIC, ROBUSTNESS AND COMPUTATIONAL ISSUES	23
Specialized sessions	
Robust methods for the analysis of Economic (Big) data Organizer and Chair: Silvia Salini	
Fast and robust seemingly unrelated regression	25
Application to the detection of customs fraud of the goodness-of- fit testing for the newcomb-benford law	30
Monitoring the robust analysis of a single multivariate sample <i>Marco Riani, Anthony C. Atkinson and Andrea Cerioli</i>	34
Bayesian nonparametric clustering Organizer: Fabrizio Ruggeri; Chair: Renata Rotondi	
A BAYSIAN NONPARAMETRIC APPROACH TO MODEL ASSOCIATION BETWEEN CLUSTERS OF SNPS AND DISEASE RESPONSES	39
A BAYESIAN NONPARAMETRIC MODEL FOR CLUSTERING AND BORROWING INFORMATION	43
Sequential clustering based on dirichlet process priors	47

Causal Inference with Complex Data Structures Organizer and Chair: Alessandra Mattei	
SHORT TERM IMPACT OF PM10 EXPOSURE ON MORTALITY: A PROPENSITY SCORE APPROACH	52
IDENTIFICATION AND ESTIMATION OF CAUSAL MECHANISMS IN CLUSTERED ENCOURAGEMENT DESIGNS: DISENTANGLING BED NETS USING BAYESIAN PRINCIPAL STRATIFICATION	54
The effects of a dropout prevention program on secondary students' outcomes	56
Clustering in Time Series Organizer and Chair: Michele La Rocca	
Probabilistic boosted-oriented clustering of time series	61
Copula-based fuzzy clustering of time series	65
Comparing multi-step ahead forecasting functions for time series clustering	69
Multiway Analysis Organizer and Chair: Giuseppe Bove	
(Interactive) visualisation of threeway data	74
Robust fuzzy clustering of multivariate time trajectories	78
ESTIMATION PROCEDURES FOR AVOIDING DEGENERATE SOLUTIONS IN CANDE-COMP/PARAFAC	82

Big Data Analysis Organizer and Chair: Donato Malerba	
TOWARDS A STATISTICAL FRAMEWORK FOR ATTRIBUTE COMPARISON IN VERY LARGE RELATIONAL DATABASES	83
Mining big data with high performance computing solutions Fabrizio Angiulli, Stefano Basta, Stefano Lodi, Gianluca Moro and Claudio Sartori	91
Enhancing big data exploration with faceted browsing Sonia Bergamaschi, Giovanni Simonini and Song Zhu	95
New Methodologies for Composite Indicators Organizer and Chair: Agostino Di Ciaccio	
Advances in composite-based path modeling for synthetic indicators Vincenzo Esposito Vinzi, Laura Trinchera and Giorgio Russolillo	100
Composite indicators modeling	102
Measuring the importance of variables in composite indicators William Becker, Michaela Saisana, Paolo Paruolo and Andrea Saltelli	104
Cluster analysis software and validation Organizer and Chair: Christian Hennig	
Adaptive choice of input parameters in robust clustering Luis A. $Garcia$ -Escudero and $Augustin\ Mayo$ -Iscar	109
Robust model-based clustering with covariance matrix constraints Pietro Coretto and Christian Hennig	113
Flexible implementation of resampling schemes for cluster validation $\it Friedrich\ Leisch$	v117
Selecting a mixture model with a clustering focus Organizer and Chair: Gilles Celeux	
Clustering in finite mixtures using an integrated completed likeli- hood criterion	122
ESTIMATION AND MODEL SELECTION FOR MODEL-BASED CLUSTERING WITH THE CONDITIONAL CLASSIFICATION LIKELIHOOD	126

On the different ways to compute the integrated completed likeli- hood criterion	130
Exploring relationships between blocks of variables Organizer and Chair: Giorgio Russolillo	
Weighted Multiblock Clustering	135
Thematic model exploration through multiple co-structure maximi- sation: method and software	139
A NEW COMPONENT-BASED APPROACH OF REGULARISATION FOR MULTIVARIATE GENERALISED LINEAR REGRESSION	144
Solicited Sessions	
Advances in Density-based clustering Organizer and Chair: Francesca Greselin	
A nonparametric clustering method for image segmentation $Giovanna\ Menardi$	150
Robust clustering for heterogenous skew data	154
Regularizing finite mixtures of gaussian distributions	154
Latent variable models for longitudinal data - Part I Organizer and Chair: Silvia Bacci	
A JOINT MODEL FOR LONGITUDINAL AND SURVIVAL DATA BASED ON AN AR(1) LATENT PROCESS	163
FINITE MIXTURE MODELS FOR MIXED DATA: EM ALGORITHMS AND PARAFAC REPRESENTATIONS	167
On the use of the contaminated gaussian distribution in hidden markov models for longitudinal data	171
Latent variable models for longitudinal data - Part II Organizer and Chair: Francesco Bartolucci	
A HIDDEN MARKOV APPROACH TO THE ANALYSIS OF INCOMPLETE MULTIVARIATE LONGITUDINAL DATA	177
LATENT MARKOV AND GROWTH MIXTURE MODELS: A COMPARISON	181

Fulvia Pennoni and Isabella Romeo	
LATENT WORTHS AND LONGITUDINAL PAIRED COMPARISONS - A MARKOV MODEL OF DEPENDENCE	185
Multivariate data analysis in environmental sciences Organizer: Fabrizio Ruggeri; Chair: Raffaele Argiento	
Multivariate downscaling for non-gaussian data	191
Preliminary results on tapering multivariate spatio temporal models for exposure to airborne multipollutants in Europe	195
Clustering macroseismic fields by statistical data depth functions Claudio Agostinelli, Renata Rotondi and Elisa Varini	199
Advanced models for tourism analysis Organizer and Chair: Stefania Mignani	
Analysing territorial heterogeneity in tourists'satisfaction towards italian destinations	204
MICRO-ECONOMIC DETERMINANTS OF TOURIST EXPENDITURE: A QUANTILE REGRESSION APPROACH	208
INEQUALITIES AND TOURISM CONSUMPTION BEHAVIOUR: A MIXTURE MODEL ANALYSIS	212
Bayesian Networks and Graphical Models in Socio-Economic Sciences Organizer and Chair: Paola Vicard	
Bayesian networks for firm performance evaluation	217
Graphical model using copulas for measurement error modeling $Daniela\ Marella\ and\ Paola\ Vicard$	221
Time Series in Clustering Organizer and Chair: Michele La Rocca	
Parsimonious clustering of time series	226
Dynamic time warping-based fuzzy clustering for spatial time series Pierpaolo D'Urso, Marta Disegna and Riccardo Massari	230
Periodical feature based time series clustering	234

Big Data Analysis Organizer and Chair: Donato Malerba	
Interactive machine learning with R	239
Workload estimation for a call center	243
PREDICTION IN OLIVE OIL TRADE USING REGRESSION MODELS ON TEMPORAL DATA NETWORK	245
Advances in Ordinal and Preference Data Organizer and Chair: Antonio D'Ambrosio	
Measuring consensus in the setting of non-uniform qualitative scales José L. García-Lapresta and David Pérez-Román	s250
ACCURATE ALGORITHMS FOR CONSENSUS RANKING DETECTION	255
Logistic regression trees for ordinal and preference data	259
Case studies in data science from Ligurian companies Organizer and Chair: Delio Panaro	
STATISTICAL METHODS FOR THE ANALYSIS OF OSTREOPSIS OVATA BLOOM EVENTS FROM METEO-MARINE DATA	262
Data mining for optimal gambling	266
A FRAUD DETECTION ALGORITHM FOR ONLINE BANKING	271
Does directors' background matter? firm performance, board features and financial reporting reliability	275

Modeling ordinal data Organizer and Chair: Maurizio Carpita	
Posterior predictive model checks for assessing the goodness of fit of bayesian multidimensional irt models	280
International tourism in Italy: A bayesian network approach Federica Cugnata and Giovanni Perucca	284
Clustering upper level units in multilevel models for ordinal data Leonardo Grilli, Agnese Panzera and Carla Rampichini	288
Functional data analysis for environmental data Organizer and Chair: Tonio Di Battista	
CLUSTERING SPATIALLY DEPENDENT FUNCTIONAL DATA: A METHOD BASED ON THE CONCEPT OF SPATIAL DISPERSION FUNCTION OF A CURVE Elvira Romano, Antonio Balzanella and Rosanna Verde	292
Two case studies on object oriented spatial statistics	296
Inference on functional biodiversity tools	298
Advances in quantile regression Organizer and Chair: Cristina Davino	
M-QUANTILE REGRESSION: DIAGNOSTICS AND PARAMETRIC REPRESENTATION OF THE MODEL	303
QUANTILE REGRESSION: A BAYESIAN ROBUST APPROACH	307
A comparison among estimators for linear regression methods $Marilena\ Furno\ and\ Domenico\ Vistocco$	311
Handling heterogeneity among units in quantile regression Cristina Davino and Domenico Vistocco	315
Directional Data Organizer and Chair: Giovanni C. Porzio	
Small biased circular density estimation	320
A DEPTH-BASED CLASSIFIER FOR CIRCULAR DATA	32/

Giuseppe Pandolfo	
Nonparametric estimates of the mode for directional data	328
Recent developments in statistical analysis of network data Organizer and Chair: Domenico De Stefano	
Game theory and network models for the reconstruction of archaeological networks	331
A MODEL FOR CLUSTERING A SPATIAL NETWORK WITH APPLICATION TO LOCAL LABOUR SYSTEM IDENTIFICATION	335
On the sampling distributions of the ML estimators in Network ef-	220
FECT MODELS	339
CORRESPONDENCE ANALYSIS WITH DOUBLING FOR TWO-MODE VALUED NET-WORKS	343
Current challenges in clustering and classification of biomedical data Organizer and Chair: Adalbert F.X. Wilhelm	
SEMANTIC MULTI CLASSIFIER SYSTEMS FOR THE DETECTION OF AGING RELATED	
PROCESSES	348
EMOTION RECOGNITION IN HUMAN COMPUTER INTERACTION USING MULTIPLE CLASSIFIER SYSTEMS	349
Ensemble of selected classifiers	352
Contributed papers	
A GENERALIZED DISTANCE FOR INFERENCE IN FUNCTIONAL DATA	354
LONG GAPS IN MULTIVARIATE SPATIO-TEMPORAL DATA: AN APPROACH BASED ON FUNCTIONAL DATA ANALYSIS	359
Effects on curve clustering of different transformations of chronological textual data	363
A note on the reliability of a classifier	366

Robustified Classification of multivariate functional data Francesca Ieva and Anna M. Paganoni	370
Size control of robust regression estimators	374
THE MOVEMENTS OF EMOTIONS: AN EXPLORATORY CLASSIFICATION ON AFFECTIVE MOVEMENT DATA	378
Electre tri-machine learning approach to the record linkage problem Valentina Minnetti and Renato De Leone	1382
Quality of classification approaches for the quantitative analysis of international conflict $\dots \dots \dots$	387
The rtclust procedure for robust clustering	391
What are the true clusters?	396
A NOVEL MODEL-BASED CLUSTERING APPROACH FOR MASSIVE DATASETS OF SPATIALLY REGISTERED TIME SERIES. WITH APPLICATION TO SEA SURFACE TEMPERATURE REMOTE SENSING DATA	399
Big data classification: simulations in the many features case $Claus\ Weihs$	403
From big data to information: statistical issues through examples . Silvia Biffignandi and Serena Signorelli	407
BIG DATA MEET PHARMACEUTICAL INDUSTRY: AN APPLICATION ON SOCIAL MEDIA DATA	411
Defining the subjects distance in Hierarchical cluster analysis by Copula Approach	416

Supervised classification of defective crankshafts by image analysis Beatriz Remeseiro, Javier Tarrío-Saavedra, Mario Francisco-Fernández, Manuel G. Penedo, Salvador Naya and Ricardo Cao	420
Archetypal analysis for data-driven prototype identification Giancarlo Ragozini, Francesco Palumbo and Maria R. D'Esposito	424
Principal component analysis of complex data and application to climatology	428
Sparse exploratory multidimensional irt models	432
Iterative factor clustering for categorical data reconsidered $Alfonso\ Iodice\ D$ 'Enza, $Angelos\ Markos\ and\ Francesco\ Palumbo$	437
Testing antipodal symmetry of circular data	442
How to define deviance residuals in multinomial regression Giovanni Romeo, Mariangela Sciandra and Marcello Chiodi	446
Diagnostic tools for gamlss fitted objects	451
Bayesian regression analysis with linked and duplicated data <i>Andrea Tancredi, Rebecca Steorts and Brunero Liseo</i>	455
A SEMI-PARAMETRIC FAY-HERRIOT-TYPE MODEL WITH UNKNOWN SAMPLING VARIANCES	460
Posterior distributions from optimally b-robust estimating functions and approximate bayesian computation	464
MCA BASED COMMUNITY DETECTION	468
Classifying social roles by network structures	472
A MULTILEVEL HECKMAN MODEL TO INVESTIGATE FINANCIAL ASSETS AMONG OLD PEOPLE IN EUROPE	476
Optimal pricing using bayesian semiparametric price response models Winfried J. Steiner, Anett Weber, Stefan Lang and Peter Wechselberger	480

Monetary transmission models for banking interest rates Laura Parisi, Paolo Giudici, Igor Gianfrancesco and Camillo Giliberto	484
Estimating the effect of prenatal care on birth outcomes Emiliano Sironi and Massimo Cannas	490
RECURSIVE PARTITIONING: AN APPROACH BASED ON THE WEIGHTED KEMENY DISTANCE	494
Why to study abroad? An example of clustering	498
A Graphical copula-based tool for detecting tail dependence Roberta Pappadà, Fabrizio Durante and Nicola Torelli	502
CLASSIFICATION MODELS AS TOOLS OF BANKRUPTCY PREDICTION - POLISH EX- PERIENCE	506
The relationship between individual price response of Beer consumers and their demographic/psychographic characteristics $Friederike\ Paetz$	510
The ensemble conceptual clustering of symbolic data for customer loyalty analysis	514
Insert here consumers' perceptions of corporate social responsibilities and willingness to pay: A partial least squares	519
Inspecting the quality of italian wine through causal reasoning Eugenio Brentari, Maurizio Carpita and Silvia Golia	521
Exploring socio-economic factors associated with adherence to the mediterranean diet: a multilevel approach	525
Big data and 'social' reputation: a financial example Paola Cerchiello	529
Bayesian networks for stock picking	533
Portfolio selection with lasso algorithm	537
Sunspot in economic models with externalities	540

ROBUST CLUSTERING FOR HETEROGENOUS SKEW DATA

Luis Angel García-Escudero, ¹ Francesca Greselin² and Agustin Mayo-Iscar¹

ABSTRACT: The existing robust methods for model-based classification and clustering deal with elliptically contoured components. Here we introduce robust estimation for mixtures of skew-normal, by the joint usage of trimming and constraints. The model allows to fit heterogeneous skew data with great flexibility.

KEYWORDS: Clustering, robust estimation, skew data.

1 Introduction

In recent years, empirical evidences of asymmetric departures from normality in some real subpopulations suggested the introduction of skewed components in the classical mixture model approach. Therefore, increasing attention has been devoted to mixtures of multivariate skew normal and skew t, as well as to mixtures of normal-inverse-Gaussian distributions, shifted asymmetric Laplace distributions, generalized hyperbolic distributions and hierarchical mixtures (mixtures of mixtures), to capture asymmetric shapes in components. Most of these parametric families of skew distributions are clearly related, as it has been described in a careful review by Lee & McLachlan (2013a). In this paper we address robust estimation of mixtures of skew normal distributions. Our interest for the Finite Mixtures of Canonical Fundamental Skew Normal (FM CFUSN) has been motivated by the aim of adapting to the Gaussian case the methodology developed for Finite Mixtures of Canonical Fundamental Skew t (FM CFUST) in Lee & McLachlan (2014), based on skew distributions originated in Arellano-Valle & Genton (2005). By mimicking the CFUST properties, CFUSN has the special appealing of including as particular cases the restricted and unrestricted Skew Normal, respectively rMSN and uMSN. The great flexibility of CFUSN is due to its parameters, which allow to separately govern location, scale, correlation, and skewness. We will show,

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herein, that the FM CFUSN offers a very tractable model for many situations of departures from symmetry, being analytically and computationally easier than the FM CFUST. Taking advantage of its robust estimation, the model becomes even more adaptable to real phenomena. Indeed, robustness is often required by many datasets available nowadays, where populations are noisy, have some non-symmetric features, and could contain outliers asymmetrically disposed around the data clusters.

The first step to achieve robustness and obtain good breakdown properties for the estimators, is based on a *trimming procedure* incorporated along the iterations of the EM algorithm. The key idea is that a small portion of observations, which are highly unlikely to occur under the current fitted model assumption, is discarded from contributing to the parameter estimates. The same methodology has been proven to be very effective when addressing robust clustering for Gaussian and *t* mixtures models (see Neykov *et al.*, 2007; Gallegos & Ritter, 2009; García-Escudero *et al.*, 2014). Furthermore - and this is the second step - we implement a *constrained ML estimation* for the component covariances, aiming at reducing spurious solutions and preventing singularities of the likelihood, along the lines of Ingrassia & Rocci (2007).

Monte Carlo experiments show that bias and MSE of the estimator in several cases of contaminated data are dramatically inflated, while they return to be comparable to results obtained on skew data without noise, when the combined effect of trimming and constrained estimation is applied. Further, it can be shown that the estimator resists the influence of all classes of contaminating observations, as far as the outliers appear in a proportion lower than the trimming level α . As a final remark, we want to stress that the joint usage of trimming and constrained estimation allow to set the underlying mathematical and statistical problems as well posed ones.

2 Methodology

We consider the ML estimation for a g-component mixture model in which a random sample $(\mathbf{Y}_1, \dots, \mathbf{Y}_n)$ follows a mixture of CFUSNs. The probability density function can be written as

$$\mathbf{Y}_{j} \sim \sum_{i=1}^{g} \pi_{i} f(\mathbf{y}_{j} | \mu_{i}, \Sigma_{i}, \Delta_{i}), \quad \pi_{i} \geq 0, \quad \sum_{i=1}^{g} \pi_{i} = 1,$$

where $f(\cdot)$ denotes a CFUSN density, $\Theta = (\theta_1, \dots, \theta_g)$ with $\theta_i = (\pi_i, \mu_i, \Sigma_i, \Delta_i)$ are the unknown parameters of component i, and π_i are the weights of the groups.

To define the location-scale variant of the CFUSN distribution, let $(\mathbf{Y}_0, \mathbf{Y}_1)$ be a p+q multivariate normal r.v., such that

$$\begin{bmatrix} \mathbf{Y}_0 \\ \mathbf{Y}_1 \end{bmatrix} \sim \mathcal{N}_{q+p} \begin{pmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \mathbf{I}_q & 0 \\ 0 & \Sigma \end{bmatrix} \end{pmatrix} \tag{1}$$

where Σ is a positive definite scale matrix and 0 is a vector of zeros with appropriate dimension. Then, given a $p \times q$ matrix Δ , we obtain the stochastic representation for \mathbf{Y} , i.e. $\mathbf{Y} = \mu + \Delta |\mathbf{Y}_0| + \mathbf{Y}_1$, that follows the CFUSN distribution with density given by

$$f(\mathbf{y}; \mu, \Sigma, \Delta) = 2^q \phi_p(\mathbf{y}; \mu, \Omega) \Phi_q(\Delta^T \Omega^{-1}(\mathbf{y} - \mu); 0, \Lambda)$$

where $\Omega = \Sigma + \Delta \Delta^T$, $\Lambda = \mathbf{I}_p - \Delta^T \Omega^{-1} \Delta$, and as usual $\phi_p(\mathbf{y}; \mu, \Sigma)$ denotes the p-dimensional density of the multivariate Gaussian with mean μ and scale Σ evaluated at \mathbf{y} , while $\Phi_q(\cdot)$ denotes the cumulative distribution function.

In the ML approach, we consider the following *trimmed* log-likelihood function (see Neykov *et al.*, 2007; Gallegos & Ritter, 2009; García-Escudero *et al.*, 2014)

$$\mathcal{L}_{trim} = \sum_{i=1}^{n} z(\mathbf{y}_i) \log \left[\sum_{g=1}^{G} \phi_p(\mathbf{y}_i; \mu_g, \Omega_g) \Phi_q(\Delta_g^T \Omega_g^{-1}(\mathbf{y}_i - \mu_g); 0, \Lambda_g) \pi_g \right]. \quad (2)$$

By $z(\cdot)$ we denote a 0-1 trimming indicator function that indicates whether observation \mathbf{y}_i is trimmed off: $z(\mathbf{y}_i)$ =0, or not: $z(\mathbf{y}_i)$ =1. A fixed fraction α of observations, whose contributions to the likelihood are lower than their α -quantile, can be unassigned by setting $\sum_{i=1}^n z(\mathbf{y}_i) = [n(1-\alpha)]$ at each E-step, hence they do not contribute to the parameter estimation.

Further, we want to deal with the unboundedness of the target function \mathcal{L}_{trim} when no constraints are imposed on the scatter parameters. In this case, the defining problem is ill-posed because the log-likelihood tends to ∞ when either $\mu_g = \mathbf{y}_i$ and $|\Sigma_g| \to 0$. As a trivial consequence, the EM algorithm can be trapped into non-interesting local maximizers, called "spurious" solutions, and the result of the EM algorithm strongly depends on its initialization. For this reason, we set a constraint on the maximization in (2), concerning the eigenvalues $\{\lambda_l(\Sigma_g)\}_{l=1,\dots,d}$ of the scatter matrices Σ_g , by imposing

$$\lambda_{l_1}(\Sigma_{g_1}) \le c \ \lambda_{l_2}(\Sigma_{g_2})$$
 for every $1 \le l_1 \ne l_2 \le d$ and $1 \le g_1 \ne g_2 \le G$.

as in Ingrassia & Rocci (2007); García-Escudero *et al.* (2008); Gallegos & Ritter (2009). It is important to remark that the parameters related with location

and skewness are not affected by the constrained maximization applied to the covariance matrices.

As usual, among many runs of the EM algorithm, we select the parameters given by the best final likelihood. To complete a synthetic description of the algorithm, our proposal for the initialization of the EM is to take a small random subsample for each component and to draw, for each observation in it, a random variate \mathbf{Y}_0 from $\mathcal{N}_p(0,\mathbf{I}_p)$. By applying multivariate regression on the selected observations in each component, and using the corresponding vectors $|\mathbf{Y}_0|$, we get an estimation of the model parameters (i.e. μ_g , Σ_g and Δ_g) for the g-th component. Finally, initial estimations for the group weights are drawn from a multinomial random variable.

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