Constraints on input (E)

What is the input?

- lamp spectrum : $\{ b_i \}$
- mask transmittance :

$$h_i(x_1, x_2) := f(\cdot,\cdot) \cdot \exp[j\, g_i(\cdot,\cdot)]$$

Then : $v_i(x_1, x_2) = b_i \cdot h_i(\cdot)$

We prefer smooth phase masks:

$$\sum_i \left[\left(\frac{\partial g_i}{\partial x_1} \right)^2 + \left(\frac{\partial g_i}{\partial x_2} \right)^2 \right] \leq \text{Const.}$$

U_{ad} := admissible input set

$$= \underbrace{H^1(G_0) \times \ldots \times H^1(G_0)}_{I \text{ times}}$$

It can be shown that:

$\underline{v} \in U_{ad}$ has a smooth phase $\underline{g}(\cdot)$

Then :

$$E(\underline{v}) := a_1 \| \underline{v} \|^2_{U_{ad}} + a_2 \| \underline{\nabla} g \|^2_{L^2}$$

How to optimise?

$$\text{ADJ.} \longrightarrow \nabla J$$

If we were given the functional gradient $\nabla J(\cdot)$ then a minimising input $\underline{v}^m(\cdot)$ would satisfy:

$$\langle \nabla J(\underline{v}) \mid \underline{v} - \underline{v}^m \rangle \geq 0$$

$$\forall\, \underline{v} \text{ s.t. } \| \underline{v} - v^m \|_{V_{ad}} \leq a_m$$

$$a_m > 0.$$

It can be shown that $\nabla J(\cdot)$ is related to the solution $\underline{q}(\cdot)$ of the adjoint system

To arrive at the latter, time discretisation is essential ...

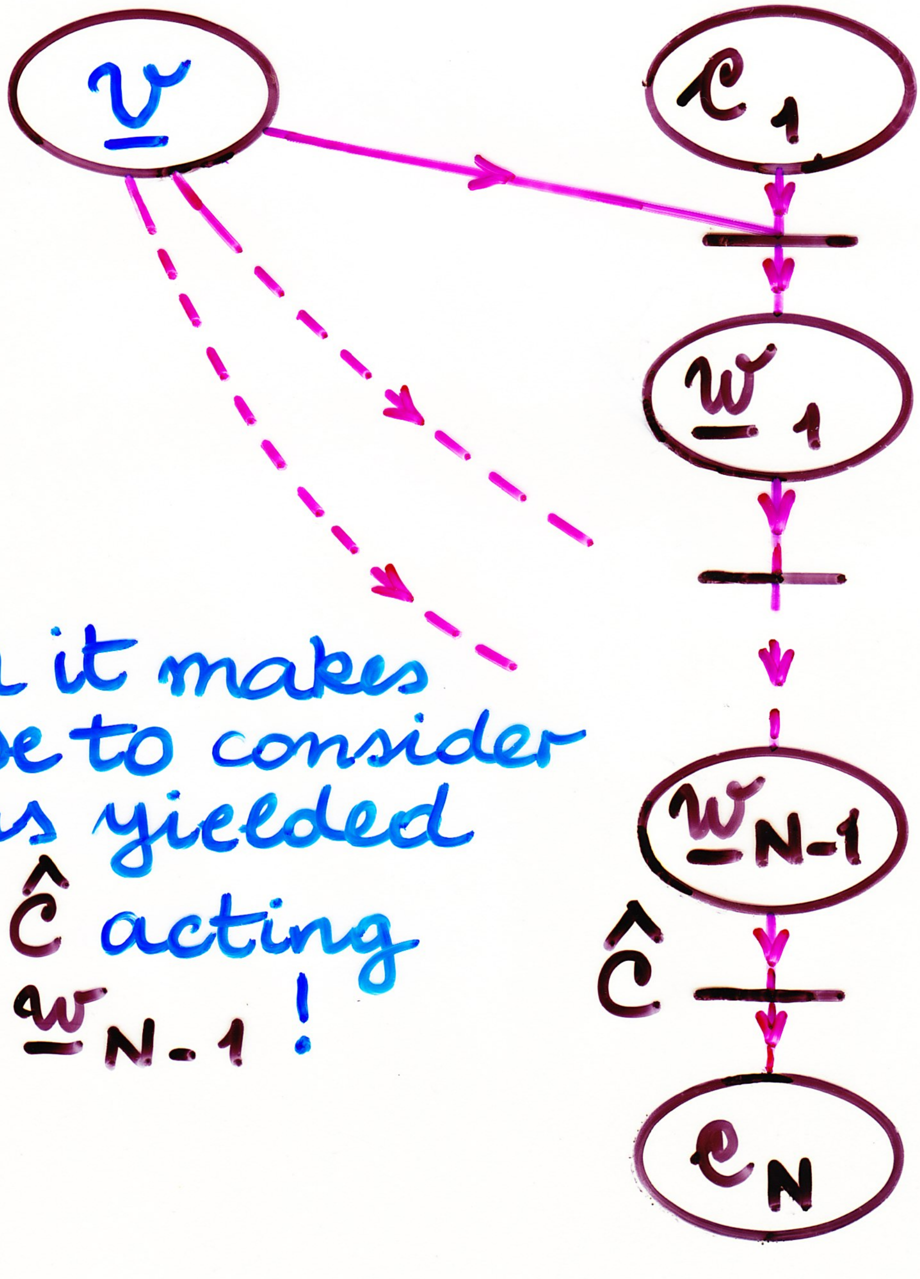
DISCRETE TIME

c+$\underline{w}$-equations

$t_n := (n-1)T/(N-1)$

$1 \leq n \leq N$,

$c(\underline{x}, t) \Rightarrow c(\underline{x}, t_n)$



then it makes sense to consider c_N as yielded by $\hat{C}$ acting on $\underline{w}_{N-1}$!

OUTPUT MAP

$$\underline{v} \longrightarrow \boxed{\hat{A}} \xrightarrow{\underline{w}_{N-1}} \boxed{\hat{C}} \xrightarrow{c_N}$$

$$c(\underline{x}, t_N) = C(\underline{x}, t_{N-1}) \cdot$$
$$\cdot \exp\left(-\sum_i s_i \cdot |w_i(x, t_{N-1})|^2 \cdot t_s\right)$$
$$= \hat{C}\, \underline{w}(\underline{x}, t_{N-1})$$

Introduction of $\hat{C}$ leads to the adjoint system

$$\begin{cases} \left(\nabla^2 + k_i^2(\cdot)\right)^{\dagger} q_i(\cdot) = \\ \qquad = -2 \cdot (c_N - c_d) \cdot t_s \cdot s_i \cdot w_i^{\dagger}(\underline{x}, t_{N-1}) \\ q_i\Big|_{G_0 \cup G_x} = 0 \\ \partial_r q_i - j k_i(\cdot)\, q_i \Big|_S = 0 \end{cases}$$