AXIOMATIC SYSTEM THEORY & OPTICAL IMAGES

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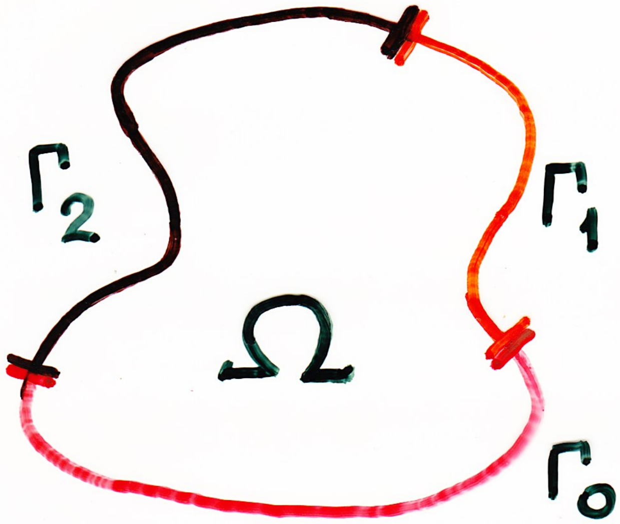
IMAGE 3-D '79
STRASBOURG


SYSTEM
REAL
3-D
IMAGE
x∈Ω⊂ℝ³; 0≤t≤T
ψ:Ω×[0,T]→W([0,T],H)
(x,t)→ψ(x,t)∈ℂ
→|ψ(x,t)|²

DISTRIBUTED PARAMETER SYSTEM

$$(U, \Psi, Y; A_\Omega; A_\Gamma; B_\Omega, B_\Gamma; C)$$



$$\Omega \subset \mathbb{R}^3$$

$$\bigcup_{i=0}^{M} \Gamma_i = \Gamma = \partial\Omega$$

$$A_\Omega(x,t)\psi(x,t) = B_\Omega(x)u(x,t)$$

$$x \in \Omega;\ t \geqslant 0;\ \psi \in \Psi;\ u \in U_\Omega$$

$$A_\Gamma(x,t)\psi(x,t) = B_\Gamma(x)v(x,t)$$

$$x \in \Gamma;\ v \in U_\Gamma$$

$$U_\Omega \times U_\Gamma := U;\quad (u;v) := w$$

$$\psi(x,0) = f_0(x);\ \frac{\partial}{\partial t}\psi(x,0) = f_1(x); \ldots \frac{\partial^{n-1}}{\partial t^{n-1}}\psi(x,0) = f_{n-1}(x)$$

$$C:\ \Psi \longrightarrow Y$$

$$\psi \longmapsto C\psi := y$$

CONTROLLABILITY

$$S_0 := \{\psi_0(x,T;w)\,;\, w\in U\,;\ \text{all ICs} = 0\}$$

$$\overline{S}_0 = \Psi$$

$$S := \{\psi(x,T;w)\,;\, w\in U\,;\ \text{any IC}\}$$

CONTROLLABILITY & UNIQUENESS

$$\overline{S} = \Psi \underset{\text{HBT}}{\Leftrightarrow} [\forall \psi \in \Psi,\ \langle h,\psi\rangle = 0 \Leftrightarrow h = 0]$$

GENERALIZED $W_1 \to W_2$ CONTROLLABILITY

W_1, W_2 BANACH SPACES

$$\Theta : W_1 \longrightarrow W_2$$
$$\eta \longmapsto \Theta\eta := \vartheta$$

$$\Theta(W_1) := \{\vartheta\,;\, \eta \in W_1\} \qquad \overline{\Theta(W_1)} = W_2$$

OBSERVABILITY

$$C : \Psi \longrightarrow Y \qquad \text{is one-to-one}$$
$$\psi \longmapsto C\psi := y$$

$$C \text{ linear, invertible} \Leftrightarrow \operatorname{Ker} C = 0_{\Psi}$$

OPTIMAL CONTROL

$$J(w) := P(C\psi(w); z_d) + E(w)$$

$$J(\hat{w}) = \inf_{w \in U_{ad}} J(w)$$

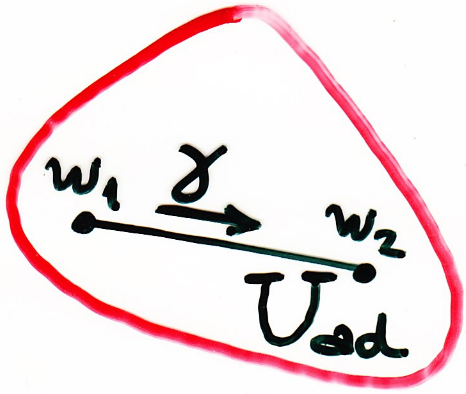
A SET OF SUFFICIENT CONDITIONS

$\overline{U_{ad}} = U_{ad} \subset U$, HILBERT SPACE

$$U_{ad} \neq \emptyset$$

$\forall w_1, w_2 \in U_{ad}; \forall \gamma, 0 \leq \gamma \leq 1$

$((1-\gamma) w_1 + \gamma w_2) \in U_{ad}$



$J(\cdot)$ CONTINUOUS FCN.

$$\lim_{\|w\| \to \infty} J(w) = +\infty$$

$\forall w_1, w_2 \in U_{ad}; \forall \theta, 0 \leq \theta \leq 1$

$J((1-\theta) w_1 + \theta w_2) \leq$

$\leq (1-\theta) J(w_1) + \theta J(w_2)$