

Endogenous Market Structures and Optimal Taxation*

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Abstract

This paper provides optimal labour and dividend income taxation in a general equilibrium model with oligopolistic competition and endogenous firms' entry. In the long run the optimal dividend income tax corrects for inefficient entry. The dividend income tax depends on the form of competition and the nature of the sunk entry costs. In particular, it is higher in market structures characterized by competition in quantities rather than those characterized by price competition. Oligopolistic competition leads to an endogenous countercyclical price markup. As a result, offsetting the distortions over the business cycle requires deviations from full tax smoothing.

1 Introduction

Recent macroeconomic literature emphasizes the importance of the creation of new firms for the propagation of business cycle fluctuations. Bilbiie, Ghironi and Melitz (2012, BGM henceforth), Jaimovich and Floetotto (2010), Colciago and Etro (2010) and Etro and Colciago (2010), among others, show that accounting for firms' dynamics helps to improve the performance of dynamic general equilibrium models in replicating the variability of the main macroeconomic variables in response

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to exogenous disturbances.¹ Most of these studies are characterized by imperfect competition in the goods market. As a result, an inefficient number of producers (and products) may arise in equilibrium, leading to welfare losses for the society. For this reason, policy measures aimed at removing market distortions could be desirable. This paper provides optimal Ramsey dividend and labour income taxation in a framework characterized by alternative, imperfectly competitive, endogenous market structures. The issue of capital and corporate taxation has been at the forefront of policy discussions. Some countries, including Spain, France, Sweden and the United States have, in recent years, reduced capital gains taxes or corporate taxes. While this study provides an analysis of the dynamic inefficiencies characterizing an economy featuring imperfect competition with endogenous entry and endogenous labour supply, the resulting model economy is a suitable environment in which to evaluate the effects of changes in corporate taxation on firm creation. Market structures are said to be endogenous since the number of producers and price markups are determined in each period. The economy features distinct sectors, each one characterized by many firms supplying goods that can be imperfectly substitutable to a different degree, taking strategic interactions into account and competing either in prices (Bertrand competition) or in quantities (Cournot competition). As in BGM (2012), the entry of a new firm into the market amounts to the creation of a new product. Sunk entry costs allow to endogenize the entry and the (stock market) value of each firm in each sector. The preferences of agents are characterized by love for variety, such that spreading a given nominal consumption expenditure over a larger number of goods leads to an increase in utility. The degree of market power, as measured by the price markup, depends endogenously on the form of competition, on the degree of substitutability between goods and on the equilibrium number of firms. Importantly, the price markup is countercyclical. During an economic boom profit opportunities attract firms into the market. This strengthens competition and, via strategic interactions, reduces price markups. An early reference on the procyclicality of the number of firms in the U.S. is Chatterjee and Cooper (1993), while a more recent one is Lewis and Poilly (2012). The countercyclicality of the price markup is consistent with the empirical findings by Bilts (1987), Rotemberg and Woodford (2000) and Galí et al. (2007). Nevertheless, aggregate profits remain strongly procyclical as in the data. As emphasized by BGM (2007), the market equilibrium is characterized

¹Early contributors to this literature are Chatterjee and Cooper (1993), Devereux et al. (1996) and Devereux and Lee (2001). More recent developments have come from Bergin and Corsetti (2008) and Faia (2012).

by two distortions: a *Labour Distortion* and an *Entry Distortion*. As in other models with an imperfectly competitive goods market, the presence of a price markup leads to a wedge between the marginal product of labour and the marginal rate of substitution between consumption and hours. Oligopolistic competition renders this wedge time varying. The *Entry Distortion* operates through the intertemporal firms creation margin and leads to an inefficient number of firms in equilibrium. A positive dividend income tax is optimal in the case of excessive entry and vice versa. To minimize the welfare cost associated with these distortions, the Ramsey optimal fiscal policy, both in the short and long run, is provided. The government levies taxes on dividend income and labour income and issues state contingent bonds to finance an exogenous stream of public spending. In the long run, the dividend income tax removes the entry distortion, as in Chugh and Ghironi (2011). The dividend income tax can be positive, negative or zero depending on the form of the entry cost and on the form of competition. A clear message emerging from our analysis is that the optimal dividend income tax rate is higher in market structures characterized by lower competition. In particular, it is higher under Cournot competition than under Bertrand or monopolistic competition. The long-run labour income tax is positive under all market structures considered. As a result, the labour distortion cannot be removed and the long-run Ramsey equilibrium is inefficient. This suggests an analogy between dividend income taxation in our framework and capital income taxation in the standard growth model with imperfect competition. Adopting the neoclassical growth model and assuming full commitment with time-varying taxes, Judd (1985) and Chamley (1986) show that the optimal policy should eliminate all distortions on investment decisions by suppressing capital taxes in the long run. This conclusion is robust to many different extensions.² Judd (1997) and (2002) augments the standard growth model to allow for imperfectly competitive product markets with a fixed number of firms. He shows that the optimal long-run capital income tax can be negative. In particular, he argues that tax policy can be used as a substitute for antitrust policy to remove the inefficiencies delivered by a monopoly in terms of capital accumulation and the output level. Labour income taxation or consumption taxation should be used to provide the long run subsidy to capital, income and to finance government spending. Such a policy delivers efficiency along the investment margin but disregards the social cost of labour distortion. Similarly, in the framework analysed in this paper, the dividend income tax, which is a form of capital taxation,

²For a review of the extensions, see Ljungqvist and Sargent (2004) and Chari and Kehoe (1999).

offsets the distortion along the intertemporal (entry) margin. Guo and Lansing (1999) extend the analysis in Judd (1997) to argue that the magnitude and sign of the capital income tax crucially depend on the degree of monopoly power. In a similar vein, our analysis suggests that higher dividend income taxes should be implemented in less competitive market structures. Coto-Martinez, Garriga and Sanchez-Losada (2007, CGS henceforth) consider an environment characterized by endogenous entry and monopolistic competition. They argue that a key assumption in Judd's result about negative long run capital income tax was that the number of firms was fixed. They show that with endogenous entry a capital income subsidy, which is equivalent to a negative dividend income tax in this paper, could lead to too many firms in equilibrium. When both endogenous entry and endogenous labour supply are considered, it is critical whether factors and dividend income can be taxed at different rates. CGS (2007) show that when this is the case, one tax instrument is used to control for the markup inefficiency and the other tax instrument controls firms' entry. Over the business cycle, Chari, Christiano and Kehoe (1991) show, in an environment similar to that in Chamley (1986), that the capital income tax rate is close to zero on average. Chugh and Ghironi (2011) show that the optimal dividend and labour income tax rates are constant in a framework with endogenous entry and monopolistic competition. This is not the case in oligopolistic market structures. Due to the countercyclicality of the price markup, the distortions affecting the economy are time varying. Thus, counteracting these distortions requires non-constant tax rates. In particular, the dividend income tax rate is procyclical, meaning that it decreases during expansions, whereas the labour income tax rate is countercyclical. These findings are consistent with the discussions about the cyclicity of optimal subsidies in BGM (2007) and Etro (2009). Etro (2009), in particular, characterizes the cyclicity of optimal ad valorem sales subsidies and optimal labour income subsidies in a model similar to the one adopted here. However, both Etro (2009) and BGM (2007) assume that the government has access to lump sum taxes to finance the optimal subsidies. This possibility is ruled out in our analysis. Since revenues are raised with distortionary taxes, the long-run inefficiency of the Ramsey equilibrium is unavoidable and the level of activity is below the efficient one. Besides the form of competition, the form of the sunk entry costs also matters for optimal taxation purposes. For this reason, together with alternative forms of competition, two forms of the entry costs are considered. One features a constant entry cost measured in units of output, and the other one features entry costs in terms of labour. In the latter framework, building a new firm requires hiring workers. As a result, the entry

cost depends on the real wage. Since the latter is set as a markdown over the marginal product of labour, with the markup depending on the degree of competition in the goods market, changes in the extent of competition also affect the magnitude of the entry costs. The present framework features two models as special cases in the entry literature that also focus on optimal taxation problems: CGS (2007) and Chugh and Ghironi (2011). CGS (2007) consider an environment characterized by monopolistic competition under constant sunk entry costs. As mentioned above, they show that endogenous entry fundamentally affects the optimal dividend tax policy. Further, they find that tax smoothing is optimal. Chugh and Ghironi (2011) consider a framework with monopolistic competition and sunk entry cost in terms of labour.³ In this case the optimal long-run dividend income tax is zero and taxes are constant over the business cycle. By neglecting strategic interactions and considering the appropriate form of the entry costs, the provided framework reduces to either one of these models. For this reason, it can be regarded as a general framework to study optimal taxation problems under various forms of imperfect competition in a dynamic setting. It should be noted that in all the versions of the model we consider, a reduction in dividend income taxes spurs an increase in entry rates in the short run which, in turn, leads to an increase in the steady state number of producers. This is consistent with the empirical evidence in Da Rin et al. (2011), who consider a firm-level database with entry data of several million European companies between 1997 and 2004. They find an economically sizeable and statistically significant positive effect of a reduction of corporate taxation on entry rates. Colciago and Rossi (2011) introduce endogenous market structures in the Mortensen-Pissarides (1994) model of search in the labour market. This allows a simultaneous evaluation of the effects of corporate taxation on the creation of jobs and firms and the quantification of the macroeconomic effects of structural reforms in product and labour markets, an issue of central interest during the Great Recession. To conclude, it should be noted that this work extends to a dynamic general equilibrium setting a well-established literature on imperfect competition and optimal taxation in static contexts. Myles (1989) studies the design of optimal commodity taxes under imperfect competition, while Myles (1996) studies the combination of ad valorem and specific taxes that induce Ramsey pricing. De Nicolò and Matteuzzi (2000) consider homogenous Cournot oligopolies and show that ad valorem taxes are, in the case of constant marginal costs, welfare dominant with respect to specific taxes. More

³In their framework, setting up a new firm requires workers. As a result, the cost of entry depends on the real wage.

recently, Lewis (2010) studies the design of optimal labour and dividend income taxes under oligopolistic competition and endogenous entry in a static environment. She shows how optimal tax rates depend upon the degree of substitutability between goods. The remainder of the paper is as follows. Section 2 presents the model, Section 3 defines the market equilibrium, Section 4 characterizes the efficient allocation, Section 5 provides the Ramsey optimal fiscal policy and Section 6 concludes. The technical details are in the Appendix.

2 The Model

The economy features a continuum of atomistic sectors, or industries, on the unit interval. Each sector is characterized by different firms producing a good in different varieties and using labour as the only input. In turn, the sectoral goods are imperfect substitutes for each other and are aggregated into a final good. Households use the final good for consumption and investment purposes. Oligopolistic competition and endogenous firms' entry are modelled at the sectoral level. At the beginning of each period, N_{jt}^e new firms enter into sector $j \in (0, 1)$, while at the end of the period a fraction $\delta \in (0, 1)$ of market participants exits from the market for exogenous reasons.⁴ As a result, the number of firms in a sector N_{jt} , follows the equation of motion:

$$N_{jt+1} = (1 - \delta)(N_{jt} + N_{jt}^e) \quad (1)$$

where N_{jt}^e is the number of new entrants in sector j at time t . Following BGM (2012), I assume that new entrants at time t will only start producing at time $t + 1$ and that the probability of exit from the market, δ , is independent of the period of entry and identical across sectors. The assumption of an exogenous constant exit rate is adopted for tractability but it also has empirical support. Using U.S. annual data on manufacturing, Lee and Mukoyama (2007) find that, although the entry rate is procyclical, annual exit rates are similar across booms and recessions. Alternative forms of competition between the firms within each sector are considered below. In particular, the focus is on the traditional monopolistic competition setting and the approach based on oligopolistic competition developed by Jaimovich and Floetotto (2008) and Etro and Colciago (2010). As in Ghironi and Melitz (2005) and BGM (2012), who gave new life to the interesting literature on the role of entry in macroeconomic models, sunk entry costs are introduced to endogenize the number of firms in each sector. The nature and form of the entry

⁴As discussed in BGM (2012), if macroeconomic shocks are small enough, $N_{j,t}^e$ is positive in every period. New entrants finance their entry on the stock market.

costs will be specified below, where alternative specifications will also be considered. The household side is standard. They supply labour to firms and choose how much to save in riskless bonds and in the creation of new firms through the stock market.

2.1 Firms and Technology

The final good is produced according to the function

$$Y_t = \left[\int_0^1 Y_{jt}^{\frac{\omega-1}{\omega}} dj \right]^{\frac{\omega}{\omega-1}} \quad (2)$$

where Y_{jt} denotes the output of sector j and ω is the elasticity of substitution between any two different sectoral goods. The final good producer behaves competitively. In each sector j , there are $N_{jt} > 1$ firms producing differentiated goods that are aggregated into a sectoral good by a CES (constant elasticity of substitution) aggregating function defined as

$$Y_{jt} = \left[\sum_{i=1}^{N_{jt}} y_{jt}(i)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \quad (3)$$

where $y_{jt}(i)$ is the production of good i in sector j and $\theta > 1$ is the elasticity of substitution between sectoral goods. As in Etro and Colciago (2010), a unit elasticity of substitution between goods belonging to different sectors was assumed. This allows realistic separation of limited substitutability at the aggregate level and high substitutability at the disaggregate level. Each firm i in sector j produces a differentiated good with the following production function

$$y_{jt}(i) = A_t h_{jt}^c(i) \quad (4)$$

where A_t represents technology that is common across sectors and evolves exogenously over time, while $h_{jt}^c(i)$ is the labour input used by the individual firm for the production of the final good. The unit intersectoral elasticity of substitution implies that nominal expenditure, EXP_t , is identical across sectors. Thus, the final producer's demand for each sectoral good is

$$P_{jt} Y_{jt} = P_t Y_t = EXP_t. \quad (5)$$

where P_{jt} is the price index of sector j and P_t is the price of the final good at period t . Denoting with $p_{jt}(i)$ the price of good i in sector j , the demand faced by the producer of each variant is

$$y_{jt}(i) = \left(\frac{p_{jt}(i)}{P_{jt}} \right)^{-\theta} Y_{jt} \quad (6)$$

where P_{jt} is defined as

$$P_{jt} = \left[\sum_{i=1}^{N_{jt}} (p_{jt}(i))^{1-\theta} \right]^{\frac{1}{1-\theta}} \quad (7)$$

Using (6) and (5), the individual demand of good i can be written as a function of aggregate expenditure,

$$y_{jt}(i) = \frac{p_{jt}^{-\theta}}{P_{jt}^{1-\theta}} EXP_t \quad (8)$$

As technology, the entry cost and the exit probability are identical across sectors, in what follows the index j is disregarded to consider a representative sector.

2.2 Households

Consider a representative agent with utility:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t - v \frac{H_t^{1+1/\varphi}}{1+1/\varphi} \right\} \quad v, \varphi \geq 0 \quad (9)$$

where $\beta \in (0, 1)$ is the discount factor, H_t are the hours worked and C_t is the consumption of the final good. The representative agent enjoys labour and dividend income. The household maximizes (9) by choosing hours of work and how much to invest in bonds and risky stocks. The timing of investment in the stock market is as in BGM (2012) and Chugh and Ghironi (2011). At the beginning of period t , the household owns x_t shares of a mutual fund of the N_t firms that produce in period t , each of which pays a dividend d_t . Denoting the value of a firm with V_t , it follows that the value of the portfolio held by the household is $x_t V_t N_t$. During period t , the household purchases x_{t+1} shares in a fund of these N_t firms as well as the N_t^e new firms created during period t , to be carried into period $t+1$. Total stock market purchases are thus $x_{t+1} V_t (N_t + N_t^e)$. At the very end of period t , a fraction of these firms disappears from the market.⁵ Following the production and sales of the N_t varieties in the imperfectly competitive goods markets, firms distribute the dividend d_t to households. The household's total dividend income is thus $D_t = x_t d_t N_t$, which is taxed at the rate τ_t^d . The variable

⁵Due to the Poisson nature of exit shocks, the household does not know which firms will disappear from the market, so it finances the continued operations of all incumbent firms as well as those of the new entrants.

w_t is the market real wage, and τ_t^l is the tax rate on labour income. The household's holdings of the state-contingent one-period real government bond that pays off in period t are B_t and B_{t+1}^j are the end-of-period holdings of government bonds that pay off in state j in period $t+1$, which has a purchase price of $1/R_t^j$ in period t . The Flow budget constraint of the household is

$$\sum_j \frac{1}{R_t^j} B_{t+1}^j + C_t + x_{t+1} V_t (N_t + N_t^e) = (1 - \tau_t^l) w_t H_t + B_t + x_t V_t N_t + (1 - \tau_t^d) x_t d_t N_t$$

The FOCs (first-order conditions) for the household problem are represented by a standard Euler equation for bond holdings

$$\frac{1}{C_t} = \beta R_t^j \frac{1}{C_{t+1}^j}$$

an asset pricing equation

$$V_t = \beta(1 - \delta) E_t \frac{C_t}{C_{t+1}^j} [(1 - \tau_{t+1}^d) \pi_{t+1}(\theta, N_{t+1}) + V_{t+1}] \quad (10)$$

and the condition for optimal labour supply

$$v C_t H_t^{\frac{1}{\varphi}} = (1 - \tau_t^l) w_t$$

2.3 Endogenous Entry

Upon entry firms face a sunk cost, defined as f_t . In each period entry is determined endogenously to equate the value of firms to the entry costs. In what follows two popular forms of the entry cost, f_t , are considered. *Form 1*, adopted, inter alia, by Jaimovich and Floetotto (2008) and CGS (2007), features a constant entry cost measured in units of output, $f_t = \psi$. *Form 2*, adopted by BGM (2007), features an entry cost equal to η/A_t units of labour, with $\eta > 0$. Note that under this specification, technology shocks affect the productivity of the workers that produce goods and also of the workers that create new businesses.

2.4 Government

The government faces an exogenous expenditure stream $\{G_t\}_{t=0}^{\infty}$ in real terms. To finance this stream it issues real state contingent bonds, B_t^j , where the superscript j refers to the state of nature, and collects taxes on labour and dividend income. Its period-by-period budget constraint is given by

$$\sum_j \frac{1}{R_t^j} B_{t+1}^j + \tau_t = G_t + B_t \quad (11)$$

where $\tau_t = \tau_t^l w_t H_t + \tau_t^d N_t d_t$ are total tax revenues. The government consumes the same index of goods faced by the household and optimizes the composition of its expenditure across goods.⁶ Public spending evolves exogenously over time.

2.5 Strategic Interactions

In each period, the same expenditure for each sector EXP_t is allocated across the available goods according to the standard direct demand function derived from the expenditure minimization problem of the household and the government. It follows that the direct individual demand faced by a firm, $y_t(i)$, can be written as

$$y_t(i) = Y_t \left(\frac{p_t(i)}{P_t} \right)^{-\theta} = \frac{p_t(i)^{-\theta}}{P_t^{1-\theta}} Y_t P_t = \frac{p_t(i)^{-\theta} EXP_t}{P_t^{1-\theta}} \quad i = 1, 2, \dots, N_t \quad (12)$$

Inverting the direct demand functions, the system of inverse demand functions can be derived:

$$p_t(i) = \frac{y_t(i)^{-\frac{1}{\theta}} EXP_t}{\sum_{i=1}^{N_t} y_t(i)^{\frac{\theta-1}{\theta}}} \quad i = 1, 2, \dots, N_t \quad (13)$$

which will be useful in the remainder of the analysis. Firms cannot credibly commit to a sequence of strategies, therefore their behaviour is equivalent to maximize current profits in each period taking as given the strategies of the other firms. A main interest of this study is in the evaluation of the efficiency of equilibria characterized by popular forms of competition by firms, such as competition in prices and quantities. Firms take as given their marginal cost of production and the aggregate nominal expenditure.⁷ Under different forms of competition, we obtain equilibrium prices satisfying

$$p_t(i) = \mu(\theta, N_t) \frac{W_t}{A_t} \quad (14)$$

where $\frac{W_t}{A_t}$ is the marginal cost and $\mu(\theta, N_t) > 1$ is the markup function. In the next sections, the markup functions under alternative forms of market competition are characterized.

⁶Hence it follows that $g_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\theta} G_t$.

⁷Of course, both of them are endogenous in general equilibrium but it is reasonable to assume that firms do not perceive marginal costs and aggregate expenditure as being affected by their choices.

2.5.1 Price Competition

Consider competition in prices. In each period, the gross profits of firm i can be expressed as:

$$\Pi_t [p_t(i)] = \frac{\left[p_t(i) - \frac{W_t}{A_t} \right] p_t(i)^{-\theta} EX P_t}{\left[\sum_{j=1}^{N_t} p_t(j)^{1-\theta} \right]} \quad (15)$$

Firms compete by choosing their prices. We consider two alternative approaches to this problem. The first one is the traditional monopolistic competition approach, which neglects strategic interactions between firms. The second one is the Bertrand approach, where strategic interactions are taken into consideration. The outcome of profit maximization under monopolistic competition is well known. Each firm i chooses the price $p_t(i)$ to maximize profit taking as given the price of the other firms, neglecting the effect of their price choice on the sectoral price index. The symmetric equilibrium price is $p_t = \mu^{MC}(\theta) W_t/A_t$, which is associated with the constant price markup $\mu^{MC}(\theta) = \frac{\theta}{(\theta-1)}$. The latter does not depend on the extent of competition but just on the elasticity of substitution between goods. Under Bertrand competition, each firm i chooses the price $p_t(i)$ to maximize profit taking as given the price of other firms. The first-order condition for any firm i is:

$$p_t(i)^{-\theta} - \theta \left(p_t(i) - \frac{W_t}{A_t} \right) p_t(i)^{-\theta-1} = \frac{(1-\theta)p_t(i)^{-\theta} \left(p_t(i) - \frac{W_t}{A_t} \right) p_t(i)^{-\theta}}{\sum_{i=1}^{N_t} p_t(i)^{1-\theta}}$$

Note that the term on the right-hand side is the effect of the price strategy of a firm on the price index: higher prices reduce overall demand, therefore firms tend to set higher markups compared to monopolistic competition. The symmetric equilibrium price p_t must satisfy

$$p_t = \mu^B(\theta, N_t) \frac{W_t}{A_t}$$

where the markup reads as

$$\mu^B(\theta, N_t) = \frac{1 + \theta(N_t - 1)}{(\theta - 1)(N_t - 1)} \quad (16)$$

As discussed in more detail in Etro and Colciago (2010), the markup is decreasing in the degree of substitutability between products θ and in

the number of firms. Importantly, when $N_t \rightarrow \infty$ the markup tends to $\mu^{MC}(\theta)$, the standard one under monopolistic competition.⁸

2.5.2 Quantity Competition

Consider now competition in quantities in the form of Cournot competition. Using the inverse demand function (13), the profit function of a firm i can be expressed as a function of its output $y_t(i)$ and the output of all the other firms:

$$\begin{aligned} \Pi_t[y_t(i)] &= \left[p_t(i) - \frac{W_t}{A_t} \right] y_t(i) = \\ &= \frac{y_t(i)^{\frac{\theta-1}{\theta}} EXP_t}{\sum_{j=1}^{N_t} y_t(j)^{\frac{\theta-1}{\theta}}} - \frac{W_t y_t(i)}{A_t} \end{aligned} \quad (17)$$

Assume now that each firm chooses its production $y_t(i)$ taking as given the production of the other firms. The first-order conditions:

$$\left(\frac{\theta-1}{\theta} \right) \frac{y_t(i)^{-\frac{1}{\theta}} EXP_t}{\sum_i y_t(i)^{\frac{\theta-1}{\theta}}} - \left(\frac{\theta-1}{\theta} \right) \frac{y_t(i)^{\frac{\theta-2}{\theta}} EXP_t}{\left[\sum_i y_t(i)^{\frac{\theta-1}{\theta}} \right]^2} = \frac{W_t}{A_t}$$

for all firms $i = 1, 2, \dots, N_t$ can be simplified imposing the symmetry of the Cournot equilibrium. This generates the individual output:

$$y_t = \frac{(\theta-1)(N_t-1)A_t EXP_t}{\theta N_t^2 W_t} \quad (18)$$

Substituting into the inverse price, one obtains the equilibrium price $p_t = \mu^C(\theta, N_t) \frac{W_t}{A_t}$, where

$$\mu^C(\theta, N_t) = \frac{\theta N_t}{(\theta-1)(N_t-1)} \quad (19)$$

is the markup under competition in quantities. For a given number of firms, the markup under competition in quantities is always larger than

⁸Since total expenditure EXP_t is equalized between sectors, we assume that it is also perceived as given by the firms. Under the alternative hypothesis that the sum between public and private consumption, $C_t + G_t$, is perceived as given, we would obtain the higher markup:

$$\tilde{\mu}^B(\theta, N_t) = \frac{\theta(N_t-1)}{(\theta-1)(N_t-1)-1}$$

which leads to similar qualitative results. This case would correspond to the equilibrium markup proposed by Yang and Heijdra (1993).

the one obtained before under competition in prices, as is well known for models of product differentiation (see, for instance, Vives, 1999). Note that the markup is decreasing in the degree of substitutability between products θ and in the number of competitors. Finally, only when $N_t \rightarrow \infty$ the markup tends to $\mu^{MC}(\theta)$, the markup under monopolistic competition.

3 Market Equilibrium

This section contains the conditions characterizing the market equilibrium (ME, henceforth). Merging the household flow budget constraint with the government budget leads to

$$Y_t^c + N_t^e V_t = w_t H_t + N_t \pi_t \quad (20)$$

where $Y_t^c = C_t + G_t$ denotes the sum between private and public consumption of the final good. Note that the sum between labour income and profits income equals aggregate GDP. The Euler equation for firms' shares reads as

$$V_t = \beta(1 - \delta) E_t \left(\frac{C_{t+1}}{C_t} \right)^{-1} \left((1 - \tau_{t+1}^d) \pi_{t+1} + V_{t+1} \right) \quad (21)$$

while the set of Euler equations for bond holdings provides the definition of the stochastic discount factor as $\beta E_t \left(\frac{C_{t+1}}{C_t} \right)^{-1}$. The real wage can be derived from the equilibrium pricing relation as

$$w_t = \frac{A_t p_t}{\mu(\theta, N_t) P_t} = \frac{\rho_t}{\mu_t} A_t$$

where in the symmetric equilibrium $\rho_t = \frac{p_t}{P_t} = N_t^{1/(\theta-1)}$. The first-order condition for labour supply is

$$v C_t H_t^{\frac{1}{\varphi}} = \frac{(1 - \tau_t^l)}{\mu_t} \rho_t A_t \quad (22)$$

Also we must consider the equation determining the dynamics of the number of firms

$$N_{t+1} = (1 - \delta) (N_t + N_t^e) \quad (23)$$

It remains to impose the entry condition and the clearing of the market. To do so, the analysis differentiates according to the form of the entry cost. *Form 1*. When the latter is measured in constant units of output, the labour input is entirely employed for the production of the final good;

thus, the clearing of the labour market requires $H_t = N_t h_t^c$. The demand faced by firm i reads as $y_t = \frac{Y_t}{N_t \rho_t}$. In this case, firm i 's profits are

$$\pi_t = \left(1 - \frac{1}{\mu_t}\right) \rho_t y_t = \left(1 - \frac{1}{\mu_t}\right) \frac{Y_t}{N_t}$$

Aggregating profits over firms and summing to labour income delivers GDP as $\rho_t A_t H_t$. Since the entry condition is simply $V_t = \psi$, the resource constraint (24) becomes

$$Y_t^c + N_t^e \psi = \rho_t A_t H_t \quad (24)$$

Note that GDP here coincides with the production of the final good. The Euler equation for firms' share translates into

$$\psi = \beta(1 - \delta) E_t \left(\frac{C_{t+1}}{C_t} \right)^{-1} \left((1 - \tau_{t+1}^d) \left(1 - \frac{1}{\mu_{t+1}}\right) \frac{Y_t}{N_{t+1}} + \psi \right) \quad (25)$$

Form 2. When the entry cost is measured in units of labour, the economy amounts to one that features two sectors, one where $N_t^e \frac{\eta}{A_t}$ units of labour are used to produce new firms and the other one where $N_t h_t^c = H_t - N_t^e \frac{\eta}{A_t}$ units of labour are used to produce the final good. This implies that the set-up of a new firm reduces the labour input available for the production of the final good. In this setting, the individual demand faced by firms i is $y_t = \frac{Y_t^c}{N_t \rho_t}$ and firm level profits can be written as⁹

$$\pi_t = \left(1 - \frac{1}{\mu_t}\right) \rho_t y_t = \left(1 - \frac{1}{\mu_t}\right) \frac{Y_t^c}{N_t}$$

Aggregating profits over firms and summing to labour income delivers GDP as $GDP_t = \left(1 - \frac{1}{\mu_t}\right) Y_t^c + \frac{\rho_t}{\mu_t} A_t H_t$. Also, recall that the entry condition implies $V_t = f_t = \frac{\eta}{A_t} w_t = \eta \frac{\rho_t}{\mu_t}$. In this case, the resource constraint reads as

$$Y_t^c + N_t^e \eta \rho_t = \rho_t A_t H_t \quad (26)$$

⁹To see this, note that

$$P_t (C_t + G_t) = \int_0^{N_t} p_{it} y_{it} di$$

in a symmetric equilibrium it follows that

$$P_t (C_t + G_t) = N_t p_t y_t$$

thus

$$y_t = \frac{P_t (C_t + G_t)}{p_t N_t} = \frac{(C_t + G_t)}{\rho_t N_t}.$$

The Euler equation (21) for the value of the firm reduces, instead, to

$$\eta \frac{\rho_t}{\mu_t} = \beta(1 - \delta) E_t \left(\frac{C_{t+1}}{C_t} \right)^{-1} \left((1 - \tau_{t+1}^d) \left(1 - \frac{1}{\mu_{t+1}} \right) \frac{Y_{t+1}^c}{N_{t+1}} + \eta \frac{\rho_{t+1}}{\mu_{t+1}} \right) \quad (27)$$

Appendix A reports the full set of equilibrium conditions defining the ME for both forms of the entry costs.

4 Efficient Equilibrium

This section outlines a scenario where a benevolent social planner (SP) maximizes households' lifetime utility by choosing quantities directly. In doing this, the SP is subject to the same technological constraints described in the previous sections. The SP maximizes (9) with respect to $\{C_t, N_{t+1}, N_t^e, H_t\}_{t=0}^{\infty}$. The choice is subject to two constraints. The first one is given by the dynamics of the number of firms, equation (23), and the second one is the resource constraint, which is represented by equation (26) in the case of Form 2 of the entry costs and by equation (24) when entry costs are measured in terms of output. The SP takes into account the effect of the number of varieties, N_t , on the relative price, ρ_t , which is a primitive of the problem. The FOC with respect to H_t is independent of the form of the entry cost and reads as

$$\chi C_t H_t^{\frac{1}{\varphi}} = \rho_t A_t \quad (28)$$

This condition simply states that the SP equates the marginal rate of substitution between hours and consumption, the left-hand side, to the marginal product of labour, the right-hand side, which depends on the number of varieties in the economy. The SP's Euler equation depends instead on the form of the entry cost. In the case of Form 1, it reads as

$$\psi = \beta E_t (1 - \delta) \frac{C_t}{C_{t+1}} \left[\epsilon(N_{t+1}) \frac{Y_{t+1}}{N_{t+1}} + \psi \right] \quad (29)$$

where $\epsilon(N_{t+1}) = \frac{\rho_{N,t+1} N_{t+1}}{\rho_{t+1}}$ is the benefit of variety in elasticity form and $\rho_{N,t+1}$ is the derivative of the benefit of variety with respect to the number of firms. Under CES preferences $\epsilon(N_{t+1})$ is a constant equal to $\frac{1}{\theta-1}$; hence, in the remainder, we simply denote it with ϵ . Condition (29) states that the SP will create firms up to the point where the utility cost of creating a new firm, $\frac{1}{C_t} \psi$, equals the benefit from the creation of an additional variety. The latter is given by the discounted utility value of the sum between the additional utility that obtains from increasing time $t+1$ number of varieties measured in units of the final good, namely $\epsilon \frac{Y_{t+1}}{N_{t+1}}$, and the continuation value of the marginal firm, that is ψ .¹⁰ The

¹⁰Note that $\epsilon(N_{t+1})$ is an elasticity; thus, it is a pure number.

SP's Euler equation in the case of entry costs in Form 2 reads as

$$\eta\rho_t = \beta E_t (1 - \delta) \frac{C_t}{C_{t+1}} \left[\epsilon \frac{Y_{t+1}^c}{N_{t+1}} + \eta\rho_{t+1} \right] \quad (30)$$

The interpretation of equation (30) is similar to that provided for equation (29), with two differences. The first one is that the value of a new firm in terms of the final good is not constant over time (and states of nature) but depends on the benefit of variety ρ_t . The reason is that love for variety affects the marginal productivity of labour and thus the opportunity cost of creating a new firm in terms of the final good. This also implies that whenever the number of varieties differs from the efficient one the value of a firm is distorted with respect to the efficient one. The second difference is that at time $t+1$ the creation of N_{t+1}^e new firms requires $\frac{\eta}{A_t} N_{t+1}^e$ units of labour, which are diverted from the production of the final good. As a result, the additional output that the marginal firm creates depends on the sum between private and public consumption, Y^c , and not on GDP. Appendix B reports details concerning the computation of the efficient equilibrium for both forms of the entry costs.

4.1 Market Distortions

The market allocation features two distortions. To identify them it is convenient, for the time being, to set fiscal instruments to zero, $\tau_t^d = \tau_t^l = 0$. In the next sections we reintroduce fiscal instruments and design them in order to minimize the welfare losses associated with the distortions that we are about to discuss. The first distortion is referred to as the *Labour Distortion*. In the competitive equilibrium, labour is supplied up to the point where the following condition is satisfied

$$\chi C_t H_t^{1/\varphi} = \frac{\rho_t}{\mu_t} A_t \quad (31)$$

A comparison between equation (28) and equation (31) reveals that in the decentralized equilibrium the marginal rate of substitution between hours and consumption, $\chi C_t h_t^{1/\varphi}$, is lower than the marginal rate of transformation between hours and output, $\rho_t A_t$. As in other models with an imperfectly competitive goods market, as for example in BGM (2012), this wedge is due to the presence of a price markup. Importantly, oligopolistic competition renders this wedge time varying. The second distortion involved in the decentralized allocation is an *Entry Distortion*. This wedge operates through the intertemporal firms creation margin and could lead to an inefficient number of firms in equilibrium. The number of firms could be inefficient for the following reason, as argued by CGS (2007). The entry decision of the individual firm ignores the welfare

gains associated with increased variety. This leads to inefficiently low entry. Also, the potential entrant ignores the negative effects of increased competition on the profits of other firms already in the market. The latter, known as the business stealing effect, leads to excessive entry. As a result, the equilibrium number of firms could be either inefficiently high or low, depending on which externality predominates. To illustrate the entry distortion the Euler equations in the decentralized economy will be compared with those obtained in the efficient equilibrium. Again, it is convenient to distinguish according to the form of the entry costs and to consider a non-stochastic version of the economy. *Form 1.* Consider the right-hand side of the Euler equation in the decentralized equilibrium, equation (25). The latter equals the right-hand side of the corresponding equation in the centralized equilibrium, equation (29), if

$$\left(1 - \frac{1}{\mu_{t+1}}\right) = \epsilon \quad (32)$$

Thus, efficiency holds if the so-called Lerner index equals the benefit of variety in elasticity form.¹¹ In this case the private incentive to enter the market, $\left(1 - \frac{1}{\mu_{t+1}}\right)$, is identical to the social one, and the two externalities described above cancel out. If the Lerner index is larger (lower) than the social incentive there will be excessive (too low) entry. Since $\epsilon = \frac{1}{\theta-1}$ the condition above requires the price markup to be constant over time and states of nature, $\mu_t = \mu$. As a result, it cannot be satisfied under oligopolistic competition. Note also that although monopolistic competition leads to a constant markup $\mu = \frac{\theta}{\theta-1}$, the condition for efficiency is not satisfied since $1 - \frac{1}{\mu} < \epsilon$. *Form 2.* Comparing the Euler equation in the decentralized equilibrium, equation (27), with the corresponding Euler equation in the SP equilibrium, equation (30), shows that two conditions need to be imposed to reinstate efficiency, namely

$$\mu_t \left(1 - \frac{1}{\mu_{t+1}}\right) = \epsilon, \quad (33)$$

and

$$\frac{\mu_t}{\mu_{t+1}} = 1. \quad (34)$$

¹¹The Lerner index is defined as follows:

$$\begin{aligned} LI &= \frac{p(i) - MC(i)}{p(i)} = \frac{\frac{p(i)}{P} - \frac{MC(i)}{P}}{\frac{p(i)}{P}} = \\ &= \frac{\rho - \frac{\rho}{\mu}}{\rho} = 1 - \frac{1}{\mu} \end{aligned}$$

Combining conditions (33) and (34), we recover those emphasized by BGM (2007), that is

$$\epsilon = \mu_{t+1} - 1 \text{ and } \frac{\mu_t}{\mu_{t+1}} = 1$$

which imply that the price markup should be constant over time and that the benefit of variety in elasticity form should equal the market power as measured by the net markup. Under oligopolistic competition both conditions fail. Contrary to the previous case, these conditions are satisfied under monopolistic competition, as emphasized by BGM (2007) and Chugh and Ghironi (2011). Thus, under monopolistic competition, with entry costs in Form 2, the entry margin is not distorted. Note that the economic environments in Chugh and Ghironi (2011) and CGS (2007) can be considered as special cases of the one outlined here. In particular, the present set-up collapses to that considered by CGS (2007) when entry costs are in Form 1 and strategic interactions between firms are neglected. It coincides, instead, with that considered by Chugh and Ghironi (2011) when entry costs are in Form 2 and strategic interaction are neglected. The next section outlines the fiscal policy aimed at minimizing welfare losses due to the market distortions when the government can raise revenues solely by imposing distortionary taxes on labour and dividends and issuing state contingent bonds.

5 Ramsey Optimal Fiscal policy

In this section we study the second-best tax policy in an economy without lump sum taxes. Consider the Euler equation for firms' shares, which in its general form reads as

$$f_t = \beta(1 - \delta)E_t \left(\frac{C_{t+1}}{C_t} \right)^{-1} ((1 - \tau_{t+1}^d) \pi_{t+1} + f_{t+1})$$

Expected future dividend income taxes cannot be removed from this equation using other equilibrium conditions. As a result, a pure primal approach cannot be applied to solve for the optimal policy. As emphasized by Chugh and Ghironi (2011), the set of allocations that the SP can select cannot uniquely be characterized by means of the so-called implementability constraint. Further, computing a first-order condition with respect to $E_t \tau_{t+1}^d$ would leave τ_{t+1}^d indeterminate from the point of view of time-t. To resolve this indeterminacy issue the solution proposed by Chugh and Ghironi (2011) is adopted. In particular, it is assumed that the SP chooses a state contingent schedule for the time t+1 dividend income tax rate $\tau_{j,t+1}^d$, where j indexes the state of the economy.

This schedule is in the time t information set. Also, we assume that the Ramsey Planner commits to the schedule, meaning that the state contingent tax rate is implemented with certainty at time $t+1$.¹² The Ramsey allocation is derived by solving

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t - v \frac{h_t^{1+1/\varphi}}{1+1/\varphi} \right\}$$

The choice variables are C_t , N_t , H_t , N_t^e and $\tau_{t/t+1}^d$. The allocation is restricted by four constraints; two of them depend on the form of the entry cost. The constraints, which are independent of the form of the entry cost, are equation (23), determining the dynamic of the number of firms, and the implementability constraint, which reads as

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[1 - v h_t^{\frac{1}{\varphi}} \right] = \frac{b_0}{C_0} + \frac{1}{C_0} [(1 - \tau_0^d) d_0 + V_0] N_0 s_0$$

The allocation is further restricted by equations (24) and (25) in the case of the entry cost of Form 1 and by equations (26) and (27) in the case of the entry cost in Form 2. Government expenditure $\{G_t\}_{t=0}^{\infty}$ is exogenously given. The first-order conditions for the Ramsey problem are reported in the Appendix for both forms of the entry costs. To compute Ramsey allocations I follow Linneman and Shabert (2011). That is, throughout the analysis it is assumed that the policy maker can credibly commit himself but the initial period ($t = 0$) is ignored. In deriving the Ramsey policy, the problem that the policy maker's decision rules will be different for the first period in which the policy is implemented is neglected. This is justified by the fact that the interest is in making statements about the deterministic steady state as well as about business cycle fluctuations around it, while the transition path from the initial values towards the steady state is not analysed. As is common in the Ramsey literature (see Khan, King and Wolman (2003)), it is further assumed that the initial values of the predetermined variables are equal to their values in the deterministic Ramsey steady state. This amounts to adopting the so-called *Timeless Perspective*, popularized by Woodford (2003). The dynamics in response to shocks under the Ramsey policy are obtained by solving a first-order approximation to the Ramsey first-order conditions. As shown in various contributions by Schmitt-Grohè and Uribe, in models characterized by more frictions than are in the present one, a first-order approximation to the first-order conditions delivers dynamics that are very close to the exact ones.

¹²In other words, $\tau_{t/t+1}^d$ is chosen at time t . The selected value is then implemented with certainty at time $t+1$.

5.1 Calibration

Since part of the following analysis is numerical, the calibration of structural parameters follows. The time unit is a quarter. The discount factor, β , is set to the standard value for quarterly data, 0.99, while the rate of business destruction, δ , equals 0.025 to match the U.S. empirical level of 10 per cent business destruction per year. Steady-state productivity is equal to $A = 1$. The baseline value for the entry cost is set to $\eta = \psi = 1$. This leads to a share of investment in GDP of around 12 per cent in our models. The baseline value for the intrasectoral elasticity of substitution is $\theta = 6$, which is in line with the typical calibration for monopolistic competition and delivers markup levels within the empirically relevant range.¹³ Turning to fiscal parameters, the ratio of government spending over GDP equals 0.22, as estimated by Schmitt-Grohè and Uribe (2005). For the sake of calibration, we set the steady state labour income tax rate to 20 per cent and the steady state dividend income tax rate to 30 per cent and assume that the government has lump sum taxes available to balance its budget. These values represent the mean labour income tax rate and the mean dividend income tax rate in the US over the period 1947:Q1-2009:Q4, as reported by Chugh and Ghironi (2011). When computing the Ramsey steady state and Ramsey optimal policy we assume that lump sum taxes are not available and fix the ratio of government debt to output equal to 0.5 on an annual basis, in line with the U.S. post-war average. In what follows, equilibrium allocations under Cournot, Bertrand and monopolistic competition for each entry cost configuration are compared. This is done while holding parameters fixed in order to understand the role of the different market structures. To this end, the following calibration strategy for the utility parameter v is adopted. The value of v is such that steady state labour supply is equal to one under monopolistic competition. In this case, the Frish elasticity of labour supply reduces to φ , to which we assign a value of four, as in King and Rebelo (2000). Next, the values of v and φ are held constant under both Bertrand and Cournot competition. In all the versions of the models we consider, a reduction in dividend income taxes spurs an increase in entry rates in the short run, which, in turn, leads to an increase in the steady state number of producers. Under the baseline calibration, a 1 per cent reduction in the dividend income tax rate leads to an impact increase in the entry rate of about 0.4 per cent in all the ver-

¹³Oliveira, Martins and Scarpetta (1999) provide estimates of price markups for US manufacturing industries over the period 1970–1992. In broad terms, most of the sectoral markups defined over value added are in the range of 30–60 per cent; when they are defined over gross output they are in the range of 5–25 per cent. In the latter case, high markups of over 40 per cent are observed in a few sectors.

sions of the model considered.¹⁴ The elasticity of the long-run number of producers to the long-run dividend income taxes ranges from 1 per cent in the case of Cournot competition under entry costs in Form 2 to 1.7 per cent under Bertrand competition under entry costs in Form 1. These features of the model are consistent with the empirical evidence in Da Rin et al. (2011), who consider a firm-level database with entry data of several million European companies between 1997 and 2004. They find an economically sizeable and statistically significant positive effect of a reduction of corporate taxation on entry rates.¹⁵ There are two exogenous processes in the economy, namely those for government spending and for technology. They are both assumed to be AR (1) processes in log deviations from the steady state:

$$\begin{aligned}\log \frac{G_t}{G} &= \rho_g \log \frac{G_{t-1}}{G} + \varepsilon_t^g \\ \log \frac{A_t}{A} &= \rho_a \log \frac{A_{t-1}}{A} + \varepsilon_t^a\end{aligned}$$

The autoregressive coefficient for the technology process is $\rho_a = 0.979$ and the standard deviation of the disturbance, σ_a , is 0.0072, as in the RBC model by King and Rebelo (2000). The parameterization of the government spending process follows Chari and Kehoe (1999) in setting $\rho_g = 0.97$ and $\sigma_g = 0.027$.

5.2 Ramsey Steady State

As discussed above, the market equilibrium is cursed by two distortions. The *Labor Distortion* is due to a wedge between the within period marginal rate of substitution between hours and consumption and the marginal rate of transformation between hours and output. Considering the tax rate on labor income, the steady state wedge is given by $\frac{1-\tau^l}{\mu}$.¹⁶ Efficiency requires a labor income subsidy such that $1 - \tau^l = \mu$. However, absent lump sum taxes and with the need to finance Government spending, the subsidy cannot be provided.¹⁷ As a result the Ramsey Planner

¹⁴The entry rate is measured by the number of entering firms as a percentage of the total number of firms in each period. The impact change of the entry rate in the face of a reduction in the dividend income tax rate is obtained numerically, simulating the effect of a 1 per cent unexpected reduction in the tax rate.

¹⁵Specifically, they find that a 10 per cent reduction in taxation implies a 2.64 per cent increase in the entry rate.

¹⁶To see this consider the first order condition for labor supply in the ME, that is equation (22).

¹⁷A labor income subsidies $\tau^l < 0$ can only occur if the Government holds assets to the extent that taxes are not necessary to finance public spending.

cannot remove the labor distortion, and hours will be lower in the Ramsey steady state with respect to those worked in the long run allocation reached by the SP.

On the contrary, by using the dividend income tax, the Ramsey Planner removes the *Entry Distortion*. This result was first identified by Chugh and Ghironi (2011). In what follows it is shown that the level of the optimal dividend income tax rate differs according to the form of competition and to the form of the entry cost. Proposition 1 provides the main result of this section.

Proposition 1 *Optimal long-run dividend income taxes depend on the form of the entry costs*

$$(i) \text{ Form 1: } \tau^d = 1 - \frac{\epsilon}{1 - \frac{1}{\mu}}$$

$$(ii) \text{ Form 2: } \tau^d = 1 - \frac{\epsilon}{\mu - 1}$$

Proof. In the Appendix. ■

Consider condition (i). As mentioned above when the private incentive to create a new firm, as measured by the Lerner Index, that is $\left(1 - \frac{1}{\mu}\right)$, is larger than the social incentive to introduce a new variety, ϵ , there would be excessive entry. In this case it is optimal to tax profits. A profit income subsidy is, instead, optimal in the opposite situation. In the Appendix it is shown that under the optimal dividend income tax policy, the steady state version of the Planner Euler equation for the number of firms is

$$\psi = \beta(1 - \delta) \left[\epsilon \frac{\rho AH}{N} + \psi \right]$$

which is identical to the steady state counterpart of equation (29), i.e. the Euler equation obtained in the efficient equilibrium. This implies that the Planner implements the efficient level of units of effective labor per firm, $\frac{\rho AH}{N}$. Recall, however that the Ramsey Planner cannot remove the labor distortion. For this reason steady state hours will be lower in the Ramsey steady state with respect to the those observed in the long run allocation reached by the SP. As a result, the number of firms is lower than the optimal one even if the inefficiency along the entry margin is removed. Notice that in the standard neoclassical growth model the Ramsey Planner would target the efficient ratio between capital and hours. This further emphasizes the analogy between the stock of capital

in the neoclassical model and the stock of firms in the economy we have outlined. The optimal dividend income tax rate under Bertrand competition is

$$\tau_{\text{Bertrand}}^d = \frac{1}{N} - \frac{1}{\theta - 1}$$

while under Cournot competition we obtain

$$\tau_{\text{Cournot}}^d = \frac{1}{N} - \frac{1}{(\theta - 1)^2}$$

Notice that the optimal tax rate in the case of monopolistic competition is

$$\tau_{\text{Monopolistic}}^d = -\frac{1}{\theta - 1},$$

Under oligopolistic competition the optimal dividend income tax could take the form of a subsidy, depending on the parameterization of the model. On the contrary, removing the entry distortion under monopolistic competition requires a subsidy no matter the parameterization of the model. This means that monopolistic competition always leads to a suboptimal steady state number of firms. Figure 1 displays, in the case of entry costs in Form 1, the Ramsey steady state number of firms, hours, output per firm and optimal tax rates as a function of the entry cost under Bertrand, Cournot and monopolistic competition. Allocations are compared to the efficient ones. First, for any given value of the entry cost, the Ramsey Planner implements the same allocation for hours and the number of firms across market structures.

These differ from the efficient ones. As mentioned above, since the Planner cannot remove the labor distortion, hours and the number of firms are lower than their efficient counterpart. The Ramsey Planner targets the efficient level of units of effective labor per firm. This, however, requires a different combination of optimal taxes across market structures. In the case of Cournot competition the market allocation would lead to an inefficiently large number of firms, as a result restoring efficiency along the entry margin requires a positive dividend income tax. On the contrary, as in CGS (2007), the number of firms under monopolistic competition would be too low and dividend income should be subsidized to promote entry. Bertrand competition falls between these two cases. As stated by Vives (1984), Bertrand competition can be regarded as a more competitive market structure with respect to Cournot and for this reason it is judged as more efficient.¹⁸ This analysis suggests that, for given entry costs, the dividend income tax should be

¹⁸Vives (1984) provide the following intuitive explanation to support this view. In

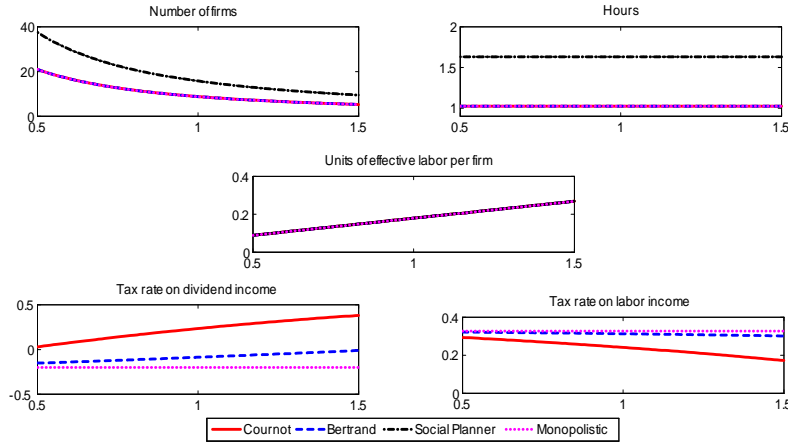


Figure 1: Ramsey Steady State under Entry Costs in Form 1. Entry cost (ψ) on the horizontal axis.

lower in markets characterized by more competitive market structures. The opposite instead holds for the labor income tax. Due to the need of financing the exogenous level of public expenditure the optimal labor income tax is instead higher under monopolistic competition, where dividend income was more heavily subsidized, with respect to other market structures.

Condition (ii) is isomorphic to that obtained by BGM (2007) and Chugh and Ghironi (2011) under the case of monopolistic competition. In this case profits should be taxed whenever the net markup exceeds the benefit of variety in elasticity form. However, under monopolistic competition condition (ii) is automatically satisfied, since the benefit of variety in elasticity form equals the net markup and markups are constant, and the market equilibrium displays efficiency along the entry margin. This is not the case under oligopolistic competition. Under Bertrand competition the optimal tax rate is

$$\tau_{\text{Bertrand}}^d = \frac{1}{N}$$

while under Cournot competition is

$$\tau_{\text{Cournot}}^d = \frac{\theta}{N + \theta - 1}$$

Cournot competition each firm expects the others to cut prices in response to price cuts, while in Bertrand competition the firm expects the others to maintain their prices; therefore Cournot penalizes price cutting more. One should expect Cournot prices to be higher than Bertrand prices.

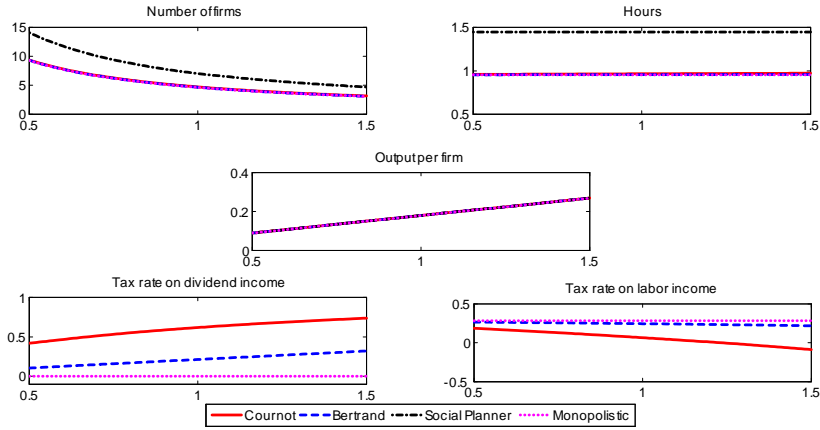


Figure 2: Ramsey Steady State under Entry Costs in Form 2. Entry cost (η) on the horizontal axis.

Hence entry costs in Form 2 always imply a positive dividend income tax rate under oligopolistic competition. Figure 2 displays, for the case of entry costs in Form 2, the Ramsey Steady State number of firms and hours together with optimal tax rate as a function of the entry cost. In this case allocations differ across market structures. The reason is that, under entry costs in Form 2, the price markup affects the market value of a firm. As a result the initial stock of wealth, which affects the implementability constraint, depends on the form of the market structure.¹⁹ As above, since the Labor distortion cannot be removed, hours and the number of firms are lower with respect to their efficient counterpart. In the Appendix it is shown that the Ramsey Planner targets the efficient level of output per firm, $\frac{Y^c}{N}$.²⁰

¹⁹Recall that under the *Timeless Perspective* the initial values of the predetermined variables are equal to their values in the deterministic Ramsey steady state.

²⁰As shown by Chugh and Ghironi (2011) the Ramsey Planner can also remove the entry distortion by using an entry subsidy instead of the dividend income tax. Define τ^s the entry subsidy such that the net entry cost is $(1 - \tau_t^s) f_t$. It can be shown that optimal subsidies depend on the form of the entry cost as follows

$$(i) \text{ Form 1: } \tau^s = 1 - \frac{1 - \frac{1}{\mu}}{\epsilon}$$

$$(ii) \text{ Form 2: } \tau^s = 1 - \frac{\mu - 1}{\epsilon}$$

The Ramsey Planner will resort to an entry tax in the case of excessive entry or to a subsidy in the case of inefficiently low entry.

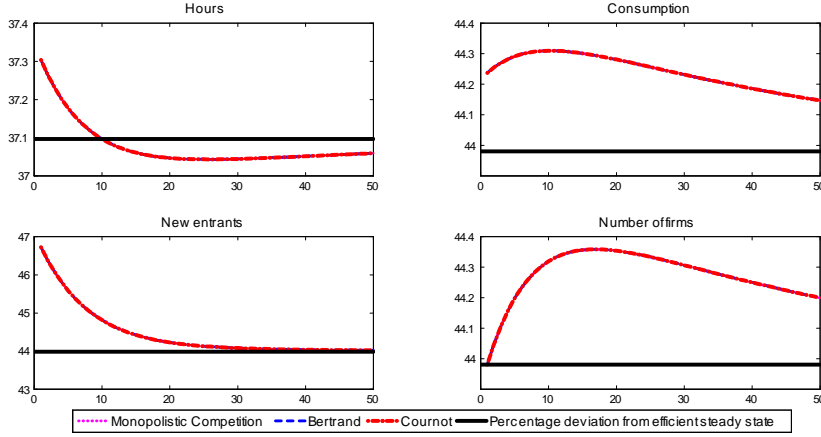


Figure 3: Entry costs in Form 1. Percentage deviations between efficient and second best dynamics in response to a technology shock. Horizontal lines refer to steady state percentage deviations.

5.3 Ramsey Dynamics and Optimal Tax Volatility

This section compares the efficient equilibrium allocation to the Ramsey allocation and shows the business cycle implications of the Ramsey optimal policy in response to technology shocks. As mentioned above, dynamics are obtained by solving a first-order approximation to the equilibrium conditions.

Consider variable x . Define

$$\tilde{x}_t = \frac{x_t^{eff} - x_t^R}{x^{eff}}$$

as the deviation between the value assumed by x_t in the efficient equilibrium, x_t^{eff} , and that assumed by the same variable in the Ramsey equilibrium, x_t^R , where x^{eff} is the value of x at the efficient steady state. Deviations are expressed as a percentage of efficient steady state values.

Figure 3 depicts the dynamics of \tilde{H}_t , \tilde{N}_t , \tilde{C}_t and \tilde{N}_t^e in response to a one standard deviation technology shock under entry costs in Form 1. Dashed and dash-dotted lines refer to Ramsey dynamics under Bertrand and Cournot competition respectively, and dotted lines to the case of Ramsey dynamics under monopolistic competition. Horizontal continuous lines in each panel represent deviations between the first best steady state and the Ramsey steady state as a percentage of the first best steady state, that is $\tilde{x}^{ss} = \frac{x^{eff} - x^R}{x^{eff}}$, where x^R is the value of x at the Ramsey steady state.

Notice that deviations between the efficient and the Ramsey equilib-

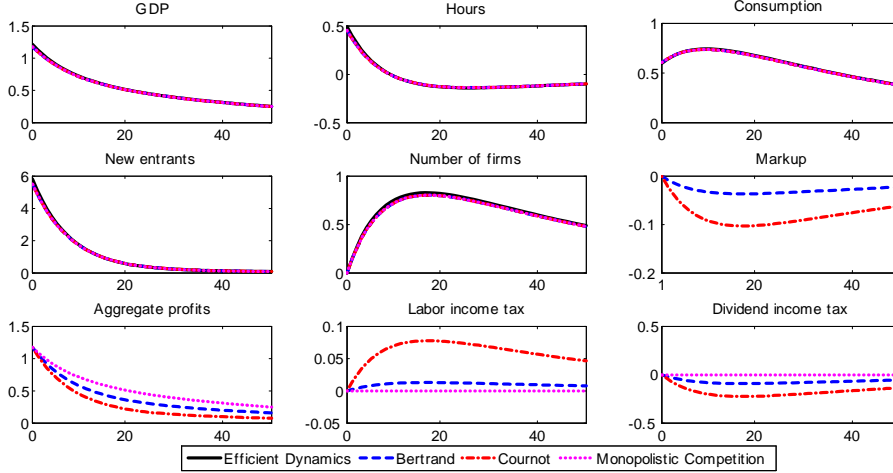


Figure 4: Entry Costs in form 1. Response of the main macroeconomic variables to a one standard deviation shock to technology. Solid lines refer to the SP allocation, dashed and dash-dotted lines refer to Ramsey dynamics under Bertrand and Cournot respectively and dotted lines to the case of Ramsey dynamics under monopolistic competition.

ria, as well as steady state deviations, are identical across market structures. The Figure shows that the bulk of the difference between the first and second best allocation is due to the steady state difference, which is not affected by the shock. Since the Planner must raise revenues using distortionary taxes, the long-run inefficiency of the Ramsey equilibrium is unavoidable, and the level of activity is below the efficient one. While the deviations of the main macroeconomic variables from their first best values do not depend on the form of the market structure, the dynamics of tax rates in response to the technology shock do.

To show this, and to further understand the transmission of technology shocks implied by the model economy, Figures 4 depicts percentage deviations from the respective steady states of key variables in response to a one standard deviation technology shock under entry costs in Form 1. For tax rates we report deviations from the steady state level in percentage points. Continuous lines refer to the efficient equilibrium, dashed and dash-dotted lines refer to Ramsey dynamics under Bertrand the Cournot competition respectively, and dotted lines to the case of Ramsey dynamics under monopolistic competition.

The technology shock creates expectations of future profits which lead to the entry of new firms in the market. This is so under both the Ramsey and the efficient equilibrium. Under monopolistic compe-

tition the markup is constant along the business cycle. As can be seen from Figure 4, this results in constant tax rates. This is not the case under Oligopolistic competition. Under Bertrand and Cournot competition the entry of new firms leads to higher competition which, in turn, leads to a countercyclical price markup. A non constant price markup implies that the inefficiency wedges in the economy are also non constant. The Ramsey Planner partially offsets the effects of changes in the price markup on the inefficiency wedges by adjusting the tax rates. To see this, consider the optimal response of the labor income tax rate, τ_t^l . As mentioned above, in the ME there exists an inefficiency wedge between the marginal rate of substitution between hours and consumption and the corresponding marginal rate of transformation. Considering the tax rate on labor income, this is given by $\frac{1-\tau_t^l}{\mu_t}$.²¹ Efficiency requires $1 - \tau_t^l = \mu_t$, that is a labor income subsidy which, in response to shocks, moves in opposite direction with respect to the price markup. However, absent lump sum taxes and with the need to finance Government spending, the subsidy cannot be provided. Nevertheless, the analysis shows that τ_t^l offsets the effects of changes in the price markup on the inefficiency wedge. In particular, when the markup decreases, due to higher competition, the labor income tax rate increases. As discussed in Bilbiie, Ghironi and Melitz (2007) and Etro (2009) a lower tax rate should be adopted in periods/states with a lower number of producers.²² The Figure shows that the optimal response of the labor income tax rate is symmetric, with respect to the horizontal axis, to the response of the price markup. Similar reasoning explains the dynamics of the optimal dividend income tax rate. As competition increases the price markup reduces, leading to a lower private incentive to create a new firm. To align the private to the social incentive, which is constant, the Ramsey Planner reduces the dividend income tax rate. Changes in the tax rates are mild, but stronger under Cournot competition where the markup is characterized by a higher elasticity to the number of firms with respect to Bertrand. The Ramsey policy under entry costs in Form 1 is, thus, characterized by a countercyclical labor income tax rate and by a procyclical dividend income tax.

Figure 5 displays the dynamics of \tilde{H}_t , \tilde{N}_t , \tilde{C}_t and \tilde{N}_t^e in response to a one standard deviation technology shock under entry costs in Form 2. Lines have the same meaning as in the Figure 3. With respect to the case

²¹This can be understood by considering the first order condition for labor supply in the ME, that is equation (22).

²²Etro (2009) also considers a tax rate on sales. He argues that the tax rate should be countercyclical.

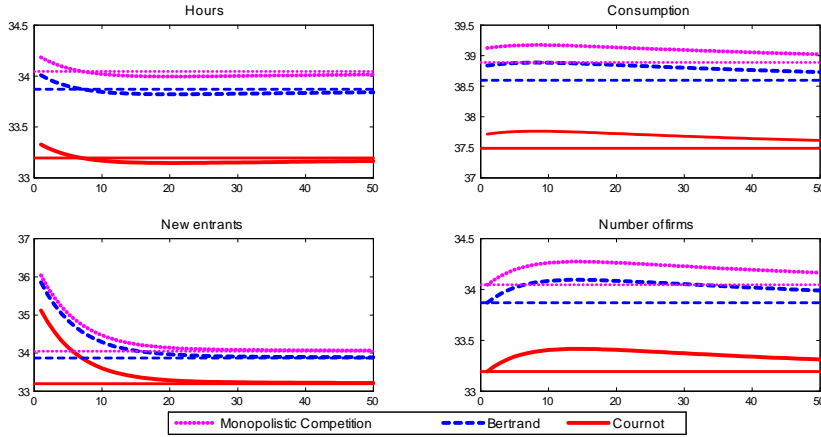


Figure 5: Entry costs in form 2. Percentage deviations between efficient and second best dynamics in response to a technology shock. Horizontal lines refer to steady state percentage deviations.

of entry costs in Form 1, allocations differ across market structures.²³ For both hours and consumption the largest deviation with respect to first best can be observed under monopolistic competition. This is due to the fact that the deviation between the Ramsey steady state and the efficient steady state is larger under monopolistic competition with respect to other market structures.

Figure 6 portrays percentage deviations from the respective steady state of key variables in response to a one standard deviation technology shock under entry costs in Form 2. Lines have the same meaning as in Figure 4. Previous considerations extend to this case. A relevant difference is that offsetting the distortion along the entry margin requires an initial increase in the dividend income tax rate which is reverted after few periods.

Notice that similar dynamics, although quantitatively less sizeable, can be observed in the case of a Government spending shock.²⁴ The variability of the main macroeconomic variables when the system is contemporaneously perturbed by a technology and a government spending shock is analysed next. Table 1 displays the mean (\bar{x}), the coefficient of

²³In the previous subsection it was suggested that the initial stock of wealth differs across market structures under entry costs in Form 2. The initial stock of wealth affects the implementability constraint, leading to a difference in the allocations across market structures.

²⁴Dynamics in response to a Government spending shock are not reported. Notice, however, that, aggregate consumption drops in response to a Government spending shock under all the market structures considered.

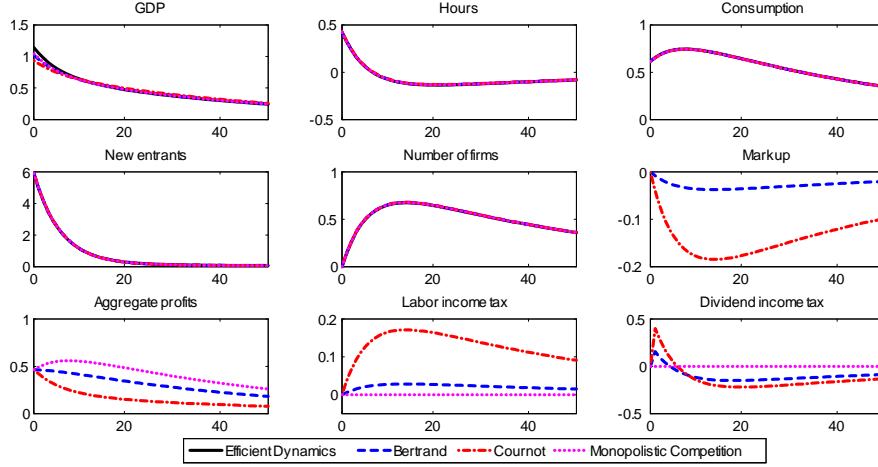


Figure 6: Entry Costs in form 2. Response of the main macroeconomic variables to a one standard deviation shock to technology. Solid lines refer to the social planner allocation, dashed and dash-dotted lines refer to Ramsey dynamics under Bertrand and the Cournot respectively and dotted lines to the case of Ramsey dynamics under monopolistic competition.

variation ($\sigma(x)/\bar{x}$) and the correlation with output ($Cor(Y_t, x_t)$) of a number of variables of interest under the Ramsey dividend and labour income taxation policy in the case of entry costs in Form 1. For tax rates, the standard deviation in percentage points ($\sigma(x)$) is reported instead of the coefficient of variation. Table 2 reports the same statistics for the case of entry costs in Form 2.

Under entry costs in Form 1, the variability of the main macroeconomic variables under the optimal policy is identical across the market structures. However, this is reached by means of a different fiscal policy. As expected from Figure 4, although under monopolistic competition taxes are constant, this is not the case under oligopolistic competition. Tax rates are more volatile under Cournot competition than under Bertrand, with the dividend income tax being more variable than the labour income tax rate. Recall that the elasticity of the price markup to the number of firms is higher under Cournot. As a result, minimizing the welfare cost of the distortions over the business cycle requires more variable taxes when firms compete in quantities.

Under entry costs in Form 2, allocations and volatilities are no longer identical across market structures. Interestingly, while the overall variability characterizing the economy in response to shocks, as measured

x	Y	C	H	N	N^e	τ_t^l	τ^d
Monopolistic competition							
\bar{x}	1.58	1.01	1.02	8.83	0.22	0.32	-0.2
$\sigma(x)/\bar{x}$	1.79	0.88	1.15	0.61	7.49	0	0
$Cor(Y_t, x_t)$	1	0.60	0.87	0.4	0.95	0	0
Bertrand Competition							
\bar{x}	1.58	1.01	1.02	8.83	0.22	0.31	-0.08
$\sigma(x)/\bar{x}$	1.79	0.88	1.15	0.61	7.49	0.02	0.07
$Cor(Y_t, x_t)$	1	0.60	0.87	0.4	0.95	0.11	-0.4
Cournot Competition							
\bar{x}	1.58	1.01	1.02	8.83	0.22	0.24	0.23
$\sigma(x)$	1.79	0.88	1.15	0.61	7.49	0.06	0.17
$Cor(Y_t, x_t)$	1	0.60	0.87	0.4	0.95	0.12	-0.4

Table 1: Mean, standard deviations and correlations with output of main-macro variables under alternative market structures. Shocks are to productivity and government spending. Entry Costs in Form 1

x	Y	C	H	N	N^e	τ_t^l	τ^d
Monopolistic competition							
\bar{x}	1.27	0.85	0.95	4.64	0.11	0.28	0
$\sigma(x)/\bar{x}$	1.60	0.91	1.10	0.61	7.90	0	0
$Cor(Y_t, x_t)$	1	0.60	0.85	0.50	0.93	0	0
Bertrand Competition							
\bar{x}	1.27	0.86	0.96	4.66	0.12	0.24	0.21
$\sigma(x)/\bar{x}$	1.57	0.91	1.09	0.61	7.90	0.02	0.26
$Cor(Y_t, x_t)$	1	0.60	0.85	0.50	0.93	0.21	0.60
Cournot Competition							
\bar{x}	1.25	0.87	0.96	4.68	0.12	0.07	0.62
$\sigma(x)$	1.46	0.91	1.08	0.61	7.91	0.15	0.59
$Cor(Y_t, x_t)$	1	0.60	0.85	0.53	0.91	0.24	0.72

Table 2: Mean, standard deviations and correlations with output of main-macro variables under alternative market structures. Shocks are to productivity and government spending. Entry costs in Form 2

by the standard deviation of output, is higher under entry cost in Form 1, the variability of tax rates is higher under entry costs in Form 2. In particular, the standard deviation of the dividend income tax under Cournot competition is sizeable.

6 Conclusions

This study proposes an economy where the degree of market power, as measured by the price markup, depends endogenously on the form of competition, on the degree of substitutability between goods and on the number of firms. Imperfect competition leads to distortions in both the goods and the labour market and in both the short and the long run. The optimal long-run dividend income corrects for inefficient entry, and it is higher in market structures characterized by lower competition. In particular, it is higher under Cournot competition than under Bertrand or monopolistic competition. The labour distortion cannot be removed by the Ramsey Planner. As a result, hours worked and the number of firms/products in the Ramsey steady state are lower than their efficient counterparts. As mentioned in the text, the resulting steady-state distortion is large. Whereas optimal taxes over the business cycle are constant under monopolistic competition, this is not the case in an oligopolistic market structure. Also, the effect of alternative forms of sunk entry costs for the design of optimal taxation has been considered. The resulting framework features as special cases two models in the entry literature that also focus on optimal taxation problems in the case of endogenous dynamics of the number of firms. Coto Martinez et al. (2007) consider an environment characterized by monopolistic competition under constant sunk entry costs. Chugh and Ghironi (2011) consider a framework with monopolistic competition and sunk entry cost in terms of labour. By neglecting strategic interactions and considering the appropriate form of the entry costs our model reduces to either one of these models. For this reason, it can be regarded as a general framework to study optimal taxation problems under various forms of imperfect competition. The analysis could be extended in various dimensions. One aspect we neglect is the asymmetry between market competitors in terms of both size and the probability of exit from the market. Haltiwanger et al. (2010) show that younger firms are more likely to exit from the market than more mature firms. Implementing these features in the model and studying their effects on optimal taxation is left for future research.

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Appendix

Appendix A. Market Equilibrium

6.0.1 Entry Costs in Form 1

Given the exogenous processes $\{A_t, G_t\}_{t=0}^{\infty}$ and processes $\{\tau_t^d, \tau_t^l\}_{t=0}^{\infty}$, the Market Equilibrium (ME) consists of an allocation $\{C_t, H_t, N_t, N_t^e, B_{t+1}\}_{t=0}^{\infty}$ which satisfies the first order condition for labor supply, equation (22), the Government budget, equation (11), the dynamics of the number of firms, equation (23), the definition of the price markup and the definition of the love for variety, the resource constraint (24) and the Euler equation (25) .

To compute the steady state of the ME, notice that the steady state

number of entrants is $N^e = \frac{\delta}{(1-\delta)}N$ and that the Euler equation for shares implies $V = \frac{\beta(1-\delta)(1-\tau^d)}{[1-\beta(1-\delta)]}\pi$.

Steady state profits are given by²⁵

$$\pi = \rho y - wh = \rho \left(1 - \frac{1}{\mu}\right) y = \rho \left(1 - \frac{1}{\mu}\right) \frac{Y}{N\rho} = \left(1 - \frac{1}{\mu}\right) \frac{Y}{N},$$

hence the share of profits over output reads as

$$\frac{\pi N}{Y} = 1 - \frac{1}{\mu}$$

and the value of firms over output is

$$\frac{NV}{Y} = \frac{\beta(1-\delta)(1-\tau^d)}{[1-\beta(1-\delta)]} \left(1 - \frac{1}{\mu}\right)$$

Investment over output is

$$\frac{VN^e}{Y} = \frac{V}{Y} \frac{\delta}{(1-\delta)} N = \frac{NV}{Y} \frac{\delta}{(1-\delta)} = \delta \frac{\beta(1-\tau^d) \left(1 - \frac{1}{\mu}\right)}{[1-\beta(1-\delta)]}$$

The share of consumption over output is

$$\frac{C}{Y} = 1 - g_y - \frac{N^e \psi}{Y}$$

Finally to get the ratio of labor income over output recall that

$$\frac{wH}{Y} = 1 - \frac{\pi N}{Y}$$

In order to fix v we assume that $H=1$.²⁶ In this case

$$v = (1 - \tau^l) \frac{\frac{w}{Y}}{\frac{C}{Y}}$$

where both ratios are known. To compute the number of firms notice that

$$V = \frac{\beta(1-\delta)(1-\tau^d)}{[1-\beta(1-\delta)]}\pi$$

²⁵Notice that this is the main difference wrt to cost 2 since in that case profits depend on Y^c .

²⁶As mentioned in the section on calibration I fix $H=1$ under monopolistic competition and obtain the corresponding value of v . Under oligopolistic competition I consider the value of v so obtained and compute the corresponding value of H .

Imposing the entry condition and substituting for individual profits

$$N = \frac{\beta(1-\delta)(1-\tau^d)\left(1-\frac{1}{\mu}\right)\rho}{(1-\tau^s)[1-\beta(1-\delta)]\psi} AH$$

the solution to this equation delivers the number of firms at the steady state. This allows to compute all the other variables. For a given H the number of firms at the steady state is larger the higher the markup, hence is larger under oligopolistic competition.

Entry Costs in Form 2

In the case of entry costs in form 2 the definition of the Market Equilibrium (ME) differs from that provided in the case of entry costs in form 1 for two equations. The first one is the aggregate resource constraint, which in this case is given by equation (26), the second one is the Euler equation, which is given by equation (27). The steady state level of individual profits is

$$\pi = \rho y - wh = \left(\rho - \frac{w}{A}\right) y = \rho \left(1 - \frac{1}{\mu}\right) y = \left(1 - \frac{1}{\mu}\right) \frac{(C+G)}{N} = \left(1 - \frac{1}{\mu}\right) \frac{Y^c}{N}$$

As a result

$$\frac{\pi N}{Y^c} = \left(1 - \frac{1}{\mu}\right)$$

To obtain the share of investment over consumption output notice that

$$V = \frac{\beta(1-\delta)(1-\tau^d)}{[1-\beta(1-\delta)]} \left(1 - \frac{1}{\mu}\right) \frac{Y^c}{N}$$

and

$$\frac{VN^e}{Y^c} = (1-\tau^d) \frac{\delta\beta}{[1-\beta(1-\delta)]} \left(1 - \frac{1}{\mu}\right) = (1-\tau^d) \frac{\delta(\mu-1)}{\mu(r+\delta)}$$

To compute shares over aggregate output recall that

$$1 = \frac{Y^c}{Y} + \frac{N^e V Y^c}{Y} = \frac{Y^c}{Y} \left[1 + \delta \frac{\beta(1-\tau^d)}{[1-\beta(1-\delta)]} \left(1 - \frac{1}{\mu}\right) \right]$$

thus

$$\frac{Y^c}{Y} = \left[1 + (1-\tau^d) \delta \frac{\beta}{[1-\beta(1-\delta)]} \left(1 - \frac{1}{\mu}\right) \right]^{-1}$$

which implies that the share of private consumption over output is

$$\frac{C}{Y} = \frac{Y^c}{Y} - g_y$$

Since $\frac{WH}{Y} + \frac{\Pi}{Y} = 1$ we can compute the ratio between labor income and GDP as

$$\frac{WH}{Y} = 1 - \frac{\pi N Y^c}{Y^c Y}$$

Given H, the latter leads to

$$v = \frac{(1 - \tau^l) wH}{CH^{1+\frac{1}{\varphi}}} = \frac{(1 - \tau^l) \frac{wH}{Y}}{\frac{C}{Y} H^{1+\frac{1}{\varphi}}}$$

Labor market equilibrium requires

$$\begin{aligned} H &= H_t^C + H_t^E = Nh + \frac{\eta}{A} N^e = N \frac{y}{A} + \frac{\eta}{A} N^e \\ &= \frac{Y^c}{\rho A} + \frac{\eta}{A} N^e \end{aligned}$$

thus $N^e = \frac{AH}{\eta} - \frac{Y^c}{\rho\eta}$ and

$$N = \frac{(1 - \delta) AH}{\delta \eta} - \frac{(1 - \delta) Y^c}{\delta \eta \rho}$$

Next consider

$$V = \frac{\beta(1 - \delta)(1 - \tau^d)}{[1 - \beta(1 - \delta)]} \left(1 - \frac{1}{\mu}\right) \frac{Y^c}{N}$$

substituting for the entry condition delivers

$$\frac{Y^c}{\eta\rho} = \frac{1}{\mu} \frac{[1 - \beta(1 - \delta)]}{\beta(1 - \delta)(1 - \tau^d) \left(1 - \frac{1}{\mu}\right)} N$$

Substituting the latter into the equation of motion for the number of firms delivers an equation that can be solved for N

$$N = \frac{\frac{(1-\delta) AH}{\delta \eta}}{\left(1 + \frac{1}{\delta} \frac{[1-\beta(1-\delta)]}{\beta(1-\tau^d)(\mu-1)}\right)}$$

Aa above a higher markups leads to a higher number of firms in equilibrium for any given H.

Appendix B. Efficient Equilibrium

Entry Costs in Form 1

The social Planner problem reads as

$$\max_{\{C_t, N_{t+1}, N_t^e, H_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t - v \frac{H_t^{1+1/\varphi}}{1+1/\varphi} \right\}$$

s.t.

$$C_t + G_t + N_t^e \psi = \rho_t A_t H_t$$

and

$$N_{t+1} = (1 - \delta) (N_t + N_t^e)$$

We attach the Lagrange Multiplier λ_t to the first constraint and the multiplier σ_t to the second one. First order conditions are as follows

$$C_t : \frac{1}{C_t} = \lambda_t$$

$$N_{t+1} : \sigma_t = \beta E_t \lambda_{t+1} \rho_{N,t+1} A_t H_t + \beta E_t \sigma_{t+1} (1 - \delta)$$

$$N_t^e : \lambda_t \psi = (1 - \delta) \sigma_t$$

$$H_t : v H_t^{1/\varphi} = \lambda_t \rho_t A_t$$

Combining the first and the third condition delivers

$$\frac{1}{C_t} \frac{\psi}{(1 - \delta)} = \sigma_t$$

Substituting the latter into the third condition we obtain

$$\psi = (1 - \delta) \beta E_t \frac{C_t}{C_{t+1}} (\rho_{N,t+1} A_t H_t + \psi)$$

which can be written as

$$\psi = (1 - \delta) \beta E_t \frac{C_t}{C_{t+1}} \left(\epsilon \frac{Y_t}{N_t} + \psi \right)$$

finally the FOC with respect to hours can be written as

$$v H_t^{1/\varphi} C_t = \rho_t A_t$$

To obtain the steady state we have the following equations

$$C + G + N^e \psi = \rho A H$$

$$N = (1 - \delta) (N + N^e)$$

$$\psi = (1 - \delta) \beta \left(\epsilon \frac{Y}{N} + \psi \right)$$

$$vH^{1/\varphi}C = \rho A$$

Consider the resource constraint

$$\frac{C}{Y} = 1 - g_y - \frac{N^e \psi}{Y} = 1 - g_y - \frac{\psi}{Y} \frac{\delta}{1 - \delta} N$$

or

$$\frac{C}{Y} = 1 - g_y - \psi \frac{\delta}{1 - \delta} \frac{N}{Y}$$

From the third equation

$$\frac{Y}{N} = \frac{(1 - (1 - \delta) \beta) \psi}{(1 - \delta) \beta \epsilon}$$

Combining we obtain

$$\frac{C}{Y} = 1 - g_y - \frac{\delta}{1 - \delta} \frac{(1 - \delta) \beta \epsilon}{(1 - (1 - \delta) \beta)}$$

Next consider equation

$$H^{1+1/\varphi} = \frac{1}{vC} \rho AH = \frac{Y}{vC}$$

hence we have H as

$$H = \left(\frac{Y}{vC} \right)^{\frac{1}{1+1/\varphi}}$$

Next we want to compute N. Notice that

$$\frac{N}{\rho} = \frac{(1 - \delta) \beta \epsilon}{(1 - (1 - \delta) \beta) \psi} AH$$

given $\rho = N^{\frac{1}{\theta-1}}$ it follows

$$N^{\frac{\theta-2}{\theta-1}} = \frac{(1 - \delta) \beta \epsilon}{(1 - (1 - \delta) \beta) \psi} AH$$

or

$$N = \left[\frac{(1 - \delta) \beta \epsilon}{(1 - (1 - \delta) \beta) \psi} AH \right]^{\frac{\theta-1}{\theta-2}}$$

Entry Costs in form 2

The Social Planner problem can be written as follows

$$\max_{\{C_t, N_{t+1}, N_t^e, H_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t - v \frac{H_t^{1+1/\varphi}}{1+1/\varphi} \right\}$$

s.t.

$$C_t + G_t + N_t^e \eta \rho_t = \rho_t A_t H_t$$

and

$$N_{t+1} = (1 - \delta) (N_t + N_t^e)$$

We attach the Lagrange Multiplier λ_t to the first constraint and the multiplier σ_t to the second one. First order conditions are as follows

$$C_t : \frac{1}{C_t} = \lambda_t$$

$$N_{t+1} : \sigma_t = \beta E_t \lambda_{t+1} \rho_{N,t+1} (A_{t+1} H_{t+1} - N_{t+1}^e \eta) + \beta E_t \sigma_{t+1} (1 - \delta)$$

$$N_t^e : \lambda_t \eta \rho_t = (1 - \delta) \sigma_t$$

$$H_t : v H_t^{1/\varphi} = \lambda_t \rho_t A_t$$

Substituting the first condition into the third delivers

$$\frac{1}{(1 - \delta) C_t} \eta \rho_t = \sigma_t$$

Substituting the latter and the definition of λ_t into the other equations we are left with

$$\eta \rho_t = (1 - \delta) \beta E_t \frac{C_t}{C_{t+1}} \left[\rho_{N,t+1} (A_{t+1} H_{t+1} - N_{t+1}^e \eta) + \eta \rho_{t+1} \right] \quad (35)$$

and

$$v H_t^{1/\varphi} C_t = \rho_t A_t$$

Since $A_t H_t - N_t^e \eta = \frac{C_t + G_t}{\rho_t}$, equation (35) can be rewritten as

$$\eta \rho_t = (1 - \delta) \beta E_t \frac{C_t}{C_{t+1}} \left[\rho_{N,t+1} \frac{C_{t+1} + G_{t+1}}{\rho_{t+1}} + \eta \rho_{t+1} \right]$$

or

$$\eta \rho_t = (1 - \delta) \beta E_t \frac{C_t}{C_{t+1}} \left[\epsilon \frac{C_{t+1} + G_{t+1}}{N_{t+1}} + \eta \rho_{t+1} \right]$$

To find the steady state we can consider the following equations

$$\eta \rho = (1 - \delta) \beta \left[\epsilon \frac{Y^c}{N} + \eta \rho \right]$$

$$\begin{aligned}
vH^{1/\varphi}C &= \rho A \\
Y^c + N^e\eta\rho &= \rho AH \\
N &= (1 - \delta)(N + N^e)
\end{aligned}$$

From the first one

$$\frac{Y^c}{\eta\rho} = \frac{(1 - (1 - \delta)\beta)}{(1 - \delta)\beta\epsilon} N$$

The aggregate resource constraint implies

$$\frac{Y^c}{\eta\rho} = \frac{AH}{\eta} - \frac{\delta}{1 - \delta} N$$

Combining

$$N = \frac{\frac{AH}{\eta}}{\frac{(1 - (1 - \delta)\beta)}{(1 - \delta)\beta\epsilon} + \frac{\delta}{1 - \delta}}$$

we get N as a function of H. Notice that we have repeatedly used the steady state version of the equation of motion for the number of firms. Consider again the aggregate resource constraint

$$C + G + N^e\eta\rho = \rho AH$$

or

$$\frac{C}{\rho AH} = 1 - g_y - \frac{N^e\eta\rho}{\rho AH} = 1 - g_y - \eta\rho \frac{\delta}{1 - \delta} \frac{N}{\rho AH}$$

then

$$vH^{1/\varphi} \frac{C}{\rho AH} = \frac{\rho A}{\rho AH}$$

delivers H implicitly as a function of N

$$vH^{1/\varphi} \left(1 - g_y - \eta\rho \frac{\delta}{1 - \delta} \frac{N}{\rho AH} \right) = \frac{1}{H}$$

The latter is equivalent to

$$vH^{1+1/\varphi} \left(1 - g_y - \eta\rho \frac{\delta}{1 - \delta} \frac{N}{\rho AH} \right) = 1$$

Next substitute for N as a function of H in the round bracket and

$$H = \left\{ v \left[1 - g_y - \frac{\delta}{1 - \delta} \left(\frac{(1 - (1 - \delta)\beta)}{(1 - \delta)\beta\epsilon} + \frac{\delta}{1 - \delta} \right)^{-1} \right] \right\}^{-\frac{\varphi}{(1 + \varphi)}}$$

Appendix C. The Implementability Constraint

This Appendix follows closely Arsenau and Chugh (2012) and Chugh and Ghironi (2012). Consider the household flow budget constraint (in the symmetric equilibrium)

$$\sum_j \frac{1}{R_t^j} B_{t+1}^j + V_t(N_t + N_t^e)x_{t+1} + C_t = (1 - \tau_t^l) w_t H_t + B_t + [(1 - \tau_t^d) d_t + V_t] N_t x_t$$

Multiply both sides by $\beta^t u_c(c_t)$

$$\begin{aligned} & \sum_j \beta^t u_c(c_t) \frac{1}{R_t^j} B_{t+1}^j + \beta^t u_c(c_t) V_t(N_t + N_t^e)x_{t+1} + \beta^t u_c(c_t) C_t \\ &= \beta^t u_c(c_t) (1 - \tau_t^l) w_t H_t + \beta^t u_c(c_t) b_t + \beta^t u_c(c_t) [(1 - \tau_t^d) d_t + V_t] N_t x_t \end{aligned}$$

and sum over dates starting from $t=0$, where all term are understood as in expectation as of time 0

$$\begin{aligned} & \sum_{t=0}^{\infty} \sum_j \beta^t u_c(c_t) \frac{1}{R_t^j} B_{t+1}^j + \sum_{t=0}^{\infty} \beta^t u_c(c_t) V_t(N_t + N_t^e)x_{t+1} + \sum_{t=0}^{\infty} \beta^t u_c(c_t) C_t \\ &= \sum_{t=0}^{\infty} \beta^t u_c(c_t) (1 - \tau_t^l) w_t H_t + \sum_{t=0}^{\infty} \beta^t u_c(c_t) b_t + \sum_{t=0}^{\infty} \beta^t u_c(c_t) [(1 - \tau_t^d) d_t + V_t] N_t x_t \end{aligned}$$

The euler equation for bonds implies $u_c(C_t) = \beta R_t^j u_c(C_{t+1}^j)$, using this in the first term on the LHS

$$\begin{aligned} & \sum_{t=0}^{\infty} \sum_j \beta^{t+1} u_c(C_{t+1}^j) B_{t+1}^j + \sum_{t=0}^{\infty} \beta^t u_c(C_t) V_t(N_t + N_t^e)x_{t+1} + \sum_{t=0}^{\infty} \beta^t u_c(C_t) C_t \\ &= \sum_{t=0}^{\infty} \beta^t u_c(c_t) (1 - \tau_t^l) w_t H_t + \sum_{t=0}^{\infty} \beta^t u_c(C_t) b_t + \sum_{t=0}^{\infty} \beta^t u_c(C_t) [(1 - \tau_t^d) d_t + V_t] N_t x_t \end{aligned}$$

Notice that the term $\sum_j u_c(C_{t+1}^j) B_{t+1}^j$ can be understood as the payoff of a risk free bond. As such we can cancel out the first summation on the LHS with the respective terms in the second summation in the RHS, leaving just time 0 terms

$$\begin{aligned} & \sum_{t=0}^{\infty} \beta^t u_c(c_t) V_t(N_t + N_t^e) + \sum_{t=0}^{\infty} \beta^t u_c(c_t) C_t \\ &= \sum_{t=0}^{\infty} \beta^t u_c(c_t) (1 - \tau_t^l) w_t H_t + u_c(c_0) b_0 + \sum_{t=0}^{\infty} \beta^t u_c(c_t) [(1 - \tau_t^d) d_t + V_t] N_t \end{aligned}$$

Notice that the clearing of the asset market implies $x_t = 1$ at all t . Considering that $\frac{u_h(h_t)}{u_c(c_t)} = - (1 - \tau_t^l) w_t$ leads to

$$\begin{aligned} & \sum_{t=0}^{\infty} \beta^t u_c(c_t) V_t (N_t + N_t^e) + \sum_{t=0}^{\infty} \beta^t u_c(c_t) C_t + \sum_{t=0}^{\infty} \beta^t u_h(h_t) H_t \\ &= u_c(c_0) b_0 + \sum_{t=0}^{\infty} \beta^t u_c(c_t) [(1 - \tau_t^d) d_t + V_t] N_t \end{aligned}$$

Next consider

$$u_c(C_t) V_t = E_t \beta (1 - \delta) u_c(C_{t+1}) [(1 - \tau_{t+1}^d) d_{t+1} + V_{t+1}]$$

and plug it into the first summation in the LHS

$$\begin{aligned} & \sum_{t=0}^{\infty} \beta^{t+1} (1 - \delta) u_c(C_{t+1}) [(1 - \tau_{t+1}^d) d_{t+1} + V_{t+1}] (N_t + N_t^e) x_{t+1} + \\ & + \sum_{t=0}^{\infty} \beta^t u_c(c_t) C_t + \sum_{t=0}^{\infty} \beta^t u_h(h_t) h_t \\ &= u_c(c_0) b_0 + \sum_{t=0}^{\infty} \beta^t u_c(c_t) [(1 - \tau_t^d) d_t + V_t] N_t x_t \end{aligned}$$

considering that

$$(N_t + N_t^e) = \frac{N_{t+1}}{1 - \delta}$$

it follows

$$\begin{aligned} & \sum_{t=0}^{\infty} \beta^{t+1} u_c(C_{t+1}) [(1 - \tau_{t+1}^d) d_{t+1} + V_{t+1}] N_{t+1} + \\ & + \sum_{t=0}^{\infty} \beta^t u_c(c_t) C_t + \sum_{t=0}^{\infty} \beta^t u_h(h_t) h_t \\ &= u_c(c_0) b_0 + \sum_{t=0}^{\infty} \beta^t u_c(c_t) [(1 - \tau_t^d) d_t + V_t] N_t \end{aligned}$$

Simplifying the first summation on the LHS with the second in the RHS delivers the implementability constraint

$$E_0 \sum_{t=0}^{\infty} \beta^t [u_c(C_t) C_t + u_h(H_t) H_t] = u_c(C_0) B_0 + u_c(C_0) [(1 - \tau_0^d) d_0 + V_0] N_0$$

where we reintroduced the expectation operator.

Appendix D. The Ramsey Problem

Entry Costs in form 1. Includes proof of result (i) in Proposition 1.

The Ramsey problem reads as

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, H_t)$$

subject to

$$N_{t+1} = (1 - \delta)(N_t + N_t^e) : \lambda_{1t}$$

$$C_t + G_t + N_t^e \psi = \rho_t A_t H_t : \lambda_{2t}$$

$$\psi u_{ct} = \beta(1 - \delta) E_t u_{ct+1} \left((1 - \tau_{t+1}^d) \left(1 - \frac{1}{\mu} \right) \frac{C_{t+1} + G_{t+1}}{N_{t+1}} + \psi \right) : \lambda_{3t}$$

$$E_0 \sum_{t=0}^{\infty} \beta^t E_0 [u_{ct} C_t + u_{ht} H_t] = u_{c0} B_0 + u_{c0} [(1 - \tau_0^d) d_0 + V_0] N_0 : \xi$$

Where λ_{it} define the Lagrange multipliers respectively attached to each constraint and ξ is the (constant) lagrange multiplier attached to the implementability constraint.

The choice variables are C_t , N_t , H_t , N_t^e , and $\tau_{t/t+1}^d$. Following Ljungqvist and Sargent (2004) I define

$$V(C_t, H_t, \xi) = u(C_t, H_t) + \xi (u_{ct} C_t + u_{ht} H_t)$$

and

$$\Omega = u_{c0} B_0 + u_{c0} [(1 - \tau_0^d) d_0 + V_0] N_0$$

As a result the Lagrangian function can be written as

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} V(C_t, H_t, \xi) + \lambda_{1t} [(1 - \delta)(N_t + N_t^e) - N_{t+1}] \\ \quad + \lambda_{2t} (\rho_t A_t H_t - C_t - G_t - N_t^e \psi) + \\ \lambda_{3t} \left[\psi u_{ct} - \beta(1 - \delta) E_t u_{ct+1} \left((1 - \tau_{t+1}^d) \left(1 - \frac{1}{\mu_{t+1}} \right) \frac{\rho_{t+1} A_{t+1} H_{t+1}}{N_{t+1}} + \psi \right) \right] \end{array} \right\} - \xi \Omega$$

The first order conditions for periods $t \geq 1$ are

$$C_t : V_c(C_t, H_t, \xi) - \lambda_{2t} + \lambda_{3t} u_{cct} \psi - \lambda_{3t-1} (1 - \delta) u_{cct} \left((1 - \tau_{t/t-1}^d) \left(1 - \frac{1}{\mu_t} \right) \frac{\rho_t A_t H_t}{N_t} + \psi \right) = 0$$

$$\begin{aligned} N_{t+1} : \lambda_{1t} + \beta(1 - \delta) \lambda_{3t} \frac{E_t u_{ct+1} \left(1 - \tau_{t+1}^d \right) A_{t+1} H_{t+1}}{N_{t+1}} & \left[\left(1 - \frac{1}{\mu_{t+1}} \right) \frac{\rho_{N_{t+1}} N_{t+1} - \rho_{t+1}}{N_{t+1}} \right. \\ & \left. + \frac{\mu_{N_{t+1}} \rho_{t+1}}{\mu_{t+1}^2} \right] \\ & = \beta(1 - \delta) E_t \lambda_{1t+1} + \beta E_t \lambda_{2t+1} \rho_{N_{t+1}} A_{t+1} H_{t+1} \end{aligned}$$

$$\tau_{t+1/t}^d : (1 - \delta)\beta^{t+1}\lambda_{3t}E_0u_{ct+1} \left(1 - \frac{1}{\mu_{t+1}}\right) \frac{\rho_{t+1}A_{t+1}H_{t+1}}{N_{t+1}} = 0$$

$$N_t^e : \lambda_{1t}(1 - \delta) = \lambda_{2t}\psi$$

$$H_t : V_h(C_t, H_t, \xi) + \lambda_{2t}\rho_t A_t - \lambda_{3t-1}(1 - \delta)u_{ct} \left(1 - \tau_{t/t-1}^d\right) \left(1 - \frac{1}{\mu_t}\right) \frac{\rho_t A_t}{N_t} = 0$$

Since

$$V(C_t, H_t, \xi) = u(C_t, H_t) + \xi(u_{ct}C_t + u_{ht}H_t)$$

it follows

$$V_c(C_t, H_t, \xi) = u_c(C, H) + \xi u_{cct}C_t + \xi u_{ct} = \frac{1}{C_t} - \xi \frac{1}{C_t^2}C_t + \xi \frac{1}{C_t} = \frac{1}{C_t}$$

and

$$V_h(C_t, H_t, \xi) = -vH_t^{1/\varphi} \left[\xi \left(\frac{1 + \varphi}{\varphi} \right) + 1 \right]$$

Consider now the steady state. The FOC with respect to $\tau_{t+1/t}^d$ reads as

$$\tau_{t+1/t}^d : (1 - \delta)\beta^{t+1}\lambda_3 \left(1 - \frac{1}{\mu}\right) \frac{C + G}{N} u_c = 0$$

The latter implies that at the steady state λ_3 is equal to zero. In the Ramsey steady state the firms entry condition does not restrict the allocation. As a result we can write the steady state version of the FOCs as

$$C_t : \lambda_2 = u_c$$

$$H_t : V_h(C, H, \xi) + u_{ct}\rho A = 0$$

$$N_t^e : \lambda_1 = \frac{\psi}{(1 - \delta)} u_c$$

$$N_{t+1} : \beta\rho_N AH = [1 - \beta(1 - \delta)] \frac{\psi}{(1 - \delta)}$$

The FOC with respect to N can be written as

$$\psi = \beta(1 - \delta) [\rho_N AH + \psi]$$

Since

$$\rho_N = \frac{1}{\theta - 1} \frac{\rho}{N} = \epsilon \frac{\rho}{N}$$

it follows

$$\psi = \beta(1 - \delta) \left[\epsilon \frac{\rho AH}{N} + \psi \right]$$

The euler equation for asset implies that

$$\psi = \beta(1 - \delta) \left((1 - \tau^d) \left(1 - \frac{1}{\mu} \right) \frac{\rho AH}{N} + \psi \right)$$

For the latter two equations to be consistent with each other it has to be the case that

$$\epsilon = (1 - \tau^d) \left(1 - \frac{1}{\mu} \right)$$

or

$$\tau^d = 1 - \frac{\epsilon}{\left(1 - \frac{1}{\mu} \right)}$$

which proves point (i) in proposition 1. Importantly, the dividend income tax differs from zero also under monopolistic competition. Substituting the optimal dividend income tax into the Euler equation for shares we get

$$\psi = \beta(1 - \delta) \left(\epsilon \frac{Y}{N} + \psi \right)$$

Which implies

$$\frac{Y}{N} = \frac{[1 - \beta(1 - \delta)]}{\beta(1 - \delta)\epsilon} \psi$$

Notice that

$$\frac{C}{Y} = 1 - g_y - \frac{\delta}{1 - \delta} \psi \frac{N}{Y}$$

Next consider the implementability constraint, which can be written as

$$\Omega = \frac{Y}{C} \left(\frac{B}{Y} + \frac{\epsilon}{\left(1 - \frac{1}{\mu} \right)} \frac{\Pi}{Y} + \frac{VN}{Y} \right)$$

where $\frac{B}{Y}$ is exogenously given and $\frac{C}{Y}$ has been computed above. The Euler equation with respect to assets implies

$$\frac{VN}{Y} = \frac{\beta(1 - \delta)}{[1 - \beta(1 - \delta)]} \epsilon$$

and also we know

$$\begin{aligned} \frac{\Pi}{Y} &= \left(1 - \frac{1}{\mu} \right) \\ \Omega &= \frac{Y}{C} \left(\frac{B}{Y} + \epsilon + \frac{VN}{Y} \right) \\ H &= \left[\frac{1 - (1 - \beta)\Omega}{v} \right]^{\frac{\varphi}{1+\varphi}} \end{aligned}$$

Finally given H and recalling that

$$\frac{Y}{N} = \frac{[1 - \beta(1 - \delta)](\theta - 1)}{\beta(1 - \delta)} \psi$$

it follows

$$N = \left[\frac{\beta(1 - \delta)\epsilon}{[1 - \beta(1 - \delta)]\psi} AH \right]^{\frac{\theta-1}{\theta-2}}$$

Entry Costs in Form 2. Includes proof of result (ii) in Proposition 2.

In this case the Lagrangian is

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} V(C_t, H_t, \xi) + \lambda_{1t} [(1 - \delta)(N_t + N_t^e) - N_{t+1}] \\ \quad + \lambda_{2t} (\rho_t A_t H_t - C_t - G_t - N_t^e \eta \rho_t) + \\ \quad \eta \frac{\rho_t}{\mu_t} u_{ct} + \\ \lambda_{3t} \left[-\beta(1 - \delta) u_{ct+1} \left(\left(1 - \tau_{t+1}^d/t\right) \left(1 - \frac{1}{\mu_{t+1}}\right) \frac{C_{t+1} + G_{t+1}}{N_{t+1}} + \eta \frac{\rho_{t+1}}{\mu_{t+1}} \right) \right] \end{array} \right\} - \xi \Omega$$

As above, the choice variables are C_t , N_t , H_t , N_t^e , and $\tau_{t/t+1}^d$. The first order conditions are

$$\begin{aligned} C_t : & V_c(C_t, H_t, \xi) + \lambda_{3t} \eta \frac{\rho_t}{\mu} u_{cct} \\ & - \lambda_{3t-1} \left[(1 - \delta) u_{cct} \left(\left(1 - \tau_{t/t-1}^d\right) \left(1 - \frac{1}{\mu}\right) \frac{C_t + G_t}{N_t} + \eta \frac{\rho_t}{\mu} \right) + \right. \\ & \left. + (1 - \delta) u_{ct} \left(1 - \tau_{t/t-1}^d\right) \left(1 - \frac{1}{\mu}\right) \frac{1}{N_t} \right] \\ & = \lambda_{2t} \end{aligned}$$

$$N_{t+1} : \lambda_{1t} +$$

$$\begin{aligned} & \beta(1 - \delta) \lambda_{3t} E_t u_{ct+1} \left[+ \frac{\left(1 - \tau_{t+1}^d\right)}{\mu_{t+1}} \left(\frac{\mu_{N;t+1}}{\mu_{t+1}} - \frac{\mu_{t+1} - 1}{N_{t+1}} \right) \frac{Y_{t+1}^c}{N_{t+1}} \right] \\ & + \beta \eta (1 - \delta) \lambda_{3t} E_t u_{ct+1} \left(\frac{\rho_{Nt+1} \mu_{t+1} - \mu_{Nt+1} \rho_{t+1}}{\mu_{t+1}^2} \right) \\ & = \beta(1 - \delta) E_t \lambda_{1t+1} + \beta E_t \lambda_{2t+1} \rho_{Nt+1} (A_{t+1} H_{t+1} - N_{t+1}^e \eta) + \\ & + \beta \eta E_t \lambda_{3t+1} u_{ct+1} \left(\frac{\rho_{Nt+1} \mu_{t+1} - \mu_{Nt+1} \rho_{t+1}}{\mu_{t+1}^2} \right) \end{aligned}$$

$$H_t : V_h(C_t, H_t, \xi) + \lambda_{2t} \rho_t A_t = 0$$

$$N_t^e : \lambda_{1t}(1 - \delta) = \lambda_{2t}\eta\rho_t$$

$$\tau_{t+1/t}^d : (1 - \delta)\beta^{t+1}E_t u_{ct+1}\lambda_{3t} \left(1 - \frac{1}{\mu_{t+1}}\right) \frac{C_{t+1} + G_{t+1}}{N_{t+1}} = 0$$

Notice that

$$\rho_{tN} = \frac{1}{\theta - 1} \frac{\rho_t}{N_t}$$

$$\mu_t^C = \frac{\theta N_t}{(\theta - 1)(N_t - 1)}$$

$$\mu_{Nt}^C = \frac{\theta(\theta - 1)(N_t - 1) - (\theta - 1)\theta N_t}{(\theta - 1)^2(N_t - 1)^2} = \frac{\theta(N_t - 1) - \theta N_t}{(\theta - 1)(N_t - 1)^2} = -\frac{\theta}{(\theta - 1)(N_t - 1)^2}$$

$$\mu_t^B = \frac{1 + \theta(N_t - 1)}{(\theta - 1)(N_t - 1)}$$

$$\mu_{Nt}^B = \frac{\theta(\theta - 1)(N_t - 1) - (\theta - 1)[1 + \theta(N_t - 1)]}{(\theta - 1)^2(N - 1)^2} = \frac{-1}{(\theta - 1)(N - 1)^2}$$

Given $\lambda_3 = 0$ we can write the steady state system as above

$$C_t : V_c(C, H, \xi) = \lambda_2$$

$$N_{t+1} : \lambda_1 = \beta [(1 - \delta)\lambda_1 + \lambda_2\rho_N(AH - N^e\eta)]$$

$$H_t : V_h(C, H, \xi) + \lambda_2\rho A = 0$$

$$N_t^e : \lambda_1(1 - \delta) = \lambda_2\eta\rho$$

Since $\lambda_1 = \frac{\eta\rho}{(1-\delta)}V_c(C, H, \xi)$ and $-\frac{V_h(C, H, \xi)}{V_c(C, H, \xi)} = \rho A$ and given the definitions of V_c and V_h we get

$$\frac{1}{\beta} = (1 - \delta) \left[1 + \epsilon \frac{1}{\eta\rho} \frac{Y^c}{N} \right]$$

Evaluating the Euler equation for assets at the steady state implies

$$\frac{1}{\beta} = (1 - \delta) \left(1 + \frac{(1 - \tau^d)(\mu - 1)Y^c}{\eta\rho N} \right)$$

For the two to be consistent it has to be the case that

$$(1 - \tau^d)(\mu - 1) = \epsilon$$

which proves point (ii) in Proposition 1. Notice that in the monopolistic competition case this implies $\tau^d = 0$. The Euler equation for assets evaluated at the steady state reads as

$$\frac{1}{\beta} = (1 - \delta) \left[1 + \frac{\epsilon}{\rho\eta} \frac{Y^c}{N} \right]$$

then

$$\frac{Y^c}{\eta\rho} = \frac{1 - \beta(1 - \delta)}{(1 - \delta)\beta\epsilon} N$$

The aggregate resource constraint implies

$$\frac{Y^c}{\eta\rho} = \frac{AH}{\eta} - N^e$$

using the equation for the dynamics of the number of firms

$$\frac{Y^c}{\eta\rho} = \frac{AH}{\eta} - \frac{\delta}{1 - \delta} N$$

Combing the latter two equations

$$N = \frac{\frac{AH}{\eta}}{\frac{(1 - (1 - \delta)\beta)}{(1 - \delta)\beta\epsilon} + \frac{\delta}{1 - \delta}} = \frac{\frac{(1 - \delta) AH}{\delta} \frac{\eta}{\eta}}{1 + \frac{(1 - (1 - \delta)\beta)}{\delta\beta\epsilon}}$$

we get N as a function of H . This also implies that we can compute the markup, under both Cournot and Bertrand, as a function of H . Recall that it has to be the case that

$$Y = wH + \Pi$$

since

$$wH = \frac{\rho}{\mu} AH; \quad \Pi = \left(1 - \frac{1}{\mu}\right) Y^c$$

it follows

$$Y = \frac{\rho}{\mu} AH + \left(1 - \frac{1}{\mu}\right) Y^c$$

using the aggregate resource constraint $Y^c = \rho AH - \eta\rho N^e$ we obtain

$$\begin{aligned} Y &= \frac{\rho}{\mu} AH + \left(1 - \frac{1}{\mu}\right) (\rho AH - \eta\rho N^e) \\ &= \rho AH - \rho A \left(\frac{\mu - 1}{\mu}\right) \frac{\eta}{A} N^e \end{aligned}$$

Also notice

$$1 = \left(1 - \frac{1}{\mu}\right) \frac{Y^c}{Y} + \frac{1}{\mu} \frac{\rho AH}{Y}$$

To compute $\frac{Y^c}{Y}$ and $\frac{C}{Y}$ consider the euler equation for assets

$$V = \frac{\beta(1 - \delta) \frac{\epsilon}{\mu} Y^c}{1 - \beta(1 - \delta) N}$$

which implies

$$\frac{VN^e}{Y^c} = \frac{\beta(1-\delta)\frac{\epsilon}{\mu}N^e}{1-\beta(1-\delta)N} = \frac{\delta\beta\frac{\epsilon}{\mu}}{1-\beta(1-\delta)}$$

this allows to compute $\frac{Y^c}{Y}$ as follows

$$Y^c + N^eV = Y$$

then

$$\frac{Y^c}{Y} = 1 - \frac{N^eV}{Y^c} \frac{Y^c}{Y}$$

and finally

$$\frac{Y^c}{Y} = \left(1 + \frac{VN^e}{Y^c}\right)^{-1}$$

From the latter we get $\frac{C}{Y}$ as

$$\frac{C}{Y} = \frac{Y^c}{Y} - \frac{G}{Y}$$

Knowing $\frac{Y^c}{Y}$ we can determine $\frac{\rho AH}{Y}$

$$\frac{\rho AH}{Y} = \mu \left[1 - \left(1 - \frac{1}{\mu}\right) \frac{Y^c}{Y}\right]$$

The FOC for hours

$$-\frac{V_h(C, H, \xi)}{V_c(C, H, \xi)} = \rho A$$

substituting the definitions of variables

$$\frac{vH_t^{1/\varphi} \left[\xi \left(\frac{1+\varphi}{\varphi} \right) + 1 \right]}{\frac{1}{C}} = \rho A$$

or

$$H^{1/\varphi} = \frac{\frac{1}{v} \frac{\rho A}{C}}{\left[\xi \left(\frac{1+\varphi}{\varphi} \right) + 1 \right]}$$

Multiplying both sides by H, the latter is equivalent to

$$H = \left[\frac{\frac{\rho AH}{Y} \frac{Y}{C}}{v \left[\xi \left(\frac{1+\varphi}{\varphi} \right) + 1 \right]} \right]^{\frac{\varphi}{1+\varphi}}$$

Hence H is both a function of H and ξ . Next consider the implementability constraint

$$\Omega = \frac{Y B}{C Y} + (1 - \tau^d) \frac{\pi N}{C} + \frac{V N}{C}$$

As a result

$$\Omega = \frac{Y}{C} \left(\frac{B}{Y} + (1 - \tau^d) \frac{\Pi}{Y} + \frac{V N}{Y} \right) = \frac{Y}{C} \left(\frac{B}{Y} + \frac{\epsilon}{(\mu - 1) Y} \frac{\Pi}{Y} + \frac{V N}{Y} \right)$$

where $\frac{B}{Y}$ is given and $\frac{C}{Y}$ is a function of H . Also from

$$V = \frac{\beta (1 - \delta) \frac{\epsilon}{\mu} Y^c}{1 - \beta (1 - \delta) N}$$

we get

$$\frac{V N}{Y} = \frac{\beta (1 - \delta) \frac{\epsilon}{\mu} Y^c}{1 - \beta (1 - \delta) Y}$$

and

$$\frac{\Pi}{Y} = \left(1 - \frac{1}{\mu} \right) \frac{Y^c}{Y}$$

Hence we can compute Ω as a function of H . Next using the steady state version of the implementability constraint we get

$$\frac{1}{1 - \beta} [1 - v H^{1+1/\varphi}] = \Omega$$

or

$$1 - v H^{1+1/\varphi} = (1 - \beta) \Omega$$

which implies

$$H = \left[\frac{1 - (1 - \beta) \Omega}{v} \right]^{\frac{\varphi}{1+\varphi}}$$

which is a function solely of H and can be solved numerically. Given the value H we can determine the lagrange multiplier ξ

$$\xi = \frac{\varphi}{1 + \varphi} \left[\frac{\frac{1}{v} \frac{\rho A H Y}{Y C}}{H^{\frac{1+\varphi}{\varphi}}} - 1 \right]$$

Recall that N can be computed as

$$N = \frac{\frac{A}{\eta} \frac{1-\delta}{\delta}}{1 + \frac{(1-(1-\delta)\beta)}{\delta\beta\epsilon}} H$$

which allows to compute the price markup at thus the Ramsey steady state. Also it implies a a value for ρ . Since

$$\eta \frac{\rho}{\mu} A = \frac{\beta (1 - \delta) \frac{\epsilon}{\mu} Y^c}{1 - \beta (1 - \delta) N}$$

we get

$$Y^c = \frac{\eta \rho A N}{\mu} \frac{1 - \beta (1 - \delta)}{\beta (1 - \delta) \frac{\epsilon}{\mu}}$$

In particular notice that

$$\tau^d = 1 - \frac{\epsilon}{(\mu - 1)}$$

and

$$\tau^l = 1 - \frac{v C H^{\frac{1}{\varphi}}}{w}$$