

Contributions from Factor Components to the Gini,  
Bonferroni and Zenga inequality indexes: an application to  
income data from EU countries

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## Abstract

In this work we apply a new approach to assess contributions from factor components to income inequality. The new approach is based on the insight that most (synthetic) inequality indexes may be viewed as (weighted) averages of point inequality indexes, which measure inequality between population subgroups identified by income. Assessing the contribution of factor components to point inequality indexes is usually an easy task and, using these contributions, it is straightforward to define contributions to the corresponding (synthetic) overall inequality indexes as well. As we shall show through an analysis of income data from Eurostat's European Community Household Panel Survey (ECHP), the approach based on point inequality indexes gives rise to readily interpretable results, which, we believe, is an advantage over other methods that have been proposed in literature.

**Keywords:** point inequality index, synthetic inequality index, factor components, Gini index, Bonferroni index, Zenga index

## 1 Introduction

When Gini (1914) first proposed what later became the virtually most widely used inequality index, he set out from the fact that the cumulative income share  $q_i$  of the  $i$  less fortunate members of a given population can never exceed the corresponding cumulative population share  $p_i = i/N$ . Based on this insight, Gini (1914) measured the degree of inequality suffered by the  $i$  poorest population members through the ratio

$$G_i := \frac{p_i - q_i}{p_i}, \quad i = 1, 2, \dots, N. \quad (1)$$

and defined the nowadays widely used synthetic inequality index

$$\begin{aligned} G' &:= \frac{\sum_{i=1}^{N-1} G_i \times p_i}{\sum_{i=1}^{N-1} p_i} \\ &= \frac{\sum_{i=1}^{N-1} (p_i - q_i)}{\sum_{i=1}^{N-1} p_i} \end{aligned} \quad (2)$$

as weighted average of the first  $N - 1$  point inequality indexes  $G_i$  (note that  $G_N = 0$  in every income distribution) with weights given by the cumulative population shares  $p_i$ . It is well-known that  $G'$  can be linked to the graph with the Lorenz curve (Lorenz 1905): in fact, the numerator in (2) equals

$N$  times the so called concentration area, while the denominator is equal to  $N$  times the area of the triangle with vertices in  $(0, 0)$ ,  $((N - 1)/N, 0)$  and  $(1, 1)$ .<sup>1</sup>

Some years after Gini (1914), Bonferroni (1930) proposed another inequality index based on point inequality indexes which compare the mean income  $\bar{M}_i$  of the  $i$  poorest population members with the mean income  $M$  of the whole population:

$$V_i := \frac{M - \bar{M}_i}{M}, \quad i = 1, 2, \dots, N.$$

Differently from Gini, Bonferroni however attached the same weight to all point inequality indexes  $V_i$  (except for  $V_N$  which is zero in every income distribution) and defined the synthetic inequality index named after him simply by

$$V' := \frac{1}{N - 1} \sum_{i=1}^{N-1} V_i.$$

As pointed out by De Vergottini (1940), the point inequality indexes  $G_i$  and  $V_i$  are the same and thus the synthetic Gini and the Bonferroni indexes differ only because of the different weighting schemes. This result prompted some criticism for Gini's inequality index: why would one want to attach larger weights to point inequality indexes that refer to comparisons between almost identical population subgroups?<sup>2</sup> Going one step further, one could however also ask: why to compare income of almost identical population subgroups?

The inequality index  $I$  proposed by Zenga (2007a) provides an answer to both the above issues. In fact, if all population members have different incomes,  $I$  is defined as the unweighted average of the point inequality indexes

$$I_i := \frac{\overset{+}{M}_i - \bar{M}_i}{\overset{+}{M}_i}, \quad i = 1, 2, \dots, N,$$

where  $\overset{+}{M}_i$  denotes the mean income of the  $N - i$  richest population members for  $i = 1, 2, \dots, N - 1$ , and  $\overset{+}{M}_N$  is defined as the income of the richest population member. On the other hand, if there

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<sup>1</sup>If no population member is allowed to have negative income and the whole population income is concentrated in the hands of a single population member, then the Lorenz curve is given by the two line segments that join the points  $(0, 0)$ ,  $((N - 1)/N, 0)$  and  $(1, 1)$ .

<sup>2</sup>As the subscript  $i$  increases,  $p_i$  does increase as well and the population subgroup given by the  $i$  poorest population members gets larger and larger until it includes all population members.

are two or more population members that have the same income, the inequality index  $I$  does no longer involve all point inequality indexes  $I_i$ , because Zenga (2007a), as opposed to Gini (1914) and Bonferroni (1930), attaches the same point inequality index  $I_i$  to population members with the same total income. Thus, in populations where the income variable takes on  $k$  ( $2 \leq k \leq N$ ) different values

$$y_1 < y_2 < \dots < y_j < \dots < y_k$$

with respective frequencies  $n_1, n_2, \dots, n_j, \dots, n_k$ , the synthetic  $I$  index is defined by

$$I := \frac{1}{N} \sum_{j=1}^k I_{N_j} \times n_j$$

where  $N_j := \sum_{j'=1}^j n_{j'}$ .<sup>3</sup>

Despite the fact that the inequality index  $I$  has been introduced in the literature since a relatively short time, several research papers have already been published about it and its underlying point inequality indexes  $I_i$ . To cite a few, there are Zenga (2007b), Zenga (2008) and Greselin et al. (2013) which illustrate some applications to real income distributions; Poliscchio (2008), Poliscchio and Porro (2008), Porro (2008) and Porro (2011) which deal with properties of the curve defined by the point inequality indexes  $I_i$  and its relation with the Lorenz curve; and Greselin and Pasquazzi (2009), Greselin et al. (2010), Langel and Tillé (2012), Antal et al. (2011) and Greselin et al. (2014) which analyze inferential problems related to the  $I$  index. As for decomposition rules, Radaelli (2008) proposes a subgroups decomposition for the point inequality indexes  $I_i$  and the synthetic  $I$  index that has been applied to real income data in Radaelli (2007), Radaelli (2008b) and Greselin et al. (2009) and that has been compared with a subgroups decomposition rule for Gini's index in Radaelli (2010). The factor components decomposition rule we are going to describe below and to apply in the following sections has been originally proposed in Zenga et al. (2012) and has been extended to the Gini and Bonferroni indexes in Zenga (2013).

To view the synthetic Gini, Bonferroni and Zenga indexes as averages of point inequality indexes does not only suggest straightforward interpretations for their meaning, but does also suggest a straightforward method to measure the contributions from factor components to inequality. In fact,

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<sup>3</sup>In large populations where only a few population members have equal incomes there is no practical difference between the value of  $I$  and of  $I' := \frac{1}{N} \sum_{i=1}^N I_i$  (see the example in Zenga et al. (2012)).

if total income  $Y$  is given by the sum

$$X_1 + X_2 + \cdots + X_s + \cdots + X_c$$

of the incomes from  $c$  factor components, then the contribution from factor component  $X_s$  to the point inequality index  $V_i = G_i$  is simply given by

$$C_i(X_s) := \frac{M(X_s) - \bar{M}_i(X_s)}{M(Y)},$$

where  $M(\cdot)$  and  $\bar{M}_i(\cdot)$  denote the means of the variable between parentheses in the whole population and among the  $i$  population members with smallest total income  $Y$ , respectively. Similarly, the contribution to the point inequality index  $I_i$  is given by

$$B_i(X_s) := \frac{\bar{M}_i^+(X_s) - \bar{M}_i^-(X_s)}{\bar{M}_i^+(Y)},$$

where the meaning of  $\bar{M}_i^+(\cdot)$  is analogous to that of  $\bar{M}_i^-(\cdot)$ . Now, since

$$\sum_{s=1}^c C_i(X_s) = G_i = V_i, \quad i = 1, 2, \dots, N, \quad (3)$$

and

$$\sum_{s=1}^c I_i(X_s) = I_i, \quad i = 1, 2, \dots, N,$$

the contributions from factor component  $X_s$  to  $G'$ ,  $V'$  and  $I$  can be simply defined by

$$G'(X_s) := \frac{\sum_{i=1}^{N-1} C_i(X_s) \times p_i}{\sum_{i=1}^{N-1} p_i},$$

$$V'(X_s) := \frac{1}{N-1} \sum_{i=1}^{N-1} C_i(X_s),$$

and

$$I(X_s) := \frac{1}{N} \sum_{j=1}^k B_{N_j}(X_s) \times n_j,$$

respectively. However, due to the fact that population members with the same total income  $Y$  might have different incomes from some factor components, the definitions of  $C_i(X_s)$  and  $B_i(X_s)$  might not be unique at subscript values  $i$  different from  $N_1, N_2, \dots, N_k$  and this non-uniqueness problem might

also affect  $G'(X_s)$  and  $V'(X_s)$ .<sup>4</sup> One possible solution is to replace the non-unique contributions  $C_i(X_s)$  by their average value over all possible values that can be attained by changing the order of the population members with the same total income  $Y$ . This solution does obviously preserve the sums in (3), and thus it makes sure that  $G'(X_s)$  and  $V'(X_s)$  are uniquely defined as well. However, if there are many population members with the same total income  $Y$ , it could be computationally quite burdensome and thus we shall adopt a different solution in the present work: we simply apply slight modifications to the definitions of  $G'$  and  $V'$  in order to make sure that they involve only point inequality indexes  $G_i = V_i$  at the subscript values  $i = N_1, N_2, \dots, N_k$  where the contributions  $C_i(X_s)$  are uniquely defined. As we shall see, the resulting inequality indexes as well as the relevant point inequality indexes and contributions will then depend only on the joint distribution function of total income  $Y$  and the factor components  $X_s$ , which makes their computation an easy task. In place of  $G'$  we shall consider<sup>5</sup>

$$G := \frac{\sum_{j=1}^k G_{N_j} \times g_j}{\sum_{j=1}^k g_j},$$

where

$$g_j := N_j \times \frac{n_j + n_{j+1}}{2N^2}, \quad j = 1, 2, \dots, k-1,$$

and

$$g_k := N_k \times \frac{n_k}{2N^2},$$

and in place of  $V'$ , we shall consider<sup>6</sup>

$$V := \frac{1}{N} \sum_{j=1}^k V_{N_j} \times n_j.$$

Since the contributions  $C_i(X_s)$  are uniquely defined at  $i = N_1, N_2, \dots, N_k$ , it follows that the corresponding contributions  $G_i(X_s)$  and  $V_i(X_s)$ , and moreover the contributions to the synthetic inequality indexes  $G$  and  $V$  defined by

$$G(X_s) := \frac{\sum_{j=1}^k C_{N_j} \times g_j}{\sum_{j=1}^k g_j}$$

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<sup>4</sup> $I(X_s)$  is not affected by this problem, because it depends only on the contributions  $B_i(X_s)$  at  $i = N_1, N_2, \dots, N_k$ .

<sup>5</sup>It is not difficult to show that the numerator in the definition of  $G$  equals  $N$  times the concentration area, while the denominator is always equal to  $N/2$ , i.e.  $N$  times the area of the triangle with vertices in  $(0, 0)$ ,  $(1, 0)$  and  $(1, 1)$ .

<sup>6</sup>Note that  $V \leq V'$ , since  $V_1 > V_2 > \dots > V_N$ , and since in the definition of  $V$  the point inequality indexes  $V_i$  are replaced by  $V_{N_j}$  for  $N_{j-1} < i \leq N_j$ ,  $j = 1, 2, \dots, k$ .

and

$$V(X_s) := \frac{1}{N} \sum_{j=1}^k C_{N_j} \times n_j,$$

respectively, are uniquely defined as well.

Dividing the *absolute* contributions  $C_i(X_s)$  and  $B_i(X_s)$  by their respective point inequality indexes yields *relative* contributions with a very neat interpretation. In fact,

$$\begin{aligned} \omega_i(X_s) &:= \frac{C_i(X_s)}{V_i} = \frac{C_i(X_s)}{G_i} \\ &= \frac{M(X_s) - \bar{M}_i(X_s)}{M(Y) - \bar{M}_i(Y)} \end{aligned}$$

means that  $\omega_i(X_s) \times 100\%$  of the difference between the mean population income and the mean income of the  $i$  poorest population members is due to factor component  $X_s$ , and an analogous interpretation can be attached to

$$\begin{aligned} \beta_i(X_s) &:= \frac{B_i(X_s)}{I_i} \\ &= \frac{\bar{M}_i^+(X_s) - \bar{M}_i(X_s)}{\bar{M}_i^+(Y) - \bar{M}_i(Y)}. \end{aligned}$$

Actually the interpretations of  $\omega_i$  and  $\beta_i$  for  $i = 1, 2, \dots, N - 1$  can also be interchanged since, as pointed out by Zenga (2013), their values are always the same. This perhaps unexpected result follows immediately from the fact that

$$\frac{N-i}{N} \left( \bar{M}_i^+(\cdot) - \bar{M}_i(\cdot) \right) = M(\cdot) - \bar{M}_i(\cdot), \quad i = 1, 2, \dots, N - 1.$$

As for  $\omega_N$  and  $\beta_N$ , it is worth noting that  $\omega_N$  is always zero, while  $\beta_N \neq 0$  in general.

Dividing  $G(X_s)$ ,  $V(X_s)$  and  $I(X_s)$  by their respective synthetic inequality indexes yields relative contributions to overall inequality in the distribution of  $Y$  as measured by  $G$ ,  $V$  and  $I$ :

$$\begin{aligned} \lambda(X_s) &:= \frac{G(X_s)}{G} \\ &= \frac{\sum_{j=1}^k C_{N_j}(X_s) \times g_j}{\sum_{j=1}^k C_{N_j} \times g_j} \\ &= \frac{\sum_{j=1}^k \omega_{N_j}(X_s) \times G_{N_j} \times g_j}{\sum_{j=1}^k G_{N_j} \times g_j}, \end{aligned} \tag{4}$$

$$\begin{aligned}
\omega(X_s) &:= \frac{V(X_s)}{V} \\
&= \frac{\sum_{j=1}^k C_{N_j}(X_s) \times n_j}{\sum_{j=1}^k V_{N_j} \times n_j} \\
&= \frac{\sum_{j=1}^k \omega_{N_j}(X_s) \times V_{N_j} \times n_j}{\sum_{j=1}^k V_{N_j} \times n_j},
\end{aligned} \tag{5}$$

and

$$\begin{aligned}
\beta(X_s) &:= \frac{B(X_s)}{I} \\
&= \frac{\sum_{j=1}^k B_{N_j}(X_s) \times n_j}{\sum_{j=1}^k I_{N_j} \times n_j} \\
&= \frac{\sum_{j=1}^k \beta_{N_j}(X_s) \times I_{N_j} \times n_j}{\sum_{j=1}^k I_{N_j} \times n_j}.
\end{aligned} \tag{6}$$

Since  $\omega_i = \beta_i$  except for  $i = N$ , it turns out that differences among the relative contributions  $\lambda(X_s)$ ,  $\omega(X_s)$  and  $\beta(X_s)$  are roughly speaking only due to the different weighting schemes in (4), (5) and (6).

As pointed out in Zenga et al. (2012), it is instructive to compare the relative contributions  $\omega_i(X_s)$  and  $\beta_i(X_s)$  and their weighted averages  $\lambda(X_s)$ ,  $\omega(X_s)$  and  $\beta(X_s)$  with the share  $\gamma(X_s)$  of factor component  $X_s$  on total population income. In fact, in the hypothetical case, the so-called *scale transformation hypothesis*, where  $X_s = \gamma(X_s) \times Y$  for all population members, one would have

$$\bar{M}_i(X_s) = \gamma(X_s) \bar{M}_i(Y) \quad \text{and} \quad \bar{M}_i^+(X_s) = \gamma(X_s) \bar{M}_i^+(Y)$$

for all  $i = 1, 2, \dots, N$ , so that

$$\omega_i(X_s) = \beta_i(X_s) = \gamma(X_s), \quad i = 1, 2, \dots, N-1, \quad \beta_N = \gamma(X_s)$$

and therefore

$$\lambda(X_s) = \omega(X_s) = \beta(X_s) = \gamma(X_s).$$

In real income distributions one should obviously expect that  $X_s \neq \gamma(X_s) \times Y$  for most population members, but since the deviations  $X_s - \gamma(X_s) \times Y$  must sum to zero, the scale transformation hypothesis provides a useful benchmark against which to compare the actual distribution of factor components. For illustrative purposes we shall next describe two types of deviations from the scale transformation hypothesis that are helpful for the interpretation of the relative contributions.



First, we shall consider the case where  $X_s < \gamma(X_s) \times Y$  for population members with low total income  $Y$ , say  $Y < y_{j^*}$ , and  $X_s \geq \gamma(X_s) \times Y$  for population members with  $Y \geq y_{j^*}$ . It is not difficult to show that in this case

$$\bar{M}_i(X_s) < \gamma(X_s)\bar{M}_i(Y) \quad \text{and} \quad \dot{M}_i(X_s) > \gamma(X_s)\dot{M}_i(Y)$$

for  $i = 1, 2, \dots, N$ , so that

$$\omega_i(X_s) = \beta_i(X_s) > \gamma(X_s), \quad i = 1, 2, \dots, N - 1, \quad \beta_N > \gamma(X_s)$$

and therefore

$$\lambda(X_s) > \gamma(X_s), \quad \omega(X_s) > \gamma(X_s) \quad \text{and} \quad \beta(X_s) > \gamma(X_s).$$

Factor components such that  $\lambda(X_s)$ ,  $\omega(X_s)$  and/or  $\beta(X_s)$  exceed  $\gamma(X_s)$  should thus be regarded as having an exacerbating impact on inequality.

The second case is opposite to the first one. It occurs when  $X_s > \gamma(X_s) \times Y$  for population members such that  $Y < y_{j^*}$ , and  $X_s \leq \gamma(X_s) \times Y$  for population members with  $Y \geq y_{j^*}$ . In this case,

$$\bar{M}_i(X_s) > \gamma(X_s)\bar{M}_i(Y) \quad \text{and} \quad \dot{M}_i(X_s) < \gamma(X_s)\dot{M}_i(Y)$$

for  $i = 1, 2, \dots, N$ , so that

$$\omega_i(X_s) = \beta_i(X_s) < \gamma(X_s), \quad i = 1, 2, \dots, N - 1, \quad \beta_N < \gamma(X_s)$$

and therefore

$$\lambda(X_s) < \gamma(X_s), \quad \omega(X_s) < \gamma(X_s), \quad \text{and} \quad \beta(X_s) < \gamma(X_s).$$

In view of the latter inequalities, one should regard factor components such that  $\lambda(X_s)$ ,  $\omega(X_s)$  and/or  $\beta(X_s)$  are smaller than  $\gamma(X_s)$  as having a mitigating impact on inequality.

As the above discussion shows, the approach based on point inequality indexes gives rise to meaningful and readily interpretable results, which, we believe, is its main advantage over other methods that have been proposed in the literature (see e.g. Rao 1969, Fei et al. 1978, Shorrocks 1982, Shorrocks 1983, Lerman and Yitzhaki 1984, Lerman and Yitzhaki 1985, Radaelli and Zenga 2005). In the following sections we shall apply this approach to data from the 2001 wave of the European Community Household Panel (henceforth ECHP) in order to assess its outcome on real data.

The rest of this work is thus organized as follows. In Section 2 we shall provide some general information about the ECHP and about the income data therein. We shall define four income components whose contributions to inequality as measured by the point and synthetic inequality indexes  $G$ ,  $V$  and  $I$  will be assessed in Section 3. Conclusions and final remarks end this work in Section 4.

## 2 ECHP income data

The European Community Household Panel (ECHP) is a multi-purpose annual longitudinal survey covering the time span between 1994 and 2001. Its aim is to provide comparable information from EU countries. It is centrally designed and coordinated by EuroStat and covers topics such as demographics, labor force behavior, income, health, education and training, housing, migration, etc.. The objective of the ECHP is to represent the population of the EU at individual and household level. More information about this survey may be found in the accompanying documentation (see ECHP 1996a, ECHP 1996b and ECHP 2003).

In the present work we analyze data about household income from the Users' Database (UDB) referring to the 2001 wave of the ECHP. Information on income is collected very detailed in the ECHP questionnaire. Some of the income components are collected at household level, while others are collected for each individual in sample households. In order to have complete information at both household and individual level, household income components are shared among its members aged over 16 and personal income components are aggregated for the whole household. To be specific, income components collected at household level are: property and rental income, social assistance and housing allowances. All other income components are collected individually among persons aged over 16 who reside in sample households. As for taxes, some of the income components are collected net and others gross of taxes. To allow for the computation of comparable net values, the survey provides net/gross ratios (variable  $HIO20$  in the Household-file of the UDB<sup>7</sup>) for each household.

Below we shall apply the formulae of Section 1 to evaluate the contributions from several income factor components to inequality in the distribution of total net household income (variable  $HI100$ ).

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<sup>7</sup>Except for the country-specific informations provided in Table 2 in the next Section, all other variables listed in this work are included in the Household-file

To avoid excessive scattering of the contributions among a large number of income factor components, we shall aggregate the latter into four main components:

- **Wage and salary income** ( $X_1$  :=variable *HI111*). This income factor component includes wages and salary payments and any other form of pay for work as an employee or apprentice.
- **Self-employment income** ( $X_2$  :=variable *HI112*). This includes any income from self-employment such as own business, professional practice or farm, working as free-lance or sub-contractor, providing services or selling goods on own account.
- **Other income components** ( $X_3$  :=the sum of variables *HI121*, *HI122*, *HI123* and *HI140*). This includes capital income (variable *HI121*), income from property and rents (variable *HI122*), private transfers (variable *HI123*) and adjustments for within household non-response (variable *HI140*).
- **Social transfers** ( $X_4$  :=variable *HI130*). This includes unemployment related benefits, pension or benefit relating to old-age or retirement, survivor's pension or benefits for widows or orphans, family related benefits, benefits relating to sickness or invalidity, education related allowances and any other social benefits.

Except for the samples from France and Finland, the variables *HIxxx* in the UDB contain amounts of income net of taxes. For households where these variables are filled,<sup>8</sup> the reported net values are consistent in the sense that

$$\begin{aligned}
 &\text{net household income } (Y := HI100) := \\
 &:= \text{wage and salary income } (X_1 := HI111) + \\
 &\quad + \text{self employment income } (X_2 := HI112) + \\
 &\quad + \text{other income components} \\
 &\quad (X_3 := HI121 + HI122 + HI123 + HI140) + \\
 &\quad + \text{social transfers income } (X_4 := HI130).
 \end{aligned}$$

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<sup>8</sup>The variables referring to the income components are always filled if the net household income variable *HI100* is filled. However, for all countries except for Luxembourg, there are some (very few) households where the value of the net household income variable is missing.

For households belonging to the samples from France and Finland, the variables  $HI111$ ,  $HI112$ ,  $HI130$ ,  $HI121$ ,  $HI122$  and  $HI123$  report gross values, which must be converted into net values through multiplication by variable  $HI020$  (the household net/gross ratio), while all other variables  $HIxxx$  do still contain net values. Thus, for the households included in the samples from France and Finland,

$$\begin{aligned}
& \text{net household income } (Y := HI100) := \\
& := \text{wage and salary income } (X_1 := HI111) + \\
& \quad + \text{self employment income } (X_2 := HI020 \times HI112) + \\
& \quad + \text{other income components} \\
& \quad (X_3 := HI020 \times (HI121 + HI122 + HI123) + HI140) \\
& \quad + \text{social transfers income } (X_4 := HI020 \times HI130).
\end{aligned}$$

## 2.1 Estimation from survey data

As virtually every modern survey, the ECHP provides weights for the computation of estimates for population statistics. Following the suggestion in [7], we used the cross-sectional household weights provided in the household file of the UDB (variable  $HG004$ ) in the computation of all estimates concerning the household populations in the various countries. In the ECHP each household with completed household interview has his own non negative cross-sectional household weight  $HG004$ , and these weights are scaled in such way that their sum over all interviewed households equals the number  $N^*$  of interviewed households in each country. Since we are however concerned with the distribution of income and since we excluded from the analysis the households for which the total net household income variable (variable  $HI100$ ) is not filled, we rescaled the cross-sectional household weights (variable  $HG004$ ) to make sure that in each country their sum equals the number  $N$  of households for which the net household income variable is filled. Table 2 reports  $N^*$ ,  $N$  and the ratio  $\theta$  between the sum of the original cross-sectional household weights provided by the ECHP for households where the total net income variable  $HI100$  is not filled and  $N^*$  in the denominator. Note that while  $(N^* - N)/N^*$  is even larger than 10% in Sweden,  $\theta$  does not exceed 2% in any country.

In order to account for the rescaled cross-sectional household weights in the estimates provided in the next section, we used plug-in estimators: since all population statistics introduced in Section 1 depend only on the joint population distribution function for total income  $Y$  and the factor

Table 1: Sample sizes in the 2001 wave of the ECHP

Country	$N^*$	$N$	$(N^* - N)/N^*$	$\theta$
Ireland	1760	1757	0.002	0.001
Denmark	2283	2279	0.002	0.001
Belgium	2362	2342	0.008	0.010
Luxembourg	2428	2428	0.000	0.000
Austria	2544	2535	0.004	0.002
Finland	3115	3106	0.003	0.002
Greece	3916	3895	0.005	0.006
Portugal	4614	4588	0.006	0.005
UK	4819	4779	0.008	0.009
Netherlands	4851	4824	0.006	0.005
Spain	4966	4950	0.003	0.003
Sweden	5680	5085	0.105	0.020
France	5345	5247	0.018	0.015
Italy	5606	5525	0.014	0.012
Germany	5563	5559	0.001	0.003

components  $X_s$ , we simply replaced the frequencies  $n_j$  by the sums  $h_j$  of the rescaled cross-sectional households weights of sample households with total income  $Y = y_j$ , and the cumulated frequencies  $N_j$  by the corresponding cumulated sums  $H_j := \sum_{j'=1}^j h_{j'}$ . Note that in this way the point inequality indexes  $G_i$ ,  $V_i$  and  $I_i$  and their corresponding contributions might correspond to non integer values  $i$  with range between 0 and  $N$ . To free the notation from its dependence on different values of  $N$  in different countries we shall henceforth report subscript values  $p$  between 0 and 1 in place of  $i$ . For example, we shall indicate by  $G_p$ ,  $0 < p < 1$ , the point inequality index  $G_{H_j}$  at the smallest (possibly non integer) cumulated weight  $H_1, H_2, \dots, H_j, \dots, H_k$  such that  $N \times p \leq H_j$ .

### 3 Inequality in the distribution of net total household income in EU countries

In this Section we analyze the contributions from factor components to inequality in the distribution of total net household income in the countries included in the 2001 wave of the ECHP. All estimates account for the rescaled cross-sectional household weights as defined in the previous section. Table 2 reports for each country the population size  $pop$ , the number of households  $hh$  and the average household size  $pop/hh$  as from the Country-file included in the UDB provided by EuroStat. Along with those statistics, Table 2 reports also the sample sizes  $N$  (as defined in Section 2), the estimates  $Median(Y)$  and  $M(Y)$  (in Euro) for the median and the mean, respectively, of the distributions of total net household income, and finally the estimates  $G$ ,  $V$  and  $I$  for the inequality indexes of Gini, Bonferroni and Zenga, respectively. The countries are ordered according to  $G$  from Denmark ( $G = 0.302$ ) to Portugal ( $G = 0.402$ ).

Next, we shall now proceed to analyze how the four factor components defined in Section 2 affect inequality. The following analysis will be based on the results reported in Table 3.

- **Wage and salary income**, with shares  $\gamma(X_1)$  between 0.482 in Greece and 0.680 in Denmark, accounts for the largest share on total income  $Y$  in all 15 countries. To understand how this factor component affects inequality, we first observe that the contributions  $\lambda(X_1)$ ,  $\omega(X_1)$  and  $\beta(X_1)$  are clearly larger than  $\gamma(X_1)$ . *Wage and salary income* does thus contribute more to overall inequality than it would do under the scale transformation hypothesis. To assess the impact on inequality at different levels  $p$  of the income distribution, we shall next examine the relative contributions  $\beta_p(X_1)$ : we find that  $\beta_p(X_1) > \gamma(X_1)$  for all countries for all values of  $p$  reported in Table 3, and that the trend of  $\beta_p(X_1)$  is quite similar in all countries:  $\beta_p(X_1)$  tends to increase for  $0 < p \leq 0.25$  and to decrease for  $p > 0.75$ . For the interpretation of the relative contributions, recall that  $\beta_p(X_1)$  is the ratio between  $\bar{M}_p^+(X_1) - \bar{M}_p^-(X_1)$  and  $\bar{M}_p^+(Y) - \bar{M}_p^-(Y)$ . In Italy, for example,  $\beta_{0.50}(X_1) = 0.661$  indicates that the difference between the means of *wage and salary income* among the households belonging to the lower half of the income distribution and those belonging to the upper half is equal to 0.661 times the difference between the corresponding means of total income  $Y$ .

Table 2: General information about countries included in the 2001 wave of the ECHP

Country	$pop \times 10^{-3}$	$hh \times 10^{-3}$	$Pop/hh$	$N$	$Median(Y)^a$	$M(Y)^a$	$G$	$V$	$I$
Denmark	5.368	2.456	2.19	2279	33561	34597	0.302	0.435	0.646
Netherlands	15.773	6.889	2.29	4824	22331	24788	0.303	0.428	0.643
Luxembourg	0.433	0.172	2.52	2428	38333	44729	0.304	0.414	0.631
Austria	7.986	3.3	2.42	2535	25058	28543	0.328	0.456	0.672
France	57.949	24.523	2.36	5247	24408	28053	0.329	0.457	0.674
Sweden	8.663	4.576	1.89	5085	20389	23651	0.331	0.459	0.677
Germany	81.569	37.711	2.16	5559	24554	28486	0.336	0.460	0.679
Italy	57.388	21.967	2.61	5525	18179	21210	0.342	0.471	0.688
Finland	5.12	2.381	2.15	3106	21067	24801	0.350	0.481	0.697
Belgium	10.263	4.278	2.4	2342	25558	30374	0.354	0.472	0.694
United Kingdom	59.063	25.564	2.31	4779	26893	32151	0.369	0.499	0.717
Greece	10.354	3.993	2.59	3895	12208	14853	0.382	0.517	0.734
Ireland	3.839	1.291	2.97	1757	25457	30685	0.388	0.524	0.740
Spain	39.137	13.281	2.95	4950	16810	21453	0.399	0.526	0.745
Portugal	10.024	3.391	2.96	4588	12362	15661	0.402	0.530	0.749

<sup>a</sup> The estimates for the median and the mean of the household income distributions are expressed in Euros. They have been obtained using the fixed conversion rates for Germany, Denmark, Netherlands, Luxembourg, France, UK, Ireland, Italy, Greece, Spain, Portugal and Austria and using the conversion rates for the year 2001 as given in the Country-file of the ECHP for Belgium, Finland and Sweden.

- **Self-employment income.** The share  $\gamma(X_2)$  of *self-employment income* on total population income may vary a lot from country to country. In fact, it ranges from as low as  $\gamma(X_2) = 0.018$  in Sweden, to  $\gamma(X_2) = 0.210$  in Greece. Apart from Greece, the list of countries with shares  $\gamma(X_2)$  well above 0.1 includes Italy ( $\gamma(X_2) = 0.162$ ), Spain ( $\gamma(X_2) = 0.145$ ), Ireland ( $\gamma(X_2) = 0.137$ ) and Portugal ( $\gamma(X_2) = 0.124$ ). The contributions  $\lambda(X_2)$ ,  $\omega(X_2)$  and  $\beta(X_2)$  do clearly exceed  $\gamma(X_2)$  in all countries except for Sweden, indicating that also this factor component exacerbates overall inequality in the distribution of total income  $Y$ . The relative contributions  $\beta_p(X_2)$  are, except for Sweden, clearly larger than  $\gamma(X_2)$  at all levels of  $p$  reported in Table 3, and they tend to increase as  $p$  gets larger. In many countries the increasing trend is quite marked starting

from  $p = 0.5$ .

- **Other income components.** The share of income from this factor component is about  $\gamma(X_3) = 0.050$  in all countries except for Belgium and the United Kingdom, where  $\gamma(X_3) = 0.108$  and  $\gamma(X_3) = 0.132$ , respectively. The contributions  $\lambda(X_3)$ ,  $\omega(X_3)$  and  $\beta(X_3)$  do slightly exceed  $\gamma(X_3)$  in most countries, indicating that, like the former two factor components, the *other income components* tend to exacerbate inequality as well. The largest contributions  $\lambda(X_3)$ ,  $\omega(X_3)$  and  $\beta(X_3)$  are observed in those countries where the share  $\gamma(X_3)$  is also largest, i.e. Belgium and the United Kingdom. Inspection of the relative contributions  $\beta_p$  reveals an increasing trend in most countries. In some countries like Belgium, Finland, Sweden and the United Kingdom the increasing trend is quite marked in the final part of the income distribution (i.e. for  $p \geq 0.75$ ).
- **Social transfers,** with shares  $\gamma(X_4)$  between 0.190 in Ireland, and 0.323 in Sweden, is the second largest factor component in all considered countries. As expected, the relative contributions  $\lambda(X_4)$ ,  $\omega(X_4)$  and  $\beta(X_4)$  are clearly smaller than  $\gamma(X_4)$ , confirming that this factor component has an offsetting impact on inequality. In Belgium, Denmark, Ireland, Luxembourg and the United Kingdom some of the relative contributions  $\lambda(X_4)$ ,  $\omega(X_4)$  and/or  $\beta(X_4)$  are even negative. As for the relative contributions  $\beta_p(X_4)$ , they are constantly smaller than  $\gamma(X_4)$  in all considered countries and they exhibit a decreasing trend in the initial part of the income distribution up to  $p = 0.50$ , and are thereafter almost constant, except for Sweden, where the decreasing trend holds on until  $p = 0.75$ , and for Denmark, where  $\beta_p(X_4)$  increases after  $p = 0.500$ .

## 4 Conclusions

Existing literature about inequality index decomposition by factor components (Rao 1969, Fei et al 1978, Pyatt et al. 1980, Shorrocks 1982, Shorrocks 1983, Lerman and Yitzhaki 1984, Lerman and Yitzhaki 1985, Radaelli and Zenga 2005) has mainly been focused on (synthetic) overall inequality indexes. Only recently Zenga et al (2012) explored a new approach: given that the synthetic inequality index proposed in Zenga (2007) is defined as average value of point inequality indexes, the authors



define first the contributions from factor components to the point inequality indexes, and then the contributions to the synthetic inequality index simply as average values of the contributions to the point inequality indexes. An interesting and attractive feature of this approach is that it allows for a more detailed analysis of the sources of inequality because of the easy and straightforward link between the contributions to the point inequality indexes and to the synthetic index. In addition to the approach based on point inequality indexes, Zenga et al (2012) introduce also a useful benchmark situation, the so-called *scale transformation hypothesis*, against which to compare the actual distribution of factor components in order to determine in which direction they do affect inequality in distribution of total income. Under the *scale transformation hypothesis* it is assumed that for each household the shares of income from the factor components equal their corresponding shares on total population income. Factor components with contributions to inequality larger than their theoretical value under the *scale transformation hypothesis* should be deemed to have an exacerbating impact on inequality in the distribution of total income, while otherwise their impact should be considered as inequality-offsetting.

In a later work (Zenga 2013) extended the approach based on point inequality indexes to other two notable inequality indexes which are defined as average values of point inequality indexes: the Gini and the Bonferroni indexes. In this latter work it is shown that for a given factor component, the *relative* contributions to the point inequality indexes underlying the Gini, Bonferroni and Zenga indexes are all the same, and thus that the *relative* contributions to these three synthetic indexes are weighted averages, with different weights, of a unique set of relative contributions to point inequality indexes. In a further work Arcagni and Zenga (2014) employed the approach based on point inequality indexes to obtain a decomposition rule for the inequality index  $\xi$  proposed by Zenga (1984) as well.

In the present work we applied the decomposition rules based on point inequality indexes for the Gini, Bonferroni and Zenga indexes to household income data from the 2001 wave of the European Community Household Panel. We considered four factor components: "wage and salary income", "self-employment income", "other incomes" and "social transfers". The outcome shows that the former two factor components exhibit larger contributions to the point and synthetic inequality indexes than they would have under the *scale transformation hypothesis* and thus they should be considered as inequality-exacerbating factor components. The observed contributions to the three

synthetic inequality indexes from the *other income components*, which include roughly speaking capital income, income from property and rents and private transfers, do also exceed their theoretical values under the *scale transformation hypothesis* in most countries, although to a lesser extent. Detailed analysis of the contributions to the point inequality indexes in the latter countries shows that the impact on inequality from the *other income components* gets larger as smaller population shares of high-income households are compared with the rest of the household population. The *social transfers* factor component, on the other hand, is in most countries the only factor component with an offsetting impact on inequality in the sense that its contributions to the point and synthetic inequality indexes are smaller than under the *scale transformation hypothesis*. It is worth noting that the *relative* contributions tend to decrease in the initial part of the income distribution, where small population shares of low-income households are compared with the rest of the household population.

Table 3: Contributions to inequality from income factor components

Austria										
$\gamma(\cdot)$	$\beta_{0.05}(\cdot)$	$\beta_{0.10}(\cdot)$	$\beta_{0.25}(\cdot)$	$\beta_{0.50}(\cdot)$	$\beta_{0.75}(\cdot)$	$\beta_{0.90}(\cdot)$	$\beta_{0.95}(\cdot)$	$\lambda(\cdot)$	$\omega(\cdot)$	$\beta(\cdot)$
$X_1$	0.622	0.740	0.834	0.857	0.817	0.811	0.761	0.834	0.818	0.818
$X_2$	0.061	0.063	0.086	0.094	0.118	0.111	0.132	0.097	0.087	0.095
$X_3$	0.032	0.006	0.025	0.031	0.046	0.056	0.056	0.036	0.028	0.033
$X_4$	0.284	0.191	0.103	0.018	0.019	0.022	0.050	0.033	0.066	0.053
	$p = 0.05$	$p = 0.10$	$p = 0.25$	$p = 0.50$	$p = 0.75$	$p = 0.90$	$p = 0.95$	$G$	$V$	$I$
$G_p, V_p$	0.810	0.751	0.630	0.460	0.280	0.147	0.091	0.328	0.456	-
$I_p$	0.817	0.770	0.694	0.630	0.609	0.633	0.664	-	-	0.672
Belgium										
$\gamma(\cdot)$	$\beta_{0.05}(\cdot)$	$\beta_{0.10}(\cdot)$	$\beta_{0.25}(\cdot)$	$\beta_{0.50}(\cdot)$	$\beta_{0.75}(\cdot)$	$\beta_{0.90}(\cdot)$	$\beta_{0.95}(\cdot)$	$\lambda(\cdot)$	$\omega(\cdot)$	$\beta(\cdot)$
$X_1$	0.550	0.672	0.776	0.787	0.696	0.544	0.431	0.720	0.731	0.692
$X_2$	0.071	0.081	0.103	0.130	0.172	0.230	0.301	0.149	0.122	0.152
$X_3$	0.108	0.129	0.139	0.150	0.172	0.219	0.260	0.163	0.149	0.166
$X_4$	0.271	0.118	-0.019	-0.067	-0.040	0.007	0.008	-0.032	-0.001	-0.009
	$p = 0.05$	$p = 0.10$	$p = 0.25$	$p = 0.50$	$p = 0.75$	$p = 0.90$	$p = 0.95$	$G$	$V$	$I$
$G_p, V_p$	0.787	0.733	0.635	0.488	0.310	0.182	0.126	0.354	0.472	-
$I_p$	0.795	0.752	0.699	0.656	0.643	0.689	0.742	-	-	0.694
Denmark										
$\gamma(\cdot)$	$\beta_{0.05}(\cdot)$	$\beta_{0.10}(\cdot)$	$\beta_{0.25}(\cdot)$	$\beta_{0.50}(\cdot)$	$\beta_{0.75}(\cdot)$	$\beta_{0.90}(\cdot)$	$\beta_{0.95}(\cdot)$	$\lambda(\cdot)$	$\omega(\cdot)$	$\beta(\cdot)$
$X_1$	0.680	0.762	0.928	1.018	0.961	0.773	0.675	0.945	0.908	0.890
$X_2$	0.051	0.061	0.079	0.095	0.110	0.175	0.217	0.102	0.086	0.108
$X_3$	0.043	0.039	0.040	0.036	0.038	0.067	0.064	0.039	0.038	0.042
$X_4$	0.226	0.138	-0.047	-0.149	-0.109	-0.014	0.044	-0.086	-0.032	-0.040
	$p = 0.05$	$p = 0.10$	$p = 0.25$	$p = 0.50$	$p = 0.75$	$p = 0.90$	$p = 0.95$	$G$	$V$	$I$
$G_p, V_p$	0.816	0.748	0.619	0.437	0.240	0.120	0.075	0.302	0.435	-
$I_p$	0.823	0.768	0.684	0.608	0.558	0.577	0.619	-	-	0.646

Finland													
$\gamma(\cdot)$	$\beta_{0.05}(\cdot)$	$\beta_{0.10}(\cdot)$	$\beta_{0.25}(\cdot)$	$\beta_{0.50}(\cdot)$	$\beta_{0.75}(\cdot)$	$\beta_{0.90}(\cdot)$	$\beta_{0.95}(\cdot)$	$\lambda(\cdot)$	$\omega(\cdot)$	$\beta(\cdot)$			
$X_1$	0.612	0.676	0.808	0.814	0.809	0.707	0.608	0.791	0.772	0.758			
$X_2$	0.070	0.078	0.094	0.110	0.137	0.172	0.211	0.119	0.103	0.117			
$X_3$	0.053	0.053	0.065	0.075	0.099	0.161	0.231	0.090	0.075	0.098			
$X_4$	0.266	0.193	0.034	0.000	-0.045	-0.040	-0.051	0.001	0.050	0.027			
		$p = 0.05$	$p = 0.10$	$p = 0.25$	$p = 0.50$	$p = 0.75$	$p = 0.90$	$p = 0.95$	$G$	$V$	$I$		
$G_p, V_p$	0.821	0.763	0.666	0.500	0.298	0.157	0.098	0.350	0.481	-			
$I_p$	0.829	0.781	0.727	0.666	0.629	0.649	0.685	-	-	0.697			
France													
$\gamma(\cdot)$	$\beta_{0.05}(\cdot)$	$\beta_{0.10}(\cdot)$	$\beta_{0.25}(\cdot)$	$\beta_{0.50}(\cdot)$	$\beta_{0.75}(\cdot)$	$\beta_{0.90}(\cdot)$	$\beta_{0.95}(\cdot)$	$\lambda(\cdot)$	$\omega(\cdot)$	$\beta(\cdot)$			
$X_1$	0.604	0.695	0.780	0.830	0.815	0.777	0.744	0.803	0.777	0.779			
$X_2$	0.063	0.069	0.083	0.090	0.111	0.148	0.182	0.100	0.087	0.102			
$X_3$	0.044	0.030	0.036	0.036	0.041	0.036	0.038	0.038	0.036	0.037			
$X_4$	0.290	0.207	0.101	0.045	0.033	0.0390	0.036	0.060	0.100	0.082			
	$p = 0.05$	$p = 0.10$	$p = 0.25$	$p = 0.50$	$p = 0.75$	$p = 0.90$	$p = 0.95$	$G$	$V$	$I$			
$G_p, V_p$	0.823	0.749	0.623	0.458	0.284	0.153	0.096	0.329	0.457	-			
$I_p$	0.831	0.768	0.688	0.628	0.614	0.643	0.679	-	-	0.674			
Germany													
$\gamma(\cdot)$	$\beta_{0.05}(\cdot)$	$\beta_{0.10}(\cdot)$	$\beta_{0.25}(\cdot)$	$\beta_{0.50}(\cdot)$	$\beta_{0.75}(\cdot)$	$\beta_{0.90}(\cdot)$	$\beta_{0.95}(\cdot)$	$\lambda(\cdot)$	$\omega(\cdot)$	$\beta(\cdot)$			
$X_1$	0.529	0.622	0.687	0.712	0.648	0.555	0.454	0.667	0.667	0.641			
$X_2$	0.090	0.109	0.128	0.161	0.212	0.268	0.327	0.178	0.151	0.183			
$X_3$	0.076	0.065	0.091	0.096	0.128	0.129	0.133	0.108	0.095	0.105			
$X_4$	0.305	0.204	0.094	0.031	0.012	0.048	0.085	0.047	0.088	0.071			
	$p = 0.05$	$p = 0.10$	$p = 0.25$	$p = 0.50$	$p = 0.75$	$p = 0.90$	$p = 0.95$	$G$	$V$	$I$			
$G_p, V_p$	0.797	0.738	0.628	0.467	0.292	0.162	0.105	0.336	0.460	-			
$I_p$	0.805	0.758	0.692	0.637	0.622	0.658	0.698	-	-	0.679			

Greece										
$\gamma(\cdot)$	$\beta_{0.05}(\cdot)$	$\beta_{0.10}(\cdot)$	$\beta_{0.25}(\cdot)$	$\beta_{0.50}(\cdot)$	$\beta_{0.75}(\cdot)$	$\beta_{0.90}(\cdot)$	$\beta_{0.95}(\cdot)$	$\lambda(\cdot)$	$\omega(\cdot)$	$\beta(\cdot)$
$X_1$	0.482	0.539	0.649	0.681	0.654	0.588	0.587	0.647	0.627	0.621
$X_2$	0.210	0.226	0.250	0.220	0.217	0.251	0.280	0.233	0.233	0.239
$X_3$	0.058	0.054	0.052	0.065	0.074	0.100	0.098	0.070	0.062	0.071
$X_4$	0.250	0.181	0.050	0.034	0.054	0.061	0.035	0.050	0.079	0.069
	$p = 0.05$	$p = 0.10$	$p = 0.25$	$p = 0.50$	$p = 0.75$	$p = 0.90$	$p = 0.95$	$G$	$V$	$I$
$G_p, V_p$	0.878	0.821	0.706	0.532	0.336	0.182	0.116	0.382	0.517	-
$I_p$	0.884	0.836	0.762	0.695	0.669	0.689	0.723	-	-	0.734

Ireland										
$\gamma(\cdot)$	$\beta_{0.05}(\cdot)$	$\beta_{0.10}(\cdot)$	$\beta_{0.25}(\cdot)$	$\beta_{0.50}(\cdot)$	$\beta_{0.75}(\cdot)$	$\beta_{0.90}(\cdot)$	$\beta_{0.95}(\cdot)$	$\lambda(\cdot)$	$\omega(\cdot)$	$\beta(\cdot)$
$X_1$	0.615	0.720	0.784	0.755	0.691	0.611	0.588	0.732	0.742	0.715
$X_2$	0.137	0.153	0.167	0.200	0.240	0.311	0.361	0.214	0.189	0.219
$X_3$	0.059	0.063	0.067	0.072	0.071	0.075	0.056	0.068	0.067	0.066
$X_4$	0.190	0.064	-0.019	-0.028	-0.002	0.003	-0.004	-0.014	0.002	0.000
	$p = 0.05$	$p = 0.10$	$p = 0.25$	$p = 0.50$	$p = 0.75$	$p = 0.90$	$p = 0.95$	$G$	$V$	$I$
$G_p, V_p$	0.847	0.820	0.734	0.548	0.336	0.178	0.111	0.388	0.524	-
$I_p$	0.853	0.835	0.786	0.708	0.669	0.683	0.713	-	-	0.740

Italy										
$\gamma(\cdot)$	$\beta_{0.05}(\cdot)$	$\beta_{0.10}(\cdot)$	$\beta_{0.25}(\cdot)$	$\beta_{0.50}(\cdot)$	$\beta_{0.75}(\cdot)$	$\beta_{0.90}(\cdot)$	$\beta_{0.95}(\cdot)$	$\lambda(\cdot)$	$\omega(\cdot)$	$\beta(\cdot)$
$X_1$	0.505	0.586	0.688	0.661	0.631	0.542	0.479	0.642	0.639	0.621
$X_2$	0.162	0.182	0.197	0.220	0.229	0.306	0.347	0.226	0.208	0.230
$X_3$	0.041	0.022	0.040	0.053	0.063	0.080	0.098	0.055	0.045	0.054
$X_4$	0.293	0.211	0.075	0.067	0.077	0.072	0.077	0.077	0.108	0.095
	$p = 0.05$	$p = 0.10$	$p = 0.25$	$p = 0.50$	$p = 0.75$	$p = 0.90$	$p = 0.95$	$G$	$V$	$I$
$G_p, V_p$	0.831	0.769	0.644	0.480	0.297	0.159	0.098	0.342	0.471	-
$I_p$	0.838	0.787	0.707	0.648	0.626	0.653	0.685	-	-	0.689

Luxembourg											
	$\gamma(\cdot)$	$\beta_{0.05}(\cdot)$	$\beta_{0.10}(\cdot)$	$\beta_{0.25}(\cdot)$	$\beta_{0.50}(\cdot)$	$\beta_{0.75}(\cdot)$	$\beta_{0.90}(\cdot)$	$\beta_{0.95}(\cdot)$	$\lambda(\cdot)$	$\omega(\cdot)$	$\beta(\cdot)$
$X_1$	0.635	0.733	0.771	0.859	0.882	0.847	0.737	0.674	0.843	0.825	0.812
$X_2$	0.042	0.054	0.060	0.065	0.078	0.115	0.143	0.155	0.092	0.077	0.091
$X_3$	0.050	0.063	0.062	0.064	0.071	0.085	0.125	0.169	0.079	0.071	0.085
$X_4$	0.273	0.151	0.107	0.012	-0.032	-0.047	-0.005	0.002	-0.015	0.028	0.012
		$p = 0.05$	$p = 0.10$	$p = 0.25$	$p = 0.50$	$p = 0.75$	$p = 0.90$	$p = 0.95$	$G$	$V$	$I$
$G_p, V_p$		0.714	0.662	0.563	0.423	0.269	0.144	0.089	0.304	0.414	-
$I_p$		0.725	0.685	0.632	0.594	0.596	0.626	0.658	-	-	0.631

Netherlands											
	$\gamma(\cdot)$	$\beta_{0.05}(\cdot)$	$\beta_{0.10}(\cdot)$	$\beta_{0.25}(\cdot)$	$\beta_{0.50}(\cdot)$	$\beta_{0.75}(\cdot)$	$\beta_{0.90}(\cdot)$	$\beta_{0.95}(\cdot)$	$\lambda(\cdot)$	$\omega(\cdot)$	$\beta(\cdot)$
$X_1$	0.635	0.693	0.788	0.871	0.883	0.839	0.804	0.743	0.856	0.831	0.827
$X_2$	0.038	0.045	0.046	0.057	0.074	0.098	0.118	0.160	0.080	0.065	0.080
$X_3$	0.058	0.034	0.049	0.068	0.064	0.060	0.048	0.052	0.062	0.059	0.059
$X_4$	0.269	0.228	0.117	0.004	-0.020	0.003	0.031	0.045	0.002	0.045	0.034
		$p = 0.05$	$p = 0.10$	$p = 0.25$	$p = 0.50$	$p = 0.75$	$p = 0.90$	$p = 0.95$	$G$	$V$	$I$
$G_p, V_p$		0.804	0.714	0.589	0.423	0.253	0.136	0.088	0.303	0.428	-
$I_p$		0.812	0.735	0.656	0.595	0.576	0.611	0.657	-	-	0.643

Portugal											
	$\gamma(\cdot)$	$\beta_{0.05}(\cdot)$	$\beta_{0.10}(\cdot)$	$\beta_{0.25}(\cdot)$	$\beta_{0.50}(\cdot)$	$\beta_{0.75}(\cdot)$	$\beta_{0.90}(\cdot)$	$\beta_{0.95}(\cdot)$	$\lambda(\cdot)$	$\omega(\cdot)$	$\beta(\cdot)$
$X_1$	0.629	0.716	0.750	0.778	0.747	0.745	0.747	0.714	0.755	0.754	0.746
$X_2$	0.124	0.131	0.141	0.148	0.148	0.121	0.088	0.108	0.133	0.137	0.132
$X_3$	0.034	0.035	0.037	0.039	0.048	0.059	0.071	0.093	0.052	0.045	0.055
$X_4$	0.214	0.117	0.073	0.035	0.057	0.075	0.094	0.085	0.061	0.065	0.067
		$p = 0.05$	$p = 0.10$	$p = 0.25$	$p = 0.50$	$p = 0.75$	$p = 0.90$	$p = 0.95$	$G$	$V$	$I$
$G_p, V_p$		0.869	0.821	0.710	0.544	0.364	0.216	0.142	0.402	0.530	-
$I_p$		0.874	0.836	0.765	0.705	0.696	0.733	0.768	-	-	0.749

## Spain

	$\gamma(\cdot)$	$\beta_{0.05}(\cdot)$	$\beta_{0.10}(\cdot)$	$\beta_{0.25}(\cdot)$	$\beta_{0.50}(\cdot)$	$\beta_{0.75}(\cdot)$	$\beta_{0.90}(\cdot)$	$\beta_{0.95}(\cdot)$	$\lambda(\cdot)$	$\omega(\cdot)$	$\beta(\cdot)$
$X_1$	0.573	0.634	0.679	0.715	0.686	0.625	0.585	0.491	0.655	0.668	0.643
$X_2$	0.145	0.159	0.173	0.181	0.196	0.222	0.255	0.319	0.210	0.192	0.212
$X_3$	0.061	0.058	0.065	0.068	0.081	0.104	0.142	0.162	0.093	0.080	0.091
$X_4$	0.221	0.149	0.084	0.036	0.037	0.049	0.018	0.028	0.042	0.060	0.053
		$p = 0.05$	$p = 0.10$	$p = 0.25$	$p = 0.50$	$p = 0.75$	$p = 0.90$	$p = 0.95$	$G$	$V$	$I$
$G_p, V_p$		0.857	0.803	0.703	0.543	0.360	0.210	0.144	0.399	0.526	-
$I_p$		0.863	0.819	0.759	0.703	0.692	0.727	0.766	-	-	0.745

## Sweden

	$\gamma(\cdot)$	$\beta_{0.05}(\cdot)$	$\beta_{0.10}(\cdot)$	$\beta_{0.25}(\cdot)$	$\beta_{0.50}(\cdot)$	$\beta_{0.75}(\cdot)$	$\beta_{0.90}(\cdot)$	$\beta_{0.95}(\cdot)$	$\lambda(\cdot)$	$\omega(\cdot)$	$\beta(\cdot)$
$X_1$	0.609	0.658	0.718	0.839	0.829	0.900	0.865	0.827	0.845	0.803	0.817
$X_2$	0.018	0.003	0.000	0.013	0.013	0.007	0.016	0.024	0.011	0.010	0.012
$X_3$	0.050	0.055	0.058	0.070	0.083	0.105	0.139	0.156	0.092	0.077	0.091
$X_4$	0.323	0.284	0.224	0.078	0.075	-0.012	-0.020	-0.006	0.052	0.110	0.080
		$p = 0.05$	$p = 0.10$	$p = 0.25$	$p = 0.50$	$p = 0.75$	$p = 0.90$	$p = 0.95$	$G$	$V$	$I$
$G_p, V_p$		0.835	0.753	0.624	0.465	0.282	0.154	0.100	0.331	0.459	-
$I_p$		0.842	0.772	0.688	0.635	0.611	0.645	0.689	-	-	0.677

## United Kingdom

	$\gamma(\cdot)$	$\beta_{0.05}(\cdot)$	$\beta_{0.10}(\cdot)$	$\beta_{0.25}(\cdot)$	$\beta_{0.50}(\cdot)$	$\beta_{0.75}(\cdot)$	$\beta_{0.90}(\cdot)$	$\beta_{0.95}(\cdot)$	$\lambda(\cdot)$	$\omega(\cdot)$	$\beta(\cdot)$
$X_1$	0.556	0.633	0.668	0.737	0.787	0.734	0.670	0.588	0.741	0.722	0.709
$X_2$	0.076	0.085	0.091	0.104	0.117	0.145	0.161	0.193	0.127	0.112	0.130
$X_3$	0.132	0.138	0.145	0.162	0.178	0.187	0.222	0.244	0.181	0.167	0.178
$X_4$	0.236	0.144	0.096	-0.003	-0.082	-0.066	-0.053	-0.025	-0.049	-0.001	-0.017
		$p = 0.05$	$p = 0.10$	$p = 0.25$	$p = 0.50$	$p = 0.75$	$p = 0.90$	$p = 0.95$	$G$	$V$	$I$
$G_p, V_p$		0.838	0.784	0.680	0.514	0.322	0.178	0.115	0.369	0.499	-
$I_p$		0.844	0.802	0.739	0.679	0.655	0.684	0.721	-	-	0.717

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