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Financial Modelling

Contributions to  
Management Science

Lorenzo Peccati · Matti Virén  
Editors

# Financial Modelling

Recent Research



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## PREFACE

The volume collects a selection of papers presented and discussed during the two Meetings held in 1992 of the EURO Working Group on Financial Modelling. In April the works were held in Cogne (Aosta Valley - Italy) and in November in Turku (Finland).

The Group was founded eight years ago and at present is formed by some hundreds of people from over ten European countries and from the United States.

The unusually high rhythm of two Meetings per years has been always kept, with the exception of one of the first years. This reveals the strong vitality of this community. The wide variety of papers presented and discussed, together with the originality of their approach and of the results, also witnesses the quality of the work the Group is doing in Finance.

There are more than one way to work in this fastly growing field. A largely diffused approach is mainly oriented in building theories to be cast within some general economic paradigm. If some simplifications are needed to get perfect theoretical coherence with the preferred paradigm, they are easily accepted. The most diffuse approach within the Group, although attentive to general theories, tries sometimes to build workable models where many relevant details of the reality are captured even if the price is not to adhere to some general theory. This does not mean, of course, that the Group is against general paradigms. This simply means that general paradigms are welcome as long as they are useful, but need not to be fully respected when the reality appears clearly in contrast with their assumptions.

In our mind this is a sort of *incipit* for a *Manifesto* of a European way to think of Finance. European banks, insurance companies and other financial intermediaries are expected in the next decades to bet more and more efficiently over two partially distinct gaming-tables: on the wide and efficient world financial markets but also on the non-necessarily wide, inefficient and thin local markets. For the first gaming-table the standard theory of Finance provides a robust guide to the decision maker. Too often the problems common on the second gaming table are (im)pudically hidden under the familiar categories of "imperfections", "irrationalities" and "noises" and the task to cope with them is blindly committed to the otherwise powerful use of some standard brownian motion. A European way to Finance should take seriously account of these simple points. A gap between standard Finance theory and common financial reality must be covered with a bridge. The bricks of the bridge are (at least partially) provided twice per year by the components of our Group.

The papers in this volume cover a wide range of topics. Thus, the papers deal with insurance and risk management, stock market behavior, taxation and market

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**SINGLE AND PERIODIC PREMIUMS FOR  
GUARANTEED EQUITY-LINKED LIFE INSURANCE  
UNDER INTEREST-RATE RISK:  
THE "LOGNORMAL + VASICEK" CASE <sup>(1)</sup>**

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**ABSTRACT**

Interest-rate risk, while significantly affecting the pricing of almost all life-insurance products, has been up to now disregarded in the analysis of equity-linked policies for which a minimum-amount-guaranteed provision operates. The purpose of the present paper is to build on the work of Brennan and Schwartz(1976,1979a,b) and Delbaen(1990) to show how uncertainty in interest rates influences both single and periodic premiums for equity-linked life insurance. To this end, we consider a model in which the unit price of the fund to which benefits are referred follows a lognormal process, while the spot rate of interest is described as in Vasicek(1977), and we employ the martingale approach to contingent-claims pricing introduced by Harrison and Kreps(1979) to obtain pricing formulae for guaranteed equity-linked policies that account for interest-rate risk. The paper includes a detailed comparative static analysis of our extended formulae, as well as some numerical examples.

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<sup>(\*)</sup> also at Department of Economics, University of Chicago, Chicago II, USA.



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# Financial Modelling

Recent Research

With 40 Figures

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# Expectations and News in an Imitative Stock-Market<sup>1</sup>

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## 1 Introduction

A basic starting point, in describing the literature about the causes of stock market prices movements, is the trivial evidence that the analysis is roughly split in two main parts: the theoretical and the empirical one, with clear problems of communication.

On the one hand, the bedrock belief of a host of academics and scholars, the efficient market hypothesis (EMH), assumes that a stock-market is (almost) always efficient and rational, even if a growing part of the same contemporary theoretical analysis is devoted to discuss the realism of this hypothesis. On the other hand, the literature counts on a bulk of empirical works on the irrationality and on the difficulties in finding the rationale of the behaviour of the agents.

The EMH is usually presented in three versions: the weak, the semistrong, the strong versions are based on the common assumption that it is impossible to beat the market, regularly, by using information, because the prices, immediately incorporate process at best all the available information.

The only differences between the three version concern the quantity and quality of information processed:

“a weak form which considers only information gathered from historical price movements; a semistrong form, in which prices reflect all information that is public; a strong form, in which both private and public information is supposed to be reflected in the market price.” See PHLIPS (1988) p. 244.

<sup>1</sup>The Authors are deeply indebted with an anonymous referee for useful remarks and suggestions that have improved this research. The responsibility of the authors for the remaining mistakes is full and exclusive.

The EMH hypothesis lays the emphasis on information, on signals, which can modify the expectations about the fundamental value of the stocks and on the aftermaths on their prices.

A common view within the EMH wing is centered around the assumed rationality of all the agents in processing signals and news. By contrast, at the other extreme, many analysts, economists, investment advisors, or journalists, who are expected to predict the financial market, stress on the psychological dimension in processing the available information, particularly during the market's crises and crashes, or, on the other side, in the manic phase. We think that it is worthwhile to overcome the shortcomings of the two wings, trying to integrate both empirical and theoretical results.

More in details, our approach rests on:

- (a) the rejection of the EMH's view of the stock market as a community of almost similar agents as concerns the access to information, and the capability to process it. Our contention is that agents are very different. Information is a commodity (see ARROW (1984), p. 138):

“I will think rather of information as a general descriptive term for an economically interesting category of goods which has not hitherto been accorded much attention by economic theorists.”.

Some agents can use far and wide all information available, other can only seize the signals represented by the investment behaviour of the smart-money investors. Imitation in the marketplace is the behavioural consequence of the information shortage, both of the availability of signals and of the capability to process them correctly;

- (b) the hypothesis of the immediate discount by the professional traders of any change in expectations of the price of the stocks, brought about by news;
- (c) the rejection of the too simple equations:
- (I) difficulties to understand the behaviour of the agents in the marketplace = irrationality, “madness” of the market
  - (II) psychological variables = irrationality
  - (III) irrationality = incomprehensible, inscrutable issue;
- (d) the interdisciplinary analysis, where the cognitive psychology, supported by empirical inquiries, has a role in finding some typical aspects of the cognitive behaviour of the agents in processing news and in modifying their expectations. We have recently had the possibility to find a substantial agreement between our approach and two non formalized monographies in psychological economics of the beginning of the Eighties: see MAITAL (1982) and SMITH (1981).

In light of our own beliefs, we are now approaching a core problem: the modelling of the impact of news on an imitative stock-market. Under some simplifying assumptions, a particular attention will be dedicated to the formalization of some fundamental psychological features of the issue.



The sketch of the paper is the following.

In sec. 2 we shall briefly remind the main points behind the framework of the imitative approach (FERRARI, LUCIANO AND PECCATI (1993)), with an equation describing the endogenous formation of the call epochs sequence, that we shall adopt as departure point. Sec. 3 is devoted to introduce the main hypotheses of the present model as concerns the arrival of signals to the agents. Sec. 4 recalls the main results from psychology concerning the way human beings react to the stimuli they receive. Sec. 5 offers an overview of a general model incorporating the signalling process introduced before. Owing to the fact that this general model appears to be untreatable with analytical tools, in sec. 6 we describe a special version and give account of the results of some numerical simulations we have made on it. Sec. 7 gathers the main conclusions stemming from the simulations.

Paper concerning partially related problems with reference to different market model are: KYLE (1989), where rational informed and uninformed agents interact, HALTIWANGER WALDMAN (1985), where the utility of the result for informed and uninformed agents turns down to depend (negatively) on the number of agents that choose similarly and irreversibly and the more recent VIVES (1992) where the speed of revelation of information is focused. In our framework,

1. not necessarily the agents act on the basis of strict standard rules of rationality, but follow schemes described in GOTA PECCATI (1993),
2. what is irreversible is the structure of behaviour but not the behaviour in itself
3. we emphasize the effect of a random process of information arrival instead of studying the revelation process.

## 2 The imitative approach

The general framework of the model is, in essence, the same of the paper FERRARI LUCIANO PECCATI (1993) (see also FERRARI PECCATI (1989), LUCIANO (1989), CORNAGLIA (1993), LUCIANO (1993), CORNAGLIA (1991), an GOTA PECCATI (1993)), characterized by institutionally different kinds of agents: smart investors (P.T.: "professional traders" in the model), and nonprofessional ones (N.P.T.), who imitate the P.T.'s in their investment decisions, and a neuter agent (S.: the "specialist" in the model), whose aim is to clear the market making a price on the stock.

In that model the only relevant piece of news for the agents was the initial price, inherited at the beginning of a trading day from the previous session. Starting from this initial price, the market mechanism determines a sequence of prices at which a sequence of transactions is made. All the P.T.'s observe these prices and assume their decisions. The information coming from the prices is processed through a comparison with the personal opinion about the fundamental value (f.v.) of the asset. The difference between the current price and f.v., assumed *constant during the trading day*, is one of the key variables for the order placing policy of the agents. We assumed no other signal during the negotiations.

Summarizing and more in details, in the previous model there are only three types of agents:

- (i) The Specialist (S.) who is obliged to make a price on the stock, his aim is to clear the market, not to make profits.
- (ii) The Professional Traders (P.T.); this kind of agent continuously negotiates with the specialist to make profits every time there is a difference between the current price of the security and his opinion on its fundamental value (f.v.). The professional trader pays constant attention to the information flow arriving to the market, he has a good capability to process all the news relevant for the market. Precisely he is able to transform heterogeneous signals into adequate stock prices revisions induced by the incoming news. Because of that he continuously changes his expectations on the dynamics of the fundamental value of the stock. This does not imply that his position is always that of the "winner"; we simply assume that these abilities are present in a P.T. with a larger extent than in the trades we shall shortly describe in the next item.
- (iii) The Non-Professional Traders (N.P.T.), who haven't the availability of information and the capability, by processing it, to change their expectations on the f.v.. The non-professional traders, aware of their weak position in the market, can only, rationally, try to imitate the professional investor's decisions to trade. Eventually the non-professional traders, far from the continuous negotiations and quotations of the market, enter the market, buying or selling, before the beginning of the trading day (t.d.), placing irrevocable orders.

The fundamental equations of the model are the following:

$$B_n - B_{n-1} = -\beta [p_{n-1} - h] \quad (1)$$

$$p_n = k + \eta [Q + B_{n-1}] \quad (2)$$

$$p_0 = p \quad (3)$$

$$q_m = -\alpha_m [p - r] + \gamma_m B^* \quad (m = 1, 2, 3, \dots, M) \quad (4)$$

$$Q = -A[p - r] + \Gamma B^* ; A = \sum_{m=1}^n \alpha_m ; \Gamma = \sum_{m=1}^n \gamma_m \quad (5)$$

In equation 1, describing the behaviour of the P.T.,  $B_n$  is the total amount of orders placed in the  $n$ -th epoch of the trading day,  $\beta$  is the positive reaction parameter to the difference between the last price, in the epoch  $t_{n-1}$ , made by the S. and  $h$ , namely the f.v. in the opinion of the P.T. The total amount of transactions by the P.T. in the t.d. is:

$$B^* \stackrel{d}{=} \lim_{n \rightarrow +\infty} B_n \text{ (if this limit exists).}$$

In equation (2) we formalize the behaviour of the S., where  $\eta$  is a positive parameter and  $k$  represents the value of the F.V. in the opinion of this agent. In equation (2) there is also the variable  $Q$ , which can be described (in equation (5)) as the summation of the  $q_m$  from equation 4.

In our hypotheses there are  $M$  N.P.T., labelled with  $m = 1, 2, 3, \dots, M$ , each of them enters the market by an order whose value,  $q_m$ , on one hand, is function result of the difference between the f.v., in the opinion of this agent, and the listed price



of the previous t.d. and on the other hand is a fraction of the total transactions of the P.T. in the last trading day. The parameters  $\alpha$ ,  $\gamma$  and  $\Gamma$  are positive.

The starting price  $p_0$  of the t. d. is the listed price of the previous negotiations. The equilibrium values of  $B_n$  and  $p_n$  are:

$$B^* = \frac{h - k}{\eta} - Q \quad (6)$$

$$p^* = h \quad (7)$$

The equilibrium is stable iff (FERRARI LUCIANO PECCATI (1993))  $|1 - \eta\beta| < 1$ .

Some results, obtained in this oversimplified framework, deserve interest, but this approach can be sophisticated to get: a large part of the empirical evidence suggests that trading days without impact on the negotiations of relevant news are rare.

Because of this, our concern, in the present paper, is with the effect of the arrival of news in an imitative stock-market. Our aim is to elaborate the framework of a more general model, which comprises the previous, no-signals one, as a special case. For the sake of the simplicity in this model (and in the previous one) only the dynamics of a single stock is analysed. In the previous model of an imitative stock-market we drew attention both on the short-run equilibrium price (the listed price for a single trading day), and on the long-run price. In this model the long-run equilibrium will remain out of view.

### 3 The basic hypotheses of the model

We define a "signal" as the unit of information available by the P.T. The representative agent changes his expectations on the dynamics of the f.v. according to the signals, as soon as possible. We assume that there is no setback in processing signals. In other words, the financial market must be seen as a place where an incessant stream of news changes continuously expectations for returns and capital gains.

Signals and, generally speaking, information differ under many aspects. The most important is the tremendous qualitative heterogeneity, signals can concern all kind of events. In the first place data regularly supplied on macroeconomic variables should be considered. See LOISTL LANDES at p. 136. They also write at p. 157:

... the variation of a company's quotations is explained 45 to 75 per cent by the data of the annual report of that company.

Other relevant signals come from one-off events like wars, riots, political shocks, tax cuts, etc., both, chains of different, but related events, or other single very different and idiosyncratic contingencies.

In the model we overlook all these qualitative aspects, in fact we assume the capability of the agents for integrating all these aspects of information into their judgement. Let  $h$  be the f.v. at the arrival of a signal. The agent processes it and updates his opinion about the f.v. from  $h$  to  $h + \Delta h$ .

In the model the only relevant aspects of signals are:

- (a) the *sign* of each information, namely if the signal implies a positive or negative  $\Delta h$ ;
- (b) the *intensity* or the market impact of news, namely their effect on  $|\Delta h|$ ;
- (c) the effect of the whole set of signals on the *expectation* of a *trend*;
- (d) related to the previous point, the *timing* of signals, namely the reciprocal position in time of the news.

We assume that the third and the fourth points are important for the psychological climate of the market, particularly for understanding phenomena as the investor's jitters, panic, depressed mood, mania, etc.

More in details, in the present model, only the P.T. has access to the news, relevant for the market. Another line of attack can be discerned: only the P.T., mainly in comparison with the N.P.T.'s, has the scientific tools for integrating information into a judgement on f.v.. In our model, these views do coincide.

Similar problems have been studied recently in CENCI CERQUETTI (1991) and (1992). These interesting results are obtained with the assumption of an exogenous deterministic process of evolution for the f.v.. With this paper we try to study the case of a random process influencing this evolution.

During the daily stock-market negotiations, the P.T. changes his opinion about the f.v. ( $h + \Delta h$ ) according to the variations of his expectations, brought about by signals. A sequence of calls is made at times  $t_1, t_2, \dots, t_n, \dots$  and in the occasion of each call  $t_n$  the S. fixes the market price  $p_n$ . The difference between the last equilibrium price ( $p_{n-1}$ ) and the updated opinion about the f.v. presses the P.T. to trade.

During the trading day there can be:  $s = 0, 1, 2, 3, 4, \dots$  signals, different in their sign and in their intensity. They arrive at epochs  $z_1, z_2, \dots, z_s, \dots$ . We can also assume that  $s$  has a finite maximum value. In fact there can be little if any room for doubting that the informative channels have a maximum capacity, and the capability to process the information is bounded. Anyway the possibility of an infinity of signals could be accepted in some cases to get approximations easy to handle.

We assume that the number of signals per trading day is a random variable. We assume also that the arrival time of *each* signal, during the fixed time span of the t.d., is a random variable. Each signal, as previously stressed, generates an expectation and consequently a  $\Delta h$ , different in sign and intensity. We assume that the  $\Delta h$ 's are independent random variables. Obviously the values  $\Delta h < -h$  are absurd. To avoid this setback and to avoid to change the normal probability distribution in another distribution, difficult to handle, we assume a sufficiently small variance.

Many types of distributions appear to be interesting. This point is touched tentatively in the conclusions section.

In the numerical simulations we made normality for  $\Delta h$  was assumed, although densities like the one portrayed in the figure 1 appears to be interesting to grasp the non-neutral randomness of the sequence of signals during — say — the downswing phase of the economic cycle, when values  $\Delta h < 0$  are more probable.



#### 4 Further hypotheses on signals

It is well known that, at present, there is no specific psychological study on the impact of news in a marketplace, except for the phenomena of the "rumor". Nevertheless it seems worthy and reasonable to make some informed guesses. We think that it is important, at this stage of evolution of the theory, to suggest some new hypotheses also as a starting point for further empirical researches.

It is important to outline that signals cannot be interpreted as "atomic" stimuli without any mutual interference. It is intuitive that there can be a difference in reacting to the same (sign and intensity) stimuli between the case of separate and far in time signals, that appear to be disposed in a neuter and well balanced sequence, and the case where the same stimuli are disposed in a non-neuter sequence. For instance, at the beginning of the negotiations two negative signals, very close each other, can strengthen a pessimistic mood in the formation of the following expectations.

It is necessary to keep in mind that the quality and the quantity of information, in comparison with the role it plays in the financial markets, are always scarce. Because of this fact, the agents, under the continuous stress represented by their role of decision makers, in very short lapses of time, try always to foresee beyond the simple "atomic" events. The "hunger" of news in financial markets is particularly glaring in rumor phenomena. We cite from KOENIG (1985), p. 157:

What does play a big part on Wall Street is rumor as a substitute for news. In a population where information is essential what is not available is often invented. Because information is critical everyone is operating with a high-power antenna. In this tense atmosphere the population is sensitized to a bit of news here, a piece of report there. People under stress need closure, order, a feeling of completion; they need to put pieces together so that they can make a sense of the whole.

In other words, what is called the mood of the market can be interpreted as a sort of mutual reinforcement of the signals, maybe different in quality and intensity, but homogeneous in the sign. We think that this attitude about the sequences of news affecting the expectations and the same processing of the further signals, far beyond the content of each event, can be captured and formalized in some important aspects. As a basis of every further analysis on this issue, may be worth to lay emphasis on two remarkable achievements of the cognitive psychology:

- (a) the empirical and experimental evidence of a judgement mechanism, consisting in the spontaneous comparison of the new cognitive stimuli with an adaptation level, a sort of "anchor" cognitive content, resulting from the average of the intensities of the previous similar stimuli. See HELSON (1947) and (1964).

This author analysed the phenomena consisting in the difference of sensations brought about by the same cognitive stimuli in different conditions. Helson observed in his experiments a spontaneous comparison of the new stimuli with an adaptation level, namely a sort of average of previous similar stimuli, changing with the accruing of similar experiences. This "anchor" estimate represents a sort of neuter point, like the zero point in the centigrade scale.

For instance if an individual, who must value the height of some other persons, has a continuous and durable experience, prior the valuation, of short persons (for instance, 1.65 meters in average height) will value a 1.70 meter person as tall. To the contrary, he will value the same person short, if he had an experience, of long duration, of taller persons.

See also EISER (1980).

- (b) the empirical and experimental evidence of a misconception of chance, not limited to naive subjects, according to which (KAHNEMAN TVERSKY (1982))

"People expect a sequence of events generated by a random process will represent the essential characteristics of that process even when the sequence is short".

Kahneman and Tversky immediately further write:

In considering tosses of a coin for heads or tails, for example, people regard the sequence H-T-H-T-T-H to be more likely than the sequence H-H-H-T-T-T, which does not appear random, and also more likely than the sequence H-H-H-H-T-H, which does not represent the fairness of the coin. Thus, people expect that the essential characteristics of the process will be represented not only globally in the entire sequence, but also locally in each of its parts.

and also (at p. 7):

Another consequence of the belief in local representativeness is the well-known gambler's fallacy. After observing a long run of red on the roulette wheel, for example, most people erroneously believe that black is now due, presumably because the occurrence of black will result in a more representative sequence than the occurrence of an additional red.

Thus it is reasonable, assuming that agents (P.T.), after a sequence, dense of signals, have the spontaneous tendency

- (a) to interpret the sequence of signals, consistent in sign, as nonrandom, even if it is very short and actually random.
- (b) to evaluate cognitively (and emotionally) all the new signals on the basis of the previous, measuring the new  $\Delta h_{j+1}$  under the influence of the last  $\Delta h_j$ 's ( $j = 1, 2, \dots, n$ ), as a sort of "anchor" value. An investor, pressed by time and compelled to decide, somewhat intuitively could find a "trend", namely the expectation of future events of the same sign, even if the events are merely random. As concerns the emotional attitude, this "trend" can be seen as the pessimistic/optimistic mood. Eventually, we draw the reader's attention on the enormous advantage that the early finding of an actual trend can offer versus the simple processing of each single signal. A large evidence suggests that this mechanism is in a tight relationship with time. The less the time



to process the information and to decide how to trade, the less the capability to react in a non-emotional, non-spontaneous way. The less the time between signals, the more perceivable are the relationship between them.

The formalization of this functional relationship with time can be drawn from a lot of experimental studies in related topics (See BORING (1950) and HILGARD BOWER (1987)).

## 5 The general theoretic model

It is time to formalize all the focal points on the variations of the f.v. and their effect on the negotiations of the stock, after the occurrence of a signal. We shall stress, mainly, on the modifications, brought about by the news, on the f.v. of the P.T.. The most general equation, on the P.T.'s behaviour, (the key equation of the previous and present model) is now the following:

$$B_n - B_{n-1} = b[p_{n-1}, G(t_n)]. \quad (8)$$

We assume  $B_n$  as the total amount of orders placed within the  $n$ -th epoch of the trading day,  $b: \mathbb{R}^2 \rightarrow \mathbb{R}$  as an appropriate function and  $G(t_n)$  as a stochastic process, whose realizations give the time evolution of the values.

Needless to say that equation 8 has its initial conditions. A general set of models, consistent with this equation, can be explicitly dealt with by considering the generic process  $G(t_n)$  in the following way:

1. by defining the signal arrival's instant  $z_j$  as a random variable (the number of signals per day is also a random variable);
2. by associating, given the instants  $z_j$ , the random values  $\Delta h_j$ 's.

The value of  $G(t_n)$  at  $t_n$  will be:  $G(t_n) = H_j$ ; where  $\hat{j} = \max\{j | z_j < t_n\}$ .

$H_j$  is recursively defined by the equation:

$$H_j = L(H_{j-1}, \Delta h_j, z_j - z_{j-1}) \quad (9)$$

If in the time lapse  $(t_{n-1}, t_n)$  don't fall  $z_j$  values, it follows that:  $G(t_n) = G(t_{n-1})$ .

The realizations of the stochastic process  $G(t_n)$  are piecewise constant with "jumps" at the epochs  $z_j$ . The amount of these variations is brought about by the  $\Delta h_j$  with a correction, which is a function of the length of the time lapse between two consecutive "jumps". The time instants  $t_n$  are functionally related by the equation:

$$t_n = \Phi(\tau - t_{n-1}, z_j, \Delta h_j) \quad (10)$$

where  $\tau$  denotes the t.d. length.

Moreover we (obviously) assume that:  $\lim_{n \rightarrow +\infty} t_n = \tau$ .

Summarizing, equation 1 can be written as:

$$B_n - B_{n-1} = b[p_{n-1}, \Delta h_j] \quad (11)$$

where  $\hat{j} = \max\{j | z_j < t\}$  and the  $z_j$ 's are the epochs of the random occurrence of the signals (also their amount is a random variable) during the trading day. Focusing on the term  $H_j$  (see the equation 9), the function  $L$  is assumed monotone, decreasing and convex with respect to its last argument.

## 6 Special cases and numerical simulations

In the present paper our concern is with a some special cases. Typically they result from specializing and itemizing the general model, portrayed by equation 8.

The framework of these models follows:

$$B_n - B_{n-1} = -\beta [p_{n-1} - (h + H_j)]; \beta > 0 \quad (12)$$

$$H_j = \Delta h_j + H_{j-1} \phi(z_j - z_{j-1}) \quad (13)$$

with  $\phi$  monotone, decreasing, convex and such that  $\phi(0) = 1$ .

$$H_0 = 0, \Delta h_0 = 0 \quad (14)$$

$$\Delta h \sim N(0, \sigma^2) \quad (15)$$

$$p_n = k + \eta [Q + B_n]; k > 0; h \geq 0; Q \geq 0 \quad (16)$$

$$q_m = -\alpha_m (p - r) + \gamma_m B^*; m = 1, 2, 3, \dots, M \quad (17)$$

$$Q = -A(p - r) + \Gamma B^*; A = \sum_{m=1}^M \alpha_m; \Gamma = \sum_{m=1}^M \gamma_m \quad (18)$$

$$p_0 = p \quad (19)$$

$$t_n = t_{n-1} + \left[ \frac{1}{1 + D_{n-1}} \right] (\tau - t_{n-1}) \quad (20)$$

$$D_{n-1} = \theta \cdot |p_n - p_{n-1}|; \theta > 0 \quad (21)$$

Equation 16 describes the behaviour of the S.. For more details and for the meaning of the constants  $Q$  and  $q$  (equations: 17, 18) see the appendix.

The initial condition  $p_0 = p$  equates the starting price of the trading day to the listed price of the previous trading day.

The equations 20 and 21 determine the reaction time of agents after each call and, therefore, the distance between one call and the ensuing one. A general analysis of this topic is available in FERRARI LUCIANO PECCATI (1993).

On the other side of the model, there can be two alternative lines of attack to the  $z_j$ 's:

- (a) given the maximum  $s$ , i.e., the maximum finite number of signals per trading day and given the random value  $s$  for the specific trading day,  $s$  numbers  $z_j$  ( $1 \leq j \leq s$ ) are drawn in independent trials, as independent random variables with probability density function (p.d.f.)  $f(z_j) = 1/\tau$  ( $0 \leq z_j \leq \tau$ );
- (b) the lengths of the time spans  $z_j - z_{j-1}$  are assumed to be independent random variables with the same exponential p.d.f.

An important result is common for all the realizations of  $G(t_n)$  in the subset of models, previously outlined:  $G(t_n)$  can be interpreted as a sum of at random number of dependent random variables:

$$G(t_n) = h + \Delta h_j + \sum_{j=1}^{\hat{j}} \Delta h_j \prod_{i=j-1}^{\hat{j}} \phi(z_i - z_{i-1}); \hat{j} = \max j | z_j < t_n \quad (22)$$



An important step forward, up to this point, seems to show the frequencies of the different final equilibrium values of  $p_n$ , by numerical simulation. Our goal is to enquire on the qualitative shape of the p.d.f. of the equilibrium value for  $p_n$ .

The simulation actually was performed by software tools which simulated, several thousand times, the stochastic path of all the variables of the model (12) — (21), by choosing the option (a) for the  $z_j$ 's and with appropriate initial conditions.

In particular, in figure 2.1 are graphed the different percentages of the equilibrium prices, given the following initial conditions and parameters:

$p_0 = 50$
$B_0 = 0$
$0 \leq s \leq 5$
$k = 50$
$h = 50$
$Q = 20$
$\sigma^2 = 1$
$\beta = .3$
$\eta = .4$
$\theta = 1.5$
total number of simulations = 80607

In figure 2.2 are graphed the results of the same kind of numerical simulation, with only one different parameter:

$$1 \leq s \leq 2$$

and with total number of simulations = 12794.

Caution must be exercised in the interpretation of the results, but it seems plausible that the p.d.f. of the equilibrium price appears to be nearly normal with  $\sigma^2$  directly functionally related to the maximum capacity of the information channels.

Another exploration we have made concerns the way the prices at subsequent epochs are related. We have computed the standard autocorrelation coefficients between prices with lags of 1,2,3,4,...,10 periods. We got declining values starting from approx. .9 for the lag 1 till to approx. .38 in the case of 10 periods lag. This can give a rough idea about the effect on prices paths of the random flow of information.

## 7 Conclusions and further research

The purpose of this paper is to analyze the listed price for a single trading day, under the random pressure and influence of news. Commenting on this set of highly simplified models, in our opinion, the most important conclusions are four.

1. The erratic fluctuations of the day to day movements in prices of the stocks, similar, in the opinion of many scholars, to the Brownian motion, can be interpreted as the partial result of the stream of all the news into the marketplace. Caution must be exercised, but this approach provides an interesting and even fascinating window on further empirical and theoretical inquiries

about the fundamental characteristics of the mechanisms of using information in the marketplace and on their influence on prices, until now poorly known.

2. The approach, based on signals, supplies some interesting improvements with respect to the previous formalizations of imitation in the stock market. In the original models the N.P.I.'s clearly entered the market always "irrationally", because only the P.T. could make profits. Indeed the f.v. of the N.P.I.'s was irrelevant in the equilibrium price determination, except in the extreme situation of equality with the f.v. of the P.T. In the present model, instead, the equilibrium price is a random variable, thus neither the P.T., nor the N.P.T. can a priori beat the market in the medium term (few trading days).
3. In the present model the closeness between the stock market and the economic cycle can be, partially, formalized by different shapes of the normal p.d.f. of the random variable  $\Delta h$ , as previously outlined.
4. One place where the model surely applies is the microanalysis of the behaviour of agents during negotiations. Undoubtedly it seems worthwhile an empirical verification of some interesting results of the models:
  - (a) Commenting on these results, an important point is that, in the sequences the first  $\Delta h$ 's have, by far, the most important impact on the final variation of the F. V. and on the equilibrium price. The more the signals, the more important are the first news in the trading day. The first signals influence the mood of the further negotiations, unless the following signals are, by far, outstanding in their effect on  $\Delta h$ 's.
  - (b) If the first signals have an high intensity, they have an interesting asymmetric effect: if the last news have the same sign, then they brought about volatility of prices; on the contrary if the last signals have a different sign then prices are sticky in their moving to the previous equilibrium price. There is an interesting debate on the volatility of the stock market. Current debate often falls on the measures of the variance of speculative stock prices, against simple models of market efficiency. Some alternative models have been provided, involving, for example, the economic culture of people as a key variable to allow, in models, more variation in prices, see SHILLER (1990). Underlying this position there is the opinion that the signals processed during negotiations are exactly those necessary to forecast dividends of the next epochs, processed by perfectly rational agents, without distortions. Our contention is that, in light of these beliefs, the a posteriori measures of volatility confute the "naive" view of the role of information, not the possibility to refer the volatility to the random stream of information. In our opinion it is important to overcome the too simple opinion about the mechanisms of processing information, we mean the "atomic" view of signals. A more realistic view of the role and mechanisms of processing news in the market can overcome the interpretation of the measures of volatility "as implying that prices show too much variation to be explained in terms of the random arrival of new information about the fundamental determinants of price" (SHILLER (1990), p. 131)



- (c) In general, the more the signals in the trading day the less probable the values of the equilibrium price close to  $h$ .

Under the simple hypothesis that the signals have non-positive expected values it is easy to prove that  $G(t_n)$  is a supermartingale. Further research is needed to try the application of limit theorems for (super)martingales. See, for instance HALL HEYDE (1980).

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## APPENDIX

We consider the process generating the sequence of the call epochs  $\{t_n\}$ . We specialize slightly equation(3) by dropping the direct dependence of the inter-call time lapse on the signal process.

We assume that two consecutive call epochs  $t_n$  and  $t_{n+1}$  are related by a difference equation of the type:

$$t_{n+1} - t_n = \psi(\delta_n, \tau - t_n) \quad (23)$$

where  $\delta_n = |p_n - p_{n-1}|$ . This equation states that:

**A. 1** the generic time span between calls  $h$  depends only on the (absolute) value of the two last stock prices and on the residual time of the market session;

or, in symbols, and omitting the indices:

$$h = \psi(\delta, \tau - t)$$

A further assumption concerns the type of dependence of  $h$  on the most recent price variation:

**A. 2**  $h'_\delta/h$  depends only on  $\delta$ , and it is the same whatever the residual time of the session  $\tau - t$ .

This assumption allows us to write:

$$h = \psi_1(\delta)\psi_2(\tau - t)$$

Qualitative properties of  $\psi_1$  that appear to be acceptable are:

**A. 3**  $\psi_1$  is positive, decreasing, convex and equal to 1 in the origin

In our simulation model we adopted the specification:

$$\psi_1(\delta) = \frac{1}{1 + \delta}$$

Now we turn to the dependence on  $\tau - t$ . A reasonable assumption could be:

**A. 4** the relative variation of  $h$  is inversely proportional to the residual time:

$$h'_t \propto \frac{h}{\tau - t}$$

this implies  $\psi_2(\tau - t) = (\tau - t)^m$  or:

$$h = \psi_1(\delta)(\tau - t)^m \quad (24)$$

with  $m > 0$ .

Another reasonable requirement is that

**A. 5** if the price is in equilibrium then no more transaction is made:

$$\delta = 0 \Rightarrow h = \tau - t \quad (25)$$

In this case  $\psi_1(0) = 1$  and consequently from (24) we get  $h = (\tau - t)^m$ . This is compatible with 25 iff  $m = 1$  or:

$$h = \psi_1(\delta)(\tau - t)$$

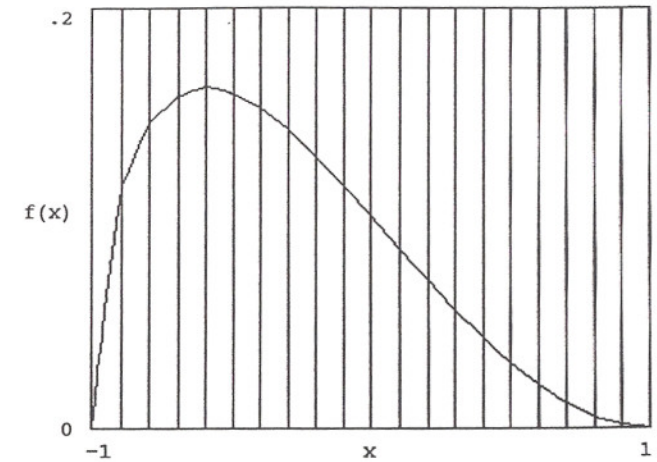


Figure 1



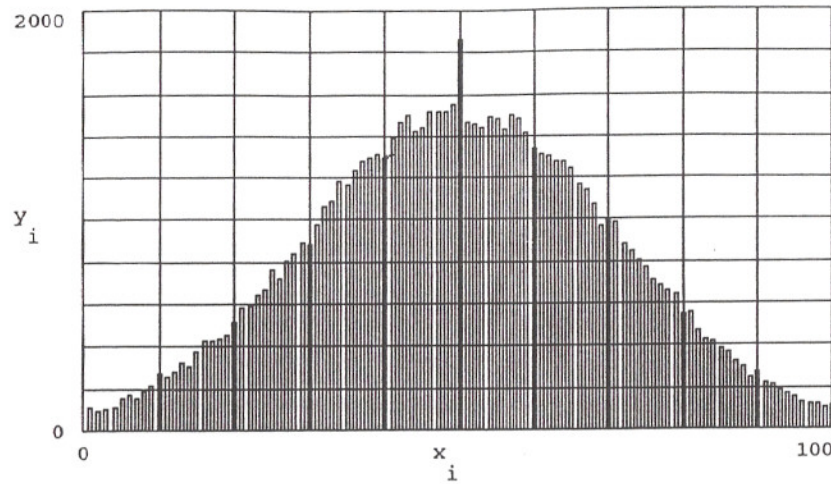


Figure 2.1

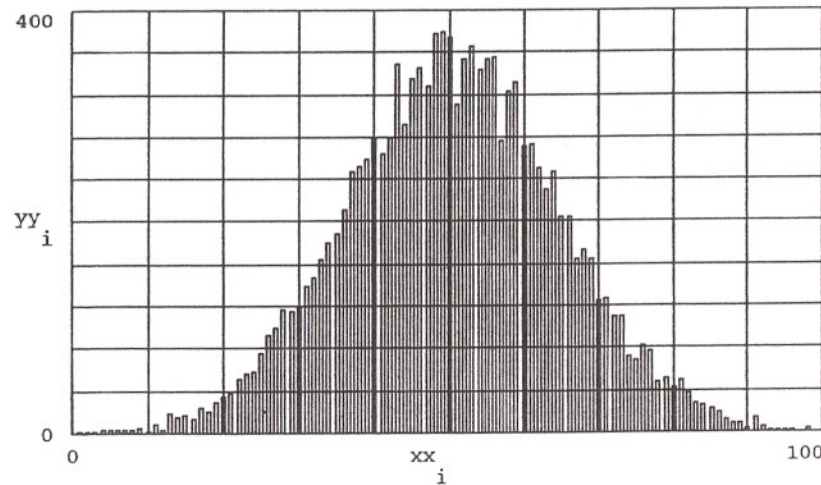


Figure 2.2

## An Artificial Adaptive Speculative Stock Market

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The model presented in this paper considers the interaction among traders acting in a financial market. Each agent bases her evaluation of the asset on an (agent-specific) interpretation of an (partially agent-specific) information set. The evaluation process of an agent is simulated by a neural network, which can be interpreted as a nonlinear way to process the available information. It is shown in the paper that, given our assumptions on the structure of the market, each agent learns from her experience and updates her "model of the world" according to the distance between the actual average opinion of the market and her forecast of the same variable. We study the long-run distribution of the behavior of the market, and show the importance that history may have in determining the long-run equilibrium. We also study the performances of various agents in terms of wealth, and show the importance of learning ability. The market shows long periods of calm followed by sudden burst of volatility, when the agents revise their average opinion.

## 1. Introduction

This paper builds upon the consideration that the availability of powerful computers coupled with the discovery of new computational models and methods (some of which, like neural networks, will be described later on) now allows researchers to explicitly use models based on interactions among heterogeneous agents. This is particularly useful since often technical problems force economists to adopt the representative agent fiction to study situations which, in practice, can only be understood by considering many interacting agents. For example the debate about the relationship between stock prices and the present value of fundamentals originated by the work of Shiller (1981) may be interpreted as an attempt to understand financial markets on the basis of the valuation given by a single representative agents.

The strong assumptions shared by representative agent models contrast deeply with our