The fund-flow approach. A critical survey*

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Abstract

The fund-flow approach to production theory was proposed by Nicholas Georgescu-Roegen almost half a century ago and it provides a description of production as a process unfolding in time and entailing a temporal coordination between different elements. Despite the initial interest, the approach has seldom received attention by mainstream economists and it is not yet well known. The paper critically surveys Georgescu-Roegen's model, together with: i) the later developments and modifications; ii) the recent criticisms focused on its limitations due to the instrumental assumption of constant efficiency for funds. The paper concludes by discussing the limits and comparative merits and suggesting possible applications.

Keywords: Fund-flow model; Georgescu-Roegen; Production theory; Returns to scale; Technical coefficients.

JEL Classification: B29; B59; C67; D24; L23; O33.

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1 Introduction

The fund-flow model of production was developed by Georgescu-Roegen (1969, 1970, 1971); it is based on the distinction between *funds* – "the *agents* of a process" of production – and *flows* – the elements "which are *used* or *acted upon* by the agents" (1971, p.230) – and treats production as a process unfolding in historical time.

The model has proved well-suited to analyze the actual organization of production, which requires temporal coordination and interaction between its elements, and it has been taken up by some authors who have extended or partially modified the original framework to make it more operational. In particular, Tani (1986) has provided a finer analytical description of the conditions for the *line production*. Morroni (1992) and Piacentini (1995) have developed synthetic representations of the production process consistent with the fund-flow framework. Piacentini (1995) has analyzed *time-explicit cost functions*.

The approach has also recently attracted a number of criticisms, because of its inadequate treatment of the problem of capital utilization (Kurz and Salvadori, 2003) and the lack of operational conclusions (Lager, 2000).

In the paper, I first review Georgescu-Roegen's original model with the analytical refinements by Tani (1988) and Mir-Artigues and González-Calvet (2007) (Section 2). Then, I critically survey the later developments and extensions (Section 3). Finally, I discuss the criticisms, together with the pros and cons of the approach (Section 4). Section 5 concludes with some final remarks and suggesting possible applications.

2 Standard framework

The fund-flow model was presented by Nicholas Georgescu-Roegen in the mid-sixties at the *Conference of the International Economic Association* (Rome, 1965), and originally aimed at illustrating the harm caused by the "blind symbolism" of the mainstream theory of production.[1]

Since then, it has appeared in some of his subsequent works with only minor modifications (Georgescu-Roegen, 1969, 1970, 1971, 1990).

2.1 Process as Change and differences among funds, flows and stocks

According to Georgescu-Roegen, the fund-flow model is an *analytical-descriptive* method to study the process of production. Indeed, one should first realize that "process is a particularly baffling concept, for process is Change or is

[1] Georgescu-Roegen's contributions to economic analysis are not limited to the fund-flow model. For an account of his life and contributions see, for instance, Maneschi and Zamagni (1997).

nothing at all". As such, a process can never be defined, but only analytically delimited. "No analytical boundary, no analytical process" (1971, p.211). These boundaries must be both *spatial* – delimiting the frontier of the process – and *temporal* – determining its duration.

By taking such boundaries as a datum, one can record at each instant in time the elements that cross them, entering or leaving the process. Georgescu-Roegen distinguishes them in:

- *funds*: elements that enter *and* leave the process, providing certain services over a certain period. They are never physically incorporated in the product. E.g. workers, land, and capital equipment in the production process of shoes.[2]

- *flows*: elements that either enter *or* leave the process, but not both, i.e. "elements which appear only as input or only as output" (Georgescu-Roegen, 1970, p.4). Some flows enter the process and are then "incorporated" in the product (e.g. energy and leather in shoe production); whereas some others only leave it (the shoes and waste generated by the production activity).

A fund is not a stock: while a stock can be accumulated or decumulated in one single instant, the use of a fund, i.e. its decumulation, requires time. To give an example, a bag of twenty candies is a stock: you can make twenty children happy today, tomorrow or make one children happy for twenty days. An electric bulb lasting one thousand hours is a fund: you cannot use it to light one thousand rooms for an hour at the same time.

While all stocks accumulate or decumulate in a flow, not all flows imply an increase or reduction in a stock (e.g. electricity). Georgescu-Roegen strongly criticizes the distinction between stocks and flows as commonly meant in economics and crystallized in the so-called Fisher's (1896) dictum: "stock relates to a *point* in time, flow to a *stretch* of time". According to him, the mistake implied by this definition was the consequence of the "original sin" of mainstream economics: the adoption of a mechanicist perspective, where "Change consists of locomotion and nothing else". For him, a better definition of flow is "a stock spread out over an interval of time" (Georgescu-Roegen, 1971, p.223), where the stock is the "*quantum* of substance".

As for the difference between flows and fund services, Georgescu-Roegen stresses that no confusion can arise since the latter are expressed in terms of substance $\times$ time, whereas the former in terms of substance/time.

[2]The classification is process-related: a fund in one process may well be a flow in another. Georgescu-Roegen considers a fund also the bulk of semi-processed goods (*process-fund*). The inclusion has been criticized by Landesmann and Scazzieri (1996b) for the merely passive role of these goods in the process, but, as noted by Mir-Artigues and González-Calvet (2007), their exclusion might create problems of internal consistency in the representation of line productions. On this point see also Section 3.1.

2.2 Analytical representation of the production process

Having identified the spatial and temporal boundaries of the process, one can analytically describe production by referring to the temporal patterns of entrance and exit of the "substances" crossing these boundaries. In this representation processes develop over time, and the punchline of the approach is the explicit consideration of the temporal dimension of production.[3]

In particular, given a process temporally delimited from 0 to T ($t \in [0, T]$), by denoting with $I_k(t)$ ($O_k(t)$) a function of time expressing the cumulative amount of the element k that has entered (left) the process from 0 to t, the process can be analytically represented by the following vector of functions:

$$\left(-I_1(t)_0^T, \ldots, -I_K(t)_0^T;\ O_1(t)_0^T, \ldots, O_K(t)_0^T\right) \tag{1}$$

A more compact representation is:

$$\left(F_1(t)_0^T, \ldots, F_K(t)_0^T\right) \tag{2}$$

where:

$$F_k(t) = O_k(t) - I_k(t). \tag{3}$$

The flow elements are identically represented by Eq. (1) and (2), since the cumulative output (input) functions of all the inflows (outflows) elements are identically nihil and therefore redundant. As for funds instead, the value of $F_k(t)$ (≤ 0) returns the degree of operation in the process of the fund k.[4] To emphasize such difference, for funds Eq. (3) can be denoted with $U_k(t)$ ($\equiv F_k(t)$), and Eq. (2) rewritten as:

$$\left(F_1(t)_0^T, \ldots, F_M(t)_0^T,\ U_1(t)_0^T, \ldots, U_J(t)_0^T\right) \equiv \left(\mathbf{F}(t)_0^T, \mathbf{U}(t)_0^T\right) \tag{4}$$

where the first M elements are flows and the others funds ($M + J = K$).

In order to maintain a symmetry with the flow coordinates, for funds one might also use the cumulative amount of their services:[5]

$$S_j(t) = \int_0^t U_j(\tau)\ d\tau \tag{5}$$

and represent the production process as:

$$\left(F_1(t)_0^T, \ldots, F_M(t)_0^T,\ S_1(t)_0^T, \ldots, S_J(t)_0^T\right) \equiv \left(\mathbf{F}(t)_0^T, \mathbf{S}(t)_0^T\right). \tag{6}$$

[3] An attempt to model time-specific analysis within a neoclassical framework is Winston (1982). An explicit consideration of the time profile of in- and out-flows can be found also in Frisch's (1964) *phase diagrams*, although such diagrammatic tools do not enter in his core analytical framework.

[4] If $F_k(t) = 0$ at time t the fund k is not in operation. A negative value indicates that the fund is actively involved in the process.

[5] Given that $U_j(t)$ is only piecewise continuous, the integral should be defined piece by piece.

Eq. (1), (2), (4) and (6) are all alternative analytical representations.

Assuming that the elements are ordered in such a way that the first element is the outflow of the output of interest (e.g. the outflow of shoes in the production process of shoes), the "catalogue of all *feasible* and *not-wasteful* recipes" (Georgescu-Roegen, 1971, p.236) can be represented by the following functional, i.e. a relation from a set of functions to a function:

$$\begin{aligned} Q(t)_0^T &= \Psi\left[F_2(t)_0^T, \dots, F_M(t)_0^T,\ U_1(t)_0^T, \dots, U_J(t)_0^T\right] \equiv \\ &= \Psi\left[\mathbf{F}_{-1}(t)_0^T, \mathbf{U}(t)_0^T\right] \end{aligned} \tag{7}$$

where $Q(t) \equiv F_1(t)$.

2.3 Elementary process and production systems

If one defines the *elementary process*, as "the process by which every unit of the product – a single piece of furniture or a molecule of gasoline – is produced" (Georgescu-Roegen, 1971, p.5), i.e. a process such that $Q(t) = 0$ for each $t \in [0, T)$ and $Q(T) = 1$, she soon realizes that most of the involved funds remain idle or underutilized during a great part of the process.

Georgescu-Roegen identifies three possible, not mutually exclusive, temporal arrangements of the elementary processes: (i) *in series* – or *in sequence* (e.g. Mir-Artigues and González-Calvet, 2007) or *in succession* (e.g. Piacentini, 1995): elementary processes activated one after the other with no overlap in time; (ii) *in parallel*: elementary processes carried out simultaneously, i.e. started at the same time and repeated once completed; (iii) *in line*: elementary processes activated one after the other with some predetermined lag δ ($\leq T$), so that they only partially overlap.[6]

Given that each elementary process releases one unit of output every T units of time, with elementary processes activated in series the *scale of the process*, i.e. the amount of output per unit of time, is $1/T$.

In the arrangement in series there are two possible sources of inefficiencies. First, when indivisibilities exist for funds, some funds may be underutilized. So, for instance, if your oven can accommodate 100 biscuits and you employ it for one biscuit only, you are actually using 1/100 of its capacity. Second, if the effective use of funds within the elementary process is not continuous, some funds may remain idle for some time. In the previous example, if the production process of biscuits lasts one hour and you use the oven only to cook them, let's say for 20 minutes, the oven is actually idle for 40 minutes.

An arrangement in parallel can remove the first source of inefficiency. More precisely, let κ_j^* be the maximum number of elementary processes that

[6] Some authors (e.g. Tani, 1988; Mir-Artigues and González-Calvet, 2007) also distinguish a *conjoined activation* (or *functional processes* or *job shop processes*), in which, because funds are characterized by a certain degree of versatility and *different* elementary processes have some tasks in common, the funds jump between the stages of the different processes.

a unit of a j-type fund can simultaneously process, in order to remove any excess capacity the number of elementary processes simultaneously activated must be equal to the *least common multiple* ($\bar{n}$) of the κ_j^*s for all the funds involved in the process. The *size of the process*, i.e. the number of elementary processes simultaneously activated, must be therefore equal to $\bar{n}$ or a multiple of it, with a minimum number of j-type funds employed equal to $\bar{n}/\kappa_j^*$.

While parallel production can actually deal with the first kind of inefficiency, it cannot address the second: the possible existence of periods of idleness for the funds. To cope with it, one needs an arrangement in line. More precisely, given an elementary process involving J different types of funds, with $d_{j1}, d_{j2}, \dots, d_{jh} \in [0, T]$ be the durations of the intervals of time in which the j-type fund is effectively used in the process, in order to completely remove the idleness of all the funds, one must activate T/δ elementary processes starting at *cycle time* intervals δ, that is the *greatest common measure* (or *divisor* in case of integers) of the d_{ji}s.[7] To implement such line production one needs $\theta_j = \sum_i d_{ji}/\delta$ units of j. Figure 1 provides an example of such arrangement for the case of a simple process lasting 34 hours and involving only a fund fully used for three times, with time intervals respectively of 12, 4 and 10 hours. In this case the cycle time is equal to 2, in the *stabilized line process*[8] there are 17 elementary processes acting simultaneously and 13 units of the fund continuously used.

Given that δ is the maximum value compatible with the continuous use of funds, T/δ is the minimum size at which this condition holds. With this size, the output per unit of time is $1/\delta$.

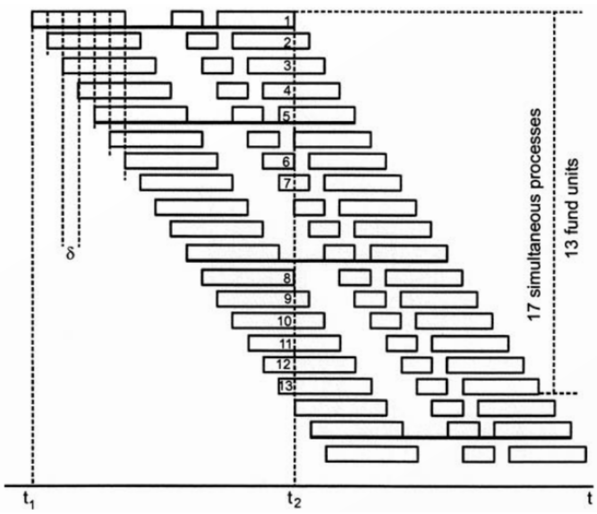
To get rid of both the sources of inefficiency – underutilization and idleness – one can resort to the parallel activation of $\bar{n}$ processes every δ units of time, or, alternatively, to an arrangement in line of the processes with cycle time $\delta/\bar{n}$ (Tani, 1986).

It follows that the *minimum efficient size* of the process is $T\bar{n}/\delta$, whereas $\bar{n}/\delta$ its *minimum efficient scale*. At every scale/size not multiple of these values the overall efficiency decreases. This is nothing but the formal expression of the so-called *multiple principle* (or *Babbage's (1835) factory principle*), according to which "efficiency reversals over certain ranges of increases in production levels can only be avoided if the scale increases take place in discrete jumps" (Landesmann, 1986, p.309).

Finally, it is important to note that, in an arrangement in line, the

[7]The measure may also be a non integer number. A necessary condition is that the d_{ji}s and T are all *commensurable* numbers, i.e. their ratios are all rational numbers. Commensurability between two numbers a and b is in fact a necessary and sufficient condition for the existence of some real number c, and integers m and n, such that $a = m \times c$ and $b = n \times c$. Furthermore, the requirement of commensurability is not as stringent as it may first appear. As stressed by Tani (1988), it is always possible to operate the process in line with an *ad hoc* lengthening of some stage.

[8]An arrangement in line with cycle time δ of an elementary process of duration T is stabilized after $(T/\delta - 1)$ cycles – or $(T - \delta)$ units of time.



Source: Mir-Artigues and González-Calvet (2007).

Figure 1: A line process with cycle time δ

different operations performed in the production process can always be assigned to the different funds to reach their *full specialization*, i.e. a division of the different phases among funds where each one performs a different operation. In fact, although this is not the only possible division compatible with the continuous use of the funds in the process,[9] it is always feasible (Tani, 1986; Morroni, 1992). Fund specialization seems therefore to arise quite naturally from the arrangement in line and it is thus another factor behind the efficiency enhancing effect of the line production.[10]

2.4 Indivisibility, decomposability and minimum efficient scale

The existence of an efficient scale of production and the multiple principle in the fund-flow model derives from the presence of both indivisibilities of production elements and rigidities of the time profile of their use.

On the one side, the fund-flow approach stresses how the presence of *limitational inputs* (Georgescu-Roegen, 1935, 1966) – i.e. inputs transformed in strict proportions during the production process – implies low substitutability among production elements. On the other side, it makes a distinction between

[9]If the durations of the periods of activity *and* idleness are all commensurable quantities, it is always possible to find a solution in which each unit performs all the operations of the elementary process (Tani, 1986).

[10]In particular, as far as labor is concerned, as first stressed by Smith (1776), specialization allows to speed up learning-by-doing and make mechanization easier. Moreover, it also allows to "separate tasks according to the degree of skill or strength required" by the funds (*Babbage's principle*).

indivisibility of production elements and *process indivisibility*, emphasizing how the divisibility of production elements is a necessary but not sufficient condition for the divisibility of the process, and this is in turn a necessary though not sufficient condition for constant returns to scale.[11]

A process is deemed indivisible if it is not possible to activate processes of smaller scale with the same proportions of inputs and outputs. In the fund-flow model, the definition must also consider the temporal pattern of production and restated as follows: a process $(\mathbf{F}(t), \mathbf{U}(t))$ is divisible if there is a $\eta > 1$ such that $\left(\frac{1}{\eta}\mathbf{F}(t), \frac{1}{\eta}\mathbf{U}(t)\right)$ is also a feasible process (Mir-Artigues and González-Calvet, 2007).

Besides the indivisibility of production elements and a scale-dependent nature of many processes,[12] there is another source of process indivisibility related with the arrangement in line. Indeed, in case of line production systems, the organized process has only a quite limited range of efficient activation scales, even when all its elements are perfectly divisible. These rigidities come from the need to satisfy the time profile of the activation of the funds in the elementary process.

The temporal dimension in the model allows also to distinguish the character of divisibility from that of *decomposability* (or *fragmentability*) of processes. An elementary process is decomposable if one can identify G *subprocesses* (or *stages*) of length T_g $(g = 1, \ldots, G)$ that can be separately activated; or, in formal terms, an elementary process $(\mathbf{F}(t), \mathbf{U}(t))$ is decomposable if there are G (> 1) subprocesses $(\mathbf{F}_g(t), \mathbf{U}_g(t))$, not all necessarily of the same length, such that (Tani, 1976; Mir-Artigues and González-Calvet, 2007):

$$\sum_g (\mathbf{F}_g(t), \mathbf{U}_g(t)) \equiv (\mathbf{F}(t), \mathbf{U}(t)) \qquad t \in [0, T].$$

To analyze the consequences of this feature on the minimum efficient scale/size of processes and input requirements, let us consider a generic decomposable elementary process whose subprocesses employ different fund elements each, and denote with δ_g the cycle time associated with the minimum

[11] As for production elements, it is possible to further distinguish: i) *technical indivisibility*, when a particular commodity cannot be divided, once it is exchanged, into amounts usable for production or consumption; ii) *economic indivisibility*, when one cannot exchange less than a given unit of a particular commodity (Morroni, 1992). A particular form of economic indivisibility is the *temporal indivisibility*, when one cannot acquire less than a given time of availability of a given fund, no matter the amount of services actually provided. This is indeed the prerequisite of the impact on costs of fund idleness: if it could be possible to pay for the availability of the funds only when they were actually employed, the existence of idleness would not increase costs.

[12] "All individual processes whether in biology or technology follow exactly the same pattern: beyond a certain scale some collapse, others explode, or melt, or freeze. In a word, they cease to work at all. Below another scale, they do not even exist" (Georgescu-Roegen, 1976, p.288).

efficient scale of activation in line of the subprocess g. Since δ_g is the greatest common measure of the intervals of fund activity in the subprocess g, while δ the greatest common measure of those intervals for the whole process, and the former intervals are a subset of the latter because of the assumption that each phase employs different types of funds, it follows that $\delta_g = \eta\ \delta$, with $\eta \geq 1$. Hence, each subprocess cannot have a minimum efficient scale greater than the whole process.

Given that the minimum requirements of type j funds employed in the subprocess g are $\theta_j^g = \sum_i d_{ji}/\delta_g$, we have:

$$\frac{\theta_j^g}{\theta_j} = \frac{\sum_i d_{ji}/\delta_g}{\sum_i d_{ji}/\delta} = \frac{\delta}{\delta_g} = \frac{1}{\eta},$$

Hence, even when the process in line associated with the complete elementary process is not divisible, the processes in line associated with the different subprocesses can be divisible and hence performed at a lower scale by different production units (Tani, 1976, 1986).[13]

2.5 Stabilized line productions and production functions

It is interesting to study the conditions under which the fund-flow model and the standard representation of production by means of production functions converge. As pointed out by Georgescu-Roegen, this happens in the limiting case of a *continuous stabilized line production*. In this case, since production can be treated as instantaneous and funds used continuously, we have:

$$S_j(t) = \theta_j \cdot t \qquad F_m(t) = f_m \cdot t \qquad Q(t) = q \cdot t$$

where $f_m = F_m(\delta)/\delta$ is the flow rate of the inflow m in each cycle time and q is the flow rate of output.

The functional (7) then becomes:

$$(q\ t)_0^t = \Psi\left[(f_2 t)_0^t, \ldots, (f_M t)_0^t,\ \theta_1, \ldots, \theta_J\right]$$

which is "a very special functional: first, every function involved in it depends upon a single parameter and, second, the value of t is entirely arbitrary." (Georgescu-Roegen, 1970, p.6). Hence, production processes can be expressed with the function:

$$q = \Phi(f_2, \ldots, f_M, \theta_1, \ldots, \theta_J) \tag{8}$$

[13] As stressed by Tani (1986), the decomposability of the elementary process is not a sufficient condition for the decomposability of the line process. What is necessary and sufficient is that, when in a phase of the elementary process a j-type fund is employed, the durations of the uses are all multiples of $c_j\delta$, where c_j is the number of elementary processes that can simultaneously use the same unit of the fund in the whole process arranged in line. This condition is always satisfied when each phase uses different types of funds.

or, alternatively, with the function:

$$Q = \Theta(F_2, \dots, F_M, S_1, \dots, S_J) \tag{9}$$

or:

$$Q = \Lambda(F_2, \dots, F_M, \theta_1, \dots, \theta_J) \tag{10}$$

which closely resemble neoclassical production functions.

However, the "tacit presumption that the forms" (8) and (9) (or (10)) "are equivalent implies that returns to scale must be constant" (1970, p.2). Indeed, we have:

$$\begin{aligned} t\ \Phi(f_2, \dots, f_M, \theta_1, \dots, \theta_J) = t \cdot q = Q = \Theta(F_2, \dots, F_M, S_1, \dots, S_J) = \\ = \Theta(f_2 \cdot t, \dots, f_M \cdot t, \theta_1 \cdot t, \dots, \theta_J \cdot t) \end{aligned}$$

Since this relation must be true for any t, it must hold also for $t = 1$. Hence:

$$\begin{aligned} \Phi(f_2, \dots, f_M, \theta_1, \dots, \theta_J) = \Theta(f_2, \dots, f_M, \theta_1, \dots, \theta_J) \\ (= \Lambda(f_2, \dots, f_M, \theta_1, \dots, \theta_J)) \end{aligned}$$

It follows that $\Phi(.) \equiv \Theta(.)$ $(\equiv \Lambda(.))$ and it is a linear homogeneous function.

But, concludes Georgescu-Roegen, "this does not mean that the factory process operates with constant returns to scale" and the "analytical imbroglio" behind production functions is thus brought to light: the homogeneity of the function results from "the tautology that if we double the time during which a factory works, then the quantity of every flow element and the service of every fund will also double. The issue of returns to scale pertains, instead, to what happens if the fund elements are doubled" (1970, p.7).

Therefore, a better representation of the process should make time explicit also in this case:

$$Q = \Theta(F_2, \dots, F_M, S_1, \dots, S_J; t) \tag{11}$$

or

$$Q = \Lambda(F_2, \dots, F_M, \theta_1, \dots, \theta_J; t) \tag{12}$$

3 Developments and modifications

By taking into account the period of production and the temporal patterns of input use, the fund-flow model allows the analysis of historical time in production models, and the economic decisions involved in production properly emerge as far more complex than just choosing the right combination of inputs. Nonetheless, because of its detailed description of the process, the model soon becomes too demanding, both in terms of analytical tractability and data requirement for practical uses, and hardly deliver useful generalizations or operational conclusions.[14]

To overcome these problems, some scholars have modified the original formulation, to reduce the analytical complexity while maintaining the basic insights. In particular, Piacentini (1995, 1997) has analyzed the effects of the different arrangements of the elementary process on average total cost, by assuming that the elementary process itself can be divided into phases (Section 3.1). Morroni (1992) and Piacentini (1995) have developed synthetic and operational representations of technologies consistent with the fund-flow approach (Section 3.2). Mir-Artigues and González-Calvet (2007) have borrowed tools from operational research and production management to study the basic features of line production (Section 3.3).[15]

3.1 Time-explicit cost functions and time-saving innovation

The different temporal arrangements of production processes, explicitly modeled in a fund-flow approach, clearly affect production costs. The cumulative cost incurred for fund services and inflows are:

$$E(t) = -\left(\sum_m p_m F_m(t) + \sum_{j=1}^{J} w_j S_j(t)\right)$$

where p_m is the price of the inflow m and w_j the price paid for the services of the j-type fund.

If the payments are made when the inputs enter the process, one needs to capitalize/discount them, as they refer to different moments in time. With a constant rate of interest (r), the total cost incurred at t is:

$$\begin{aligned} C(t) &= \int_0^t \mathrm{e}^{r(t-\tau)}\, dE(\tau) = \\ &= -\left(\sum_m p_m \int_0^t \mathrm{e}^{r(t-\tau)} F'_m(\tau)\, d\tau + \sum_{j=1}^{J} w_j \int_0^t \mathrm{e}^{r(t-\tau)} U_j(\tau)\, d\tau\right) \end{aligned} \quad (13)$$

This is the approach chosen by Tani (1986, Ch.9) and Zamagni (1993, Ch.8). However, this level of detail soon leads to intractability. Therefore, Piacentini (1989, 1995) makes some simplifying assumptions and studies the effects of the different possible arrangements of production processes on

[14]As pointed out by Mir-Artigues and González-Calvet (2007), the other criticism, namely that the detailed representation of production processes pertains to the domain of engineering and it is therefore outside the scope of economics, is captious and cannot be accepted.

[15]The model developed by Scazzieri (1993) and Landesmann and Scazzieri (1996a,b), where production process is conceived as a network of tasks, also strongly builds on Georgescu-Roegen's approach.

average costs. In particular, he assumes that production processes can be broken down into a sequence of *phases*, so to assume that the limiting case of a continuous stabilized line production (Section 2.5) holds in each of them. Then, he goes on studying the impact on average costs of the different forms of possible arrangements of elementary processes when there are temporal indivisibilities in the payment schedule for funds.

For an elementary process of length T_e with a single product, in case of *activation in series* the average cost, i.e. the ratio of total costs (C) over output (Q) in a given reference period (H) (e.g. total number of hours per annum), is:

$$c_s = \frac{C}{Q} = \frac{C}{H\frac{1}{T_e}} = \frac{\sum_j \sigma_j + \frac{H}{T_e}\sum_m p_m a_m}{H\frac{1}{T_e}} = \frac{T_e}{H}\sum_j \sigma_j + \sum_m p_m a_m \tag{14}$$

where σ_j is the (exogenous) cost of the availability of the fund j for the reference period (the year), $a_m = -F_m(T_e)$ is the technical coefficient for the inflow m and p_m its unit price.

If there are indivisible funds, the average cost corresponding to the *activation in parallel* that removes all the excess capacities is:

$$c_p = \frac{C}{\frac{H}{T_e}\bar{n}} = \frac{\sum_j \frac{\bar{n}}{\kappa_j^*}\sigma_j}{\frac{H}{T_e}\bar{n}} + \sum_m p_m a_m = \frac{T_e}{H}\sum_j \frac{1}{\kappa_j^*}\sigma_j + \sum_m p_m a_m \tag{15}$$

where κ_j^* is the maximum number of elementary processes that a unit of fund j can process at the same time and $\bar{n}$ the least common multiple of the κ_j^*s. Because $\kappa_j^* \geq 1$ for all $j \in \{1, \dots, J\}$, it follows that $c_p \leq c_s$.[16]

Finally, in case of *activation in line*, the average cost that corresponds to the temporal organization of elementary processes that eliminates the idle time for all the funds is:

$$c_l = \frac{C}{H\frac{1}{\delta}} = \frac{\delta}{H}\sum_j \theta_j \sigma_j + \sum_m p_m a_m \tag{16}$$

where $\theta_j = \sum_i d_{ji}/\delta$ are the units of j needed to perform the T_e/δ elementary processes in the stabilized line production. By denoting with d_j ($= \sum_i d_{ji} \leq$

[16] Piacentini (1995) also works out for each fund a *parameter of saturation* (Ω_j), equal to the number of maximum (κ_j^*) over actual (κ_j) number of elementary processes one unit of the j-type fund simultaneously processes, and he claims that the average unit cost in case of non-full capacity operation (c_p') is directly proportional to c_p, with a constant of proportionality equal to the maximum of such parameters. However, when the process is not fully constrained by the "bottleneck condition", the claim is false. In general:

$$c_p' = \frac{\sum_j \frac{\sigma_j}{\sum_j \sigma_j}\frac{1}{\kappa_j}}{\sum_j \frac{\sigma_j}{\sum_j \sigma_j}\frac{1}{\kappa_j^*}} c_p \leq \max_j (\Omega_j) c_p$$

i.e., although c_p' is directly proportional to c_p, the constant of proportionality is equal to the ratio between the weighted harmonic means of κ_j^* and κ_j.

T_e) the time of effective utilization of the fund j in the elementary process, Eq. (16) can be rewritten as:

$$c_l = \frac{T_e}{H} \sum_j \frac{d_j}{T_e} \sigma_j + \sum_m p_m a_m \qquad (17)$$

Since $d_j \leq T_e$ for all $j \in \{1, \ldots, J\}$, it follows that $c_l \leq c_s$.

This last effect is different from the previous one, because it comes from a more efficient use of resources in time rather than from the traditional effect of scale. Piacentini (1995) refers to it as *temporal economies*, emphasizing that, "although activation in line doubtless implies high volumes of production, these are the result of a higher speed of "throughput" of inputs within the process rather than of "scale" meant as aggregation of productive capacity at a given moment of time." (1995, p.476).[17]

These are the effects of the different possible arrangements of elementary processes on average costs. Now, making the hypothesis of a process split into its component phases, leaving aside inflows, denoting with σ_i the cost of the availability of the bundle of funds needed in the phase i, and with t_i the time needed to perform such phase, the average cost of each phase is:

$$c_i = t_i \frac{\sigma_i}{H} \qquad (18)$$

But the unit cost of a process performing in succession the different phases will be $\sum_i c_i$ only in case of *balanced production*, i.e. $t_i = t_j$ for each i, j. In the more general case in which $t_i \neq t_j$ for some i, j, the cost is:

$$c = \sum_i \frac{\sigma_i}{H / \max_i(t_i)} = t_{\max} \sum_i \frac{\sigma_i}{H} \qquad (19)$$

In this situation the pace of production is constrained by the productivity of the slowest phase: "for phases upstream of the bottleneck, accumulation of a stock of unfinished products which cannot be further processed would be wasteful; phases downstream, on the other hand, are directly constrained by the bottleneck." (Piacentini, 1995, p.476).

Moreover, with respect to the unit cost of each phase (Eq. (18)), two possible and *distinct* sources of cost reduction can be identified: i) a decrease in the price/quantity of the input bundle needed to perform the phase (σ_i); ii) a reduction in the phase production time (t_i).

Accordingly, one can classify (process) innovations in two broad classes: i) *input-saving*; and ii) *time-saving*.[18] As pointed out by Piacentini (1997),

[17] Also in the case of activation in line, Piacentini (1995) works out the relation between the average unit cost for the case of "perfect" vs. imperfect coordination, and also in this case his argument is not flawless. Indeed, no simple relation exists between the unit costs for the case of smooth and non smooth operations.

[18] The distinction is in Piacentini (1997), who however identifies three kinds of innovation: i) *capital-saving*; ii) *labor-saving*; and iii) *time-saving*.

learning-by-doing should be properly viewed as a source of time-saving, rather than input-saving technical progress.

Moreover, from Eq. (19) it follows that a time-saving innovation in a single phase is actually effective only if it falls on the slowest phase and as long as it does not create a new bottleneck, i.e. a new phase that becomes the most lengthy one.

Finally, as for input flows, besides the cost of the flows embodied in each unit of output ($\sum_m p_m a_m$), one should also consider the *opportunity cost* of the *circulating capital* (or *process-fund*), i.e. the value of semi-processed goods that must already be available when a stabilized process in line is started and still remain, as work in progress, when it is stopped. While the temporal dimension of the process does not affect input flows as such, it affects instead the volume of the process-fund and so the extent of the opportunity cost associated with it.[19] Such cost should be included as a component of the average cost and clearly depends on time.[20]

The presence of both aspects, physical inputs and time, in the relation between process-fund-related economies and the length of the production process has led Morroni (1992) to distinguish a third form of innovation: *organizational-inventory-saving*. This includes all the innovations that reduce the quantity of semi-finished products in *organizational inventories*, i.e. breaks that are not an integral part of the process, but depend on the actual organization of production. This type of technical change is intermediate between time-saving and input-saving innovations because it allows a decrease in both duration and input quantities.

This classification properly emphasizes the difference between: organizational innovations that reduce working capital without altering the process time; and the other innovations that instead shorten this time. However, on the one hand, a shortening of the production length can decrease *per se* the process-fund; on the other hand, the main channel through which time-saving technical change impacts on costs is by increasing the *speed of rotation* of flows, so augmenting output in the given reference period. It seems therefore better referring to all the improvements that reduce the length of the process only as time-saving innovations, treating the possible related reduction in the working capital as a by-product.

Morroni's account of the time profile of the production process (summarized in Table 1) is nonetheless very useful to identify the possible sources of time-saving innovations and their nature. Indeed, organizational changes

[19] For instance, with the simplifying assumption that all flows enter the process at the beginning of each phase, the process-fund is $\sum_{i,m} p_m a_{mi} t_i$, where a_{mi} is the coefficient of inflow m in phase i.

[20] "Any increase in t_i owing to "waiting time" or miscoordination will proportionally increase "work-in-process" cost. The significance of organizational improvements such as "just in time" operation, where the required inputs become available exactly at the moment of their active immission in the process, is clearly evidenced" (Piacentini, 1995, p.479).

Table 1: The time profile of production in Morroni (1992)

Time ⟶

<table>
<tr><td colspan="7">Response Time[a]</td></tr>
<tr><td rowspan="5">Time to collect customer orders and transfer them to production lines</td><td colspan="5">Gross Duration</td><td rowspan="5">Time to deliver the product</td></tr>
<tr><td rowspan="4">Delivery time of raw materials</td><td colspan="4">Duration</td></tr>
<tr><td rowspan="3">Pauses for shift regulations, working conditions and seasonal variations</td><td colspan="3">Working Time</td></tr>
<tr><td rowspan="2">Organizational inventories[b]</td><td colspan="2">Process Time</td></tr>
<tr><td>Technical inventories[c]</td><td>Net Process Time[d]</td></tr>
</table>

[a] "This is the time lapse between the order being received and the delivery of the finished product" (Morroni, 1992, p.73). In a pure *pull* system – i.e. production started upon activation by demand downstream – it is always higher than gross duration; in a *push* system – i.e. production executed before demand and finished products stocked in warehouses – it can be significantly lower than duration.

[b] Breaks due to: i) differences in the productive capacity of individual process phases; ii) internal movement times.

[c] Breaks due to periods of time in which semi-finished goods lie in inventories for maturing or settling, as an integral part of the process.

[d] *Net Process Time* (or *Gross Machine Time*): the time needed to produce a unit of output, excluding all interruptions apart from i) loading; ii) set-up; and iii) maintenance breaks of funds.

Financial Time: period of time between the payment for raw materials and the sale of the product.

mostly impact on *duration* and *working time*, via the reduction of organizational inventories, but they seldom affect *net process time* or *process time*, which are instead reduced mainly by technical innovations. On the contrary, the *gross duration* of the process is influenced also by factors, such as delivery time of raw materials, that can be under the control of different agents and are strongly affected by improvements in transportation and communications. Finally, *response time*, i.e. "the time lapse between the order being received and the delivery of the finished product" (Morroni, 1992, p.73), strongly depends on the actual organization of production. So, for instance, in a pure *pull* system, where production starts upon activation by demand downstream, the response time is necessarily higher than the gross duration. On the contrary, in a *push* system, i.e. a system in which production is executed before demand arises and the finished products are stocked in warehouses, the response time is not directly related to the gross duration, at least in the short run when the demand is satisfied using the accumulated stocks, and may be significantly smaller than the latter.

3.2 Synthetic and operational representations of technologies in a fund-flow approach

Piacentini's device of a logical breakdown of the production process into phases, for which it is then reasonable the assumption of a continuous stabilized line production, turns out to be particularly useful in a synthetic representation of the production process and of technical progress, consistent with a fund-flow approach. As said in Section 2.5, in this case one can avoid using functionals and describe the phases only by means of flow rates, fund units and time durations, thus employing a model of production resembling the input-output framework.

In order to analytically represent an elementary process made up of I phases, Piacentini (1987, 1989, 1995, 1996) specifies three elements: i) a vector of production times by phase, $(t_1, \dots, t_I)$; ii) a flows/phases matrix $\mathbf{f}$, whose generic element f_{mi} is the flow rate per unit of time of the outflow (inflow) m in the phase i; iii) a funds/phases matrix $\boldsymbol{\Theta}$, where the element θ_{ji} measures the units of fund j employed in phase i:

$$\mathbf{t} = \begin{pmatrix} t_1 & \dots & t_I \end{pmatrix} \quad \mathbf{f} = \begin{pmatrix} f_{11} & \dots & f_{1I} \\ \vdots & \ddots & \vdots \\ f_{M1} & \dots & f_{MI} \end{pmatrix} \quad \boldsymbol{\Theta} = \begin{pmatrix} \theta_{11} & \dots & \theta_{1I} \\ \vdots & \ddots & \vdots \\ \theta_{J1} & \dots & \theta_{JI} \end{pmatrix} \qquad (20)$$

This representation "allows our recipe of the production process to be enhanced by means of information on the temporal scanning of inflows and outflows of the process, while traditional information on limitational input/output ratios is preserved" (1995, p.472).[21]

[21] Piacentini (1987, 1997) also extends his analytical framework to *multi-product opera-*

With the same aim, i.e. to simplify and operationalize the fund-flow model, a different conceptual scheme is put forward by Morroni (1992, 1996, 1999). While the building block of Piacentini's analysis is the concept of "phase", Morroni instead relies on the notion of *stage* of a decomposable process (Section 2.4).

In his empirical analysis, Morroni (1992) summarizes the relevant information of production processes by means of two tables, detailed at the level of the single stage: i) the *quantitative-temporal matrix* $\mathbf{A}_{pt}$ (Table 2), which shows, for a *given total process time*, "the dated input and output flows, and fund services, required by an elementary technical unit (or a chain of elementary technical units) to produce one economically indivisible unit of the product emerging from an organized elementary process" (1992, p.86-87); ii) the *organizational scheme* (Table 3), that "summarizes or develops data provided by the production matrix", also giving "further information on the time profile and the dimension of scale of the elementary process considered" (1992, p.93).[22]

With respect to this framework, it should be stressed that, apart from an explicit consideration of the process-fund in the last three rows, the quantitative-temporal matrix suggested by Morroni (1992) is ultimately no different from a traditional input-output matrix at a stage level. But, on the one side, the choice of the stage as the "atom" of the analysis does not justify in itself the validity of such representation: nothing assures that a decomposition of the process in its constituent stages actually reduces its complexity.[23] On the other side, time as such does not enter directly in the representation: first of all, the index of the matrix is actually the only reference to time in the quantitative-temporal matrix – too little to justify the adjective "temporal" in the name; second, the real description of the time profile of production is in the Block B of the organizational scheme, but it has no direct connections with the previous description of production in terms of flows and fund services. The only place where time enters in the picture is the separate account of organizational inventories, which indirectly measures the extent of the unbalances among the stages in the production process. Definitely too little.

Nonetheless, Morroni's analytical framework can provide researchers with a detailed picture of production processes which can be very useful in the microeconomic analysis of process innovations.

tions.

[22]Morroni's (1992; 1996) studies are one of the few "real" empirical applications based on the fund-flow approach. He has also contributed to develop a computer program – *Kronos Production Analyser* (Moriggia and Morroni, 1993; Morroni and Moriggia, 1995) – to help researchers collecting, organizing and analyzing production data in a way consistent with his theoretical framework.

[23]Nevertheless, the framework does not necessarily require such decomposition and it is adopted only when it simplifies the analysis and/or provides further information. I am indebted to one of the referees for having suggested this point.

Table 2: Example of a quantitative-temporal matrix for a simple process made of three stages

Production elements	Intermediate Stage (IS) 1	2	3	Total
Output IS1	a_{11}	$-a_{12}$		0
Waste IS1	a_{21}			a_{21}
Output IS2		a_{32}	$-a_{33}$	0
Waste IS2		a_{42}		a_{42}
Output IS3			a_{53}	a_{53}
Waste IS3			a_{63}	a_{63}
Input IS1	$-a_{71}$			$-a_{71}$
Input IS1, IS2, IS3	$-a_{81}$	$-a_{82}$	$-a_{83}$	$-(a_{81} + a_{82} + a_{83})$
⋮				⋮
Services of fund IS2		$-s_{12}$		$-s_{12}$
Services of fund IS1, IS3	$-s_{21}$		$-s_{23}$	$-(s_{21} + s_{23})$
⋮				⋮
Organization inventories	$-a_{K+1,1}$	$-a_{K+1,2}$	$-a_{K+1,3}$	$-a_{K+1,4}$
Technical inventories	$-a_{K+2,1}$	$-a_{K+2,2}$	$-a_{K+2,3}$	$-a_{K+2,4}$
Elements in progress	$-a_{K+3,1}$	$-a_{K+3,2}$	$-a_{K+3,3}$	$-a_{K+3,4}$

Table 3: The organizational scheme

A	*Output*[a]	1	Internal production per day
		2	External production per day
		3	Production sold per day
B	*Time*	1	Net process time
		2	Process time
		3	Working time
		4	Duration
		5	Response time
		6	Gross duration
C	*Labour*	1	(who) Number of workers by occupation, shifts, sex, age, education
		2	(how) Tasks, jobs and skills by occupation
		3	(where) Employees/machine ratio
D	*Plant*	1	Machines by type (number, time and intesity of use)
		2	Adaptability (variations in quantity produced)
		3	Flexibility (variations in product mix, minimum batch)
E	*Demand*	1	Quantitative variations
		2	Qualitative variations
F	*Quality of products in progress*	1	Average incidence of defective intermediate products

Source: Morroni (1992).

[a]Each row of the scheme is divided into $I + 1$ columns. Each of the first I columns provides information for the correspondent intermediate stage. The last column gives information for the process as a whole.

3.3 Fund-flow model and production management

The analysis of the conditions for an arrangement in line of the production process to reduce idle times and increase fund productivities in the fund-flow model shows strong connections with some of the issues that *production* (and *inventory*) *management* usually treats in an engineer-oriented perspective (e.g. Vonderembse and White, 1991). Within the field of *operational research*, the latter commonly deals with the problem of time optimization of line processes. This has spurred some scholars to explore these analysis in trying to find some useful crossing between the two fields, and, in particular, to take advantage of the results of the latter in developing time-explicit economic models of production.

Some references to production management are in Piacentini (1997), who addresses the issue of *optimal lot-sizing* (Nahmias, 2008) in analyzing the relation between production costs and switching times. However, the first seminal analysis of the differences and similarities in the treatment of production processes in the fund-flow approach and production management can be found in Mir-Artigues and González-Calvet (2007).

The authors focus on the issue of *assembly line design* in production management (e.g. Scholl, 1999), where the problem of reduction of process duration and idle times for funds is usually treated as a problem of assignment of a given set of *tasks* (I), temporally ordered on the base of a *precedence graph*, to a number of *workstations*.

In this *balancing problem* – usually quite complex to solve and that requires sophisticated mathematical algorithms for its solution – one must first consider the *feasibility problem* for each proposed assembly line and then its optimization. The outcome is a certain assignment of the tasks to a certain number of workstations ($J \leq I$), performing the related tasks in a common interval of time, the *cycle time* (c). This interval comprises both the *service time* (ς_j) and the *balancing delay time* ($c - \varsigma_j$), where the former is divided in two components: the *effective working interval* (or *transformation work time*) and the *non-processing time*, i.e. the time required to move tooling, load and unload jigs, move materials across workstations, etc.[24]

This seems in many respects the natural framework to study the bottlenecks emerging in interlinked activities, with the related inducement

[24] With respect to each design, one can then compute the *balance delay ratio*, i.e. an index of relative efficiency:

$$BR = 1 - \frac{\sum_j \varsigma_j}{cJ}$$

and the *smoothness index*, i.e. an index of the degree of homogeneity in the distribution of work between the different workstations:

$$SX = \sqrt{\sum_j (c - \varsigma_j)^2}.$$

mechanisms of innovation emphasized by Rosenberg (1976), and probably most of its fruitful applications to economic theory have yet to come, although, needless to say, these engineer-oriented models are sometimes too complex and specific, with very few useful generalizations or strong economic implications.

4 Limits and comparative merits

4.1 On the "sameness" of funds

In Georgescu-Roegen's model there is a crucial assumption, namely, that each fund-element that leaves a production process is the *same* element that has entered it, or, at least, that we can treat it so by assuming that its level of efficiency is kept constant over the production cycles.

Such an "heroic" step was taken by Georgescu-Roegen because of "the merits of the fiction", which "are beyond question" (1971, p.229).[25]

In fact, he plainly considered the analytical possibility of representing the used funds – i.e. tired workers and worn-out equipment – as by-products of the production process, treating them as different commodities and so reducing fixed to circulating capital, but decided not to follow this representation. As he put it:

> ... an analytical picture in which the same worker (or the same tool) is split into two elements would undoubtedly complicate matters beyond description. The reason why these complications have not upset the various other analytical models currently used in natural and social sciences is that the issue of qualitative change of qualitative change has been written off *ab initio* by various artifices. ... (Nevertheless) we should expect an economist the make room in his analytical representation of a production process for ... the wear and tear. ... But in doing so he resorts to evaluating depreciation in money terms according to one of the conventional rules set up by bookkeepers. The solution is not only arbitrary, but also logically circuitous: it presupposes that prices and the interest rate, which in fact are influenced by production, are independent of it.
>
> An inspection of the basic models of production (in real terms) reveals however, that none includes the tired worker or the used tool among their coordinates. In addition to the formal complications already mentioned, there are other reasons which

[25] Funny enough, this idea of "capital equipment *being kept as a constant fund by the very process in which it participates*" (1971, p.229, emphasis in original) is a blatant violation of the Entropy Law, whereas much of Georgescu-Roegen's work can be read instead as an attempt to make economic theory consistent with this law. And he was aware of this.

> command the economist to avoid the inclusion of these elements in his analytical representations of a process. The economist is interested first and last in *commodities*. ...
>
> Even though there is no fast and general rule for determining what is and what is not a commodity, by no stretch of imagination could we say that tired workers and used tools are commodities. They certainly are *outputs* in every process, yet the aim of the economic production is not to produce tired workers and worn-out equipment. Also, with a few exceptions – used automobiles and used dwellings are the most conspicuous ones – no used equipment has a market in the proper sense of the word and, hence, no 'market price'. Moreover, to include tired workers and used tools among the products of industry would invite us to attribute a cost of production to such peculiar commodities. Of course, the suggestion is nonsense. Economics cannot abandon its commodity fetishism any more than physics can renounce its fetishism of elementary particle or chemistry can renounce that of molecule. (Georgescu-Roegen, 1971, p.217-218)

It is worth making some comments on this long quotation. First of all, the above claim that no economic model included tired workers or used tools among the products is actually false. As pointed out by Kurz and Salvadori (2003), there were models that allowed for used tools in a joint-product framework, namely those by von Neumann (1945) and Sraffa (1960), and Georgescu-Roegen knew for sure the former (see Georgescu-Roegen, 1966, p.311).

Second, apart from arbitrariness and logical circularity in the solution to the problem of depreciation, the arguments Georgescu-Roegen provides for ruling out used machineries or tired workers are rather weak. In particular, he argues that tired workers and used tools cannot be considered commodities because i) their production is not the aim of the process; ii) they have no proper market and thus no market price; iii) they have no "real" cost of production. However, all these arguments can be disproved. First, as stressed by Mir-Artigues and González-Calvet (2007), even if no production process is actually meant at the production of used equipment for sale, they are nevertheless by-products for which there is always a second-hand market or a scrap market. Even when no such market exists, there are yet book values. As regards the supposed production cost we are invited to attribute to the used equipment only because they are included among the outputs, it suffices to notice that, among the output of each production process there are almost always outflows (e.g. waste or other emissions) which may have a value, positive or negative, although no attached "production cost".

The arguments put forward by Georgescu-Roegen to support his "fiction" are therefore unconvincing. Nevertheless, this fiction, with the related partial equilibrium framework that sustains it, has also some merits, which, far from being "beyond question", should be plainly discussed.

In what follows, I first survey the main drawbacks of the fund-flow model, as pointed out by Kurz and Salvadori (2003) and Lager (2000), and mostly coming from the assumption of a perennial maintenance of the original efficiency for funds.

4.2 Limitations of the model

The limitations of the fund-flow model connected with the crucial assumption of "economic invariableness" for funds have been throughly analyzed by Kurz and Salvadori (2003).

They stress that, if such invariableness must be understood as keeping each and every durable means of production at the original level of efficiency, it may be both technically unfeasible and economically unviable. Moreover, it excludes *ipso facto* from the analysis important issues concerning fixed capital, namely: i) the choice of the economic lifetime of a durable means of production; ii) the choice of its pattern of utilization over time.

First, it is only by assuming a decreasing or changing efficiency profile in capital goods that issues of *premature truncation*, i.e. the possibility of a machine becoming economically obsolete before the end of its technically feasible lifetime, can arise.

Second, the hypothesis of a constant efficiency profile impinges upon the possibility to analyze the optimal patterns of utilization of durable capital goods. Indeed, given the assumption of constant efficiency for funds, in the fund-flow model the maximum degree of utilization consistent with a given endowment is always the optimal one. However, it might well be the case that the optimal degree of utilization differs from the maximal one, since it depends on the efficiency profile of durable goods and the time variability of input and output prices.

In fact, when the constant efficiency hypothesis does not hold, the fund-flow model may fail to identify the cost-minimizing technique, and this is readily shown by the authors using the von Neumann-Sraffa approach to fixed capital. In this approach, a *flow-flow* description of technology is adopted and each process k is represented as:

$$\begin{pmatrix} \mathbf{a}_k \\ l_k \end{pmatrix} \rightarrow \mathbf{b}_k \tag{21}$$

where the vectors $\mathbf{a}_k, \mathbf{b}_k \in \mathcal{R}^N$ are, respectively, inputs and outputs of the N products in the process and the scalar l_j is the labor input. The fixed capital is reduced to circulating capital by treating the old machines left at the end of each period as different goods from the ones that entered production at the beginning of the period. And the available processes are represented in a compact form as:

$$\begin{pmatrix} \mathbf{A} \\ \mathbf{l} \end{pmatrix} \to \mathbf{B} \tag{22}$$

where $\mathbf{A} = (\mathbf{a}_1 \ldots \mathbf{a}_K)$, $\mathbf{l} = (l_1 \ldots l_K)$, and $\mathbf{B} = (\mathbf{b}_1 \ldots \mathbf{b}_K)$.

4.3 Fund-flow model special case of the flow-flow model

Kurz and Salvadori (2003) concludes that the fund-flow approach is more restrictive than the von Neumann-Sraffa approach, because the former cannot properly deal with the problems of "fund depreciation".

The point is taken up by Lager (2000), who starts from an extension of the latter to deal with production processes lasting more the one period. In particular, he represents a production process as follows:

$$\left(\begin{pmatrix} \mathbf{a}_{k\,0} \\ l_{k\,0} \end{pmatrix}, \begin{pmatrix} \mathbf{a}_{k\,1} \\ l_{k\,1} \end{pmatrix}, \ldots, \begin{pmatrix} \mathbf{a}_{k,T_k-1} \\ l_{k,T_k-1} \end{pmatrix} \right) \to (\mathbf{b}_{k\,1}, \mathbf{b}_{k\,2}, \ldots, \mathbf{b}_{k\,T_k}) \tag{23}$$

where $\mathbf{a}_{k\,t}$ ($\mathbf{b}_{k\,t}$) is the vector of the inflows (outflows) in the process k in period t, $l_{k\,t}$ is the labor input during the same period, the process lasts T_k periods and, since production requires time, $\mathbf{a}_{k\,T_k} = \mathbf{b}_{k\,0} = \mathbf{0}$.

Each process is therefore described as a series of dated quantities of inflows and outflows in a discrete time environment, and the previous Eq. (21) can be considered a special case of Eq. (23). Lager calls the general case a *flow-input flow-output* process, and notes that any generic flow-input flow-output process lasting T_k periods can be always broken down into $(T_k - 1)$ *point-input point-output* processes of unit duration by introducing additional intermediate goods connecting the time series of the processes:

$$\underbrace{\begin{pmatrix} \mathbf{a}_{k\,0} \\ \mathbf{0} \\ l_{k\,0} \end{pmatrix} \to \begin{pmatrix} \mathbf{b}_{k\,1} \\ \mathbf{e}_1\lambda_1 \end{pmatrix}}_{k_1}, \underbrace{\begin{pmatrix} \mathbf{a}_{k\,1} \\ \mathbf{e}_1\lambda_1 \\ l_{k\,1} \end{pmatrix} \to \begin{pmatrix} \mathbf{b}_{k\,2} \\ \mathbf{e}_2\lambda_2 \end{pmatrix}}_{k_2}, \quad \ldots, \quad \underbrace{\begin{pmatrix} \mathbf{a}_{k,T_k-1} \\ \mathbf{e}_{T_k-1}\lambda_{T_k-1} \\ l_{k,T_k-1} \end{pmatrix} \to \begin{pmatrix} \mathbf{b}_{k\,T_k} \\ \mathbf{0} \end{pmatrix}}_{k_{T_k}} \tag{24}$$

where $\mathbf{e}_i$ is a vector of dimension $(T_k - 1)$ with the ith element equal to one and all the other elements to zero. Eq. (24) is considered by Lager an equivalent vertically disintegrated point-input point-output representation of (23).

In this framework, Georgescu-Roegen's fund-flow approach is represented as the special case in which fixed capital lasts "forever". In particular, by assuming that the vectors of inputs and outputs are ordered in such a way that the first elements are, respectively, circulating capital ($\mathbf{a}_k^c$) and "real" outputs ($\mathbf{b}_k^q$), while the others are fixed capital inputs ($\mathbf{a}_k^f$) and used machines

$(\mathbf{b}^f_k)$, Eq. (23) can be rewritten as:

$$\left(\begin{pmatrix}\mathbf{a}^c_{k\,0}\\ \mathbf{a}^f_{k\,0}\\ l_{k\,0}\end{pmatrix},\ldots,\begin{pmatrix}\mathbf{a}^c_{k,T_k-1}\\ \mathbf{a}^f_{k,T_k-1}\\ l_{k,T_k-1}\end{pmatrix}\right)\to\left(\begin{pmatrix}\mathbf{b}^q_{k\,1}\\ \mathbf{b}^f_{k\,1}\end{pmatrix},\ldots,\begin{pmatrix}\mathbf{b}^q_{k\,T_k}\\ \mathbf{b}^f_{k\,T_k}\end{pmatrix}\right) \tag{25}$$

where the idea of perennial machines is captured by the following two conditions:

$$\sum_{\tau=1}^{t}(\mathbf{b}^f_{k\,\tau}-\mathbf{a}^f_{k,\tau-1})\leqq\mathbf{0}\qquad\forall\ t\in\{1,2,\ldots,T-1\} \tag{26}$$

$$\sum_{\tau=1}^{T}(\mathbf{b}^f_{k\,\tau}-\mathbf{a}^f_{k,\tau-1})=\mathbf{0} \tag{27}$$

Condition (26) states that the total amount of machines listed among the outputs during the interval $[1,t]$ cannot be greater than the amount of them entering the process within the interval $[1,t-1]$, while condition (27) imposes that all the machines entered the process actually will leave it at the end.

Lager also considers the possible alternative representation in which funds elements are described by their services:

$$\left(\begin{pmatrix}\mathbf{a}^c_{k\,0}\\ \theta_{k\,0}\\ l_{k\,0}\end{pmatrix},\ldots,\begin{pmatrix}\mathbf{a}^c_{k,T_k-1}\\ \theta_{k,T_k-1}\\ l_{k,T_k-1}\end{pmatrix}\right)\to\left(\mathbf{b}^q_{k\,1},\ldots,\mathbf{b}^q_{k\,T_k}\right) \tag{28}$$

where the vector $\theta_{k\,t}$ represents the units of funds ("perennial" capital goods) employed in the interval $[t,t+1]$:

$$\theta_{k\,t}=\sum_{\tau=0}^{t}\mathbf{a}^f_{k\,\tau}-\sum_{\tau=1}^{t}\mathbf{b}^f_{k\,\tau} \tag{29}$$

Lager emphasizes that the previous representation clearly reveals that in the fund-flow model fixed capital is treated like Ricardian land, i.e. a natural resource with "*original* and *indestructible* powers", and this implies that its price is determined by the present value of the rental rates paid for it.

4.4 Comparative merits of the fund-flow approach

From the previous analysis it seems quite natural to conclude that the fund-flow approach should be abandoned in favor of the flow-flow one, and indeed this is explicitly stated by Kurz and Salvadori, who write: "we cannot identify any aspect which can be tackled using the latter, but not the former, this is enough to decide in favor of the flow-flow approach" (2003, p. 499).

In what follows I instead try to argue that the former claim is untenable. First, the fund-flow model cannot be considered a specific case of Lager's

(2000) specification of the von Neumann-Sraffa approach. Second, i) the von Neumann-Sraffa approach suffers from serious limitations too; and ii) these limitations are radically different from the ones of the fund-flow approach. This implies that, contrary to what stated by Kurz and Salvadori, one can identify important aspects of production effectively dealt in a fund-flow model, and that a flow-flow one is instead not able to cope with. Third, the limitations of the fund-flow approach emphasized by Kurz and Salvadori are not so disruptive as they claim and can be easily overcome.

Let us start with Lager's (2000) claim that the fund-flow model is only a specific case of his own specification of the von Neumann-Sraffa approach. As we saw, Lager's model treats what he calls flow-input flow-output processes as a discrete set of vectors of dated inputs and outputs (Eq. (23)). In fact, given the discrete nature of the model, in case of processes entailing practically continuous flows (e.g. electricity, emissions, etc.), this is a very rough approximation of reality. In this case, what is actually recorded is:

$$a_{kj,t} = I_{kj}(t+1) - I_{kj}(t) = \int_t^{t+1} I'_{kj}(\tau)\ d\tau \tag{30}$$

$$b_{kj,t} = O_{kj}(t+1) - O_{kj}(t) = \int_t^{t+1} O'_{kj}(\tau)\ d\tau \tag{31}$$

where $I_{kj}(t)$ ($O_{kj}(t)$) is the cumulative input (output) of j in process k at time t and $I'_{kj}(t)$ ($O'_{kj}(t)$) the correspondent instantaneous rate of flow.

One might reply that the approximation could be made less severe by reducing the interval of the "discrete jump". And this is true, but, at the same time, it is also true that, the less the time span of the jump, the greater the number of elements needed to describe the process in Lager's representation. In particular, there is an exponential increase in complexity, since halving the interval entails, for each process, doubling the number of vectors of inputs and outputs and also doubling the elements in each vector aimed at capturing the "durable means of production", given that each durable element leaving a stage may in principle be treated as a different element with respect to the one that has entered it. Moreover, the less the time span of the interval in the point-input point-output process, the less untenable Georgescu-Roegen's hypothesis of sameness for funds, and thus the more the latter model is a suitable description of the process stage.

Strictly speaking, a description of production as a series of point-input point-output processes is not even a representation in terms of flows, because in order to represent a flow one has to consider an *interval* of time – even if infinitesimal – and not a *point* in time.[26]

[26] This is explicitly, though quite incidentally, recognized by Lager himself, when he says that in the von Neumann-Sraffa models inputs and outputs are measured as "stocks at a point in time" (1997, p.370).

Hence, Lager's suggestion should be retain as a very rough approximation of reality. In fact, if his model had to be applied literally, it would not be just "hardly (to) find data for a rigorous application" (Lager, 2000, p.249), but simply impossible. What one can do is instead to conceive each point observation as an approximation of the cumulated inputs (or outputs) between two point observations (Eq. (30) and (31)). And this is the way these models are usually interpreted.

When we move from the description of the single process to the description of the whole system and the interdependencies among the different processes, as in Kurz and Salvadori (2003), there is another important assumption to be considered: all processes must have the same (unit) duration. When this is not the case, "processes of longer duration (have) to be broken down into single processes of unit duration introducing if necessary intermediate products as additional goods" (von Neumann, 1945, p.2). If we interpret input-output coefficients in these models simply as an ex post accounting of intersectoral transactions, this temporal rescaling does not raise any issue. But if we instead assume constant returns to scale – as we need to if we want to apply linear algebra to solve the problem of the choice of technique or to find the operation intensities of the different processes (see, for instance, Kurz and Salvadori, 1995, 2003) – the temporal rescaling can generate inconsistencies in all the cases in which the processes cannot be fragmented.[27]

This is not to mention the related but distinct assumption of divisibility (of both elements and processes) behind the hypothesis of constant returns to scale. As we saw, the fund-flow model clearly shows that the relation between inputs and outputs is hardly constant and not even continuous.

In fact, on a closer examination, it shows something more, namely, that the relation captured by an input-output coefficient a_{ij} – i.e. the quantity of commodity i "used up" per unit of output j – can be considered quite stable if i is a flow in Georgescu-Roegen's sense, i.e. an input to be physically incorporated in the product, given that these inputs are usually limitational inputs, but not so when i is instead a fund, which is never physically

[27]Let us note in passing that this reduction of all the processes to the same duration, so excluding *ipso facto* from the analysis issues connected with temporal coordination, makes extremely difficult to understand the benefit involved in the arrangements in line.

A hint of the conceptual problems involved in capturing the very same idea of line production systems within a von Neumann-Sraffa framework is provided by Kurz and Salvadori (2003) themselves when they quote Georgescu-Roegen (1970):

> He added that "the economics of production reduces to two commandments: first, produce by the factory system (i.e. by arrangement in parallel) and, second, let the factory operate around the clock" (Georgescu-Roegen, 1970, p.8). (Kurz and Salvadori, 2003, p.498)

Here the explanatory note in brackets (i.e. by arrangement in parallel), which has been added by the authors, is actually wrong: the peculiarity of a factory system is not an arrangement in parallel, but in line!

incorporated in the product.

The "technical" coefficients of funds are not stable, because they crucially depend on the speed of rotation of flows; a piece of information utterly ignored in the input-output framework. The fund-flow model shows that this speed of rotation is affected by the actual arrangement of production processes in time, and the possibility to implement such an arrangement is in turn affected by the overall scale of the organized process.

Kurz and Salvadori claim that their analytical framework "does not do away with Georgescu-Roegen's important distinction between the 'agents of a process' of production ... and its flow elements", because there is "no presumption that by analytically reducing fixed capital to circulating, the former becomes substitutable against the elements of circulating capital as conventionally defined" (2003, p.496). But the point is not the complementarity between circulating and fixed capital, but rather the instability of the derived coefficients for the latter. And such instability is not the result of processes of "substitution" between circulating and fixed capital, as commonly meant in economics, but rather of changes in the speed of rotation.

But that is not all. There is a particular fund – or stock, given its debatable classification (e.g. Landesmann and Scazzieri, 1996b) – missing in the von Neumann-Sraffa representation of production processes: the process-fund. When a new process in line is activated the "pipeline" has to be fulfilled. After that, the duration of production processes is greatly reduced. This strongly affects input-output coefficients. One might say that, given the long-run perspective of the analysis, one could look only at the coefficients prevailing in the stabilized processes. Nonetheless, it is important to note that this stock (or fund) cannot be treated as any other stock, that can be reduced without altering the functioning of the system in the steady-state, but must be maintained above a certain level, although this entails a cost. A reduction of the duration of production processes, besides reducing input-output coefficients of funds in a given period, can actually reduce this stock. Such decrease is *per se* a benefit, but a flow-flow approach fails to capture this aspect.

Finally, what is really needed for a fund-flow approach to work is not an hypothesis of "perennial" maintenance of the original efficiency for funds, as stated by its critics. What we need to assume is simply that the hypothesis of "sameness" for funds holds for a *certain period of time* or until a *certain level of wear and tear*, so we can treat them as the same good within that period or below that threshold. If these "goods" were actually treated as different after this period or above this level nothing would change in the analytical apparatus of the model.

Clearly, there is some degree of arbitrariness in choosing the period or the level, but this arbitrariness is simply connected with the discrete nature of the choice, and it is not higher and possibly lower than the arbitrariness present in all the models based on a von Neumann-Sraffa representation.

5 Concluding remarks

The fund-flow approach, put forward by Nicholas Georgescu-Roegen almost half a century ago, makes the temporal structure of production explicit and initiates the formal analysis of the patterns of coordination among the factors of production in economics. Georgescu-Roegen identifies the different possible arrangements of processes in time and formally studies the relation between labor division and production efficiency. In this respect, he realizes that the assembly line and the factory system, which allow to strongly reduce the idleness of inputs, "deserves to be placed side by side with money as the two most fateful *economic* innovations for mankind" (Georgescu-Roegen, 1970, p.8, emphasis added).

This paper has been intended at critically and, as far as possible, exhaustively reviewing the contributions centered around the fund-flow approach.

I have first summed up Georgescu-Roegen's original formulation with the analytical refinements by Tani (1986), emphasizing the important implications of the model for production theory, namely: the discontinuities in the relation between average cost and output; the difference between input divisibility and process divisibility; the notion of process decomposability.

I have then dealt with some suggested extensions and modifications of the original framework, mostly intended at "operationalizing" the model, in particular: the time-explicit cost functions of Piacentini (1995); and the synthetic representations of production consistent with a fund-flow approach put forward by Piacentini (1995, 1997) and Morroni (1992).

The analysis of the optimal temporal arrangement of production processes is also the subject of production and inventory management. Some scholars (e.g. Mir-Artigues and González-Calvet, 2007) have tried to engage these fields in the fund-flow approach. This looks quite promising and seems the natural framework to study the bottlenecks induced by process innovations in interlinked activities.

Finally, I have coped with some potentially disruptive criticisms recently raised against the approach by Kurz and Salvadori (2003) and Lager (2000). They emphasize the inadequate treatment of the problem of durable goods utilization in Georgescu-Roegen's framework, because of the "fiction" of a constant efficiency for funds, and claim that the fund-flow approach is just a special case of the von Neumann-Sraffa approach, which is always superior.

As I argued, the fund-flow model cannot be considered a specific case of the von Neumann-Sraffa approach and the latter suffers from serious and radically different limitations, namely: the discrete treatment of time; the non technical nature of "technical" coefficients; the strong instability of these coefficients when worked out for funds; the lack of crucial pieces of information, such as the speed of rotation and the process-fund. It means that there are important aspects of the production process that a flow-flow approach cannot cope with, while a fund-flow one can.

Hence, although not suitable to analyze the reciprocal influences between economic sectors or the optimal pattern of utilization of durable goods, the fund-flow approach can give us invaluable insights on the organizational aspects of production processes, as processes unfolding in time and requiring coordination between their elements. One can analyze aspects such as the temporal coordination among the phases and the different patterns of activation, that are related with different scales and continuity to process operations.

In particular, the approach can help enhancing our understanding of the possible sources and forms of technical change; and, in this respect, it seems particularly important the conceptual category of time-saving technical change put forward by Piacentini (1995). But the approach could probably prove useful also in the analysis of the organization of labor, the payment structure, the industrial dynamics and the firm behavior in oligopolistic markets. Indeed, all these aspects are intimately linked to the organization of production in time, which is in turn the cause and effect of consumer demand, market structure and technology.

Finally, a time-explicit theory of production can enhance also our understanding of the role played by the actual organization of production on the macroeconomic effects of demand and supply shocks, and in the dynamics of their transmission across sectors and countries. Just to give an example, some economists have argued that the "great trade collapse" occurred between the third quarter of 2008 and the second quarter of 2009 was mainly the result of the just-in-time organization of the vertically integrated production networks, that can rapidly transmit demand shocks across countries and sectors (e.g. Baldwin, 2009). However, no one have so far tried to formalize such insights and the reason is probably that the mainstream production theory is not able to cope with these time-related issues. In this respect, Morroni's (1992) framework, with its rigorous analysis of the time profile of the production process under the different organization settings, could prove very useful.

Likely, some of the most useful applications of the fund-flow approach have yet to come.

References

Babbage, C. (1835). *On the Economy of Machinery and Manufactures* (4th ed.). London: Knight.

Baldwin, R. (2009). The great trade collapse: What caused it and what does it mean? *VoxEU.org 27 November 2009.*

Fisher, I. (1896). What is capital. *Economic Journal VI*, 514.

Frisch, R. (1964). *Theory of production.* Kluwer Academic Publishers.

Georgescu-Roegen, N. (1935). Fixed coefficients of production and the marginal productivity theory. *The Review of Economic Studies 3*(1), 40–49.

Georgescu-Roegen, N. (1966). *Analytical economics: issues and problems.* Harvard University Press.

Georgescu-Roegen, N. (1969). Process in farming vs. process in manufacturing: A problem of balanced development. In U. Papi and C. Nunn (Eds.), *Economic problems of agriculture in Industrial Societies*, pp. 497–533. London: MacMillan.

Georgescu-Roegen, N. (1970). The economics of production. *American Economic Review 60*(2), 1–9.

Georgescu-Roegen, N. (1971). *The Entropy Law and the Economic Process.* Cambridge: Harvard University Press.

Georgescu-Roegen, N. (1976). *Energy and economic myths: institutional and analytical economic essays.* New York: Pergamon Press.

Georgescu-Roegen, N. (1990). The production process and dynamic economics. In M. Baranzini and R. Scazzieri (Eds.), *The economic theory of structural change.* Cambridge: Cambridge University Press.

Kurz, H. D. and N. Salvadori (1995). *Theory of production: a long-period analysis.* Cambridge: Cambridge University Press.

Kurz, H. D. and N. Salvadori (2003). Fund-flow versus flow-flow in production theory: Reflections on Georgescu-Roegen's contribution. *Journal of Economic Behavior & Organization 51*, 487–505.

Lager, C. (1997). Treatment of fixed capital in the Sraffian framework and in the theory of dynamic input-output models. *Economic Systems Research 9*(4), 357–73.

Lager, C. (2000). Production, prices and time: a comparison of some alternative concepts. *Economic Systems Research 12*(2), 231–253.

Landesmann, M. A. (1986). Conceptions of technology and the production process. In M. Baranzini and R. Scazzieri (Eds.), *Foundations of Economics*, Number 281-310. London: Blackwell.

Landesmann, M. A. and R. Scazzieri (1996a). Introduction. Production and economic dynamics. In M. A. Landesmann and R. Scazzieri (Eds.), *Production and Economic Dynamics.* Cambridge University Press.

Landesmann, M. A. and R. Scazzieri (1996b). The production process: description and analysis. In M. A. Landesmann and R. Scazzieri (Eds.), *Production and Economic Dynamics*. Cambridge University Press.

Maneschi, A. and S. Zamagni (1997). Nicholas Georgescu-Roegen, 1906-1994. *The Economic Journal 107*(442), 695–707.

Mir-Artigues, P. and J. González-Calvet (2007). *Funds, Flows and Time. An Alternative Approach to the Microeconomic Analysis of Productive Activities*. Berlin: Springer.

Moriggia, V. and M. Morroni (1993). Sistema automatico Kronos per l'analisi dei processi produttivi. caratteristiche generali e guida all'utilizzo nella ricerca applicata. Quaderni del Dipartimento di Matematica, Statistica, Informatica e Applicazioni 23, Università degli Studi di Bergamo.

Morroni, M. (1992). *Production Process and Technical Change*. Cambridge: Cambridge University Press.

Morroni, M. (1996). Un'applicazione del modello fondi-flussi alia produzione di apparecchiature per telecomunicazioni. University of Pisa.

Morroni, M. (1999). Production and time: a flow-fund analysis. In K. Mayumi and J. M. Gowdy (Eds.), *Bioeconomics and Sustainability. Essays in Honor of Nicholas Georgescu-Roegen*, pp. 194–228. Aldershot: Edward Elgar.

Morroni, M. and V. Moriggia (1995). *Kronos Production Analyser. Versione 2.1. Manuale d'uso*. Pisa: Edizioni ETS.

Nahmias, S. (2008). *Production and operations analysis* (6th ed.). MacGraw-Hill.

Piacentini, P. (1987). Costi ed efficienza in un modello di produzione a flusso lineare. *Economia Politica 4*(3), 381–405.

Piacentini, P. (1989). Coordinazione temporale ed efficienza produttiva. In S. Zamagni (Ed.), *Le teorie economiche della produzione*, pp. 163–179. Bologna: II Mulino.

Piacentini, P. (1995). A time-explicit theory of production: analytical and operational suggestions following a fund-flow approach. *Structural Change and Economic Dynamics 6*(4), 461–483.

Piacentini, P. (1996). Processi produttivi ed analisi dei costi di produzione. In A. Romagnoli (Ed.), *Teoria dei processi produttivi. Uno studio sull'unità tecnica di produzione*, pp. 91–106. Torino: Giappichelli.

Piacentini, P. (1997). Time-saving and innovative processes. In G. Antonelli and N. De Liso (Eds.), *Economics of Structural and Technological Change*, pp. 170–183. London: Routledge.

Rosenberg, N. (1976). *Perspective on technology.* Cambridge: Cambridge University Press.

Scazzieri, R. (1993). *A theory of production. Tasks, processes and technical practices.* Oxford: Clarendon Press.

Scholl, A. (1999). *Balancing and Sequencing of Assembly Lines* (2nd ed.). Heildelberg: Physica-Verlag.

Smith, A. (1776). *An Inquiry into the Nature and Causes of the Wealth of Nations.* London: Methuen.

Sraffa, P. (1960). *Production of Commodities by Means of Commodities.* Cambridge: Cambridge University Press.

Tani, P. (1976). La rappresentazione analitica del processo di produzione: alcune premesse teoriche al problema del decentramento. *Note Economiche IX*(4-5), 124–141.

Tani, P. (1986). *Analisi microeconomica della produzione.* Roma: Nuova Italia Scientifica.

Tani, P. (1988). Flows, funds, and sectorial interdependence in the theory of production. *Political Economy. Studies in the Surplus Approach IV*(1), 3–21.

von Neumann, J. (1945). A model of general equilibrium. *Review of Economic Studies 9*, 1–9.

Vonderembse, M. A. and G. P. White (1991). *Operation Management. Concepts, Methods, and Strategies* (2nd ed.). St. Paul: West Publishing Company.

Winston, G. (1982). *The timing of economic activities.* Cambridge: Cambridge University Press.

Zamagni, S. (1993). *Economia politica. Corso di microeconomia* (3rd ed.). Rome: La Nuova Italia Scientifica.