

DEPARTMENT OF ECONOMICS UNIVERSITY OF MILAN - BICOCCA

WORKING PAPER SERIES

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No. 19 - March 1999

Dipartimento di Economia Politica Università degli Studi di Milano - Bicocca <u>http://dipeco.economia.unimib.it</u>

Dynamic Seigniorage Models Revisited. Should Fiscal Flexibility and Conservative Central Bankers Go Together?

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First version July '97

This version February'99

Abstract

This paper presents a dynamic seigniorage model where excessive debt levels persist in steady state, causing a permanent inflation bias. Discretionary monetary responses to shocks are too interventionist because they do not take into account the role of debt policy, which spreads part of the adjustment onto future periods. Institutional design should contemplate the appointment of weight-conservative central bankers. The central bank preferences should be more conservative the more the government is willing to delay the adjustment of expenditures following a supply shock. The combination of fiscal intervention and a zero inflation rule describes how members of a monetary union might react to asymmetric shocks. The costs of this regime are negligible if the discount factor is small and seigniorage losses are limited.

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Acknowledgement: the paper greatly benefited from conversations with E. Colombo, G. Femminis, M.Lossani, A. Muscatelli and P.Natale. The usual disclaimer applies.

1. Introduction

It is often argued that inflation has a fiscal root. Empirical research identifies optimal tax considerations as a determinant of inflation differences across countries (Campillo and Miron, 1996). Furthermore, an important strand of literature sees distortionary taxes as the source of timeinconsistency in the conduct of monetary policy (Alesina and Tabellini, 1987). Therefore it seems a little surprising that the recent debate on monetary institutions, i.e. the controversy between performance-based contracts à la Walsh and weight conservatism à la Rogoff, has neglected the consequences of debt policy for institutional design (Walsh, 1995; Svensson, 1997; Muscatelli, 1998). This is probably due to the fact that dynamic seigniorage models provide a framework for the analysis of time-inconsistency in monetary policy games that is not entirely satisfactory. Consider for instance the work of Jensen (1994), who investigates the link between debt accumulation and monetary policy. Jensen's model suggests that inflation is a temporary phenomenon because in the long run re-invested budget surpluses will earn the income necessary to completely finance the public expenditure target, removing the need for distortionary sources of revenue¹. Unfortunately empirical evidence suggests that governments are apparently unwilling to accumulate such large surpluses. The persistence of excessive debt levels may be explained with the politico-economic equilibria that arise from the intergenerational conflict analysed in Cuckierman and Meltzer (1989), where the accumulation of public debt allows bequest-constrained individuals to raise their consumption levels at the expenses of future generations.

This paper presents a dynamic seigniorage model that accounts for the policymaker's aversion to a policy of debt reduction. By doing so we obtain a more realistic description of debt and inflation in the long run, showing that positive debt and inflation levels persist in steady state if the policymaker is sufficiently averse to debt reduction. Within this framework we are able to analyse both the consequences of political shifts – i.e. changes in the relative preference for public expenditures – and

¹ Obstfeld (1991) and Van der Ploeg (1995) obtain a similar result using different models.

the performance of monetary regimes, presenting an alternative view on the desirability of countercyclical monetary policies when fiscal policy is also used for stabilisation purposes.

We find that political shifts unambiguously affect steady state equilibria. For instance, debt increases if the median voter becomes relatively less concerned with the benefits of public expenditures. This result is explained as follows. When choosing the optimal debt policy, the policymaker takes into account both the political cost of reducing debt and the long-term increase in expenditures which originates from a permanent reduction in debt service. If the perceived benefits from expenditures fall, the incentive to reduce debt will obviously be weaker. Thus we provide an interpretation of the surge in debt levels following the election of right-wing governments which is alternative to strategic debt models (Persson and Svensson, 1989).

Turning to the analysis of monetary policy, we find that the inflation bias is inversely related to current debt levels, and time-dependent - due to the sluggish convergence of debt to steady state. Another result concerns the effects of discretionary stabilisation policies, when both the fiscal authority and the central bank react to supply shocks². In this case uncoordinated monetary policy overreacts to disturbances because it does not internalise the role of debt, which spreads part of the adjustment over future expenditures. In fact the strength of the optimal monetary response to shocks is positively related to the policymaker's discount factor. The more the policymaker is willing to use debt as a shock absorber, the less monetary intervention is needed.

These results bear important implications for the design of monetary institutions. First of all, we analyse delegation schemes. We find that inflation targets are ill-suited to correct the time-varying component of the inflation bias. Furthermore, targets cannot limit the excess sensitivity of monetary policy to shocks. To improve welfare, institutional design should contemplate the assignment of targets to weight-conservative Central Banks. Second, we analyse the performance of a simple zero-

 $^{^{2}}$ Countercyclical policies are often conceived as the exclusive domain of monetary policy, which provides a more flexible instrument. In fact what matters is whether fiscal policy is sufficiently flexible to respond to shocks. Empirical evidence suggests that OECD governments have made substantial use of their debt policies (Sorensen and Yoshua 1998).

inflation rule when the economy is subject to supply shocks. The combination of fiscal intervention and zero-inflation rule presents a stylised account of how members of a monetary union might react to asymmetric shocks. We find that even such extreme form of conservatism is preferred to discretionary monetary stabilisation policy if a small amount of domestic money holdings limits the loss of seigniorage revenues <u>and</u> if the fiscal authority relies heavily on debt as a shock-absorber. The paper is organised as follows. In section 2 we present the model and derive the steady state solution for debt levels and expenditures. In section 3 we identify the precommitment monetary

policy rule and define the inflation bias. In section 4 we evaluate the performance of alternative institutional arrangements. Section 5 concludes.

2. The model

To begin with, we define the aggregate supply function

$$\boldsymbol{y}_{t} = (\boldsymbol{p} - \boldsymbol{p}^{e})_{t} + \boldsymbol{e}_{t} - \boldsymbol{t}_{t}$$
(1)

where output deviations from the socially optimal level, y_t , depend on distortionary taxes³, t_t , a shock e_t , independently distributed with zero mean and finite variance s_e^2 , and inflation surprises $(p - p^e)_t$ where p_t^e defines the rational expectation of inflation.

In each period public expenditures, G_t , are financed by means of public debt, distortionary taxes and seigniorage revenues. Hence the government budget constraint is

$$D_t = (1+r)D_{t-1} + \left(G - t - \hat{k}p\right)_t$$
(2)

where D_t denotes the stock of government debt at the end of period *t*, *r* is the *real* rate of interest⁴. and \hat{k} defines the effect of inflation on seigniorage revenues, as in Beetsma et al. (1997).

³ Following Alesina and Tabellini (1987) we define t as a tax rate on the total revenue of firms.

⁴ To limit analytical complexities we assume that r is constant and government debt is fully indexed, as in Jensen (1994), Beetsma and Bovenberg (1997).

Jensen (1994) postulates an intertemporal version of the loss function presented in Alesina and Tabellini (1987), where it is assumed that losses are quadratic in output, inflation and expenditure deviations from a desired target, \tilde{G} . He shows that in equilibrium inflation and tax distortions disappear because the government is able to build up a stock of negative debt. In other words, reinvested budget surpluses will earn the income necessary to finance expenditures, removing the need for distortionary sources of revenue. Output distortions and inflation therefore persist only during the transition, and are positively related to current debt levels. Convergence to a non-distortionary steady state is ruled out only if the policymaker discounts the future so heavily that the process of debt accumulation becomes unstable. The model offers useful insights on the relationship between debt dynamics and inflation, but the steady state properties appear counterfactual. The zero-inflation equilibrium implies that current generations altruistically bear the costs of running budget surpluses in order to relieve future generations from the burden of distortionary taxation. This outcome might hold in a purely neo-Ricardian world where generations are altruistically linked through bequests, so that the intertemporal distribution of deficits only responds to efficiency considerations. However, Cukierman an Meltzer (1989) show that if some individuals are bequest constrained – i.e. they would like to borrow from future generations leaving negative bequests - fiscal policy may be biased towards excessive debt accumulation. In fact the public debt policy allows bequest-constrained individuals to raise their consumption levels at the expenses of future generations. This happens because deficits are used to subsidise the consumption of bequest-constrained agents, whereas debt will partly substitute capital in the portfolio of non bequest-constrained individuals. Extending the analysis of Cuckierman and Meltzer to account for distortionary taxation and time-inconsistency in monetary policy would quickly render their model intractable. To the contrary, the Jensen model may be easily modified to capture the essence of the contribution of Cuckierman and Meltzer and to obtain the persistence of public debt in equilibrium. Consider the following intertemporal loss function:

$$W_{t} = \sum_{s=t}^{\infty} \boldsymbol{b}^{s-t} \boldsymbol{L}_{s}^{G}$$

$$L_{s}^{G} = \frac{1}{2} \left[(\boldsymbol{y})_{s}^{2} + \boldsymbol{k}_{1} \left(\boldsymbol{G}_{s} - \boldsymbol{\tilde{G}} \right)_{s}^{2} + \boldsymbol{k}_{2} \boldsymbol{p}_{s}^{2} + \boldsymbol{k}_{3} \left(\boldsymbol{\tilde{D}} - \boldsymbol{D}_{s} \right) \right]$$
(3)

where **b** is a discount factor. As in Jensen (1994), we assume that the per-period loss function is quadratic in output, inflation and expenditure deviations from target, \tilde{G} . In addition, the linear term⁵ $k_3(\tilde{D}-D_s)$ implies that running a budget surplus in order to reduce the stock of debt carried over to the next periods entails a political cost for the fiscal authority⁶

To close the model we need further assumptions about policy regimes. In this section we discuss the case of full discretion, where the policymaker and the central bank are characterised by identical preferences, and neither is able to precommit. The Bellman equation is⁷:

$$V(D_{t-1}) = \min_{p_t, t_t, D_t} \left\{ \frac{1}{2} \left[y^2 + k_1 \left(G - \tilde{G} \right)^2 + k_2 p^2 + k_3 \left(\tilde{D} - D \right) \right]_t + b V(D_t) \right\}$$
(4)

where inflation, taxes and the stock of debt carried over to the next period are the policy instruments. To solve the model we derive the first-order conditions.

$$\boldsymbol{k}_{T}(\boldsymbol{G}-\boldsymbol{\tilde{G}})_{t}-\boldsymbol{k}_{3}+\boldsymbol{V}_{D}(\boldsymbol{D}_{t})\boldsymbol{b}=\boldsymbol{0} \tag{5}$$

The intertemporal condition (5), states that in each period D_t is set at the level where the marginal costs⁸ from a policy of debt reduction must equal the discounted value of marginal benefits originating from the greater availability of resources for public spending in the future:

⁵ For sake of analytical tractability we do not consider quadratic deviations of debt from target.

⁶ One may think of \tilde{D} as the level of debt which would emerge if non-distortionary taxes were available in a world where bequest-constrained individuals affect politico-economic equilibria.

$$V_{D} = \frac{\P L_{t+1}^{G}}{\P D_{t}} = -k_{1} (1+r) E_{t} \Big(G_{t+1} - \tilde{G} \Big)$$
(6)

Substituting (6) into (5) and forwarding we obtain the general form of the Euler equation which defines the optimal relationship between expected and current expenditures for any future period:

$$E_t \left(G_{t+j} - \widetilde{G} \right) = \left[\left(G_t - \widetilde{G} \right) - \frac{k_3}{k_j} \right] \left[(1+r) b \right]^{-j}, \quad \forall j \ge 0,$$
(7)

If $k_3 > 0$ the ratio of current to future expenditures increases relative to the case discussed in Jensen (1994).

Then we turn to the static conditions that determine the optimal mix of revenues in each period.

$$-(\mathbf{y})_t + \mathbf{k}_t \left(\mathbf{G} - \tilde{\mathbf{G}} \right)_t = 0 \tag{8}$$

$$(\mathbf{y})_t + \hat{\mathbf{k}}\mathbf{k}_t \left(\mathbf{G} - \tilde{\mathbf{G}}\right)_t + \mathbf{k}_2 \mathbf{p}_t = 0 \tag{9}$$

Condition (8) equates the marginal benefits of a tax-financed increase in expenditures with the marginal costs of higher taxes. Condition (9) equates the perceived marginal costs and benefits of inflation, where the latter include both seigniorage-financed increases in public expenditure and the higher output generated by monetary surprises.

The next step is the definition of the wage setters' forecast rule. In modelling the game between the policy-maker and the private sector we confine our attention to Markov-perfect equilibria, where game history affects current actions of the players only through the state variables. The only concern

 ⁷ We closely follow here Jensen's solution method (Jensen, 1994).
 ⁸ Raising expenditures is obviously beneficial as long as they are below target.

to the private sector here is to prevent monetary surprises⁹, therefore the best inflation forecast rule is:

$$\boldsymbol{p}_t^e = \boldsymbol{E} \boldsymbol{p}_t \tag{10}$$

Using eq.(1), (8), (9) and (10) we obtain the open loop solutions for taxes and inflation:

$$\boldsymbol{t}_{t}^{d} = \boldsymbol{e}_{t} + (\boldsymbol{p} - \boldsymbol{p}^{e})_{t} - \boldsymbol{k}_{1} (\boldsymbol{G} - \boldsymbol{\tilde{G}})_{t} = \boldsymbol{e}_{t} - \boldsymbol{k}_{1} (\boldsymbol{G} - \boldsymbol{\tilde{G}})_{t} - (\boldsymbol{1} + \boldsymbol{\hat{k}}) \frac{\boldsymbol{k}_{1}}{\boldsymbol{k}_{2}} [(\boldsymbol{G} - \boldsymbol{\tilde{G}})_{t} - \boldsymbol{E} (\boldsymbol{G} - \boldsymbol{\tilde{G}})_{t}] (11)$$
$$\boldsymbol{p}_{t}^{d} = -(\boldsymbol{1} + \boldsymbol{\hat{k}}) \frac{\boldsymbol{k}_{1}}{\boldsymbol{k}_{2}} (\boldsymbol{G} - \boldsymbol{\tilde{G}})_{t}$$
(12)

The tax rate rises with the public expenditures gap (eq. 11), and fully incorporates the output shifts due to supply shocks and monetary surprises. As a result inflation responds only to the expenditure gap (eq.12). However supply shocks do have an impact on inflation through their effect on current expenditures. To obtain the optimal level of expenditures under discretion, G_t^d , we proceed as follows. Having imposed the standard no-Ponzi-Game condition:

$$\lim_{t \to \infty} (1+r)^{-t} D_t = 0 \tag{13}$$

the intertemporal budget constraint takes the form¹⁰:

$$\left(G-\widetilde{G}\right)_{t}+E_{t}\sum_{s=t+1}^{\infty}(1+r)^{s-t}\left(G-\widetilde{G}\right)_{s}+\frac{1+r}{r}\widetilde{G}+(1+r)D_{t-1}=\left(\hat{k}p+t\right)_{t}+E_{t}\sum_{s=t+1}^{\infty}(1+r)^{s-t}\left(\hat{k}p+t\right)_{s}$$
(14)

Substituting (7), (11), (12) into (14) yields:

⁹ The focus on Markov-perfect equilibria is standard in monetary policy games (Van der Ploeg, 1995; Svensson, 1997) For a more extensive characterisation see Lockwood and Philippopoulos (1994). This assumption is obviously implicit in Jensen (1994)

¹⁰ The term $\frac{1+r}{r}\tilde{G}$ appears in (14) because we have chosen to express current and future expenditures as deviations from the target.

$$\boldsymbol{G}_{t}^{d} - \boldsymbol{\widetilde{G}} = -\left(\boldsymbol{W}^{d}\right)^{-1} \left\{ \left(1 + r\right) \left(\frac{\boldsymbol{\widetilde{G}}}{r} + \boldsymbol{D}_{t-1}\right) - \frac{\boldsymbol{k}_{3}}{\boldsymbol{k}_{1}\left[\left(1 + r\right)^{2}\boldsymbol{b} - 1\right]} \right\} + \left(\boldsymbol{Q}^{d}\right)^{-1}\boldsymbol{e}_{t}$$
(15)

where:

$$W^{d} = \frac{\left(1 + k_{1} + (1 + \hat{k})\hat{k}\frac{k_{1}}{k_{2}}\right)(1 + r)^{2}b}{\left[(1 + r)^{2}b - 1\right]}; Q^{d} = W^{d} + (1 + \hat{k})\frac{k_{1}}{k_{2}}$$

Solving eq.(2) for G^d , p^d , t^d yields:

$$D_{t}^{d} = \frac{D_{t-1}^{d}}{(1+r)b} - \frac{b(1+r)-1}{b(1+r)r}\tilde{G} + \frac{k_{3}}{k_{1}}\frac{b(1+r)^{2}-1}{b(1+r)^{2}} - \left\{1 - \left(Q^{d}\right)^{-1}\left[1 + k_{1} + \hat{k}\left(1 + \hat{k}\right)\frac{k_{1}}{k_{2}}\right]\right\}e_{t}$$
(16)

which is stable if

$$(1+r)b > 1.$$
 (17)

In the rest of the paper we shall assume that (17) holds.

The sensitivity of expenditures to the current debt burden (eq.15) may be interpreted as follows. An increase in $(1+r)D_{t-1}$ must be matched by an increase in the present value of current and expected

primary surpluses, which is measured by
$$-W^{d}(G^{d} - \tilde{G})$$
. The term $\left[\frac{(1+r)^{2}b}{(1+r)^{2}b-1}\right]^{-1}$ defines the

proportion of adjustment which is implemented immediately. As long as (17) holds, the impact of $(1+r)D_{t-1}$ on G_t falls with the discount factor, because a greater share of the debt burden is shifted onto the future. Moreover, the reduction in current expenditures necessary to balance the intertemporal budget constraint is inversely related to the strength of the response, in each period, of

taxes and inflation to the expenditures gap. The term $\left[1 + k_1 + (1 + \hat{k})\hat{k}\frac{k_1}{k_2}\right]$ describes the sensitivity

of the primary balance to a change in G_t .

Debt policy is also used to cushion expenditures from supply shocks. Observe that expenditures are more sensitive to debt changes than to supply shocks, i.e. $W^d < Q^d$. This result is intuitively explained as follows. An adverse supply shock requires a tax reduction that drains resources otherwise available for public spending. However, the unexpected fall in expenditures triggers an inflation surprise that stimulates output, raises seigniorage revenues and limits the tax reduction (eq.11). The term $(1+\hat{k})\frac{k_1}{k_2}$ describes how the inflation surprise weakens the impact of adverse supply shocks on expenditures. The relevance of this effect is inversely related to the size of the discount factor. In fact when **b** is relatively low the response of current expenditures to shocks is limited because the government shifts a relatively large part of the adjustment onto future periods. In steady state we get:

$$D_{\infty}^{d} = -\frac{\tilde{G}}{r} + \frac{k_{3}}{k_{1}(1+r)[b(1+r)-1]}$$
(18a)

$$\left(G_{\infty}^{d} - \tilde{G}\right) = -\frac{k_{3}}{k_{1}\left(1 + r\right)\left[1 + k_{1} + \hat{k}\left(1 + \hat{k}\right)\frac{k_{1}}{k_{2}}\right]}$$
(18b)

 D_{∞}^{d} rises with k_{3} . If the latter term is sufficiently strong, positive debt levels persist in equilibrium. Only if $k_{3} = 0$ inflation and distortionary taxes disappear in equilibrium. Comparing (7) to (18a,b) it is easy to see that the policymaker's reluctance to reduce debt leads to higher current expenditures but causes an inefficient equilibrium in steady state. We are also able to assess the implications of political shifts on debt accumulation. For instance, if society assigns less importance to the welfare system and is relatively more concerned with the distortionary effects of higher taxes - i.e. both \tilde{G} and k_1 fall - the policymaker will cut taxes and expenditures but accumulate debt. The latter effect takes place because the policymaker will be less concerned with the long-term reduction of resources available for public expenditures.

3. Monetary precommitment

To assess the welfare implications of a discretionary regime we obviously need to define a precommitment rule as the benchmark case. Eq.(10) shows that any systematic attempt to offset output distortions by means of monetary surprises simply raises inflation expectations. Thus, absent supply shocks, the optimal policy rule should simply equate the marginal costs of higher inflation to the marginal benefits in terms of greater expenditures¹¹. On the other hand, optimal tax considerations intuitively suggest that both monetary and fiscal policies should adjust in response to shocks. In fact eq.(15) shows that monetary policy flexibility allows to reduce the sensitivity of expenditures to supply shocks. Therefore we assume that the central bank is able to precommit to the following policy rule:

$$\boldsymbol{p}_{t}^{c} = -\hat{\boldsymbol{k}}\frac{\boldsymbol{k}_{t}}{\boldsymbol{k}_{2}}\boldsymbol{E}(\boldsymbol{G}-\boldsymbol{\tilde{G}})_{t} - \boldsymbol{f}\left[(\boldsymbol{G}-\boldsymbol{\tilde{G}})-\boldsymbol{E}(\boldsymbol{G}-\boldsymbol{\tilde{G}})\right]_{t} \quad (19)$$

The term $\left[-\hat{k}\frac{k_1}{k_2}E(G-\tilde{G})_t\right]$ implies that on average monetary policy will only provide the optimal

amount of seigniorage revenue necessary to finance the expenditures forecast¹², neglecting the effect of tax distortions on output. The optimal monetary response to expenditure shocks, i. e. parameter f, should strike a balance between the need to stabilise expenditures by adjusting seigniorage, and

¹¹ This point is also discussed in Beetsma and Bovenberg (1997).

¹² $E(G - \tilde{G})_t$ is the rational expectation of expenditures available at the time when inflation expectations are formed.

the increase in inflation volatility that such policy would inevitably cause. To identify the optimal (precommitment) inflation rate, the inflation bias and the optimal value of f we proceed as follows. From (11), (14) and (19) we get the solutions for the tax rate, expenditures and debt.

$$\boldsymbol{t}_{t}^{d} = \boldsymbol{e}_{t} - \boldsymbol{k}_{1}(\boldsymbol{G}_{t} - \boldsymbol{\tilde{G}}) - \boldsymbol{f} \left[\left(\boldsymbol{G} - \boldsymbol{\tilde{G}}\right)_{t} - \boldsymbol{E} \left(\boldsymbol{G} - \boldsymbol{\tilde{G}}\right)_{t} \right]$$

$$\boldsymbol{G}_{t}^{c} - \boldsymbol{\tilde{G}} = -\left(\boldsymbol{W}^{c}\right)^{-1} \left[\left(1 + r\right) \left(\frac{\boldsymbol{\tilde{G}}}{r} + \boldsymbol{D}_{t-1} \right) - \frac{\boldsymbol{k}_{3}}{\boldsymbol{k}_{1} \boldsymbol{r}} \right] + \left(\boldsymbol{Q}^{c}\right)^{-1} \boldsymbol{e}_{t}$$
(20)
(21)

where $W^{c} = \left(1 + k_{1} + \hat{k} \frac{k_{1}}{k_{2}}\right) \left[\frac{(1+r)^{2} b}{(1+r)^{2} b - 1}\right]; Q^{c} = W^{c} + f$

$$D_{t}^{c} = \frac{D_{t-1}}{(1+r)b} - \frac{b(1+r)-1}{b(1+r)r}\tilde{G} + \frac{k_{3}}{k_{1}r}\frac{b(1+r)^{2}-1}{b(1+r)^{2}} - \left\{1 - \left(Q^{c}\right)^{-1}\left(1+k_{1}+\hat{k}f\right)\right\}e_{t} \quad (22)$$

Using (11), (12), (15) (19) and (21) we get the inflation bias in each period:

$$E\{p^{d} - p^{c}\} = \left\{ \left[\frac{(1+\hat{k})}{1+k_{1} + (1+\hat{k})\hat{k}\frac{k_{1}}{k_{2}}} - \frac{\hat{k}}{1+k_{1} + \hat{k}\frac{k_{1}}{k_{2}}} \right] \left[\frac{(1+r)^{2}b - 1}{(1+r)^{2}b} \right] \left[(1+r)\left(\frac{\tilde{G}}{r} + D_{t-1}\right) - \frac{k_{3}}{k_{1}[(1+r)^{2}b - 1]} \right] \right\}$$

$$(23)$$

The first r.h.s. term of (23) shows that the inflation bias is less than proportional to the differences in the sensitivity of inflation to the output gap. In fact the larger amount of seigniorage revenues available under discretion allows the policymaker to raise expenditures. This, in turn, mitigates the inflation bias. The second r.h.s. term of (23) is already familiar: it defines the policymaker's willingness to adjust the current primary balance in response to the former accumulation of debt. The

weaker it is, the smaller are both the expenditure gap and the inflation bias. Finally, the third r.h.s. term of (23) shows that past accumulation of debt widens the expenditure gap and exacerbates the inflation bias. Moreover, it is interesting to observe that the policymaker aversion to a reduction of debt generates a smaller current inflation bias. This happens because the larger k_3 the greater the level of current expenditures. However the following discussion will show that this causes a larger inflation bias in steady state.

Comparing (16) and (22) it is easy to see that the precommitment rule cannot affect the level of debt in steady state: eq. (18a) still holds. On the other hand the precommitment rule lowers seigniorage revenues and reduces the level of expenditures in steady state:

$$\left(G_{\infty}^{c} - \tilde{G}\right) = -\frac{k_{3}}{k_{1}\left(1 + r\right)\left[1 + k_{1} + \hat{k}\frac{k_{1}}{k_{2}}\right]}$$
(24)

As a consequence, the inflation bias in steady state is:

$$E\{p_{\infty}^{d} - p_{\infty}^{c}\} = \left\{\frac{(1+\hat{k})}{\left[1+k_{1}+(1+\hat{k})\hat{k}\frac{k_{1}}{k_{2}}\right]} - \frac{\hat{k}}{\left(1+k_{1}+\hat{k}\frac{k_{1}}{k_{2}}\right)}\right\}\frac{k_{3}}{k_{2}(1+r)} (25)$$

Substituting (19), (20) and (21) into the welfare function and minimising its expected value with respect to f we get the optimal monetary response to stochastic shifts in expenditures¹³:

$$f^* = \frac{(1+k_1)}{W^c} \frac{k_1}{k_2}$$
(26)

The optimal monetary response to supply shocks is weaker than under discretion. The intuition behind this result is as follows. We have already shown that under discretion the monetary response

to unexpected shifts in expenditures triggers a fiscal reaction. This, in turn, reduces the sensitivity of current taxes and expenditures to shocks. Under a balanced budget rule the latter effect would always offset the welfare losses deriving from the greater volatility of inflation. Unfortunately, in this case the policy rule (12) ignores the role of debt policy, which spreads part of the adjustment over future expenditures and limits the need for monetary intervention. Therefore a discretionary monetary policy is too "interventionist". The problem gets more serious when the discount factor is relatively low and the policymaker is relatively more willing to adjust the debt level rather than current expenditures. This explains why f^* falls as $(1+r)^2 \mathbf{b} \to 1$.

4. Implications for institutional design

Let us now turn to the analysis of policy design. Suppose the policymaker delegates monetary policy to an independent central bank, held accountable by means of a performance-based contract which includes an explicit inflation target¹⁴. The central banker's loss function may be written as:

$$W_t^b = \sum_{s=t}^{\infty} \boldsymbol{b}^{s-t} \boldsymbol{L}_s^b$$

$$L_s^b = \frac{1}{2} \left[\boldsymbol{y}_s^2 + \boldsymbol{k}_1 \left(\boldsymbol{G}_s - \boldsymbol{\tilde{G}} \right)^2 + \boldsymbol{k}_2 \left(\boldsymbol{p}_s - \boldsymbol{p}_s^T \right)^2 \right]$$
(27)

The policymaker and the central bank minimise (3) and (27) respectively¹⁵. The open loop solutions for debt policy and the tax rate, defined in eq. (7) and (11), still hold. By contrast eq. (12) becomes:

¹⁵ We assume that the fiscal and monetary authorities act non-co-operatively, confining our analysis to Markov equilibria, characterised by a combination of t_t , p_t , D_t such that *i*) p_t , D_t minimise (4) taking p_t as given; *ii*) p_t minimises (27) taking p_t , D_t as given.

¹³ See the Appendix for a proof.

¹⁴ It would be straightforward to show that the results presented in this section would hold even if the central bank were not concerned with the level of public expenditures. Furthermore, our conclusions do not depend on the specific features of the contract between the bank and her political principal, provided that such contract cannot be state contingent.

$$\boldsymbol{p}_{t}^{b} = \boldsymbol{p}_{t}^{T} - \left(1 + \hat{\boldsymbol{k}}\right) \frac{\boldsymbol{k}_{t}}{\boldsymbol{k}_{z}} \left(\boldsymbol{G} - \boldsymbol{\widetilde{G}}\right)_{t}$$
(28)

From eq. (25) it is intuitively obvious that setting

$$\boldsymbol{p}^{T} = -\boldsymbol{E} \left\{ \boldsymbol{p}_{\infty}^{d} - \boldsymbol{p}_{\infty}^{c} \right\}$$
⁽²⁹⁾

removes the inflation bias in steady state. Unfortunately, the inflation bias is path dependent because debt is adjusted to supply shocks. It is widely acknowledged that it is practically impossible to encode fully state-contingent rules into a monetary constitution. Thus the inflation target can only remove the steady state component of the inflation bias, leaving the history-dependent component entirely unaffected¹⁶. By the same token it would be straightforward to show that the responses of expenditures, taxes, inflation and debt to supply shocks are identical to the case of discretion, eq.(11), (12), (15) and (16).

Consider instead what would happen if monetary policy were delegated to a weight-conservative central bank¹⁷, characterised by the following loss function:

$$W_{t}^{B} = \sum_{s=t}^{\infty} \boldsymbol{b}^{t-s} \boldsymbol{L}_{s}^{B}$$

$$L_{s}^{B} = \frac{1}{2} \left[(\boldsymbol{y}_{s})^{2} + \boldsymbol{a}\boldsymbol{k}_{1} (\boldsymbol{G}_{s} - \boldsymbol{\tilde{G}})^{2} + \boldsymbol{g}\boldsymbol{k}_{2} (\boldsymbol{p}_{s})^{2} \right]$$

$$0 \le \boldsymbol{a} < 1; \boldsymbol{g} > 1;$$
(30)

In this case the closed-loop solution for inflation is:

$$\boldsymbol{p}_{t}^{B} = -\frac{\left(1+a\hat{k}\right)k_{1}}{gk_{2}}\left(\boldsymbol{G}_{t}^{B}-\boldsymbol{\tilde{G}}\right) = \frac{\left(1+a\hat{k}\right)k_{1}}{gk_{2}}\left\{\left(\boldsymbol{W}^{B}\right)^{-1}\left[\left(1+r\right)\left(\frac{\boldsymbol{\tilde{G}}}{r}+\boldsymbol{D}_{t}\right)-\frac{k_{3}}{k_{1}r}\right]-\left(\boldsymbol{Q}^{B}\right)^{-1}\boldsymbol{e}_{t}\right\}$$
(31)

where

¹⁶ Svensson (1997) reaches the same conclusion assuming that output is autocorrelated.

¹⁷ Observe that (30) accounts for the case where expenditures do not enter the Bank's objective function, a = 0, and even with the possibility that price stability is the only objective for monetary policy, $g \to \infty$

$$W^{B} = \left[1 + k_{1} + (1 + a\hat{k})\hat{k}\frac{k_{1}}{gk_{2}}\right]\left[\frac{(1 + r)^{2}b}{(1 + r)^{2}b - 1}\right]; Q^{B} = W^{B} + (1 + a\hat{k})\frac{k_{1}}{gk_{2}}$$

A simple example will show that weight conservatism may improve welfare. For instance, suppose that $\frac{1+a\hat{k}}{g} = \hat{k}$, then $W^{B} = W^{c}$. In this case the policy response to current debt levels coincides

with the precommitment solution. From this point of view weight-conservatism is unambiguously preferred to an inflation-targeting regime. On the other hand, both regimes generate suboptimal responses to supply shocks. In the appendix we show that the response of a goal independent central bank is preferred for relatively low values of b. This happens because the more the policymaker is willing to use debt as a shock absorber the less monetary intervention is needed. It would be straightforward to show that assigning a target to a weight-conservative central banker would always improve welfare relative to the two alternatives discussed above.

Let us now turn to the analysis of a simple zero-inflation rule. Beetsma et al. (1996) have pointed out that, absent supply shocks, a zero-inflation rule dominates discretion if \hat{k} is sufficiently small. Extending their analysis to account for supply shocks, we are able to show that such rule may generate second order losses if <u>both</u> \hat{k} and the discount factor are sufficiently small. This requires that seigniorage is a relatively unimportant source of revenues and that the government relies heavily on debt policy as a buffer against supply shocks. The same argument obviously carries over to the comparison with delegation schemes.

4. Conclusions

This paper presents a model where governments are unable to eradicate inflationary pressures by means of an appropriate debt policy. The choice of monetary regime has no impact on steady state debt levels, which are crucially influenced by the government reluctance to engage in a policy of debt reduction. On the other hand, in steady state public expenditures are constrained by the availability of seigniorage revenues.

Turning to the analysis of policy regimes, we find that "pure" inflation targets are unable to remove the path-dependent component of the inflation bias and generate monetary responses to shocks which are too interventionist. Thus institutional design should contemplate the assignment of targets to weight-conservative Central Banks. The government willingness to use debt policy as a buffer to stabilise expenditures crucially affects the choice of central bank preferences: the latter should be more conservative the more interventionist is debt policy.

These results suggest that worries about the "excessive" independence and conservatism of the European Central Bank might be overstated if the "Stability and Growth Pact" will allow EMU members to implement a countercyclical debt policy. Since seigniorage revenues in Europe have become negligible long before the start of EMU, the argument obviously carries over to the case of asymmetric disturbances. In fact our conclusions about the performance of a simple zero-inflation rule suggest that the adverse effects caused by the renounce to a national monetary policy may be relatively unimportant if policymakers are willing and able to use their fiscal instruments to stabilise output and expenditures. Therefore EMU members should quickly correct their long-term fiscal stances and bring structural deficits closer to balance. If this does not happen, the Stability Pact will become a straightjacket on national fiscal policies and the potential gains from fiscal stabilisation will remain out of reach.

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Appendix

To determine the optimal monetary response to stochastic shifts in expenditures we proceed as follows. Taking the expected value of the loss function (3):

$$E\{W_{t}^{c}\} = E\left\{\frac{1}{2}\sum_{s=t}^{\infty} \boldsymbol{b}^{s-t}\left[\left(y_{s}^{c}\right)^{2} + k_{1}\left(G_{s}^{c} - \widetilde{G}\right)^{2} + k_{2}\left(\boldsymbol{p}_{s}^{c}\right)^{2} + k_{3}\left(\widetilde{D} - D_{s}^{c}\right)\right]\right\} = E\left\{\frac{1}{2}\sum_{s=t}^{\infty} \boldsymbol{b}^{s-t}\frac{1}{2}k_{3}\left(\widetilde{D} - D_{s}^{c}\right)\right\} + E\left\{\frac{1}{2}\sum_{s=t}^{\infty} \boldsymbol{b}^{s-t}\frac{1}{2}k_{3}\left(\widetilde{D} - D_{s}^{c}\right)\right\}$$

$$+\frac{1}{2}\left[\frac{(1+r)^{2} \mathbf{b}}{(1+r)^{2} \mathbf{b}-1}\right]\left\{\frac{\left[\left(k_{1}+k_{1}^{2}+\left(\hat{k}\right)^{2}\frac{k_{1}^{2}}{k_{2}}\right)\right]\left[\left(1+r\right)\left(\frac{\tilde{G}}{r}+D_{t-1}\right)-\frac{k_{3}}{k_{1}\left[(1+r)^{2} \mathbf{b}-1\right]}\right]^{2}+\frac{\left(k_{1}+k_{1}^{2}+k_{2} \mathbf{f}^{2}\right) \mathbf{s}_{e}^{2}}{\left(\mathbf{Q}^{\circ}\right)^{2}}\right]$$

where we know from eq. (22) that the expected path of debt, i.e. the term $E\left\{\sum_{s=t}^{\infty} \boldsymbol{b}^{s-t} \boldsymbol{k}_{3}\left(\boldsymbol{\tilde{D}} - \boldsymbol{D}_{s}^{c}\right)\right\}$, is

independent from f.

Observe that
$$\frac{\partial E\{W_t^c\}}{\partial f} = \frac{W^c j k_2 - (k_1 + k_1^2)}{(Q^c)^3}$$
. Hence $\frac{\partial E\{W_t^c\}}{\partial f} < 0$ if $f = 0$ and $\frac{\partial E\{W_t^c\}}{\partial f} \to 0$ from above

as $f \rightarrow \infty$

Therefore
$$E\{W_t^c\}$$
 is minimised¹⁸ at $f^* = \frac{(1+k_1)}{W^c} \frac{k_1}{k_2}$

To show that the monetary response to shocks implemented by a weight-conservative central banker may be preferred to the corresponding monetary policy carried out under an inflation targeting regime we proceed as follows.

Under discretion the expected loss amounts to

$$E\left\{W_{t}^{d}\right\} = E\left\{\sum_{s=t}^{\infty} \boldsymbol{b}^{s-t} \frac{1}{2}\left[\left(y_{s}^{d}\right)^{2} + k_{1}\left(\boldsymbol{G}_{s}^{d} - \widetilde{\boldsymbol{G}}\right)^{2} + k_{2}\left(\boldsymbol{p}_{s}^{d}\right)^{2} + k_{3}\left(\widetilde{\boldsymbol{D}} - \boldsymbol{D}_{s}^{d}\right)\right]\right\} = E\left\{\frac{1}{2}\sum_{s=t}^{\infty} \boldsymbol{b}^{s-t} k_{3}\left(\widetilde{\boldsymbol{D}} - \boldsymbol{D}_{s}^{d}\right)\right\} + E\left\{\sum_{s=t}^{\infty} \boldsymbol{b}^{s-t} k_{3}\left(\widetilde{\boldsymbol{D}} - \boldsymbol{D}_{s}^{d}\right)^{2}\right\}$$

¹⁸ These results obviously hold if $(1+r)\mathbf{b} > 1$

$$+\frac{\left(k_{1}+k_{1}^{2}+\left(1+\hat{k}\right)^{2}\frac{k_{1}^{2}}{k_{2}}\right)}{2}\left[\frac{(1+r)^{2}b}{(1+r)^{2}b-1}\right]\left\{\left(W^{d}\right)^{-2}\left[\left(1+r\right)\left(\frac{\tilde{G}}{r}+D_{t-1}\right)-\frac{k_{3}}{k_{1}\left[\left(1+r\right)^{2}b-1\right]}\right]^{2}+\left(Q^{d}\right)^{-2}\boldsymbol{s}_{e}^{2}\right\}$$

If monetary policy is delegated to an independent central bank, eq. (26), whose preferences are such

that $\frac{1+a\hat{k}}{g} = \hat{k}$, the policymaker's expected loss function amounts to:

$$E\left\{W_{t}^{B}\right\} = E\left\{\sum_{s=t}^{\infty} \boldsymbol{b}^{s-t} \frac{1}{2}\left[\left(y_{s}^{B}\right)^{2} + k_{1}\left(\boldsymbol{G}_{s}^{B} - \boldsymbol{\tilde{G}}\right)^{2} + k_{2}\left(\boldsymbol{p}_{s}^{B}\right)^{2} + k_{3}\left(\boldsymbol{\tilde{D}} - \boldsymbol{D}_{s}^{B}\right)\right]\right\} = E\left\{\frac{1}{2}\sum_{s=t}^{\infty} \boldsymbol{b}^{s-t}k_{3}\left(\boldsymbol{\tilde{D}} - \boldsymbol{D}_{s}^{B}\right)\right\} + E\left\{\sum_{s=t}^{\infty} \boldsymbol{b}^{s-t}k_{3}\left(\boldsymbol{\tilde{D}} - \boldsymbol{D}_{s}^{B}\right)\right\}$$

$$+\frac{\left(k_{1}+k_{1}^{2}+\hat{k}^{2}\frac{k_{1}^{2}}{k_{2}}\right)}{2}\left[\frac{(1+r)^{2}b}{(1+r)^{2}b-1}\right]\left\{\left(W^{c}\right)^{-2}\left[(1+r)\left(\frac{\tilde{G}}{r}+D_{t-1}\right)-\frac{k_{3}}{k_{1}\left[(1+r)^{2}b-1\right]}\right]^{2}+\left(Q^{B}\right)^{-2}s_{e}^{2}\right\}$$

where
$$Q^{B} = W^{c} + \hat{k} \frac{k_{1}}{k_{2}}$$

 $E\{W_{t}^{d}\} - E\{W_{t}^{B}\} = \frac{1}{2} \left[\frac{(1+r)^{2} b}{(1+r)^{2} b-1} \right] \left\{ \frac{\left[\frac{k_{1} + k_{1}^{2} + (1+\hat{k})^{2} \frac{k_{1}^{2}}{k_{2}} - \frac{k_{1} + k_{1}^{2} + \hat{k}^{2} \frac{k_{1}^{2}}{k_{2}}}{(W^{c})^{2}} - \frac{k_{1} + k_{1}^{2} + \hat{k}^{2} \frac{k_{1}^{2}}{k_{2}}}{(W^{c})^{2}} \right] \left[(1+r) \left(\frac{\tilde{G}}{r} + D_{t-1} \right) - \frac{k_{3}}{k_{1} [(1+r)^{2} b-1]} \right]^{2} \right\} + \frac{1}{2} \left[\frac{(1+r)^{2} b}{(1+r)^{2} b-1} \right] \left\{ \frac{\left[\frac{k_{1} + k_{1}^{2} + (1+\hat{k})^{2} \frac{k_{1}^{2}}{k_{2}} - \frac{k_{1}^{2} + \hat{k}^{2} \frac{k_{1}^{2}}{k_{2}} - \frac{k_{2}^{2} + \hat{k}^{2} - \frac{k_{2}^{2}}{k_{2}} - \frac{k_{2}^{2} + \hat{k}^{2} - \frac{k_{2}^{2}}{k_{2}} - \frac{k_{2}^{2} + \hat{k}^{2} - \frac{k_{2}^{2}}{k_{2}} - \frac{k_{2}^{2} + \frac{k_{2}^{2} + \frac{k_{2}^{2}}{k_{2}} - \frac{k_{2}^{2} + \frac{k_{2}^{2}}{k_{2}} - \frac{k_{2}^{2} + \frac{k_{2}^{2}}{k_{2}} - \frac{k_{2}^{2}}{k_{2}} - \frac{k_{2}^{2} + \frac{k_{2}^{2}}{k_{2}} - \frac{k_{2}^{2}}{k_{2}} - \frac{k_{2}^{2} + \frac{k_{2}^{2}}{k_{2}} - \frac{k_{2}^{2} + \frac{k_{2}^{2}}{k_{2}} - \frac{k_{2}^{2} + \frac{k_{2}^{2}}{k_{2}} - \frac{k_{2}^{2} + \frac{k_{2}^{2}}{k_{2}} - \frac{k_{2}^{2} + \frac{k_{2}^{2}}{k_{2}} - \frac{k_{2}^{2} + \frac{k_{2}^{2}}{k_{2}} - \frac{k_{2}^{2}}{k_{2$

It would be straightforward to show that the sign of the term in the first curly bracket is unambiguously positive. On the other hand the sign of the term in the second curly bracket is positive if

$$+\left[\frac{(1+r)^{2} \mathbf{b}}{(1+r)^{2} \mathbf{b}-1}\right]^{2} \left\{ \left[\left(1+k_{1}+(1+\hat{k})\hat{k}\frac{k_{1}}{k_{2}}\right)(1-\hat{k}^{2})+(1+k_{1})\hat{k}\right] \left(1+k_{1}+\hat{k}\frac{k_{1}}{k_{2}}\right) \right\} + 2\left[\frac{(1+r)^{2} \mathbf{b}}{(1+r)^{2} \mathbf{b}-1}\right] \left\{ \left(1+k_{1}+\hat{k}\frac{k_{1}}{k_{2}}\right) \left(1+2\hat{k})\hat{k}\frac{k_{1}}{k_{2}}-1\right] - \left(1+\hat{k}\left(1+k_{1}+\hat{k}^{2}\frac{k_{1}}{k_{2}}\right)\hat{k}^{2}\frac{k_{1}}{k_{2}}\right) + -\left(1+2\hat{k}\right)\frac{k_{1}}{k_{2}}(1+k_{1}) > 0 \right\} \right\}$$

Since the monetary response to shocks under inflation targeting corresponds to the case of discretion, this implies that monetary policy delegation to weight-conservative central banker characterised by (30) is always preferred to an inflation-targeting regime if the term $\left[\frac{(1+r)^2 \mathbf{b}}{(1+r)^2 \mathbf{b}-1}\right]$

is sufficiently large.

Turning to the case of a zero inflation rule

$$E\{W_{t}^{d}\}-E\{W_{t}|p=0\}=$$

$$\left[\frac{(1+r)^{2}b}{(1+r)^{2}b-1}\right]\left\{\left[\frac{\left(k_{1}+k_{1}^{2}+(1+\hat{k})^{2}\frac{k_{1}^{2}}{k_{2}}\right)}{(W^{d})^{2}}-\frac{1}{\left[\frac{(1+r)^{2}b}{(1+r)^{2}b-1}\right]^{2}(1+k_{1})}\right]\left[(1+r)\left(\frac{\tilde{G}}{r}+D_{t-1}\right)-\frac{k_{3}}{k_{1}\left[(1+r)^{2}b-1\right]}\right]^{2}$$

$$+\left[\frac{(1+r)^{2}b}{(1+r)^{2}b-1}\right]\left\{\frac{\left(k_{1}+k_{1}^{2}+(1+\hat{k})^{2}\frac{k_{1}^{2}}{k_{2}}\right)}{(Q^{d})^{2}}-\frac{1}{\left[\frac{(1+r)^{2}b}{(1+r)^{2}b-1}\right]^{2}(1+k_{1})}s_{e}^{2}\right\}$$

The term in the first curly brackets is positive if \hat{k} is sufficiently small, as in Beetsma et al. (1997). The term in the second curly brackets is positive if

$$+\left[\frac{(1+r)^{2} \mathbf{b}}{(1+r)^{2} \mathbf{b}-1}\right]^{2} \left\{ \left[\frac{\hat{k}\left(1+k_{1}+\frac{k_{1}}{k_{2}}\right)}{\left(\mathbf{W}^{d}\right)^{2}}+\hat{k}^{2} \frac{k_{1}}{k_{2}}-(1+k_{1})\right]\right\} -2\left[\frac{(1+r)^{2} \mathbf{b}}{(1+r)^{2} \mathbf{b}-1}\right] \left\{ \left(1+k_{1}+\hat{k}\left(1+\hat{k}\right)\frac{k_{1}}{k_{2}}\right)\right\} -\left(1+\hat{k}\right)\frac{k_{1}}{k_{2}} > 0$$

This requires both that $\left[\frac{(1+r)^2 \mathbf{b}}{(1+r)^2 \mathbf{b}-1}\right]$ be sufficiently large and that

$$\hat{k} < \left\{ \left[\left(1 + k_1 + \frac{k_1}{k_2} \right)^2 + 4(1 + k_1) \frac{k_1}{k_2} \right]^{\frac{1}{2}} - \left(1 + k_1 + \frac{k_1}{k_2} \right) \right] \left(2 \frac{k_1}{k_2} \right)^{-1} \right\}$$