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Abstract

In this paper we develop an OLG model with heterogeneous agents, money and bequests, introducing occupational choice and financing constraints when capital markets are imperfect. We show how, under appropriate conditions, all the moments of the distribution are affected by changes in money growth. More precisely, if capital markets are imperfect and heterogeneous agents are liquidity constrained, investment in fixed capital is not efficient and aggregate wages and profits depend on the availability of loanable funds. An increase in money growth may imply a more efficient aggregate investment. Therefore aggregate product and wealth positively depend on an acceleration in money growth.

1 Introduction

Macroeconomic theory generally focuses on the aggregate consequences of monetary policy, without considering its distributional effects. In a previous paper (Longaretti and Delli Gatti, 2004) we have shown that, in an OLG model with money and bequests, if agents were homogeneous, that is in a representative agent economy, monetary policy would be superneutral: output, consumption and wealth (of the representative agent) would be independent of money growth. On the contrary if agents differed from one another as far as income and wealth are concerned, the average levels of output, consumption and wealth would still be independent of money growth but the individual levels of the same variables would indeed be affected by changes in the rate of growth of money. In other words, the first moment of the distribution of income/wealth would be invariant to changes in money growth (i.e. the distribution is mean preserving) but the latter would affect the second and higher moments. In fact by increasing the rate of money growth, agents who are relatively poor in income/wealth become wealthier whereas relatively rich agents become less wealthy. However, the relative ranking is not reversed. As a consequence, while the first moments of the distributions of output, consumption and wealth do not depend on money, higher moments are influenced by the rate of money growth. If the variance is thought of as a rough measure of inequality, then inequality is decreasing with money growth. In other words income and wealth distribution is the source of a *financial accelerator* since monetary policy has clear asymmetric effects.

In this paper we extend the original framework introducing occupational choice and financing constraints when capital markets are imperfect. More precisely we assume the income heterogeneity is the result of an occupational choice: agents may choose wether being workers or entrepreneurs at a fixed cost. If there is no credit for the entrepreneurial investment, when agents are liquidity constrained, there exists a gap between the potential and the effective entrepreneurial share of the population. The occupational choice is therefore no more efficient. We show that in this context under appropriate conditions on the parameters, money is not superneutral, in the sense that all the moments of the distribution are affected by changes in money growth. The higher money growth the more efficient the occupational choice and the higher the average income and wealth. There exists a level of money growth that allows all the potential entrepreneurs to effectively undertake the investment project. Once the effective equals the potential share of the population, money only affects the second and higher moments of the distribution of income and wealth and is superneutral at the aggregate (and average) level.

The paper is organized as follows. In Section 2 we present and discuss the main features of an OLG economy with money and bequests. Section 3 focuses on occupational choice. Section 4 concludes.

2 The environment

For the sake of simplicity, we assume that population is constant and consists of N young and N old individuals (of the previous generation) per period. The i-th individual has (real) wealth (measured in units of output) ω_{it} when young. Output is perishable and therefore wealth cannot be stored to be consumed in the future. For simplicity, preferences are uniform across individuals and the young do not receive utility from consumption. Assuming intergenerational altruism, the well behaved utility function is $U = U(c_{it+1}, b_{it+1})$ where c_{it+1} is consumption of the agent when old and b_{it+1} is bequest of the old to the young (wealth of the young). In a monetary economy, in order to consume when old, the young at time t sells its output to the old of the previous generation at the price P_t in exchange for money M_{it}:

$$M_{it} = P_t \omega_{it}$$

or

$$\frac{M_{it}}{P_t} = \omega_{it} \tag{1}$$

Aggregating across individuals we get:

$$\frac{M_t}{P_t} = \Omega_t \tag{2}$$

where $M_t \equiv \sum_{i=1}^{N} M_{it}$ is the aggregate demand for money and $\Omega_t \equiv \sum_{i=1}^{N} \omega_{it}$ is aggregate wealth. Wealth can be decomposed in income y_{it} which the individual gets taking part in the production process as a worker or an entrepreneur (see below) and bequest b_{it}[1]

$$\omega_{it} = y_i + b_{it} \tag{3}$$

Money is a means of payment and a store of value which can be carried on from one period to the next in order to buy goods. When old, the agent spends the money received when young M_{it} plus a money transfer proportional to the *average money holding* $h_t \equiv \frac{H_t}{N}$, where H_t is the aggregate money supply. Assuming that there is equilibrium on the money market in t, i.e. aggregate supply H_t is equal to aggregate demand M_t, we can define the individual money transfer as:

$$T_{it+1} = \mu h_t = \frac{\mu M_t}{N} \tag{4}$$

$0 < \mu < 1$. The transfer is uniform across individuals while money balances are not necessarily the same for each and every agent.

The old spend money to buy consumption goods and leave a bequest to the young:

$$M_{it+1} = M_{it} + T_{it+1} = M_{it} + \mu h_t = P_{t+1}\left(c_{it+1} + b_{it+1}\right) \tag{5}$$

Dividing by P_{t+1} and substituting (1) into (5) we obtain the lifetime budget constraint:

$$RMB_i \equiv \frac{M_{it} + T_{it+1}}{P_{t+1}} \equiv \frac{M_{it}}{P_t}\frac{P_t}{P_{t+1}} + \frac{\mu h_t}{P_{t+1}} \equiv \theta_{t+1}\left(\omega_{it} + \mu\frac{h_t}{P_t}\right) = c_{it+1} + b_{it+1} \tag{6}$$

where RMB_i stands for real money balances of the old, $\theta_{t+1} \equiv \frac{P_t}{P_{t+1}} \equiv \frac{1}{1+\pi_{t+1}}$ is the real rate of return of money (π_{t+1} is inflation in t+1). According to (6) real money balances are spent either on consumption goods or bequest.

[1]Matter of factly, a bequest is a monetary transfer from the old to the young whose value in nominal terms is B_{it}.Therefore $b_{it} = B_{it}/P_t$ is the real money transfer (i.e. the money transfer at constant prices) from the old to the young.

Equilibrium on the money market is brought about by $M_t = H_t$ so that $\dfrac{M_t}{P_t} = \dfrac{H_t}{P_t} = \Omega_t$. Dividing by N, we get $\dfrac{h_t}{P_t} = \bar{\omega}_t$ where $\bar{\omega}_t$ is average wealth[2]. Therefore, the real money balances of the old can be written as $\theta_{t+1}(\omega_{it} + \mu\bar{\omega}_t)$ and the lifetime budget constraint becomes:

$$\theta_{t+1}(\omega_{it} + \mu\bar{\omega}_t) = c_{it+1} + b_{it+1} \tag{7}$$

In order to derive close form solutions, let's assume preferences are represented by a Cobb-Douglas utility function:

$$U = c_{it+1}^{(\gamma)} b_{it+1}^{(1-\gamma)} \tag{8}$$

with $0 < \gamma < 1$. Maximizing (8) subject to (7) yields:

$$c_{it+1} = \gamma\theta_{t+1}(\omega_{it} + \mu\bar{\omega}_t) \tag{9}$$

$$b_{it+1} = (1-\gamma)\,\theta_{t+1}(\omega_{it} + \mu\bar{\omega}_t) \tag{10}$$

Thanks to the Cobb-Douglas utility function, both consumption and bequest are proportional to RMB_i.

Aggregate transfers (to the old) in t+1 is $H_{t+1} - H_t = \sum_{i=1}^{N} T_{it+1} = N\mu\frac{H_t}{N} = \mu H_t$. Hence the supply of money in t+1 is $H_{t+1} = H_t(1+\mu)$. Thanks to equilibrium on the money market in t $H_{t+1} = M_t(1+\mu)$.

Equilibrium on the money market in t+1 is brought about by $M_{t+1} = H_{t+1}$ or $P_{t+1}\Omega_{t+1} = M_t(1+\mu)$. Dividing by P_t, recalling (2) and rearranging we get:

$$\frac{P_{t+1}}{P_t}\Omega_{t+1} = \frac{M_t}{P_t}(1+\mu) = \Omega_t(1+\mu)$$

or, dividing by N,

$$\frac{P_{t+1}}{P_t}\bar{\omega}_{t+1} = \bar{\omega}_t(1+\mu)$$

[2]Thus, the money transfer is proportional to average nominal wealth: $T_{it+1} = \mu h_t = P_t\bar{\omega}_t$.

and finally

$$\frac{P_{t+1}}{P_t} \equiv 1 + \pi_{t+1} = \frac{1+\mu}{1+g_{t+1}} \tag{11}$$

where g_{t+1} is the rate of growth of aggregate (and average) wealth: $g_{t+1} \equiv \frac{\Omega_{t+1}}{\Omega_t} = \frac{\bar{\omega}_{t+1}}{\bar{\omega}_t}$. If (11) holds, equilibrium in the goods market is assured[3].

Using (11), (10) becomes:

$$b_{it+1} = \frac{1-\gamma}{1+\mu}\frac{\bar{\omega}_{t+1}}{\bar{\omega}_t}\left(\omega_{it} + \mu\bar{\omega}_t\right) \tag{12}$$

Let us focus now on equation (3). For the moment income y_i can be conceived of as non-inherited wealth. In other words it can be thought of as an exogenous variable. Later on, we will specify it as income earned by workers (wages) and entrepreneurs (profits).

The distribution of income across agents is the *primary* distribution, while the distribution of wealth is *secondary*, i.e. derived from the former by adding the bequest. As it will become clear in a moment, also the distribution of bequest is secondary, i.e. derived from the distribution of income.

Averaging (3) one gets

$$\bar{\omega}_t = \bar{y} + \bar{b}_t \tag{13}$$

where $\bar{\omega}_t$, $\bar{y}$ and $\bar{b}_t$ are average wealth, income and bequests respectively. We can carry on the dynamic analysis in terms of income or bequest. Substituting (3) into (12) we obtain the law of motion of wealth:

$$\omega_{it+1} = y_i + \frac{1-\gamma}{1+\mu}\frac{\bar{\omega}_{t+1}}{\bar{\omega}_t}\left(\omega_{it} + \mu\bar{\omega}_t\right) \tag{14}$$

[3] Aggregating the budget constraints, it turns out that the sum of aggregate consumption and aggregate bequest must be equal to the aggregate real money balances of the old, i.e. $\theta_{t+1}\left(\Omega_t + \mu N h_t\right) = \sum_i c_{it+1} + \sum_i b_{it+1}$. But $h_t = \bar{\omega}_t$ and $Nh_t = N\bar{\omega}_t = \Omega_t$.Therefore: $\theta_{t+1}\left(1+\mu\right)\Omega_t = C_{t+1} + B_{t+1}$.If (11) holds true, then $\Omega_{t+1} = \theta_{t+1}\left(1+\mu\right)\Omega_t$. Substituting this expression into the previous one yields $\Omega_{t+1} = \sum_i c_{it+1} + \sum_i b_{it+1}$. But income is part of total wealth, i.e. $\Omega_{t+1} = Y_{t+1} + \sum_i b_{it+1}$.Hence $Y_{t+1} = \sum_i c_{it+1}$ which is the equilibrium condition on the goods market.

Averaging (14) we obtain

$$\bar{\omega}_{t+1} = \bar{y} + (1-\gamma)\,\bar{\omega}_{t+1}$$

which simplifies to

$$\bar{\omega}_{t+1} = \frac{\bar{y}}{\gamma}$$

which is constant over time. Therefore we can write

$$\bar{\omega} = \frac{\bar{y}}{\gamma} \tag{15}$$

The law of motion of wealth is:

$$b_{it+1} = \frac{1-\gamma}{1+\mu}\frac{\bar{y}+\bar{b}_{t+1}}{\bar{y}+\bar{b}_t}\left(y_i + b_{it} + \mu\bar{y} + \mu\bar{b}_t\right) \tag{16}$$

There is a *mean field effect* at work, here: individual bequest in t+1 depends not only on individual bequest in t but also on average bequest. The mean field effect captures non-strategic interaction between the individual agent and the rest of the population proxied by the average agent (see Aoki 1996 and the references therein).

Averaging (16) we get that average bequest is constant over time

$$\bar{b} = \frac{1-\gamma}{\gamma}\bar{y} \tag{17}$$

Finally it is easy to prove that average consumption is

$$\bar{c} = \bar{y} \tag{18}$$

3 Workers and entrepreneurs

In what follows we consider income heterogeneity as the result of an occupational choice made by heterogeneous agents. Since birth, the i-th agent is endowed with an investment project whose return is ρ_i. The return is distributed as a uniform random variable with support (0,1).When young, they make an occupational choice, which consists in being a worker or an entrepreneur. The young population, therefore, consists of workers and entrepreneurs. Both types of agents supply workhours to the "production sector" which produces output and sells it to the old of the previous generation against money. This generates the income of the young, i.e. the wage of the worker, which is fixed at w, or the profit of the entrepreneur. In order to become an entrepreneur an individual must pay a fixed cost r. Therefore, the i-th individual, if entrepreneurs, gets profit $\pi_i = \rho_i - r$.Workers have the same income w, while entrepreneurs' income is differentiated. The old receive also the money transfer. Finally, as in the previous case, the young receives also a bequest b_{it}. Output therefore is $\omega_{it}^w = w + b_{it}$ and $\omega_{it}^e = \pi_i + b_{it} = \rho_i + b_{it} - r$ for the worker and the entrepreneur respectively.

Preferences are represented by $U = (c_{it+1})^{\gamma} (b_{it+1})^{1-\gamma}$. Therefore the young do not consume. They exchange their wealth (income and bequest) for money: $P_t \omega_{it}^j = M_{it}^j$, $j = w, e$. The old receive money transfers ($T_{it+1} = \mu h_t$) and spend their money to consume and leave a bequest. LBC is (see section 2): $\theta_{t+1} \left(\omega_{it}^j + \mu \frac{h_t}{P_t} \right) = c_{it+1} + b_{_{it+1}}$where $\theta_{t+1} \left(\omega_{it}^j + \mu \frac{h_t}{P_t} \right)$ are real money balances of the old.

Since, given the preferences, indirect utility is

$$U^j = \left(\gamma \theta_{t+1} \left(\omega_{it}^j + \mu \frac{h_t}{P_t} \right) \right)^{\gamma} \left[(1-\gamma) \theta_{t+1} \left(\omega_{it}^j + \mu \frac{h_t}{P_t} \right) \right]^{1-\gamma}$$

or

$$U^j = (\gamma)^{\gamma} (1-\gamma)^{1-\gamma} \theta_{t+1} \left(\omega_{it}^j + \mu \frac{h_t}{P_t} \right)$$

and the real rate of return on money and money transfers are uniform across the population, the occupational choice depends on the relative magnitude of income obtained when young as worker or entrepreneur.

The individual becomes entrepreneur if $\pi_{it} + b_{it} = \rho_i - r + b_{it} \geq w + b_{it}$ or $\rho_i \geq w + r \equiv \widehat{\rho}$, i.e. if his return is high enough to yield a profit higher than the wage. $\widehat{\rho}$ is the minimum return on his project the individual must have in order to become an entrepreneur. It turns out, quite simply, that the minimum return for the entrepreneur is equal to the sum of the wage and the fixed cost. Since the return is distributed as a uniform random variable with support (0,1), it is clear that $w + r \equiv \widehat{\rho}$ is also the share of workers in the population. This share is constant and independent of money growth.

The profit of the entrepreneur π_i is distributed as a uniform random variable with support $(w = \widehat{\rho} - r, 1 - r)$ where $1 - r$ is the maximum profit (by the assumption above). Therefore average income of the entrepreneur is

$$\bar{\pi} = \frac{1 + w - r}{2} \tag{19}$$

Within class inequality is zero for the workers (they all receive the same wage), $V(\pi_i) = \frac{(1-r-w)^2}{12}$. Average income is

$$\bar{y} = w\widehat{\rho} + \bar{\pi}(1 - \widehat{\rho}) = \frac{w^2 + 2wr + 1 - 2r + r^2}{2} \tag{20}$$

The variance of income for the population as a whole is $V(y_i) = (1 - \widehat{\rho}) \frac{(1-r-w)^2}{12}$.

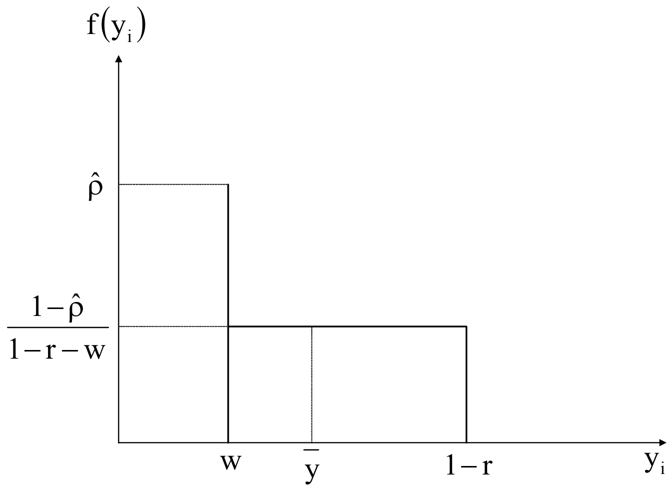

Figure 1

Let us define now the laws of motion. Recalling, as shown above, that $\frac{h_t}{P_t} = \bar{\omega}_t$ where $\bar{\omega}_t$ is average wealth of the whole economy, and $\bar{\omega} = \bar{y} + \bar{b}$ where $\bar{y}$ is average income and $\bar{b}$ average bequest. Average bequest is

$$\bar{b} = b^w \widehat{\rho} + \bar{b}^e (1 - \widehat{\rho})$$

Each and every worker has the following law of motion of wealth:

$$b^w_{t+1} = \frac{1-\gamma}{1+\mu}\left(w + b^w_t + \mu\bar{y} + \mu\bar{b}\right) \tag{21}$$

The steady state of (21) is

$$b^w = \frac{1-\gamma}{\mu+\gamma}\left(w + \mu\bar{y} + \mu\bar{b}\right) \tag{22}$$

The individual with $\rho_i \geq w + r \equiv \widehat{\rho}$ has the following law of motion of wealth:

$$b^e_{it+1} = \frac{1-\gamma}{1+\mu}\left(\pi_i + b^e_{it} + \mu\bar{y} + \mu\bar{b}\right) \tag{23}$$

whose steady state is

$$b^e_i = \frac{1-\gamma}{\mu+\gamma}\left(\pi_i + \mu\bar{y} + \mu\bar{b}\right) \tag{24}$$

Averaging (23) we obtain the law of motion of the average wealth of the entrepreneur

$$\bar{b}^e_{t+1} = \frac{1-\gamma}{1+\mu}\left(\bar{\pi} + \bar{b}^e_t + \mu\bar{y} + \mu\bar{b}\right) \tag{25}$$

whose steady state is:

$$\bar{b}^e = \frac{1-\gamma}{\mu+\gamma}\left(\bar{\pi} + \mu\bar{y} + \mu\bar{b}\right) \tag{26}$$

Plugging (17) into (21) and (23) we get:

$$b^w_{t+1} = \frac{1-\gamma}{1+\mu}\left(w + b^w_t + \frac{\mu}{\gamma}\bar{y}\right) \tag{27}$$

and

$$b^e_{it+1} = \frac{1-\gamma}{1+\mu}\left(\pi_i + b^e_{it} + \frac{\mu}{\gamma}\bar{y}\right) \tag{28}$$

and the steady states are

$$b^{w*} = \frac{1-\gamma}{\mu+\gamma}\left(w + \frac{\mu}{\gamma}\bar{y}\right) \tag{29}$$

$$b_i^{e*} = \frac{1-\gamma}{\mu+\gamma}\left(\pi_i + \frac{\mu}{\gamma}\bar{y}\right) \tag{30}$$

$$\bar{b}^{e*} = \frac{1-\gamma}{\mu+\gamma}\left(\bar{\pi} + \frac{\mu}{\gamma}\bar{y}\right) \tag{31}$$

As to the impact of money growth on bequest, we know that an increase in the rate of money growth is beneficial for the relatively poor and detrimental for the relatively wealthy. It is clear that the wage is always lower than average income: therefore an increase in money growth always boosts the wealth of workers. As to entrepreneurs, some of them – those with a return which falls in the range $\overline{\overline{\rho}} = \overline{y}+r > \rho_i > w+r = \widehat{\rho}$ – gain from an acceleration in monetary expansion, the others – whose return falls in the range $1+r > \rho_i > \overline{\overline{\rho}}$ – loose. On average, however, money growth does not affect wealth (output and consumption). Therefore money is superneutral on average but is not superneutral at the individual level. As a consequence, while the first moments of the distributions of wealth, consumption and bequest do not depend on money, higher moments are influenced by the rate of money growth. In particular, monetary policy has clear asymmetric effects: small entrepreneurs bear the brunt of a deceleration of monetary expansion while big entrepreneurs gain from it. However, average entrepreneurial income is necessarily greater than average economy-wide income. Therefore on average, entrepreneurs gain from a deceleration of money growth.

In figure 2 (see Longaretti and Delli Gatti 2004) we plot the primary distribution (income distribution) on the horizontal axis and the secondary distribution (bequest distribution) on the vertical axis. Moreover we plot the income-bequest line which is upward sloping and parametrized at the rate of money growth. As μ increases the income-bequest line rotates clockwise around $\left(\overline{y}, \overline{b}\right)$. Projecting the primary distribution on the income-bequest line, we get the secondary distribution. In the figure it is clear the distributional effects of money: as μ increases, the primary distribution does not change, whereas the secondary distribution is mean preserving, whereas its variance decreases. If the variance is thought of as a rough measure of inequality, we conclude that inequality decreases as increases.

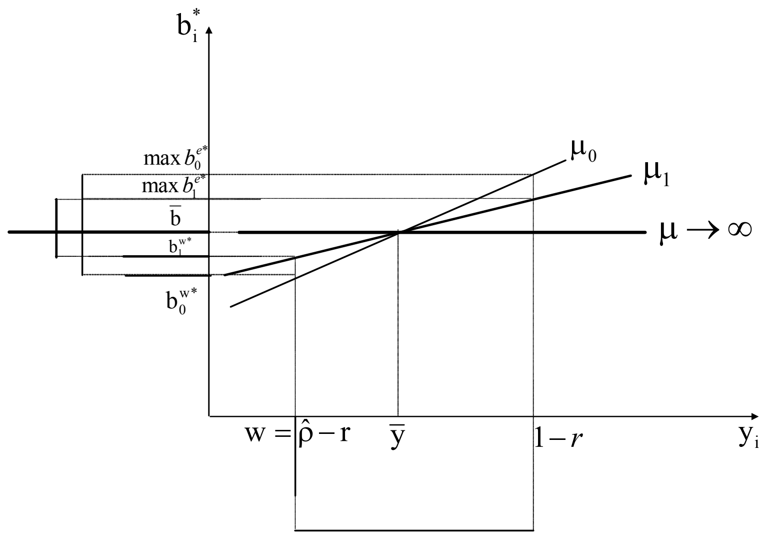

Figure 2

4 Financing constraints

In this section we consider a variant of the environment described in the previous one. Suppose that capital markets are imperfect in the sense that there is no credit market to carry on investment. In this case the agent with a return higher than $\widehat{\rho}$ is a potential entrepreneur who can actually carry on his project if and only if he is self-financed, i.e. if $b_{it} - r > 0$. A financially constrained entrepreneur, i.e. an agent whose return is higher than $\widehat{\rho}$ but whose internal funds are insufficient to pay for the fixed cost $(b_{it} - r < 0)$ must necessarily revert to the condition of worker. In this context, therefore, an entrepreneur must not only have a relatively high return but also relatively wealthy. A potential entrepreneur who cannot afford incurring the fixed cost r falls behind in the social ladder and is lumped together with the working class.

We can envisage two different scenarios. The first happens when $b^{w*} = \frac{1-\gamma}{\mu+\gamma}\left(w + \frac{\mu}{\gamma}\overline{y}\right) \geq r$ i.e. $w \geq r\frac{\mu+\gamma}{1-\gamma} - \frac{\mu}{\gamma}\overline{y}$ and is depicted in figure 3. In this case the economy consists only of self-financed individuals and the absence of a credit market does not prevent the implementation of all the entrepreneurs' investment projects. As a consequence, all the results of the previous section are confirmed.

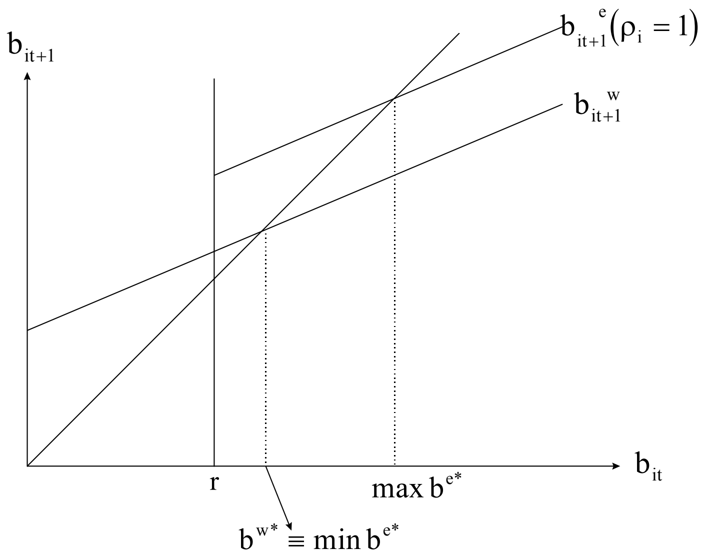

Figure 3

A different and more interesting scenario happens, symmetrically, when $b^{w*} = \frac{1-\gamma}{\mu+\gamma}\left(w + \frac{\mu}{\gamma}\bar{y}\right) < r$, i.e.

$$w < r\frac{\mu+\gamma}{1-\gamma} - \frac{\mu}{\gamma}\bar{y} \tag{32}$$

and is depicted in figure 4.

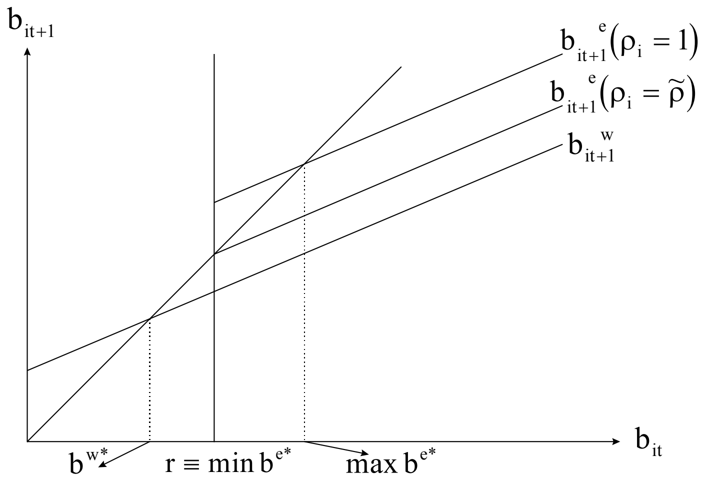

Figure 4

In this case there are some potential entrepreneurs who must give up the investment project, because of financial constraints, and be workers. As a consequence not all the agents whose project has a return high enough to become entrepreneurs actually become entrepreneurs in the steady state. Potential entrepreneurs with a return such that $\frac{1-\gamma}{\mu+\gamma}\left(\rho_i - r + \frac{\mu}{\gamma}\overline{y}\right) < r$, i.e.

$$\rho_i < \frac{1+\mu}{1-\gamma}r - \frac{\mu}{\gamma}\overline{y} = \widetilde{\rho} \tag{33}$$

will never catch up with the self financed entrepreneurs. In the steady state the wealth of entrepreneurs falls in the range $(r, \max b^e_i)$. r is the steady state of the entrepreneur with efficiency $\rho_i = \widetilde{\rho}$. $\max b^e_i$ is the steady state of the entrepreneur with maximum return $\rho_i = 1$ whose law of motion is

$$b^e_{it+1} = \frac{1-\gamma}{1+\mu}\left(1 - r + b^e_{it} + \frac{\mu}{\gamma}\overline{y}\right)$$

Therefore

$$\max b^{e*} = \frac{1-\gamma}{\mu+\gamma}\left(1 - r + \frac{\mu}{\gamma}\overline{y}\right)$$

The average wealth of the entrepreneurs therefore is:

$$\bar{b}^e = \frac{\min b^{e*} + \max b^{e*}}{2} = \frac{1}{2}\left(\frac{\mu + 2\gamma - 1}{\mu + \gamma} r + \frac{1-\gamma}{\mu+\gamma}\left(1 + \frac{\mu}{\gamma}\overline{y}\right)\right)$$

where average income is

$$\overline{y} = w\widetilde{\rho} + \overline{y}^e \left(1 - \widetilde{\rho}\right) = w\widetilde{\rho} + \left(\bar{\rho}^e - r\right)\left(1 - \widetilde{\rho}\right)$$

and

$$\bar{\rho}^e = \frac{1 + \widetilde{\rho}}{2}$$

Therefore

$$\overline{y} = \frac{1}{2} - r + \widetilde{\rho}\left(w + r\right) - \frac{\widetilde{\rho}^2}{2} \tag{34}$$

From (33) it comes out:

$$\overline{y} = \frac{1+\mu}{1-\gamma}\frac{\gamma}{\mu} r - \frac{\gamma}{\mu}\widetilde{\rho} \tag{35}$$

Let us define $A = \frac{1+\mu}{1-\gamma}\frac{\gamma}{\mu} r$, $B = \frac{\gamma}{\mu}$, $C = \frac{1}{2} - r$ and $D = (w + r)$. (34) and (35) therefore become:

$$\overline{y} = A - B\widetilde{\rho} \tag{36}$$

$$\overline{y} = C + D\widetilde{\rho} - \frac{\widetilde{\rho}^2}{2} \tag{37}$$

Note that, according to (37), $\overline{y}$ is a concave non monotonic function of $\widetilde{\rho}$, which presents a maximum in $\widetilde{\rho} = \widehat{\rho} = w + r$. It is clear that the *effective* entrepreneurial share of the population must be non-greater than the *potential* one. That is $1 - \widetilde{\rho} \leq 1 - \widehat{\rho}$. Therefore we have to focus only on the decreasing part of (37), corresponding to $\widetilde{\rho} \geq \widehat{\rho}$. According to (36), $\overline{y}$ is a linear decreasing function of $\widetilde{\rho}$. (36) and (37) represent a system in two unknowns: $\overline{y}$ and $\widetilde{\rho}$. Graphically speaking the solution to the system is represented by the intersection between the straight decreasing line and the decreasing part of the parabola. Notice that *a priori* we could have no, one or multiple equilibria.

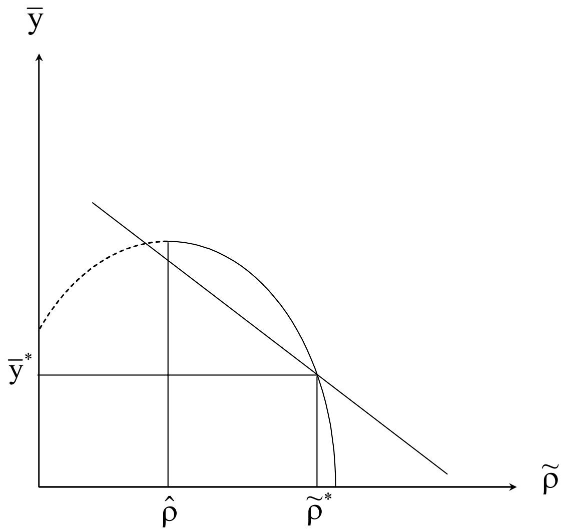

Figure 5

It is interesting now to pass analyzing the effects of money growth. For this purpose, first notice that (37) is not affected by μ, whereas it is (36). Actually we can write

$$\overline{y} = A(\mu) - B(\mu)\rho \tag{38}$$

where $\frac{\partial A}{\partial \mu} < 0$ and $\frac{\partial B}{\partial \mu} < 0$.

Graphically speaking therefore μ reduces the intercept of the straight line in figure 5 and makes it flatter. The effects of an increase in μ on the equilibrium average income and the equilibrium effective entrepreneurial share of the population are therefore *a priori* ambiguous, namely it may happen either that $\overline{y}^*$ increases and $\widetilde{\rho}^*$ decreases or the other way around. In figure 6 we plot the former case and in figure 7 we plot the case of two equilibria. In this case both the effects of an increase in μ on $\overline{y}^*$ and on $\widetilde{\rho}^*$ may occur.

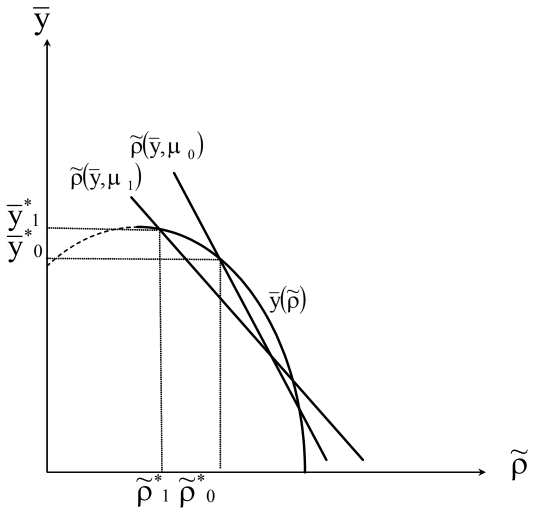

Figure 6

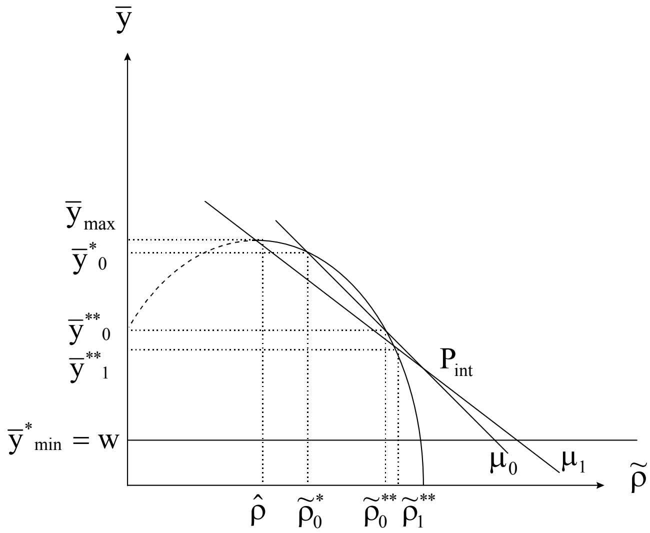

Figure 7

What we have sketched is relevant for consideration about individual and aggregate superneutrality of money.

Proposition 1 *When capital markets are incomplete, money may be non-superneutral. Actually as far as average income is affected by μ, monetary* *policy is superneutral neither at individual nor at aggregate level.*

In fact

$$\frac{\partial \overline{b}^*}{\partial \mu} = \frac{1-\gamma}{\gamma} \frac{\partial \overline{y}^*}{\partial \mu} \tag{39}$$

$$\frac{\partial \overline{c}^*}{\partial \mu} = \frac{\partial \overline{y}^*}{\partial \mu} \tag{40}$$

$$\frac{\partial \overline{\omega}^*}{\partial \mu} = \frac{1}{\gamma} \frac{\partial \overline{y}^*}{\partial \mu} \tag{41}$$

In the appendix we show a qualitative taxonomy of the effects of μ on $\overline{y}$ and on $\widetilde{\rho}$. Reasonable and to some extent necessary restrictions on the parameters anyway allow us to uniquely conclude that:

Proposition 2 *an acceleration in money growth increases the entrepreneurial share of the population and, as a consequence the average income. Moreover there exists a critical level of money growth that allows the economy to shift from scenario 2 to scenario 1. This is* $\mu_{\max} = 2\gamma \frac{\gamma r - w + w\gamma}{1-\gamma-2r+w^2-w^2\gamma+2wr-2wr\gamma+r^2-r^2\gamma}$.

This proposition can be seen using the income-bequest line that projects the primary distribution on the secondary one as we did in the previous section. In figure 8. It can be seen how the mean and the variance of the primary distribution increases as μ increases. This comes from the more efficient occupational choice. As μ increases, a higher share of the population can effectively undertake the entrepreneurial investment project. On the other hand the distributional effects on bequests work as in the perfect capital markets framework.

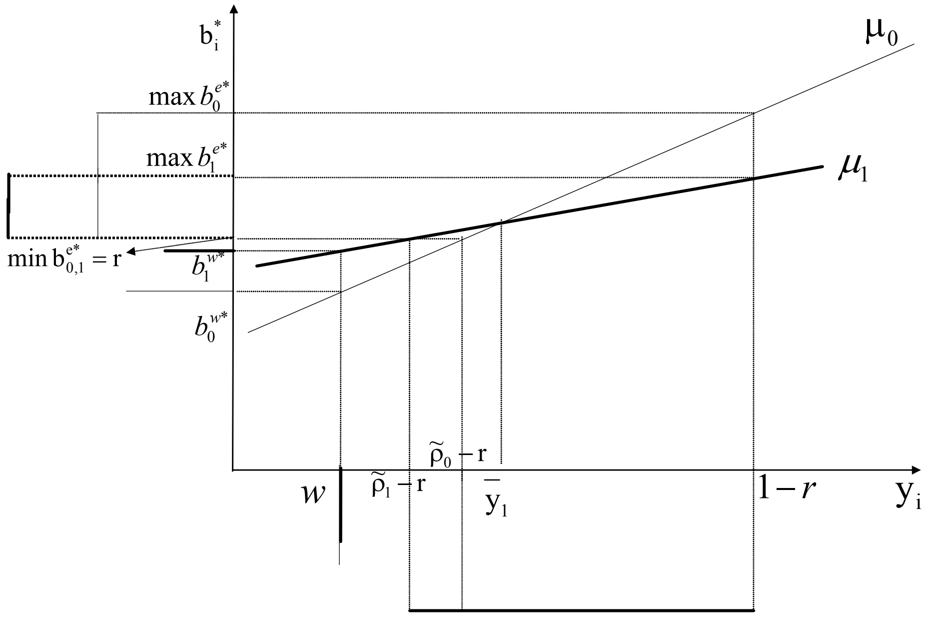

Figure 8

Proposition 3 *Once scenario 1 is reached, $\overline{y}$ does not change anymore as μ increases since the effective entrepreneurial share of the population coincides with the potential one, that is not affected by money growth. The unique effect that monetary policy may have then is at the individual level, by reducing the variance of the distribution of wealth and bequests, without affecting the first moments of the distributions themselves.*

The average income in scenario 1 is, in other words, the maximum average income and is equal to:

$$\overline{y}_{\max} = \frac{1}{2} - r + \frac{(w+r)^2}{2}$$

It is independent on μ. As μ increases, the money transfer effect allow the population to converge to a unique level of bequest, whatever their occupation:

$$\lim_{\mu \to +\infty} \frac{1-\gamma}{\mu+\gamma}\left(y_i + \frac{\mu}{\gamma}\overline{y}_{\max}\right) = \lim_{\mu \to +\infty} \frac{1-\gamma}{\mu+\gamma} y_i + \frac{1-\gamma}{\left(1+\frac{\gamma}{\mu}\right)\gamma}\overline{y}_{\max} = \frac{1-\gamma}{\gamma}\overline{y}_{\max}$$

5 Conclusions

In this paper we have developed a model that investigates the distributional and aggregate effects of monetary policy. In a previous paper (Longaretti and Delli Gatti, 2004) we have shown that, in an OLG model with money and bequests, if agents differed from one another as far as income and wealth are concerned, the average levels of wealth, consumption and bequests would still be independent of money growth but the individual levels of the same variables would indeed be affected by changes in the rate of growth of money. In other words, while the first moments of the distributions of wealth, consumption and bequests do not depend on money, higher moments are influenced by the rate of money growth. This is due to a mean field effect which is at work that captures non-strategic interaction between the agent and the population, proxied by the average agent, and makes money non-superneutral at the individual level.

In this paper we have extended the original framework introducing occupational choice and financing constraints. We have shown that, under appropriate conditions, money is not superneutral, in the sense that all the moments of the distribution are affected by changes in money growth.

Heterogeneous agents as far as income and wealth are concerned make an occupational choice. They decide if being entrepreneurs of workers. If capital markets are incomplete and there is no credit for the entrepreneurial investment, investment itself may be inefficient and the effective entrepreneurial share of the population may differ from the potential one. Some potential entrepreneurs must give up the investment and become workers. In such an eventuality, monetary policy may be a way to solve this underinvestment. An acceleration in money growth, in this context, increases the entrepreneurial share of the population and, as a consequence, the average income. In other words, the money transfer effects may be sufficiently high to allow the whole population to be self-financed, so that the absence of a credit market does not prevent the implementation of all the entrepreneurs' investment projects. Once the potential and effective entrepreneurial share of the population coincide, monetary policy is effective only at the individual level and no more at the average and aggregate level. An increase in money growth reduces the variance of the distribution of wealth and bequests, without affecting the first moments of the distributions themselves. As in Longaretti and Delli Gatti (2002), this result confirms the stylized fact that small entrepreneurs bear the brunt of a deceleration of monetary growth.

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Appendix

Let us focus on scenario 2. Remind that it comes from the condition $w < \frac{\mu+\gamma}{1-\gamma}r - \frac{\mu}{\gamma}\overline{y}$, that is $\overline{y} < \frac{\mu+\gamma}{1-\gamma}\frac{\gamma}{\mu}r - \frac{\gamma}{\mu}w = \overline{\overline{y}}$. $\overline{\overline{y}}$ represents the value reached by the straight line of equation $\overline{y} = \frac{1+\mu}{1-\gamma}\frac{\gamma}{\mu}r - \frac{\gamma}{\mu}\widetilde{\rho}$, when $\widetilde{\rho} = \widehat{\rho} = w + r$. Graphically speaking this means that equilibrium average income must be lower than $\overline{\overline{y}}$.

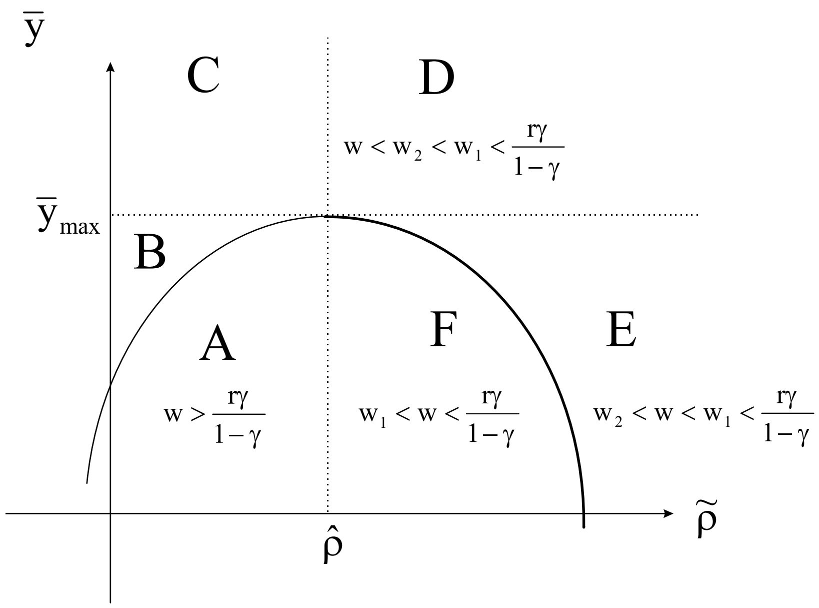

Figure A.1

As μ increases, the straight line rotates around a point (P_{int}) whose coordinates do not depend on μ:

$$P_{int}\begin{cases}\widetilde{\rho}_{int}=\frac{A(\mu_1)-A(\mu_0)}{B(\mu_1)-B(\mu_0)}=\frac{r}{1-\gamma}\\ \overline{y}_{int}=\frac{r\gamma}{1-\gamma}\end{cases}$$

The taxonomy of the effects of μ on $\overline{y}$ and on $\widetilde{\rho}$ changes as P_{int} lays in areas A, B, C, D, E or F (see figure A.1).

Moreover notice that:

$\widetilde{\rho}_{int}>\widehat{\rho}$ if $w<\frac{\gamma}{1-\gamma}r$

$\widetilde{\rho}_{int}<\widehat{\rho}$ if $w>\frac{\gamma}{1-\gamma}r$

This, together with the condition for scenario 2, implies that

$\widetilde{\rho}_{int}>\widehat{\rho}$ if $w<\frac{\gamma}{1-\gamma}r$ and $w<\frac{\gamma}{1-\gamma}r+\left(\frac{\mu}{1-\gamma}r-\frac{\mu}{\gamma}\overline{y}\right)$. Therefore, scenario 2 and $\widetilde{\rho}_{int}>\widehat{\rho}$ contemporarily hold if $\frac{\mu}{1-\gamma}r-\frac{\mu}{\gamma}\overline{y}\leq 0$, that is if $\overline{y}\geq\frac{r\gamma}{1-\gamma}\equiv\overline{y}_{int}$. Graphically speaking it means that, if $\widetilde{\rho}_{int}>\widehat{\rho}$, in order to be in scenario 2, *if μ is positive* we have to select the equilibrium average income that lays above $\overline{y}_{int}$. Symmetrically, if $\widetilde{\rho}_{int}<\widehat{\rho}$, in order to be in scenario 2, we have to select the equilibrium that lays below $\overline{y}_{int}$. But in this case $w>\frac{\gamma}{1-\gamma}r$ and $\overline{y}<\frac{\gamma}{1-\gamma}r$, therefore $\overline{y}<w$, that is impossible, since w is the minimum average income, corresponding to the case in which the whole population is entrapped into poverty and is composed by workers. We conclude that $\widetilde{\rho}_{int}$ may never be lower than $\widehat{\rho}$.

Let's look for the values of w for which the intersection point lies above the parabola. Formally we have to solve the following:

$\frac{r\gamma}{1-\gamma}\geq\frac{1}{2}-r+(r+w)\frac{r}{1-\gamma}-\frac{\left(\frac{r}{1-\gamma}\right)^2}{2}$. This is true for

$$w\leq\frac{-\frac{1}{2}+r-\frac{r^2}{1-\gamma}+\frac{1}{2}\frac{r^2}{(1-\gamma)^2}+r\frac{\gamma}{1-\gamma}}{r}(1-\gamma)\equiv w_1$$

Let us now study the relation between w_1 and $\frac{\gamma}{1-\gamma}r$.

It is possible to verify that $\begin{cases}w_1<\frac{\gamma}{1-\gamma}r\text{ for any }r\neq 1-\gamma\\ w_1=\frac{\gamma}{1-\gamma}r\text{ for }r=1-\gamma\end{cases}$. Graphically speaking this means that the intersection point may lay above the parabola only when $\widetilde{\rho}_{int}>\widehat{\rho}$.

Note that, as said above, if $\widetilde{\rho}_{int}>\widehat{\rho}$, equilibrium average income must be greater than $\overline{y}_{int}$, therefore no equilibrium is associated with a positive rate of money growth if P_{int} lays in D.

The conditions in order to P_{int} lays in E can be summarized as

$$w_2 < w < w_1 < \frac{r\gamma}{1-\gamma}$$

The conditions in order to P_{int} lays in F can be summarized as

$$w_1 < w < \frac{r\gamma}{1-\gamma}$$

P_{int} cannot lay in A, C and in B.

As far as the results we got till now, what we can conclude is only a taxonomy of the qualitative effects of a change of money growth.

If P_{int} lays in F, an increase in money growth implies an increase of the effective entrepreneurial share of the population and, as a consequence, an increase of the equilibrium average income. As μ reaches the value $\mu_{\max} = 2\gamma \frac{\gamma r - w + w\gamma}{1-\gamma-2r+w^2-w^2\gamma+2wr-2wr\gamma+r^2-r^2\gamma}$, the effective share of the population coincides with the potential one and scenario 2 changes into scenario 1. In this case the equilibrium average income is equal to $\overline{y}_{\max} = \frac{1}{2} - r + \frac{(w+r)^2}{2}$ and the increase in money growth only reduces the variance of the distribution of wealth and bequests.

If P_{int} lays in E and $\mu < \mu_{\max}$, there exist two equilibria and an increase in money growth implies opposite effects on the two equilibria (look at figure 6).

If instead P_{int} lays in E and $\mu > \mu_{\max}$, there exists a unique equilibrium and an increase in money growth reduces the effective entrepreneurial share of the population and the equilibrium average income.

In what follows we study the probability P_{int} effectively lays in F or in E. In the following pictures, we plot $\frac{\gamma r}{1-\gamma}$ (the straight line), w_1 (the solid curve) and w_2 (the dash curve), with respect to r, according to different values of γ. F correspond to the area between the straight line and the solid curve, whereas E to the area between the solid and the dash curve. The first picture corresponds to $\gamma = 0.6$, the second to $\gamma = 0.4$, the third to $\gamma = 0.3$

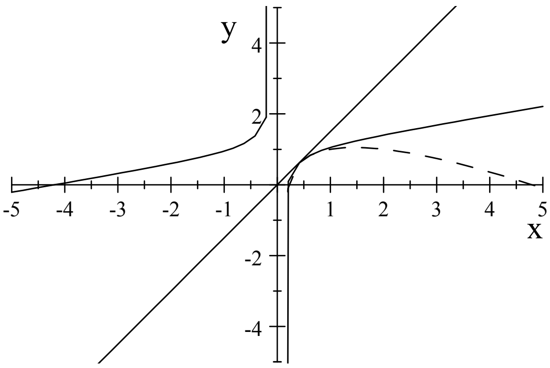

Figure A.2

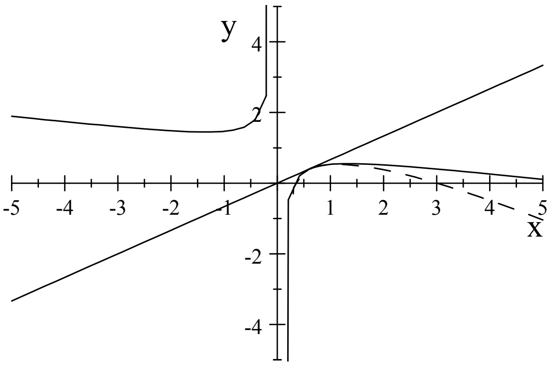

Figure A.3

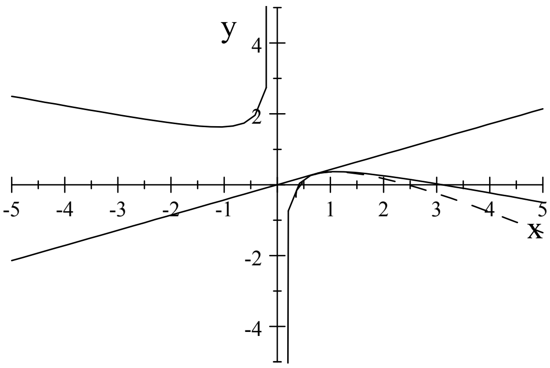

Figure A.4

We can see how the probability P_{int} lays in E reduces as γ decreases, but we can say more if we impose a restriction on r. Actually r represents the cost on the entrepreneurial project. In order the net return from the investment is positive at least for the most efficient entrepreneur, r must be lower than 1. If we look at the previous graphs with this new restriction then area E almost disappears, moreover the net return from the investment for the most efficient entrepreneur must be not only positive, but also greater that the return the agent gets becoming a worker, that is w. Actually, if this does not hold, the potential entrepreneurial share of the population is zero. Formally $\widehat{\rho} \equiv w + r < 1$, that implies $r < 1 - w$. A sufficiently high level of w therefore implies that the probability P_{int} lays in E is zero.