UNIVERSITA' DEGLI STUDI DI MILANO
BICOCCA
AUDENTES FORTUNA IUVAT

DEPARTMENT OF ECONOMICS
UNIVERSITY OF MILAN - BICOCCA

WORKING PAPER SERIES

Revisiting Public Debt and Inflation: Fiscal Implications of an Independent Central Banker

Patrizio Tirelli

No. 31 - January 2001

Dipartimento di Economia Politica
Università degli Studi di Milano - Bicocca
http://dipeco.economia.unimib.it

Revisiting Public Debt and Inflation: Fiscal Implications of an Independent Central Banker.

P.Tirelli
(Universita' di Milano Bicocca and
Glasgow University)

25 November 2000

Abstract

The mainstream literature on monetary policy games under output persistence posits that: a) monetary regimes do not affect real variables in the steady state; b) optimal institutional design should entirely remove the inflation bias. We show that neither result necessarily holds if output persistence originates from debt dynamics and distortionary taxation. First, monetary delegation induces a strategic use of debt policy affecting steady-state distortions. Second, the reduction of such distortions may require monetary institutions that tolerate an inflation rate above the socially optimal level.

Corresponding address:
Patrizio Tirelli
Dipartimento di Economia Politica
Facolta' di Economia
Universita' di Milano Bicocca
Piazza dell'Ateneo Nuovo, 1
20126 Milano
E-mail: patrizio.tirelli@unimib.it

Acknowledgements: the paper greatly benefited from conversations with F. Carmignani, M. Catenaro, E. Colombo, G. Femminis, M.Lossani, A. Muscatelli and P.Natale. The usual disclaimer applies.
An earlier version of this paper was presented at the 1999 Meeting of the Econometric Society, with the title "Dynamic Seigniorage Models Revisited: Should Fiscal Flexibility and Conservative Central Bankers Go Together?"

1. Introduction

It is often argued that inflation has a fiscal root. Empirical research in fact identifies optimal tax considerations as a determinant of inflation differences across countries (Campillo and Miron, 1996). Furthermore, an important strand of the literature sees distortionary taxes as the source of time-inconsistency in the conduct of monetary policy (Alesina and Tabellini, 1987). Therefore, one may ask why the recent debate on monetary institutions — i.e. the controversy between performance-based contracts *à la* Walsh and weight conservatism *à la* Rogoff — has so far neglected the consequences of debt policy for institutional design (Walsh, 1995; Svensson, 1997; Muscatelli, 1998).

The answer probably is that models describing the interdependence between fiscal and monetary policy do not provide a convincing characterisation of the economy in steady state, either because the policymakers' time horizon is arbitrarily shortened or because obtained outcomes are counterfactual. For instance, the strategic interdependence between the fiscal policymaker and the central bank is sometimes addressed within a two-period framework (Beetsma and Bovenberg 1997, 1999a,b). Alternatively, the work by Jensen (1994) extends the Alesina and Tabellini (1987) model to the case of debt accumulation within an infinite-horizon framework. He argues that inflation is a temporary phenomenon because in steady state re-invested budget surpluses will earn the income required to completely finance the public expenditure target. This, in turn, removes the need for distortionary sources of revenue[1]. Unfortunately empirical evidence suggests that governments are apparently unwilling to accumulate such large surpluses.

[1] Obstfeld (1991) and Van der Ploeg (1995) obtain a similar result using different models.

In this paper we present a modified version of the Jensen's model, where it is assumed that political incentives bias the policymaker's preferences against a policy of debt reduction. By doing so we obtain a more realistic description of the long-run relationship between debt and inflation, showing that positive levels of both variables may persist in the steady state. We also identify the optimal monetary policy rule in response to supply shocks, challenging the results obtained in popular models where output exhibits persistence. For instance, Svensson (1997) shows that output persistence induces the policymaker to accept higher inflation volatility in order to limit the additional future effects of current supply shocks. Hence the optimal inflation response is larger when such persistence factors are accounted for. Svensson's argument is reversed in our model, because we derive persistence from the intertemporal redistribution of tax distortions induced by the debt policy. In this case, debt policy spreads over time the burden of adjustment, reducing the optimal volatility of inflation relative to the case where persistence is absent — i.e. under a balanced budget rule. Hence, debt-induced intertemporal factors therefore call for a more conservative monetary stance.

The main contribution of the paper is a reassessment of monetary regimes based on policy delegation to an independent central bank. The literature on games between the fiscal authority and the central bank has emphasised that delegation leads to the inefficient use of debt policy. Beetsma and Bovenberg (1999b) show that, if the inflation target is non-contingent, the fiscal player will use debt strategically to influence next-period inflation, while taking current inflation as given. As a result, inflation ends up being inefficiently redistributed to the current period. Although such an effect appears in our model as well, our conclusions are rather different. By extending the two-period model of Beetsma and Bovenberg to an infinite horizon, we find that the "cautious" debt policy raises the volatility of the inflation bias but has long-term beneficial effects, since distortions are reduced in the steady state. By the same token, a further reduction in steady-state distortions may be achieved if the central bank's inflation target is larger than the socially optimal inflation rate. A recent wave of research on the effects of monetary regimes on unionised labour markets has pointed out that conservative central banks raise wage claims if unions are inflation averse[2] (Cukierman and Lippi, 1999; Guzzo and Velasco, 1999; Lippi, 1999). We extend the same argument to the interaction between the central bank and an inflation averse fiscal policymaker. We also find that the central bank implements "myopic" policy responses to shocks. This happens because debt policy is taken as given and the intertemporal effects of monetary policy actions are consequently ignored. The purpose of institutional design is therefore twofold. First, it should limit fiscal distortions in steady state. Thus, if constitutional constraints on fiscal policy cannot entirely remove fiscal distortions, the inflation target should induce the central bank to tolerate a steady-state rate of inflation above the socially optimal level. Second, institutional design should optimise policy responses to shocks. We show that this requires a certain degree of weigh conservatism in the central bank loss function.

The remainder of the paper is organised as follows. Section 2 presents the model, deriving both the policy rules and the steady state solution in a regime of full discretion. Section 3 evaluates the performance of alternative institutional arrangements. Section 4 concludes sketching some possible extensions to the analysis of monetary and fiscal policies within the European Monetary Union.

[2] For an alternative view, see Coricelli *et al.* (2000).

2. The model

Let us consider the following supply function

$$y_t = -t_t + e_t + p_t - p_t^e \qquad (1)$$

where output deviations from the socially optimal level, y_t, depend on distortionary taxes[3], t_t, a shock e_t, independently distributed with zero mean and finite variance s_e^2, and inflation surprises $p_t - p_t^e$, where p_t^e defines expected inflation.

In each period public expenditures, G_t, are financed by means of public debt and distortionary taxes. Hence the government budget constraint can be written as[4]:

$$D_t = (1+r)D_{t-1} + G_t - t_t \qquad (2)$$

where D_t denotes the stock of government debt at the end of period t and r is the *real* rate of interest[5].

Let us now turn to the definition of the policymaker's preferences. Consider the following intertemporal loss function[6]:

---

[3] Following Alesina and Tabellini (1987) we define t as a tax rate on the total revenue of firms.

[4] All variables are normalised by non-distortionary output, as in Beetsma and Bovenberg (1997).

[5] To limit analytical complexities we assume that r is constant and government debt is fully indexed, as in Jensen (1994) and Beetsma and Bovenberg (1997). The r.h.s. term of eq. (2) does not include seigniorage revenues. It is well known that in modern economies the limited amount of domestic money holdings relative to GDP severely constrains the possibility of raising anticipated seigniorage revenues. For sake of simplicity we therefore neglect this component of the budget constraint. None of our results would significantly change if we modeled seigniorage revenues. The proof of this claim is available upon request.

[6] Quadratic formulations of the loss function may look unduly restrictive. However (3) may be viewed as an acceptable approximation to a more general utility function. With this justification, the policymaker's loss functions is assumed to be quadratic even in models that explicitly model the representative agent's preferences (Rotemberg and Woodford, 1997, 1999; Dixit and Lambertini, 2000).

$$
\begin{aligned}
W_t &= \sum_{s=0}^{\infty} b^{\,s} L^G_{t+s} \\
L^G_{t+s} &= \frac{1}{2}\left[y^2_{t+s} + k_1\left(G_{t+s} - \tilde{G}\right)^2 + k_2\left(p_{t+s} - \tilde{p}\right)^2 + k_3\left(D_{t+s} - \tilde{D}\right)^2\right]
\end{aligned}
\tag{3}
$$

where b is the discount factor. The terms $\tilde{p}$, $\tilde{G}$ and $\tilde{D}$ define respectively the policymaker's targets for inflation, public expenditures and debt. The assumption that the loss function is quadratic in output, expenditures and inflation is standard in the literature since the seminal contribution of Alesina and Tabellini (1987). The inclusion of a quadratic term in debt is perhaps more controversial and requires some discussion, although it can be found in Tabellini (1986).

The argument is better understood discussing what happens when it is postulated that $k_3 = 0$ and the policymaker retains full discretion in the conduct of both fiscal and monetary policy, as in Jensen (1994). In this case re-invested budget surpluses build up a stock of negative debt in the steady state, earning the income necessary to entirely finance the desired level of expenditures. As a result, tax distortions and inflation disappear. Therefore, Jensen's formulation leads to predictions which are unable to explain the observed persistence of debt, tax distortions and inflation in the long run. Moreover, it is not suitable for our purposes, as we aim to investigate whether choice of the monetary regime affects debt and tax distortions in the steady state.

It is intuitively obvious that Jensen's result cannot hold if the policymaker pursues a non-negative debt target, as in (3). The persistence of excessive debt levels — which cause steady-state tax distortions — has several explanations. For instance it may be seen as the consequence of electoral competition when policymakers disagree about the composition or the level of public expenditures (Alesina and Tabellini, 1989; Persson and Svensson 1989). Another strand of literature emphasises the role of intergenerational conflict (Cukierman and Meltzer, 1989). The zero-distortions steady state equilibrium implies that current generations bear the costs of running budget surpluses in order to relieve future generations from the burden of distortionary taxation. This outcome might hold in a world where generations are altruistically linked through bequests, so that the intertemporal distribution of deficits only responds to efficiency considerations. Yet, fiscal policy may be biased towards excessive debt accumulation if some individuals are bequest constrained — i.e. they would like to borrow from future generations leaving negative bequests. In fact, the public debt policy allows bequest-constrained individuals to raise their consumption levels at the expenses of future generations. This happens because deficits are used to subsidise the consumption of bequest-constrained agents, whereas debt will partly substitute capital in the portfolio of non bequest-constrained individuals.

The loss function (3) may be interpreted as follows. The target $\tilde{D}$ defines the level of debt which would emerge if non-distortionary taxes were available in a world where bequest-constrained individuals affect politico-economic equilibria[7]. Moreover, k_3 can be interpreted as the political cost of tolerating debt deviations from $\tilde{D}$. This is an admittedly rough-and-ready way to incorporate adverse political incentives into the policymaker's behaviour[8] and to obtain the persistence of inefficient tax distortions in steady-state equilibrium. However, even this simple modification of the loss function is sufficient to investigate the independent effect of monetary policy delegation schemes on such distortions.

[7] The literature sometime assumes that adverse political incentives lower the policymaker's discount factor (Beetsma and Bovenberg, 1997). Making the same assumption here would not reverse the Jensen's conclusion that fiscal distortions are only a transitory phenomenon, unless the discount factor were so low that the dynamic system would become unstable.

[8] Extending the Cuckierman and Meltzer framework to account for distortionary taxation and time–inconsistency in monetary policy would quickly render their model unsuitable for the analysis of monetary regimes. By the same token, explicitly modeling electoral incentives as in Alesina and Tabellini (1988) would unnecessarily complicate the algebra unless one made the additional assumption that central bank policies affect electoral outcomes. Exploring such an hypothesis is beyond the scope of this paper.

To close the system we need further assumptions about policy regimes. In the following sections we model the specific features of such regimes and discuss the corresponding solutions.

3. Full discretion

In this section we discuss the case of full discretion, where the policymaker controls all policy instruments and is unable to precommit.

3.1 Systematic policies

To sharpen the argument about the persistence of fiscal distortions in steady state, our analysis begins applying the Jensen's solution method to a deterministic version of our model. Following Jensen (1994), we assume that current debt policy decisions affect the future economy, whereas current taxes and inflation only affect the present state of the economy. The static first-order conditions that determine taxes and inflation in each period are as follows:

$$-y_t + k_1 g_t = 0 \tag{4}$$

$$y_t + k_2 \left(p_t - \tilde{p}\right) = 0 \tag{5}$$

where we set $g_t = \left(G_t - \tilde{G}\right)$ to simplify notation. Condition (4) equates the marginal benefits of a tax-financed increase in expenditures to the marginal costs of higher taxes. Condition (5) equates the marginal costs of inflation to the perceived benefits in terms of output expansion following a monetary surprise.

Using (1), (4), (5) and imposing the rational expectations constraint:

$$p_t^e = E\left(p_t\right) \tag{6}$$

where E defines the rational expectations operator, we obtain the open loop solutions for taxes and inflation[9]:

$$E(t_t) = -k_1 E(g_t) \tag{7}$$

$$E(p_t) = \tilde{p} - \frac{k_1}{k_2} E(g_t) \tag{8}$$

Taxes are proportional to the expenditure gap. As a result, output distortions in each period amount to $k_1 g_t$ and inflation deviations from the optimal level are proportional to such distortions.

Let us now turn to the intertemporal first order condition for debt policy:

$$k_1 g_t + k_3 \left(D_t - \tilde{D}\right) - bk_1 (1+r) E_t g_{t+1} = 0 \tag{9}$$

The stock of debt carried over to the next period, D_t is set at the level[10] where the current marginal effect of a debt increase must equal the discounted value of the marginal effects originating from the smaller amount of resources available for public spending in the future.

Assumption 1. *The policymaker's discount factor is such that* $b > \frac{1}{(1+r)^2}$.

The model has two solutions for the expenditure gap and the stock of debt. If the policymaker's discount factor is relatively low, the intertemporal budget constraint is satisfied only if fiscal policy follows a balanced-budget rule (proof in Appendix I). This trivial solution is ruled out only if Assumption 1 holds.

---

[9] We introduce the expectations operator in (7) and (8) because the solutions for the deterministic model coincide with the expected values of the solutions in the stochastic model to be discussed in section 2.2.

[10] Following Jensen, D_t is a control variable, hence the transition equation does not include the state at *t*-1 (Jensen, 1994 p. 769). *A fortiori* this argument holds for the determination of D_{t+1}.

Proposition 1. *Tax distortions and the inflation rate are state dependent. Tax distortions and the inflation bias persist in steady state (Proof in Appendix I).* .

The solutions for public expenditures and debt are as follows:

$$E(d_t) = \frac{(1+r)}{\Theta} E(d_{t-1}) \tag{10}$$

$$E(g_t) = g_{ss} - \frac{\Theta - 1}{\Theta(1+k_1)}(1+r)E(d_{t-1}) \tag{11}$$

$$D_{ss} = \left\{ \tilde{D}\frac{k_3(1+k_1)}{k_1} - \tilde{G}\frac{(1+r)}{r}[(1+r)b - 1] \right\} [\Theta - (1+r)]^{-1} \tag{12}$$

$$g_{ss} = -(\tilde{D}r + \tilde{G})\frac{k_3}{k_1}[\Theta - (1+r)]^{-1} \tag{13}$$

where $d_t = (D_t - D_{ss})$, $\Theta = (1+r)^2 b + \frac{k_3}{k_1}(1+k_1)$ and the subscript *ss* defines steady-state values.

Equation (10) defines debt dynamics, which are stable if:

$$\Theta > (1+r) \tag{14}$$

Assumption 2. *Condition (14) holds.*

Equations (12) and (13) identify the steady-state levels of debt and expenditures, D_{ss} and g_{ss} respectively. Given assumption 2, it is easy to see that in steady state the stock of debt is positive if $k_3\tilde{D}$ is sufficiently large[11]. Moreover, for $k_3 > 0$, steady-state expenditures are always below target.

---

[11] If $k_3 = 0$ the model is unstable unless $(1+r)b > 1$. In this case the only stable equilibrium implies that debt is negative in steady state, as in Jensen (1994).

Turning to the analysis of deviations from the steady state, the sensitivity of expenditures to the current debt burden (eq.11) may be interpreted as follows. A change in $(1+r)E(d_{t-1})$ must be matched by a symmetric adjustment in the present value of current and expected primary surpluses, which is measured by $\left\{-(1+k_1)\left(\frac{\Theta-1}{\Theta}\right)^{-1}E(g_t)\right\}$. The term $\frac{\Theta-1}{\Theta}$ defines the proportion of the adjustment[12] implemented immediately. Such a proportion falls with the discount factor but increases with k_3, since the greater aversion to the use of debt policy calls for a stronger reaction of current expenditures. Moreover, the variation in current expenditures required to balance the intertemporal budget constraint is inversely related to the strength of the tax response to the expenditures gap in each period, as measured by the term k_1.

Substitution of (13) into (8) yields the steady-state rate of inflation:

$$p_{ss} = \tilde{p} + \frac{k_3}{k_2}\left(\tilde{D}r + \tilde{G}\right)\left[\Theta - (1+r)\right]^{-1} \tag{15}$$

where $\frac{k_3}{k_2}\left(\tilde{D}r + \tilde{G}\right)\left[\Theta - (1+r)\right]^{-1}$ defines the steady-state inflation bias[13]. Furthermore, substitution of (11) into (8) shows that expected inflation is state dependent and driven by debt dynamics.

$$E(p_t) = p_{ss} + \frac{k_1}{k_2}\frac{\Theta-1}{\Theta(1+k_1)}(1+r)E(d_{t-1}) \tag{16}$$

2.2 Countercyclical policies

---

[12] Observe that $\frac{Q-1}{Q} > 0$ is a necessary condition for stability. It is interesting to observe that the stability condition (14) can be reinterpreted as a ceiling to the proportion of adjustment shifted onto the future.

[13] Since systematic monetary policies cannot affect output distortions, the first best monetary rule should induce expectations such that $E(p) = \tilde{p}$.

<u>Proposition 2.</u> *Monetary responses to shocks affect the future state of the economy. Intertemporal factors induce a more conservative monetary policy stance (Proof in Appendix II).*

Taking into account that expected values of taxes, public expenditures and inflation are determined according to equations (6), (11) and (16), the policy rules may be defined as follows

$$p_t = E(p_t) + me_t \tag{17}$$

$$t_t = E(t_t) - k_1[g_t - E(g_t)] + e_t + p_t - E(p_t) \tag{18}$$

$$g_t = E(g_t) + \frac{\Theta - 1}{\Theta}\frac{(1+m)}{(1+k_1)}e_t \tag{19}$$

$$d_t = \frac{(1+r)}{\Theta}d_{t-1} - \frac{(1+m)}{\Theta}e_t \tag{20}$$

where m is an undetermined coefficient definin the monetary response to supply shocks.

Applying Jensen's method, we obtain[14]:

$$\hat{m} = -\frac{k_1}{k_2\left[(1+k_1)\hat{\Omega} + \frac{k_1}{k_2}\right]} \tag{21}$$

where $\hat{\Omega} = \frac{\Theta}{\Theta - 1}$.

From equation (20) it is easy to see that unanticipated monetary policy responses to shocks affect output and, consequently, the tax rate and debt dynamics. This result falsifies Jensen's guess that the future state of the economy is independent from current monetary policies. The value of m that minimises the policymaker's loss function (3) is:

---

[14] The proof for (21) is in Appendix I.

$$\tilde{m} = -\frac{k_1}{k_2\left[(1+k_1)\tilde{\Omega} + \frac{k_1}{k_2}\right]} \tag{22}$$

where $\tilde{\Omega} = \dfrac{\Theta^2}{(\Theta-1)^2 + \frac{k_3}{k_1}(1+k_1)}$

Equations (21) and (22) differ unless the policymaker adheres to a balanced budget, when both $\tilde{\Omega}$ and $\hat{\Omega}$ converge to 1. Furthermore, since $\tilde{\Omega}$ is inversely related to k_3, monetary policy is increasingly sensitive to shocks as the policymaker becomes more averse to debt volatility. Finally, some tedious algebra shows that the monetary rule unambiguously becomes more "conservative" – i.e. $\tilde{\Omega} > \hat{\Omega}$ – when intertemporal effects are taken into account. If we compare equation (22) with the limiting case of a balanced-budget rule, it is easy to see that the strength of the optimal policy response to shocks is conditional to the proportion of adjustment the policymaker is willing to spread onto the future.

Finally, observe that the policy responses to shocks are obtained under the assumption that political incentives affect the policymaker's use of debt policy. It would be straightforward to show that when such incentives do not matter and the policymaker is identified with the social planner — i.e. $k_3 = 0$ — the optimal monetary response becomes unambiguously more conservative.

4. Implications for institutional design

Suppose monetary policy is delegated to an independent central bank, whose loss function may be written as:

$$W_t^B = \sum_{s=0}^{\infty} b^{\,s} L_{t+s}^B$$
$$L_{t+s}^B = \frac{1}{2}\left[y_{t+s}^2 + g k_2 \left(p_{t+s} - p_{t+s}^T\right)^2 \right] \tag{23}$$

where p_{t+s}^T is the inflation target assigned to the bank. Observe that in (23) parameter $g > 0$ accounts for idiosyncratic central bank aversion to inflation without necessarily implying weight-conservatism. The policymaker and the central bank minimise (3) and (23) respectively. We assume that the fiscal and monetary authorities act non-cooperatively.

<u>Definition 1.</u> *In the game between the fiscal policymaker and the central bank, a Markov equilibrium is characterised by a combination of t_t, p_t, D_t such that i) t_t, D_t minimise (3) taking p_t as given; ii) p_t minimises (23) taking t_t, D_t as given.*

Let us start with the analysis of monetary policy. By taking debt as given, the central bank ignores the intertemporal effects of monetary policy responses to supply shocks. Therefore the first order condition for monetary policy is static:

$$y_t + g k_2 \left(p_t - p_t^T\right) = 0 \tag{24}$$

The first order condition for the tax instrument is identical to (4), hence the open-loop rules for taxes and inflation are as follows:

$$t_t = -k_1 g_t + e_t + p_t^B - E\left(p_t^B\right) \tag{25}$$

$$p_t^B = p_t^T - \frac{k_1}{g k_2} g_t \tag{26}$$

The analysis of debt policy requires a careful discussion. Beetsma and Bovenberg (1997) point out that delegation to an independent central bank induces strategic use of the debt policy, in order to influence next period expected inflation while current inflation is taken as given. If expected future inflation is excessively high, the fiscal authority cuts down the amount of debt-financed expenditures. This policy reduces future tax distortions and inflation expectations, but increases current levels of taxes and inflation[15]. The first order condition for debt policy in period t is:

$$k_1 g_t + k_3\left(D_t - \tilde{D}\right) + ß\,\frac{\partial E_t L_{t+1}^G}{\partial D_t} = 0 \tag{27}$$

where

$$\frac{\partial E_t L_{t+1}^G}{\partial D_t} = E_t\left(y_{t+1}\right)\frac{\partial E_t\left(y_{t+1}\right)}{\partial D_t} + k_1 E_t\left(g_{t+1}\right)\frac{\partial E_t\left(g_{t+1}\right)}{\partial D_t} + k_2 E_t\left(p_{t+1}^B - \tilde{p}\right)\frac{\partial E_t\left(p_{t+1}^B\right)}{\partial D_t} =$$
$$k_1\left(1+k_1\right)E_t\left(g_{t+1}\right)\frac{\partial E_t\left(g_{t+1}\right)}{\partial D_t} + k_2 E\left(p_{t+1}^T - \frac{k_1}{g k_2} g_{t+1} - \tilde{p}\right)\left[\frac{\partial E_t\left(p_{t+1}^T\right)}{\partial D_t} - \frac{k_1}{g k_2}\frac{\partial E_t\left(g_{t+1}\right)}{\partial D_t}\right]$$

Next-period expected loss is obviously affected by the monetary policy regime. Observe that the difference with the case of full discretion arises because the policymaker correctly anticipates the effects of his debt policy on future inflation but does not internalise the consequences for current inflation.

To solve the model we must identify the derivatives $\frac{\partial E_t\left(p_{t+1}^T\right)}{\partial D_t}, \frac{\partial E_t\left(g_{t+1}\right)}{\partial D_t}$. From equation (2) we know that:

$$g_{t+1} = D_{t+1} - (1+r)D_t + g_t - t_{t+1} \tag{28}$$

Therefore, substituting the tax rule (25) into (28) and taking expected values we get:

$$\frac{\partial E\left(g_{t+1}\right)}{\partial D_t} = -\left(1+r\right)\left(1+k_1\right)^{-1} \tag{29}$$

To obtain $\frac{\partial E_t\left(p_{t+1}^T\right)}{\partial D_t}$ we obviously need to define the inflation target.

Assumption 3. *The inflation target is non-contingent*

[15] Equations (25) and (26) confirm that a fall in expenditures is matched by an increase in taxes.

$$p^{T}_{t+s} = p^{T}* \qquad \forall s \geq 0 \tag{30}$$

State-contingent targets would provide the flexibility necessary to optimise the transition to steady state. However, the assumption that the inflation target is revised period by period is often criticised. For instance, Beetsma and Jensen (1999) argue that if it is possible to adjust a target before expectations are formed, it must also be possible to revise it later on, undermining the credibility of the target. Thus, in the following we analyse the performance of non-contingent targets[16].

Substituting (30) into (27) we obtain:

$$\frac{\partial E_t\left(L^{G}_{t+1}\right)}{\partial D_t} = -k_1 \left\{ E_t\left(g_{t+1}\right) r - \frac{\left(p^{T}* - \tilde{p}\right)}{?\left(1+k_1\right)} \right\} \left(1+r\right) \tag{31}$$

where $\quad r = \left(1 + \frac{k_1}{?^2 k_2 \left(1+k_1\right)}\right)$

The fiscal policymaker now perceives that the target cannot affect the stochastic component of the inflation bias and restrains his debt policy whenever expected inflation is too high and vice versa.

Proposition 3. *The choice of central bank preferences affects steady-state fiscal distortions if, in each period, inflation is imperfectly controlled. A trade-off exists in the steady state between the inflation bias and distortions. Complete elimination of the steady-state inflation bias increases distortions relative to the case of discretion (Proof in Appendix III).*

---

[16] In Appendix V we show that our results are robust to the inclusion of a state-contingent component into (30)

The solutions for debt, expenditures and inflation are as follows:

$$d_t* = \frac{(1+r)}{\Theta*} d_{t-1}* - \frac{(1+m*)}{\Theta*} e_t \tag{32}$$

$$g_t* = E(g_t*) + \frac{\Theta*-1}{\Theta*} \frac{(1+m*)}{(1+k_1)} e_t \tag{33}$$

$$E(g_t*) = g_{ss}* - \frac{\Theta*-1}{\Theta*(1+k_1)} (1+r) d_{t-1}* \tag{34}$$

$$D_{ss}* = \left\{ \tilde{D} \frac{k_3}{k_1} - \frac{\tilde{G}}{r} \left[(1+r)^2 br - (1+r) \right] - \frac{(p^T* - \tilde{p})}{r\, g(1+k_1)} \left[(1+r)^2 b - 1 \right] \right\} \left[Q* - (1+r) \right]^{-1} \tag{35}$$

$$g_{ss}* = -\frac{k_3}{k_1} \frac{(\tilde{D}r + \tilde{G})}{[\Theta* - (1+r)]} + \frac{(p^T* - \tilde{p})\left[(1+r)^2 b - 1 \right]}{[\Theta* - (1+r)]\, g(1+k_1)^2} \tag{36}$$

$$p_t = E(p_t*) + m* e_t \tag{37}$$

$$E(p_t*) = p^T* - \frac{k_1}{d g k_2} E(g_t*) \tag{38}$$

where:

$$\Theta* = \Theta + \left[(1+r)^2 b - 1 \right] \frac{k_1}{g^2 k_2 (1+k_1)} ; m* = -\frac{k_1}{d g k_2 \left[(1+k_1) \frac{\Theta*}{\Theta*-1} + \frac{k_1}{d g k_2} \right]}$$

The new pattern of debt accumulation (coefficient $\frac{(1+r)}{\Theta*}$ in equation 32) is clearly determined by the perceived impact of debt policy on next period loss. Relative to the case of discretion (equation 10), the fiscal policymaker now restrains his debt policy in order to reduce the future deviation of inflation from $\tilde{p}$. Beetsma and Bovenberg (1999b) show that in a two-period model this has adverse effects in the short term, but are unable to investigate steady states. By extending their analysis to the infinite horizon, we find that strategic debt policy raises the volatility of the inflation bias (equation 38) but has long-term beneficial effects: distortions are reduced in steady state[17] (equation 36) because $\Theta^* > \Theta$. A further reduction in steady-state distortions is obtained if the central bank inflation target is larger than the socially optimal inflation rate. “Liberal” central bankers – such that $g < 1$ and/or $p^{T*} > \tilde{p}$ – induce the fiscal policymaker to restrain debt accumulation in order to bring future expected inflation closer to the socially optimal level. This, in turn, has beneficial effects on output distortions in steady state. Thus a trade-off exists between the elimination of the inflation bias and the reduction in output distortions. By contrast, if the central banker is target-conservative, as in Svensson (1997), steady state distortions increase. Specifically, if the non-contingent target is designed to remove the steady-state inflation bias:

$$p^{T}* = \tilde{p} - \frac{k_1}{g k_2} g_{ss}* \tag{39}$$

the steady-state solution for public expenditures is obtained substituting (39) into (36)

$$g_{ss}* = -\frac{k_3}{k_1} \frac{\left(\tilde{D}r + \tilde{G}\right)}{\left\{\left[\Theta * -(1+r)\right] - \frac{k_1\left[(1+r)^2 b - 1\right]}{g^2 k_2 (1+k_1)^2}\right\}} \tag{40}$$

Proposition 4. *For any given value of , strategic debt policy strengthens* $|m*|$ *and the sensitivity of public expenditures to shocks.*

Once more, this happens because the fiscal policymaker anticipates the consequences of debt policy on future inflation volatility, but neglects the implications for current inflation.

4.1 Optimal monetary institutions

[17] In the limiting case where $g = 0$ the “inflation scare” induces the fiscal policymaker to keep debt at the socially optimal level, therefore the inflation bias is eliminated in steady state. However the same inflation scare would inhibit any fiscal response to shocks, hence this extreme case cannot achieve the first best.

In our framework, the first-best is described by the non-distortionary steady state discussed in Jensen (1994). To eliminate the inefficient accumulation of debt in steady state, it would be obvious to advocate a constitutional constraint on fiscal policy – the endorsement of a negative debt target. However, as pointed out in Beetsma and Bovenberg (1998), in practice it may be difficult to adjust the preferences of the fiscal policymaker and even constitutional constraints can at best mitigate the adverse-incentives faced by the fiscal policymaker. For instance, the Maastricht Treaty requires that EMU members pursue 60% debt-to-GDP target ratios, which are obviously far from solving the issue discussed here. In this section we investigate the independent role of monetary institutions in limiting fiscal distortions[18]. At this regard, optimal central bank design should minimise the expected value of a loss function where political incentives do not matter, as in the following:

$$
\begin{aligned}
&E\{W_t\} = \sum_{s=0}^{\infty} b^{\,s} E_t\{L_{t+s}^G\} \\
&L_{t+s}^G = \frac{1}{2}\left[y_{t+s}^2 + k_1\left(G_{t+s} - \tilde{G}\right)^2 + k_2\left(p_{t+s} - \tilde{p}\right)^2\right]
\end{aligned}
\tag{41}
$$

The non-contingent inflation target and the degree of central bank aversion to inflation should minimise (41), subject to (25), (33), (34), (36), (37) and (38).

Proposition 5. *Optimal monetary institutions require that in steady state the Central bank tolerates an inflation rate above the socially optimal level. A certain degree of central bank weight conservatism is desirable to optimise policy responses to shocks (proof in Appendix IV).*

Due to the nature of the loss function (41), it is never optimal to choose a corner solution such as the complete elimination of the inflation bias in steady state. The choice of g also affects policy responses to shocks. In section 2.2 we have shown that political incentives lead to debt

---

[18] See the Appendix for a proof of the results discussed in this section.

policies which are too timid and monetary policies which are too active. Setting $g > 1$ would shift all policy rules in the right direction.

Finally, we briefly discuss the role of state-contingent targets. Consider the following formulation:

$$p^T_{t+1} = p^{T} * + a \frac{k_1}{g k_2} E_t\left(g_{t+1}\right) \tag{42}$$

The target is composed of two distinct terms. The first, $p^T *$, is constant. The second, $a \frac{k_1}{g k_2} E_t\left(g_{t+1}\right)$, is state dependent and implies that the target is conditional upon the current stock of debt. The coefficient a in (42) may be interpreted as the strength of the inflation target response to the change in inflation expectations caused by debt policy.

Proposition 6. *If the inflation target in each period is conditional upon past debt levels, the revision rule for the target should optimise the transition to the steady state, minimising the variance of the inflation bias. To retain some influence on steady-state distortions, the socially optimal inflation target should not completely sterilise the impact of current debt policy on future inflation, as advocated in Beetsma and Bovenberg (1999b)(Proof in Appendix V).*

4.Conclusions

Popular models of monetary policy games interpret institutional design as a means to remove the inflation bias and to optimise policy responses to shocks.

We argue that a trade-off exists between inflation and fiscal distortions, therefore challenging the view that monetary institutions should be designed to completely remove the inflation bias. In fact, our model assigns a more ambitious task to monetary institutions, and calls for a softer attitude towards inflation. We believe that further research should apply this approach to the EMU case, where fiscal policy is decentralised and each national policymaker retains some degree of flexibility in the use of debt policy, regardless of the Stability and Growth Pact. Given our results, it is reasonable to expect that ECB preferences should be adjusted to affect debt accumulation, conditional on the game played by national fiscal authorities. This suggests that the emphasis on price stability should be reconsidered.

References

Alesina, A. and G. Tabellini, (1987), Rules and Discretion with Noncoordinated Monetary and Fiscal Policies, *Economic Inquiry*, 619-30.

Alesina, A. and G. Tabellini, (1989), A Positive Theory of Fiscal Deficits and Government Debt, *Review of Economic Studies, 57*, 403-414.

Beetsma, R. and A. L.Bovenberg, (1997), Central Bank Independence and Public Debt Policy, *Journal of Economic Dynamics and Control*, 21, 873-894.

Beetsma, R.M.W.J. and A.L. Bovenberg, (1998), Monetary Union without Fiscal Co-ordination May Discipline Policymakers, *Journal of International Economics*, 45, 239-258.

Beetsma, R. and A. L.Bovenberg, (1999a), Does monetary Unification Lead to Excessive Debt Accumulation?, *Journal of Public Economics*, 74, 299-325.

Beetsma, R. and A. L.Bovenberg, (1999b), Inflation Targets and Debt Accumulation in a Monetary Union, Tilburg CentER Discussion Paper: 99108.

Beetsma, R., A. L.Bovenberg and H.Jensen, (1994), Note on "Loss of Monetary Discretion in a Simple Dynamic Policy Game", *Journal of Economic Dynamics and Control*, 20, 1797-1800.

Beetsma, R. and and H.Jensen, (1999), Optimal Inflation Targets, "Conservative" Central Banks, and Linear Inflation Contracts: Comment, *American Economic Review*, 89, 342-347.

Campillo, M. and J. Miron, (1996), Why Does Inflation Differ Across Countries?, NBER WP No5540, April 1996.

Coricelli, F., A. Cukierman and A. Dalmazzo, (2000), Monetary Institutions, Monopolistic Competition, Unionized Labor Markets and Economic Performance, WP http://www.econ-pol.unisi.it/scdbanc/coricelli.shtml.

Cukierman, A., and A. Meltzer, A. (1989), A Political Theory of Government Debt and Deficits in a Neo-Ricardian Framework, *American Economic Review*, 79, 713-748.

Cukierman, A., and F. Lippi, (1999), Central Bank Independence, Centralisation of Wage Bargaining, Inflation and Unemployment – Theory and Some Evidence, *European Economic Review*, 43, 1395-1434.

Dixit, A. and L. Lambertini, (2000), Fiscal Discretion Destroys Monetary Commitment, WP http://econweb.sscnet.ucla.edu/lambertini/papers/eea2000.pdf.

Engwerda, J., B. van Aarle and J. Plasmans, (1998) Fiscal Policy Interaction in the EMU, Tilburg CentER Discussion Paper: 98113.

Guzzo, V. and A. Velasco, (1999), The Case for a Populist Central Bank, *European Economic Review*, 43, 1317-1344.

Jensen, H. (1994), Loss of Monetary Discretion in a Simple Dynamic Policy Game, *Journal of Economic Dynamics and Control*, 18, 763-779.

Lippi, F. (1999), Strategic Monetary Policy with Non-Atomistic Wage Setters: a Case for Non-Neutrality, CEPR Discussion Paper: 2218

Lockwood, B., and A. Philippopoulos, (1994), Inside Power, Unemployment Dynamics and Multiple Inflation Equilibria, *Economica*, **61**, 59-77.

Muscatelli, V.A. (1998). Inflation Contracts and Inflation Targets under Uncertainty: Why we Might Need Conservative Central Bankers?, *Economica*, forthcoming.

Obstfeld, M., (1991), Dynamic Seigniorage Theory: An Exploration, *CEPR* WP No.519.

Persson, T., and L. Svensson, Why a Stubborn Conservative Would Run a Deficit: Policy with Time-Inconsistent Preferences, *Quarterly Journal of Economics*, 325-45.

Rogoff, K., (1985), The Optimal Degree of Commitment to an Intermediate Monetary Target, *Quarterly Journal of Economics*, 100, 1169-1189.

Tabellini, G., (1986) Money, Debt and Deficits in a Dynamic Game. *Journal of Economic Dynamics & Control*. Vol. 10 (4), p 427-42.

Rotemberg, J. and M. Woodford, (1997) An Optimization-based Econometric Framework for the Evaluation of Monetary Policy, NBER Macroeconomic Annual, 12, 297-346.

Rotemberg, J. and M. Woodford, (1999) Interest-Rate Rules in an Estimated Sticky-Price Model, in J. Taylor, (ed.) Monetary Policy Rules, Chicago: University of Chicago Press.

Svensson, L., (1997), Optimal Inflation Targets, “Conservative” Central Banks and Linear Inflation Contracts, *American Economic Review*.

Van der Ploeg, F., (1995), Political Economy of Monetary and Budgetary Policy, *International Economic Review*, 36, 427-439.

Walsh, C., (1995), Optimal Contracts for Central Bankers, *American Economic Review*, 85, 150-167.

Appendix I. Solution of the model following Jensen's method

The Bellman equation is:

$$E\{V(D_{t-1})\} = \min_{p_t, t_t, D_t} \left\{ \frac{1}{2}\left[y_t^2 + k_1 g_t^2 + k_2 (p_t - \tilde{p})^2 + k_3 (D_t - \tilde{D})^2 \right] + ßV(D_t) \right\} \tag{A.1}$$

Inflation, taxes and the stock of debt carried over to the next period are the policy instruments. Equation (A.1) is minimised subject to (1) and (2). Recall the first-order static conditions:

$$-y_t + k_1 g_t = 0 \tag{4}$$

$$y_t + k_2 (p_t - \tilde{p}) = 0 \tag{5}$$

which are obtained under the assumption that current tax and inflation decisions do not affect the future state of the economy (Jensen 1994).

Using (1), (4) and (5) we get the open-loop solutions for taxes and expenditures:

$$t_t = e_t + p_t - p_t^e - k_1 g_t \tag{A.2}$$

$$p_t = \tilde{p} - \frac{k_1}{k_2} g_t \tag{A.3}$$

Assuming rational expectations, from equation (A.3) we get

$$p_t^e = Ep_t = \tilde{p} - \frac{k_1}{k_2} E\{g_t\} \tag{A.4}$$

Recall the intertemporal condition for debt policy:

$$k_1 g_t + k_3 (D_t - \tilde{D}) - bk_1 (1 + r) E_t g_{t+1} = 0 \tag{9}$$

The latter implies that $V_D(D_t) = -k_1(1 + r)E_t g_{t+1}$.

Forwarding (9) we obtain the optimal relationship between current and expected expenditures for any future period:

$$E_t\left(g_{t+s}\right)=\left[g_t+\frac{k_3}{k_1}\left(D_t-\tilde{D}\right)\right]\left[(1+r)ß\right]^{-s}, \qquad \forall s \geq 0, \tag{A.5}$$

Using (2), (A.2), (A.3) and (A.4) equation (A.5) can be rearranged as:

$$E_t\left(g_{t+s}\right)= \\ \left\{\left[1+\frac{k_3}{k_1}\left(1+k_1\right)\left(1+\frac{k_1}{k_2}\right)\right]g_t+\frac{k_3}{k_1}\left(1+k_1\right)\left[(1+r)D_{t-1}+\tilde{G}-\frac{k_1}{k_2}E_t\left(g_t\right)-e_t-\tilde{D}\right]\right\}\left[(1+r)b\right]^{-s} \tag{A.6}$$

To obtain the closed-form solution for expenditures we impose the standard no Ponzi-Game condition:

$$\lim_{t\to\infty}(1+r)^{-t}D_t=0 \tag{A.7}$$

The intertemporal budget constraint[19] is:

$$g_t-t_t+E_t\left\{\sum_{s=1}^{\infty}\left(g_{t+s}-t_{t+s}\right)(1+r)^{-s}\right\}+\frac{1+r}{r}\tilde{G}+(1+r)D_{t-1}=0 \tag{A.8}$$

The solution for g_t is obtained substituting (A.2), (A.3), (A.4) and (A.6) into (A.8). Observe that if $(1+r)^2 b<1$, the discount factor b is so low that the intertemporal budget constraint is satisfied only if the policymaker adopts a balanced budget rule. In this case we get:

$$g_t=E\left(g_t\right)+\frac{e_t}{\left(1+k_1\right)+\frac{k_1}{k_2}} \tag{A.9}$$

[19] The term $\frac{1+r}{r}\tilde{G}$ appears in (A.8) because we have chosen to express current and future expenditures as deviations from the target.

$$E(g_t) = -\left\{(1+r)D_{t-1} + \tilde{G} - \tilde{D}\right\}(1+k_1)^{-1} \tag{A.10}$$

Substituting (A.2), (A.3), (A.4), (A.9), (A.10), into (2) we get the solutions for debt policy:

$$D_t = \tilde{D} \tag{A.11}$$

By contrast, if $(1+r)^2 b > 1$, the solutions for public expenditures and debt are:

$$g_t = E(g_t) + \frac{e_t}{(1+k_1)\frac{\Theta}{\Theta - 1} + \frac{k_1}{k_2}} \tag{A.12}$$

$$E(g_t) = -\left\{\frac{\Theta - 1}{\Theta}(1+r)D_{t-1} + \tilde{Z}\right\}(1+k_1)^{-1} \tag{A.13}$$

where: $\Theta = (1+r)^2 b + \frac{k_3}{k_1}(1+k_1)$ and $\tilde{Z} = \frac{\tilde{G}}{\Theta}\left\{\left[\frac{(1+r)^2 b - 1}{r}\right] + \Theta - 1\right\} - \frac{\tilde{D}}{\Theta}\frac{k_3}{k_1}(1+k_1)$

$$D_t = E(D_t) - \left\{1 - \frac{1 + k_1 + \frac{k_1}{k_2}}{(1+k_1)\frac{\Theta}{\Theta - 1} + \frac{k_1}{k_2}}\right\} e_t \tag{A.14}$$

$$E(D_t) = \frac{(1+r)D_{t-1}}{\Theta} - \tilde{Z} + \tilde{G} \tag{A.15}$$

The steady state solution for debt (equation 12) is derived from (A.15). Substituting (12) into (A.13) gives the steady state solution for public expenditures (equation 13). Then equations (A.13) and (A.14) are easily rearranged to obtain (10) and (11).

Substituting (A.12) into (A.3) we obtain the monetary response to shocks $\hat{m}$ (equation 21). By the same token (A.14) can be rearranged to obtain equation (20) under $m = \hat{m}$. We briefly comment on the logic of this result. An adverse supply shock requires a tax reduction that drains resources otherwise available for public spending. However, the tax reduction cannot entirely stabilise output, because the policymaker must find a balance between the two conflicting goals of expenditures and output stabilisation. As shown in (A.3) inflation reacts to output distortions which are proportional to the expenditures gap. Hence, the unanticipated fall in expenditures triggers an inflation surprise that limits the output loss and the need to reduce taxes[20]. The term $\frac{k_1}{k_2}$ in equation (A.12) describes how the inflation surprise weakens the impact of adverse supply shocks on expenditures and ultimately on output. Equation (A.14) shows that monetary policy responses to shocks affect debt dynamics through their impact on taxes. Therefore, the stochastic component of monetary policy does affect the future state of the economy. This result is inconsistent with the assumption that debt accumulation is invariant to current tax and inflation policies. In fact Jensen's guess would be confirmed only if one could obtain that $\hat{m} = 0$, when debt policy would react to shocks exactly as it reacts to past accumulation of debt. Unfortunately this is not the case, therefore Jensen's method is unsuitable for the analysis of supply shocks.

Appendix II. Solution for $\tilde{m}$

To identify $\tilde{m}$ (equation 22), we assume that that the monetary rule is defined as in equation (17). Then we derive the value of m that minimises the expected value of the policymaker's loss function. This is easily done taking the expected value of the loss function (3) [21]:

$$E\{W_t\} = E\left\{\frac{1}{2}\sum_{s=0}^{\infty} ß^s \left[y_{t+s}^2 + k_1 g_{t+s}^2 + k_2 (p_{t+s} - \tilde{p})^2 + k_3 \left(D_{t+s} - \tilde{D}\right)^2\right]\right\} \qquad \text{(A.16)}$$

From (1), (7), (11), (A.2), (A.3), (19) and assuming that the economy is in steady state[22], we obtain:

---

[20] Eq. (A.2) shows that any inflation surprise turns into a change in tax revenues.

[21] The optimal policy rule is defined before shocks are observed. Such solution strategy is clearly inappropriate to identify the systematic component of monetary policy because it would obtain time-inconsistent policies. We use it here because, by definition, the rational expectations constraint does not matter for the identification of the monetary response to supply shocks.

[22] This assumption simplifies notation for the deterministic component of the welfare loss, but is irrelevant for the identification of $\tilde{m}$.

$$E(W_t) = \left\{ \begin{array}{l} k_1(1+k_1)\left[g_{ss}^2 + \left(\frac{\Theta-1}{\Theta}\right)^2 \frac{(1+m)^2}{(1+k_1)^2} s_e^2\right] + \\ + k_2\left[\left(\frac{k_1}{k_2}\right)^2 g_{ss}^2 + m^2 s_e^2\right] + k_3\left[\left(D_{ss}-\tilde{D}\right)^2 + \left(\frac{1+m}{\Theta}\right)^2 s_e^2\right] \end{array} \right\} \frac{1}{1-b} \tag{A.17}$$

It is straightforward to show that (22) minimises (A.17).

Appendix III. Solutions under a non-contingent inflation target

Substituting equation (32) into (27) the intertemporal first order conditions becomes:

$$E_t\{g_{t+s}\} = \left[g_t + \frac{k_3}{k_1}\left(D_t - \tilde{D}\right)\right](r)^{-1}[(1+r)b]^{-s} + \frac{p^T * - \tilde{p}}{g(1+k_1)r}, \quad \forall s \geq 0, \tag{A.18}$$

where

$$r = 1 + \frac{k_1}{g^2 k_2 (1+k_1)}$$

Using (7), the intertemporal budget constraint (A.8) may be rewritten as

$$g_t + (1+k_1)E_t\left\{\sum_{s=1}^{\infty}(1+r)^{-s} g_{t+s}\right\} + \frac{1+r}{r}\tilde{G} + (1+r)D_{t-1} = t_t \tag{A.19}$$

where, from (A.18),

$$E_t\left\{\sum_{s=1}^{\infty}(1+r)^{-s} g_{t+s}\right\} = \left[g_t + \frac{k_3}{k_1}\left(D_t - \tilde{D}\right)\right](r)^{-1}\left\{\left[(1+r)^2 b - 1\right]\right\}^{-s} + \frac{p^T * - \tilde{p}}{r\, g(1+k_1)r} \tag{A.20}$$

Substituting (2), (25), (26) and the expected value of (26) into (A.20) we obtain:

$$g_t = E(g_t) + \frac{e_t}{(1+k_1)\frac{\Theta *}{\Theta * - 1} + \frac{k_1}{g k_2}} \tag{A.21}$$

$$E(g_t) = -\left\{\frac{\Theta * - 1}{\Theta *}(1+r)D_{t-1} + \tilde{Z} *\right\}(1+k_1)^{-1} \tag{A.22}$$

where: $\Theta^* = \Theta + \left[(1+r)^2 b - 1\right] \dfrac{k_1}{g^2 k_2 (1+k_1)}$

and $\tilde{?}* = \dfrac{\tilde{G}}{T*}\left\{\dfrac{\left[(1+r)^2 ß - 1\right]}{r} + T* - 1\right\} - \dfrac{\tilde{D}}{T*}\dfrac{k_3}{k_1}(1+k_1) + \dfrac{p^T * - \tilde{p}}{?(1+k_1)T*}\dfrac{\left[(1+r)^2 ß - 1\right]}{r} r$

Substituting (25), (26), the expected value of (26), (A.21) and (A.22) into (2) we get the solutions for debt dynamics:

$$D_t = E(D_t) - \left\{ 1 - \frac{1 + k_1 + \dfrac{k_1}{gk_2}}{(1+k_1)\dfrac{Q*}{Q*-1} + \dfrac{k_1}{gk_2}} \right\} e_t \tag{A.23}$$

$$E(D_t) = \frac{(1+r)D_{t-1}}{\Theta *} - \tilde{Z}* + \tilde{G} \tag{A.24}$$

From (A.24) we can obtain steady-state debt as a function of the inflation target:

$$D_{ss}* = \frac{(\tilde{G} - \tilde{Z}*)\Theta*}{\Theta* - (1+r)} \tag{A.25}$$

Substituting (A.25) into (A.22) we get:

$$g_{ss}* = -\left\{ \frac{\tilde{G}[(\Theta*-1)(1+r)] - r\Theta*\tilde{Z}*}{[\Theta*-(1+r)]} \right\}(1+k_1)^{-1} \tag{A.26}$$

Equations (A.25) and (A.26) may be rearranged as follows:

$$D_{ss}* = \left\{ \tilde{D}\frac{k_3}{k_1} - \frac{\tilde{G}}{r}\left[(1+r)^2 br - (1+r)\right] - \frac{(p^T * - \tilde{p})}{r\,g(1+k_1)}\left[(1+r)^2 b - 1\right] \right\} [Q* - (1+r)]^{-1} \tag{A.27}$$

$$g_{ss}* = -\frac{k_3}{k_1}\frac{(\tilde{D}r + \tilde{G})}{[\Theta* - (1+r)]} + \frac{(p^T * - \tilde{p})\left[(1+r)^2 b - 1\right]}{[\Theta* - (1+r)]\,g(1+k_1)^2} \tag{A.28}$$

Appendix IV. The optimal inflation bias is positive

In steady state the value of (41) is

$$E(W_t)=\left\{\begin{array}{l} k_1(1+k_1)\left[g_{ss}^2+\left(\dfrac{\Theta-1}{\Theta}\right)^2\dfrac{(1+m)^2}{(1+k_1)^2}s_e^2\right]+ \\ +k_2\left[(p_{ss}-\tilde{p})^2\left(\dfrac{k_1}{k_2}\right)^2 g_{ss}^2+m^2 s_e^2\right]\end{array}\right\}\frac{1}{1-b} \tag{A.29}$$

where g_{ss} is defined in (36) and p_{ss} is easily obtained from (36) and (38). We focus on the case where the inflation target is the social planner's control variable. Taking the first-order derivative we get

$$\frac{\partial E(W_t)}{\partial p^T *}=\left\{\begin{array}{l}\dfrac{2k_1(1+k_1)\left[b(1+r)^2-1\right]g_{ss}}{[\Theta*-(1+r)]g(1+k_1)^2}+ \\ +2k_2\dfrac{(p_{ss}-\tilde{p})}{[\Theta*-(1+r)]}\left[1-\left(\dfrac{k_1}{g^2k_2}\right)\dfrac{\left[b(1+r)^2-1\right]}{[\Theta*-(1+r)](1+k_1)^2}\right]\end{array}\right\}\frac{1}{1-b} \tag{A.30}$$

If the inflation target is determined according to (39), we obtain $p_{ss}-\tilde{p}=0$. From (40) we know that in this case $g_{ss}<0$, therefore $\dfrac{\partial E(W_t)}{\partial p^T *}<0$: it is never optimal to entirely remove the inflation bias in steady state.

Appendix V. Solutions under a state-contingent inflation target

If the target is defined as in (42), condition (A.18) becomes

$$E_t\{g_{t+s}\}=\left[g_t+\frac{k_3}{k_1}\left(D_t-\tilde{D}\right)\right](r_a)^{-1}[(1+r)b]^{-s}+\frac{p^T*-\tilde{p}}{g(1+k_1)r_a},\quad \forall s\geq 0, \tag{A.31}$$

where

$$r_a = 1 + (1 - a)\frac{k_1}{g^2 k_2 (1 + k_1)}$$

The solutions for debt, expenditures and inflation are as follows:

$$d_t^a = \frac{(1 + r)}{Q\,*} d_{t-1}^a - \frac{(1 + m^a)}{Q^a} e_t \tag{A.32}$$

$$g_t^a = E(g_t^a) + \frac{Q^a - 1}{Q^a} \frac{(1 + m^a)}{(1 + k_1)} e_t \tag{A.33}$$

$$E(g_t^a) = g_{ss}^a - \frac{Q^a - 1}{Q^a (1 + k_1)} (1 + r) d_{t-1}^a \tag{A.34}$$

$$D_{ss}^a = \left\{ \tilde{D} \frac{k_3}{k_1} - \frac{\tilde{G}}{r} \left[(1 + r)^2 b r_a - (1 + r) \right] - \frac{(p^T * - \tilde{p})}{r g (1 + k_1)} \left[(1 + r)^2 b - 1 \right] \right\} \left[Q^a - (1 + r) \right]^{-1} \tag{A.35}$$

$$g_{ss}^a = -\frac{k_3}{k_1} \frac{(\tilde{D} r + \tilde{G})}{\left[Q^a - (1 + r) \right]} + \frac{(p^T * - \tilde{p})\left[(1 + r)^2 b - 1 \right]}{\left[Q * - (1 + r) \right] g (1 + k_1)^2} \tag{A.36}$$

$$p_t^a = E(p_t^a) + m^a e_t \tag{A.37}$$

$$E(p_t^a) = p^T * - (1 - a) \frac{k_1}{g k_2} E(g_t *) \tag{A.38}$$

where:

$$Q^a = Q + (1 - a)\left[(1 + r)^2 b - 1 \right] \frac{k_1}{g^2 k_2 (1 + k_1)} ; m* = -\frac{k_1}{g k_2 \left[(1 + k_1) \frac{Q^a}{Q^a - 1} + \frac{k_1}{g k_2} \right]}$$

If $a = 1$ we obviously obtain that $\frac{\partial E_t (p_{t+1}^T)}{\partial D_t} = -\frac{\partial E_t (g_{t+1})}{\partial D_t}$ and $\frac{\partial E_t (p_{t+1}^B)}{\partial D_t} = 0$. As a consequence (27) is identical to (9), the inflation bias is entirely removed and the solutions for the expected levels of expenditures, taxes and debt are the same as under discretion. However, for $0 < a < 1$ the solutions are qualitatively identical to those obtained under a non-contingent target. Observe that even if a state-contingent target is feasible, imperfect control of the inflation bias is necessary to obtain a reduction of fiscal distortions in steady state.