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Abstract

Developing a model with a switching mechanism, we show how complex dynamics can be generated even though heterogeneity arises among agents with the same trading rules (fundamentalists). We assume that there are two experts which are imitated by other operators. We show that (i) market instability and periodic, or even, chaotic price fluctuations can be generated; (ii) conditions exist under which an expert can drive another expert out of the market; (iii) two experts can survive when the dynamic system either generates a period doubling bifurcation around an attractor or when an homoclinic bifurcation leads to the merging of the two attractors (i.e. Dieci et al., 2001); (iv) a central role is played by the reaction to misalignment of both market makers and agents.

JEL: C61, G11, G12, D84
Keywords: mathematical economics, chaos, heterogeneous interacting agents, financial markets.

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1 Introduction

In the last decade, starting from empirical works which showed how financial market agents employ different strategies (i.e. Taylor and Allen 1992), an increasing number of theoretical works have been dedicated to analyzing the dynamics of financial markets due to interactions of heterogeneous agents.

These works specifically try to examine whether certain observable features of financial markets, such as high volatility or unpredictability, can be caused by agents' heterogeneity.

In the canonical models (Day and Huang, 1990; Chiarella, 1992), the theory of complex dynamics is determined by interactions between traders with different trading rules (for a complete survey see Hommes, 2006). Three typical investor strategies are usually considered**.** Fundamentalists look at the distance between current price and a fundamental value which is extracted from information related to the economic system. Chartists work out observed price patterns from the past, trying to take advantage of the bull and bear market. Lastly, there are the noise traders who do not rely on a specific economic thought, but on factors such as rumours (for a complete survey see Hommes, 2006). Contrary to the efficient market hypothesis, speculators may cause instability in the market. The basic idea is that if averages fail to cancel out differences in beliefs, prices may deviate from the economic fundamentals. Different patterns of instability can be traced by starting from different assumptions. In Day and Huang (1990) and Chiarella (1992) price fluctuations are related to non linearity in the chartists trading rule and a central role is played by the weights assigned to fundamentalists and chartists because while the former has a stabilizing effect, the latter introduces instability to the system. In contrast, in Brock and Hommes (1997, 1998), nonlinearity depends on switching the process from a fundamental to a chartist trading rule and vice versa. The basic idea is that agents moving from one strategy to another in search of better accuracy regarding prediction generate fluctuations that may destabilize the market. Price deviations may be triggered by differences in beliefs and amplified by evolutionary dynamics between different schemes.

Therefore in these models a central role is played by differences in trading rules. Particularly according to Hommes (2006 pag.55): "Sophisticated traders, such as fundamentalists or rational arbitrageurs typically act as a *stabilizing force*, pushing prices in the directions of the RE fundamental value. Technical traders, such as feedback traders, trend extrapolators and contrarians typically act as a *destabilizing force*, pushing prices away from the fundamental".

Developing a model with a switching mechanism, we show how complex dynamics can be generated even though heterogeneity arises among agents *with the same trading rule*. We assume that there are two experts who act as fundamentalists[1], and are imitated by other operators. Moreover agents can switch from one expert to the other following an adaptive belief system (Brock and Hommes 1997). Mainly, agents' switch is driven by experts' ability, approximated by the distance between fundamental value and price. A very simple switch mechanism, based on square error, is employed: the less the margin of square error, the higher the quota of agents that emulate that expert. The switching mechanism has a central role in our story since there is an endogenous relation between the quote of agents that follow each expert and price level. We show that (i) market instability and periodic, or even, chaotic price fluctuations can be generated; (ii) conditions exist under which an expert can drive another expert out of the market; (iii) two experts can survive when the dynamic system either generates a period doubling bifurcation around an attractor or when an homoclinic bifurcation leads to the merging of the two attractors (i.e. Dieci et al., 2001); (iv) a central role is played by the reaction to misalignment of both market makers and agents.

After presenting the model in section two, we will analyse the results when agents are homogeneous in section three. We will show how the linear map generated guarantees a monotonically or oscillatory convergence for a particular set of parameters. In section four we will discuss the conditions necessary for existence, the stability of fixed points and the insurgence of a flip bifurcation. Widening our analysis, we will show the circumstances, under which an expert is cancelled out by the other, and the appearance of a periodic, or even, chaotic, price fluctuation. Finally section six provides brief concluding remarks ad suggestions for further research.

2 The Model

As stated above, opposed to a stylized asset pricing model, we have two types of agents: market makers and fundamentalists. The former mediate in transactions, setting prices in reply to excess demand (supply), while the latter, believing that prices move towards their fundamental values, buy assets that are undervalued and sell them when they are overvalued. We assume that there are two experts who act as fundamentalists, and are imitated by other operators. As in Brock and Hommes (1998) and in Hommes et al. (2005) we explore a model with two assets: one risky and one risk free. A perfectly elastic supply and a gross return (R>1) characterize the risk-free asset. Moreover, a price ex-dividend (X_t) and a (stochastic) dividend process (y_t) represent the key elements of the risky asset. Let i=1,2 be the two experts, the following formula expresses their wealth:

$$W_{i,t+1} = RW_{i,t} + (X_{t+1} + y_{t+1} - RX_t)q_{i,t} \quad (1)$$

[1] Recently using a variation of the minority game, Ferreira et al. (2005) analyzed the interaction among speculators who disagree about fundamental prices.

where the fundamentalist i acquires in time t shares of risky asset $q_{i,t}$. Given wealth expectations ($E_{i,t}$) and a constant variance over time ($V_{i,t} = \sigma^2$), traders maximize the demand for shares $q_{i,t}$, solving the following:

$$Max_{z_{i,t}} \left\{ E_{i,t}(W_{i,t+1}) - \frac{a}{2} V_{i,t}(W_{i,t+1}) \right\} \tag{2}$$

where a is the strictly positive constant risk aversion for both investors. Hence the investor i demands an amount $q_{i,t}$ following:

$$q_{i,t} = \frac{E_{i,t}(X_{t+1} + y_{t+1} - RX_t)}{aV_{i,t}(X_{t+1} + y_{t+1} - RX_t)} = \frac{E_{i,t}(X_{t+1} + y_{t+1} - RX_t)}{a\sigma^2} \tag{3}$$

We assume that they have a common correct expectations on dividends ($E_{i,t}(y_{t+1}) = E_t(y_{t+1}) = \bar{y}$) and future prices ($E_{i,t}(X_{t+1}) = E_t(X^*_{t+1}) = F_i$). It is worth noting that F_i represent the benchmark fundamental values detected by the experts analyzing economic factors, such as macroeconomic indexes (Hommes, 2001). We are aware that the assumption of *common expectations on dividend* is restrictive. However, the qualitative dynamic behaviour of the model is not modified but this assumption[2]. Hence, equation (3) can be rewritten as follows:

$$q_{i,t} = \alpha(F_i - P_t) \tag{3b}$$

where $P_t = RX_t - y_{t+1}$ and $\alpha = \frac{1}{a\sigma^2}$ is the coefficient reaction of the investors as related to misalignment, and negatively related to risk aversion. For simplicity and to avoid losing generality, we assume $F_1 \le F_2$.

The price of the asset does not follow a mechanism such as the walrasian auctioneer but a market maker mechanism where out of equilibrium exchanges are possible. Particularly, market makers apply the following rule:

$$P_{t+1} = P_t + \beta\left[w_{t+1}q_{1,t+1} + (1 - w_{t+1})q_{2,t+1}\right] \tag{4}$$

The function points out the relation between the excess quantity (demanded or supplied) and the price change where β is the speed of adjustment to change quantity demanded and w_{t+1} is the proportion of agents that imitate *expert 1* and depend on the switching mechanism. According to (4) with a positive (negative) excess demand market makers dismiss their inventory increasing (decreasing) the price.

[2] It is to show that different beliefs alter mainly the halfway steady states, without having any impact on dynamics, because it is unstable and circumscribe just the basins of attractions of coexistent attractors.

The switching process is based on the heterogeneity in expertise, represented by both the distance between the fundamentals and P_t, between the two experts. Particularly, agents are more likely to imitate the expert whose prediction is closer to P_t. Let SE_t^i to be the squared errors of the two experts:

$$SE_t^1 = (F_1 - P_t)^2$$

$$SE_t^2 = (F_2 - P_t)^2$$

Using an adaptive rational mechanism [Brock and Hommes, 1997, 1998], w_{t+1} can be viewed as a frequency:

$$w_{t+1} = 1 - \frac{(F_1 - P_t)^2}{(F_1 - P_t)^2 + (F_2 - P_t)^2} \tag{5}$$

Similarly to Kaizoji (2003) and He and Westerhoff (2005) the switching mechanism is based on the accuracy of prediction. However their mechanism is built looking at differences between chartists and fundamentalist. Mainly in Kaizoji (2003) agents choose chartist strategy "according to the difference between the squared prediction errors of each strategy". While in He and Westerhoff (2005) the larger deviation of current price from fundamental values the greater is the quota of agents that follow a chartist's strategy.

In our case it is worth noting that the quota of agents that follow expert i depends on the relative distance between the corresponding fundamental value, F_i and the current price. Moreover this mechanism is real clear-cut: when the fundamental value F_i is equal to current price, in the next period all agents follow the corresponding expert. This implies that the quota varies from zero to one.

Substituting (3b) and (5) in (4) the following dynamic price equation is achieved:

$$P_{t+1} = P_t + \alpha\beta \left\{ \frac{(F_1 - P_t)(F_2 - P_t)\left[F_1 + F_2 - 2P_t\right]}{(F_1 - P_t)^2 + (F_2 - P_t)^2} \right\} \tag{6}$$

which is a one dimensional nonlinear map. As in our model, it is inspired by Day and Huang (1990); and, as in He and Westerhoff (2005) and Dieci et al (2001), the dynamic is triggered by a cubic one-dimensional map following the switching mechanism.

3 Homogeneous Fundamentalists: uniqueness, existence and stability

Given that we want to show how simple heterogeneity can generate complex dynamics, it is worth starting our analysis exploring the case when there is complete homogeneity ($F_1 = F_2 = F$)[3].

Proposition 1. When $F_1 = F_2 = F$ there is a unique fixed point: $P = F$. This steady state is globally stable if and only if $\alpha\beta < 1$. There are period-two cycles if and only if $\alpha = \frac{1}{\beta}$. Finally, a divergence to infinity arises if $\alpha\beta > 1$.

Proof.

For $F_1 = F_2 = F$, equation [6] can be re-written as:

$$P_{t+1} = P_t + 2\alpha\beta(F - P_t) \tag{7}$$

which is a linear map. A steady state condition is implied, particularly when $P_{t+1} = P_t = P^*$ and then $P^* = P^* + \alpha\beta(F - P^*)$. That is when $P^* = F$. The fundamental value is the only steady state that comes out from the system. Moreover, the derivate of equation [7] valued in F is

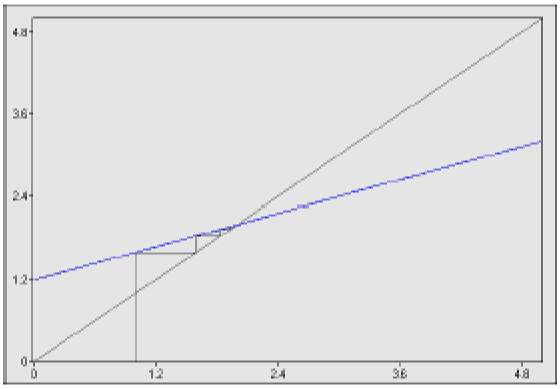
$$\left.\frac{d(P_{t+1})}{dP_t}\right|_{P_t = F} = 1 - 2\alpha\beta \tag{8}$$

hence the steady state is globally stable if and only if $\alpha\beta < 1$.

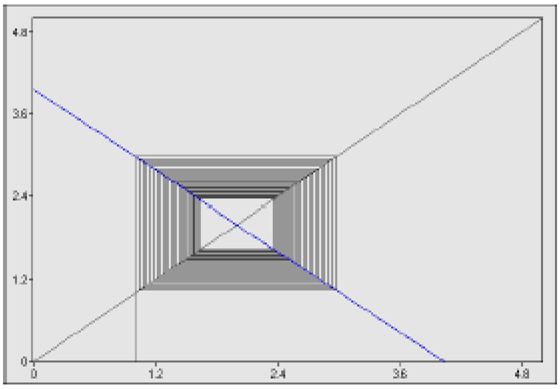
With homogeneity we have a uniquely steady state, F ; from a dynamical point of view the behaviour of the model is quite simple. In Figure 1, setting $F = 2$, $\alpha = 1$ and $P_0 = 1$ we show the dynamic of equation (7) with different values of the market maker reaction coefficient, β . Figure 1a and 1b respectively show the case in which there is global stability with monotonic or oscillatory convergence.

[3] Inequality in the reaction coefficients does not change the following analysis.

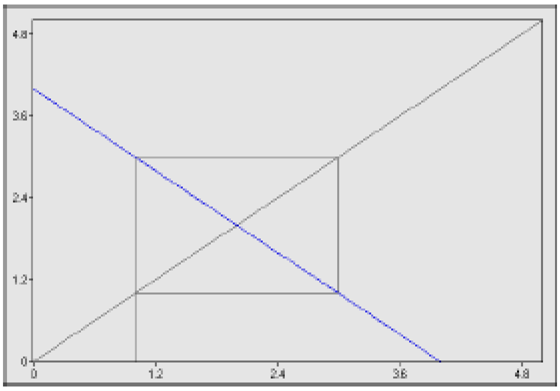
Figure 1 – Dynamics with Homogeneity



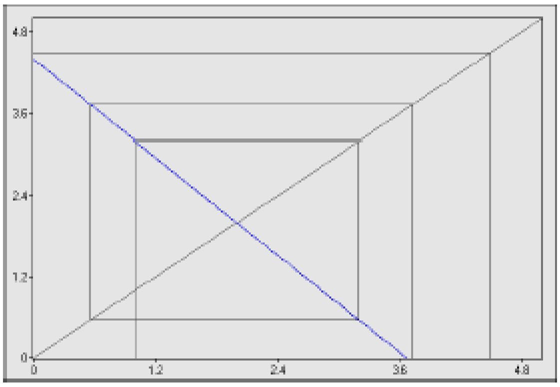
1.a) $\beta = 0.3$



1.b) $\beta = 0.99$



1.c) $\beta = 1$



1.d) $\beta = 1.1$

Note 1 - $F = 2$, $\alpha = 1$ **and** $P_0 = 1$

In Figure 1c we show the particular set of parameters that determines a period-two cycle and finally in Figure 1d how the divergence to infinity is reported.

Therefore the linearity guarantees a monotonically or oscillatory convergence for a particular set of parameters. Specifically convergence is guaranteed both by market makers and by all agents who do not overreact to misalignment. Otherwise, if one of the reaction coefficients is too high a divergence to infinity occurs. Furthermore, only if the map is perpendicular to the intersecting line, can there be two period cycles.

4 Heterogeneous Fundamentalists: Steady States and Local Analysis

Differently from what we have seen in the last paragraph, even a simple heterogeneity gives rise to new steady states and to a richer variety of dynamical behaviour. In this section we explore the existence of steady states and local stability by devoting it to the analysis of global dynamics.

Let $P_{t+1} = P_t = P^*$ be the steady state of the map (6). Hence, it has to satisfy:

$$\frac{(F_1 - P^*)(F_2 - P^*)\left[F_1 + F_2 - 2P^*\right]}{(F_1 - P^*)^2 + (F_2 - P^*)^2} = 0 \tag{9}$$

Applying straightforward algebra, the fundamental values of the two experts, F_1 and F_2, are always steady states. In addition there is a third steady state represented, equal to:

$$P^* = \frac{F_2 + F_1}{2} \tag{10}$$

which is exactly the arithmetic means of the fundamental values; P^* is encompassed between the two fundamental values: $F_1 < P^* < F_2$.

Finally, given equation (5), in the steady states F_i (with $i = 1,2$) we have the share of agents that follow the fundamental value F_j (where $i \neq j$) decreases to zero: all agents follow only one fundamentalist. The coexistence of both groups is possible with the third steady state. Particularly the quote of agents that follow the expert F_1 is equal to $w(P^*) = \frac{(F_1)^2}{(F_1)^2 + (F_2)^2}$. It is worth noting that this share is positively related with F_1 and negatively with F_2. However, as discussed below, this steady state is not relevant since it is always instable.

Proposition 2. Given the map (6), the steady states F_1 and F_2 are stable if $\alpha\beta < 2$. On the other hand the steady state P^* is always unstable.

Proof. The derivative of the dynamic equation (6) is equal to:

$$\frac{dP_{t+1}}{dP_t} = 1 + \alpha\beta\left[\frac{2(F_1 - P_t)^2(F_2 - P_t)^2 - (F_2 - P_t)^4 - (F_1 - P_t)^4 - 2\left[(F_1 - P_t)^2 + (F_2 - P_t)^2\right](F_1 - P_t)(F_2 - P_t)}{\left[(F_1 - P_t)^2 + (F_2 - P_t)^2\right]^2}\right]$$

(11)

Using straightforward algebra to evaluate the (11) in the fixed points F_i we achieve:

$$\left.\frac{dP_{t+1}}{dP_t}\right|_{P_t = F_i} = 1 + \alpha\beta\left[\frac{(F_1 - F_2)(F_2 - F_1)}{(F_1 - F_2)^2}\right] = 1 - \alpha\beta \tag{12}$$

then we have local stability for:

$\beta\alpha < 2$.

On the other hand, the instability of P^* can be revealed evaluating (11) in P^*. The result is:

$$\left.\frac{dP_{t+1}}{dP_t}\right|_{P_t=P^*} = 1 + \alpha\beta \qquad (13)$$

Since both β and α are positive the derivate (13) is more than 1: therefore, there is always instability.

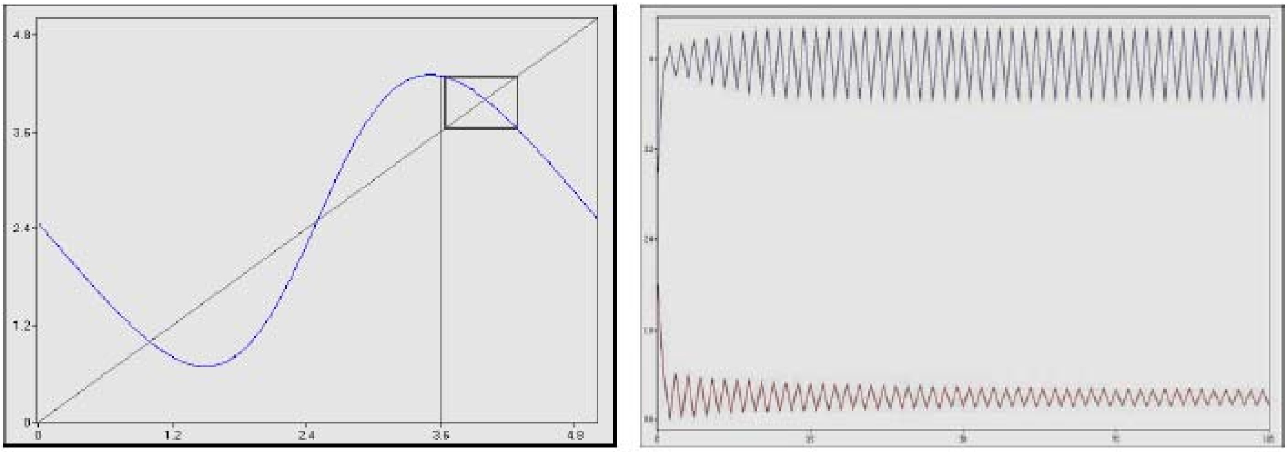
The stability properties of these two steady states do not depend on the fundamental values themselves but rather on both reaction coefficients α and β. This result indicates that a limited feedback to misalignments and narrow market maker reaction can stabilize the fundamental value. Therefore turbulences in the market are generated by an overreaction to market misalignments and by an overreaction of market makers to excesses in demand (supply).

Proposition 3. The map (6) exhibits in F_i a period doubling bifurcation when $\alpha\beta = 2$.

Proof. The dynamic map (6) satisfies the canonical conditions required for the flip bifurcation (period doubling) [J.Hale and Kocak H, Dynamic and Bifurcation, 1991, Springer-Verlag]. Indeed $\alpha\beta = 2$ are non-hyperbolic points for (6), when $\alpha\beta < 2$ the fixed points F_i are attracting, when $\alpha\beta > 2$ they are repelling. Hence there is a change in the nature of dynamics when $\alpha\beta = 2$. Specifically a unique asymptotically stable periodic point of period two arises.

To shed some light on what really happen in the market, Figure (2a) and (2b) report a case in which $F_1 = 1$, $F_2 = 4$, $\beta = 1$ and $\alpha = 2$. Given equation (12) we already know that these parameters produce a period doubling bifurcation. Economically, starting from an excess in demand ($P_o < F_2$), agent overreaction leads to a large price increase in such a way that the price becomes higher than F_2. An excess in demand is transformed into an excess in supply. Even in this case, given a high α, agents that follow expert 2 supply a bulky quantity that leads the price down, particularly less than F_2. Hence the system fluctuates between excess of demand and excess of supply at round F_2.

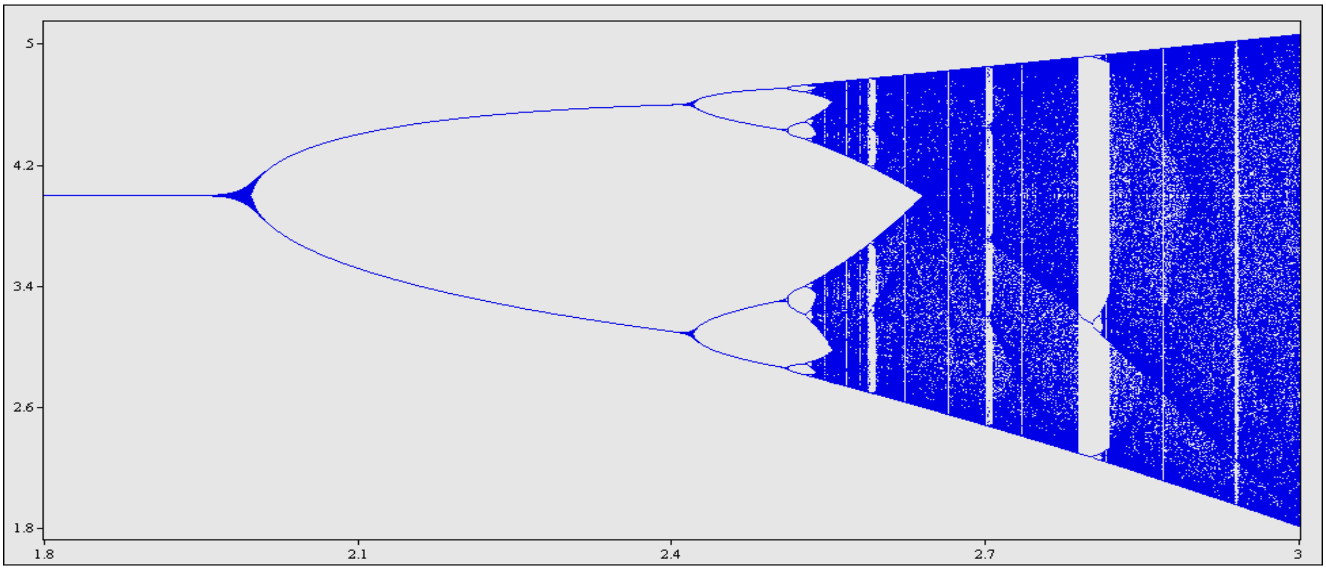
Figure 2 – Cycle of period two (a) and time series with cycle of period two around the steady state (b)



Note 2 - $F_1 = 1$, $F_2 = 4$, $\beta = 1$ **and** $\alpha = 2.1$

Figure [2b] shows that this behaviour is symmetric: it happens around each of the fundamental values. To clarify the dynamics that depend on α, we have reported in Figure (3), a bifurcation diagram. It shows different values of price for different values of α, particularly between (1.8 and 3). It is easy illustrated that from stability we first move to a period of doubling bifurcation and afterwards to one of chaos.

Figure 3 – Bifurcation diagram for α between 1.8 and 3



Note 3 - $F_1 = 1$, $F_2 = 4$, $\beta = 1$ **and** $\alpha = [1.8,3]$

Finally we can state that the map is symmetric in relation to P^* and dynamically all qualitative changes (bifurcations, stability/instability, etc.) around the fixed points, F_1 and F_2, occur due to the same set of parameters.

5 Global Analysis

In this section we explore the global dynamics of the map. Globally the dynamical behaviour is very simple and when the market maker reaction is low we have a monotonic convergence to the fundamental values.

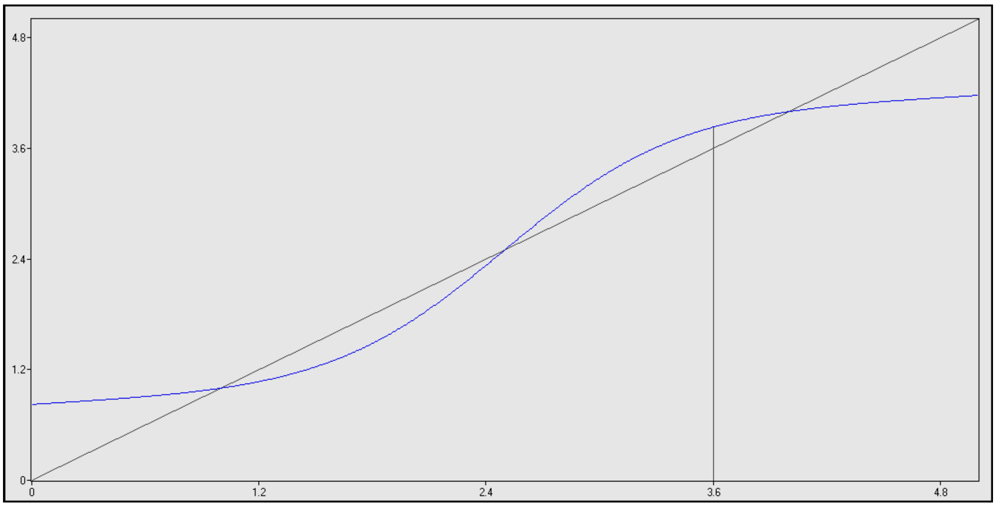
Proposition 4. For a low enough value of β, $\bar{\beta}$, the initial conditions belonging to the interval $[0, P^*)$ converge at the lowest fundamental value F_1; alternatively when the initial conditions lie in the interval (P^*, ∞), they converge to the highest fundamental value F_2.

Proof. Rewriting equation (11) as

$$\frac{d(P_{t+1})}{dP_t} = 1 + \beta D = 1 + \alpha\beta R \qquad (14)$$

where $D = \alpha R = \alpha f(F_1, F_2)$, it is straightforward that for each combination of parameters (α, F_1, F_2), a value $\bar{\beta}$ exists in such a way that D is always equal to more than zero. Hence the map of the dynamical system is monotonic and therefore invertible as shown in Figure [4].

Figure 4 – Map (6) with a monotonic dynamics

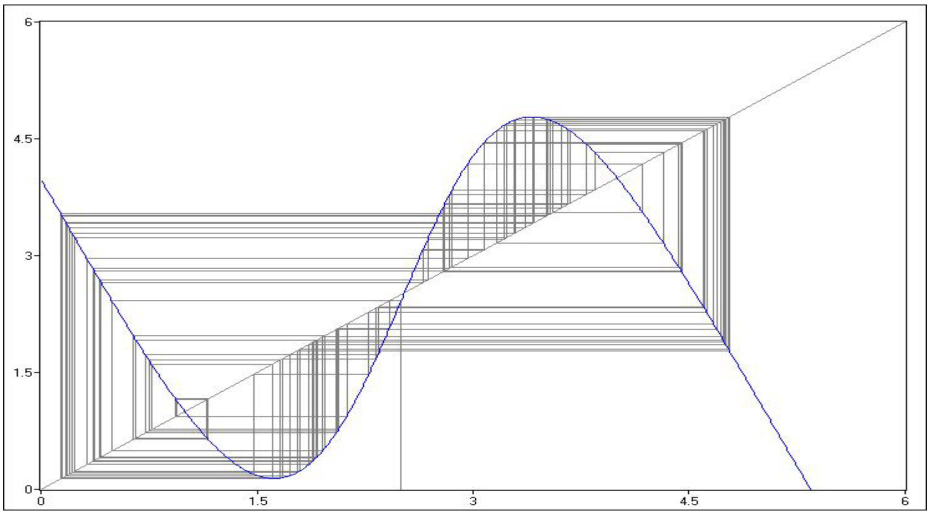


Note 4 – $F_1 = 1$, $F_2 = 4$, $\beta = 1$ **and** $\alpha = 1$

It is easy to demonstrate that in this case the initial conditions belonging to the interval $[0, P^*)$ converge at the lowest fundamental value F_1; alternatively when the initial conditions lie in the interval (P^*, ∞), they converge to the highest fundamental value F_2.

By using numerical simulations we now explore the particular route to homoclinic bifurcation. Particularly we set up parameters $\alpha = 1$, $F_1 = 1$, and $F_2 = 4$. Increasing the reaction coefficients of market makers at $\beta = 2$ a period-doubling bifurcation arises and there are two symmetric stable cycles of period two. According to Dieci et al. (2001), flip bifurcation does not affect the global structure of the basins: cycles of period two simply substitute the two steady states. However, further growth of β leads initially to a new attractive period-four cycles, which is followed by a two symmetric chaotic attractors. Finally for $\beta \approx 2.92$ a homoclinic bifurcation emerges (figure [5]). The new structure of the basins produced implies the synthesis between the basins of the two fundamental values: bull and bear price fluctuations appear.

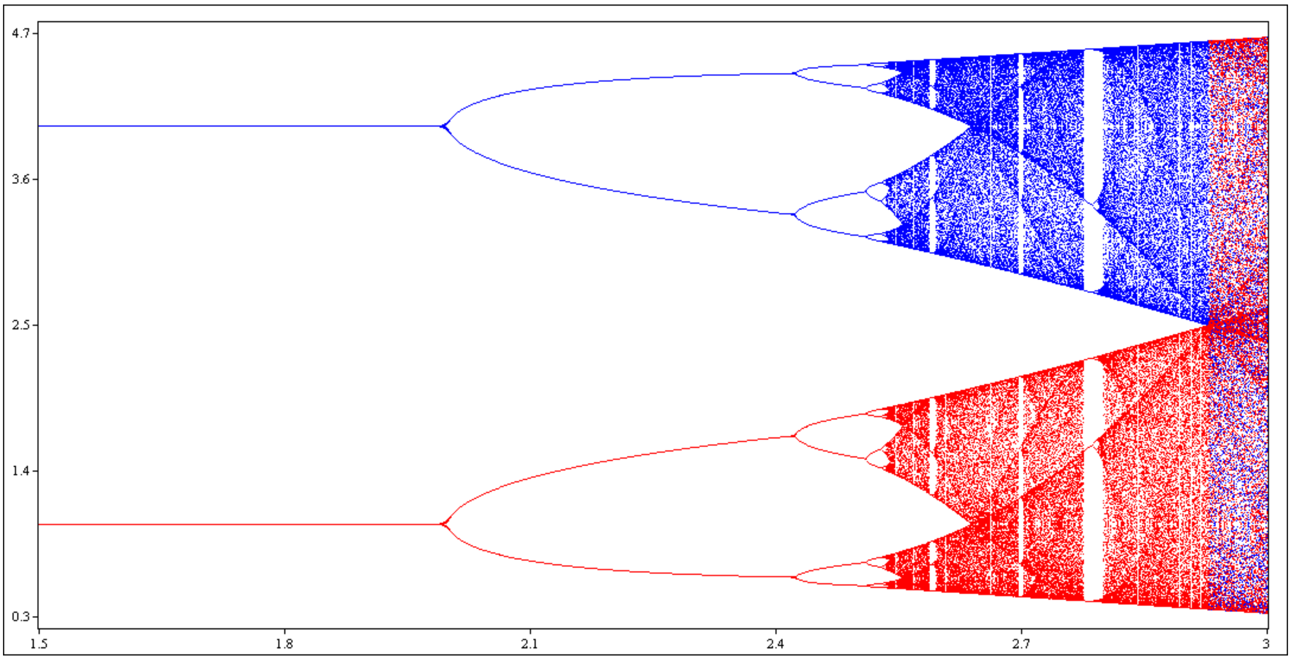
Figure 5 – Homoclinic Bifurcation



Note 5 - $F_1 = 1$, $F_2 = 4$, $\beta = 3$ **and** $\alpha = 1$

As shown, different authors have illustrated (i.e. He and Westeroff, 2005; Dieci et al., 2001), that homoclinic bifurcation occurs when a local maximum and minimum are mapped in the unstable steady state P^*. As shown even by Figure 6, which represents the bifurcation diagram for β the two symmetric chaotic attractors merge into a single one.

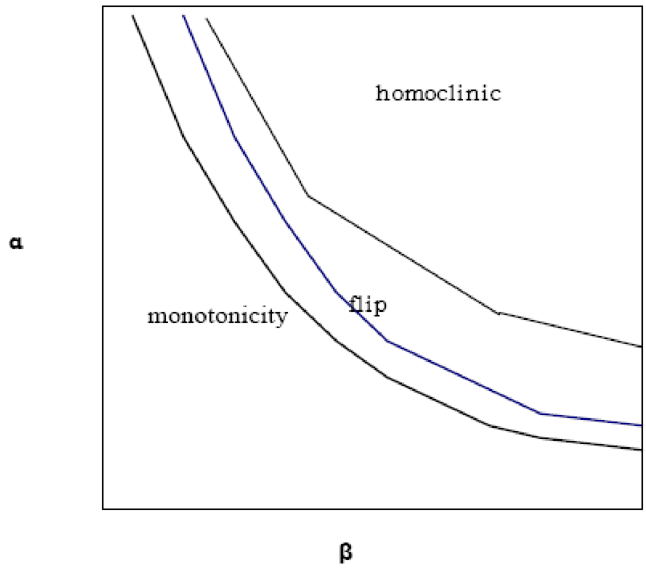
Figure 6 – Bifurcation Diagram around two attractors, for β between 1.5 and 3



Note 6 $F_1 = 1$, $F_2 = 4$, $\alpha = 1$

Finally, holding $F_1 = 1$ and $F_2 = 4$ both local and global behaviour of map (6) are qualitatively reported in figure (7) varying both reaction coefficients, α and β, which cause qualitative changes in the dynamics. Particularly, given equation (14) there is a monotonic convergence if $\alpha < \frac{1}{\beta R}$. Otherwise there is a flip bifurcation if $\alpha = \frac{2}{\beta}$.

Figure 7 – Quality Stability Region with $F_1 = 1$ and $F_2 = 4$



Finally, the homoclinic bifurcation achieved by numerical estimation raises the local maximum and the local minimum relapses into the repelling steady state P^*.

6 Conclusion and Further research

This paper covers price dynamics in financial markets. Unlike canonical models (based on heterogeneity), we focus on agents with the same trading rules (fundamentalists) where heterogeneity depends on different fundamental values and reaction coefficients to misalignments. In addition we employ a very simple switch mechanism.

We show that (i) market instability and periodic, or even, chaotic price fluctuations can be generated; (ii) conditions exist under which an expert can drive another expert out of the market; (iii) two experts can survive when the dynamic system either generates a period doubling bifurcation around an attractor or when an homoclinic bifurcation leads to the merging of the two attractors (i.e. Dieci et al., 2001); (iv) a central role is played by the reaction to misalignment of both market makers and agents.

Heterogeneity in financial markets has been developed in various models which have aided in explaining financial market dynamics. A further development of our model should aim at using a different algorithm, such as that of Brook-Hommes (1997), which can always foresee the coexistence of both experts. Moreover, since our simple switch mechanism is based on the distance between current price and fundamental values, it would be interesting to analyze the dynamics generated by heterogeneity in the case of profitability based imitative process.

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