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STACKELBERG COMPETITION WITH ENDOGENOUS ENTRY

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Abstract

This paper analyzes market structures where leaders have a first mover advantage and entry by the followers is endogenous. The strategy of the leaders is always more aggressive than the strategy of the followers independently from strategic substitutability or complementarity. Under quantity competition, the leader produces more than any other firm and I determine the conditions for entry deterrence to be optimal (high substitutability and constant or decreasing marginal costs). Under price competition, the leader sets a lower price than each follower, just the opposite than with an exogenous number of firms. In contests the leader invests more than each follower. In all these cases a leadership improves the allocation of resources compared to the Nash equilibrium with endogenous entry.

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1 Introduction

This paper studies market structures where one or more firms have a first mover advantage in the sense of Stackelberg (1934) on the other firms, but entry in the market is endogenous, characterizes their equilibrium and performs welfare analysis for some of the applications. These market structures emerge in many sectors which are substantially competitive (a fringe of firms is ready to enter whenever there is a profitable opportunity), but where some incumbent firms have a competitive advantage over the followers (because of technological, historical or legal reasons, or just because entry was not possible at an earlier stage) and choose their strategies before them.[1]

While standard models of symmetric competition have been widely studied in the presence of endogenous entry in a Marshallian tradition, traditional models with incumbent firms facing a competitive fringe of entrants have been mostly limited to the analysis of markets with homogenous goods and entry deterrence, for instance in the theories of limit pricing and of contestable markets (Baumol, Panzar and Willig, 1982). More recently, Vives (1988) has analyzed games with sequential entry of multiple firms, but without endogenizing the entry process (see also Anderson and Engers, 1992, 1994). This paper is an attempt to provide a general characterization of Stackelberg equilibria with endogenous entry of followers.

The analysis delivers a simple result: leaders facing endogenous entry are always aggressive compared to the followers, in the sense that they always produce more under competition in quantities or they set lower prices under competition in prices, while this is not necessarily the case when the number of firms is exogenous. With a fixed number of firms the leader is mainly concerned about the reactions of the other firms to its own choices, but these reactions are opposite according to whether strategic substitutability or complementarity holds. However, when entry is endogenous, the leader is mainly concerned about the effect of its own choices on the entry decision: an accomodating behaviour would be ineffective because (the induced) entry would make it unprofitable, while an aggressive behaviour limits entry and spreads a small mark up over a large market share.

[1] As well known, the commitment of the leaders to a strategy may not be credible in the long run. Nevertheless, such a commitment represents a credible advantage in markets with a short horizon or when strategies are costly to change. For instance, in some seasonal markets firms choose their production level at the beginning of the season and it is hard to change such a strategic choice afterward. In other markets, prices are sticky in the short run because the information to reoptimize is costly or because a price change can induce adverse reputational effects on the perception of the customers: being the first mover in the price choice provides the leader with a credible commitment in the short run. In patent races, a preliminary investment in research and development represents a solid commitment to an innovation strategy.

While I develop the main analysis in a general framework that nests a number of models, I also study in detail some of these models, starting from competition in quantities. When goods are imperfect substitutes and the average cost function is U-shaped market leaders facing endogenous entry produce more than their followers but entry occurs in equilibrium. However, when goods are homogenous and marginal costs are constant, the leader finds it optimal to increase output until no one of the potential entrants can obtain profits in the market. Nevertheless, in both cases effective or potential entry induce the leader to choose an extremely low price, and welfare is higher compared to the corresponding Cournot equilibrium with endogenous entry. In models with competition in prices, the impact of endogenous entry is even more radical. While leaders facing an exogenous number of followers choose a higher price than their competitors, leaders facing endogenous entry always choose a lower price than each other firm and obtain positive profits (enjoying a first mover advantage). I analyze the cases of Logit and Dixit-Stiglitz demands and verify that welfare is also higher under Stackelberg competition with endogenous entry: again, the lower price of the leaders improves the allocation of resources. Finally, also different kinds of contests are nested in our general model. In these models we have leaders facing endogenous entry that invest more than any other firm: between the different applications, I will focus on patent races, where it emerges again that the allocation of resources is improved when one of the firms acts as a leader and entry is endogenous.

The analysis of Stackelberg competition with endogenous entry is closely related with three older theoretical frameworks. The first is the literature on entry deterrence associated with the socalled Bain-Modigliani-Sylos Labini framework. The inital contributions by Bain (1956), Sylos Labini (1956) and Modigliani (1958) took in consideration the effects of entry on the behaviour of market leaders, but they were not developed in a coherent game theoretic framework and were substantially limited to the case of competition with perfectly substitute goods and constant or decreasing marginal costs (which not by chance, as we will see, are sufficient conditions for entry deterrence).

The second is the dominant firm theory, which tries to explain the pricing decision of a market leader facing a competitive fringe of firms taking as given the price of the leader (see Viscusi *et al.*, 2005, Ch. 6). Assuming that the supply of this fringe is increasing in the price, the demand of the leader is total market demand net of this supply. The profit maximizing price of the firm is above marginal cost but constrained by the competitive fringe. The dominant firm theory provides interesting insights on the behaviour of market leaders under competitive pressure, and this work tries to provide an alternative game-theoretic foundation for its results.

The third is the theory of contestable markets by Baumol, Panzar and Willig (1982), which is mostly focused on markets for homogenous goods without sunk costs of entry, and it shows that the possibility of "hit and run" strategies by potential entrants is compatible only with an equilibrium price equal to the average cost: in other words, a single potential entrant is enough to insure a competitive behaviour of the incumbent. An interesting feature of this equilibrium is that it corresponds to the one emerging under price competition where the incumbent sets its price before the entrants. In this paper I will try to examine more general situations in which the goods are not necessarily homogenous, and the incumbent leader and the followers can compete both in prices and quantities. In such a case, only a potentially large set of competitors that guarantees that entry can be regarded as endogenous induces a competitive behaviour of the leader.

These results lead to a few policy implications. The general aggressive behaviour of the market leaders facing endogenous entry suggests that large market shares and large extraprofits for these leaders can be the consequence of a competitive behaviour induced by the competitive pressure of the entrants. Therefore, in the field of antitrust policy, and in particular in investigations concerning abuse of dominance, a preliminary examination of the entry conditions is crucial to verify whether large market shares of the leaders can be a sympthom of dominance.

The paper is organized as follows. Section 2 presents the general model and derives the main general results, Section 3-5 study in detail applications to models of competition in quantities, prices and in contests, and develop a welfare analysis for each application. Section 6 discusses some extensions and Section 7 provides some policy implications. Section 8 concludes. Proofs and further extensions are left in the Appendix.

2 The model

In this section I will introduce a general model of market structure and study Stackelberg equilibria with and without endogenous entry.

Consider many identical firms which are potential entrants in a market. Each firm i active in the market chooses a single strategic variable $x_i \in \mathbf{X} \subset \mathbb{R}_+$ where the close set $\mathbf{X}$ is a strategy space. If n firms enter in the market, a set of strategies delivers the net profit function for firm i:

$$\pi_i = \Pi\left(x_i, X_{-i}\right) - K \tag{1}$$

where the effects or spillovers induced by the strategies of the other firms on firm i's profits are summarized by $X_{-i} = \sum_{j=1, j\neq i}^{n} h(x_j)$ for some function $h : \mathbf{X} \to \mathbb{R}_+$ which is assumed continuous, differentiable, positive and increasing, and $K \geq 0$ is a fixed cost of production. I assume that the function $\Pi : \mathbf{X} \times \mathbb{R}_+ \to \mathbb{R}_+$ is twice differentiable and quasiconcave in x_i with an optimal strategy $x(X_{-i})$ for any level of spillovers X_{-i}, while spillovers are assumed to exert a negative effect on profits, $\Pi_2 < 0$, which, as we will see, is a necessary condition for having free entry equilibria.[2]

[2]Subscripts denote derivatives. In order to focus on interesting issues I also assume that

In general, it could be that Π_{12} is positive, so that we have *strategic complementarity* (SC), or negative so that we have *strategic substitutability* (SS). I will define strategy x_i as aggressive compared to strategy x_j when $x_i > x_j$ and as accomodating when the opposite holds: notice that a more aggressive strategy by one firm reduces the profits of the other firms. Most of the commonly used models of oligopolistic competition are nested in our general specification: as we will see, these include all symmetric models of competition in quantities, models of competition in prices with Logit and Dixit-Stiglitz demand functions, standard patent races, contests, rent seeking games, and others.

One of the firms is a leader in the sense that is a first mover in the choice of the strategy.[3] The other firms are the followers and the entry of these is endogenous. Since a generic zero profit condition will determine the endogenous number of firms, I will refer to the equilibrium as to a free entry equilibrium. In particular, I will define a Stackelberg Equilibrium with Free Entry (SEFE) as a subgame perfect equilibrium of the game with the following simple sequence of moves:

1) in the first stage, a leader, firm L, enters, pays the fixed cost F and chooses its own strategy, say x_L;

2) in the second stage, after knowing the strategy of the leader, all potential entrants simultaneously decide "in" or "out": if a firm decides "in", it pays the fixed cost K;

3) in the third stage all the followers that have entered choose their own strategy x_i (hence, the followers play in Nash strategies between themselves).

In the last stage, the choice of each follower has to satisfy the first order condition:

$$\Pi_1(x_i, X_{-i}) = 0 \tag{2}$$

where $X_{-i} = \sum_{j \neq i} h(x_j) + h(x_L)$. In this kind of games, given the number of firms, a pure-strategy equilibrium exists if the reaction functions are continuous or do not have downward jumps (see Vives, 1999). Unfortunately this may not be the case due to the presence of fixed costs, but weak conditions for existence have been studied for many applications.[4] In this general framework I will just assume existence of a unique symmetric equilibrium where all the followers

$K \in (0, \Pi[x(\chi), \chi])$ where $\chi = h(x(0))$, which just means that if one firm is choosing the monopolistic strategy (the optimal one given inactivity of the others), entry is profitable: the market is not a natural monopoly.

[3] The exogeneity of this leadership can be a realistic description for markets with established dominant firms, or where entry at an earlier stage was not possible for technological or legal reasons, for liberalized markets that were once considered natural monopolies or those where intellectual property rights play an important role. Later I will extend the model to multiple leaders and endogenous leadership. I am extremely thankful to a referee for pointing out this issue.

[4] For instance, see Amir and Lambson (2000) on Cournot games with perfectly substitute goods and Vives (1999) for a survey. In general, under SC there are only symmetric equilibria but there may be more than one, while under SS there is a unique symmetric equilibrium but there may be other asymmetric equilibria.

choose the same strategy x_F (this happens in all our examples and, in general, under a standard contraction condition, $\Pi_{11} + (n-2)h'(x_F)\left|\Pi_{12}\right| < 0$).[5]

Therefore, I will simply focus on a particular symmetric equilibrium for the second stage of the game. This will be characterized by the first order condition:

$$\Pi_1\left[x_F, (n-2)h(x_F) + h(x_L)\right] = 0 \tag{3}$$

where I used the fact that the spillovers perceived by any follower are $X_{-F} = (n-2)h(x_F)+h(x_L)$. Subscripts F and L will denote the representative follower and the leader in the symmetric equilibrium through all the paper.

2.1 Exogenous entry

When the number of firms is exogenous, the standard results on Stackelberg duopolies easily generalize. Given the number of firms n, total differentiation of (3) implies that the reaction functions of the followers have a slope $dx_F/dx_L \propto \Pi_{12}\left(x_F, X_{-F}\right)$. Therefore, an increase in the strategy of the leader x_L makes the followers more aggressive under the assumption of SC ($\Pi_{12} > 0$) and more accommodating under SS ($\Pi_{12} < 0$). Consequently, in case of an interior solution, the optimality condition for the leader can be easily derived as:

$$\Pi_1^L\left(x_L, X_{-L}\right) = \frac{\Pi_2^L \Pi_{12}(n-1)h'(x_F)h'(x_L)}{\Pi_{11} + (n-2)h'(x_F)\Pi_{12}}$$

whose right hand side has the sign of $\Pi_{12}^L\left(x_L, X_{-L}\right)$. We then obtain the traditional result for which $x_L < x_F$ under SC and $x_L > x_F$ under SS: Stackelberg competition with a fixed number of firms implies that the leader is aggressive compared to each follower under SS and accommodating under SC.[6] We now turn to the case where the number of firms is endogenous.[7]

2.2 Endogenous entry

Now, let us consider endogenous entry. I will now provide an intuitive and constructive argument to characterize the SEFE which will be useful in the

[5] This always holds for $n = 2$. With more than one follower, weaker conditions for uniqueness are available for particular models. Nevertheless, one should keep in mind that whenever asymmetric equilibria can emerge, as under quantity competition with strong increasing returns (see Amir and Lambson, 2000), the predictive power of our selected equilibrium would be diminished.

[6] The ambiguity of these results on the optimal behaviour of a leader is even deeper when reaction functions are not monotonic, that is when SS holds in some regions and not in others.

[7] As usual in this literature I will simplify the analysis considering n as a real number when larger than 2. A general treatment is more complex but the spirit of the result is unchanged under regularity conditions. Anyway, as a referee noticed, the following analysis can be interpreted as focusing on two extreme cases: one is a duopoly model in which entry can possibly be deterred and the other is a model with one leader and many small firms in which integer constraints can be ignored. In Etro (2007) I discuss how to derive the exact equilibria taking the integer constraint on n into account.

applications of the next section, leaving formal proofs to Appendix A.

Imagine that some followers enter in the market in equilibrium. Then, in the last stage we still have the first order equilibrium condition for the followers (3). Under our assumptions, the profits for the followers are decreasing in the number of entrants,[8] therefore in the second stage the followers enter in the market until there are positive profits to be made, and one can impose the free entry condition:

$$\Pi\left[x_F, (n-2)h(x_F) + h(x_L)\right] = K \tag{4}$$

The system (3)-(4) can be thought of as determining the behavior of the followers in the second and third stages, namely as determining x_F and n as functions of the leader's first stage action. But we can also look at these two equations in a different way: they can be solved for the two unknowns (x_F, X_{-F}) which only depend on the fixed cost of production and not on the strategy of the leader. Given (x_F, X_{-F}), there is a unique locus of (x_L, n) pairs that satisfy the equilibrium relation $X_{-F} = (n-2)h(x_F) + h(x_L)$. In other words, the strategy of the followers is independent from the strategy of the leader, while their number must change with the latter.[9]

Let us now move to the first stage and study the choice of the leader. As long as entry takes place, the perceived spillovers of the leader can be written as:

$$X_{-L} = (n-1)h(x_F) = X_{-F} + h(x_F) - h(x_L) \tag{5}$$

which depends on x_L only through the last term, since we have just seen that the pair (x_F, X_{-F}) does not depend on x_L. We can use this result to verify when entry of followers takes place or not. It is immediate that entry does not occur for any strategy of the leader x_L above a cut-off $\bar{x}_L$ such that $n = 2$ or, substituting in (5), such that:

$$X_{-F} = h(\bar{x}_L) \tag{6}$$

which clearly implies $\bar{x}_L \geq x_F$. Entry occurs whenever $x_L < \bar{x}_L$. In such a case, the leader chooses the optimal strategy to maximize:

$$\pi_L = \Pi^L[x_L, X_{-F} + h(x_F) - h(x_L)] - F \tag{7}$$

[8] Indeed we have:

$$\frac{d\Pi}{dn} = \frac{\Pi_2 \Pi_{11} h(x_F)}{\Pi_{11} + (n-2)h'(x_F)\Pi_{12}} < 0$$

since $\Pi_2 < 0$ and the denominator is negative under the stability assumption. In case profits have a positive limit for n increasing, a free entry equilibrium requires high enough fixed costs.

[9] The invariance property $(dx_F/dx_L = 0)$ is quite important since it shows that what matters for the leader is not the reaction of each single follower to its strategy, but the effect on entry. This is exactly the opposite of what happens in the Stackelberg equilibrium of the previous section: when entry is exogenous the leader takes as given the number of followers and looks at the reaction of their strategies to its own strategy; when entry is endogenous the leader takes as given the strategies of the followers and looks at the reaction of their number to its own strategy.

which delivers the first order condition:[10]

$$\Pi_1^L \left[x_L, (n-1)h(x_F)\right] = \Pi_2^L \left[x_L, (n-1)h(x_F)\right] h'(x_L) \tag{8}$$

In this case the equilibrium values for x_L, x_F and n are given by the system of three equations (3)-(4)-(8).

In general, the profit function perceived by the leader is an inverted U relation in x_L for any strategy below the entry deterrence level $\bar{x}_L$, and it takes positive values just for $x_L > x_F$. Beyond the cut-off $\bar{x}_L$, it is downward sloping (under the assumption that the market is not a natural monopoly). Consequently, the entry deterring strategy is optimal only if it provides higher profits than the locally optimal strategy characterized by (8). If we are just interested in the qualitative behaviour of the firms, we can conclude as follows:

PROPOSITION 1. **A SEFE always implies that the leader is aggressive compared to each follower, and each follower either does not enter or chooses the same strategy as under Nash competition with free entry.**

Before turning to the applications of this result, we briefly comment on the comparative statics of the equilibrium. The impact of a generic parameter affecting the profit functions is quite complicated and intuitions are hard to grasp, but we can make some useful progress focusing on changes in the fixed cost. It turns out that the results are typically the opposite if SS or SC holds. For simplicity, let us assume $\Pi_{22} \geq 0$, which will hold in our examples.[11] We obtain:

PROPOSITION 2. **Consider a SEFE where entry of followers tUnder strategic substitutability, a) if $\Pi_{12}h'(x_F) > \Pi_{11}$, the strategy of each firm is increasing and the number of firms is decreasing in K, b) otherwise, the strategy of entrants (leader) is increasing (decreasing) in K. Under strategic complementarity, c) if $\Pi_{12}^L < (=)\Pi_{22}^L \Pi_1^L / \Pi_2^L$, the strategy of entrants and their number are decreasing while the strategy of the leader is increasing in (independent from) K, d) otherwise, the strategy of each firm is decreasing in K.**

We will now verify these results in models of quantity and price competition and in contests and we will also discuss the welfare implications of the specific applications.

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[10]Notice that the second order condition is:

$$D^L = \Pi_{11}^L - 2\Pi_{12}^L h'(x_L) - \Pi_2^L h''(x_L) + \Pi_{22}^L h'(x_L)^2 < 0$$

that I assume to be satisfied at the interior optimum.

[11]$\Pi_{22} > 0$ in the case of quantity competition and perfectly substitute goods as long as demand is convex, in our examples of price competition and in standard patent races.

3 Competition in quantities

In this section I apply the general results on the SEFE to models of competition in quantities. Let us consider a market where each firm i chooses the output x_i and faces an inverse demand $p_i = p(x_i, X_{-i})$ which is decreasing in both arguments. This implies that goods are substitutes, but not necessarily homogenous - of course, in case of homogenous goods the inverse demand could be written as $p = p(X)$ for all firms, with X total output. If the cost function is an increasing function $c(x_i)$, the profit for firm i is:

$$\pi_i = x_i p\left(x_i, X_{-i}\right) - c(x_i) - K \tag{9}$$

with $X_{-i} = \sum_{j \neq i}^{n} h(x_j)$, $p_1 < 0$ and $p_2 < 0$. We can verify that the associated profit function is nested in our general form (1) under simple regularity conditions.

To characterize the SEFE, let us start by looking at the last stages. Assume first that the output of the leader is low enough that entry occurs in equilibrium (we will verify later on when this is the case). The equilibrium first order condition for the followers and the endogenous entry condition are:

$$p\left(x_F, X_{-F}\right) + x_F p_1(x_F, X_{-F}) = c'(x_F)$$

$$x_F p\left(x_F, X_{-F}\right) = c(x_F) + K$$

and they pin down the production of the followers x_F and their spillovers X_{-F} independently from the production of the leader. Consequently, the profits of the leader can be rewritten as:

$$\begin{aligned} \pi_L &= x_L p\left(x_L, X_{-L}\right) - c(x_L) - K \\ &= x_L p\left[x_L, X_{-F} + h(x_F) - h(x_L)\right] - c(x_L) - K \end{aligned}$$

whose maximization delivers the optimality condition:

$$p(x_L, X_{-L}) + x_L\left[p_1(x_L, X_{-L}) - p_2(x_L, X_{-L})h'(x_L)\right] = c'(x_L) \tag{10}$$

Comparing the equilibrium optimality conditions of the leader and the followers, and using the fact that goods are substitutes ($p_2 < 0$), it follows that the leader produces more than each follower. For these conditions to characterize the equilibrium it must be that the alternative strategy of entry deterrence provides lower profits to the leader. As intuitive, this happens when marginal costs are strongly increasing in the output or there is little substitutability between goods.

In particular, when goods are homogenous, the inverse demand is simply $p(X)$, and the cost function is convex, the equilibrium condition for the leader boils down to an equation between the price and its marginal cost. In such a case, the equilibrium is fully characterized by the following conditions:

$$p(X) = \frac{c'(x)}{1 - 1/\epsilon} = \frac{c(x) + F}{x} = c'(x_L) \tag{11}$$

where the first equality is a traditional mark up rule for the followers (with ϵ elasticity of demand), the second equality is the endogenous entry condition, and the third one defines the pricing rule of the leader. Notice that while the followers produce below the optimal scale (defined by the equality between marginal and average cost), the leader produces above this scale and obtains positive profits thanks to the increasing marginal costs.

We now provide two simple examples of SEFE in models of competition in quantities where entry occurs.

Example 1: U-shaped cost function Consider homogenous goods and linear demand $p = a - X$ for all firms, where $X = \sum_{j=1}^{n} x_j$ is total output. Imagine that the average cost function is U-shaped, and in particular that the cost function is quadratic. Hence, the profit function for firm i is:

$$\Pi(x_i, X_{-i}) = x_i(a - x_i - X_{-i}) - \frac{dx_i^2}{2} \tag{12}$$

Consider the last stage. Given the production of the leader x_L, the equilibrium output for the entrants is $x_F = (a - x_L)/(n + d)$, and the associated profits are $\Pi^F = (2 + d)(a - x_L)^2/2(n + d)^2$. Under free entry, the zero profit condition delivers the number of firms $n = (a - x_L)\sqrt{(2 + d)/2K} - d$ and the production of each follower:

$$x_F = \sqrt{\frac{2K}{2 + d}} \tag{13}$$

Hence total production is $X = a - (1 + d)\sqrt{2K/(2 + d)}$, which is independent from the leader's production. The gross profit function of the leader in the first stage, as long as there is entry, that is for $n > 2$ or $x_L < a - \sqrt{2K(2 + d)}$, is:

$$\Pi^L(x_L, X_{-L}) = (1 + d)\sqrt{\frac{2K}{2 + d}}x_L - \frac{d}{2}x_L^2$$

which is concave in x_L and maximized by:

$$x_L = \frac{1 + d}{d}\sqrt{\frac{2K}{2 + d}} \tag{14}$$

Accordingly, the output of the leader is always higher than the output of the followers. The equilibrium number of firms is $n = a\sqrt{(2 + d)/2K} - 1/d - d + 1$. This number is larger than two when d is positive and large enough, the size of the market (a) is large enough (or the fixed costs are small enough); if this is not the case we have an equilibrium with entry deterrence. Notice that under Cournot competition with free entry each firm would produce the same as x_F derived above, and also total production would be the same, but the number of firms would be larger by $1/d$. For instance, if $d = 1$ Stackelberg competition eliminates one entrant and replaces its production with the leader producing the double than any other firm.

Since we know from Mankiw and Whinston (1986) that the Cournot equilibrium with free entry is characterized by too many firms producing too little, it is intuitive that Stackelberg competition with free entry improves the allocation of resources inducing a reduction in the number of firms. This result holds for any demand function as long as goods are homogenous, since our model implies always that total production is the same under Stackelberg and Cournot competition when entry is free, but a leader produces more than the followers and there are less firms in the Stackelberg case. Hence, the associated reduction in wasted fixed costs comes back in form of profits for the leader, which increases welfare. In conclusion, consumer surplus is the same, but welfare is higher under Stackelberg competition with endogenous entry:

Proposition 3. **Consider quantity competition with homogenous goods. Under endogenous entry, as long as there is entry of some followers, Stackelberg competition in quantities is always Pareto superior with respect to Cournot competition.**

Example 2: Product differentiation The second example is a linear model with product differentiation, where inverse demand is now $p_i = a - x_i - b_i X_{-i}$ with $b \in (0,1)$,[12] and c is the constant marginal cost. Profits for firm i are now given by:

$$\Pi(x_i, X_{-i}) = x_i \left(a - x_i - bX_{-i}\right) - cx_i \tag{15}$$

Under Stackelberg competition, as long as substitutability between goods is limited enough (b is small) there are entrants producing $x_F = (a - bx_L - c)/[2 + b(n-2)]$, which simplifies to:

$$x_F = \sqrt{K} \tag{16}$$

when we endogenize the number of firms under free entry. The profit function perceived by the leader is now:

$$\Pi^L\left(x_L, X_{-L}\right) = x_L \left[(2-b)\sqrt{K} - (1-b)x_L\right]$$

that is maximized when the leader produces:

$$x_L = \frac{2-b}{2(1-b)}\sqrt{K} \tag{17}$$

[12]This can be derived from the following quadratic utility of a representative agent:

$$U = a\sum_{i=1}^{n} C_i - \frac{1}{2}\left[\sum_{i=1}^{n} C_i^2 + b\sum_i \sum_{j \neq i} C_i C_j\right] + C_0$$

where C_i is consumption of good i and C_0 is the numeraire.

Again, the production of the leader is higher than the production of the followers. Moreover, the leader will offer its good at a lower price than the followers, namely:

$$p_L = c + \left(1 - \frac{b}{2}\right)\sqrt{K} < p_F = c + \sqrt{K} \tag{18}$$

The consequence is that entry of followers is reduced. Since consumers value product differentiation in such a model the welfare consequences are complex. Nevertheless, one can verify that the reduction in the price of the leader more than compensates the reduction in the number of varieties, and consumer surplus is strictly increased by the leadership.[13] Therefore, in this case the consumers strictly gain from the aggressive pricing strategy of the leader even if this induces some firms to exit and reduces the number of varieties provided in the market.

These examples have shown cases in which entry occurs, and shows that the leader is always more aggressive than each follower. Notice that in both examples the comparative statics with respect of the fixed cost was following case a) of Prop. 2.

Let us now move to the equilibria where entry deterrence takes place. When goods are homogenous or highly substitute, or when the marginal cost is decreasing, constant or not too much increasing, the optimality for the leader implies a corner solution with entry deterrence. In our general formulation this requires:

$$x_F p(x_F, \bar{x}_L) = c(x_F) + K \quad \Longleftrightarrow \quad \bar{x}_L = X_{-F} - x_F \tag{19}$$

Notice that the entry deterring output must be decreasing in the fixed cost, since this cost helps the leader to exclude the rivals, and it must approach the average variable cost when the fixed cost (of entry for the followers) tends to zero.

In the case of general demand functions for homogenous goods, we can actually find a simple sufficient condition for entry-deterrence which just depends on the shape of the cost function:

[13]Using the quadratic utility that generates this demand function, in equilibrium we have:

$$U = Y + \frac{1}{2}\left[\sum_{i=1}^{n} x_i^2 + b\sum_i \sum_{j\neq i} x_i x_j\right]$$

where Y is the exogenous income of the representative agent.The gain in consumer surplus from the presence of a leader when entry is endogenous is:

$$\Delta U = \frac{b(2-b)F}{8(1-b)} > 0$$

and the gain in welfare is $\Delta W = \Delta U + \pi_L$. I am thankful to Nisvan Erkal and Daniel Piccinin for insightful discussions on this point.

PROPOSITION 4. **Consider quantity competition with homogenous goods. Whenever marginal costs of production are constant or decreasing, Stackelberg competition in quantities with endogenous entry always delivers entry-deterrence with only the leader in the market.**

To verify this result, I will provide another example.

Example 3: Linear demand and constant marginal costs Consider the simplest model of quantity competition, with linear demand $p = a - X$, which derives from a quadratic utility, and constant marginal costs c. The profits of a generic firm i are given by:

$$\Pi(x_i, X_{-i}) = x_i (a - x_i - X_{-i}) - cx_i \tag{20}$$

In this case, already mentioned in Etro (2006a), the SEFE is characterized by entry deterrence with the leader producing:

$$\bar{x}_L = a - c - 2\sqrt{K} \tag{21}$$

and obtaining positive profits. The limit price in this SEFE $p = c+2\sqrt{K}$ is above the equilibrium price under Cournot competition with free entry. Nevertheless, welfare is higher because the profits of the leader (associated with the savings in fixed costs) are enough to compensate for the lower consumer surplus (associated with the lower production).[14]

Notice that this limit price is above the one emerging in case of Stackelberg competition in prices, which would lead to a price equating average cost as in the theory of contestable markets.[15] In both cases, the price converges to the marginal cost when fixed costs disappear.

4 Competition in prices

The role of price leadership is often underestimated for two main reasons. The first is that commitments to prices are hardly credible when it is easy and

[14]Indeed, adopting the standard definition of welfare, in the Cournot equilibrium with free entry this is:

$$W^C = \frac{(a - c - \sqrt{K})^2}{2}$$

while in the SEFE it is:

$$W^S = \frac{X^2}{2} + \pi_L = \frac{(a-c)^2}{2} - 3K$$

It can be verified that welfare is higher in the Stackelberg case for any $K < 4(a-c)^2/49$, which always holds when the market is not a natural monopoly, that is for $K < (a-c)^2/16$.

[15]While we confined the analysis of SEFE to well-behaved profit functions, the general concept applies also to the case of price competition with a leader and free entry. In such a case the equilibrium requires a limit pricing by the incumbent satisfying $p = a - x = c + F/x$, and corresponds to the equilibrium of the contestable market theory. In this sense, our SEFE generalizes that theory.

relatively inexpensive to change prices. While this is true for long term commitments, it is also true that short term commitments can be credible in most markets. In particular, when a price change requires substantial information whose acquisition is costly and when it can exert adverse reputational effects on the perception of the customers, a commitment to a fixed price can be reasonable. Moreover, when a price change induces quick entry into the market or exit from it, a commitment to a fixed price may have a stronger rationale: while this is inconsistent with a situation in which the number of firms is exogenous, the price commitment becomes a relevant option exactly when entry in the market is endogenous.

The second reason for which a price leadership may poorly describe the behaviour of market leaders is probably more pervasive and it relies on the absence of a first mover advantage in simple models of competition in prices. For instance, in standard duopolies, a price leader obtains less profits than its follower, and for this reason neither one or the other firm would like to be leaders: there is actually a second mover advantage. As we will see, this result disappears and the first mover advantage is back exactly when entry in the market is endogenous.

I will focus on a large class of models of price competition with imperfect substitutability between goods where the direct demand can be written as:

$$D_i = D\left[p_i, \sum_{j=1, j\neq i}^{n} g(p_j)\right] \tag{22}$$

with $D_1 < 0$, $D_2 < 0$, $g(p) > 0$ and $g'(p) < 0$, which implies that demand of good i decreases in p_i and increases in any p_j with $j \neq i$. For consistency with our definitions, let us define the strategic variable as $x_i = 1/p_i$, and $h(x) = g(1/x)$. Then, assuming for simplicity a constant marginal cost c, we can write profits as:

$$\pi_i = (p_i - c)\, D\left(p_i, \textstyle\sum_{j\neq i} g(p_j)\right) - K = \left(\frac{1}{x_i} - c\right) D\left(\frac{1}{x_i}, X_{-i}\right) - K$$

where $X_{-i} = \sum_{j\neq i} g(p_j) = \sum_{j\neq i} h(x_j)$. Clearly, also this model is nested in our general formulation (1) under simple regularity conditions. While the direction of the strategic effect is not obvious, SC holds in most models of competition in prices: this implies that leaders facing exogenous entry tend to be accomodating setting higher prices than their followers.

Before analyzing the SEFE of this model, we present a few examples of well known demand functions that belong to the class defined above. A first example is given by the Logit demand:

$$D_i = \frac{Ne^{-\lambda p_i}}{\sum_{j=1}^{n} e^{-\lambda p_j}} \tag{23}$$

with $N > 0$ and $\lambda > 0$. This demand belongs to our class of demand functions after setting $g(p) = \exp(-\lambda p)$, which satisfies $g'(p) < 0$. Anderson *et al.* (1992) have shown that this demand is consistent with a representative agent maximizing the utility:

$$U = C_0 - \left(\frac{1}{\lambda}\right) \sum_{j=1}^{n} C_j \ln\left(\frac{C_j}{N}\right) \tag{24}$$

when $\sum_{j=1}^{n} C_j = N$ and $-\infty$ otherwise (total consumption for the n goods is exogenous), under the budget constraint $C_0 + \sum_{j=1}^{n} p_j C_j = Y$, with C_0 as the numeraire. This interpretation allows to think of $1/\lambda$ as a measure of the variety-seeking behavior of the representative consumer.

Other important cases derive from the class of demand functions introduced by Dixit and Stiglitz (1977) and derived from the maximization of a utility function of a representative agent as $U = u\left[C_0, V\left(\sum_{j=1}^{n} C_j^{\theta}\right)\right]$ under the budget constraint $C_0 + \sum_{j=1}^{n} p_j C_j = Y$, where C_0 is the numeraire, $u(\cdot)$ is quasilinear or homothetic, $V(\cdot)$ is increasing and concave, and $\theta \in (0, 1]$ parametrizes the substitutability between goods. For instance, consider the utility function:

$$U = C_0^{\alpha} \left[\sum_{j=1}^{n} C_j^{\theta}\right]^{\frac{1}{\theta}} \tag{25}$$

with $\theta \in (0, 1)$ and $\alpha > 0$. In this case the constant elasticity of substitution between goods is $1/(1-\theta)$ and increases in θ. Demand for each good $i = 1, ..., n$ can be derived as:

$$D_i = \frac{p_i^{-\frac{1}{1-\theta}} Y}{(1+\alpha)\left(\sum_{j=1}^{n} p_j^{-\frac{\theta}{1-\theta}}\right)} \tag{26}$$

which belongs to our general class after setting $g(p) = p^{-\frac{\theta}{1-\theta}}$, that of course satisfies $g'(p) < 0$. Similar demand functions and related models of price competition have been widely employed in many fields where imperfect competition plays a crucial role, including the new trade theory, the newkeynesian macroeconomics, the new open macroeconomy and the endogenous growth theory.

Let us move now to the characterization of the SEFE in the general case and in these examples. To focus on the most interesting situations, I will assume that product differentiation is such that entry deterrence is never desirable for the leader. Denoting with p_F and p_L the prices of the followers and the leader, the optimality condition for the followers and the endogenous entry condition are:

$$D\left(p_F, X_{-F}\right) + (p_F - c) D_1\left(p_F, X_{-F}\right) = 0$$

$$\left(p_F - c\right) D\left(p_F, X_{-F}\right) = K$$

and they pin down the price of the followers p_F and their spillovers $X_F = (n-1)g(p_F)$, so that the profit of the leader becomes:

$$\begin{aligned}\pi_L &= (p_L - c)D\left[p_L, (n-1)g(p_F) - g(p_L)\right] - K = \\ &= (p_L - c)D\left[p_L, X_{-F} + g(p_F) - g(p_L)\right] - K\end{aligned}$$

Profit maximization delivers the equilibrium condition:

$$D(p_L, X_{-L}) + (p_L - c)\left[D_1(p_L, X_{-L}) - D_2(p_L, X_{-L})g'(p_L)\right] = 0 \tag{27}$$

which implies a lower price p_L than the price of the followers, since the last term on the left hand side is now negative. This is a crucial result by itself since we are quite use to associate price competition with accommodating leaders setting higher prices than the followers: this standard outcome collapses under endogenous entry. Moreover, the leader is now obtaining positive profits, while each follower does not gain any profits: the first mover advantage is back.

Example 4: Logit demand Consider the Logit demand (23). Using our transformation of variables $p_i = 1/x_i$ we obtain the gross profits:

$$\Pi(x_i, X_{-i}) = \frac{Ne^{-\lambda/x_i}}{e^{-\lambda/x_i} + X_{-i}}\left(\frac{1}{x_i} - c\right) \tag{28}$$

where $X_{-i} = \sum_{j \neq i} e^{-\lambda/x_j}$. Let us characterize the SEFE. First of all, as usual, let us look at the stage in which the leader as already chosen its price p_L and the followers enter and choose their prices. Their first order condition can be written as:

$$p_i = c + \frac{1}{\lambda(1 - D_i/N)}$$

where the demand on the right hand side depends on the price of the leader and all the other prices as well. However, under free entry we must have also that the markup of the followers exactly covers the fixed cost of production, hence $D_i(p_i - c) = K$. If the price of the leader is not too low or the fixed cost not too high, there is indeed entry in equilibrium and we can solve these two equations for the demand of the followers and their prices in symmetric equilibrium:

$$p_F = c + \frac{1}{\lambda} + \frac{K}{N}, \quad D_F = \frac{\lambda KN}{N + \lambda K} \tag{29}$$

Notice that they do not depend on the price chosen by the leader. The profits perceived by the leader are now:

$$\pi_L = (p_L - c)D_L - K = \frac{(p_L - c)e^{-\lambda p_L}D_F}{e^{-\lambda p_F}} - K$$

where we could use our previous results to substitute for p_F and D_F. Finally, profit maximization by the leader provides its equilibrium price:

$$p_L = c + \frac{1}{\lambda} < p_F \tag{30}$$

which is lower than the price of each follower and independent from the fixed cost (Prop 2.c applies). Moreover, using the microfoundation pointed out by Anderson *et al.* (1992) in terms of the quasilinear utility (24), one can show that this equilibrium is Pareto efficient compared to the correspondent Bertrand equilibrium with free entry: the reduction in the price of the leader reduces entry, leaves unchanged consumer surplus and increases firms' profits, inducing an increase in total welfare (see the Appendix).

Example 5: Dixit-Stiglitz demand In the case of the isoelastic demand (26) derived above from the utility function (25), using our transformation of variables $p_i = 1/x_i$, we obtain the gross profits:

$$\Pi(x_i, X_{-i}) = \frac{x_i^{\frac{1}{1-\theta}} \left(\frac{Y}{1+\alpha}\right)}{x_i^{\frac{\theta}{1-\theta}} + X_{-i}} \left(\frac{1}{x_i} - c\right) \tag{31}$$

where $X_{-i} = \sum_{j \neq i} x_j^{\theta/(1-\theta)}$. In the SEFE we obtain the following equilibrium prices:

$$p_L = \frac{c}{\theta} \qquad p_F = \frac{cY}{\theta\left[Y - K(1+\alpha)\right]} \tag{32}$$

where of course the leader applies a lower mark up than each follower. Notice that again the price of the followers increases in the fixed cost, while the price of the leader is independent (Prop 2.c applies). It can be verified that in any version of the Dixit-Stiglitz model where $1/(1-\theta)$ is the constant elasticity of substitution between goods and c is the marginal cost of production, as long as entry is endogenous, the leader will choose the price $p_L = c/\theta$ and the followers will choose a higher price. Indeed, free entry pins down the price index that is perceived by the leader, whose optimization problem is of the following kind:

$$\max (p_L - c) D_L \propto (p_L - c) p_L^{-\frac{1}{1-\theta}}$$

which always delivers the price above. As a consequence, the leader produces more than each follower and the number of followers is reduced compared to the exact Dixit-Stiglitz equilibrium with free entry. Once again, however, consumer surplus is not changed because the price index is unaffected. Since the leader obtains positive profits, welfare is increased overall (see the Appendix).

We can summarize the results obtained from these two examples as follows:

Proposition 5. **In a model of price competition with Logit demand or Dixit-Stiglitz demand and endogenous entry, a leader sells its variety at a lower price than the entrants, inducing a Pareto improvement in the allocation of resources.**

In all these models we can verify the existence of an unambiguous ranking of market structures from a welfare point of view. Indeed, from the best to the worst case for welfare we have: 1) free entry with a leader; 2) free entry without a leader; 3) barriers to entry without a leader; 4) barriers to entry with a leader.

5 Contests

Another application of our model concerns contests where firms compete to obtain a prize: for instance this could be a patent on new technologies in a patent race, a prize in a principal-agent model, or a generic rent in a rent seeking contest. There are many ways to model such a situation, but here I will focus on the patent race introduced by Loury (1979) where firms choose an up-front investment x_i to obtain an innovation according to a Poisson process. Technically, this is a patent race in the continuum with arrival rates of innovation given by functions $h(x)$ with $h(0) = 0$, $h'(x) > 0$ and $h''(x) \gtrless 0$ for $x \lessgtr \hat{x}$ for some $\hat{x} > 0$. Hence, given the interest rate r and the value of the patent V, the expected gross profit for firm i is:

$$\pi_i = \frac{h(x_i)V}{r + h(x_i) + X_{-i}} - x_i - K \tag{33}$$

which is again a particular case of our general model and implies SS (at the turning point).

Imagine that one firm has a first mover advantage and invests before the other firms in R&D.[16] In such a case, the first order conditions and the free entry condition for the followers imply a symmetric equilibrium between them with investment implicitly given by:

$$h'(x_F) = \frac{h(x_F)}{x_F + K}\left(\frac{V}{V - x_F - K}\right) \tag{34}$$

Let us focus on the case where entry takes place in equilibrium. Using the free entry condition, the net profit of the leader can then be rewritten as:

$$\pi_L = \frac{h(x_L)V}{[r + h(x_L) + X_{-L}]} - x_L - F = \frac{h(x_L)(x_F + K)}{h(x_F)} - (x_L + K)$$

from which the first order condition:

$$h'(x_L) = \frac{h(x_F)}{x_F + K} \tag{35}$$

[16] Notice that there is not a time-inconsistency issue here, since after the investment is made, there cannot be a chance to change it. See Etro (2004, 2008) for discussions.

defines an interior maximum when $h''(x_L) < 0$: it follows that the investment of the leader is higher than the investment of the followers. If we imagine that the innovation as also a positive value, we can analyze the equilibrium from a welfare point of view. Since the aggregate probability of innovation is unchanged by the presence of a leader, but the number of firms and total R&D investment are reduced, welfare must improve, as formalized below:

Proposition 6. **A first mover firm in a contest with endogenous entry invests more than any other firm and creates a Pareto improvement in the allocation of resources compared to Nash competition.**

Also in this case entry deterrence may occur when the marginal productivity of the investment has a lower bound (for instance when $h(x)$ is linear).

6 Extensions

The results of the previous sections can be extended in many directions to be able to describe market structures in a more realistic way. This section will consider a few: introducing a technological asymmetry between the leader and the followers, extending the model to multiple leaders and endogenizing the same leadership *status* (Appendix B and C discuss further extensions, allowing for multiple strategies and introducing simple forms of heterogeneities).

6.1 Asymmetries between leader and followers

Weaker forms of our basic result, the general aggressive behaviour of leaders facing free entry, emerge even when we introduce technological differences between firms. Here I will focus on a simple exogenous asymmetry between the leader and the followers to verify under which conditions the leader is still aggressive. I assume that the leader has the profit function:

$$\pi_L = \Pi^L(x_L, X_{-L}, y) - K$$

where $y > 0$ is a new parameter specific to the leader. The basic assumptions are $\Pi_3^L \equiv \partial \Pi^L / \partial y > 0$ and $\Pi^L(x, X, 0) = \Pi^i(x, X)$. A first mover advantage is often associated with some asymmetry between the leader and the followers. For instance, it is natural to link the first mover advantage with some technological or market advantage, as a lower marginal cost $c(y)$ with $c'(y) < 0$. In general, under asymmetry we obtain a strategy of the leader which depends on y, $x_L = x_L(y)$, and therefore the number of entrants, but not their individual strategy, also depends on y. One can show:

Proposition 7. **An asymmetric SEFE implies that the leader is aggressive whenever $\Pi_{13}^L \geq \Pi_{23}^L(\Pi_1^L/\Pi_2^L)$ or y is small enough.**

The intuition is the following: an increase in the advantage of the leader (that is in y) induces a higher incentive to aggressiveness if it raises the marginal benefit from it more than the change in its marginal cost. Indeed the sufficient condition could be rewritten as $\partial(\Pi_1^L/\Pi_2^L)/\partial y \leq 0$, that is the marginal rate of substitution between x_L and X_{-L} should be decreasing in y. If this condition does not hold, it means that $x'_L(y) < 0$, hence for a great enough y (a strong enough asymmetry) the leader will be accommodating ($x_L(y) < x_F$).

To exemplify how one can apply this result, notice that the leader with a lower marginal cost than its followers will always be aggressive because under competition in quantities we have $\Pi_{13}^L > 0$ and $\Pi_{23}^L = 0$, and under competition in prices we have $\Pi_{13}^L > 0$ and $\Pi_{23}^L = -D_2\left(p_L, X_{-L}\right)c'(y) < 0$. Similarly one can examine other kinds of exogenous asymmetries (on the demand side, in the financial structure, in complementary markets, and so on) and verify how the incentives of the leader to be aggressive are changed. Another application was developed in Etro (2004) where I extended patent races similar to those of Section 5 to the case in which the incumbent monopolist has a flow of current profits: while under Nash competition and free entry this incumbent would not participate to the race, when the same incumbent has the leadership in the contest, its investment is higher than that of any other firm.[17] One can verify that we are in the case in which the asymmetry does not affect the strategy of the leader.[18]

6.2 Multiple leaders

Until now we considered a simple game with just one leader playing in the first stage. Here we will consider the case in which multiple leaders play simultaneously in the first stage. Hence the timing of the game becomes the following: 1) in the first stage, m leaders simultaneously choose their own strategies; 2) in the second stage, potential entrants decide whether to enter or not; 3) in the third stage each one of the $n-m$ followers that entered chooses its own strategy. Later on we will discuss how to endogenize m.

We should consider two different situations: one in which entry of followers is not deterred in equilibrium and one in which the leaders deter entry. Consider first the case in which the number of leaders m is small enough, or the cost of deterrence is large enough that entry of followers takes place in equilibrium. In

[17]Many real world innovations are obtained by dominant firms, especially in high-tech sectors and pharmaceutical sectors, where leading companies (like Microsoft, Intel, Pfizer or Merck) feature the highest R&D-turnover ratios. See Segerstrom (2007) for a related discussion.

[18]If the leader has a flow of current profits y, its objective function becomes:

$$\pi_L\left(y\right) = \frac{h(x_L)V + y}{r + h(x_L) + X_{-L}} - x_L - K$$

The independence of the leader'strategy from y is just an immediate consequence of Prop. 7 since $\Pi_{13}^L = \Pi_{23}^L(\Pi_1^L/\Pi_2^L) = -h'(x_L)/\left[r + h(x_L) + X_{-L}\right]^2 < 0$.

such a case, the behaviour of the leaders can be characterized in a similar fashion to our basic analysis. Moreover, contrary to what happens when the number of firms n in the market is exogenous (when the number of leaders m affects their strategic interaction, their strategies and their profits), with endogenous entry, the number of leaders does not affect their strategies and their profits:

PROPOSITION 8. **In a SEFE with m leaders, as long as there is endogenous entry of some followers, each leader is aggressive compared to each follower and its strategy and profits are independent from m.**

This confirms the spirit of our results with a single leader. Now each one of the leaders behaves in an aggressive way compared to the followers and also independently from the other leaders: even the profit of each leader is not affected by the number of leaders, while the number of entrants is clearly decreased by an increase in the number of leaders.

The situation is more complicated if there is entry deterrence in equilibrium. In the case of an exogenous number of firms entry deterrence is a sort of public good for the leaders, which introduces free-riding issues in their behavior. Gilbert and Vives (1986) have analyzed this issue in a model with m leaders facing a potential entrant, while Tesoriere (2006) has extended our model to analyze m leaders facing endogenous entry in a linear model of competition in quantities: while multiple equilibria can emerge, total output is equal or larger than the entry deterrent output in all of them.

6.3 Endogenous leadership

After developing a Stackelberg model with multiple leaders and endogenous entry of followers, it is natural to question what happens when the entry of leaders is endogenous as well.

The simplest way to endogenize the number of leaders is by adding an initial stage of the game where firms decide simultaneosly whether to become a leader or not. Any firm can make an investment, say L, which provides the status of leader in the market, while any firm that does not invest can only enter in the market as a follower: in other words, commitment to strategies is costly. As Prop. 8 suggests, as long as there is entry of followers, it must be that all leaders obtain the same level of positive profits (which is independent from the number of leaders m). Hence, if the investment needed to become leader is small enough, there must be always incentives to invest to become leaders when this does not deter entry of followers. Then, consider the largest number of leaders compatible with some entry, say $M < n$. Given this number of leaders, another firm may invest in leadership and subsequently engage in Nash competition with the other leaders only (entry of followers is now deterred by construction). If such an entry is profitable, the equilibrium must imply only leaders in the market and an endogenous number $m^* > M$ derived from a free entry condition with a fixed cost $K + L$ (clearly this happens whenever the cost of leadership is zero or small enough). If this is not the case, the only equilibrium implies $m^* = M$ firms investing in leadership and a residual competitive fringe of $n - M$ followers: once again, as Prop. 8 still implies, all leaders would be aggressive compared to each follower.

Another interesting situation emerges when entry is sequential, so that there is a hierarchical leadership. While a general treatment of sequential games is complex, Vives (1988) and Anderson and Engers (1992, 1994) have fully characterized sequential competition in quantities with linear costs and isoelastic demand, and with an exogenous number of firms. Their analysis makes clear that in case of endogenous entry the only possible equilibrium would imply entry deterrence, a result confirmed by the analysis developed more recently by Tesoriere (2006).

7 Policy implications

The endogenous entry approach is based on the idea that entry in a market is a rational decision based on the profitable opportunities that are available in a market. In this paper I adopted this approach to study the behaviour of market leaders in the choice of their strategies, but the same approach can be used to study preliminary strategic commitments by market leaders,[19] horizontal mergers[20] and even cartels[21] in markets where entry is endogenous. For this reason, this approach could be fruitful to examine policy issues in markets where entry is relatively easy, fast and sensible to the profitable opportunities, and it can be regarded as endogenous at least in the medium term.

The main field of application is antitrust policy. The study of strategic interactions in markets with endogenous entry may allow to build a bridge between the two main approaches to antitrust: on one side the Chicago school, that has focused its informal analysis on the role of free entry in constraining incumbent firms, but has largely neglected the role of strategic interactions between firms (see Posner, 2001), and on the other side the post-Chicago approach, that has focused on the strategic interactions between incumbents and entrants mostly without emphasizing the role of endogeneity of the market structure (for a survey see Motta, 2004).

The analysis of the behaviour of leaders in markets where entry is endogenous is potentially relevant for the field of antitrust analysis which concerns abuse of dominance (or monopolisation). In particular, it allows to re-examine the same concepts of market power and dominance, which are too often automatically associated with a large market share of the leading firm. Our analysis has shown that leaders tend to be aggressive, or more aggressive compared to their

[19]See Etro (2006) and Etro (2007, Ch. 2) on predatory pricing, bundling, price discrimination, vertical restraints and other commitments that are relevant for antitrust purposes.

[20]See Davisdon and Mukherjee (2007), Erkal and Piccinin (2007) and Etro (2007, Ch. 2).

[21]See Etro (2007, Ch. 3).

followers, exactly when entry is endogenous: in such a case, aggressive strategies are associated with large market shares and positive profits for the leaders, but also with low prices constrained by the behaviour of the followers.

In the basic model with homogenous goods, U-shaped cost functions and competition in quantities, the equilibrium price equates the marginal cost of the entrants augmented with a mark up depending on the elasticity of demand, and it also equates their average cost of production, because entry exhausts all the profit opportunities. At the same time, the leader produces to equate its marginal cost to the price level: since the followers produce below the efficient scale of production and the leader above it, the leader manages to obtain a larger market share and positive profits. When we introduce product differentiation, the leader keeps producing more than the other firms and also sells its product at a lower price, under both competition in quantities and in prices. Similar results emerge when competition is for the market, through investments in R&D to obtain new products. In such a case, leaders invest more than the other firms when entry in this competition is endogenous. Consequently, in case of sequential innovations the same persistence of the leadership can be associated with high contestability rather than with persistent market power.[22]

Our welfare analysis of markets with endogenous entry has emphasized that, in all these cases, the presence of a leader induces an increase in welfare calculated as the sum of consumer surplus and profits. Moreover, when entry is not entirely deterred, consumer surplus is either increased or left unchanged by the presence of a leader. Therefore, entry has a crucial role in disciplining the behaviour of market leaders. Finally, when entry can be regarded as endogenous, for instance when we are mainly concerned about the medium-long run and entry is feasible in this horizon, a large market share of the leader should not be a symptom of market power, but of an aggressive strategy induced by the competitive pressure of the entrants. Therefore, in the field of antitrust policy, and in particular in investigations concerning abuse of dominance, a preliminary examination of the entry conditions is crucial to verify whether large market shares of the leaders can be a symptom of dominance or just of competitive pressure on the leaders.

Another field of application of the endogenous entry approach concerns trade policy for exporting firms. As well known from the theory of strategic trade policy, a government may tax or subsidize domestic firms that are active in international markets for profit shifting reasons: such a policy allows to turn the domestic firm into a Stackelberg leader, and to increase the net profits for the country. For instance, when a domestic firm competes against a foreign competitor in a third market, it is typically optimal to tax exports under strategic complementarity and to subsidize exports under strategic substitutability: the

[22] In Etro (2007) I applied these arguments to the software market, arguing that the endogeneity of entry in the competition in the market justifies the aggressive pricing policy, the large market share and the extra profits of its leader, and the endogenity of entry in the competition for the market justifies the persistence of its leadership.

reason is that the Stackelberg leader is accommodating in the former case and aggressive in the latter. When entry in the international market is endogenous, the same principle applies, but it is always optimal to subsidize exports, since this induces the desired aggressive behaviour. In conclusion, the ambiguity of the policy implications emerging with an exogenous number of firms is solved when entry in international markets is regarded as endogenous, and the optimal unilateral policy is always an export subsidy.[23]

8 Conclusion

This article examined the behaviour of firms with a first mover advantage over their competitors in the choice of the market strategy. A general result emerging in the presence of endogenous entry is that leaders tend to behave in an aggressive way, in particular they choose lower prices and higher output than their followers. The study of the effects of endogenous entry on the behaviour of the firms, on their commitments to adopt different strategies and the analysis of the effects of these strategies on consumers could be fruitfully investigated in the future. Finally, it would be interesting to verify some of the results obtained above and in the related literature on endogenous entry in empirical or experimental analysis.

[23]The optimal export subdisy reproduces the Stackelberg equilibrium with endogenous entry where the domestic firm acts as a leader (see Etro, 2007, Ch.3). The same principle applies to international R&D contests: in such a case, it is always optimal to subsidize R&D when entry is endogenous (Etro, 2008).

References

AMIR, R. and LAMBSON, V., 2000, "On the Effects of Entry in Cournot Markets", *Review of Economic Studies*, 67, pp. 235-54

ANDERSON, S., DE PALMA, A. and THISSE, J.F., 1992, *Discrete Choice Theory of Product Differentiation*, The MIT Press, Cambridge

ANDERSON, S. and ENGERS, M., 1992, "Stackelberg versus Cournot Oligopoly Equilibrium", *International Journal of Industrial Organization*, 10, 1, pp. 127-35

ANDERSON, S. and ENGERS, M., 1994, "Strategic Investment and Timing of Entry", *International Economic Review*, 35, 4, pp. 833-53

BAUMOL, W. J., PANZAR, E. and WILLIG, R.D., 1982, *Contestable Markets and the Theory of Industrial Structure*, Harcourt Brace Jovanovich, San Diego

BAIN, J., 1956, *Barriers to New Competition. Their Character and Consequences in Manufacturing Industry*, Harvard University Press, Cambridge

DAVIDSON, C. and A., MUKHERJEE, 2007, "Horizontal Mergers with Free Entry", *International Journal of Industrial Organization*, 25, 1, pp. 157-172

DIXIT, A., 1980, "The Role of Investment in Entry-Deterrence", *The Economic Journal*, 90, pp. 95-106

DIXIT, A. and STIGLITZ, J., 1977, "Monopolistic Competition and Optimum Product Diversity", *The American Economic Review*, 67, pp. 297-308

ERKAL, N. and D. PICCININ, 2007, Horizontal Mergers with Free Entry in Differentiated Oligopolies, mimeo, University of Melbourne

ETRO, F., 2004, "Innovation by Leaders", *The Economic Journal*, April, 101, 495, pp. 281-303

ETRO, F., 2006, "Aggressive Leaders", *The RAND Journal of Economics*, Vol. 37, Spring, pp. 146-54

ETRO, F., 2007, *Competition, Innovation, and Antitrust. A Theory of Market Leaders and its Policy Implications*, Springer-Verlag, New York and Berlin, forthcoming

ETRO, F., 2008, "Growth Leaders", *Journal of Macroeconomics*, forthcoming

GILBERT, R. and VIVES, X., 1986, "Entry Deterrence and the Free Rider Problem", *Review of Economic Studies*, 53, 1, pp. 71-83

LOURY, G., 1979, "Market Structure and Innovation", *The Quarterly Journal of Economics*, March, pp. 429-36

MANKIW, G. and WHINSTON, M., 1986, "Free Entry and Social Inefficiency", *The RAND Journal of Economics*, 17, 1, pp. 48-58

MODIGLIANI, F., 1958, "New Developments on the Oligopoly Front", *Journal of Political Economy*, 66, 3, June, pp. 215-32

MOTTA, M., 2004, *Competition Policy. Theory and Practice*, Cambridge, Cambridge University Press

POSNER, R., 2001, *Antitrust Law*, University of Chicago Press, Chicago

SEGERSTROM, P., 2007, "Intel Economics", *International Economic Review*, 48, 1, pp. 247-80

SHUBIK, M., 1980, *Market Structure and Behavior*, Cambridge, Harvard University Press

STACKELBERG, H. VON, 1934, *Marktform und Gleichgewicht*, Springer-Verlag, Vienna and Berlin

SYLOS-LABINI, P., 1956, *Oligopolio e Progresso Tecnico*, Giuffrè, Milan

TESORIERE, A., 2006, "Endogenous Timing with Free Entry", CORE Discussion Paper 06/93

VISCUSI, W. K., HARRINGTON, J. and VERNON, J., 2005, *Economics of Regulation and Antitrust*, The MIT Press, Cambridge

VIVES, X., 1988, "Sequential Entry, Industry Structure and Welfare", *European Economic Review*, 32, pp. 1671-87

VIVES, X., 1999, *Oligopoly Pricing. Old Ideas and New Tools*, The MIT Press, Cambridge

Appendix

Part A: Proofs

Proof of Prop 1: The system (3)-(4) defines the impact on x_F and n of changes in x_L. Totally differentiating the system we have:

$$\begin{bmatrix} dx_F \\ \\ dn \end{bmatrix} = -\frac{1}{\Delta} \begin{bmatrix} \Pi_2 h(x_F) & -\Pi_{12} h(x_F) \\ \\ -(n-2)\Pi_2 h'(x_F) & \Pi_{11} + (n-2)\Pi_{12} h'(x_F) \end{bmatrix} \begin{bmatrix} \Pi_{12} h'(x_L) dx_L \\ \\ \Pi_2 h'(x_L) dx_L \end{bmatrix}$$

where $\Delta = \Pi_{11}\Pi_2 h(x_F)$ and $\Pi_{11} + (n-2)\Pi_{12} h'(x_F) + \Pi_2 h(x_F) < 0$ (under the contraction condition in case of SC), which implies stability. It follows that:

$$\frac{dx_F}{dx_L} = 0 \quad \frac{dn}{dx_L} = \frac{-h'(x_L)}{h(x_F)} < 0 \quad \frac{dX_{-F}}{dx_L} = 0 \quad \frac{dX_{-L}}{dx_L} = -h'(x_L) < 0$$

which shows that the strategy of the followers is independent from the one of the leader. Since this holds also for $x_L = x_F$, that replicates the Nash equilibrium with endogenous entry, in a SEFE any active follower adopts the same strategy as in the corresponding Nash equilibrium.

At the entry stage, entry of at least one follower takes place for any $x_L < \bar{x}_L$, where $\bar{x}_L$ is such that:

$$\Pi\left[x(h(\bar{x}_L)), h(\bar{x}_L)\right] = K$$

and the profit of the leader is:

$$\pi_L = \left\{ \begin{array}{ccc} \Pi^L\left[x_L, (n-1)h(x_F)\right] - K & if & x_L < \bar{x}_L \\ \Pi^L(x_L, 0) - K & if & x_L > \bar{x}_L \end{array} \right\}$$

Therefore, the optimal strategy is given by x_L^* that satisfies the first order condition:

$$\Pi_1^L\left[x_L^*, (n-1)h(x_F)\right] = \Pi_2^L\left[x_L^*, (n-1)h(x_F)\right] h'(x_L^*)$$

if it is smaller than $\bar{x}_L$ and such that:

$$\Pi^L\left\{x_L^*, (n-1)\, h(x_F)\right\} > \Pi^L\left(\bar{x}_L, 0\right)$$

Otherwise the global optimum is the corner solution $\bar{x}_L$. We will finally show that in equilibrium $x_L > x$ always. In case of corner solution, this is trivial. Consider the case of an interior solution x_L^* as defined above. Assume that $x_L^* \leq x_F$; then it must be that $X_{-F} = (n-2)h(x_F) + h(x_L^*) \leq (n-1)h(x_F) = X_{-L}$, which implies $\Pi(x_L^*, X_{-L}) \leq \Pi(x_L^*, X_{-F})$ from the assumption $\Pi_2 < 0$. But the optimality of x_F and the free entry condition imply $\Pi(x_L^*, X_{-F}) < \Pi(x_F, X_{-F}) = K$. From these inequalities it follows that $\Pi(x_L^*, X_{-L}) < K$, which implies negative profits for the leader, contradicting the optimality of the interior solution. This implies that the profit function of the leader must have a global optimum larger than x_F. Q.E.D.

PROOF OF PROPOSITION 2. The effect of a change in the fixed cost on the strategy and the number of followers are:

$$\frac{dx_F}{dK} = \frac{[-\Pi_{12}]}{[\Pi_{11}\Pi_2]} \quad \frac{dn}{dK} = \left[\frac{\Pi_{11} + (n-2)\Pi_{12}h'(x_F)}{\Pi_{11}\Pi_2 h(x_F)}\right] + \frac{\partial n}{\partial x_L}\frac{\partial x_L}{\partial K}$$

The first derivative has the opposite sign of Π_{12}. The second has a first negative term (under the contraction condition when $\Pi_{12} > 0$) and a second ambiguous term. It follows that $d[X_{-F} + h(x_F)]/dK = [\Pi_{11} - h'(x_F)\Pi_{12}]/\Pi_{11}\Pi_2 h(x_F)$. Totally differentiating (8) we have:

$$\frac{\partial x_L}{\partial K} = -\frac{\left[\Pi_{12}^L - h'(x_L)\Pi_{22}^L\right][\Pi_{11} - \Pi_{12}h'(x_F)]}{D^L\Pi_{11}\Pi_2 h(x_F)}$$

where $D^L < 0$ from the assumption that the second order condition is satisfied. It follows that:

$$\frac{dn}{dK} \propto \left[\Pi_{11} + (n-2)\Pi_{12}h'(x_F) + \frac{h'(x_L)}{h(x_F)D^L}[\Pi_{11} - \Pi_{12}h'(x_F)]\left[\Pi_{12}^L - h'(x_L)\Pi_{22}^L\right]\right]$$

The result follows immediately after noticing from (8) that $h'(x_L) = \Pi_1^L/\Pi_2^L$. Q.E.D.

PROOF OF PROP 3: In a SEFE the number of firms is n^S and each active follower produces x. In a Cournot equilibrium with free entry, the number of firms is n^C and each one produces the same as x by Prop. 1, with welfare:

$$W^C = \int_0^{n^C x} p(j)dj - n^C[c(x) + K] = \int_0^{n^C x} p(j)dj - p(n^C x)n^C x$$

where we used the zero profit condition $p(n^C x)x = c(x) + K$. Under SEFE, the number of firms n^S satisfies the zero profit condition:

$$p\left[x_L + (n^S - 1)x\right]x = c(x) + K$$

which implies the same total production in the two cases $x_L + (n^S - 1)x = n^C x$. Hence the welfare will be:

$$\begin{aligned} W^S &= \int_0^{x_L+(n^S-1)x} p(j)dj - (n^C - 1)c(x) - c(x_L) - n^S K = \\ &= \int_0^{n^C x^C} p(j)dj - p(n^C x)n^C x + \left[x_L + (n^S - 1)x\right]p(n^C x) \\ &\quad -(n^S - 1)c(x) - c(x_L) - n^S K \\ &= W^C + x_L p\left[x_L + (n^S - 1)x\right] - c(x_L) - K = W^C + \pi_L > W^C \end{aligned}$$

which proves the claim. Q.E.D.

PROOF OF PROP 4: Adopt a generic cost function $c(x)$ with $c''(x) \leq 0$. Imagine an equilibrium without entry deterrence. The zero profit condition, stated in the proof of Prop. 3, determines the total production and, therefore, it determines also the inverse demand at the level:

$$p[x^C(n^S-1)+x_L] = \frac{K+c(x)}{x}$$

where x is always the equilibrium production of the followers, which corresponds to the equilibrium production in the Cournot equilibrium with free entry. Then, the profit function of the leader becomes:

$$\Pi^L(x_L) = x_L p[x(n^S-1)+x_L] - c(x_L) = x_L\left[\frac{K+c(x)}{x}\right] - c(x_L)$$

with:

$$\Pi^{L\prime}(x_L) = \frac{K+c(x)}{x} - c'(x_L) > 0 \qquad \Pi^{L\prime\prime}(x_L) = -c''(x_L) \geq 0$$

since $p(\cdot) > c'(x) > c'(x_L)$ for any $x_L > x$. Accordingly, the leader always gains from increasing its production all the way to the level at which entry is deterred. This level satisfies the zero profit condition for $n^S = 2$, that is $p\left(x+\bar{x}_L\right) = \left[K+c(x)\right]/x$. Since the right hand side is also equal to $p(nx)$ by the zero profit condition in the Cournot equilibrium with free entry (see the proof of Prop. 3), it follows that the entry deterrence strategy is exactly $\bar{x}_L = (n^C-1)x$. Q.E.D.

PROOF OF PROP 5: Total expenditure $\bar{Y}$ for the representative agent is given by an exogenous part Y and the net profits of the firms $\sum_{i=1}^n \pi_i$, which is zero in the Nash-Bertrand equilibrium with endogenous entry, but equal to the positive profits of the leader π_L in the Stackelberg equilibrium with endogenous entry. The welfare comparison derives from the calculation of indirect utilities (24) for the Logit model and (25) for the Dixit-Stiglitz model in both cases. Labeling with $W(\bar{Y})$ the indirect utility in function of total expenditure $\bar{Y}$, in the Logit case we have for both equilibria:

$$W(\bar{Y}) = \bar{Y} + \frac{N}{\lambda}\ln\left(1+\frac{N}{\lambda K}\right) - N(1+\lambda c) - \lambda K$$

and in the Dixit-Stiglitz case we also have for both equilibria:

$$W(\bar{Y}) = \frac{\theta\left(\alpha\bar{Y}\right)^{\alpha}\left[\bar{Y}-K(1+\alpha)\right]}{c\left(1+\alpha\right)^{1+\alpha}}\left[\frac{(1-\theta)\bar{Y}}{(1+\alpha)K}+\theta\right]^{\frac{1-\theta}{\theta}}$$

Since they are both increasing in total expenditure, the utility of the representative agent must be higher under Stackelberg competition with endogenous entry. Q.E.D.

PROOF OF PROPOSITION 6. Clearly (34) and (35) show that $h'(x_L) < h'(x_F)$ and hence $x_L > x_F \equiv x$. Net profits for the leader are:

$$\pi_L = \frac{(x_L+K)(x+K)}{h(x)}\left[\frac{h(x_L)}{x_L+K} - \frac{h(x)}{x+K}\right]$$

Imagine that the social value of the innovation is V^*. Under Nash competition with n^N firms investing x each, welfare can be expressed as:

$$W^N = \frac{n^{nN} h(x) V^*}{r + n^N h(x)} - n^N x - n^N K$$

Under Stackelberg competition with a leader investing x_L and $n^S - 1$ followers investing x, using the fact that $n^N h(x) = h(x_L) + (n^S - 1)h(x)$, we have an increase in welfare:

$$\begin{aligned} W^S &= \frac{\left[h(x_L) + (n^S - 1)h(x)\right] V^*}{r + h(x_L) + (n^S - 1)h(x)} - x_L + (n^S - 1)x - n^S K \\ &= W^N + \frac{(x + K)(x_L + K)}{h(x)} \left[\frac{h(x_L)}{x_L + K} - \frac{h(x)}{x + K}\right] > W^N \end{aligned}$$

Notice that the second term is $\pi_L > 0$. Q.E.D.

PROOF OF PROP 7: The analysis of the last stage is the same as before, and in particular $dx_F/dx_L = 0$. Now, the leader's first order condition becomes:

$$\Pi_1^L \left[x_L, X_{-F} + h(x_F) - h(x_L), y\right] = \Pi_2^L \left[x_L, X_{-F} + h(x_F) - h(x_L), y\right] h'(x_L)$$

which defines a continuous function $x_L = x_L(y)$. It follows that:

$$x_L'(y) \propto \Pi_{13}^L \left[x_L, X_{-F} + h(x_F) - h(x_L), y\right] - \Pi_{23}^L \left[x_L, X_{-F} + h(x_F) - h(x_L), y\right] h'(x_L)$$

Clearly, when the condition in the proposition holds, we have $x_L'(y) \geq 0$ and $x_L(y) \geq x_L(0) > x_F$ from Prop. 1. Otherwise, since $x_L(0) > x_F$, continuity implies that there is a neighborhood of $x_L(0)$ for y small enough where $x_L(0) > x_L(y) > x_F$. Q.E.D.

PROOF OF PROP 8: The analysis is similar to the basic one, but now total differentiation provides $dx_F/dx_L = 0$ and $dn/dx_L = -h'(x_L)/h'(x_F)$. Moreover we have:

$$\tfrac{dx_F}{dm} = 0, \quad \tfrac{dn}{dm} = 1 - \tfrac{h(x_L)}{h(x_F)} < 0$$

The first order conditions for each one of the leaders become:

$$\Pi_1^L (x_L, X_{-L}) = \Pi_2^L (x_L, X_{-L}) h'(x_L)$$

where $X_{-L} = (n - m)h(x_F) + (m - 1)h(x_L)$. Totally differentiating this condition and using dn/dm it follows that $dx_L/dm = 0$. The profit of each leader is not affected by the number of leaders since:

$$\frac{d\pi_L}{dm} = \Pi_2^L \left[h(x_L) - h(x_F) + h(x_F)\frac{dn}{dm}\right] = 0$$

which concludes the proof. Q.E.D.

Part B: Generalized Stackelberg games.

In this section, we generalize the model of Stackelberg competition to the case of positive spillovers and objective functions affected separately by the number of agents, and we discuss a few applications of the results. The aim is to show that the driving factors leading to the aggressive behaviour of a leader facing endogenous entry keep working in more general models than those studied in our main analysis, even if other factors may work as well. Consider the following profit function:

$$\pi_i = \tilde{\Pi}(x_i, \beta_i, n) - K \tag{36}$$

with $\tilde{\Pi}_x \gtreqless 0$ for $x \lesseqgtr x(\beta)$, $\tilde{\Pi}_{xx} < 0$, $\tilde{\Pi}_n \leq 0$ and no assumptions on $\tilde{\Pi}_\beta$ and on the cross derivatives. When the number of firms is exogenous, the traditional results on Stackelberg competition can be extended in a straightforward way. When entry is endogenous, which requires regularity conditions that guarantee that $d\tilde{\Pi}/dn < 0$, the characterization of the SEFE is more complex. In the last stage, for a given leader's strategy x_L we can derive the following comparative statics:

$$\frac{dx_F}{dx_L} = -\frac{h'(x_L)[\tilde{\Pi}_{x\beta}\tilde{\Pi}_n + \tilde{\Pi}_\beta\tilde{\Pi}_{xn}]}{\Delta} \qquad \frac{dn}{dx_L} = \frac{-h'(x_L)\tilde{\Pi}_{xx}\tilde{\Pi}_\beta}{\Delta} \tag{37}$$

where $\Delta \equiv \tilde{\Pi}_\beta\tilde{\Pi}_{xx}h(x) + \tilde{\Pi}_n\tilde{\Pi}_{xx} + (n-2)h'(x)[\tilde{\Pi}_n\tilde{\Pi}_{x\beta} + \tilde{\Pi}_{xn}\tilde{\Pi}_\beta] > 0$ is the determinant of the equilibrium conditions in the last stage (profit maximization $\tilde{\Pi}_x = 0$, and free entry $\tilde{\Pi} = K$). The effect of a change in the leader's strategy on the followers'strategy is ambiguous and the equilibrium condition for the leader' strategy can be derived as:

$$\tilde{\Pi}^L_x = \frac{h'(x)h'(x_L)(n-1)[\tilde{\Pi}_{x\beta}\tilde{\Pi}_n + \tilde{\Pi}_\beta\tilde{\Pi}_{xn}]\tilde{\Pi}^L_\beta}{\Delta} + \frac{h'(x_L)\tilde{\Pi}_{xx}\tilde{\Pi}_\beta\left[\tilde{\Pi}^L_\beta h(x) + \tilde{\Pi}^L_n\right]}{\Delta} \tag{38}$$

The sign of the right hand side determines whether the leader will be aggressive or not. Of course, for $\tilde{\Pi}_n = \tilde{\Pi}_{xn} = 0$ we are back to the basic case examined in the text. In case of negative spillovers ($\tilde{\Pi}_\beta h'(x) < 0$) the second term on the right is always negative and we can conclude that the leader is aggressive whenever $\tilde{\Pi}_{x\beta} < -\tilde{\Pi}_\beta\tilde{\Pi}_{xn}/\tilde{\Pi}_n$, while an accommodating behavior can only emerge in case of strong strategic complementarities.

To exemplify the way to use this result, we will employ models of product differentiation already adopted by Erkal and Piccinin (2007) in the study of mergers in markets with endogenous entry. Let us consider a generalized model of product differentiation that derives from the following utility function:

$$U = a\sum_{i=1}^{n} C_i - \frac{1}{2}\left[\sum_{i=1}^{n} C_i^2 + b\sum_i\sum_{j\neq i} C_iC_j\right] + C_0$$

where C_i is consumption of good i and C_0 is the numeraire. Goods are homogenous when $b = 1$ and they are imperfectly substitutable otherwise. These preferences generate the system of inverse demand functions of Example 2. However, inverting the system, we obtain the direct demand functions:

$$D_i = \frac{a - p_i + \frac{b}{1-b}\sum_{j \neq i}(p_j - p_i)}{1 + b(n-1)}$$

It can be verified that the profit function associated with this case is not nested in the framework of the text, but it is nested in the general framework (36). Nevertheless, it is well behaved and it is decreasing in the number of firms for given strategies. Since prices are strategic complements, the Stackelberg equilibrium with an exogenous number of firms is characterized by a higher price for the leader compared to the followers. However, the SEFE is characterized by a lower price for the leader compared to the followers. Moreover, the price of the leader is below the equilibrium price in the Nash equilibrium with free entry, while the price of the followers is above it and the number of products is reduced.[24] In the long run, prices turn into strategic substitutes: the reduction in the price of the leader induces the followers to increase their prices.

Consider now the Shubik (1980) demand, derived from the utility function:

$$U = a\sum_{i=1}^{n} C_i - \frac{n}{2(1+\mu)}\left[\sum_{i=1}^{n} C_i^2 + \frac{\mu}{n}\left(\sum_{i=1}^{n} C_i\right)^2\right]$$

with $\mu > 0$ representing the degree of substitutability between goods (with perfect homogeneity for $\mu \to \infty$). We can derive the direct and inverse demand functions for firm i as:

$$D_i = \frac{1}{n}\left[a - p_i(1+\mu) + \frac{\mu}{n}\sum_{j=1}^{n} p_j\right], \quad p_i = a - \frac{1}{1+\mu}\left(nx_i + \mu\sum_{j=1}^{n} x_j\right)$$

Of course, the associated profit functions are not nested in our basic model because they depend on the number of firms. However, they are nested in the generalized version (36), which allows us to derive conclusions on the behaviour of market leaders

[24] Assume zero marginal costs. The optimality condition of the followers and the endogenous entry condition imply the following equilibrium relation between the price of the followers p_F and the number of firms n:

$$p_F = \sqrt{\frac{F(1-b)[1+b(n-1)]}{[1+b(n-2)}}$$

A reduction in the price of the leader p_L reduces entry and, according to this relation, it increases the price of the followers. The profit of the leader is:

$$\pi_L = \frac{p_L}{1+b(n-1)}\left[a - p_L + \frac{b(n-1)}{1-b}(p_F - p_L)\right] - F$$

where both n and p_F depend on p_L. Since $\partial\pi_L/\partial n\lfloor_{p_L=p_F} < 0$, it is optimal for the leader to reduce the number of firms compared to the Nash equilibrium with free entry.

even if we cannot explicitly solve for the equilibria with endogenous entry. Under competition in quantities we have:

$$\tilde{\Pi}(x_i, \beta_i, n) = \left[a - \frac{(n+\mu)x_i + \mu\beta_i}{1+\mu} \right] x_i - cx_i$$

where the strategic variable is output x_i and $\beta_i = \sum_{j \neq i} x_j$. Notice that $\tilde{\Pi}_\beta < 0$, $\tilde{\Pi}_n < 0$, $\tilde{\Pi}_{x\beta} < 0$ and $\tilde{\Pi}_{xn} < 0$, hence both terms on the right hand side of (38) are negative: this implies that a market leader facing endogenous entry will always be aggressive, that is produce more than each follower. This should not surprising, since according to (37), an increase in the production of the leader has now a negative impact on both the number of followers and on their production as well.

Let us switch to the case of competition in prices, in which profits can be written as:

$$\tilde{\Pi}(p_i, \beta_i, n) = \frac{(p_i - c)}{n} \left[a - p_i(1+\mu) + \frac{\mu}{n}(p_i + \beta_i) \right]$$

where the strategic variable is the price p_i and $\beta_i = \sum_{j \neq i} p_j$ (notice that we did not use the transformation of the strategic variable employed in the main text for competition in prices). We now have $\tilde{\Pi}_\beta > 0$, $\tilde{\Pi}_n < 0$, $\tilde{\Pi}_{x\beta} > 0$ and $\tilde{\Pi}_{xn} < 0$. This implies that, according to (37), an increase in the price of the leader increases the price of the followers (strategic complementarity in action), but it also promotes entry: the former effect increases the profits of the leader, the latter reduces them. As a consequence, on the right hand side of (38) the first term is negative and the second one can be positive. In this case, the leader facing endogenous entry would reduce its price and the followers would reduce their prices as well (prices are strategic complements in both the short and long run). As a consequence the number of varieties provided in the market would decrease. Nevertheless, consumer surplus would strictly increase because of the generalized reduction in prices.[25]

A final application of the generalized model can be obtained reinterpreting a simple model with heterogenous firms. Imagine that the profit function for firm i is $\pi_i = A(i)\Pi(x_i, \beta_i) - F$, where $A(i)$ is a parameter of profitability (or a multiplicative shock to profitability) which differs across firms and decreases in the ordering of entry of the firms, which is indexed by i. Notice that the optimality condition for each follower, $\Pi_1(x, \beta) = 0$, is independent from the specific value of A, while entry occurs until the n^{th} firm obtains zero profits, that is $A(n)\Pi(x, \beta) = K$. Then, the model is isomorphic to the generalized model with profit function $\tilde{\Pi}(x_i, \beta_i, n) = A(n)\Pi(x_i, \beta_i)$, and one can verify that the leader is not necessarily aggressive. In particular we now have $dx/dx_L \gtrless 0$ if $\Pi_{12} \gtrless 0$: when SS holds the aggressiveness of the leader is strengthened, while under SC, this aggressiveness is dampened and it may even be reversed. Intuitively, the leader takes in consideration that a more aggressive strategy will reduce entry as usual, but such a change will also increase the profitability of the marginal firm, which changes the strategic interaction between the followers.

[25] These result derive from joint work with Nisvan Erkal and Daniel Piccinin.

Part C: Multiple strategies

Now, I will outline how a weaker version of the result on aggressive leaders generalizes when firms choose multiple strategic variables. Imagine that each firm chooses a vector of $K \geq 1$ strategic variables $x_i = [x_{i1}, x_{i2}, ..., x_{iJ}] \in \mathbb{R}_+^{\mathbb{J}}$, and its well behaved profit function can be written as:

$$\pi_i = \Pi \left[\mathbf{x}_i; \sum_{j \neq i} h(\mathbf{x}_j) \right] - K$$

with $h : \mathbb{R}_+^{\mathbb{J}} \to \mathbb{R}_+$ differentiable and increasing in all its arguments. Examples are models of multimarket competition, competition in quality and price, patent races with multiple investments and so on. Clearly these are very important cases in real markets. Results on the behaviour of leaders in similar markets with an exogenous number of firms are complicated and ambiguous since they depend on all the possible cross derivatives and hence on many specific properties of the markets. Nevertheless, under free entry, a weaker version of our result still holds.

Define firm i on average more aggressive than firm k if $h(x_i) > h(x_k)$. Then, in equilibrium we have a vector x for the followers which is independent from the leader'strategies, and the following equilibrium conditions:

$$\frac{\partial \Pi^L (\mathbf{x}_L, X_{-L})}{\partial x_{Lj}} = \left(\frac{\partial h(\mathbf{x})}{\partial x_{Lj}} \right) \frac{\partial \Pi^L (\mathbf{x}, X_L)}{\partial \beta_L} \leq 0 \text{ for all } j$$

This does not imply that the leader is more aggressive in all strategies, but that is always more aggressive in some strategies. Moreover, it follows that a SEFE with multiple strategic variables always implies that the leader is on average more aggressive than each follower. To prove this, denote with $x_i = [x_{i1}, x_{i2}, ..., x_{iJ}]$ the strategies of a firm i. Assume again that a symmetric equilibrium in the strategies of the followers exist. The system of $J+1$ equilibrium conditions for the second stage:

$$\frac{\partial \Pi \left[\mathbf{x}_F, (n-2)h(\mathbf{x}_F) + h(\mathbf{x}_L) \right]}{\partial x_{Fj}} = 0 \text{ for } j = 1, 2, .., J$$

$$\Pi \left[\mathbf{x}_F, (n-2)h(\mathbf{x}_F) + h(\mathbf{x}_L) \right] = K$$

pins down the vector x_F and $X_{-F} = (n-2)h(x_F) + h(x_L)$. Consequently the profit of the leader is:

$$\pi_L = \Pi^L \left[\mathbf{x}_L, (n-1)h(\mathbf{x}_F) \right] - K = \Pi^L \left[\mathbf{x}_L, X_{-F} + h(\mathbf{x}_F) - h(\mathbf{x}_L) \right] - K$$

which is maximized by the vector x_L that satisfies the system of J first order conditions:

$$\frac{\partial \Pi^L (\mathbf{x}_L, X_{-L})}{\partial x_{Lj}} = \frac{\partial h(\mathbf{x}_L)}{\partial x_{Lj}} \frac{\partial \Pi^L (\mathbf{x}_L, X_{-L})}{\partial X_{-L}}$$

where clearly $X_{-L} = (n-1)h(x_F)$. Imagine that there is such an interior equilibrium with $h(x_L) \leq h(x_F)$. Then it must be that $X_{-F} \leq X_{-L}$, which implies

$\Pi(x_L, X_{-L}) \leq \Pi(x_L, X_{-L})$ from the assumption $\Pi_2 < 0$. But the optimality of x_F and the free entry condition imply $\Pi(x_L, X_{-L}) < \Pi(x_F, X_{-F}) = K$, hence $\Pi(x_L, X_{-L}) < K$, which contradicts the optimality of the interior solution. This implies that $h(x_L) > h(x_F)$.

To see how to use this result, let us extend our basic model of price competition with the choice of a second strategy, the quality of the product. In such a model with vertical differentiation, each firm can offer a product of quality q_i at a price p_i, and the marginal cost for this quality level, $c(q_i)$, is increasing and convex in quality. Let us assume that consumers allocate their demand comparing the quality/price ratios of the different products. Then, expected profits are:

$$\pi_i = D\left[\frac{p_i}{q_i}, \sum_{j\neq i} g\left(\frac{p_j}{q_j}\right)\right][p_i - c(q_i)] - K$$

Defining $\theta_i = q_i/p_i$ as the quality/price ratio for firm i, this model satisfies our conditions with:

$$\Pi\left[\theta_i, q_i; \sum_{j\neq i} h(\theta_j, q_j)\right] = D\left[\frac{1}{\theta_i}, \sum_{j\neq i} h(\theta_j, q_j)\right]\left[\frac{q_i}{\theta_i} - c(q_i)\right]$$

where $h(\theta_j, q_j) = g(1/\theta_i)$. We cannot say whether the leader will offer a good with both a lower price and a higher quality, but only that the good of the leader will be better than the goods offered by the followers at least in one of these two dimensions. Nevertheless, we can also say that the leader will be more aggressive than the followers on average, which means, according to our definition, that $h(\theta_L, q_L) > h(\theta, q)$. But this implies $g(1/\theta_L) > g(1/\theta)$ or, using the fact that g is a decreasing function, that $\theta_L > \theta$. We can then conclude that the leader will supply a good with a better quality/price ratio than each other follower. Of course, more complex forms of multidimensional models remain to be studied.