Three Essays on RT Consumers, Habit Formation and The Business Cycle

Giorgio E. Motta

Department of Economics

University of Milan-Bicocca

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Abstract

The aim of the dissertation is to investigate the implication of limited asset market partecipation and habit formation in consumption for the monetary policy in new Keynesian DSGE models. It emerges that the combination of this two ingredients has important implication on the stability properties of the model and its performance in replicating the business cycle dynamics.

Contents

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I would like to express my gratitude to my supervisor, Professor Patrizio Tirelli, for his guidance and advice.

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Declaration

The material contained in this thesis has not been previously submitted for a degree in this or any other university.

CHAPTER 1

Money Targeting, Heterogeneous Agents and Dynamic Instability

1.1. Introduction

New Keynesian business cycle analysis is characterized by optimizing agents (households and firms), and by a number of nominal and real frictions in goods, labor and financial markets. Due to its success in replicating estimated impulse responses of key macroeconomic variables to a money supply shock, the Christiano et al. (2005, CEE henceforth) model is widely regarded as the epithome of this approach.

Following a seminal contribution by Mankiw (2000), who introduced the notion of heterogeneous consumers (savers and spenders), a second strand of New Keynesian literature emphasizes the role of non-optimizing agents, i.e. agents that adopt a rule-of-thumb and fully consume their current income (RT consumers henceforth). Gali et al (2004, 2007), and Bilbiie (2008), showed how RT consumers can substantially affect both stability and aggregate dynamics of New Keynesian business cycle models. De Graeve et al. (2010) introduce RT consumers to model financial risk premia. Empirical research cannot reject the RT consumers hypothesis. Estimated structural equations for consumption growth report a share of RT consumers ranging from 26 to 40% (Jacoviello, 2004; Campbell and Mankiw, 1989) More recent estimates of dynamic stochastic general equilibrium models (Coenen and Straub, 2005; Forni, Monteforte and Sessa, 2009) obtain estimates around 35%. Erceg, Guerrieri and Gust (2006) calibrate the share of RT consumers to 50% in order to replicate the dynamic performance of the Federal Reserve Board Global Model. Critics of the approach might argue that the empirical relevance of RT consumers is bound to gradually decline along with the development of financial markets (Bilbiie, Meier and

Müller, 2008). In fact, increasing regulation in the aftermath of the 2008 crisis is likely to increase the share of liquidity constrained households (OECD 2009).

The paper brings together this two strands of literature. More specifically, we investigate the robustness of the CEE model response to money supply shocks when a fraction of households does not participate to financial markets. Our proposed modification to the CEE model is quite simple, but has profound implications. In fact we find that the model is dynamically unstable unless the share of non-optimizing consumers falls short of 35%. In addition, the dynamic performance of the model is dramatically affected even when the share of non-optimizing agents is restricted to less than 30%, and its celebrated ability to replicate the business cycle response to a monetary shock simply vanishes.

The intuition behind our results is rather simple. Under an exogenous money supply rule, optimizing households' consumption drives money demand and interest rate dynamics. RT consumers generate a "Keynesian multiplier", weakening the link between output and the nominal interest rate. Instability arises when the wedge between output and consumption of optimizing agents is sufficiently large. Two frictions play an important role in determining instability. Nominal wage stickiness dampens the real wage response to shocks and substantially weakens the multiplier effect of RT consumption decisions. The opposite effect is induced by consumption habits, which limit optimizing consumers responses to shocks.

Atheoretical VAR models suggest that in the real world some stabilizing mechanism eventually forces the economy back to steady state when monetary policy is exogenous. Such a mechanism could be driven by fiscal policies. Indeed Andres et al.(2008) show that automatic stabilizers reduce the volatility of RT consumption. We therefore explore whether a fiscal automatic stabilizer can solve the instability problem. In the original CEE model Ricardian equivalence obtains and automatic stabilizers essentially play no role. In our framework they are quite effective in driving RT consumption. In fact we obtain that the model now is stable irrespective of the share of RT consumers, and the dynamic performance of the system closely follows the original CEE model.

The rest of the paper is organized as follows: In the next section we describe in detail the model structure, we then present the results concerning the model stability in section 3. Section 4 proposes alternative ways to regain stability of the model. Section 5 concludes.

1.2. The Model

We augment the CEE model to account for both Ricardian and RT consumers. The key distinction between the two groups concerns intertemporal optimization. Ricardian consumers' choices take into account future utility when choosing consumption and portfolio composition. Rule-of-Thumb consumers spend their whole income every period, thus they do not hold any wealth.

In the paper we maintain the financial structure defined in CEE. This implies that a cash-in-advance constraint is imposed on firms. The latter must hold money in order to finance the wage bill before production is sold. Ricardian consumers' demand for money is derived from their portfolio optimization. Money holdings of Rule-of-Thumb consumers correspond to their (firms-financed) nominal labour income, and are entirely used to finance current consumption.

1.2.1. Households preferences

We assume a continuum of households indexed by $j \in [0,1]$. RT consumers are defined over the interval $[0,\theta]$. The rest of the households, interval $(\theta,1]$ accounts for Ricardian consumers. All households share the same utility function:

$$U_t^i = E_0 \sum_{t=o}^{\infty} \beta^t \left\{ \ln\left(C_t^i - bC_{t-1}^i\right) - \frac{\psi_l}{1+\phi_l}(h_t^i)^{1+\phi_t} + \frac{\psi_q}{1-\sigma_q}(q_t^i)^{1-\sigma_q} \right\} \tag{1.1}$$

where $i : o, rt$ stands for household type, $q_t^i = \frac{Q_t}{P_t}$ represents households real money balances, C_t^i represents total individual consumption, b denotes consumption internal habits and h_t^i denotes individual labour supply.

1.2.1.1. Consumption Bundles. C_t^i is a standard consumption bundle

$$C_t^i = \left[\int_0^1 c\left(z\right)_t^{\frac{\eta-1}{\eta}} dj \right]^{\frac{\eta}{\eta-1}} \tag{1.2}$$

where η represents the price elasticity of demand for the individual goods.

$$P_t = \left(\int_0^1 p\left(z\right)_t^{1-\eta} dj \right)^{\frac{1}{1-\eta}}$$

is the aggregate consumption price index.

1.2.2. Firms

Goods markets are monopolistically competitive, and good z is produced with the following technology:

$$y_t\left(z\right) = \left(k_t\left(z\right)\right)^{\alpha} \left(h_t\left(z\right)\right)^{1-\alpha}$$

where $k_t\left(z\right)$ defines the physical capital services obtained from households (see section 2.4 below) and $h_t\left(z\right)$ is the composite labor input used by each firm z. The latter is defined as follows

$$h_t\left(z\right) = \left(\int_0^1 \left(h_t^j\left(z\right)\right)^{\frac{\alpha_w-1}{\alpha_w}} dj \right)^{\frac{\alpha_w}{\alpha_w-1}} \tag{1.3}$$

where the parameter $\alpha_w > 1$ is the intratemporal elasticity of substitution between labor inputs. For any given level of its labor demand $h_t\left(z\right)$, the optimal allocation across labor inputs implies

$$h_t^j\left(z\right) = \left(\frac{W_t^j}{W_t} \right)^{-\alpha_w} h_t^d\left(z\right) \tag{1.4}$$

where $W_t = \left(\int_0^1 \left(W_t^j\right)^{1-\alpha_w} dj \right)^{1/(1-\alpha_w)}$ is the standard wage index.

Firms are subject to a cash-in-advance constraint, i.e. they must borrow the wage bill $W_t h_t$ at the beginning of the period t and have to repay it at the end of the period at the gross interest rate R_t.

Firm z's nominal total production cost is given by

$$TC_t(z) = R_t W_t h_t(z) + (1 + R_t^k) k_t(z) \tag{1.5}$$

The real marginal costs are:

$$mc_t = \left(\frac{r_t^k}{\alpha}\right)^{\alpha} \left(\frac{w_t R_t}{(1-\alpha)}\right)^{1-\alpha} \tag{1.6}$$

where $w_t = \frac{W_t}{P_t}$ and $r_t^k = \frac{R_t^k}{P_t}$.

1.2.2.1. Sticky Prices. Price stickiness is based on the Calvo mechanism. In each period firm z faces a probability $1 - \lambda_p$ of being able to reoptimize its price. When a firm is not able to reoptimize, it adjusts its price to the previous period inflation, $(1 + \pi_{t-1}) = \frac{P_{t-1}}{P_{t-2}}$. The price-setting condition therefore is:

$$p_t(z) = (1 + \pi_{t-1})^{\gamma_p} p_{t-1}(z) \tag{1.7}$$

where $\gamma_p \in [0, 1]$ represents the degree of price indexation.

All the $1 - \lambda_p$ firms which reoptimize their price at time t will face symmetrical conditions and set the same price $\widetilde{P}_t$. When choosing $\widetilde{P}_t$ the optimizing firm will take into account that in the future it might not be able to reoptimize. In this case, the price at the generic period $t + s$ will read as $\widetilde{P}_t \Pi_{t,t+s-1}^{\gamma_p}$ where $\Pi_{t,t+s-1} = (1 + \pi_t) \ldots (1 + \pi_{t+s-1}) = \frac{P_{t+s-1}}{P_{t-1}}$.

$\widetilde{P}_t$ is chosen so as to maximize a discounted sum of expected future profits:

$$E_t \sum_{s=0}^{\infty} (\beta \lambda_p)^s \lambda_{t+s} \left(\widetilde{P}_t \Pi_{t,t+s-1}^{\gamma_p} - P_{t+s} mc_{t+s}\right) y_{t+s}(z)$$

subject to:

$$y_{t+s}(z) = Y_{t+s}^d \left(\frac{\widetilde{P}_t \prod_{t,t+s-1}^{\gamma_p}}{P_{t+s}}\right)^{-\eta} \tag{1.8}$$

where Y_t^d is aggregate demand and λ_t is the stochastic discount factor.

The F.O.C. for this problem is

$$E_t \sum_{s=0}^{\infty} (\beta\lambda_p)^s \lambda_{t+s} Y_{t+s}^d \left[\begin{array}{c} (1-\eta)\left(\Pi_{t,t+s-1}^{\gamma_p}\right)^{1-\eta} \widetilde{P}_t^{-\eta} (P_{t+s})^\eta + \\ +\eta \widetilde{P}_t^{-\eta-1} P_{t+s}^{\eta+1} mc_{t+s} \left(\Pi_{t,t+s-1}^{\gamma_p}\right)^{-\eta} \end{array} \right] = 0 \tag{1.9}$$

1.2.3. Labor market

There is a continuum of differentiated labor inputs indexed by $j \in [0,1]$. For each labor input there is a union j which monopolistically supplies the labor input j in the labor market j.

Each union sets the nominal wage, W_t^j, subject to (3.8). Each household i supplies all labour types at the given wage rate[1] and the total number of hours allocated to the different labor markets must satisfy the time resource constraint

$$h_t^i = \int_0^1 h_t^j dj = \int_0^1 \left(\frac{W_t^j}{W_t}\right)^{-\alpha_w} h_t^d dj \tag{1.10}$$

As in Galì (2007), we assume that the fraction of RT and Ricardian consumers is uniformly distributed across unions, and demand for each labour type is uniformly distributed across households. Ricardian and non-Ricardian households therefore work for the same amount of time, h_t. Individual labor income is

$$h_t^d W_t = \int_0^1 W_t^j \left(\frac{W_t^j}{W_t}\right)^{-\alpha_w} h_t^d dj \tag{1.11}$$

We posit that the union objective function is a weighted average $(1-\theta,\theta)$ of the utility functions of the two households types. This, in turn, implies that with flexible wages

$$w_t = \frac{W_t}{P_t} = \frac{\alpha_w}{\alpha_w - 1} \frac{\psi_l h_t^{\phi_l}}{\left[(1-\theta)U'\left(C_t^o - bC_{t-1}^o\right) + \theta U'\left(C_t^{rt} - bC_{t-1}^{rt}\right)\right]} \tag{1.12}$$

[1] Under the assumption that wages always remain above all households' marginal rate of substitution, households are willing to meet firms' labour demand.

where $\frac{\alpha_w}{(\alpha_w-1)}$ represents the wage markup over the average marginal rate of substitution.

1.2.4. Ricardian Households

Ricardian households maximize utility subject to the following period budget constraint.

Budget constraints in nominal terms:

$$M_{t+1} = R_t \left[M_t - Q_t + (\mu_t - 1)M_t\right] + A_{j,t} + Q_t + R_t^k u_t \bar{k}_t + \tag{1.13}$$

$$+D_t - P_t \left[i_t + c_t + a\left(u_t\right)\bar{k}_t\right] + h_t^d \int_0^1 W_t^j \left(\frac{W_t^j}{W_t}\right)^{-\alpha_w} dj \tag{1.14}$$

Where M_t is the total amount of money and Q_t represents households nominal cash balances. $R_t \left[M_t - Q_t + (\mu_t - 1)M_t\right]$ defines interest payments from firms which are subject to a cash-in-advance constraint $A_{j,t}$ and D_t are respectively the net cash flow from participating in state-contingent securities at time t and firm dividends.

Optimizing households own the physical stock of capital k_t, and choose the degree of its utilization, u_t, that rent to firms at the real rental rate r_t^k. The term $a\left(u_t\right)$ defines the real cost of using the capital stock with intensity u_t. Finally, i_t denotes time t real purchases of investment goods. The household's stock of physical capital evolves as:

$$\bar{k}_{t+1} = (1-\delta)\bar{k}_t + i_t \left[1 - S\left(\frac{i_t}{i_{t-1}}\right)\right] \tag{1.15}$$

$$k_t = u_t \bar{k}_t \tag{1.16}$$

where δ and S respectively denote the physical rate of depreciation and investment adjustment costs.

The solution for the household problem closely follows CEE. The Euler equation is

$$\lambda_t^o = \beta E_t \lambda_{t+1}^o \frac{R_{t+1}}{\pi_{t+1}} \tag{1.17}$$

where

$$\frac{1}{C_t^o - bC_{t-1}^o} - \frac{\beta b}{C_{t+1}^o - bC_t^o} = \lambda_t^o \tag{1.18}$$

Ricardian households money demand therefore depends positively on current consumption and negatively on current interest rate.

$$\psi_q (q_t)^{-\sigma_q} = (R_t - 1)\, \lambda_t^o \tag{1.19}$$

The following first order conditions describe demand functions for capital[2] and investment and the optimal degree of capital utilization.

$$P_{k',t} = \beta E_t \left\{ \lambda_{t+1}^o \frac{r_{t+1}^k u_{t+1} - a\left(u_{t+1}\right) + (1-\delta)\, P_{k',t+1}}{\lambda_t^o} \right\} \tag{1.20}$$

The first order condition for investment is

$$\lambda_t^o = E_t \left\{ \begin{array}{c} \lambda_t^o P_{k',t} \left[1 - S\left(\frac{i_t}{i_{t-1}}\right) - S'\left(\frac{i_t}{i_{t-1}}\right) \frac{i_t}{i_{t-1}} \right] + \\ +\beta \lambda_{t+1}^o P_{k',t+1} \left[S'\left(\frac{i_{t+1}}{i_t}\right) \left(\frac{i_{t+1}}{i_t}\right)^2 \right] \end{array} \right\} \tag{1.21}$$

$$r_t^k = a'\left(u_t\right) \tag{1.22}$$

[2] $P_{k',t}$ is the shadow relative price of one unit of capital with respect to one unit of consumption (Tobin's q).

Following CEE and SGU the investment adjustment cost function and the capital utilization function are given by: [3]

$$S\left(\frac{i_t}{i_{t-1}}\right) = \frac{\kappa}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2$$

$$a\left(u_t\right) = \gamma_1\left(u_t - 1\right) + \frac{\gamma_2}{2}\left(u_t - 1\right)^2$$

1.2.4.1. Loan Market Clearing. The financial sector is characterized by a financial intermediary that, at the beginning of the period, receives a money transfer $(\mu_t - 1)M_t$ from the monetary authority and $M_t - Q_t$ from Ricardian households. Part of this money stock is lent to firms, who need to finance their wage bill. The rest is redistributed to the Ricardian households. Loan market clearing requires that

$$W_t L_t = \mu_t M_t - Q_t \tag{1.23}$$

1.2.5. Rule-of-Thumb Households

As pointed out above, RT consumers neither save or borrow. It is worth to recall that RT consumers also receive an amount of money at the beginning of the period in form of wage bill and spend the whole amount of money by the end of the period. Due to the labour market monopolistic structure, these agents are entirely passive. In fact both their consumption and their within-period money holdings are determined by union's (wage) and firms (worked hours) decisions.

$$c_t^{rt} = q_t^{rt} = \frac{h_t^i \int_0^1 \left(\frac{w_t^j}{w_t}\right)^{-\alpha_w} w_t^j \; dj}{P_t} \tag{1.24}$$

[3]Function $S(\cdot)$ satisfies the following properties. $S(1) = S'(1) = 0$ and $S''(1) > 0$. These restrictions imply the absence of adjustment costs up to a first order approximation around the deterministic steady state.The function $a(\cdot)$, instead, is assumed to satisfy $a(1) = 0$ and $a'(1), a''(1) > 0$. Moreover the parameters γ_1 and γ_2 are fixed given that $a'(u) = r^k$ at steady state.

1.2.6. Sticky wages

In each period a union faces a constant probability $1-\lambda_w$ of being able to reoptimize the nominal wage. Unions that cannot reoptimize simply index their wages to lagged inflation:

$$W_t^j = W_{t-1}^j \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_w} = W_{t-1}^j \left(\pi_{t-1}\right)^{\gamma_w}$$

where γ_w stands for the degree of wage indexation. Just like firms, when choosing the current wage, $\widetilde{W}_t$, the optimizing union will anticipate that in the future it might not be able to reoptimize. In this case, the real wage at the generic period $t+s$ will read as

$$w_{t+s} = \widetilde{w}_t \prod_{k=1}^{s} \frac{\pi_{t+k-1}^{\gamma_w}}{\pi_{t+k}} \tag{1.25}$$

Following Colciago(2008), the representative union objective function is defined as

$$L^u = \sum_{s=0}^{\infty} (\beta\lambda_w)^s \left\{ \left[(1-\theta)\, U^o(C_{t+s}^o) + \theta U^{rt}(C_{t+s}^{rt}) \right] - U(h_{t+s}) \right\} \tag{1.26}$$

Where U_s^o, U_s^{rt} are defined as in (2.1). Thus the wage-setting decision maximizes a weighted average of the two household types utility functions, conditional to the probability that the wage cannot be reoptimized in the future. The relevant constraints are (3.14), (1.13), (3.26), (3.27).

The union's first-order condition is:[4]

$$\sum_{s=0}^{\infty} (\beta\lambda_w)^s \left[(1-\theta)\, \lambda_{t+s}^o + \theta \lambda_{t+s}^{rt} \right] h_{t+s}^d \left(w_{t+s}\right)^{\alpha_w} \left(\prod_{k=1}^{s} \frac{\pi_{t+k-1}^{\gamma_w}}{\pi_{t+k}} \right)^{-\alpha_w} \cdot \tag{1.27}$$

$$\cdot \left[\widetilde{w}_t \left(\prod_{k=1}^{s} \frac{\pi_{t+k-1}^{\gamma_w}}{\pi_{t+k}} \right) - \frac{\alpha_w}{(\alpha_w - 1)} \frac{\psi_l h_{t+s}^{\phi_l}}{\left[(1-\theta)\, \lambda_{t+s}^o + \theta \lambda_{t+s}^{rt} \right]} \right] = 0$$

where $\lambda_{t+s}^{rt} = \frac{1}{C_t^{rt} - bC_{t-1}^{rt}} - \frac{\beta b}{C_{t+1}^{rt} - bC_t^{rt}}$.

[4] It is worth noting that the combination of centralized wage setting and wage stickiness introduces an indirect form of consumption smoothing for RT consumers.

1.2.7. Aggregation

Aggregating budget constraints for each sector, we get the aggregate resource constraint:

$$Y_t = C_t + I_t + a\left(u_t\right) K_t$$

where

$$C_t = \int_0^1 C_t^i(j)\ dj = \int_0^\theta C_t^{rt}(j)\ dj + \int_\theta^1 C_t^o(j)\ dj = \theta C_t^{rt} + (1-\theta) C_t^o \tag{1.28}$$

$$I_t = (1-\theta)\int_\theta^1 I_t^o(j)\ dj \tag{1.29}$$

$$K_t = (1-\theta)\int_\theta^1 K_t^o(j)\ dj \tag{1.30}$$

1.2.8. Monetary Policy

We assume a passive monetary authority which follows a simple rule for the money growth rate

$$\mu_t = 0.5\mu_{t-1} + \varepsilon_t \tag{1.31}$$

where $\mu_t = \frac{M_t}{M_{t-1}}$ and ε_t is an i.i.d. exogenous shock with zero mean and standard deviation σ_ε

1.3. Stability Analysis

After standard log-linearization[5], it is possible to reduce the model to a system of just dynamic equations in the form

$$\widehat{X}_{t+1} = A^{-1}B\ \widehat{X}_t + \varepsilon_t \tag{1.32}$$

where the vector $\widehat{X}$ contains the variables of the reduced system: $\widehat{X}_t = \Big[\hat{\pi}_t \quad \hat{w}_t \quad \hat{c}_t \quad \hat{k}_{t-1} \quad \hat{m}_{t-1}$ and ε_t is a vector representing an exogenous shock, with zero mean and standard deviation σ_ε, to the money growth rate.

Given the complexity of the system, numerical methods are the only way to study its determinacy properties. In table 1 we present the parameters chosen for our baseline simulations. They follow CEE(2005) and Schmitt-Grohe, Uribe (2004) with the obvious exception of the RT consumers share, which is set at 0.5, as in Galì (2004). The parameter governing the degree of habit persistence, b, is set at 0.7, as in Boldrin et al. (2001). We calibrate the parameters γ_1 and γ_2 in order to have $\frac{a''}{a'} = 2.01$ as in Altig, et al. (2005)

[5]See Appendix A.1

Table 1

Parameter	Value	Description
θ	0.5	share of RT consumers
b	0.7	degree of habit persistence
β	$1.03^{-(0.25)}$	subjective discount factor
α	0.36	share of capital
δ	0.025	depreciation rate
η	6	price-elasticity of demand for a differentiated good
α_w	6	intratemporal elasticity of substitution between labor inputs
κ	2.48	parameter governing investment adjustment costs
λ_w	0.64	degree of wage stickiness
λ_p	0.6	degree of price stickiness
γ_1	0.0324	parameter governing capacity adjustment costs
γ_2	0.0652	parameter governing capacity adjustment costs
σ_m	10.62	money demand elasticity
ϕ_m	0.5393	preference parameter
γ_p	1	indexation on prices
γ_w	1	indexation on wages
ϕ_l	3	preference parameter
σ_ε	0.15	std. deviation of the exogenous shock

1.3.1. Results

The baseline version of the model is unstable. Stability is recovered only for $\theta \leq 0.36$ In the following we check the robustness of this result to changes in the model parameters and, at the same time, investigate the economic factors behind it. Given the size of the model, and the variety of nominal and real dynamic frictions it is very difficult to understand the mechanism through which the presence of rule of thumb consumers generates instability. To facilitate intuition, we begin with a very simple version of the model (Model 0), where capital is fixed, wages are flexible, there is no habit in consumption and no cash in advance constraint on firms. We shall use this very simple model to sketch our interpretation of the instability result, which points at the weak response of the interest rate to output and inflation when monetary policy is exogenous and some consumers are non-Ricardian. Then, we introduce frictions in the following sequence: cash-in-advance constraint, wage stickiness, investment adjustment costs and variable capacity utilization, consumption habits. We will show that our interpretation is robust to these additions, and that the effect of each friction on stability depends on how it impacts on the co-movements of nominal interest rate and output.

To simplify presentation, we consider the combinations of price stickiness and share of RT consumers (parameters λ_p,θ) that define the stability frontier for each of the versions of the model considered. Our results are summarized in Figure 1, where we show how the stability frontiers shift when new frictions are introduced.

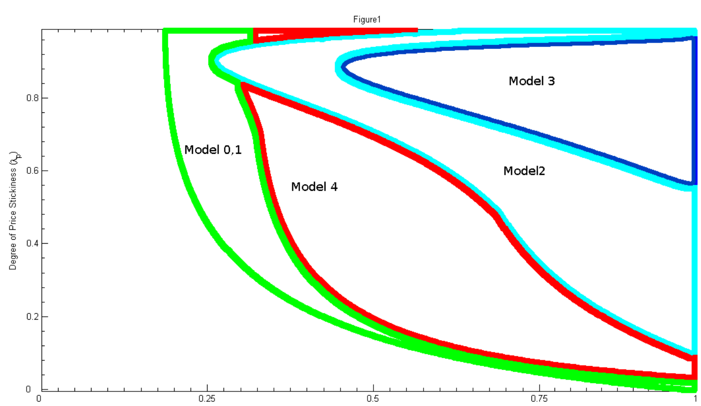

Figure 1: Determinacy Regions

1.3.1.1. Model 0. Using the relevant baseline parameters of Table 1, the model is unstable for $\theta \geq 0.23$. The threshold combinations λ_p, θ that define the stability frontiers tend to move in opposite directions: an increase in price stickiness requires a fall in the share of RT consumers.

From (2.11) and (1.19) it is easy to see that in this simple version of the model, the nominal interest rate is driven down by a money supply shock, but positively reacts to Ricardian consumption and to an inflation increase. In log-linear form, interest rate dynamics are described by

$$\hat{R}_t = (R-1)\,\hat{c}_t^o - \sigma_m\,(R-1)\,\hat{m}_t = \tag{1.33}$$

$$= (R-1)\left[\frac{\hat{y}_t}{1-\theta} - \frac{\theta\hat{c}_t^{rt}}{1-\theta}\right] + \sigma_m\,(R-1)\left[\hat{\pi}_t - \left(0.5\hat{\mu}_{t-1} + \hat{\varepsilon}_t\right)\right] + \tag{1.34}$$

$$-\sigma_m\,(R-1)\,\hat{m}_{t-1} \tag{1.35}$$

where R denotes the steady-state value of the gross nominal interest rate.

Note that the interest rate response to current inflation is very weak. In our baseline simulations $\sigma_m\,(R-1) = 0.0792$. [6] The weak interest rate response to inflation is a structural feature of a policy regime based on an exogenous money supply rule. When all agents are Ricardian this is offset by the interest rate reaction to consumption (Figure 2,solid line). By contrast, as shown in (1.33), RT consumers generate a "Keynesian multiplier effect" on the initial surge of the Ricardian households and produce a wedge between output and consumption of Ricardian consumers, the variable that drives nominal interest rates in the model. Dashed lines in Figure 2 show that even with a small share of RT consumers ($\theta = 0.2$) the link between output and the nominal interest rate is weakened and substantial differences emerge in the dynamic performance of the model.

[6] In fact, by raising σ_m to 1500 from the baseline value of 10.62 it would be possible to obtain stability.. Note, however, that σ_m is the inverse of the income elasticity of money demand, and that this would be in sharp contrast with consolidated empirical evidence and theoretical work. Several studies find an income elasticity between .5 and 1 (Choi and Oh, 2003; Knell and Stix, 2005).

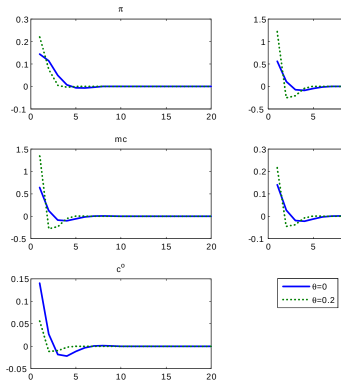

Figure 2: Responses to a Monetary Shock

Figure 3 illustrates the effect of θ on the impact responses of y, c^o, π, R and the real interest rate to the monetary shock.The distance between y and c^o is increasing in θ whereas the nominal interest rate adjustment remains constant. The growing output "multiplier effect" associated with an increasing share of RT consumers and the apparent inability of the nominal interest rate to react to the stronger output response is the key mechanism driving the incresing inflation response to θ and is the key mechanism driving the system towards instability.

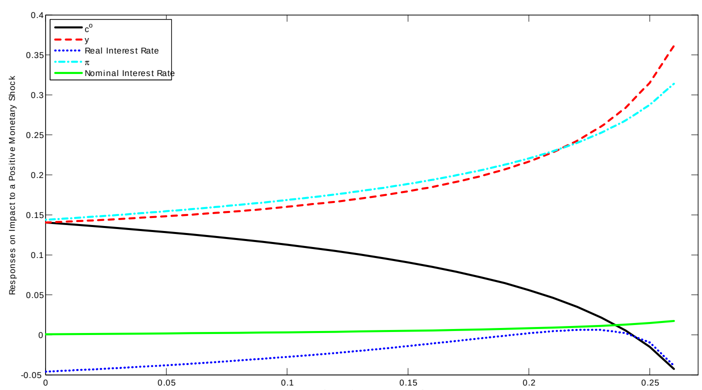

Figure 3: Impact Responses to a Monetary Shock

1.3.1.2. Model 1. Adding the cash-in-advance constraint on firms. The cash-in-advance constraint implies that, in addition to Ricardian consumers money demand, we must now consider firms demand for money, i.e. the wage bill (see eq. 1.23). From the loglinearized version of the model (see Appendix 1), money demand from optimizing consumers is given by:

$$\hat{q}_t = \frac{1}{\sigma_m}\left(\hat{c}_t^o - \frac{R}{R-1}\hat{R}_t\right) \tag{1.36}$$

The market-clearing condition in the money market is

$$\hat{h}_t + \hat{w}_t = \frac{\frac{M}{Y}}{\frac{M}{Y} - \frac{Q}{Y}}(\hat{M}_{t-1} + \hat{\mu}_t) - \frac{\frac{Q}{Y}}{\frac{M}{Y} - \frac{Q}{Y}}\hat{q}_t \tag{1.37}$$

Substituting 3.26, 1.36 into 1.37, we obtain:

$$\hat{R}_t = \frac{R-1}{R}c_t^o + \frac{R-1}{R}\sigma_m\frac{M-Q}{Q}\hat{c}_t^{rt} - \frac{R-1}{R}\sigma_m\frac{M}{Q}\left(0.5\hat{\mu}_{t-1} + \hat{\varepsilon}_t + \hat{m}_{t-1}\right) \tag{1.38}$$

Comparison between (1.38) with (1.33) shows that now the interest rate reacts to RT consumption but no longer responds to inflation. Relative to model 0, the stability frontier of the model is unaffected (Figure 1).

1.3.1.3. Model 2. Sticky wages. Wage stickiness dampens the real wage bill and limits the output multiplier effect of RT consumers (Figure 4). [7] As a result, the stability frontier of the model markedly shifts to the right (Figure 1).

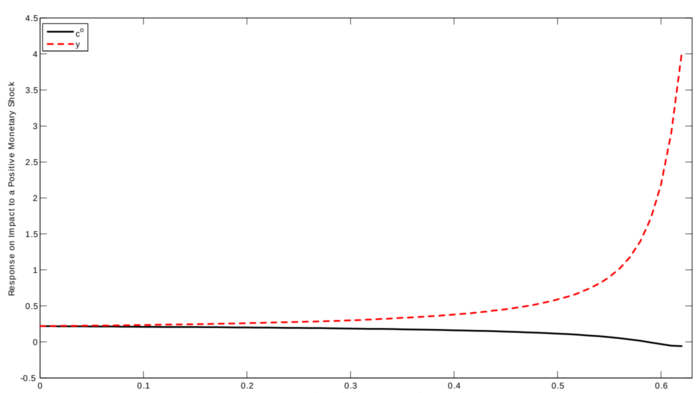

Figure 4: Impact Responses to a Monetary Shock

[7]See Colciago(2008) for a detailed discussion about the role of wage stickyness in in presence of ROT consumers.

1.3.1.4. Model 3. Endogenous capital stock. The inclusion of capital enhances the beneficial effects of wage stickiness (Figure 1). The key role is played by variable capacity utilization, which increases following the monetary shock. This, in turn, reduces labour demand and the wage bill, dampening RT consumption and its effects on marginal costs and inflation. (Figure 5).

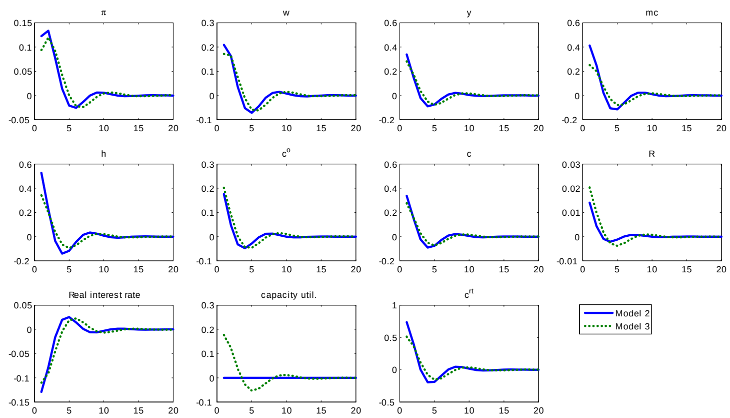

Figure 5: Responses to a Monetary Shock

1.3.1.5. Model 4. Consumption habits. We return to the full model by adding habits on consumption in households' utility functions. The stability frontier now markedly shifts to the left (Figure 1).

Habit significantly dampens Ricardian households consumption in response to the monetary shock (Figure 6a). This, in turn, limits the interest rate adjustment to the monetary shock (Figure 6b). In Figure 7 we show that habit increases the wedge between output and Ricardian consumption reaction to the shock, thus confirming our intuition about the cause of model instability.

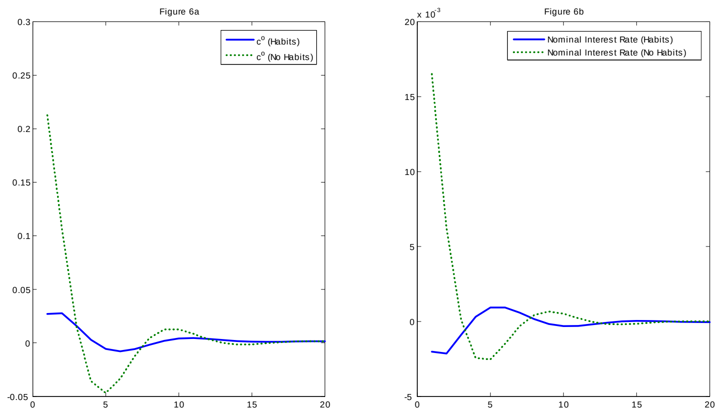

Figure 6: Responses to a Monetary Shock

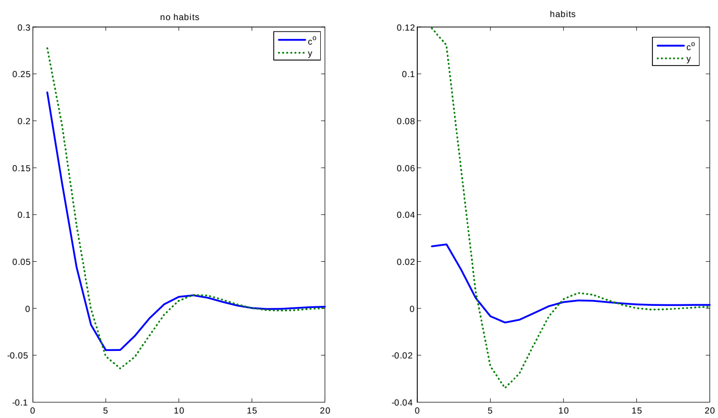

Figure 7: Responses to a Monetary Shock

1.3.1.6. Sensitivity analysis. As we pointed out in the previous section, habit persistence in consumption strongly affects model stability by dampening the interest rate response to Ricardians' consumption. Coeteris paribus, lowering b to 0.65 [8] enlarges the stability area and model's stability is guaranteed for $\theta < 0.47$ If we shift the degree of habit persistence to 0.8, that is, the value estimated in Fuhrer(2000) and Erceg et al. (2006) we see that the model is stable for $\theta \in [0, 0.19)$. Our results are robust to alternative plausible values of $\kappa \in [0.5, 5]$, $\varphi_l \in [0.5, 10]$, $\sigma_m \in [1, 100]$[9]. Given the calibration on the other parameters, changing the values for money elasticity, the Frish elasticity and

[8] $b = 0.65$ corresponds to the estimates in CEE(2005)

[9] Results available on request.

the degree of investment costs does not significantly change the threshold of constrained agents generating instability in this framework.

The intriguing role of price indexation. The last robustness check concerns wage and price indexation to past inflation. When we impose no indexation, i.e. $\gamma_p = 0$, the stability area remains almost unaffected whereas outside it the model is stable but undetermined (Figure 8).[10]

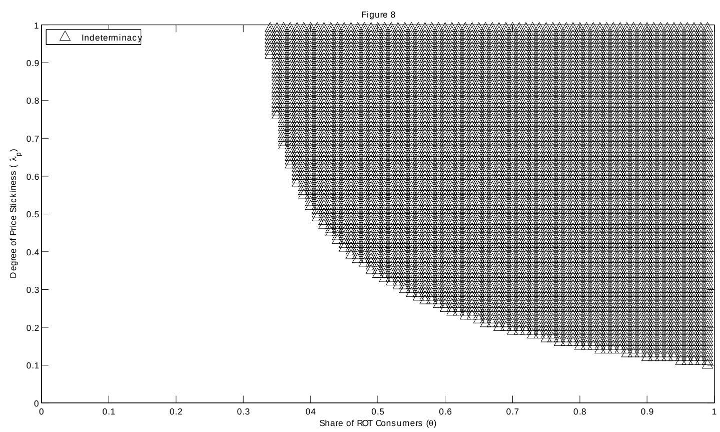

Figure 8: Determinacy Region

Our intuition is the following: as θ increases, the impact response of inflation grows with θ, whereas the real rate response is unaffected (Figure 9). The response of the factor driving forward-looking adjustment. i.e. jumps in Ricardian agents' consumption and investment, becomes weaker. As result, with indexation-induced persistence, the initial inflation increase causes further inflation growth, a wage run up and instability. Without indexation, the initial inflation surge is always reversed with an almost monotonic pattern (Figure 10). This happens because the inflation increase eventually cuts down the real wage bill and disposable income of RT consumers. This, in turn, implies that inflation reversal obtains irrespective of the real interest rate response, generating indeterminacy.

Intermediate degrees of price indexation, $0 < \gamma_p < 1$, have virtually no effect on the determinacy region which is identified by $\theta \leq 0.36$ (Figure 11). For any $\theta > 0.36$,

[10] Wage indexation plays no role in determining stability region. Simulation results available upon request.

a threshold value γ_p^* extists, such that the model turns form unstable to indetermined. Parameter γ_p^* is decreasing in θ.

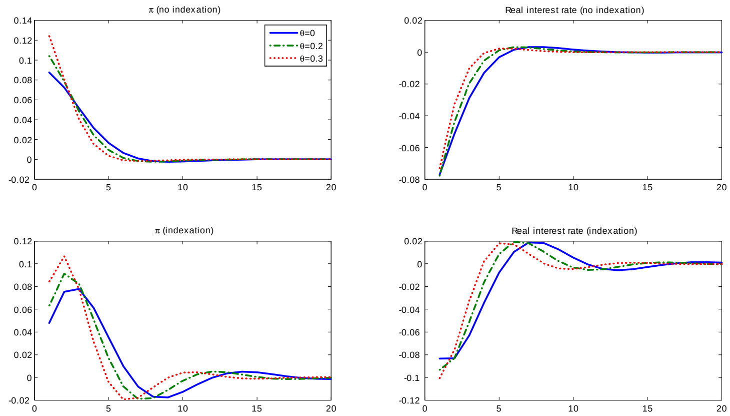

Figure 9: Responses to a Monetary Shock

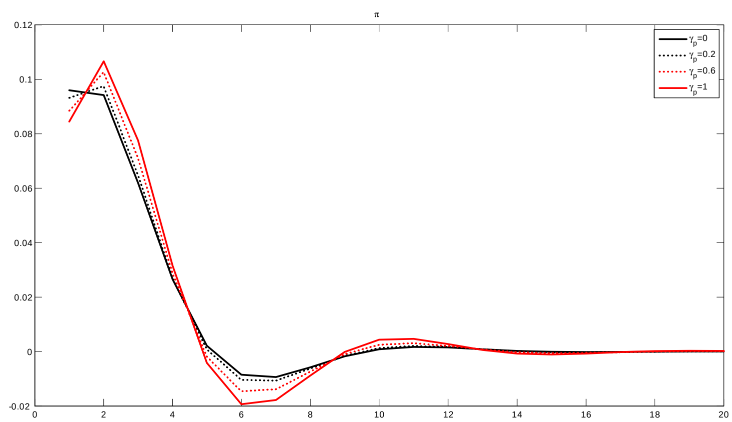

Figure 10:Responses to a Monetary Shock

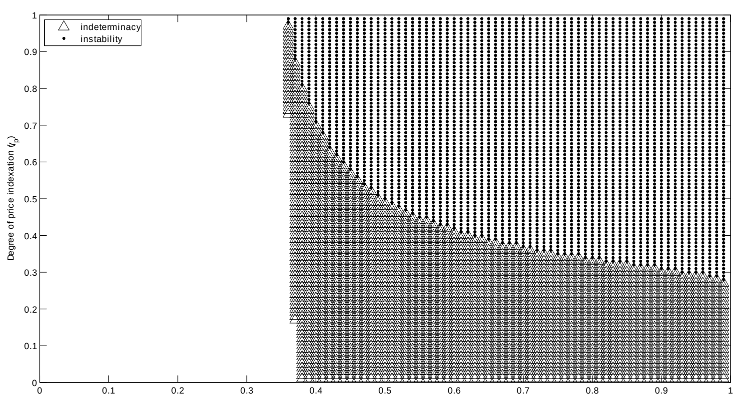

Figure 11: Determinacy Region

1.4. Dynamics

We analyze the model responses to a monetary shock when the share of RT consumers is just below the threshold which would generate instability. As shown in Figure 12, the dynamic properties of this model are upset when a relatively small share of RT consumer is considered in the economy. Aggregate consumption strongly rises on impact, and the hump-shaped dynamic response disappears. The multiplier effect of RT consumers reverses the nominal interest rate response, which turns positive on impact. Given the stronger response of nominal interest rate and wages, profits now fall on impact.

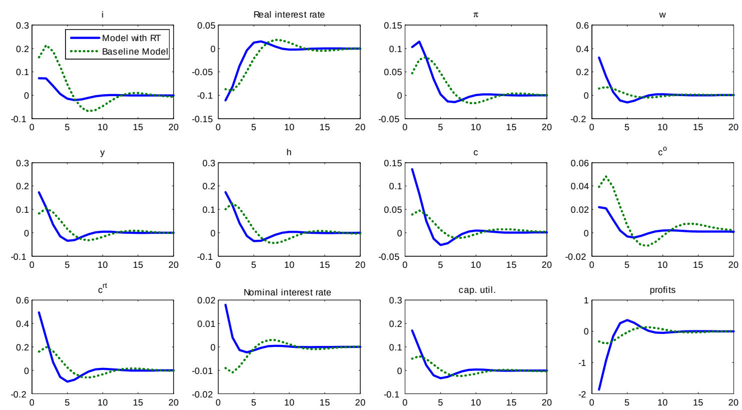

Figure 12: Responses to a Monetary Shock

1.5. Can we rescue the model?

In our model instability occurs because RT consumers do not react to the real interest rate and, given their consumption rule, weaken the interest rate response to output dynamics. Atheoretical VAR models suggest that in the real world some stabilizing mechanism eventually forces the economy back to steady state when monetary policy is exogenous.

Such a mechanism could be driven by fiscal policy that, even in the passive form of an automatic stabilizer may crucially affect RT consumption.[11] In the following we check this conjecture. To minimize modifications to the original CEE model, we assume that

[11]Automatic stabilizer are those elements of fiscal policy which vary without requirement of discretionary government actions (Auerbach and Feenberg, 2000). Van den Noord (2000), Fatas and Mihov (2001), Mabbett (2004) Dolls, Fuest and Peichl (2009) document the effectivness of automatic stabilizers.

households must pay a lump sum tax whose amount, in turn, depends on the aggregate output gap. By definition, Ricardian consumers decisions are not affected by this tax. To the contrary, RT consumption is modified as follows:

$$\hat{c}_t^{rt} = \hat{w}_t + \hat{h}_t - \widehat{tax}_t$$

where

$$\widehat{tax}_t = \gamma_y \hat{y}_t$$

As we see in Figure 13, for plausible values of γ_y ($\gamma_y = 0.55$) the instability region shifts on the right. Moreover, this latter version of the model [12] is characterized by impulse responses which are almost identical to the case of no RT consumers (Figure 14). The mechanism of this taxation is similar to the one of the keynesian multiplier on income, the fluctuations are reduced poportionally with the increase in taxes.[13]

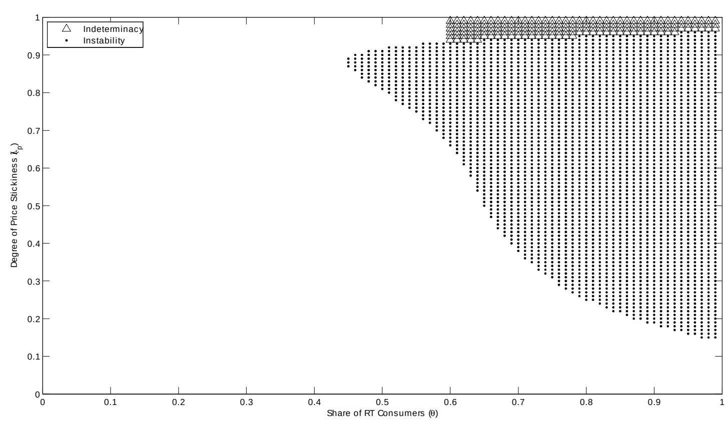

Figure 13: Determinacy Region

[12] We simulate the model with $\theta = 0.5$

[13] This result is akin to the one in Andres et al. (2008)

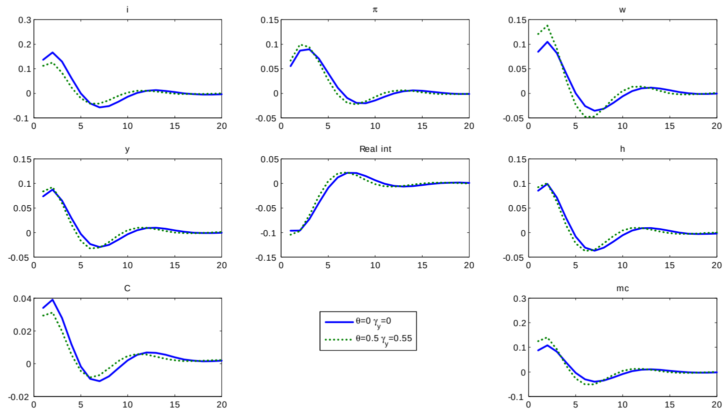

Figure 14: Responses to a Monetary Shock

1.6. Conclusion

We embodied limited asset market participation in a well known medium scale New Keynesian framework. We showed that when monetary policy is conducted following an exogenous rule on the money growth rate, the model is unstable for a limited share of Rule-of-Thumb consumers. The reason of this instability is that RT consumers behavior multiplies the response of output to a money supply shock, which cannot be restrained by the monetary rule. To restore dynamic stability we need to embed a fiscal stabilizer. This modified model maintains the dynamic performance and the consistency with empirical which characterized the original CEE framework based on a representative agent. A key result therefore is that, under limited asset market participation, macroeconomic models should explicitly account for fiscal policies, at least in the simple form of automatic stabilizers.

We found that consumption habits play a key role in driving our results. Further research should investigate how different habits specifications, i.e. external habits, could alter the model dynamic properties.

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1.7. Appendix A.1

1.7.1. log-linearized model

The stability analysis is conducted using a linearized version of the model presented above. Lower case letters from now on denote the log of the corresponding variable or their log deviations from the steady state.

Aggregate consumption is defined by:

$$\hat{C}_t = (1-\theta)\frac{c^o}{C}\hat{c}_t^o + \theta\frac{c^{rt}}{C}\hat{w}_t + \theta\frac{c^{rt}}{C}\hat{h}_t \tag{1.39}$$

The next equations describe the market clearing condition and money demand:

$$\hat{h}_t + \hat{w}_t = \frac{\frac{M}{Y}}{\frac{M}{Y}-\frac{Q}{Y}}(\hat{M}_{t-1}+\hat{\mu}_t) - \frac{\frac{Q}{Y}}{\frac{M}{Y}-\frac{Q}{Y}}\hat{Q}_t \tag{1.40}$$

$$\hat{R}_t + \frac{R-1}{R}\hat{\lambda}_t^o + \frac{R-1}{R}\sigma_m\hat{q}_t = 0 \tag{1.41}$$

Marginal costs are given by

$$\widehat{mc}_t = (1-\alpha)\left(\hat{w}_t + \hat{R}_t\right) + \alpha\hat{r}_t^k \tag{1.42}$$

The following equation combines firms' F.o.c. with respect to production factors

$$\hat{h}_t + \hat{w}_t + \hat{R}_t = \hat{k}_{t-1} + \left(1+\frac{\gamma_1}{\gamma_2}\right)\hat{r}_t^k \tag{1.43}$$

Production function is given by

$$\hat{y}_t = \alpha\hat{k}_{t-1} + \alpha\frac{\gamma_1}{\gamma_2}\hat{r}_t^k + (1-\alpha)\,\hat{h}_t \tag{1.44}$$

Aggregate resource constraint

$$\hat{y} = \frac{i}{y}\hat{\imath}_t + \frac{c}{y}\hat{c}_t + \gamma_1\frac{\gamma_1}{\gamma_2}\frac{k}{y}\hat{r}_t^k \tag{1.45}$$

RT consumption

$$\hat{c}_t^{rt} = \hat{w}_t + \hat{h}_t \tag{1.46}$$

Euler equation

$$\hat{\lambda}_t^o = \hat{\lambda}_{t+1}^o + \hat{R}_{t+1} - \hat{\pi}_{t+1} \tag{1.47}$$

Households marginal utility of consumption

$$\lambda_t^o = \frac{\beta b}{(1-\beta b)(1-b)} c_{t+1}^o - \frac{(1+\beta b^2)}{(1-\beta b)(1-b)} c_t^o + \frac{b}{(1-\beta b)(1-b)} c_{t-1}^o \tag{1.48}$$

$$\lambda_t^{rt} = \frac{\beta b}{(1-\beta b)(1-b)} c_{t+1}^{rt} - \frac{(1+\beta b^2)}{(1-\beta b)(1-b)} c_t^{rt} + \frac{b}{(1-\beta b)(1-b)} c_{t-1}^{rt} \tag{1.49}$$

Investment decisions

$$\hat{\imath}_t - \frac{1}{k(1+\beta)} \hat{P}_{k',t} - \frac{1}{(1+\beta)} i_{t-1} - \frac{\beta}{(1+\beta)} \hat{\imath}_{t+1} = 0 \tag{1.50}$$

$$\hat{\pi}_{t+1} + \beta(1-\delta)\hat{P}_{k',t+1} - \hat{P}_{k',t} = \hat{R}_{t+1} - \beta r^k \hat{r}_{t+1}^k \tag{1.51}$$

Capital accumulation

$$\hat{k}_t = (1-\delta)\hat{k}_{t-1} + \delta \hat{\imath}_t \tag{1.52}$$

Phillips Curve

$$\frac{\lambda_p}{1-\lambda_p}\left(\hat{\pi}_t - \gamma_p \hat{\pi}_{t-1}\right) = (1-\beta\lambda_p)\widehat{mc}_t + \beta\lambda_p\left(\hat{\pi}_{t+1} - \gamma_p\hat{\pi}_t\right) + \beta\frac{\lambda_p^2}{1-\lambda_p}\left(\hat{\pi}_{t+1} - \gamma_p\hat{\pi}_t\right) \tag{1.53}$$

money growth rate

$$\mu_t = m_t - m_{t-1} + \pi_t \tag{1.54}$$

Wage inflation

$$\left[\begin{array}{c} \left(\frac{1}{1-\lambda_w}+\beta\frac{\lambda_w^2}{1-\lambda_w}\right)\hat{w}_t-\beta\frac{\lambda_w}{1-\lambda_w}\hat{w}_{t+1}+ \\ -\left(\beta\lambda_w+\beta\frac{\lambda_w^2}{1-\lambda_w}\right)\hat{\pi}_{t+1}+ \\ +\left(\beta\lambda_w\gamma_w+\beta\frac{\lambda_w^2}{1-\lambda_w}\gamma_w+\frac{\lambda_w}{1-\lambda_w}\right)\hat{\pi}_t+ \\ -\frac{\lambda_w}{1-\lambda_w}\hat{w}_{t-1}-\frac{\lambda_w}{1-\lambda_w}\gamma_w\hat{\pi}_{t-1} \end{array}\right]=(1-\beta\lambda_w)\,\varphi\hat{h}_t-(1-\beta\lambda_w)\,\hat{\psi}_t \tag{1.55}$$

1.8. Appendix A.2

1.8.1. Steady State in the benchmark model

Relative to the CEE model, the presence of RT consumers influences the steady state uniquely for what concerns households individual consumption level.

From equation 1.19 and 3.23, and assuming zero inflation steady state, it holds true that

$$R=\frac{1}{\beta} \tag{1.56}$$

$$r^k=\frac{1}{\beta}-1+\delta \tag{1.57}$$

From cost minimization problem come the equations:

$$r^k = mc\,\alpha\left(\frac{k}{h}\right)^{\alpha-1} \tag{1.58}$$

$$w = \frac{mc\,(1-\alpha)\left(\frac{k}{h}\right)^{\alpha}}{R} \tag{1.59}$$

Combining the last two equation we get the real wage computed at steady state

$$mc = \frac{r^k}{\alpha}\left(\frac{k}{h}\right)^{1-\alpha} \tag{1.60}$$

$$w = \frac{r^k}{\alpha}\frac{(1-\alpha)}{(R)}\left(\frac{k}{h}\right) \tag{1.61}$$

Combining (3.38) and $mc = \frac{\eta-1}{\eta}$we get the ratio:

$$\frac{K}{h} = \left(\frac{r^k}{\alpha}\frac{\eta}{\eta-1}\right)^{\frac{1}{\alpha-1}} \tag{1.62}$$

From the production function we get

$$\frac{Y}{h} = \left(\frac{K}{h}\right)^{\alpha} \tag{1.63}$$

and as

$$\frac{I}{Y} = \delta\frac{K}{Y} \tag{1.64}$$

The aggregate resource constraint reads as:

$$Y = C + I \tag{1.65}$$

$$1 = \frac{C}{Y} + \frac{I}{Y} \tag{1.66}$$

the aggregate consumption-output ratio is given by

$$\frac{C}{Y} = 1 - \delta\frac{K}{h}\left(\frac{Y}{h}\right)^{-1} \tag{1.67}$$

The equation for the optimal wage allows us to derive the hours worked at steady state as

$$h = \left[\frac{\alpha_w - 1}{\alpha_w}\left[(1-\theta)\left(\frac{(1-\beta b)}{(1-b)}\frac{c}{C^o}\right) + \theta\left(\frac{(1-\beta b)}{(1-b)}\frac{c}{C^{rt}}\right)\right]\frac{c^{rt}}{c}\right]^{\frac{1}{(\phi_l+1)}}$$

so that

$$K = \frac{K}{h}h \tag{1.68}$$

Since RT individual consumption is given at steady state by

$$c^{rt} = w\ h$$

we can easily derive its relationship with aggregate consumption as

$$\frac{c^{rt}}{C} = \left(\frac{C}{Y}\right)^{-1} w \left(\frac{Y}{h}\right)^{-1} \tag{1.69}$$

Total consumption is the weighted average of the two groups components:

$$C = (1-\theta)\, c^{o} + \theta c^{rt} \tag{1.70}$$

From the latter, it comes straightforward

$$\frac{c^{o}}{C} = \frac{1}{1-\theta} - \frac{\theta}{1-\theta}\frac{c^{rt}}{C} \tag{1.71}$$

Optimizing households consumption at steady state is given by the sum of labour income, firms profits return of capital and returns of money rents to firms:

$$c^{o} = wh + \frac{1}{1-\theta}\left(\Pi + r^{k}K + (R-1)\, wh\right) \tag{1.72}$$

where Π are firms profits and are defined as

$$\Pi = (1 - \frac{mc}{P})y = (1 - \frac{1}{\mu})y \tag{1.73}$$

with μ representing firms markup over prices. Thus optimizing agents are the richer the higher share of RT consumers.

Aggregate consumption can be finally rewritten as

$$C = (1-\theta)\, c^{o} + \theta c^{rt} = wh + \Pi + r^{k}K + (R-1)\, wh \tag{1.74}$$

1.8.2. Steady State in the Simplest Version of the Model.

In case of no cash in advance the steady state is described by:

$$R = \frac{1}{\beta}$$

$$Y = C \tag{1.75}$$

$$Y = h^{1-\alpha} \tag{1.76}$$

$$c^{rt} = wh \tag{1.77}$$

$$\frac{w_t}{(1-\alpha)} h^{\alpha} = mc = \frac{\vartheta - 1}{\vartheta} = \frac{1}{\mu} \tag{1.78}$$

$$\frac{(1-\alpha)}{\mu} h^{-\alpha} = w \tag{1.79}$$

$$\frac{(1-\alpha)}{\mu} h^{1-\alpha} = wh \tag{1.80}$$

$$w = \frac{\frac{\alpha_w}{\alpha_w - 1} h^{\phi_l}}{\left[(1-\theta)\frac{1}{c^o} + \theta \frac{1}{c^{rt}}\right]}$$

$$C = (1-\theta) c^o + \theta c^{rt}$$

$$\frac{wh}{y} = \frac{c^{rt}}{c} = \frac{(1-\alpha)}{\mu} \tag{1.81}$$

$$\frac{c^o}{c} = \frac{1}{1-\theta} - \frac{\theta}{1-\theta} \frac{(1-\alpha)}{\mu}$$

$$\left(\frac{(1-\alpha)}{\mu} \frac{\alpha_w - 1}{\alpha_w} \left[(1-\theta)\frac{c}{c^o} + \theta \frac{c}{c^{rt}}\right]\right)^{\frac{1}{\phi_l + 1}} = h$$

CHAPTER 2

Rule-of-thumb Consumers, Consumption Habits and the Taylor Principle

2.1. Introduction

The standard New-Keynesian framework is characterized by optimizing agents (households and firms), and by a number of nominal and real frictions in goods, labor and financial markets. A remarkable strand of this literature has focused on the properties that simple and implementable interest rate rules must fulfill in order to guarantee the uniqueness of the rational expectations equilibrium and to maximize the social welfare (see Woodford(2003), Schmitt-Grohè, Uribe(2004 and 2007))

Following a seminal contribution by Mankiw (2000), who introduced the notion of heterogeneous consumers (savers and spenders), a second strand of New Keynesian literature emphasizes the role of rule-of-thumb consumers (RT consumers henceforth) which fully consume their current income and do not participate to financial markets (Galì et al. 2004, 2007). De Graeve et al. (2010) introduce RT consumers to model financial risk premia. Empirical research cannot reject the RT consumers hypothesis. Estimated structural equations for consumption growth report a share of RT consumers ranging from 26 to 40% (Jacoviello, 2004; Campbell and Mankiw, 1989) More recent estimates of dynamic stochastic general equilibrium models (Coenen and Straub, 2005; Forni, Monteforte and Sessa, 2009) obtain estimates around 35%. Erceg, Guerrieri and Gust (2006) calibrate the share of RT consumers to 50% in order to replicate the dynamic performance of the Federal Reserve Board Global Model. Critics of the approach might argue that the empirical relevance of RT consumers is bound to gradually decline along with the development of financial markets (Bilbiie, Meier and Müller, 2008). In fact, increasing regulation in the aftermath of the 2008 crisis (OECD 2009) is likely to increase the share of liquidity constrained households.

The RT consumers hypothesis bears important implications for model dynamics. Bilbiie (2008) shows that, in a world of flexible nominal wages, a low elasticity of labor supply combined with a sufficiently large share of non Ricardian agents leads to an equilibrium where an interest rate policy based on the Taylor principle cannot ensure model determinacy. The intuition behind this result is as follows. In standard models based on optimizing consumers, satisfying the Taylor principle generates a substitution effect from current to future consumption that is sufficient to rule out sunspot equilibria. By contrast, RT consumers generate a "Keynesian multiplier" effect on demand shocks that raises profits which are entirely appropriated by Ricardian agents. If the share of RT consumers is sufficiently large, this wealth effect dominates the substitution effect induced by the interest rate rule based on the Taylor principle. As a consequence, the standard monetary policy rule cannot pin down the optimizing consumers' choice to a unique equilibrium path.

Recent contributions downplay this conclusion. Colciago (2007) introduces nominal wage stickiness, finding that even a mild degree of wage stickiness dampens the Keynesian multiplier effect generated by RT consumers and restores the standard Taylor Principle even for a very large share of RT consumers. Ascari et al.(2010) show that the optimal monetary policy is almost unaffected by the presence of RT consumers as long as nominal wages are sticky.

In the paper we reconsider the issue and show that, just like wage stickiness undermines the wealth effect outlined in Bilbiie, other frictions may weaken the substitution effect induced by a policy that follows the Taylor principle. In fact, this happens when consumption habits enter the utility function. In addition, consumption habits affect the marginal rate of substitution between consumption and leisure, leading to a more rigid labor supply curve. Our simulations show that the combination of consumption habits and RT consumers has dramatic implications for model determinacy, resurrecting Bilbiie's inverted Taylor principle.

Another original contribution of the paper is the analysis of optimal operational simple rules when RT households and habit formation in consumption are taken into account. We are able to show that the higher the share of RT consumers the more important for the optimal monetary policy is the stabilization of the wage gap, the variable that drives consumption volatility for RT consumers. The combination of consumption habits and RT consumers affect the dynamic performance of the model under the optimal simple rule. Even a relatively small share of RT consumers is sufficient to generate a substantial increase in volatility. When the share of RT consumers is sufficiently large to require an inversion of the Taylor principle to preserve dynamic stability, optimal monetary policy is forced to generate some "unconventional" impulse-response functions. For instance, a favourable productivity shock is followed by an increase in inflation and by a positive output gap.

The remainder of the paper is organized as follows: In the first section we present and describe the model, in the second section we analyze the model stability properties under different specifications of labor markets. The third and fourth section describe the optimal policy problem and its implications. Section five concludes.

2.2. The Model

We consider a cashless small-scale New Keynesian model augmented for rule-of-thumb (RT) or non Ricardian consumers. We assume a continuum of households indexed by $i \in [0,1]$. As in Galì et al (2004) and (2007), households in the interval $[0,\theta]$ cannot access financial markets. The rest of the interval $(\theta,1]$ is composed by standard Ricardian households who have access to a full set of state contingent securities. The key distinction between the two groups concerns intertemporal optimization. Ricardian consumers' choices take into account future utility when choosing consumption and portfolio composition. Rule-of-Thumb consumers spend their whole income every period, thus they do not hold any wealth.

2.2.1. Households preferences

All households share the same utility function:

$$U_t^i = E_0 \sum_{t=o}^{\infty} \beta^t \left\{ \ln\left(C_t^i - bC_{t-1}^i\right) - \frac{\psi_l}{1+\phi_l} (h_t^i)^{1+\phi_t} \right\} \tag{2.1}$$

where $i : o, rt$ stands for household type, C_t^i represents total individual consumption b denotes consumption internal habits and h_t^i denotes individual labour supply.

2.2.1.1. Consumption Bundles. C_t^i is a standard consumption bundle

$$C_t^i = \left[\int_0^1 c\left(z\right)_t^{\frac{\eta-1}{\eta}} dj \right]^{\frac{\eta}{\eta-1}} \tag{2.2}$$

where η represents the price elasticity of demand for the individual goods.

$$P_t = \left(\int_0^1 p\left(z\right)_t^{1-\eta} dj \right)^{\frac{1}{1-\eta}}$$

is the aggregate consumption price index.

2.2.2. Firms

Goods are indexed by $z \in [0,1]$. Good z is produced by a monopolist with the following technology:

$$y_t\left(z\right) = h_t\left(z\right)$$

Where $h_t\left(z\right)$ is the composite labor input used by each firm z defined as follows:

$$h_t\left(z\right) = \left(\int_0^1 \left(h_t^j\left(z\right)\right)^{\frac{\alpha_w - 1}{\alpha_w}} dj \right)^{\frac{\alpha_w}{\alpha_w - 1}} \tag{2.3}$$

where the parameter $\alpha_w > 1$ is the intratemporal elasticity of substitution between labor inputs. For any given level of its labor demand $h_t(z)$, the optimal allocation of across labor inputs implies

$$h_t^j(z) = \left(\frac{W_t^j}{W_t}\right)^{-\alpha_w} h_t^d(z) \tag{2.4}$$

where $W_t = \left(\int_0^1 \left(W_t^j\right)^{1-\alpha_w} dj\right)^{1/(1-\alpha_w)}$ is the standard wage index. Firm z's nominal total production cost is given by

$$TC_t(z) = W_t h_t(z)$$

The real marginal costs are:

$$mc_t = w_t \tag{2.5}$$

where $w_t = \frac{W_t}{P_t}$ is the real wage and P_t is the consumption price index.

2.2.2.1. Sticky Prices. Price stickiness is based on the Calvo mechanism. In each period firm z faces a probability $1-\lambda_p$ of being able to reoptimize its price. When a firm is not able to reoptimize, it adjusts its price to the previous period inflation, $(1+\pi_{t-1}) = \frac{P_{t-1}}{P_{t-2}}$. The price-setting condition therefore is:

$$p_t(z) = (1+\pi_{t-1})^{\gamma_p} p_{t-1}(z) \tag{2.6}$$

where $\gamma_p \in [0,1]$ represents the degree of price indexation.

All the $1-\lambda_p$ firms which reoptimize their price at time t will face symmetrical conditions and set the same price $\widetilde{P}_t$. When choosing $\widetilde{P}_t$ the optimizing firm will take into account that in the future it might not be able to reoptimize. In this case, the price at the generic period $t+s$ will read as $\widetilde{P}_t \Pi_{t,t+s-1}^{\gamma_p}$ where $\Pi_{t,t+s-1} = (1+\pi_t)\ldots(1+\pi_{t+s-1}) = \frac{P_{t+s-1}}{P_{t-1}}$.

$\widetilde{P}_t$ is chosen so as to maximize a discounted sum of expected future profits:

$$E_t \sum_{s=0}^{\infty} (\beta\lambda_p)^s \lambda_{t+s} \left(\widetilde{P}_t \Pi_{t,t+s-1}^{\gamma_p} - P_{t+s} mc_{t+s} \right) y_{t+s}(z)$$

subject to:

$$y_{t+s}(z) = Y_{t+s}^d \left(\frac{\widetilde{P}_t \prod_{t,t+s-1}^{\gamma_p}}{P_{t+s}} \right)^{-\eta} \tag{2.7}$$

where Y_t^d is aggregate demand and λ_t is the stochastic discount factor.

The F.O.C. for this problem is

$$E_t \sum_{s=0}^{\infty} (\beta\lambda_p)^s \lambda_{t+s} Y_{t+s}^d \left[\begin{array}{c} (1-\eta) \left(\Pi_{t,t+s-1}^{\gamma_p} \right)^{1-\eta} \widetilde{P}_t^{-\eta} (P_{t+s})^{\eta} + \\ +\eta \widetilde{P}_t^{-\eta-1} P_{t+s}^{\eta+1} mc_{t+s} \left(\Pi_{t,t+s-1}^{\gamma_p} \right)^{-\eta} \end{array} \right] = 0 \tag{2.8}$$

2.2.3. Ricardian Households

Ricardian households maximize 2.1 subject to the following period budget constraint:

$$P_{t+1} B_{t+1} = R_t B_t + P_t A_{j,t} + P_t D_t - P_t C_t + h_t^d \int_0^1 W_t^j \left(\frac{W_t^j}{W_t} \right)^{-\alpha_w} dj \tag{2.9}$$

Where B_t is a riskless bond, $A_{j,t}$ and D_t are respectively the net cash flow from participating in state-contingent securities at time t and firm dividends.

The solution for the optimizing household problem is standard. The Euler equation is

$$\lambda_t^o = \beta E_t \lambda_{t+1}^o \frac{R_t}{\pi_{t+1}} \tag{2.10}$$

where

$$\frac{1}{C_t^o - bC_{t-1}^o} - \frac{\beta b}{C_{t+1}^o - bC_t^o} = \lambda_t^o \tag{2.11}$$

2.2.4. Rule-of-Thumb Households

As pointed out above, RT consumers neither save or borrow, in each period they entirely consume their labor income.

$$C_t^{rt} = h_t^d \int_0^1 \frac{W_t^j}{P_t} \left(\frac{W_t^j}{W_t} \right)^{-\alpha_w} dj \tag{2.12}$$

2.2.5. Labor market

There is a continuum of differentiated labor inputs indexed by $j \in [0,1]$. Each labor market j is monopolistically competitive and there is a union j which sets the nominal wage, W_t^j, subject to (2.4). Each household i supplies all labour types at the given wage rates [1] and the total number of hours allocated to the different labor markets must satisfy the time resource constraint

$$h_t^i = \int_0^1 h_t^j dj = \int_0^1 \left(\frac{W_t^j}{W_t} \right)^{-\alpha_w} h_t^d dj \tag{2.13}$$

As in Galì (2007), we assume that the fraction of RT and Ricardian consumers is uniformly distributed across unions, and demand for each labour type is uniformly distributed across households. Ricardian and non-Ricardian households therefore work for the same amount of time, h_t. Individual labor income is

$$h_t^d W_t = \int_0^1 W_t^j \left(\frac{W_t^j}{W_t} \right)^{-\alpha_w} h_t^d dj \tag{2.14}$$

We posit that the union objective function is a weighted average $(1-\theta, \theta)$ of the utility functions of the two households types. This, in turn, implies that with flexible wages

[1] Under the assumption that wages always remain above all households' marginal rate of substitution, households are willing to meet firms' labour demand.

where $\frac{\alpha_w}{(\alpha_w - 1)}$ represents the

$$w_t = \frac{W_t}{P_t} = \frac{\alpha_w}{\alpha_w - 1} \frac{\psi_l h_t^{\phi_l}}{\left[(1-\theta) U'\left(C_t^o - bC_{t-1}^o\right) + \theta U'\left(C_t^{rt} - bC_{t-1}^{rt}\right)\right]} \tag{2.15}$$

wage markup over the average marginal rate of substitution.

Determinacy analysis in section (2.3) below will take perfect competition in the labor market as a benchmark. In that case the individual labor supplies of the two groups will differ:

$$w_t = \frac{\psi_l \left(h_t^i\right)^{\phi_l}}{\left[U'\left(C_t^i - bC_{t-1}^i\right)\right]} \tag{2.16}$$

Note that when habits are absent, $b = 0$, the labour supply of RT consumers is constant: $h_t^{rt} = \psi_l^{-\frac{1}{1+\psi_l}}$.

2.2.5.1. Sticky wages. In each period a union faces a constant probability $1 - \lambda_w$ of being able to reoptimize the nominal wage. Unions that cannot reoptimize simply index their wages to lagged inflation:

$$W_t^j = W_{t-1}^j \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_w} = W_{t-1}^j \left(\pi_{t-1}\right)^{\gamma_w}$$

where γ_w stands for the degree of wage indexation. Just like firms, when choosing the current wage, $\widetilde{W}_t$, the optimizing union will anticipate that in the future it might not be able to reoptimize. In this case, the wage at the generic period $t+s$ will read as (in real terms)

$$w_{t+s} = \widetilde{w}_t \prod_{k=1}^{s} \frac{\pi_{t+k-1}^{\gamma_w}}{\pi_{t+k}} \tag{2.17}$$

Following Colciago(2008), the representative union objective function is defined as

$$L^u = \sum_{s=0}^{\infty} (\beta\lambda_w)^s \left\{ \left[(1-\theta)\, U^o(C^o_{t+s}) + \theta U^{rt}(C^{rt}_{t+s}) \right] - U(h_{t+s}) \right\} \tag{2.18}$$

Where U^o_s, U^{rt}_s are defined as in (2.1). Thus the wage-setting decision maximizes a weighted average of the two household types conditional to the probability that the wage cannot be reoptimized in the future. The relevant constraints are (3.14), (2.9), (3.26), (3.27).

The union's first-order condition is:

$$\begin{aligned} &\sum_{s=0}^{\infty} (\beta\lambda_w)^s \left[(1-\theta)\, \lambda^o_{t+s} + \theta\lambda^{rt}_{t+s} \right] h^d_{t+s} \left(w_{t+s}\right)^{\alpha_w} \left(\prod_{k=1}^{s} \frac{\pi^{\gamma_w}_{t+k-1}}{\pi_{t+k}} \right)^{-\alpha_w} \cdot \\ &\cdot \left[\widetilde{w}_t \left(\prod_{k=1}^{s} \frac{\pi^{\gamma_w}_{t+k-1}}{\pi_{t+k}} \right) - \frac{\alpha_w}{(\alpha_w - 1)} \frac{\psi_l h^{\phi_l}_{t+s}}{\left[(1-\theta)\, \lambda^o_{t+s} + \theta\lambda^{rt}_{t+s} \right]} \right] = 0 \end{aligned} \tag{2.19}$$

where $\lambda^{rt}_{t+s} = \frac{1}{C^o_t - bC^o_{t-1}} - \frac{\beta b}{C^o_{t+1} - bC^o_t}$. It is worth noting that the combination of centralized wage setting and wage stickiness introduces an indirect form of consumption smoothing for RT consumers.

2.2.6. Monetary Policy

We assume a monetary authority follows a rule of the type:

$$r_t = (\pi_t)^{\varphi_\pi} \tag{2.20}$$

where $r_t = R_t - 1$ is the net nominal interest rate

2.2.7. Aggregation

Aggregate consumption C_t is a weighted average of the respective variable for each household type, thus

$$C_t = \int_0^1 C_t^i(j)\, dj = \int_0^\theta C_t^{rt}(j)\, dj + \int_\theta^1 C_t^o(j)\, dj = \theta C_t^{rt} + (1-\theta) C_t^o \tag{2.21}$$

Aggregating budget constraints for each sector, after few manipulations we get the aggregate resource constraint as

$$Y_t = C_t$$

2.2.8. Steady State

As in Ascari et al.(2010) and Bilbiie (2008), we need to make the assumption of an efficient steady state in order to study the welfare properties of the economy represented by this model.

We therefore assume that at the steady state firms are taxed by the Government by a constant employment tax, τ, and then receive the money back through lump-sum transfer, $T = \tau \frac{W}{P} h$. In this case steady-state firms profits are:

$$D = Y - (1-\tau)\, h \frac{W}{P} - T$$

The efficient steady state is characterized by perfect competition and zero profits. If this is the case, it follows that $C^{rt} = C^o = C$ and all households have the same marginal rate of substitution between labour and consumption (MRS). Under the above assumption, the equilibrium wage at the steady state is give by

$$w = \frac{1}{(1-\tau)\left(1+\mu_p\right)} MPL = (1+\mu_w)\, MRS$$

where $\mu_w = \frac{\alpha_w}{\alpha_w - 1}$ and $\mu_p = \frac{\eta}{\eta - 1}$ are the markups in labour and good markets respectively.

Since $MPL = MRS = 1$ must hold at the efficient steady state, we need that

$$\tau = 1 - \frac{1}{\left(1+\mu_p\right)(1+\mu_w)}$$

The resulting value of τ will lead to zero steady state profits and to equilibria in goods and labour markets equivalent to those under perfect competition in both markets.

2.3. Stability Analysis

Given the model size, determinacy analysis requires numerical methods.

Parameters are calibrated following Christiano et al. (2005), technology process is modeled as in Schmitt-Grohe, Uribe(2007):

TABLE 1		
Parameter	Value	Description
b	0.7	degree of habit persistence
β	0.99	subjective discount factor
λ_p	0.6	price stickiness
λ_w	0.64	wage stickiness
γ_p	1	indexation on prices
γ_w	1	indexation on wages
φ_l	1	preference parameter
$\frac{\eta}{(\eta-1)}$	1.2	price mark-up
$\frac{\alpha_w}{(\alpha_w-1)}$	1.2	wage mark-up
ρ_a	0.8556	shock persistence
σ_a	1	shock std. deviation

Our model encompasses previous contributions that investigated the impact of RT consumers on the effectiveness of the Taylor principle, $\varphi_\pi > 1$ in (2.20), under different labor market structures. To facilitate comparison we first discuss the case of a perfectly competitive labor market, as in Bilbiie (2008). Then we introduce monopolistic competition under flexible wages. Finally, we consider the sticky-wage models of Colciago(2006) and Ascari, Colciago, Rossi (2010).

2.3.1. Competitive labor market

The white areas in Panel a of Figure 1 define the determinacy regions that obtains for different combinations of θ, φ_π when the labor market is competitive and consumption habits are absent. If the share of RT consumers exceeds a threshold value $\theta = 0.48$ determinacy requires an inversion of the Taylor principle: $\varphi_\pi < 1$. This broadly coincides with Bilbiie (2008) who has shown that, for a sufficiently large share of RT consumers the Taylor principle cannot rule out sunspot equilibria. The intuition behind this result is as follows. Suppose that firms form an arbitrary expectation of future price increases and therefore choose to raise the current price. The simultaneous (real) interest rate response induces a substitution effect in the consumption decisions of Ricardian households: C_t^o is such that $E_t\left\{\Delta C_{t+1}^o\right\} > 0$ (see equations 3.20, 2.11 for $b = 0$). If all consumers were ricardian, this would allow a unique $\Delta C_t^o < 0$ consistent with convergence to steady state, thus generating in t a negative output gap sufficient to rule out the initial price increase as a possible equilibrium. By contrast, in this model ricardian agents can react to the real interest rate surge by choosing $\Delta C_t^o > 0$, because RT consumers induce a "Keynesian multiplier" effect that raises profits which are entirely appropriated by ricardian agents. If this wealth effect is sufficiently strong, i.e. the share of RT consumers is sufficiently large, the choice of C_t^o such that $E_t\left\{\Delta C_{t+1}^o\right\} > 0, \Delta C_t^o > 0$ may be consistent with the rational expectation of future return to steady state. In this case C_t^o confirms the increases in current and expected inflation.

In Panel b of Figure 1 we show that determinacy regions remain almost identical when habits affect consumption utility. In fact habits substantially modify both the substitution and the wealth effects discussed above. To understand this, look at the log-linearized versions of conditions 3.20, 2.11 and of 2.16 subject to 3.26.

$$\left(\frac{1+b+\beta b^2}{(1-\beta b)(1-b)}\right) c_t^o = \left\{ \begin{array}{c} \left(\frac{b}{(1-\beta b)(1-b)}\right) c_{t-1}^o - \left(\frac{\beta b}{(1-\beta b)(1-b)}\right) c_{t+2}^o + \\ + \left(\frac{1+\beta b+\beta b^2}{(1-\beta b)(1-b)}\right) c_{t+1}^o + \hat{\pi}_{t+1}^e - \hat{R}_t \end{array} \right\} \tag{2.22}$$

$$\hat{h}_t^{rt} = \frac{\left\{ \begin{array}{c} \frac{\beta b}{(1-\beta b)(1-b)} c_{t+1}^{rt} - \left[\frac{\left(1+\beta b^2\right)}{(1-\beta b)(1-b)} - 1 \right] \hat{w}_t + \\ + \frac{b}{(1-\beta b)(1-b)} c_{t-1}^{rt} \end{array} \right\}}{\phi_h + \frac{(1+\beta b^2)}{(1-\beta b)(1-b)}} \tag{2.23}$$

From (2.22) it is easy to see that consumption habits reduce the sensitivity of ricardian consumers to real interest rate changes, weakening the substitution effect that is crucial to obtain determinacy under the Taylor principle. Equation 2.23 shows instead that habits weaken the wealth effect induced by RT consumers' choices. In fact, when $b = 0$ the labour supply of RT consumers is constant and their consumption decisions are driven by the wage rate which increases if $\Delta c_t^o > 0$. If $b > 0$, then $\hat{h}_t^{rt}$ negatively correlates with the wage rate. This happens because habits induce RT consumers to behave in a forward-looking manner, taking into account that an increase in their current income will also raise next-period habits with adverse effects on future utility. Consumption habits therefore reverse the standard labor supply reaction to a wage rate increase when consumers are non-Ricardian. As a result the Keynesian multiplier effect generated by RT consumers is now weaker.

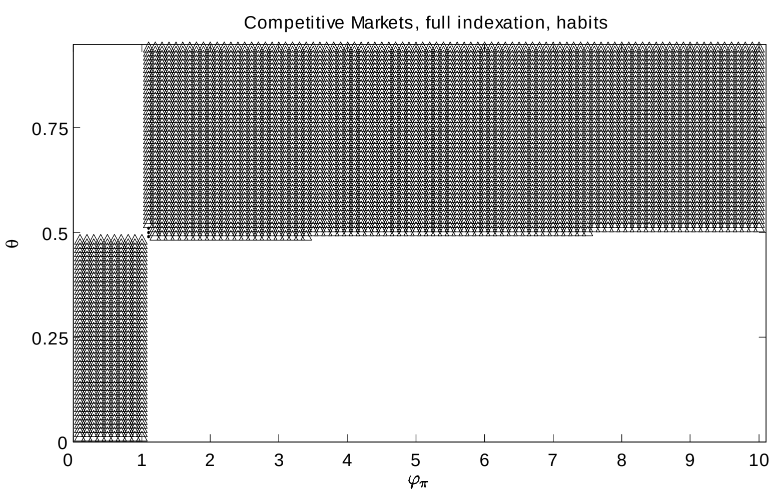

Figure 1a: Determinacy Area

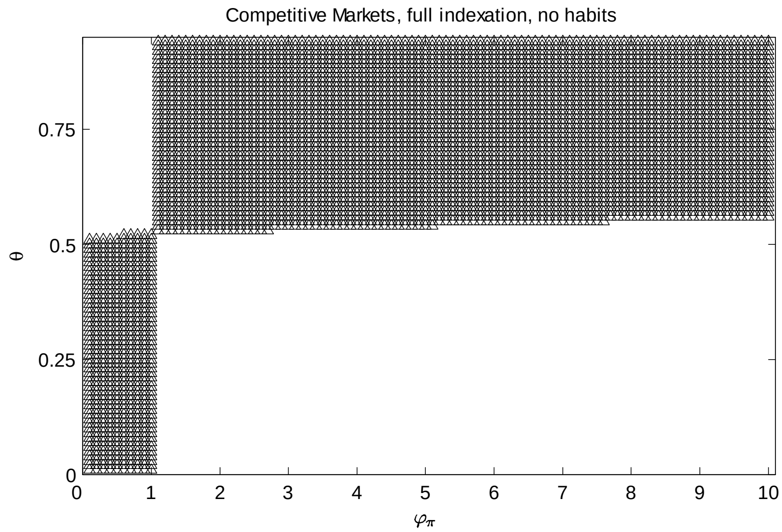

Figure 1b: Determinacy Area

2.3.2. Monopolistic wage setters

Consider now a monopolistically competitive labour market. Condition (2.15) characterizes labor market equilibrium if nominal wages are flexible. Determinacy regions for $b = 0$, $b > 0$ are reported in Figure 2, panels a and b respectively . From a comparison between panels a of Figures 1 and 2 we see that, relative to the case of no habits and perfectly competitive labor market, monopolistic competition lowers the share of RT consumers that requires an inversion of the Taylor principle. This happens because the labor supplies of the two households groups coincide and consumption choices of ricardian households directly affect the labor supply of RT consumers. As a result, the Keynesian multiplier effect induced by RT consumers is unambiguously stronger than under perfect competition

$$h_t = h_t^{rt} = \frac{\left\{ \begin{array}{c} \left(1 - \frac{\theta\left(1+\beta b^2\right)}{(1-\beta b)(1-b)}\right) w_t - \frac{(1-\theta)\left(1+\beta b^2\right)}{(1-\beta b)(1-b)} c_t^o + \frac{(1-\theta)\beta b}{(1-\beta b)(1-b)} c_{t+1}^o + \\ + \frac{\theta \beta b}{(1-\beta b)(1-b)} c_{t+1}^{rt} + \frac{(1-\theta) b}{(1-\beta b)(1-b)} c_{t-1}^o + \frac{\theta b}{(1-\beta b)(1-b)} c_{t-1}^{rt} \end{array} \right\}}{\left(\phi_h + \frac{\theta(1+\beta b^2)}{(1-\beta b)(1-b)}\right)} \tag{2.24}$$

given the expectations

$$w_t = \left(\varphi_h + \frac{(1+\beta b^2)}{(1-\beta b)(1-b)}\right) y_t$$

Introducing habits in the monopolistic competition model dramatically lowers the threshold value of θ that triggers an inversion of the Taylor principle. Relative to the perfect competition-cum-habit case, this happens because habits weaken the substitution effect triggered by real interest rate changes, but no longer induces the negative response of RT labor supply to a real wage increase.

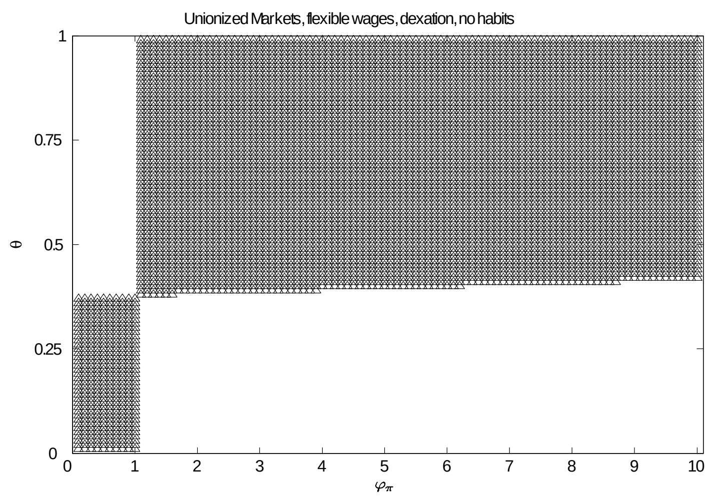

Figure 2b: Determinacy Area

2.3.2.1. Sticky wages. Colciago (2006) and Ascari, Colciago, Rossi (2010) show that, in a model without habits, wage stickiness is enough to wipe out the wealth effect identified in Bilbiie (2008), thus restoring the effectiveness of the Taylor principle (Figure 3, panel a). The intuition behind this result is very simple. Sticky wages dampen the real wage response to an aggregate demand increase and unambiguously limit the Keynesian multiplier effect of RT consumers. Panel b of Figure 3 shows that wage stickiness plays a much lesser role once consumption habits are introduced. Under our parameter calibrations, determinacy requires an inversion of the Taylor principle when the share of Rule-of-thumb consumers reaches 42%. As pointed out above, habits play their crucial role by weakening the substitution effect associated to real interest rate movements.

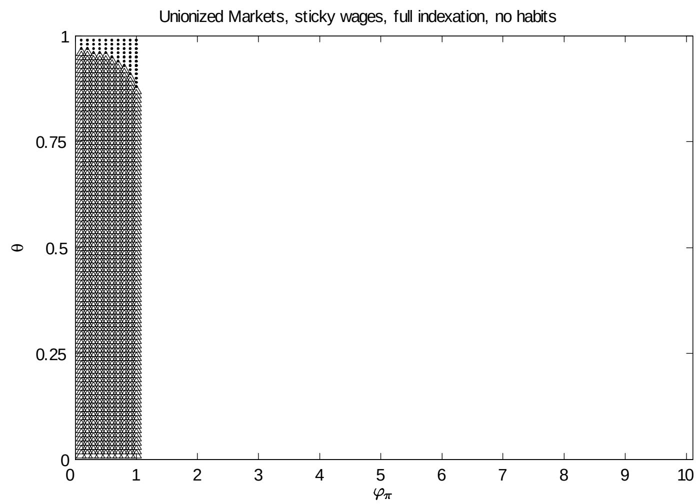

Figure 3a: Determinacy Area

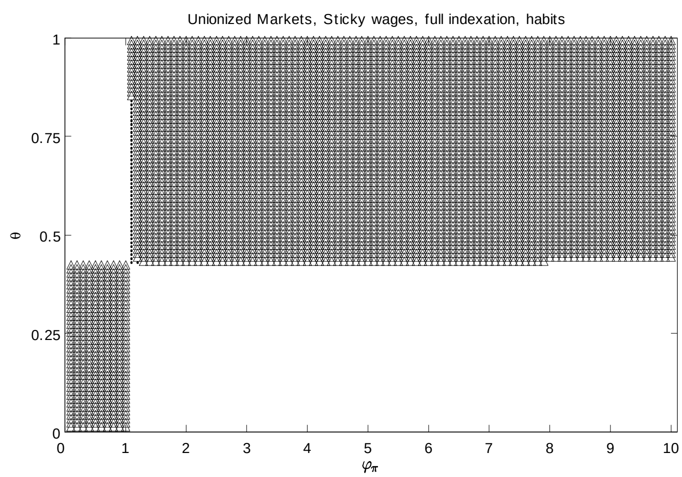

Figure 3b: Determinacy Area

2.4. Optimal Simple Implementable Rule

In this section we turn to the analysis of the optimal simple monetary policy rule as the one in (2.20) , given the determinacy constraints of the model. Our interest here is to identify a policy space that minimizes deviations from socially efficient outcomes. To this end we first identify the solution to the social planner problem.

2.4.1. Social Planner Problem

It should be noted from the outset that the two household groups have symmetrical preferences, but have different access to financial markets. As a result, from the social planner perspective, the consumption and worked hours responses to shocks should be identical for the two groups. In addition, the social planner faces an intertemporal problem due to internal habit formation.

The social planner problem can be summarized as:

$$\max_{c_t^o, c_t^{rt}, h_t^o, h_t^{rt}} E_t \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{c} \theta \left(\log \left(c_t^{rt} - b c_{t-1}^{rt} \right) - \frac{\psi_l}{1+\phi_l} \left(h_t^{rt} \right)^{1+\phi_l} \right) + \\ + (1-\theta) \left(\log \left(c_t^{o} - b c_{t-1}^{o} \right) - \frac{\psi_l}{1+\phi_l} \left(h_t^{o} \right)^{1+\phi_l} \right) \end{array} \right]$$

subject to the following constraints which represent the composition of aggregate consumption and labour supply, the aggregate resource constraint in which in equilibrium total output must be equal to total consumption and the firms' production function:

$$\theta c_t^{rt} + (1-\theta) c_t^o = c_t$$

$$\theta h_t^{rt} + (1-\theta) h_t^o = h_t$$

$$y_t = c_t$$

$$y_t = a_t h_t$$

The resulting Lagrangian is given by:

$$\max_{c_t, h_t} \mathcal{L} = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \theta \left(\log \left(c_t^{rt} - b c_{t-1}^{rt} \right) - \frac{\psi_l}{1+\phi_l} \left(h_t^{rt} \right)^{1+\phi_l} \right) + (1-\theta) \left(\log \left(c_t^{o} - b c_{t-1}^{o} \right) - \frac{\psi_l}{1+\phi_l} \left(h_t^{o} \right)^{1+\phi_l} \right) \\ -\lambda_t \left[\theta c_t^{rt} + (1-\theta) c_t^o - a_t \left(\theta h_t^{rt} + (1-\theta) h_t^o \right) \right] \end{array} \right.$$

The first order conditions to the social planner optimization problem are the following

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c_t^{rt}} &= 0 : \theta \frac{1}{\left(c_t^{rt} - b c_{t-1}^{rt} \right)} - \theta \frac{\beta b}{\left(c_{t+1}^{rt} - b c_t^{rt} \right)} = \lambda_t \theta \\ \frac{\partial \mathcal{L}}{\partial c_t^{o}} &= 0 : (1-\theta) \frac{1}{\left(c_t^{o} - b c_{t-1}^{o} \right)} - (1-\theta) \frac{\beta b}{\left(c_{t+1}^{o} - b c_t^{o} \right)} = \lambda_t (1-\theta) \\ \frac{\partial \mathcal{L}}{\partial h_t^{rt}} &= 0 : \psi \left(h_t^{rt} \right)^{\phi_l} = \lambda_t a_t \\ \frac{\partial \mathcal{L}}{\partial h_t^{o}} &= 0 : \psi \left(h_t^{o} \right)^{\phi_l} = \lambda_t a_t \end{aligned}$$

which imply that $h_t^{rt} = h_t^o$.

In loglinear terms:

$$\frac{\beta b}{(1-\beta b)(1-b)} c_{t+1}^{rt} - \frac{(1+\beta b^2)}{(1-\beta b)(1-b)} c_t^{rt} + \frac{b}{(1-\beta b)(1-b)} c_{t-1}^{rt} = \lambda_t$$

$$\frac{\beta b}{(1-\beta b)(1-b)} c_{t+1}^{o} - \frac{(1+\beta b^2)}{(1-\beta b)(1-b)} c_t^{o} + \frac{b}{(1-\beta b)(1-b)} c_{t-1}^{o} = \lambda_t$$

$$\lambda_t = \phi h_t - a_t$$

It is easy to demonstrate that $c_t^o = c_t^{rt} = c_t = y_t$ for every $b < 1$[2].

We therefore obtain:

$$\left(\phi + \frac{(1+\beta b^2)}{(1-\beta b)(1-b)}\right) y_t^* = \frac{\beta b}{(1-\beta b)(1-b)} y_{t+1}^* + \frac{b}{(1-\beta b)(1-b)} y_{t-1}^* + (\phi+1)\, a_t$$

The efficient level of output y_t^* which would have been set by a benevolent social planner is therefore the result of an intertemporal choice, it depends on past and future level of output and it is a decreasing function of habit persistence (the more we consume today, the less utility we will have tomorrow) and a function of the technological process.

The social planner finally set the efficient wage equal to the marginal productivity of labour,i.e.

$$w_t^* = a_t \tag{2.25}$$

In figure 4 we show the efficient output dynamics in response to a technology shock. The "hump-shaped" response is due to the habit formation in households' utility function. Since the RT consumers have the same preferences as optimizing households, their presence does not affect the social planner optimal behavior.

[2] This result is not surprising given the nature of the social planner problem and the identical preference scheme for the two agents' types.

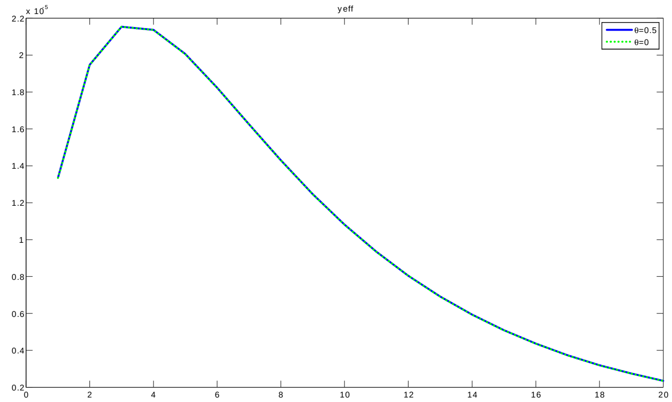

Figure 4: Social Planner Response to a Technology Shock

2.4.2. The central bank welfare function

We assume, as in Bilbiie(2008) and Ascari et al. (2010), that the central bank maximizes an average of the two groups of households utility functions weighted for their relative size. The period welfare function is therefore given by:

$$W_t = \theta \left[U\left(x_t^{rt}\right) - V(h_t^{rt})\right] + (1-\theta)\left[U\left(x_t^{o}\right) - V(h_t^{o})\right] \tag{2.26}$$

where $x_t^i = c_t^i - bc_{t-1}^i$. Moreover, given the unionized structure of the labour market, we have that $h_t^o = h_t^{rt} = h_t$ and the welfare function reads as

$$W_t = \theta U\left(x_t^{rt}\right) + (1-\theta)\, U(x_t^{o}) - V(h_t) \tag{2.27}$$

We derive the central bank loss function as a second order approximation to (2.27) around the efficient steady state. For sake of simplicity, we restrict our analysis to the no indexation case, i.e. $\gamma_p = \gamma_w = 0$.[3]. The derived loss function takes the following form[4]:

(2.28)

$$\mathcal{L} = -\frac{1}{2}\frac{(1-\beta b)}{(1-b)}\sum_{t=0}^{\infty}\beta^t \begin{bmatrix} (1-\theta)\frac{(1-b)}{(1-\beta b)}\left(x_t^o\right)^2 + \theta\frac{(1-b)}{(1-\beta b)}\left(x_t^{rt}\right)^2 + \\ +\phi y_t^2 + \frac{\alpha_w}{\kappa_w}\left(\pi_t^w\right)^2 + \frac{\eta}{\kappa_p}\left(\pi_t\right)^2 - 2\left(1+\phi\right)y_t a_t \end{bmatrix} + t.i.p. + O\left(||\xi||^3\right)$$

[3]Simulations show that indexation plays no role in determining the optimal policy Proof available upon request.

[4]Derivations are available in appendix

where $\kappa_p = \frac{(1-\beta\lambda_p)(1-\lambda_p)}{\lambda_p}$, $\kappa_w = \frac{(1-\beta\lambda_w)(1-\lambda_w)}{\lambda_w}$ and π_t^w represent the real wage inflation. All the variables in (2.28) represent deviations from the efficient steady state. Rewriting the loss function in terms of deviation of the variables from the efficient levels resulting from the social planner solution it yields

(2.29)

$$\mathcal{L} = -\frac{(1-\beta b)}{(1-b)}\frac{1}{2}\sum_{t=0}^{\infty}\beta^t \left[\begin{array}{c} \left(\phi + \frac{1}{(1-\beta b)(1-b)}\right)\left(y_t^{gap}\right)^2 + \left(2\phi + \frac{2}{(1-\beta b)(1-b)}\right) y_t^{gap} y_t^* + \\ + \frac{1}{(1-\beta b)(1-b)}\frac{\theta}{(1-\theta)}\left(w_t^{gap}\right)^2 + \frac{b^2}{(1-\beta b)(1-b)}\left(y_{t-1}^{gap}\right)^2 + \\ + \frac{2b^2}{(1-\beta b)(1-b)} y_{t-1}^{gap} y_{t-1}^* + \frac{b^2}{(1-\beta b)(1-b)}\frac{\theta}{(1-\theta)}\left(w_{t-1}^{gap}\right)^2 + \\ - \frac{2b}{(1-\beta b)(1-b)} y_t^{gap} y_{t-1}^{gap} - \frac{2b}{(1-\beta b)(1-b)} y_t^{gap} y_{t-1}^* + \\ - \frac{2b}{(1-\beta b)(1-b)} y_t^* y_{t-1}^{gap} - \frac{2b}{(1-\beta b)(1-b)}\frac{\theta}{(1-\theta)} w_t^{gap} w_{t-1}^{gap} + \\ + \frac{\alpha_w}{\kappa_w}\left(\pi_t^w\right)^2 + \frac{\eta}{\kappa_p}\left(\pi_t\right)^2 - 2\left(1+\phi\right) a_t y_t^{gap} \end{array}\right] + tip$$

The central banker problem consists in finding the interest rate response to inflation which minimizes the welfare loss function subject to the behavior of households, firms and social planner.

We study the optimal responses to a technology shock a_t searching for the coefficient on inflation which minimizes function (2.29) in the interval [-5, 5][5].

$$a_t = \rho_a a_{t-1} + \varepsilon_t$$

It is worth to notice that in (2.29) the higher the share of RT consumers the more important is the wage gap stabilization for the optimal monetary policy. When $\theta = 0$, RT consumers do not matter, (2.29) real wage gap stabilization is not an objective. The reason why wage gap stabilization is so important is that this variable drives consumption volatility for RT consumers.

[5]The restriction on the interval [-5,5] is driven by the idea that rules characterized by stronger interest rate reaction to changes in inflation are unlikely to be implemented in practice (see for further examples Ascari et al.(2010) and Schmitt-Grohe, Uribe(2004,2007) .

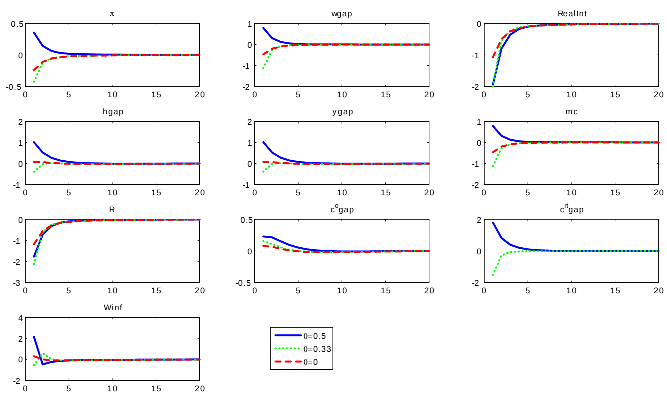

Figure 5: Responses to a Technology Shock

Figure 5 displays the dynamic responses to a technology shock when the optimal policy is implemented. Gap variables represent the deviations between the variable responses and the efficient responses resulting from the welfare optimization problem of a benevolent social planner, in a non-distorted economy.

It is clear that the presence of RT consumers in the economy significantly affects the dynamic responses to a technology shock. When only optimizing agents are present in the model (red dashed lines), the response of both nominal and real interest rate allows the policy maker to minimize both price and wage dispersion ensuring a volatility in output and consumption close to zero. Introducing a small share of RT consumers (green dotted line) which is still compatible with the Taylor principle, we observe that the fall in wage bills affect RT consumption which decreases together with the output gap. This, in turn, lowers inflation. The central banker will therefore decrease the interest rate in order to dampen inflation volatility.

Things change when we allow for a share of RT consumer which is big enough to require an inverted Taylor principle. Now, in order to obtain the dynamic stability of the economy, the central banker does not try to contract the consumption of optimizing agents when inflation increases. The Keynesian multiplier effect generated by RT consumers weakens the central banker ability to stabilize the economy. The system is characterized by a positive output gap and by an increased gap in both RT and Ricardians' consumption.

This happens because the real interest rate still responds negatively to the shock, due to the inversion of the Taylor principle. As discussed in section 3.2, habit formation in a unionized labour market dramatically increases the wage elasticity to output movements. The increase in output generated by a positive productivity shock, increases labour demand and wages. The latter responds more strongly when habits are allowed and pushes up RT consumption generating a multiplicative effect on output. The output gap is therefore markedly higher. Notice that dynamics of the real interest rate under the inverted Taylor principle is quite similar to the one characterizing the economy where the share of RT is small enough to guarantee stability under the Taylor principle.

The importance of habit persistence in magnifying the response of RT consumers and therefore the implemented monetary policy is visible by contrast in figure 6. Here we display the model's responses to the same technology shock when habit formation is not present in the households utility function. As in Ascari et al., RT consumers no longer play a significant role in determining the economy's optimal response.

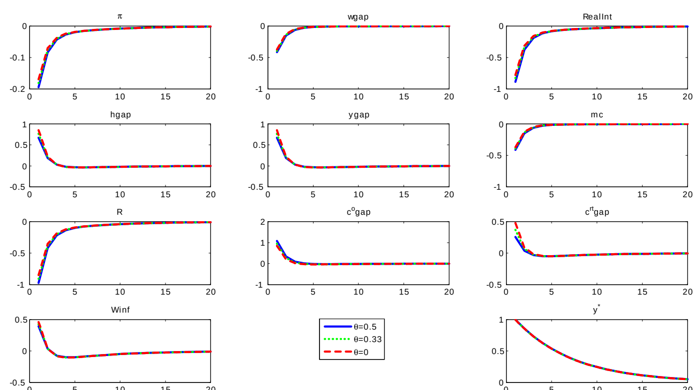

Figure 6: Responses to a Technology Shock

2.5. Conclusion

We have studied the interactions between consumption habits and RT households for what concerns both the stability of a New-Keynesian model and the optimal setup of a simple and implementable monetary rule. It emerged that when habits are taken into account, the presence of a share of financially constrained consumers cannot be ignored by the policy maker. A sufficiently large share of RT consumers requires an inversion of the Taylor principle. In addition, RT consumers affect the dynamic performance of the model under the optimal monetary policy even when the share of RT consumers is limited.

Further research will focus on a deeper analysis of the optimal policy. We are going to check the robustness of the results to different policy rules. We will also investigate how fiscal policy may contribute to stabilization. Finallly, our analysis will be extended to a medium scale new-Keynesian model accounting for capital accumulation and additional real rigidities, as in Christiano et al. (2005).

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2.6. Appendix

2.6.1. Loglinearized equilibrium conditions

2.6.1.1. Unionized Labour Market. The stability analysis is implemented using a linearized version of the model presented above. Lower case letters from now on denote the log of the corresponding variable or their log deviations from the steady state.

Aggregate consumption is defined by:

$$\hat{c}_t = (1-\theta)\frac{c^o}{c}\hat{c}_t^o + \theta\frac{c^{rt}}{c}\hat{c}_t^{rt} \tag{2.30}$$

Marginal costs are given by

$$\widehat{mc}_t = \hat{w}_t \tag{2.31}$$

Production function is given by

$$\hat{y}_t = \hat{h}_t \tag{2.32}$$

Aggregate resource constraint

$$\hat{y} = \hat{c}_t \tag{2.33}$$

RT consumption

$$\hat{c}_t^{rt} = \hat{w}_t + \hat{h}_t \tag{2.34}$$

Euler equation

$$\hat{\lambda}_t^o = \hat{\lambda}_{t+1}^o + \hat{R}_t - \hat{\pi}_{t+1} \tag{2.35}$$

Households marginal utility of consumption

$$\lambda_t^o = \frac{\beta b}{(1-\beta b)(1-b)} c_{t+1}^o - \frac{(1+\beta b^2)}{(1-\beta b)(1-b)} c_t^o + \frac{b}{(1-\beta b)(1-b)} c_{t-1}^o \tag{2.36}$$

$$\lambda_t^{rt} = \frac{\beta b}{(1-\beta b)(1-b)} c_{t+1}^{rt} - \frac{(1+\beta b^2)}{(1-\beta b)(1-b)} c_t^{rt} + \frac{b}{(1-\beta b)(1-b)} c_{t-1}^{rt} \tag{2.37}$$

Phillips Curve

$$\frac{\varepsilon_p}{1-\varepsilon_p}\left(\hat{\pi}_t - \gamma_p \hat{\pi}_{t-1}\right) = (1-\beta\varepsilon_p)\,\widehat{mc}_t + \beta\varepsilon_p\left(\hat{\pi}_{t+1} - \gamma_p\hat{\pi}_t\right) + \beta\frac{\varepsilon_p^2}{1-\varepsilon_p}\left(\hat{\pi}_{t+1} - \gamma_p\hat{\pi}_t\right) \tag{2.38}$$

Taylor Rule

$$R_t = \varphi_\pi \pi_t + \varphi_y y_t \tag{2.39}$$

Wage Setting

$$\begin{bmatrix} \left(\frac{1}{1-\varepsilon_w} + \beta\frac{\varepsilon_w^2}{1-\varepsilon_w}\right)\hat{w}_t - \beta\frac{\varepsilon_w}{1-\varepsilon_w}\hat{w}_{t+1} + \\ -\left(\beta\varepsilon_w + \beta\frac{\varepsilon_w^2}{1-\varepsilon_w}\right)\hat{\pi}_{t+1} + \\ +\left(\beta\varepsilon_w\gamma_w + \beta\frac{\varepsilon_w^2}{1-\varepsilon_w}\gamma_w + \frac{\varepsilon_w}{1-\varepsilon_w}\right)\hat{\pi}_t + \\ -\frac{\varepsilon_w}{1-\varepsilon_w}\hat{w}_{t-1} - \frac{\varepsilon_w}{1-\varepsilon_w}\gamma_w\hat{\pi}_{t-1} \end{bmatrix} = (1-\beta\varepsilon_w)\,\varphi\hat{h}_t - (1-\beta\varepsilon_w)\,\hat{\psi}_t \tag{2.40}$$

2.6.1.2. Competitive Labour Market. Under perfectly competitive markets, $\hat{h}_t \neq \hat{h}_t^o \neq h_t^{rt}$ and in detail

$$\hat{h}_t = \theta\frac{h^{rt}}{h}\hat{h}_t^{rt} + (1-\theta)\frac{h^o}{h}\hat{h}_t^o$$

where

$$\phi_h \hat{h}_t^o = \lambda_t^o + \hat{w}_t$$

$$\phi_h \hat{h}_t^{rt} = \lambda_t^{rt} + \hat{w}_t$$

2.6.2. Welfare-based Loss Function (Internal Habits)

We derive the welfare-based Loss function following step-by-step the method used in Ascari et al.

Households' utility function:

$$U_t^i = E_0 \sum_{t=o}^{\infty} \beta^t \left\{ \ln\left(X_{i,t}\right) - \frac{\psi_l}{1+\phi_l} (l_{i,t})^{1+\phi_t} \right\}$$

or

$$U_t^i = E_0 \sum_{t=o}^{\infty} \beta^t \left\{ \frac{\left(X_{i,t}\right)^{1-\sigma}}{1-\sigma} - \frac{\psi_l}{1+\phi_l} (L_{i,t})^{1+\phi_t} \right\}$$

where

$$X_t^i = C_{i,t} - bC_{i,t-1}$$

(2.41) $$W_t = \theta \left[U\left(X_{R,t}\right) - V(L_{R,t})\right] + (1-\theta)\left[U\left(X_{O,t}\right) - V(L_{O,t})\right]$$

since $L_{o,t} = L_{r,t} = L_t$

(2.42) $$W_t = \theta U\left(X_{R,t}\right) + (1-\theta)\, U\left(X_{O,t}\right) - V\left(L_t\right) =$$

(2.43) $$= U\left(X_t\right) - V\left(L_t\right)$$

remember that

$$\frac{Y_{i,t} - Y_i}{Y_i} = y_{i,t} + \frac{1}{2} y_{i,t}^2 + O[2]$$

A second order approximation of $\theta U\left(X_{R,t}\right)$ delivers

$$\theta U\left(X_{R,t}\right) \simeq \theta \left[U(X_R) + U_{X_R}(X_{R,t} - X_R) + \frac{U_{X_R X_R}}{2}\left(X_{R,t} - X_R\right)^2 \right]$$

$$\theta U\left(X_{R,t}\right) \simeq \theta \left[U\left(X_R\right) + U_{X_R}\left(X_{R,t} - X_R\right)\right] + \frac{\theta}{2}\left[U_{X_R X_R}\left(X_{R,t} - X_R\right)^2\right]$$

$$\theta \ln\left(X_{R,t}\right) \simeq \theta\left[\ln\left(X_R\right) + \frac{1}{X_R}\left(X_{R,t} - X_R\right)\right] + \frac{\theta}{2}\left[-\frac{1}{X_R^2}\left(X_{R,t} - X_R\right)^2\right]$$

$$\theta \ln\left(X_{R,t}\right) - \theta \ln\left(X_R\right) \simeq \theta \frac{\left(X_{R,t} - X_R\right)}{X_R} - \frac{\theta}{2}\frac{\left(X_{R,t} - X_R\right)^2}{X_R^2}$$

with crra

$$U_{X_{i,t}} = \frac{\left(X_{i,t}\right)^{1-\sigma}}{1-\sigma}$$

$$\theta U\left(X_{R,t}\right) \simeq \theta\left[U\left(X_R\right) + U_{X_R}\left(X_{R,t} - X_R\right)\right] + \frac{\theta}{2}\left[U_{X_R X_R}\left(X_{R,t} - X_R\right)^2\right]$$

$$\theta\left[U\left(X_{R,t}\right) - U\left(X_R\right)\right] \simeq \theta U_{X_R}\left(x_{R,t} + \frac{1}{2}x_{R,t}^2\right) X_R + \frac{\theta}{2} U_{X_R X_R} X_R^2\left[x_{R,t}^2\right]$$

$$\theta\left[U\left(X_{R,t}\right) - U\left(X_R\right)\right] \simeq \theta X_R^{1-\sigma}\left(x_{R,t} + \frac{1-\sigma}{2}x_{R,t}^2\right)$$

for Ricardians

$$(1-\theta)\ln\left(X_{O,t}\right) \simeq (1-\theta)\left[\ln\left(X_O\right) + \frac{1}{X_O}\left(X_{O,t} - X_o\right)\right] + \frac{(1-\theta)}{2}\left[-\frac{1}{X_R^2}\left(X_{R,t} - X_R\right)^2\right]$$

$$(1-\theta)\ln\left(X_{O,t}\right) - (1-\theta)\ln\left(X_O\right) \simeq (1-\theta)\frac{\left(X_{O,t} - X_O\right)}{X_O} - \frac{(1-\theta)}{2}\frac{\left(X_{O,t} - X_O\right)^2}{X_O^2}$$

$$(1-\theta)\ln\left(X_{O,t}\right) - (1-\theta)\ln\left(X_O\right) \simeq (1-\theta)\frac{\left(X_{O,t} - X_O\right)}{X_O} - \frac{(1-\theta)}{2}\frac{\left(X_{O,t} - X_O\right)^2}{X_O^2}$$

or in crra

$$(1-\theta)\left[U\left(X_{O,t}\right) - U\left(X_O\right)\right] \simeq (1-\theta) X_O^{1-\sigma}\left(x_{O,t} + \frac{1-\sigma}{2}x_{O,t}^2\right)$$

recalling that

$$X_{i,t} = C_{i,t} - bC_{i,t-1}$$

we have that it can be rewritten in terms of consumption as:

$$x_{i,t} = \frac{1}{1-b}c_{i,t} - \frac{b}{1-b}c_{i,t-1} = \frac{1}{1-b}\left(c_{i,t} - bc_{i,t-1}\right)$$

and therefore

$$x_{i,t}^2 = \frac{1}{(1-b)^2}\left(c_{i,t} - bc_{i,t-1}\right)^2$$

$X_{i,t}$ can be approximated to second order by

$$\frac{X_{i,t} - X_i}{X_i} = x_{i,t} + \frac{1}{2}x_{i,t}^2$$

so that

$$x_{i,t} + \frac{1}{2}x_{i,t}^2 = \frac{1}{1-b}\left(c_{i,t} + \frac{1}{2}c_{i,t}^2\right) - \frac{b}{1-b}\left(c_{i,t-1} + \frac{1}{2}c_{i,t-1}^2\right)$$

$$x_{i,t} = \frac{1}{1-b}\left(c_{i,t} + \frac{1}{2}c_{i,t}^2\right) - \frac{b}{1-b}\left(c_{i,t-1} + \frac{1}{2}c_{i,t-1}^2\right) - \frac{1}{2}x_{i,t}^2$$

the following equations

$$(1-\theta)\left[U\left(X_{O,t}\right) - U\left(X_O\right)\right] \simeq (1-\theta)\,X_O^{1-\sigma}\left(x_{O,t} + \frac{1-\sigma}{2}x_{O,t}^2\right)$$

$$\theta\left[U\left(X_{R,t}\right) - U\left(X_R\right)\right] \simeq \theta X_R^{1-\sigma}\left(x_{R,t} + \frac{1-\sigma}{2}x_{R,t}^2\right)$$

become

$$(1-\theta)\left[U\left(X_{O,t}\right) - U\left(X_O\right)\right] \simeq (1-\theta)\,X_O^{1-\sigma}\left(\begin{array}{c} \frac{1}{1-b}\left(c_{O,t} + \frac{1}{2}c_{O,t}^2\right) + \\ -\frac{b}{1-b}\left(c_{O,t-1} + \frac{1}{2}c_{O,t-1}^2\right) - \frac{1}{2}x_{O,t}^2 + \frac{1-\sigma}{2}x_{O,t}^2 \end{array}\right)$$

$$(1-\theta)\left[U\left(X_{O,t}\right) - U\left(X_O\right)\right] \simeq (1-\theta)\,X_O^{1-\sigma}\left(\begin{array}{c} \frac{1}{1-b}\left(c_{O,t} + \frac{1}{2}c_{O,t}^2\right) + \\ -\frac{b}{1-b}\left(c_{O,t-1} + \frac{1}{2}c_{O,t-1}^2\right) - \frac{\sigma}{2}x_{O,t}^2 \end{array}\right)$$

$$\theta\left[U\left(X_{R,t}\right) - U\left(X_R\right)\right] \simeq \theta X_R^{1-\sigma}\left(\begin{array}{c} \frac{1}{1-b}\left(c_{R,t} + \frac{1}{2}c_{R,t}^2\right) + \\ -\frac{b}{1-b}\left(c_{R,t-1} + \frac{1}{2}c_{R,t-1}^2\right) - \frac{\sigma}{2}x_{R,t}^2 \end{array}\right)$$

Approximation of $U(L_t)$ delivers

$$U(L_t) = \frac{\psi_l}{1+\phi_l}(L_t)^{1+\phi_t}$$

$$U(L_t) = U(L) + U_L(L_t - L) + \frac{U_{LL}}{2}(L_t - L)^2$$

$$\frac{\psi_l}{1+\phi_l}(L_{i,t})^{1+\phi_l} \simeq \frac{\psi_l}{1+\phi_l}(L)^{1+\phi_l} + \psi_l(L)^{\phi_l}\left(L_t - L\right) + \frac{\psi_l\phi_l}{2}(L)^{\phi_l-1}\left(L_t - L\right)^2$$

$$\frac{\psi_l}{1+\phi_l}(L_{i,t})^{1+\phi_l} - \frac{\psi_l}{1+\phi_l}(L)^{1+\phi_l} \simeq \psi_l(L)^{\phi_l}\left(L_t - L\right) + \frac{\psi_l\phi_l}{2}(L)^{\phi_l-1}\left(L_t - L\right)^2$$

$$\begin{aligned}\frac{\psi_l}{1+\phi_l}(L_{i,t})^{1+\phi_l} - \frac{\psi_l}{1+\phi_l}(L)^{1+\phi_l} \quad &\simeq \quad \psi_l(L)^{\phi_l}L_t - \psi_l\phi_l(L)^{\phi_l}L_t + \\ &+ \left(\frac{\psi_l\phi_l}{2} - \psi_l\right)(L)^{\phi_l+1} + \frac{\psi_l\phi_l}{2}(L)^{\phi_l-1}L_t^2\end{aligned}$$

$$\begin{aligned}\frac{1}{1+\phi_l}(L_{i,t})^{1+\phi_l} - \frac{1}{1+\phi_l}(L)^{1+\phi_l} \quad &\simeq \quad \left(1-\phi_l\right)(L)^{\phi_l}L_t + \frac{\phi_l}{2}(L)^{\phi_l-1}L_t^2 + \\ &+ \left(\frac{\phi_l}{2} - 1\right)(L)^{\phi_l+1}\end{aligned}$$

$$U(L_t) - U(L) = \psi_l(L)^{\phi_l}l_tL + \frac{1}{2}\psi_l(L)^{\phi_l}\phi_l l_t^2 L$$

$$U(L_t) - U(L) = U_L L l_t + \frac{1}{2}U_L L\phi_l l_t^2$$

$$U(L_t) - U(L) = U_L L\left(l_t + \frac{\phi_l}{2}l_t^2\right)$$

$$U(L_t) - U(L) = U_L L l_t + \frac{U_{LL}}{2}L^2 l_t^2$$

$$U(L_t) - U(L) = U_L L l_t + \frac{\psi_l(L)^{\phi_l}\phi_l}{2}L l_t^2$$

$$U(L_t) - U(L) = U_L L l_t + U_L L\frac{\phi_l}{2}l_t^2$$

$$U(L_t) = \frac{\psi_l}{1+\phi_l}(L_t)^{1+\phi_t}$$

$$U(L_t) = U(L) + U_L(L_t - L) + \frac{U_{LL}}{2}(L_t - L)^2$$

$$U(L_t) - U(L) = U_L L l_t + \frac{\psi_l\phi_l}{2}(L)^{\phi_l-1}l_t^2 L^2$$

$$U(L_t) - U(L) = U_L L l_t + \frac{\psi_l\phi_l}{2}(L)^{\phi_l}l_t^2 L$$

$$U(L_t) = U(L) + U_L(L_t - L) + \frac{\psi_l \phi_l}{2}(L)^{\phi_l}(L_t^2 + L^2 - 2LL_t)$$

$$U(L_t) = U(L) + U_L \frac{(L_t - L)}{L} L + \frac{U_{LL}}{2} \frac{(L_t - L)^2}{L^2} L^2$$

$$U(L_t) - U(L) = U_L L l_t + U_L L \frac{1}{2} l_t^2 + \frac{\phi_l}{2} U_L L \left(l_t^2 \right)$$

$$U(L_t) - U(L) = U_L L \left(l_t + \frac{1 + \phi_l}{2} l_t^2 \right)$$

Summing all the terms

$$\begin{aligned} W_t - W &= (1-\theta) X_O^{1-\sigma} \left(\begin{array}{c} \frac{1}{1-b} \left(c_{O,t} + \frac{1}{2} c_{O,t}^2 \right) + \\ -\frac{b}{1-b} \left(c_{O,t-1} + \frac{1}{2} c_{O,t-1}^2 \right) - \frac{\sigma}{2} x_{O,t}^2 \end{array} \right) + \\ & +\theta X_R^{1-\sigma} \left(\begin{array}{c} \frac{1}{1-b} \left(c_{R,t} + \frac{1}{2} c_{R,t}^2 \right) + \\ -\frac{b}{1-b} \left(c_{R,t-1} + \frac{1}{2} c_{R,t-1}^2 \right) - \frac{\sigma}{2} x_{R,t}^2 \end{array} \right) + \\ & -U_L L \left(l_t + \frac{1 + \phi_l}{2} l_t^2 \right) \end{aligned}$$

or, given that steady state consumption and hours worked level are identical for the two groups of agents

$$\begin{aligned} W_t - W &= (1-\theta) U_X X \left(\begin{array}{c} \frac{1}{1-b} \left(c_{O,t} + \frac{1}{2} c_{O,t}^2 \right) + \\ -\frac{b}{1-b} \left(c_{O,t-1} + \frac{1}{2} c_{O,t-1}^2 \right) - \frac{\sigma}{2} x_{O,t}^2 \end{array} \right) + \\ & +\theta U_X X \left(\begin{array}{c} \frac{1}{1-b} \left(c_{R,t} + \frac{1}{2} c_{R,t}^2 \right) + \\ -\frac{b}{1-b} \left(c_{R,t-1} + \frac{1}{2} c_{R,t-1}^2 \right) - \frac{\sigma}{2} x_{R,t}^2 \end{array} \right) + \\ & -U_L L \left(l_t + \frac{1 + \phi_l}{2} l_t^2 \right) \end{aligned}$$

$$\begin{aligned} W_t - W &= (1-\theta) X^{1-\sigma} \left(\begin{array}{c} \frac{1}{1-b} \left(c_{O,t} + \frac{1}{2} c_{O,t}^2 \right) + \\ -\frac{b}{1-b} \left(c_{O,t-1} + \frac{1}{2} c_{O,t-1}^2 \right) - \frac{\sigma}{2} x_{O,t}^2 \end{array} \right) + \\ & +\theta X^{1-\sigma} \left(\begin{array}{c} \frac{1}{1-b} \left(c_{R,t} + \frac{1}{2} c_{R,t}^2 \right) + \\ -\frac{b}{1-b} \left(c_{R,t-1} + \frac{1}{2} c_{R,t-1}^2 \right) - \frac{\sigma}{2} x_{R,t}^2 \end{array} \right) + \\ & -U_L L \left(l_t + \frac{1+\phi_l}{2} l_t^2 \right) \end{aligned}$$

From the economy production function we know that

$$l_t = y_t + d_{w,t} + d_{p,t} - a_t$$

where $d_{w,t} = \log \int_0^1 \left(\frac{W_t^j}{W_t} \right)^{-\theta_w} dj$ is the log of the wage dispersion and $d_{p,t} = \log \int_0^1 \left(\frac{P_t^i}{P_t} \right)^{-\theta_p} di$ is the log of the price dispersion. Both terms are of second order and therefore they cannot be neglected in a second order approximation. Notice that

$$l_t^2 = \left(\hat{y}_t + d_{w,t} + d_{p,t} - a_t \right)^2 = y_t^2 + a_t^2 - 2 y_t a_t$$

thus

$$\begin{aligned} W_t - W &= (1-\theta) X^{1-\sigma} \left(\begin{array}{c} \frac{1}{1-b} \left(c_{O,t} + \frac{1}{2} c_{O,t}^2 \right) + \\ -\frac{b}{1-b} \left(c_{O,t-1} + \frac{1}{2} c_{O,t-1}^2 \right) - \frac{\sigma}{2} x_{O,t}^2 \end{array} \right) + \\ & +\theta X^{1-\sigma} \left(\begin{array}{c} \frac{1}{1-b} \left(c_{R,t} + \frac{1}{2} c_{R,t}^2 \right) + \\ -\frac{b}{1-b} \left(c_{R,t-1} + \frac{1}{2} c_{R,t-1}^2 \right) - \frac{\sigma}{2} x_{R,t}^2 \end{array} \right) + \\ & -U_L L \left(\begin{array}{c} y_t + d_{w,t} + d_{p,t} - a_t + \\ + \frac{1+\phi}{2} \left(y_t^2 + a_t^2 - 2 y_t a_t \right) \end{array} \right) + tip \end{aligned}$$

or

$$
\begin{aligned}
W_t - W &= (1-\theta) X^{1-\sigma} \begin{pmatrix} \frac{1}{1-b}\left(c_{O,t} + \frac{1}{2}c_{O,t}^2\right) + \\ -\frac{b}{1-b}\left(c_{O,t-1} + \frac{1}{2}c_{O,t-1}^2\right) - \frac{\sigma}{2}x_{O,t}^2 \end{pmatrix} + \\
&+\theta X^{1-\sigma} \begin{pmatrix} \frac{1}{1-b}\left(c_{R,t} + \frac{1}{2}c_{R,t}^2\right) + \\ -\frac{b}{1-b}\left(c_{R,t-1} + \frac{1}{2}c_{R,t-1}^2\right) - \frac{\sigma}{2}x_{R,t}^2 \end{pmatrix} + \\
&-U_L L \begin{pmatrix} y_t + d_{w,t} + d_{p,t} - a_t + \frac{1+\phi}{2}y_t^2 + \\ +\frac{1+\phi}{2}a_t^2 - (1+\phi) y_t a_t \end{pmatrix} + tip
\end{aligned}
$$

$$
\begin{aligned}
W_t - W &= (1-\theta) X^{1-\sigma} \begin{pmatrix} \frac{1}{1-b}\left(c_{O,t} + \frac{1}{2}c_{O,t}^2\right) + \\ -\frac{b}{1-b}\left(c_{O,t-1} + \frac{1}{2}c_{O,t-1}^2\right) - \frac{\sigma}{2}x_{O,t}^2 \end{pmatrix} + \\
&+\theta X^{1-\sigma} \begin{pmatrix} \frac{1}{1-b}\left(c_{R,t} + \frac{1}{2}c_{R,t}^2\right) + \\ -\frac{b}{1-b}\left(c_{R,t-1} + \frac{1}{2}c_{R,t-1}^2\right) - \frac{\sigma}{2}x_{R,t}^2 \end{pmatrix} + \\
&-U_L L \begin{pmatrix} y_t + d_{w,t} + d_{p,t} - a_t + \\ +\frac{1+\phi}{2}y_t^2 - (1+\phi) y_t a_t \end{pmatrix} + tip
\end{aligned}
$$

$$
\begin{aligned}
W_t - W &= (1-\theta) U_X X \begin{pmatrix} \frac{1}{1-b}\left(c_{O,t} + \frac{1}{2}c_{O,t}^2\right) + \\ -\frac{b}{1-b}\left(c_{O,t-1} + \frac{1}{2}c_{O,t-1}^2\right) - \frac{\sigma}{2}x_{O,t}^2 \end{pmatrix} + \\
&+\theta U_X X \begin{pmatrix} \frac{1}{1-b}\left(c_{R,t} + \frac{1}{2}c_{R,t}^2\right) + \\ -\frac{b}{1-b}\left(c_{R,t-1} + \frac{1}{2}c_{R,t-1}^2\right) - \frac{\sigma}{2}x_{R,t}^2 \end{pmatrix} + \\
&-U_L L \begin{pmatrix} y_t + d_{w,t} + d_{p,t} - a_t + \\ +\frac{1+\phi}{2}y_t^2 - (1+\phi) y_t a_t \end{pmatrix} + tip
\end{aligned}
$$

$$
\begin{aligned}
W_t - W &= \sum_{t=0}^{\infty} (1-\theta) U_X X \left(\begin{array}{c} \frac{1}{1-b}\left(c_{O,t} + \frac{1}{2}c_{O,t}^2\right) + \\ -\frac{b}{1-b}\left(c_{O,t-1} + \frac{1}{2}c_{O,t-1}^2\right) - \frac{\sigma}{2}x_{O,t}^2 \end{array} \right) + \\
&\quad + \theta U_X X \left(\begin{array}{c} \frac{1}{1-b}\left(c_{R,t} + \frac{1}{2}c_{R,t}^2\right) + \\ -\frac{b}{1-b}\left(c_{R,t-1} + \frac{1}{2}c_{R,t-1}^2\right) - \frac{\sigma}{2}x_{R,t}^2 \end{array} \right) + \\
&\quad - U_L L \left(\begin{array}{c} y_t + d_{w,t} + d_{p,t} - a_t + \\ + \frac{1+\phi}{2}y_t^2 - (1+\phi)\, y_t a_t \end{array} \right) + tip
\end{aligned}
$$

$$
\begin{aligned}
W_t - W &= (1-\beta b)\sum_{t=0}^{\infty}\left[(1-\theta)\frac{1}{1-b}U_X X\left(\left(c_{O,t} + \frac{1}{2}c_{O,t}^2\right) - (1-b)\frac{\sigma}{2}x_{O,t}^2\right)\right] + \\
&\quad + (1-\beta b)\sum_{t=0}^{\infty}\left[(\theta)\frac{1}{1-b}U_X X\left(\left(c_{R,t} + \frac{1}{2}c_{R,t}^2\right) - (1-b)\frac{\sigma}{2}x_{R,t}^2\right)\right] + \\
&\quad - \sum_{t=0}^{\infty}\left[U_L L\left(y_t + d_{w,t} + d_{p,t} - a_t + \frac{1+\phi}{2}y_t^2 - (1+\phi)\, y_t a_t\right)\right] + tip
\end{aligned}
$$

$$
\begin{aligned}
W_t - W &= \frac{(1-\theta)(1-\beta b)}{1-b}U_X X\sum_{t=0}^{\infty}\left[\left(\left(c_{O,t} + \frac{1}{2}c_{O,t}^2\right) - (1-b)\frac{\sigma}{2}x_{O,t}^2\right)\right] + \\
&\quad + \frac{\theta(1-\beta b)}{1-b}U_X X\sum_{t=0}^{\infty}\left[\left(\left(c_{R,t} + \frac{1}{2}c_{R,t}^2\right) - (1-b)\frac{\sigma}{2}x_{R,t}^2\right)\right] + \\
&\quad - U_L L\sum_{t=0}^{\infty}\left[\left(y_t + d_{w,t} + d_{p,t} + \frac{1+\phi}{2}y_t^2 - (1+\phi)\, y_t a_t\right)\right] + tip
\end{aligned}
$$

$$
\begin{aligned}
W_t - W &= U_X X (1-\theta)\sum_{t=0}^{\infty}\left[\left(\frac{(1-\beta b)}{1-b}\left(c_{O,t} + \frac{1}{2}c_{O,t}^2\right) - \frac{\sigma}{2}x_{O,t}^2\right)\right] + \\
&\quad + U_X X \theta\sum_{t=0}^{\infty}\left[\left(\frac{(1-\beta b)}{1-b}\left(c_{R,t} + \frac{1}{2}c_{R,t}^2\right) - \frac{\sigma}{2}x_{R,t}^2\right)\right] + \\
&\quad - U_L L\sum_{t=0}^{\infty}\left[\left(y_t + d_{w,t} + d_{p,t} + \frac{1+\phi}{2}y_t^2 - (1+\phi)\, y_t a_t\right)\right] + tip
\end{aligned}
$$

since $U_X X = U_L L = U_C C$ and $MRS = MPL = 1$ at the efficient steady state

$$\begin{aligned} W_t - W &= X^{1-\sigma}(1-\theta)\sum_{t=0}^{\infty}\left[\left(\frac{(1-\beta b)}{(1-b)}\left(c_{O,t}+\frac{1}{2}c_{O,t}^2\right)-\frac{\sigma}{2}x_{O,t}^2\right)\right]+ \\ &\quad +X^{1-\sigma}\theta\sum_{t=0}^{\infty}\left[\left(\frac{(1-\beta b)}{(1-b)}\left(c_{R,t}+\frac{1}{2}c_{R,t}^2\right)-\frac{\sigma}{2}x_{R,t}^2\right)\right]+ \\ &\quad -\frac{(1-\beta b)}{(1-b)}X^{1-\sigma}\sum_{t=0}^{\infty}\left[\left(\begin{array}{c} y_t+d_{w,t}+d_{p,t}+ \\ +\frac{1+\phi}{2}y_t^2-(1+\phi)\,y_t a_t \end{array}\right)\right]+tip \end{aligned}$$

$$\begin{aligned} \frac{W_t - W}{U_C C} &= \frac{(1-\beta b)}{(1-b)}\left(\begin{array}{c} (1-\theta)\left(c_{O,t}+\frac{1}{2}c_{O,t}^2\right)+ \\ +\theta\left(c_{R,t}+\frac{1}{2}c_{R,t}^2\right) \end{array}\right)+ \\ &\quad -(1-\theta)\frac{\sigma}{2}x_{O,t}^2-\theta\frac{\sigma}{2}x_{R,t}^2+ \\ &\quad -\frac{(1-\beta b)}{(1-b)}\left(y_t+d_{w,t}+d_{p,t}+\frac{1+\phi}{2}y_t^2-(1+\phi)\,y_t a_t\right)+tip \end{aligned}$$

$$\begin{aligned} \frac{W_t - W}{U_C C} &= -\left(\frac{\phi}{2}\frac{(1-\beta b)}{(1-b)}\right)y_t^2-(1-\theta)\frac{\sigma}{2}x_{O,t}^2-\theta\frac{\sigma}{2}x_{R,t}^2+ \\ &\quad -\frac{(1-\beta b)}{(1-b)}\left(d_{w,t}+d_{p,t}\right)+\frac{(1-\beta b)}{(1-b)}(1+\phi)\,y_t a_t+tip \end{aligned}$$

$$\frac{W_t - W}{U_C C} = -\frac{(1-\beta b)}{(1-b)}\left[\begin{array}{c} \frac{\phi}{2}y_t^2+\theta\frac{(1-b)}{(1-\beta b)}\frac{\sigma}{2}x_{O,t}^2+ \\ +(1-\theta)\frac{(1-b)}{(1-\beta b)}\frac{\sigma}{2}x_{R,t}^2+\left(d_{w,t}+d_{p,t}\right)+ \\ -(1+\phi)\,y_t a_t \end{array}\right]+tip$$

$$\frac{W_t - W}{U_C C} = -\frac{(1-\beta b)}{(1-b)}\left[\begin{array}{c} \frac{\phi}{2}y_t^2+(1-\theta)\frac{(1-b)}{(1-\beta b)}\frac{\sigma}{2}x_{O,t}^2+ \\ +\theta\frac{(1-b)}{(1-\beta b)}\frac{\sigma}{2}x_{R,t}^2+\frac{\alpha_w}{\kappa_w}\left(\pi_t^w\right)^2+ \\ +\frac{\eta}{\kappa_p}\left(\pi_t^p\right)^2-(1+\phi)\,y_t a_t \end{array}\right]+tip$$

$$\frac{W_t - W}{U_C C} = -\frac{(1-\beta b)}{(1-b)}\left[\begin{array}{c} \frac{\phi}{2}y_t^2+(1-\theta)\frac{(1-b)}{(1-\beta b)}\frac{\sigma}{2}x_{O,t}^2+ \\ +\theta\frac{(1-b)}{(1-\beta b)}\frac{\sigma}{2}x_{R,t}^2+\frac{1}{2}\frac{\alpha_w}{\kappa_w}\left(\pi_t^w\right)^2+ \\ +\frac{1}{2}\frac{\eta}{\kappa_p}\left(\pi_t^p\right)^2-(1+\phi)\,y_t a_t \end{array}\right]+tip$$

substituting $x_{i,t}$ with its definition in terms of output and rearranging we obtain

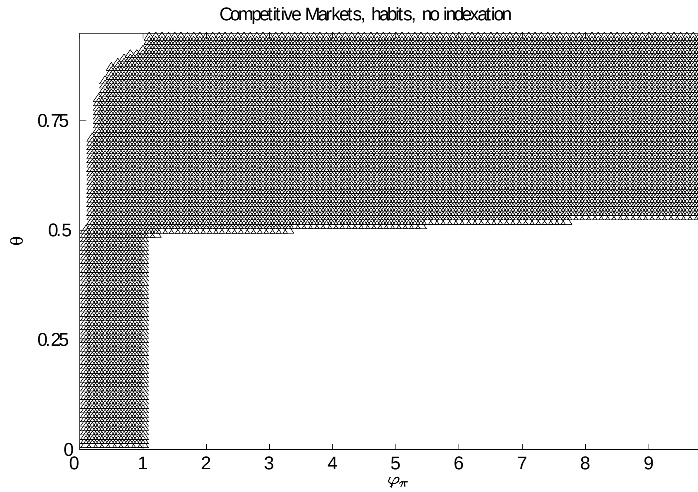
$$\frac{W_t - W}{U_C C} = -\frac{(1-\beta b)}{(1-b)}\frac{1}{2}\left[\begin{array}{c} \phi y_t^2 + \frac{1}{(1-\beta b)(1-b)}\left(\begin{array}{c} y_t^2 + \frac{\theta}{(1-\theta)} w_t^2 + \\ -\frac{2\theta}{(1-\theta)} w_t a_t \end{array}\right) + \\ + \frac{b^2}{(1-\beta b)(1-b)}\left(\begin{array}{c} y_{t-1}^2 + \frac{\theta}{(1-\theta)} w_{t-1}^2 + \\ -\frac{2\theta}{(1-\theta)} w_{t-1} a_{t-1} \end{array}\right) + \\ -\frac{2b}{(1-\beta b)(1-b)}\left(\begin{array}{c} y_t y_{t-1} + \frac{\theta}{(1-\theta)} w_t w_{t-1} + \\ -\frac{\theta}{(1-\theta)} w_t a_{t-1} - \frac{\theta}{(1-\theta)} a_t w_{t-1} \end{array}\right) + \\ + \frac{\alpha_w}{\kappa_w}\left(\pi_t^w\right)^2 + \frac{\eta}{\kappa_p}\left(\pi_t^p\right)^2 - 2\left(1+\phi\right) y_t a_t \end{array}\right] + tip$$

which can be rewritten in terms of gap variables as

$$\frac{W_t - W}{U_C C} = -\frac{(1-\beta b)}{(1-b)}\frac{1}{2}\left[\begin{array}{c} \left(\phi + \frac{1}{(1-\beta b)(1-b)}\right)\left(y_t^{gap}\right)^2 + \\ + \left(2\phi + \frac{2}{(1-\beta b)(1-b)}\right) y_t^{gap} y_t^* + \\ + \frac{1}{(1-\beta b)(1-b)}\frac{\theta}{(1-\theta)}\left(w_t^{gap}\right)^2 + \\ + \frac{b^2}{(1-\beta b)(1-b)}\left(y_{t-1}^{gap}\right)^2 + \frac{b^2}{(1-\beta b)(1-b)} 2 y_{t-1}^{gap} y_{t-1}^* + \\ + \frac{b^2}{(1-\beta b)(1-b)}\frac{\theta}{(1-\theta)}\left(w_{t-1}^{gap}\right)^2 + \\ -\frac{2b}{(1-\beta b)(1-b)} y_t^{gap} y_{t-1}^{gap} - \frac{2b}{(1-\beta b)(1-b)} y_t^{gap} y_{t-1}^* + \\ -\frac{2b}{(1-\beta b)(1-b)} y_t^* y_{t-1}^{gap} - \frac{2b}{(1-\beta b)(1-b)}\frac{\theta}{(1-\theta)} w_t^{gap} w_{t-1}^{gap} + \\ + \frac{\alpha_w}{\kappa_w}\left(\pi_t^w\right)^2 + \frac{\eta}{\kappa_p}\left(\pi_t^p\right)^2 - 2\left(1+\phi\right) a_t y_t^{gap} \end{array}\right] + tip$$

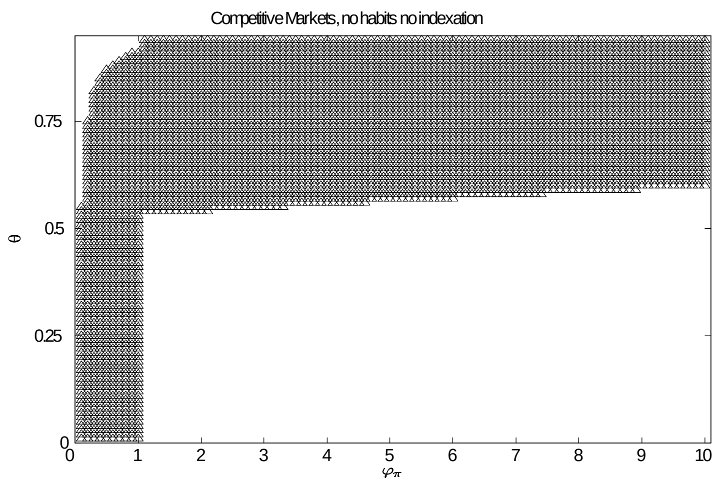
2.6.3. Robustness

Determinacy with no indexation on prices and wages ($\gamma_p = 0, \gamma_w = 0$)

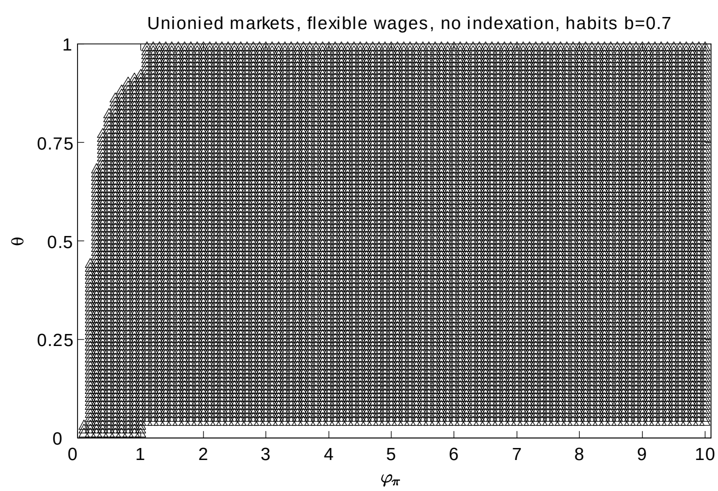


Habits in Consumption

2.6.3.1. Competitive Labour Markets.

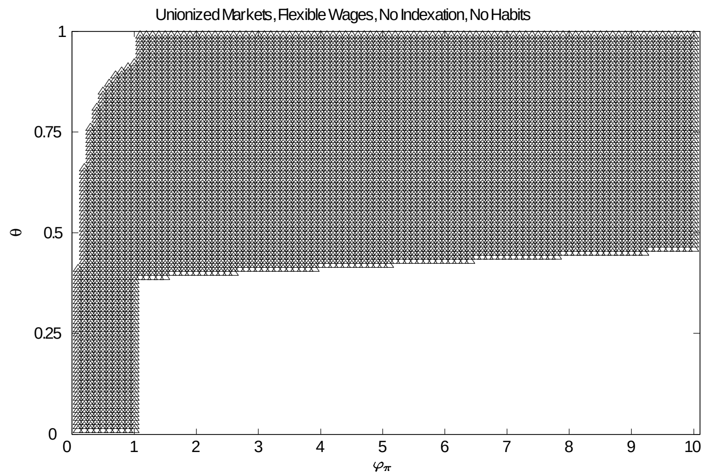


No Habits in Consumption

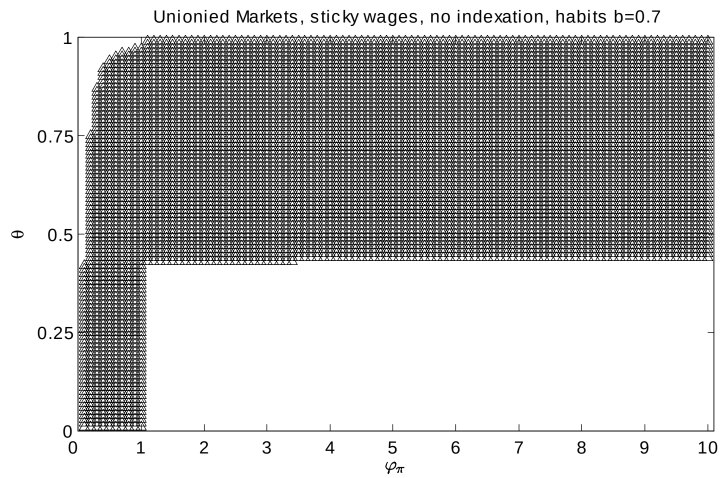


Flexible Wages, Habits in Consumption

2.6.3.2. Unionize Labour Markets.



Flexible Wages, no Habits in Consumption



Sticky Wages, Habits in Consumption

Sticky Wages, no Habits in Consumption

CHAPTER 3

External vs. Internal Habit Formation in Consumption: When it Matters.

3.1. Introduction

Habit formation in consumption has become a key ingredient in New Keynesian business cycle literature. Typically, in New Keynesian business cycle models consumption habits are introduced to generate "hump-shaped" impulse responses of output and consumption to demand and supply shocks. The empirical success of habit formation in consumption in fitting the data has been widely accepted and recognized.

However, the way habit in consumption is modeled in theoretical models is still source of debates. While consumption habits are deemed a key ingredient of modern business cycle models the modelling choice for consumption habits seems arbitrary. Some authors (Christiano, Eichenbaum and Evans (2005) ,CEE henceforth; Amato and Laubach (2004); Fuhrer (2000)) adopt "internal" habit formation (that is, complementarity between individual present consumption levels and individual past consumption levels), others (Smets and Wouters (2003), Ravn, Schmitt-Grohe, and Uribe (2006) prefer to model "external" habits (complementarity is between current individual consumption and aggregate past consumption). Dennis (2009) shows that different way to model habit formation are in fact equivalent up to a log-linear approximation for what concerns the business cycle behavior.

In the paper we show that this conclusion is not robust to the introduction of a minimal degree of agents heterogeneity. Indeed important differences arise concerning dynamics.

Following a seminal contribution by Mankiw (2000), who introduced the notion of heterogeneous consumers (savers and spenders), a second strand of New Keynesian literature emphasizes the role of non-optimizing agents, i.e. agents that adopt a rule-of-thumb and fully consume their current income (RT consumers henceforth). Gali et al (2004, 2007), and Bilbiie (2008), showed how RT consumers can substantially affect both stability and aggregate dynamics of New Keynesian business cycle models. De Graeve et al. (2010) introduce RT consumers to model financial risk premia. Empirical research cannot reject the RT consumers hypothesis. Estimated structural equations for consumption growth report a share of RT consumers ranging from 26 to 40% (Jacoviello, 2004; Campbell and Mankiw, 1989) More recent estimates of dynamic stochastic general equilibrium models (Coenen and Straub, 2005; Forni, Monteforte and Sessa, 2009) obtain estimates around 35%. Erceg, Guerrieri and Gust (2006) calibrate the share of RT consumers to 50% in order to replicate the dynamic performance of the Federal Reserve Board Global Model. Critics of the approach might argue that the empirical relevance of RT consumers is bound to gradually decline along with the development of financial markets (Bilbiie, Meier and Müller, 2008). In fact, increasing regulation in the aftermath of the 2008 crisis (OECD 2009) is likely to increase the share of liquidity constrained households.

In this paper we consider together RT consumers and consumption habit formation. We shows how the choice between internal and external habit formation leads to a completely different dynamic response to demand or supply shocks when a very small share of RT consumers are considered. The intuition is very straightforward. When habits are external, the felicity of the optimizing agents is affected by the consumption level of the rest of the population. Moreover, agents do not internalize anymore the fact that present consumption influences the agents' future utility. Ricardians' intertemporal decision are no determined by RT behavior and the link between consumption and the real interest rate is weakened. Our simulations show that this has dramatic implications for model determinacy even for a RT consumers close to 10% of the population. The model dynamics under the hypothesis of external habit formation do not seem anymore consistent with the empirical literature.

The rest of the paper is organized as follows: In the next section we describe in detail the model structure, we then present the results concerning the model dynamic performance in section 3. Section 4 concludes.

3.2. The Model

We consider a cashless version of a last generation medium-scale DSGE model a la CEE (2005) augmented for rule-of-thumb (RT) or non Ricardian consumers. The behavior of these latter agents is characterized by a simple rule of thumb: they consume their available labor income in each period, and do not save or smooth consumption over time. The key distinction between the two groups concerns intertemporal optimization. Ricardian consumers' choices take into account future utility when choosing consumption and portfolio composition. Rule-of-Thumb consumers spend their whole income every period, thus they do not hold any wealth.

3.2.1. Households

We assume a continuum of households indexed by $j \in [0,1]$. RT consumers are defined over the interval $[0,\theta]$. The rest of the households, interval $(\theta,1]$ accounts for Ricardian consumers. All households share the same utility function. We will distinguish for two alternative specification of the utility function, the first one:

$$U_t^i = E_0 \sum_{t=o}^{\infty} \beta^t \left\{ \ln \left(C_t^i - bC_{t-1}^i \right) - \frac{\psi_l}{1+\phi_l} (l_t^i)^{1+\phi_t} \right\} \tag{3.1}$$

represents the case in which habit formation in consumption is "internal", where $i : o, rt$ stands for household type, C_t^i represents total individual consumption and l_t^i denotes individual labour supply.

Otherwise, when we assume "external" habit formation in consumption, the households' utility function will be described by:

$$U_t^i = E_0 \sum_{t=o}^{\infty} \beta^t \left\{ \ln \left(C_t^i - bC_{t-1} \right) - \frac{\psi_l}{1+\phi_l} (l_t^i)^{1+\phi_t} \right\} \tag{3.2}$$

where C_{t-1} represents the level of aggregate consumption in the previous period.

3.2.1.1. Consumption Bundles. C_t^i represents the demand of a bundle of differentiated consumption goods and it is defined as

$$C_t^i = \left[\int_0^1 c_{jt}^{\frac{\eta-1}{\eta}} \, dj \right]^{\frac{\eta}{\eta-1}} \tag{3.3}$$

where η represents the price elasticity of demand for the individual goods.

At any period t each households optimally choose the consumption bundle minimizing the cost of purchasing the combination of individual goods:

$$\min_{c_{jt}} \int_0^1 p_{jt} c_{jt} \, dj \tag{3.4}$$

subject to

$$\left[\int_0^1 c_{jt}^{\frac{\eta-1}{\eta}} \, dj \right]^{\frac{\eta}{\eta-1}} \geq C_t \tag{3.5}$$

The solution of the problem is given by the following equation

$$c_{jt} = \left(\frac{p_{jt}}{P_t} \right)^{-\eta} C_t \tag{3.6}$$

where $P_t = \left(\int_0^1 p_{jt}^{1-\eta} dj \right)^{\frac{1}{1-\eta}}$ is the aggregated price index.

3.2.2. Firms

Goods markets are monopolistically competitive, and good z is produced with the following technology:

$$y_t(z) = a_t \left(k_t(z) \right)^{\alpha} \left(h_t(z) \right)^{1-\alpha}$$

where a_t represents the technology process: $a_t = \rho_a a_{t-1} + \varepsilon_t^a$ and is ε_t^a is a *i.i.d.* random variable with zero mean and variance $\sigma_{\varepsilon^a}^2$.

$k_t(z)$ defines the physical capital services obtained from households (see section 2.4 below) and $h_t(z)$ is the composite labor input used by each firm z. The latter is defined as follows

$$(3.7) \qquad h_t(z) = \left(\int_0^1 \left(h_t^j(z) \right)^{\frac{\alpha_w - 1}{\alpha_w}} dj \right)^{\frac{\alpha_w}{\alpha_w - 1}}$$

where the parameter $\alpha_w > 1$ is the intratemporal elasticity of substitution between labor inputs. For any given level of its labor demand $h_t(z)$, the optimal allocation of across labor inputs implies

$$(3.8) \qquad h_t^j(z) = \left(\frac{W_t^j}{W_t} \right)^{-\alpha_w} h_t^d(z)$$

where $W_t = \left(\int_0^1 \left(W_t^j \right)^{1-\alpha_w} dj \right)^{1/(1-\alpha_w)}$ is the standard wage index.

Firm *z's* nominal total production cost is given by

$$(3.9) \qquad TC_t(z) = W_t h_t(z) + (1 + R_t^k) k_t(z)$$

The real marginal costs are:

$$(3.10) \qquad mc_t = a_t^{-1} \left(\frac{r_t^k}{\alpha} \right)^{\alpha} \left(\frac{w_t}{(1-\alpha)} \right)^{1-\alpha}$$

where $w_t = \frac{W_t}{P_t}$ and $r_t^k = \frac{1+R_t^k}{P_t}$.

3.2.2.1. Sticky Prices. Price stickiness is based on the Calvo mechanism. In each period firm z faces a probability $1 - \lambda_p$ of being able to reoptimize its price. When a firm is not able to reoptimize, it adjusts its price to the previous period inflation, $(1 + \pi_{t-1}) = \frac{P_{t-1}}{P_{t-2}}$. The price-setting condition therefore is:

$$(3.11) \qquad p_t(z) = (1 + \pi_{t-1})^{\gamma_p} p_{t-1}(z)$$

where $\gamma_p \in [0,1]$ represents the degree of price indexation.

All the $1-\lambda_p$ firms which reoptimize their price at time t will face symmetrical conditions and set the same price $\widetilde{P}_t$. When choosing $\widetilde{P}_t$ the optimizing firm will take into account that in the future it might not be able to reoptimize. In this case, the price at the generic period $t+s$ will read as $\widetilde{P}_t\Pi_{t,t+s-1}^{\gamma_p}$ where $\Pi_{t,t+s-1} = (1+\pi_t)\ldots(1+\pi_{t+s-1}) = \frac{P_{t+s-1}}{P_{t-1}}$.

$\widetilde{P}_t$ is chosen so as to maximize a discounted sum of expected future profits:

$$E_t\sum_{s=0}^{\infty}(\beta\lambda_p)^s\lambda_{t+s}^o\left(\widetilde{P}_t\Pi_{t,t+s-1}^{\gamma_p} - P_{t+s}mc_{t+s}\right)y_{t+s}(z)$$

subject to:

$$y_{t+s}(z) = Y_{t+s}^d\left(\frac{\widetilde{P}_t\prod_{t,t+s-1}^{\gamma_p}}{P_{t+s}}\right)^{-\eta} \tag{3.12}$$

where Y_t^d is aggregate demand, λ_{t+s}^o is the marginal utility of income for Ricardian households (i.e. the value of an additional dollar for Ricardian consumers), β is the stochastic discount factor.

The F.O.C. for this problem is

$$E_t\sum_{s=0}^{\infty}(\beta\lambda_p)^s\lambda_{t+s}Y_{t+s}^d\left[\begin{array}{c}(1-\eta)\left(\Pi_{t,t+s-1}^{\gamma_p}\right)^{1-\eta}\widetilde{P}_t^{-\eta}(P_{t+s})^{\eta}+\\ +\eta\widetilde{P}_t^{-\eta-1}P_{t+s}^{\eta+1}mc_{t+s}\left(\Pi_{t,t+s-1}^{\gamma_p}\right)^{-\eta}\end{array}\right]=0 \tag{3.13}$$

3.2.3. Labor market

There is a continuum of differentiated labor inputs indexed by $j \in [0,1]$. For each labor input there is a union j which monopolistically supplies the labor input j in the labor market j.

Each union sets the nominal wage, W_t^j, subject to (3.8). Each household i supplies all labour types at the given wage rate[1] and the total number of hours allocated to the different labor markets must satisfy the time resource constraint

[1]Under the assumption that wages always remain above all households' marginal rate of substitution, households are willing to meet firms' labour demand.

$$h_t^i = \int_0^1 h_t^j dj = \int_0^1 \left(\frac{W_t^j}{W_t}\right)^{-\alpha_w} h_t^d dj \tag{3.14}$$

As in Galì (2007), we assume that the fraction of RT and Ricardian consumers is uniformly distributed across unions, and demand for each labour type is uniformly distributed across households. Ricardian and non-Ricardian households therefore work for the same amount of time, h_t. Individual labor income is

$$h_t^d W_t = \int_0^1 W_t^j \left(\frac{W_t^j}{W_t}\right)^{-\alpha_w} h_t^d dj \tag{3.15}$$

We posit that the union objective function is a weighted average $(1-\theta, \theta)$ of the utility functions of the two households types. This, in turn, implies that with flexible wages

$$w_t = \frac{W_t}{P_t} = \frac{\alpha_w}{\alpha_w - 1} \frac{\psi_l h_t^{\phi_l}}{\left[(1-\theta) U'_{C_t^o} + \theta U'_{C_t^{rt}}\right]} \tag{3.16}$$

where $\frac{\alpha_w}{(\alpha_w - 1)}$ represents the wage markup over the average marginal rate of substitution.

3.2.4. Ricardian Households

Ricardian households maximize utility subject to the following budget constraints. The Lagrangian multipliers are respectively λ_t, $\lambda_t q_t$, and $\frac{\lambda_t^o w_t}{\mu_t^o}$

Nominal budget constraints:

$$\frac{B_{t+1}}{1+R_t} + P_t \left(C_t^o + i_t\right) = B_t + A_t + \left[r_t^k u_t - a\left(u_t\right)\right] P_t k_t + h_t^d W_t + P_t \left(D_t - \tau_t\right) \tag{3.17}$$

where P_t defines the consumption price level, $h_t^d W_t$ is time t nominal labour income, A_t is the nominal net cash flow from participating at time t in the union-wide state-contingent security market, D_t represents firms' real profits. B_t denotes time t holdings of riskless nominal bonds issued by the government. R_t is the nominal interest rate on bond issued at time t. τ_t defines real lump-sum taxes.

Optimizing households own the physical stock of capital k_t, and rent it to firms at the real rental rate r_t^k. Furthermore, owners of physical capital control the degree of its utilization, u_t. The term $a(u_t)$ defines the real cost of using the capital stock with intensity u_t. Finally, i_t denotes time t real purchases of investment goods. Following Christiano et al (2005) and Schmitt-Grohe and Uribe (2005), the household's stock of physical capital evolves as:

$$\bar{k}_{t+1} = (1-\delta)\,\bar{k}_t + i_t\left[1 - S\left(\frac{i_t}{i_{t-1}}\right)\right] \tag{3.18}$$

$$k_t = u_t\bar{k}_t \tag{3.19}$$

where δ denotes the physical rate of depreciation and the function S introduces investment adjustment costs.

The solution for the household problem yields the Euler equation:

$$1 = (1+R_t)\,E_t\beta\left(\frac{\lambda_{t+1}^o}{\lambda_t^o}\right) \tag{3.20}$$

where λ_t^o represents the marginal utility of consumption for Ricardian consumers and takes two different forms depending on the habit formation type. If habits are internal we have that households internalize the fact that current consumption level may affect future utility, so that

$$\lambda_t^o = \frac{1}{c_t^o - bc_{t-1}^o} - E_t\frac{\beta b}{c_{t+1}^o - bc_t^o} \tag{3.21}$$

If habits are considered external, households' marginal utility of consumption depends only on current and past consumption level, i.e. households do not internalize the effects of current consumption levels on future utility:

$$\lambda_t^o = \frac{1}{c_t^o - bc_{t-1}} \tag{3.22}$$

Moreover, under external habits, the Ricardian equivalence and the permanent income hypothesis do not hold even for Ricardians households since their intertemporal consumption choice will now depend on lump-sum taxes and wage bills[2].

The following first order conditions describe demand functions for capital[3] and investment and the optimal degree of capital utilization.

$$P_{k',t} = \beta E_t \left\{ \lambda_{t+1}^o \frac{r_{t+1}^k u_{t+1} - a\left(u_{t+1}\right) + (1-\delta) P_{k',t+1}}{\lambda_t^o} \right\} \tag{3.23}$$

The first order condition for investment is

$$\lambda_t^o = E_t \left\{ \begin{array}{c} \lambda_t^o P_{k',t} \left[1 - S\left(\frac{i_t}{i_{t-1}}\right) - S'\left(\frac{i_t}{i_{t-1}}\right) \frac{i_t}{i_{t-1}} \right] + \\ +\beta \lambda_{t+1}^o P_{k',t+1} \left[S'\left(\frac{i_{t+1}}{i_t}\right) \left(\frac{i_{t+1}}{i_t}\right)^2 \right] \end{array} \right\} \tag{3.24}$$

$$r_t^k = a'\left(u_t\right) \tag{3.25}$$

Following Christiano *et al.* (2005) the investment adjustment cost function and the capital utilization function are given by:

$$S\left(\frac{i_t}{i_{t-1}}\right) = \frac{\kappa}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2$$

$$a\left(u_t\right) = \gamma_1 \left(u_t - 1\right) + \frac{\gamma_2}{2}\left(u_t - 1\right)^2$$

[2]In fact $c_{t-1} = \theta c_{t-1}^{rt} + (1-\theta) c_{t-1}^o = \theta(w_{t-1}h_{t-1} - \tau_{t-1}) + (1-\theta)c_{t-1}^o$

[3]$P_{k',t}$ is the shadow relative price of one unit of capital with respect to one unit of consumption (Tobin's q).

The function $S(\cdot)$ satisfies the following properties. $S(1) = S'(1) = 0$ and $S''(1) > 0$. These restrictions imply the absence of adjustment costs up to a first order approximation around the deterministic steady state. The function $a(\cdot)$, instead, is assumed to satisfy $a(1) = 0$ and $a'(1), a''(1) > 0$.

3.2.5. Rule-of-Thumb Households

As pointed out above, RT consumers neither save or borrow. Due to the labour market monopolistic structure, these agents are entirely passive. In fact both their consumption and their labour supply are determined by union's (wage) and firms (worked hours) decisions.

$$c_t^{rt} = \frac{h_t^i \int_0^1 \left(\frac{w_t^j}{w_t}\right)^{-\alpha_w} w_t^j \ dj}{P_t} - \tau_t \tag{3.26}$$

3.2.6. Sticky wages

In each period a union faces a constant probability $1 - \lambda_w$ of being able to reoptimize the nominal wage. Unions that cannot reoptimize simply index their wages to lagged inflation:

$$W_t^j = W_{t-1}^j \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\widetilde{\chi}} = W_{t-1}^j \left(\pi_{t-1}\right)^{\widetilde{\chi}}$$

where $\tilde{\chi}$ stands for the degree of wage indexation. Just like firms, when choosing the current wage, $\widetilde{W}_t$, the optimizing union will anticipate that in the future it might not be able to reoptimize. In this case, the wage at the generic period $t+s$ will read as (in real terms)

$$w_{t+s} = \widetilde{w}_t \prod_{k=1}^{s} \frac{\pi_{t+k-1}^{\widetilde{\chi}}}{\pi_{t+k}} \tag{3.27}$$

Following Colciago(2008), the representative union objective function is defined as

$$(3.28) \qquad L^u = \sum_{s=0}^{\infty} (\beta\lambda_w)^s \left\{ \left[(1-\theta)\, U^o(C^o_{t+s}) + \theta U^{rt}(C^{rt}_{t+s}) \right] - U(h_{t+s}) \right\}$$

Where U^o_s, U^{rt}_s are defined as in (3.1) or (3.2). Thus the wage-setting decision maximizes a weighted average of the two household types conditional to the probability that the wage cannot be reoptimized in the future. The relevant constraints are (3.14), (3.17), (3.26), (3.27).

The union's first-order condition is:

$$(3.29) \quad \sum_{s=0}^{\infty} (\beta\lambda_w)^s \left[(1-\theta)\, \lambda^o_{t+s} + \theta \lambda^{rt}_{t+s} \right] h^d_{t+s} \left(w_{t+s} \right)^{\alpha_w} \left(\prod_{k=1}^{s} \frac{\pi^{\tilde{\chi}}_{t+k-1}}{\pi_{t+k}} \right)^{-\alpha_w} \cdot$$

$$\cdot \left[\widetilde{w}_t \left(\prod_{k=1}^{s} \frac{\pi^{\tilde{\chi}}_{t+k-1}}{\pi_{t+k}} \right) - \frac{\alpha_w}{(\alpha_w - 1)} \frac{\psi_l h^{\phi_l}_{t+s}}{\left[(1-\theta)\, \lambda^o_{t+s} + \theta \lambda^{rt}_{t+s} \right]} \right] = 0$$

where $\lambda^{rt}_t = \frac{1}{c^{rt}_t - bc^{rt}_{t-1}} - \frac{\beta b}{c^{rt}_{t+1} - bc^{rt}_t}$ if habits are internal or $\lambda^r_t = \frac{1}{c^{rt}_t - bc_{t-1}}$. It is worth noting that the combination of centralized wage setting and wage stickiness introduces an indirect form of consumption smoothing for RT consumers.

3.2.7. Government

Government flow real budget constraint is given by

$$X_{t+1} = E_t \frac{(1+R_t)}{\pi_{t+1}} X_t + g_t - \tau_t$$

where g_t represents the government spending and X_t is the level of public debt.

We assume that taxes are set following a rule as

$$\hat{t}_t = \phi_b \hat{x}_{t-1} + \phi_g \hat{g}_t$$

where $\hat{t}_t = \frac{\tau_t - \tau}{Y}$, $\hat{x}_t = \frac{\frac{X_t}{P_t} - \frac{X_{t-1}}{P_{t-1}}}{Y}$ and $\hat{g}_t = \frac{G_t - G}{Y}$

g_t is assumed to follow an exogenous AR(1) process

$$g_t = \rho_g g_{t-1}$$

where $0 < \rho_g < 1$.[4]

3.2.8. Monetary Policy

We assume a monetary authority follows a rule of the type:

$$R_t = \varphi_\pi \pi_t + \varphi_y y_t + \varepsilon_t^p \tag{3.30}$$

where ε_t^p is a policy shock

$$\varepsilon_t^p = \rho_p \varepsilon_{t-1}^p + \varepsilon_t^{p*}$$

and is ε_t^{p*} is a *i.i.d.* random variable with zero mean and variance $\sigma^2_{\varepsilon^p}$.

3.2.9. Aggregation

Aggregate consumption C_t is a weighted average of the respective variable for each household type, thus

$$C_t = \int_0^1 C_t^i(j)\ dj = \int_0^\theta C_t^{rt}(j)\ dj + \int_\theta^1 C_t^o(j)\ dj = \theta C_t^{rt} + (1-\theta)C_t^o \tag{3.31}$$

Aggregate investment and capital stock are given respectively by

$$I_t = (1-\theta)I_t^o \tag{3.32}$$

$$K_t = (1-\theta)K_t^o \tag{3.33}$$

[4]We calibrate the model holding throughout the paper the condition for non-explosive debt dynamics which is satisfied when

$$\phi_b > \frac{\rho_g}{1+\rho_g}$$

Aggregating budget constraints for each sector, after few manipulations we get the aggregate resource constraint as

$$Y_t = C_t + I_t + a\left(u_t\right)K_t + G_t$$

3.2.10. Steady State

From equation 3.20 and 3.23, and assuming zero inflation steady state, it holds true that

$$R = \frac{1}{\beta} \tag{3.34}$$

$$r^k = \frac{1}{\beta} + \delta - 1 \tag{3.35}$$

From cost minimization problem come the equations:

$$r^k = mc\,\alpha\left(\frac{k}{h}\right)^{\alpha-1} \tag{3.36}$$

$$w = mc\left(1-\alpha\right)\left(\frac{k}{h}\right)^{\alpha} \tag{3.37}$$

Combining the last two equation we get the real wage computed at steady state

$$mc = \frac{r^k}{\alpha}\left(\frac{k}{h}\right)^{1-\alpha} \tag{3.38}$$

$$w = \frac{(1-\alpha)}{\alpha}r^k\left(\frac{k}{h}\right) \tag{3.39}$$

Combining (3.38) and $mc = \frac{\vartheta-1}{\vartheta}$we get the ratio:

$$\frac{K}{h} = \left(\frac{r^k}{\alpha}\frac{\vartheta}{\vartheta-1}\right)^{\frac{1}{\alpha-1}} \tag{3.40}$$

From the production function we get

$$\frac{Y}{h} = \left(\frac{K}{h}\right)^{\alpha} \tag{3.41}$$

and as

$$\frac{I}{Y} = \delta\frac{K}{Y} \tag{3.42}$$

The aggregate resource constraint reads as:

$$Y = C + I + G \tag{3.43}$$

$$1 = \frac{C}{Y} + \frac{I}{Y} + \frac{G}{Y} \tag{3.44}$$

the aggregate consumption-output ratio is given by

$$\frac{C}{Y} = 1 - \delta\frac{K}{h}\left(\frac{Y}{h}\right)^{-1} - \frac{G}{Y} \tag{3.45}$$

From the government budget constraint we get the ratio

$$\frac{\tau}{Y} = \left(\frac{1}{\beta} - 1\right)\frac{X}{Y} + \frac{G}{Y}$$

The presence of ROT consumers influences our steady state uniquely for what concerns aggregate consumption and its components. Since ROT individual consumption is given at steady state by

$$c^{rt} = w\ h - \tau$$

we can easily derive its relationship with aggregate output as

$$\frac{c^{rt}}{Y} = w\left(\frac{Y}{h}\right)^{-1} - \frac{\tau}{Y} \tag{3.46}$$

Total consumption is the weighted average of the two groups components:

$$C = (1-\theta)\, c^{o} + \theta c^{rt} \tag{3.47}$$

From the latter, it comes straightforward

$$\frac{c^{o}}{C} = \frac{1}{1-\theta} - \frac{\theta}{1-\theta}\frac{c^{rt}}{C} \tag{3.48}$$

3.2.11. Calibration

Parameters are mostly calibrated following CEE and Schmitt-Grohe, Uribe (2007). We calibrate the steady-state debt-output ratio to be equal to 60% and the public spending to be the 20% of total output. Setting an habit persistence parameter, b, consistent to the empirical evidence (see Fuhrer (2000), McCallum and Nelson (1999), Boldrin, Christiano and Fisher (2001) or Christiano, Eichenbaum and Evans (2005)), i.e. between 0.6 and 0.9, RT steady state marginal utility of consumption would turn negative in case of external habit formation. We avoid this inconsistency setting a lower lever of habit persistence.

(3.49)

Parameter	Value	Description
b	0.53	degree of habit persistence
β	0.99	subjective discount factor
α	0.36	share of capital
δ	0.025	depreciation rate
$\frac{\eta}{\eta-1}$	1.2	price mark-up
$\frac{\alpha_w}{\alpha_w-1}$	1.2	wage mark-up
κ	2.48	parameter governing investment adjustment costs
λ_p	0.6	price stickiness
λ_w	0.64	wage stickiness
γ_1	0.0324	parameter governing capacity adjustment costs
γ_2	0.000324	parameter governing capacity adjustment costs
γ_p	1	indexation on prices
γ_w	1	indexation on wages
φ_l	1	preference parameter
ϕ_b	0.1	taxes response to changes in public debt
ϕ_g	0.33	taxes response to changes in public spending
φ_π	1.5	policy response to changes in inflation
φ_y	0.05	policy response to changes in output
θ	0.1	share of RT consumers

3.3. Business Cycle Characteristics

As shown in Dennis (2009), under the assumption of homogeneous agents, internal and external habits produce similar model responses to shocks. We replicate Dennis's results in Figures $1-2:$ the differences between model responses under internal rather than external habits are negligible when no RT consumer are present in the economy. The intuition behind this result is that the low intertemporal elasticity of substitution

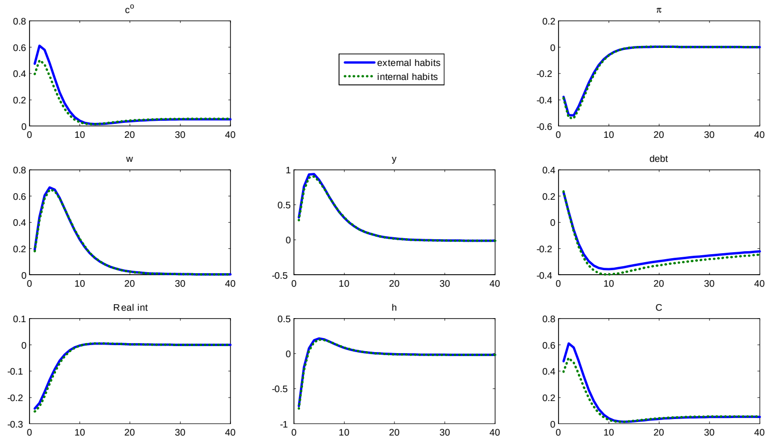


Figure 3.1. Figure 1 (technology shock)

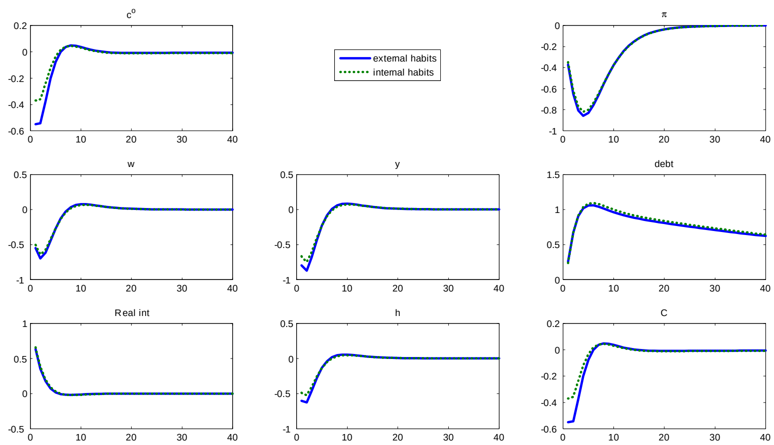


Figure 3.2. Figure 2 (monetary shock)

yields similar interactions between consumption and real interest rate regardless the habit formation type. In this case we compute a cross-correlation coefficient (when a monetary shock occurs) between Ricardian consumption (output) and real interest rate of -0.9299 (-0.8822) when habits are internal and -0.9309 (-0.8844) when habits are external.

The choice of internal rather than external habits turns crucial when we assume that a (even very small, i.e. 10%) share of the population does not participate to financial markets.

Figures $3-4$ display the model responses to different shocks under the two different habit formation specifications when RT consumers are taken into account.

The model dynamics under internal habit formation does not seem affected by the small share of RT consumer in the economy (too see this point, compare green dotted lines of figure $1-2$ with those in figure $3-4$) , but the responses to shocks under external habits changes quantitatively and qualitatively as soon as RT consumers are introduced.

The source of this alteration in model dynamics lies in the respective euler equations. Under internal habit formation the model euler equation is described by equation (3.50)[5]. Ricardians households consumption intertemporal choices are not affected by RT consumers and the resulting elasticity between consumption and real interest rate is (given the expectations): $-\frac{(1-\beta b)(1-b)}{b+(1+\beta b^2)}$ and does not depend by the share of RT consumers θ.

$$\begin{aligned}\hat{c}_t^o \;=\; & \frac{\beta b+(1+\beta b^2)}{b+(1+\beta b^2)}E_t\hat{c}_{t+1}^o+\frac{b}{b+(1+\beta b^2)}\hat{c}_{t-1}^o-\frac{\beta b}{b+(1+\beta b^2)}E_t\hat{c}_{t+2}^o+ \\ & +\frac{(1-\beta b)(1-b)}{b+(1+\beta b^2)}E_t\hat{\pi}_{t+1}-\frac{(1-\beta b)(1-b)}{b+(1+\beta b^2)}\hat{R}_t\end{aligned} \tag{3.50}$$

If we in turn consider external habit formation, RT consumers become crucial in determining Ricardian households consumption choices. The respective euler equation is given by equation (3.51)

$$\begin{aligned}\hat{c}_t^o \;=\; & \frac{b(1-\theta)}{1+b(1-\theta)}\hat{c}_{t-1}^o+\frac{b\theta\frac{c^{rt}}{C}}{\frac{c^o}{C}+b(1-\theta)\frac{c^o}{C}}\hat{c}_{t-1}^{rt}-\frac{b\theta\frac{c^{rt}}{C}}{\frac{c^o}{C}+b(1-\theta)\frac{c^o}{C}}\hat{c}_t^{rt}+ \\ & +\frac{1}{1+b(1-\theta)}E_t\hat{c}_{t+1}^o+\frac{\frac{c^o}{C}-b}{\frac{c^o}{C}+b(1-\theta)\frac{c^o}{C}}E_t\hat{\pi}_{t+1}-\frac{\frac{c^o}{C}-b}{\frac{c^o}{C}+b(1-\theta)\frac{c^o}{C}}\hat{R}_t\end{aligned} \tag{3.51}$$

It emerges that now Ricardian consumption depends on both current and past RT consumption level and its elasticity to real interest rate is increasing in the share of RT consumers θ. This property leads to model dynamics which are in sharp contrast with those under internal habit formation and, more important, with the empirical evidence (see Dennis (2009), CEE (2005), Smets and Wouters (2003)).

Intuitively, these differences stem from the basic concept of habit formation. It is indeed natural that under external habits the felicity and therefore the consumption choices

[5] Hatted variables represent log deviation from the steady state.

of Ricardian consumers crucially depend on the consumption of the RT. From this it is interesting to note that, given that RT consumers are (indirectly through the wage setting) sensitive to real interest movements, the higher the share of RT present in the economy, the higher the elasticity of Ricardian consumers to interest rate movements. This is what triggers the differences in the dynamics of the model depending on the habit formation assumed.

We focus now on the model dynamic properties when a monetary shock occurs (figure 4). The computed cross-correlation coefficients between Ricardian consumption (output) and the real interest rate under internal and external habits differ consistently and are respectively -0.9414 (-0.8758) and -0.4362 (-0.6704). The interaction between RT consumption and Ricardians consumption choices results in a dampened reaction of Ricardians consumption to changes in interest rate and therefore a lower correlation between output and interest rate.

External habit formation leads to a higher marginal utility of consumption, given the level of the latter. Wages are more elastic and a technology shock (figure 3) determines therefore an increase in consumption for optimizing agents, an increase in wages and a decrease in labour supply. A positive shock to the interest rate (figure 4) under external habits determines a fall in RT consumption which is internalize by Ricardian agents. Since current RT consumption enters in future Ricardians marginal utility positively, a fall in RT consumption makes Ricardian households more willing to consume today. The response of Ricardian agents to a monetary shock is therefore reverted together with the whole dynamics of this framework.

3.4. Conclusion

We have shown that the choice of external rather than internal habit formation become extremely important when a very low degree of heterogeneity is allowed. A medium scale New Keynesian model yields different responses to demand and supply shocks in presence of RT consumers when we consider internal rather than external habit formation.

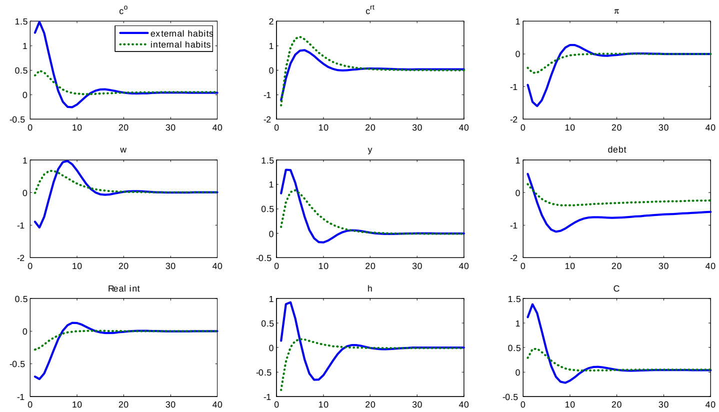


Figure 3.3. Figure 3 (Technology Shock)

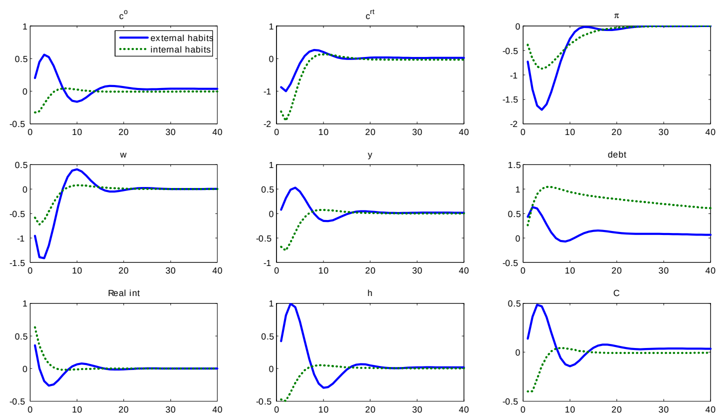


Figure 3.4. Figure 4 (Monetary Shock)

Moreover, under external habit formation, the impulse response functions generated by the model are in contrast with those we can find in the empirical literature Further research will focus on testing the robustness of our results under a generalized CRRA utility function and on studying the influence of the habit formation type for model stability. The reverted model dynamics under external habit formation induce us to believe that the determinacy properties and therefore the optimal policies derived in this framework may be strongly affected by the habits type. Finally we are going to test if automatic fiscal stabilizer may reduce the difference produced by the specific habit formation set-up and lead to a better empirical performance

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3.5. Appendix

3.5.1. Loglinearized Model

Aggregate consumption is defined by:

$$\hat{C}_t = (1-\theta)\frac{c^o}{C}\hat{c}_t^o + \theta\frac{c^{rt}}{C}\frac{wh}{c^{rt}}\hat{w}_t + \theta\frac{wh}{c^{rt}}\frac{c^{rt}}{C}\hat{h}_t - \theta\frac{c^{rt}}{C}\frac{\tau}{c^{rt}}t_t \tag{3.52}$$

Marginal costs are given by

$$\widehat{mc}_t = (1-\alpha)\,\hat{w}_t + \alpha\hat{r}_t^k \tag{3.53}$$

The following equation combines firms' F.o.c. with respect to production factors

$$\hat{h}_t + \hat{w}_t = \hat{k}_{t-1} + \left(1 + \frac{\gamma_1}{\gamma_2}\right)\hat{r}_t^k \tag{3.54}$$

Production function is given by

$$\hat{y}_t = \alpha\hat{k}_{t-1} + \alpha\frac{\gamma_1}{\gamma_2}\hat{r}_t^k + (1-\alpha)\,\hat{h}_t \tag{3.55}$$

Aggregate resource constraint

$$\hat{y} = \frac{i}{y}\hat{\imath}_t + \frac{c}{y}\hat{c}_t + \gamma_1\frac{\gamma_1}{\gamma_2}\frac{k}{y}\hat{r}_t^k + \frac{g}{y}g_t \tag{3.56}$$

RT consumption

$$\hat{c}_t^{rt} = \frac{wh}{c^{rt}}\left(\hat{w}_t + \hat{h}_t\right) - \frac{\tau}{c^{rt}}t_t \tag{3.57}$$

Euler equation

$$\hat{\lambda}_t^o = \hat{\lambda}_{t+1}^o + \hat{R}_t - \hat{\pi}_{t+1} \tag{3.58}$$

Households marginal utility of consumption under internal habits

$$\lambda_t^o = \frac{\beta b}{(1-\beta b)(1-b)} c_{t+1}^o - \frac{(1+\beta b^2)}{(1-\beta b)(1-b)} c_t^o + \frac{b}{(1-\beta b)(1-b)} c_{t-1}^o \tag{3.59}$$

$$\lambda_t^{rt} = \frac{\beta b}{(1-\beta b)(1-b)} c_{t+1}^{rt} - \frac{(1+\beta b^2)}{(1-\beta b)(1-b)} c_t^{rt} + \frac{b}{(1-\beta b)(1-b)} c_{t-1}^{rt} \tag{3.60}$$

Households marginal utility of consumption under external habits

$$\lambda_t^o = \frac{b}{\frac{c^o}{c} - b} C_{t-1} - \frac{\frac{c^o}{c}}{\frac{c^o}{c} - b} c_t^o =$$

$$\lambda_t^{rt} = \frac{b}{\frac{c^{rt}}{c} - b} C_{t-1} - \frac{\frac{c^{rt}}{c}}{\frac{c^{rt}}{c} - b} c_t^{rt}$$

Investment decisions

$$\hat{\imath}_t - \frac{1}{k(1+\beta)} \hat{P}_{k',t} - \frac{1}{(1+\beta)} i_{t-1} - \frac{\beta}{(1+\beta)} \hat{\imath}_{t+1} = 0 \tag{3.61}$$

$$\hat{\pi}_{t+1} + \beta(1-\delta)\hat{P}_{k',t+1} - \hat{P}_{k',t} = \hat{R}_{t+1} - \beta r^k \hat{r}_{t+1}^k \tag{3.62}$$

Capital accumulation

$$\hat{k}_t = (1-\delta)\hat{k}_{t-1} + \delta \hat{\imath}_t \tag{3.63}$$

Phillips Curve

$$\frac{\lambda_p}{1-\lambda_p}\left(\hat{\pi}_t - \gamma_p \hat{\pi}_{t-1}\right) = (1-\beta\lambda_p)\widehat{mc}_t + \beta\lambda_p\left(\hat{\pi}_{t+1} - \gamma_p\hat{\pi}_t\right) + \beta\frac{\lambda_p^2}{1-\lambda_p}\left(\hat{\pi}_{t+1} - \gamma_p\hat{\pi}_t\right) \tag{3.64}$$

Wage setting

$$\left[\begin{array}{c} \left(\frac{1}{1-\lambda_w}+\beta\frac{\lambda_w^2}{1-\lambda_w}\right)\hat{w}_t-\beta\frac{\lambda_w}{1-\lambda_w}\hat{w}_{t+1}+ \\ -\left(\beta\lambda_w+\beta\frac{\lambda_w^2}{1-\lambda_w}\right)\hat{\pi}_{t+1}+ \\ +\left(\beta\lambda_w\gamma_w+\beta\frac{\lambda_w^2}{1-\lambda_w}\gamma_w+\frac{\lambda_w}{1-\lambda_w}\right)\hat{\pi}_t+ \\ -\frac{\lambda_w}{1-\lambda_w}\hat{w}_{t-1}-\frac{\lambda_w}{1-\lambda_w}\gamma_w\hat{\pi}_{t-1} \end{array}\right]=\left(1-\beta\lambda_w\right)\varphi\hat{h}_t-\left(1-\beta\lambda_w\right)\left(\hat{\psi}_t\right) \tag{3.65}$$

where $\hat{\psi}_t$ represents the loglinearized weighted average of the tho households marginal utilities of consumption.

The government budget constraint is given by:

$$\hat{x}_t=\frac{g}{y}\hat{g}_t+\frac{1}{\beta}\frac{x}{y}\left(\hat{R}_{t-1}-\hat{\pi}_t\right)+\frac{1}{\beta}\hat{x}_{t-1}-\frac{\tau}{y}t_t$$