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Recent aspects of Seiberg duality:
metastable vacua, stringy instantons and M2 branes

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Contents

Introduction

Nowadays, the model which describes the fundamental interactions, in the particle physics, is the Standard Model. It is a quantum field theory, where the local gauge symmetry generates the gauge fields. These fields are the physical candidates to mediate the strong and electroweak interactions between the elementary particles. In the first case the local gauge symmetry is exact, while in the second one it is broken by the Higgs mechanism. The original Standard Model has to face the problem of the quadratic divergences of QFT which are at the origin of the so called hierarchy problem. Furthermore it seems natural to try to unify the above theories with gravity but it seems very difficult to combine QFT with general relativity, and the most promising attempt to overcome this problem is string theory. Now both the hierarchy issue as well as string theory require an additional global symmetry which ties bosons and fermions together, called supersymmetry. One can construct QFT with such a symmetry. It turns out that the vacuum state of such theories must have zero energy. From the phenomenological point of view supersymmetry has to be broken at low energy and can be restored at very high energy.

Another important aspect of supersymmetry is its relation with confining gauge theories. In many cases strongly coupled models admit a weakly coupled dual version. The use of duality has often helped the comprehension of physical phenomena. In supersymmetric gauge theories there exists a natural electric/magnetic type duality. The first historical example was the Montonen-Olive duality in $\mathcal{N}=4$. Many extensions have been studied in the following years. In this thesis we concentrate on the $\mathcal{N}=1$ case, namely the Seiberg duality [1]. This is an electric/magnetic duality relating the correlation functions of the the long distance physics of two different theories. The dual description is usually weakly coupled at low energy if the electric theory were free in the UV. Non perturbative properties of the electric theory at low energy are perturbatively calculated in the magnetic theory. Many test of validity of Seiberg duality have been performed. For example the global symmetries are the same in the dual theories, while the gauge symmetries, which are unphysical are not the same. Moreover the number of supersymmetric vacua, Witten index and superconformal indexes coincide for dual theories. Also the a-maximization has checked the consistence of duality.

Although the original duality matched two SQCD models, each one having a gauge group, Seiberg duality can be extended to theories with many gauge groups. These are the quiver gauge theories, which are associated, by the gauge/gravity correspondence, to the gauge theories living on D3 branes wrapped on Calabi-Yau (CY) singularity. In these cases all the groups are gauge groups and the matter fields are two index tensors charged under

the adjoint or the bifundamental representations of the gauge groups. The rules of duality are the same as in the SQCD case, but the number of equivalent theories is enlarged.
Recently new applications of Seiberg duality have received a large attention. Here we discuss some of them like the ISS model of metastable supersymmetry breaking, the exotic contribution of a class of stringy instanton and the $\mathrm{AdS}_4/\mathrm{CFT}_3$ correspondence.

The first application that we discuss is the existence of ISS [2] metastable vacua in SQCD. The phenomenological application of many models of supersymmetry breaking have been excluded by the constraints on the soft masses. Some of these models are recovered by requiring an hidden sector of supersymmetry breaking. In the past the study of the properties of the supersymmetry breaking hidden sectors has been problematic because many models were strongly coupled in the IR. This complication made the analysis only qualitative, and the spectra were often uncalculable. As we explained above duality is helpful in the analysis of the strongly coupled sectors.
Indeed the ISS model uses Seiberg duality to build a hidden sector of supersymmetry breaking. This is a SQCD model with N_F massive flavour of quarks. The choice of the values of N_F and N_C makes the electric theory free in the UV regime and the dual theory free in the IR. This theory has supersymmetric vacua and does not seems a viable candidate for an hidden sector. Nevertheless in the dual version of the theory some new vacua appear. These are supersymmetry breaking metastable vacua, emerging at weak coupling in the gauge theory. Seiberg duality guarantees that these vacua are also vacua of the electric theory, but the strong couplings made them too difficult to find there. Metastability can be regarded as a problem for these vacuum states. However, as long as they have long lifetime, they are phenomenologically acceptable vacua.
Metastable vacua are rather generic when the supersymmetry breaking sector is coupled with the MSSM. Supersymmetric vacua usually arise from this coupling and the supersymmetry breaking vacuum becomes metastable, if tachyons are not generated. This implies that the only difference with the ISS model is that metastability here already is accepted in the hidden sector. In this way many calculable and natural examples of supersymmetry breaking are found.

The ISS model leaves many phenomenological problems opened. It cannot be used for gauge mediating supersymmetry breaking, because R-symmetry is not broken. Here an approximate R-symmetry is preserved around the supersymmetry breaking vacuum. This symmetry forbids a gaugino mass term in the perturbative expansion of the Lagrangian, also at higher loops.
For this reason it is important to find phenomenologically appealing deformations of the ISS model. One can try to add explicit R-symmetry deformations. This usually restores supersymmetry [3] but as we will show later it is not always the case. Another possibility is that the R-symmetry is broken at loop level by the scalar potential, as in [4].
Another interesting question is the relation of the ISS model with the gauge/gravity duality. Some progress has been made by studying the interactions among systems of intersecting branes. Here Seiberg duality is the statement that the moduli space of two system of branes, obtained after an exchanging of two NS branes, coincide. Many models of metastable vacua have been worked out from system of branes [5, 6, 7, 8, 9]. Another

promising possibility comes from theories of fractional branes wrapped on CY_3 singularities. Usually the geometric deformations of the singularity add new terms to the superpotential of the underlying quiver gauge theories. Metastable vacua are ubiquitous in these models [10, 11, 12, 13].

A second application of duality that we discuss is the relation with non-perturbative exotic contribution that arise from stringy instantons.

Usually instantons provide non perturbative contributions to the gauge theories. In supersymmetry only the action for the instantonic zero modes is non vanishing. It is obtained from the ADHM construction [14]. The non perturbative contribution due to the instantons is calculated by integrating the action over the bosonic and fermionic zero modes. This field theory construction has a natural interpretation in string theory, especially in theories of branes at a singularity [15]. The dimensional reduction from ten dimensional supergravity, in a flat background, of a system of D3 branes and D(-1) branes, recovers the ADHM construction of the instantonic action of $\mathcal{N} = 4$ SYM. By orbifolding and higgsing the $\mathcal{N} = 4$. This is a dynamical phenomenon due to the cascading behaviour. Indeed the addiction of fractional branes usually breaks the conformal invariance and the theory flows to lower energy through many steps of Seiberg dualities. This flow can lead to $N = 0$ or $N = 1$ rank for some of the gauge groups of the quiver. It happens when there are zero or one fractional branes, wrapping a singularity. Non perturbative instantonic effects in these theories are associate with D(-1) branes on the D3. These branes are instantons for the corresponding gauge group. If a D(-1) is put on a stack of N D3 branes one has a gauge instanton, and the usual non perturbative contribution to the action is recovered, for example the Affleck Dine Seiberg (ADS) superpotential. However there can be one D(-1) brane on trivial nodes (where no brane or only one brane is wrapped). This is a stringy instanton, whose contribution to the superpotential, if not vanishing, is exotic. The field theory interpretation of this phenomenon is not immediate, and it resides in the cascading nature of the gauge theory. Indeed the non perturbative contribution is explained by going one step back in the cascade. This is possible because of the involutive nature of duality. The non perturbative contribution from the stringy instanton is equivalent to the classical constraint on the moduli space that has to be added to the superpotential of the dual theory.

The last application of Seiberg duality arises in the recently found AdS_4/CFT_3 correspondence, where the CFT_3 theory is a CS matter theory. The basic example is the ABJM model that describes the motion of M2 branes in the S^7/Z_k background. We restrict our interest to the dynamics of $M2$ branes at toric CY_4 singularities. The dual field theory is conjectured to be an $\mathcal{N} = 2$ CS matter theory. The brane picture shows that there are more equivalent phases of these theories.

The rules of duality for the superpotential are the same of that in four dimensions. Nevertheless, there is a difference in the transformation of the gauge groups, which depends on the CS levels [19], the CS levels are changed by duality. The brane picture is applied only to non-chiral theories, where every edge of the quiver have both an ingoing and an outgoing arrow. As in four dimensions the classical moduli space of these theories is associated to their toric diagrams. The theories obtained by displacing the branes have

the same toric diagrams. This relation is called toric duality in four dimensions, where it coincides with Seiberg duality. It seems natural to call this three dimensional duality Seiberg duality. Moreover the toric diagrams can be used also to guess some rules for the chiral case. This duality relates only a subset of three dimensional theories. This is not the end of the story because in three dimensions equivalent theories could arise from theories with different number of groups, from mirror symmetry, but we will not investigate these models.
Since this duality is a strong/weak duality, one can study metastable supersymmetry breaking in three dimensions. It was firstly done in [20]. The presence of relevant deformations is rather generic, and the perturbative analysis requires non trivial constraints on the couplings and the scales of the theory. Marginal couplings can in principle smooth these constraints. An important role is played by the R-symmetry. In all the cases where the perturbative approximation holds this symmetry is broken, explicitly or perturbatively.

The thesis is organized as follows.
In chapter 1 we give an overview of the ideas of duality that are necessary through the whole analysis. We focus on duality in the magnetic free windows, where the electric theory is UV free and the dual magnetic one is IR free. We see the mapping of the field theory deformations between the dual theories. Extensions of Seiberg duality to SQCD with adjoint matter, the KSS [21, 22, 23] duality, is also discussed. In the rest of the section we show the action of Seiberg duality in systems of intersecting branes and branes wrapped on CY singularity. In particular we see the action of the duality on a quiver and on the brane tilings.
In chapter 2 we study the connection of Seiberg duality and supersymmetry breaking. We first review the fundamental aspects of spontaneous supersymmetry breaking. Then we discuss the ISS models concentrating on the question of its embedding in the gauge mediation scenario. In the rest of the chapter we explain some extensions of the ISS models that solve some of these problems. We see that theories with adjoint matter, if deformed by some superpotential terms, lead to R-symmetry breaking metastable vacua. Also A_n quiver gauge theories have good properties for gauge mediation. We then present a systematic study of the geometric deformation that leads to metastable vacua and supersymmetry restoration in quiver gauge theory. We conclude the chapter with an analysis of spontaneous R-symmetry breaking at two loops.
In chapter 3 we analyze the connection of Seiberg duality and the exotic contribution to the superpotential from the stringy instantons. We first review the brane construction of the ADHM action for $\mathcal{N} = 4$ SYM and the extension to lower supersymmetry. Than we compute the contribution of a stringy instanton on a node of the quiver with $SU(1)$, $SP(0)$ and $SO(3)$ "gauge" groups. The gauge theories are trivial in these cases, and no contribution is expected. The fact that the contribution is non vanishing seems a pure stringy effect. We show that a connection with field theory exists, and it can be understood as a consequence of Seiberg duality.
In chapter 4 we explore duality among $\mathcal{N} = 2$ CS matter theories representing the motion

of N M2 branes on a toric CY_4 singularity. First we describe the $\mathcal{N} = 6$ ABJM theory, representing M2 on S^7/Z_k and we concentrate on Sasaki Einstein manifolds with toric CY singularities. These cases have lower supersymmetry and we study the $\mathcal{N} = 2$ situation. The rules of duality are worked out as in [19] by using the motion of the corresponding systems of branes. We find some general rules for the case of non-chiral theories, and we verify their validity with the techniques of toric geometry. We give also some examples of duality in chiral theories. We have not found general rules, but we show that in many cases, setting some of the CS levels to zero, the rules of the non chiral case still hold. We conclude the chapter with an analysis of supersymmetry breaking in three dimensions, useful to connect the duality just found with metastable vacua as in four dimensions. We conclude the discussion by proposing new applications and further developments.
After the conclusions we add some useful appendices.
In appendix A we discuss the fundamental aspects of quiver gauge theories, for ADE and for toric CY singularities. Then we explain the geometric generation of the deformation necessary for the ISS mechanism in these quivers. Details on the hierarchy among the scales of the gauge groups for the stability of the metastable vacuum are given. Moreover we show how to set some nodes of the quiver to zero, selecting the branch of the moduli space in which metastable vacua are placed. We conclude the appendix by reviewing the techniques of the quantum analysis and the calculation of the bounce action for the estimation of the lifetime of the metastable state.
In appendix B we first review the representation of spinors in 4, 6 and 10 dimensions necessary to work out the instantonic action. We illustrate the generic form of the instantonic contribution for an instanton on a $SU(1)$ gauge node. The constant contribution obtained by integrating over the bosonic zero modes is discussed. Finally we discuss the relation of our results and the ones already studied in the stringy instanton literature.
The last appendix C regards three dimensional gauge theories. We first explain the parity anomaly matching, an analog of the four dimensional t 'Hooft anomaly matching, but for a discrete parity symmetry. We evaluate the bounce action for a three dimensional potential barrier, in order to estimate the lifetime of a metastable vacuum in three dimensions. We end the appendix with a formula that calculates the Coleman Weinberg one loop potential in every dimensions.

Chapter 1

Seiberg duality

In this chapter we review the basic aspects of $\mathcal{N} = 1$ UV/IR duality. The basic example is referred to as Seiberg duality [1], where an $SU(N_c)$ "electric" gauge theory with N_f flavours of quarks possesses a dual description in terms of N_f "magnetic" flavours of quarks charged under a new $SU(N_f - N_c)$ gauge group. In the dual theory the gauge singlets, the elementary degrees of freedom, interacts with the flavours throughout superpotential term.
Duality means that the two theories describe the same physics at low momentum or at long distances, i.e. in the far infrared. Indeed, after matching the same gauge invariant operators, the Green functions become identical in the limit of zero external momenta.
Usually this duality relates the infrared physics of a strongly coupled theory with the physics of a weakly coupled theory. As a matter of fact it is not always the case, since sometimes both the electric and the magnetic theory can be strongly coupled.

In our work Seiberg duality plays a crucial role. In the case of metastable vacua the duality gives us a magnetic model weakly coupled in the infrared. In this way the model becomes calculable, and many aspects of supersymmetry breaking can be studied without the problems of strong couplings.
In the case of stringy instantons we show that the mapping of non-perturbative terms of Seiberg duality can be interpreted as an instantonic contribution also in the case of $N_F = N_C + 1$ where the dual theory does not possesses a dual gauge group.
Many aspects of 4D duality are present in 3D CFT. In $3D$ the duality group is larger than in $4D$. Nevertheless, under some restrictions and by using some rules from the brane engineering of the gauge theory, we show that a subgroup of duality inherited from $4D$ Seiberg duality still exists.

Firstly we give an introduction of Seiberg duality. We explain the details of the IR magnetic free window, we analyze the matchings of the scales, the renormalization group flow and the supersymmetric vacua. We discuss the behaviour of the non-renormalizable interactions in the dual phases, since they are common in the quiver gauge theories that play a central role in the subsequent chapters. Duality with adjoint matter (KSS duality [23]) is reviewed for its role in the study of metastable vacua. We conclude the chapter by giving the rules of Seiberg duality in systems of intersecting branes, in quiver gauge theories and in brane tilings.

1.1 Generalities of Seiberg duality

Duality is an exact symmetry, acting on the scale invariant holomorphic coupling τ in $\mathcal{N}=4$ and in certain conformal $\mathcal{N}=2$ supersymmetric gauge theories.

In asymptotically free theories the coupling constant τ runs as

$$e^{2\pi i\tau(E)} = e^{-\frac{8\pi^2}{g(E)^2}+i\theta} = \left(\frac{\Lambda}{E}\right)^{b_0} \tag{1.1}$$

and is replaced by the dynamically generated Λ. The same duality cannot act in these models. However in the abelian coulomb phase of $\mathcal{N}=2$ theories duality on the τ_{eff} has been shown in [24].

In $\mathcal{N}=1$ gauge theories one can demonstrate [1] that an electric-magnetic duality exists in $\mathcal{N}=1$ supersymmetric non-Abelian gauge theories. The archetypal theory is SQCD with $SU(N_C)$ gauge groups and N_F flavours. The anomaly free global symmetry is

$$SU(N_F)\times SU(N_F)\times U(1)_B\times U(1)_R \tag{1.2}$$

where the quarks Q and $\tilde{Q}$ transform as

	$SU(N_F)$	$SU(N_F)$	$U(1)_B$	$U(1)_R$
Q	N_F	1	1	$\frac{N_F-N_C}{N_F}$
$\tilde{Q}$	1	$\bar{N}_F$	-1	$\frac{N_F-N_C}{N_F}$

(1.3)

This theory has different behaviors for different values of N_C and N_F. In the case $N_F<N_C$ an ADS superpotential is generated and the theory does not have a ground state.

The case $N_F\geq N_c$ has a more rich dynamics. The quantum theory has a moduli space of inequivalent vacua. In the case $N_F=N_C$ the moduli space of vacua is found only after adding quantum effects, while for $N_F=N_C+1$ the classical and the quantum moduli space coincide. This last property of the moduli space still holds in the case $N_F\geq N_C$. Seiberg duality conjectures that the theory at the origin of the moduli space of mesons and baryons is in a non-Abelian Coulomb phase. This is clear for the $N_F\geq 3N_C$ theory, which is not asymptotic free in the UV. Indeed this theory is an IR free theory of interacting gluons and quarks, with a Landau pole in the UV.

The other windows in which Seiberg duality exists are $3/2N_C\leq N_F\leq 3N_c$ and $N_c+2\leq N_F\leq 3/2N_C$.

In the first case it has been argued that the theory at the origin of the moduli space is an interacting conformal field theory of quarks and gluons, and this theory has two dual descriptions. When one of them is in a weakly coupled Higgs phase its dual is in a strongly coupled confining phase. This window is referred to as the "conformal window". In the $N_F\geq 3N_C$ phase the original theory is IR free and strongly coupled in the UV.

The dual magnetic theory is then strongly coupled in the IR. This phase is referred as the "electric free window" The last possibility is $N_c + 2 \leq N_F \leq 3/2 N_C$, where the electric theory is free in the UV and the magnetic phase is weakly coupled in the IR. This window is referred as the "magnetic free window".

In all the cases the magnetic gauge group is $SU(N_F - N_C)$, and the global symmetries are the same. These last ones are indeed observables, and cannot change in the two descriptions. The gauge symmetries are redundancies more than symmetries. The two equivalent descriptions have a different number of gluons since duality make sense only in interacting scale invariant theories, where the particle interpretation is not well defined and different sets of massless interacting particle can describe the same physics.

Following the historical developments we start now by describing the rules and checks of duality in the conformal window and then we explain the behaviour in the magnetic free window.

The conformal window

The beta function must have a non-trivial zero, since the low energy theory must be scale invariant. In SQCD the NSVZ beta function is exact [25] and reads

$$\beta(g) = -\frac{g^3}{16\pi^2} \frac{3N_C - N_F(1-\gamma(g^2))}{1 - N_C \frac{g^2}{8\pi^2}} \tag{1.4}$$

where $\gamma(g^2)$ is the anomalous dimension of the mass and it is

$$\gamma(g^2) = -\frac{g^2}{8\pi^2} \frac{N_C^2 - 1}{N_C} + \mathcal{O}(g^4) \tag{1.5}$$

The Banks-Zaks fixed point exists for large N_c and $3 - \frac{N_F}{N_C} \ll 1$, and it is conjectured in all the conformal window.

The existence of a fixed point in a supersymmetric gauge theory guarantees the use of the superconformal algebra and specially the superconformal R-symmetry. The dimensions of the operators satisfy

$$D \geq \frac{3}{2}|R| \tag{1.6}$$

and the equation is saturated for chiral and anti-chiral operators, i.e. chiral operators form a ring. Moreover the R-symmetry is not anomalous and commute with the flavour symmetry $SU(N_F) \times SU(N_F)$. The gauge mesonic invariant operators have

$$D(M) = \frac{3}{2} R(Q\tilde{Q}) = 3\frac{3N_F - N_C}{N_F} \tag{1.7}$$

and the baryonic ones have

$$D(B) = D(\tilde{B}) = \frac{3N_C(N_F - N_C)}{2N_F} \tag{1.8}$$

The gauge invariant operators must be in unitary representation; for example spinless operators have $D \geq 1$, except the identity operators which has $D = 0$, and $D = 1$ for the free fields. Indeed for $D < 1$ ($D \neq 0$) a highest weight representation has negative norm states, which are not unitary, and (1.7) is inconsistent if $N_F < 3/2 N_C$

One concludes by aostulating that in the whole conformal window the theory at the origin of the moduli space is in a non Abelian Coulomb phase.

Duality

In the conformal window, by fixing N_C and lowering N_F one notes that the interactions become more strongly coupled. If there is a strongly coupled massless theory the spectrum of massless particle does not make sense. indeed the same theory has an equivalent description in terms of some new "dual" degrees of freedom. This dual theory becomes more weakly coupled as N_F decreases in the conformal window. This idea still holds in the magnetic free window, where only the dual theory make sense in the IR as a unitary theory ($D \geq 1$).

The dual theory is built as follows. In the region $N_F \leq N_C + 1$ the moduli space is not quantum corrected, and there is a vacuum at the origin of the moduli space of the gauge invariant operators, where the global symmetry is unbroken. The theory has here N_f flavors. A $SU(N_C)$ gauge group cannot preserve the unitarity of the theory, since $D \geq 1$ does not hold for all the spinless operators (except the identity). A possibility is that the dual description has the same global symmetry but a $SU(\tilde{N}_c) = SU(N_F - N_C)$ gauge symmetry. In this case both the conformal and the magnetic windows are consistent as unitary theories. The theory in the conformal window has an IR fixed point that becomes free in the magnetic window.

The gauge invariant operators in the dual description coincide with the ones of the original theory. The quarks q and $\tilde{q}$ of the dual theory are elementary. They are new fields that transform respectively under the fundamental and the anti-fundamental representation of the dual gauge group. The baryons are determined by the relations $B \sim q^{N_F - N_F}$ and $\tilde{B} \sim \tilde{q}^{N_f - N_C}$ and the dual elementary quarks transform under the global symmetries as

	$SU(N_F)$	$SU(N_F)$	$U(1)_B$	$U(1)_R$
q	$\bar{N}_F$	1	$\frac{N_C}{N_F - N_C}$	$\frac{N_C}{N_F}$
$\tilde{q}$	1	N_F	$-\frac{N_C}{N_F - N_C}$	$\frac{N_C}{N_F}$

The meson $M \sim Q\tilde{Q}$ cannot be constructed in the dual theory, and it appears as an elementary field. It is charged under the global symmetries as

$$\text{M} \quad \text{in} \quad (N_F, \bar{N}_F, 0, 2\left(\frac{N_F - N_C}{N_F}\right) \tag{1.9}$$

A superpotential term is compatible with the symmetries of the theory, and it is required by duality

$$W = \frac{1}{\hat{\Lambda}} M q \tilde{q} \tag{1.10}$$

where the trace over the color and flavor indexes is understood. The dual theory has a new meson, $N \sim q\tilde{q}$ that should make the duality inconsistent. It is not the case since it is a redundant operator, that can be absorbed in a shift of M in (1.10).

The intermediate scale $\hat{\Lambda}$ in necessary, indeed the meson in the electric theory has mass dimension two at the UV fixed point. It acquires an anomalous dimension at the IR fixed point. In the magnetic theory the meson (say M_m) is and elementary field with mass dimension one at the UV fixed point, but it flows to the same IR fixed point of the electric case. It is then necessary to relate M and M_n, and this is done by introducing the scale $\hat{\Lambda}$, via the equation $M = M_n \hat{\Lambda}$. The gauge strong coupling scale of the magnetic theory $\tilde{\Lambda}$ is related to the electric Λ scale by the relation

$$\Lambda^{3N_C - N_F} \tilde{\Lambda}^{3(N_F - N_C) - N_F} = (-1)^{N_F - N_C} \hat{\Lambda}^{N_F} \tag{1.11}$$

This scale matching relation shows that if the electric theory is strongly coupled than the magnetic theory is weakly coupled. Another important consequence of this relation is that this duality is an involution, and that the dual of the magnetic theory is the electric theory itself.

This duality is of the electric/magnetic type since, as N_F decreases, the electric theory becomes more strongly coupled and the magnetic theory becomes more weakly coupled. In the conformal window, where the theory at the origin is in an interacting non-Abelian Coulomb phase, the electric theory is more weakly coupled for $N_F > 2N_c$ while the magnetic picture is more natural for $N_F < 2N_C$.

’t Hooft anomaly matchings

Strongly coupled quantum field theories are usually described in terms of their composite fields. A non trivial check of consistency is the matching of the anomalies between the constituent degrees of freedom and the composite fields: this is also a non-trivial check of consistency for Seiberg duality. Here we review the idea of four dimensional ’t Hooft anomaly matching. In the three dimensional case no anomalies are present for the continuous symmetry, but similar checks for the discrete parity symmetry exist. We discuss it in the appendix C.2. A gauge theorie with Weyl fermions present triangular anomalies. If G is the gauge group and the fermions are in the R representation of G, the triangular Feynman graph in figure 1.1 gives the anomalous contribution

$$d_R(T^a, T^b, T^c) = \mathrm{Tr}_R \left(T^a \{T^b, T^c\} \right) \tag{1.12}$$

In the case of ’t Hooft anomaly matchings the global symmetries (that coincide between the dual theories) are slightly gauged. This gauging couples the fermions with new gauge bosons, and triangular anomalies arise.

In this case the global symmetry is $SU(N_F) \times SU(N_F) \times U(1)_R \times U(1)_B$. This new gauge theory is not anomaly free and we add to add new fermions charged under a R' representation such that the whole theory becomes anomaly free, i. e. $d_R(T^a, T^b, T^c) + d_{R'}(T^a, T^b, T^c) = 0$. The dual description describe the same low energy physics, but it has

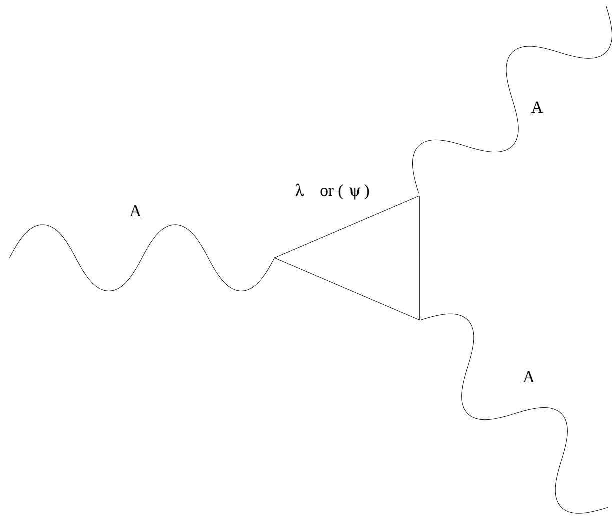


Figure 1.1: Anomalous triangular Feynman graph

a different matter content and the fermions are in a new representation $\tilde{R}$. Nevertheless the global symmetry are the same as before. Gauging the global symmetry in the magnetic theory we have to add the same new fermions R' to cancel the triangular anomalies. This implies that

$$d_R(T^a, T^b, T^c) + d_{\tilde{R}}(T^a, T^b, T^c) = 0 \tag{1.13}$$

and this is the 't Hooft anomaly matching. In SQCD the triangular anomalies in the electric and in the magnetic case match as follow

$$\begin{array}{cc} SU(N_F)^3 & N_c \\ U(1)_B SU(N_F)^2 & \frac{N_C}{2} \\ U(1)_R SU(N_F)^2 & -\frac{N^2}{2N_F} \\ U(1)_B^3 & 0 \\ U(1)_B^2 U(1)_R & -2N_C^2 \\ U(1)_B U(1)_R^2 & 0 \\ U(1)_R^3 & -\frac{2N_C^4}{N_F^2} + N_c^2 - 1 \\ U(1)_R & -N_C^2 - 1 \\ U(1)_B & 0 \end{array} \tag{1.14}$$

The magnetic free window

Also in the window $N_C + 2 \leq N_F \leq 3/2 N_C$ the magnetic theory can be described as above. However the dual theory is in a different phase. Indeed the b_0 factor of the beta function is negative

$$b_0 = 3\tilde{N}_c - N_F = 3(N_F - N_C) - N_F = 2N_F - 3N_C < 0 \tag{1.15}$$

and the theory is not asymptotically free (the added superpotential (1.10) becomes irrelevant in the IR). There are massless magnetically charged fields, and the theory is in a non-Abelian free magnetic phase. Since this dual theory is free there cannot be two different descriptions, and only the magnetic description does make sense. Indeed in the electric phase is $D < 1$ is allowed, while here $D(M) = 1$, as in every free field theory.

Mass deformation

A gauge invariant term that is often added to the SQCD is a mass term for some of quarks. We here discuss the case where only one quark is massive. A general discussion (where all the quarks remains massless) follows directly. Let's take the deformation

$$\Delta W_{ele} = m_Q Q_{N_F} \tilde{Q}^{N_F} \tag{1.16}$$

where the sum over the gauge indexes is understood.

This mass term reduces of one units the number of light quarks in the IR, and the electric theory is sent to a more strongly coupled fixed point at low energies. The new scale of the theory is Λ_L, related to the original scale Λ by the relation

$$\Lambda_L^{3N_c-(N_f-1)} = m\Lambda^{3N_c-N_F} \tag{1.17}$$

In the magnetic theory this mass is mapped into a linear deformation on the meson M, and the superpotential is modified by the term

$$\Delta W_{magn} = m M_{N_F}^{N_F} \tag{1.18}$$

The gauge group is higgsed to $SU(N_F - N_C - 1)$ with $N_F - 1$ light quarks, and at low energy the superpotential is

$$W = M_{\bar{i}}^{i} \tilde{q}_i q^{\bar{i}} \quad i, \bar{i} = 1, \ldots, N_F - 1 \tag{1.19}$$

The scale of the magnetic theory is modified from $\tilde{\Lambda}$ to $\tilde{\Lambda}_L$ by the relation

$$\tilde{\Lambda}_L^{3(N_F-N_C-1)-(N_F-1)} = -\frac{\tilde{\Lambda}^{3(N_F-N_C)-N_F}}{\langle q_{N_F} \tilde{q}^{N_F} \rangle} \tag{1.20}$$

where $\langle q_{N_F} \tilde{q}^{N_F} \rangle = -\mu m$ from the equation of motion. The magnetic theory is the dual of the massive electric theory and it is at weaker coupling at low energy. One concludes that duality is preserved by the addition of a mass term, and it relates a more strongly coupled theory to a more weakly coupled one.

The case with N_F massive flavours does not change qualitatively, and the same conclusions hold. The only difference is that the scale matching among the UV theories and their low energy limit changes.

The case $N_F = N_c + 2$

This case is slightly different because a mass term for the quarks completely higgses the magnetic gauge group.

The meson of the low energy theory is $M'^{j}_{\tilde{i}}$, where the apex refers to the flavor indexes $i, j = 1, \ldots, N_C + 1$. Similarly the quarks of the low energy theory are q' and $\tilde{q}'$, which are the the flavor that remains massless. Since at low energy $N_F = N_C + 1$ the magnetic quarks are the baryons of the low energy electric theory. The superpotential at low energy is

$$W_{eff} = \frac{1}{\Lambda_L^{2N_c-1}} M' q' \tilde{q}' \tag{1.21}$$

where Λ_L represents the scale of the low energy effective theory, which has only the light fields. The magnetic $SU(2)$ theory is completely higgsed, and the instanton contribution should be included, as in the $N_f = N_C - 1$ case. The superpotential has an additional contribution[1]

$$W_{inst} = -\frac{\det M'}{\Lambda_L^{2N_C-1}} \tag{1.22}$$

This is the superpotential for the electric theory in the $N_F = N_c + 1$ case. In that case it was due to the strong coupling effects. This derivation is in terms of instanton calculation in the weakly coupled theory. We will show that it has also an interpretation in term of stringy instanton.

Quartic deformations

Another common deformation in the electric theory is a non renormalizable term of the form

$$\Delta W_{\text{quartic}} = h \text{Tr}(Q\tilde{Q}Q\tilde{Q}) \tag{1.23}$$

The coupling h has mass dimension -1 and the operator is classically irrelevant. This non-renormalizable superpotential can be thought as arising from a renormalizable three-linear interaction of the quarks with a flavour singlet massive field.

In the magnetic window the dual theory has superpotential

$$W_{\text{magn.}} = h' m M^2 + h' M q \tilde{q} \tag{1.24}$$

where h' is marginal and m is a mass term for the meson M. This implies that integrating out this mass term we have the same superpotential as in the electric case

$$W = \frac{h'}{2m} q\tilde{q}q\tilde{q} \tag{1.25}$$

In the case $N_f = 2N_c$ duality is not dynamical, the theory is self dual. For different values of N_f and N_c self duality is a only a property of the superpotential.

[1]This contribution can be also calculate from the gluino condensation.

1.2 KSS duality

One of the first extensions of Seiberg duality was studied in [21, 22, 23]. The $SU(N_c)$ SQCD with N_f flavors of quarks is deformed by a superpotential polynomial in a field X, in the adjoint representation of the gauge group.

We here review some aspects of this duality which will be necessary for the study of metastable vacua. The simplest superpotential is

$$W = \frac{s_0}{k+1}\text{Tr}X^{k+1} \tag{1.26}$$

We shall discuss only the case of renormalizable polynomial superpotential in the adjoint field, and limit to the case of $k = 3$. The higher order polynomial terms seem irrelevant for the long distance physics in the infrared. On the contrary they strongly change the IR dynamics: they are dangerously irrelevant operators.

One can also consider a generic superpotential

$$W = \sum_{i=1}^{k} \frac{s_i}{k+1}\text{Tr}X^{k+1-i} \tag{1.27}$$

The $SU(N_c)$ and the $U(N_c)$ cases are slightly different. The linear monomial term TrX is possible in the $U(N_c)$ case but it has to be added as a Lagrange multiplier in the $SU(N_c)$ case to force the tracelessness condition on X.

Using the transformation of the complexified gauge group one can diagonalize the matrix X. For generic $\{s_i\}$ one has

$$W'(x) = \sum_{i=0}^{k-1} s_i x^{k-i} + \lambda \equiv s_0 \prod_{i=1}^{k}(x - c_i) \tag{1.28}$$

where the c_i are different from each other. In the IR the theory splits into different decoupled sets of SQCD theories, whose ground states are labeled by a sequence of integer r_i such that $r_1 \leq r_2 \leq \cdots \leq r_k$, where r_i represents the number of eigenvalues in the l-th minimum of the scalar potential. If λ is a Lagrange multiplicator, the traceless condition reads

$$\sum_{i=1}^{k} c_i r_i = 0 \tag{1.29}$$

Integrating out the massive X in each vacuum the $SU(N_c)$ [2] gauge symmetry is broken to

$$\prod_{i=1}^{k} SU(r_i) \times U(1)^{k-1} \tag{1.30}$$

[2]in the $U(N_c)$ case there is an additional $U(1)$ factor.

Note that turning on small s_i and using the fact that the theory has vacua only if and only if all $r_i \leq N_f$ one deduce that the theory has vacua if and only if

$$N_f \geq \frac{N_c}{k} \tag{1.31}$$

The magnetic theory has gauge group $SU(kN_F - N_C)$ and the matter content is constituted by the magnetic quarks, q and $\tilde{q}$, the magnetic adjoint field Y, and the mesonic gauge singlets. In this case the mesons take the form

$$(M_j)^i_{\tilde{i}} = \tilde{Q}_{\tilde{i}} X^{j-1} Q^i \quad i, \tilde{i} = 1, \ldots, N_F \quad j = 1, \ldots, k \tag{1.32}$$

The superpotential for the case (1.26) is

$$W_{magn} = \frac{\bar{s}_0}{k+1} \mathrm{Tr} Y^{k+1} + \frac{s_0}{\hat{\Lambda}^2} \sum_{j=1}^{k} M_j \tilde{q} Y^{k-j} q \tag{1.33}$$

The auxiliary scale $\hat{\Lambda}$ is related to the scales of the electric and magnetic theories by the relation

$$\Lambda^{2N_c - N_F} \tilde{\Lambda}^{2(kN_F - N_C) - N_F} = s_0^{-2N_F} \hat{\Lambda}^{2N_F} \tag{1.34}$$

Note that in this case (with only one term in the electric superpotential) the $\bar{s}_0$ in (1.33) can be choose $\bar{s}_0 = -s_0$. In the general cases it will not be always possible. Indeed if we take the dual theory of (1.27) to be

$$W = \sum_{i=0}^{k} \frac{\bar{s}_i}{k+1-i} \mathrm{Tr} X^{k+1} + \alpha(s) \tag{1.35}$$

the $\bar{s}_i = \bar{s}_i(s)$ are the magnetic coupling constant and $\alpha(s)$ is a constant. The problem of KSS duality is to find the expression of s_i and α in terms of the original magnetic variables.

The classical vacua for this magnetic theory are parameterized by some integer $\bar{r}_l$ corresponding to the eigenvalues of Y with the value $\bar{c}_l$. As in the electric case the relation

$$\sum_l \bar{r}_l = \tilde{N}_c = kN_F - N_C \tag{1.36}$$

holds. The gauge group is broken to $\prod SU(r_l) \times U(1)^{k-1}$. The magnetic multiplicities are related to the electric ones by the relation

$$\bar{r}_i = N_F - r_i \tag{1.37}$$

and it is a $1-1$ map between the electric and magnetic vacua [3]. The coincidence on the number of critical points c_i and $\bar{c}_i$ is a constraint on the dual coupling $\bar{s}_i(s)$. A possible solution is

$$\bar{s}_i = c s_i \tag{1.38}$$

[3]In the case of degeneracy the relation is n_i the relation is modified by $\bar{r}_i = n_i N_F - r_i$

with c constant. The convention of KSS is $\bar{s}_0 = -s_0$. This sets $c = -1$ and from equation (1.38) it would imply $\bar{c}_i = c_i$, that automatically satisfies the constraint on the degeneration of the singularities. In the general case the mapping (1.37) of the multiplicities is non trivial. Indeed in this case the traceless of X implies

$$\sum_{l=1}^{k} c_l r_l = 0 \tag{1.39}$$

and assuming (1.37) the traceless of Y should imply

$$\sum_{l=1}^{k} c_l (N_F - r_l) = 0 \tag{1.40}$$

which is compatible with (1.39) only if $s_1 = 0$. If it is not the case the mapping is non trivial. This mapping is constructed by considering the couplings s_i and $\bar{s}_i$ as background fields. The free energy of the model results

$$e^{-\int d^4x d^2\theta F(s_i) + cc} \equiv \langle e^{-\int d^4x d^2 W(X,s_i) F(s_i) + cc} \rangle \tag{1.41}$$

The correlation functions of the operator $\mathrm{Tr}X^j$ are given by the derivative of the free energy with respects to s_i, and they are

$$\frac{1}{k+1-i} \langle \mathrm{Tr} X^{k+1-i} \rangle = \frac{\partial F}{\partial s_i} \tag{1.42}$$

In the magnetic theory these correlation function are

$$\frac{1}{k+1-i} \langle \mathrm{Tr} Y^{k+1-i} \rangle = \frac{\partial \bar{F}}{\partial \bar{s}_i} \tag{1.43}$$

where $\bar{F}$ is defined analogously as F, and from duality we must have $\bar{F}(\bar{s}_i(s)) = F(s_i)$. This implies a set of equations for $\mathrm{Tr}X^j$ and $\mathrm{Tr}Y^j$ that read

$$\frac{1}{k+1-i} \mathrm{Tr} X^{k+1-i} = \sum_j \frac{\partial \bar{s}_j}{\partial s_j} \frac{1}{k+1-j} \mathrm{Tr} Y^{k+1-j} + \frac{\partial \alpha}{\partial s_i} \tag{1.44}$$

Solving these equations one find the mapping between $\bar{s}_i$ and s_i.

After mapping the polynomial superpotential terms for the adjoint matter, one has to built another part of the magnetic superpotential. This is the same term that appears in the pure SQCD case. The strategy is to split the magnetic theory in a decoupled set of SQCD theories, by solving the equation of motion for the adjoint fields. One than operate a Seiberg duality in each decoupled sector. The new term in W_{magn} is found such that the different SQCD sectors are not coupled. The solution to this requirement is

$$\Delta W_{magn} = \frac{1}{\hat{\Lambda}^2} \sum_{l=0}^{k-1} \sum_{j=1}^{k-l} M_j \tilde{q} Y^{k-j-l} q \tag{1.45}$$

We now skip the details of the general derivation and concentrate on the case of interest.

This is the case where $k = 2$ and the electric superpotential is

$$W_{ele} = \frac{s_0}{3}\mathrm{Tr}X^3 + \frac{s_1}{2}\mathrm{Tr}X^2 \tag{1.46}$$

The superpotential in this case has two critical points, c_1 and c_2. The gauge group is broken to

$$SU(r_1) \times SU(r_2) \times U(1) \tag{1.47}$$

The c_i are found by differentiating the superpotential and by imposing the traceless condition. One finds

$$\begin{aligned} c_1 &= \frac{s_1}{s_0}\frac{r_2}{r_1 - r_2} \\ c_1 &= \frac{s_1}{s_0}\frac{r_1}{r_2 - r_1} \end{aligned} \tag{1.48}$$

The expectation value of $\mathrm{Tr}X^2$ is

$$\langle \mathrm{Tr}X^2 \rangle = r_1 c_1^2 + r_2 c_2^2 = N_c \left(\frac{s_1}{2s_0}\right)^2 \left(\frac{N_c^2}{(r_1 - r_2)^2} - 1\right) \tag{1.49}$$

The magnetic theory gives analogous results by exchanging (s_i, c_i, r_i, X, N_c) with the set $(\bar{s}_i, \bar{c}_i, \bar{r}_i, X, \tilde{N}_c)$. The relation among the s_i and $\bar{s}_i$ are

$$s_0 = -\bar{s}_0 \quad \bar{s}_1 \tilde{N}_c = s_1 N_c \tag{1.50}$$

A last important remark for KSS duality regards the deformation of the theory by gauge invariant mesonic operators. A deformation of the form

$$W_{ele} = \tilde{Q}_{\bar{i}} m(X)_i^{\bar{i}} Q^i \tag{1.51}$$

where $m(z)_{\bar{i}}^i = \sum_j (m_j)_{\bar{i}}^i z^{j-1}$ with $j = 1, \ldots, l+1$, is mapped to the magnetic theory by the term

$$W_{magn} = \sum_{j=1}^{l+1} m_j P_j \tag{1.52}$$

where P_j is the j-th meson of the electric theory.

1.3 Seiberg duality from intersecting brane

A system of intersecting branes in type IIA string theory provides a natural description of Seiberg duality. In the simplest case the electric theory is a $SU(N_C)$ SYM gauge theory, coupled with $N_f > N_c$ massless chiral quarks. The brane realization of this theory is given in figure 1.2. There are 2 NS5 branes, N_F D6 branes and $N_F + N_C$ D4 branes.

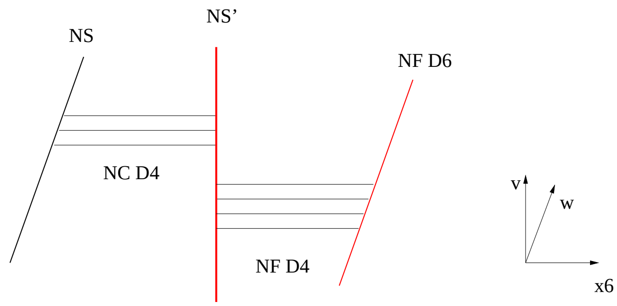


Figure 1.2: Brane realization for the electric massless SQCD in IIA

The $\mathcal{N} = 1$ SQCD gauge theory is realized by choosing the positions of the branes as follows. All the branes are stretched in the (0123) directions. One of the NS5 brane is stretched also in the (78) directions and it is located at $v, x^6 = x^9 = 0$, where $v = x^4 + ix^5$. The other NS5 is stretched in the (45) directions and it is located at x^6 =L> 0 and $w = x^9 = 0$, where $w = x^7 + ix^8$. We distinguish these two branes calling them NS and NS' respectively. The N_F D6 branes are stretched in the (789) directions and they are locate at $v = 0$ (if v is not zero it corresponds to a mass term for the quarks) and $x^6 = L' > L$. N_C D4 branes connect the two NS branes along the x^6 axis and are located at $v = w = x^9 = 0$. The other N_F D4 branes are displaced along the x^6 axis from the NS' brane and $x^6 = L'$. They are located at $v = w = x^9 = 0$. The s-rule is respected since there is no more then one D4 brane connecting the NS5 and the D6 branes.

In field theory this configuration of intersecting branes is interpreted as massless SQCD in the (0123) directions common to all the branes. The massless open string sector among the N_c D4 branes compose the $U(N_c)$ vector multiplet. The massless N_f flavours are associated to opens string among the N_C D4 and the N_F D4. The opens strings on the D6 decouples because the volume of D6 is infinite in the (789) directions. The $U(1)$ symmetry associated to a phase rotation in (78) is identified with R-symmetry. Moreover in the massless case there is a $U(1)_{45}$ rotation symmetry and a $U(N_F)^2$ flavour symmetry.

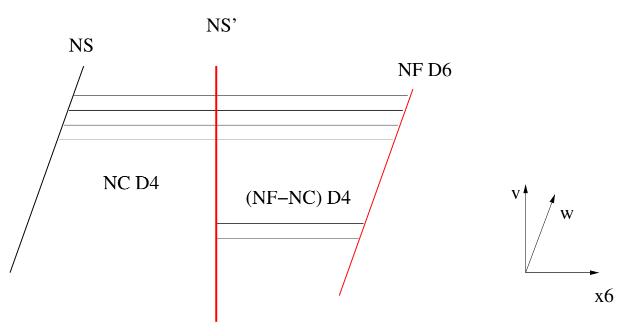


Figure 1.3: Connection of the N_C D4 branes that leaves the NS' brane

The magnetic theory is built by exchanging the NS and the NS' branes in the plane $(6, 9)$ as in figure 1.4. This brane exchange is a dynamical process, and the two branes can pass each other without meeting in this space. Indeed a FI term can be switched on in the world volume gauge theory and it corresponds to displacing the NS to respect of the NS'. After that the branes cross each other the FI is switched off.

Figure 1.3 shows what happens to the D4 branes. In the electric picture N_C among the N_F D4 extended between the NS' and N_C D6 branes can be connected to the N_C branes between the NS' and the NS. These branes leaves the NS' brane. When the NS' crosses the NS only $N_F - N_c$ D4 branes are dragged out. For this reason in the magnetic theory there are $N_F - N_C$ D4 branes connecting the NS and NS' branes.

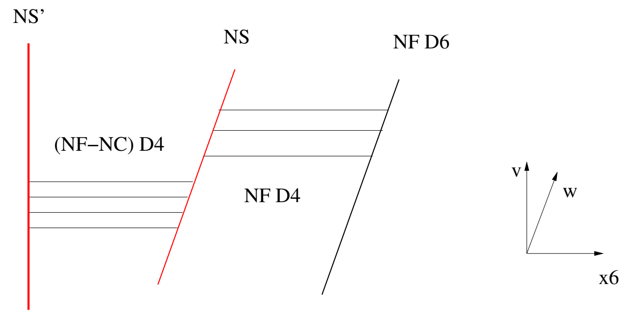


Figure 1.4: Brane realization for the magnetic SQCD in IIA

The magnetic configuration has NS' at $w = x^6 = x^9 = 0$ and NS at $v = 0, x^9 = 0$ and $0 < x^6 = L'' < L'$. The N_F D6 branes are at $x^6 = 0$ and $x^6 = L$. $N_F - N_c$ D4 branes connect the NS and the NS' along x^6 and the remaining N_F D4 extend between the NS and the D6 branes. The field theory is the $U(N_F - N_C)$ SQCD with N_F flavours, dual to the electric theory. In this case the NS and the D6 branes are parallel and we are free to move the N_F D4 branes along w. This freedom is associated to the meson, which is a new degrees of freedom of the magnetic theory. This meson M gives the superpotential $W = Mq\tilde{q}$ of the magnetic theory.

1.4 Seiberg duality on a quiver

In quiver gauge theories (see Appendix A.1 for review) Seiberg duality has a geometrical interpretation. In the conformal case, without fractional branes, the duality transformation is a map among equivalent phases. If fractional branes are added duality usually decreases the ranks of the groups and the theory displays a cascading behaviour. First we study the conformal case, then we discuss the differences in the non conformal case.

A duality transformation can be engineered only on a node of a quiver without adjoint matter. In that case indeed one should consider other examples of duality, for example SW duality in the case of $\mathcal{N} = 2$ theories or KSS duality if there is a polynomial $\mathcal{N} = 1$ superpotential for the adjoint field. We consider only nodes without adjoint matter.

The last constraint on the node that is dualized is that its flavour is $N_f = 2N_c$. This assumption is crucial in the case of toric theories. Indeed in this case duality is a map among different toric phases of the same theory of branes at singularity. With all these assumptions we can study duality on a node in a chiral on non chiral quiver.

The rules for duality follows from the example of SQCD. The dual gauge group is $SU(N_f - N_c)$, and since $N_f = 2N_c$ the dual group is still $SU(N_c)$. The magnetic quarks are generated by inverting the arrows of the fields connected with the gauge group that undergoes duality. The other fields are unchanged by duality. The other important property of duality is that the mesons of the electric theory becomes elementary degrees of freedom in the dual magnetic theory . This means that new adjoint matter is expected in the dual phase. In the case of quiver gauge theory a node is often connected with more than one flavour groups. The mesons are not only adjoint fields but also bifundamental fields. These adjoint and bifundamental fields both appear in the dual phase. In the simplest case of SQCD duality transforms a superpotential $W = 0$ into a superpotential $W = Mq\tilde{q}$.

These rules are geometric. However duality changes also the superpotential. If the node that undergoes duality is the i-th node all the operators that contain fields charged under this node in the electric superpotential will involve a meson in the magnetic phase. For example if there is a term

$$X_{ji}X_{ik}\mathcal{O}_{k,\ldots,j} \tag{1.53}$$

in the electric theory, where $\mathcal{O}_{k,\ldots,j}$ is a product of fields of the quiver with free indexes j and k and contracted over the other indexes (different from i) we have that this term is written in the dual as

$$M_{jk}\mathcal{O}_{k,\ldots,j} \tag{1.54}$$

From equation (1.53) we note that if the operator $\mathcal{O}_{j,\ldots,k}$ is made of only one field, than the dual meson is massive. Another possibility of having a dual massive meson is that the electric theory has a quartic term like

$$X_{ji}X_{ik}X_{ki}X_{ij} \to M_{jk}M_{kj} \tag{1.55}$$

In a toric theory one usually integrate out the massive mesons and obtain the dual toric phase.

The same rules that holds in the conformal case holds also in the non conformal case. The only difference is that if fractional branes are added duality is a dynamical process that lowers the degrees of freedom of the theory. In this case one has to operate a choice on the groups to be dualized: usually it is the group with the highest value of the beta function. This UV free group is usually the first to become strongly coupled in the RG flow. Duality makes it IR free and perturbatively accessible at low energy. After that other groups can become UV free and strongly coupled in the IR and another duality is needed. The theories have a cascading behaviour that can end with $\mathcal{N} = 1$ SYM (with a decoupled set of Goldstone bosons) or with more complicated supersymmetric or supersymmetry breaking theories.

1.5 Seiberg duality on the brane tiling

The study of Seiberg duality in the case of toric gauge theories is better done by the use of brane tiling (see Appendix A.1 for review). Indeed the tiling encodes all the informations necessary for duality, the structure of the gauge groups and the superpotential. As in the case of the quiver, Seiberg duality is a local transformation on the dimer, and it involves only the i-th node that undergoes duality and the next nodes.

The condition that guarantees that the theory remains in the toric phase is that only the faces with four edges can be dualized. Indeed having a conformal theory it means that only regular brane are considered. This implies that all the edge carry a contribution of$N_F/2$ to the whole flavor, and four edges give the condition $N_F = 2N_c$

Consider a face with four edges in a tiling, and draw in the internal of this face another face whose edges are parallel to the external ones. Connect then each internal vertex with one external vertex, and draw a black or with node one each new internal vertex, such that the bipartite structure of the tilings maintained. After that, cancel the external edges. The new tiling represents the Seiberg dual phase of the toric theory.

Note that for some nodes in the tiling there can be only two edges. Since a node is a superpotential term and an edge is a matter field, this implies that these nodes are identified with mass term in the superpotential. These terms have to be integrated out in the field theory. In the dimer this procedure is implemented by a cancellation of the two edges and of the internal nodes, and by the identification of the other two nodes connecting the other extrema of the edges that have been canceled out. This procedure maintains the bipartite structure and is the analog of supersymmetric integration out of the massive field in gauge theory. The four edges that compose the new internal face are the dual quarks, and the correctness of their representation under the gauge groups is guaranteed by the bipartite structure. Moreover the mesons are associated to the edges that connect the old vertexes and the new ones. As an example we show the behaviour of

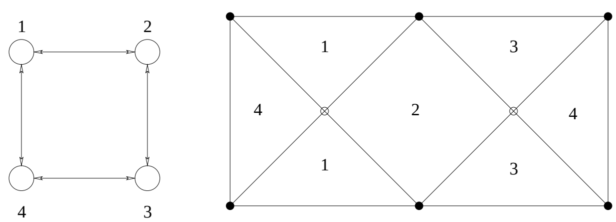


Figure 1.5: Quiver gauge theory and brane tiling for the double conifold.

Seiberg duality on the tiling of the simplest orbifold projection of the conifold, the double conifold. The superpotential of this theory is

$$W = X_{12}X_{23}X_{32}X_{21} - X_{23}X_{34}X_{43}X_{32} + X_{34}X_{41}X_{14}X_{43} - X_{41}X_{12}X_{21}X_{14} \tag{1.56}$$

The quiver and the tiling are shown in figure 1.5. Duality on node two gives a new superpotential

$$
\begin{aligned}
W = \quad & X_{41}X_{11}X_{14} - X_{21}X_{11}X_{12} + X_{12}X_{23}X_{32}X_{21} - X_{33}X_{32}X_{23} \\
+ \quad & X_{43}X_{33}X_{34} + X_{34}X_{41}X_{14}X_{43}
\end{aligned}
\tag{1.57}
$$

The new quiver and tiling for this dual phase are shown in figure 1.6.

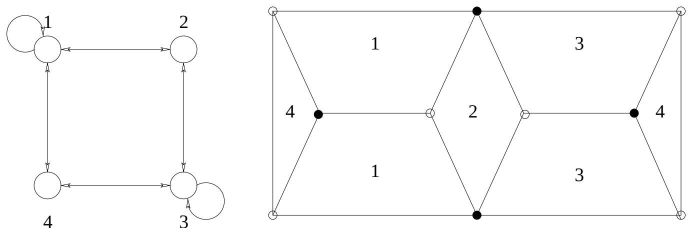


Figure 1.6: Quiver gauge theory and brane tiling for the double conifold in the dual phase.

Chapter 2

Metastable Vacua

In this section we review the basic aspects of supersymmetry breaking. Supersymmetry is broken if the scalar potential does not vanish in the ground state. This is an immediate consequence of the supersymmetry algebra, since the ground state is invariant under a supersymmetry transformation if and only if it has zero energy.

One can define a topological index, the Witten index, that signals if supersymmetry can be broken or not, under smooth variations of the parameters of the theory. Models with non zero Witten index cannot definitively break supersymmetry. Otherwise, models with a zero index, have been for a long time the only candidates for supersymmetry breaking.

In this classification two basic examples, the O'Raifeartaigh and the Fayet-Iliopoulos models, represent the simplest and basic mechanisms of supersymmetry breaking. In the first model supersymmetry breaking is based on the F-terms, and it depends on the interactions in the superpotential. In the second case supersymmetry breaking is due to the abelian factors of the gauge interactions.

The constraints imposed on the spectrum of a supersymmetry breaking theory by the supertrace impose that the mass of some superpartner is too low. These constraints can be evaded if supersymmetry is broken dynamically, for example because of non perturbative effects. This approach offers a large class of models. The problem is that the analysis of the quantum stability of the non supersymmetric states, which are often metastable, is rather complicate. This is caused by the presence of strongly coupled corrections. However $\mathcal{N} = 1$ Seiberg duality furnishes a many supersymmetric gauge theories with calculable false vacua.

2.1 Spontaneous supersymmetry breaking

In quantum field theory the minimization of the euclidean action is equivalent to the minimization of the scalar potential. Indeed a vacuum state of the scalar potential is identified with a minimum of the action, a stable configuration in the language of functional integral. Such a vacuum state must be invariant under the space-time Lorentz symmetry, and this implies that only the scalar can be non vanishing.

In a supersymmetric theory the scalar potential is written in term of the auxiliary fields and it is

$$V = K_{ij}^{-1} F_j F_i^\dagger + \frac{1}{2} D^a D_a \tag{2.1}$$

where

$$K_{ij}^{-1} = \left(\frac{\partial^2 K}{\partial \phi_i \partial \phi_j} \right)^{-1}$$

$$F_i^\dagger = \frac{\partial W(\phi)}{\partial \phi^i} \tag{2.2}$$

$$D^a = -g^a (\phi_i^\dagger (T^a)^i_j \phi^j + \xi^a) \tag{2.3}$$

The indexes i, j runs over the chiral field of the theory, the index a is over the gauge group. The term ξ^a is related to the possible $U(1)$ gauge factors.

The (2.1) scalar potential is non-negative, and if one can set contemporary to zero both the F and D terms in (2.1) then $V = 0$ is a global minimum. Otherwise there can be local mimima or only runaway directions.

The value of the vacuum energy is connected with supersymmetry. Indeed, by using the supersymmetry algebra, one can connect the Hamiltonian operator with the supersymmetry generators. One has

$$H = \frac{1}{4}(\{Q_1, Q_1^\dagger\} + \{Q_2, Q_2^\dagger\}) \tag{2.4}$$

The requirement of unbroken supersymmetry in the vacuum state $|0\rangle$ implies that $X|0\rangle = 0$. As a consequence a positive energy vacuum is a signal of supersymmetry breaking. From the discussion above in quantum field theory $\langle 0|H|0\rangle = \langle 0|V|0\rangle$. This concludes that supersymmetry is unbroken in and only if there exists a solution to the F and D term equations, i.e. the scalar potential in the vacuum state is zero.

One immediate consequence of supersymmetry breaking is the existence of a massless neutral Weyl fermion, the *Goldstino*. This particle is a fermionic degrees of freedom composite of fermions ψ_i and gaugino λ_a. In this basis the fermion mass matrix takes the form (A.40)

$$\begin{pmatrix} \langle W^{ji} \rangle & i g^a \langle \phi_l^\dagger \rangle (T^{a)}{}^l_i \\ i g^b \langle \phi_l^\dagger \rangle (T^a)^l_i & 0 \end{pmatrix} \tag{2.5}$$

If this matrix acts on the vector $v = \begin{pmatrix} \langle F^j \rangle \\ \langle D^a \rangle \end{pmatrix}$ one has

$$M v = 0 \tag{2.6}$$

The first line in (2.6) follows from $\frac{\partial V}{\partial \phi_j} = 0$, to be satisfied by a local minimum. The second line in (2.6) is satisfied by the requirement of gauge invariance. From equation (2.6) the fermionic mass matrix has at least one zero eigenvalues, there is a massless fermionic state. The presence in the spectrum of a massless fermionic particle is potentially a phenomenological problem for model building. This problem is solved by local supersymmetry. There the *Goldstino* is eaten by the graviton superpartner, the *Gravitino*. In this case the longitudinal component of the *Gravitino* is light, but no massless, and it is one of the more common LSP candidate.

After the discussion of the fundamentals of spontaneous supersymmetry breaking we show two examples. These are the O'Raifeartaigh model and the Fayet-Iliopoulos model. The first one is based on the impossibility of solving all the F terms in a theory of pure chiral fields. The second is based on a $U(1)$ gauge theory where the conditions for solving F and D terms turn out to be incompatible.

The O'Raifeartaigh model

This is a model of three pure chiral fields, with superpotential

$$W = fX + X\phi_1^2 + m\phi_1\phi_2 \tag{2.7}$$

and canonical Kahler potential $K = XX^\dagger + \phi_1\phi_1^\dagger + \phi_2\phi_2^\dagger$. The model has a $U(1)_R$ symmetry under which ϕ_1 is uncharged and ϕ_2 and X has R-charge two. The F terms equations for the fields X, ϕ_1 and ϕ_2 cannot be solved together and supersymmetry is spontaneously broken at tree level. The equations of motion are

$$\begin{aligned} F_{X^\dagger} &= f + \phi_1^2 \\ F_{\phi_1^\dagger} &= 2X\phi_1 + m\phi_2 \\ F_{\phi_2^\dagger} &= m\phi_1 \end{aligned} \tag{2.8}$$

The equation for X and ϕ_2 are incompatible, the equation for ϕ_1 fixes $\langle\phi_2\rangle = -\langle\frac{2X\phi_1}{m}\rangle$. The classical scalar potential has a moduli space of vacua with arbitrary X. One can define a parameter

$$y = \left|\frac{2f}{m^2}\right| \tag{2.9}$$

that control the position of the minimum in the ϕ_i directions. If $y \leq 1$ the minimum is at $\phi_1 = \phi_2 = 0$, and $V_{min} = |f|^2$. In the other case, where $y > 1$ the origin becomes a saddle point and the minimum is splitted at $\phi_1 = \pm\mathbf{i}\sqrt{2f(1-1/y)}$ and $\phi_2 = -2X\phi_1/m$.

The case for interest for future application is $|y| \leq 1$, and we will concentrate only on that one.

The classical scalar potential has a flat direction, the X field. One has to ensure that this flat direction does not have a tachyonic behaviour at quantum level. The one loop analysis can be performed by the study of the Coleman-Weinberg (A.46) scalar potential. The X field is considered as a background field and one studies the masses of the fluctuation for generic X. The scalar and the fermionic components of the superfield X are massless classically. While the former can acquire quantum corrections the second one is the *Goldstino* field, associated to supersymmetry breaking. The fermionic spectrum gives

$$m_{1/2} = |m|^2 + 2|X|^2 \pm 2|X|\sqrt{|m|^2 + |X|^2} \qquad (2.10)$$

The bosonic spectrum gives

$$m_0 = \eta|f| + |m|^2 + 2|X|^2 \pm \sqrt{|F|^2 + 4|X|^2(-\eta|f| + |m|^2 + |X|^2)} \qquad (2.11)$$

where $\eta = \pm 1$.

Once we know the mass spectrum we can use the CW formula for the computation of the effective potential for the X field. Since the model is R-symmetric and no field has charge different from zero or two, a general result [4] states that the pseudomodulus acquires a positive squared mass at the origin. By explicitly computation one finds that the mass term is

$$m_X = \frac{m^2}{4\pi^2 y}(-2y - (y-1)^2 \log(1-y) + (1+y)\log(1+y)) \qquad (2.12)$$

This expression is positive in the whole interval $0 < y \leq 1$, as expected.
Alternatively the computation of the mass of the pseudomodulus can be performed by using the one-loop Feynman diagrams. Since we have to calculate the mass of a classical massless field we can help us with a trick. If the scale of supersymmetry breaking is tuned to $f = 0$ the model has a supersymmetric vacuum at the origin, and the X field is not quantum corrected. Since it is massless it remains massless at every order in perturbation theory.

We have two models, the one with $f = 0$ and the one with $f \neq 0$ with the same interactions, the same field content, but a different spectrum. Since the mass of the field X is corrected only in the non supersymmetric case, the difference between the mass in the non supersymmetric case and the mass in the supersymmetric case coincide with the mass in the non supersymmetric case. Trivially we can write the equation

$$m_X = m_X^{\text{Non-Susy}} - m_X^{\text{Susy}} \qquad (2.13)$$

This trivial equation tell us that we can reduce the number of diagrams that contribute to the one loop computation for the X mass. We now proceed by calculating m_X with (2.13). We already know from [4] that the minimum is at $X = 0$. The masses of the fields are

$$m_{1/2}^2 = 0, m^2, m^2$$
$$m_0^2 = 0, 0, m^2 - 2f, m^2, m^2, m^2 + 2f$$

The only diagrams contributing involve only scalars (the diagram involving the fermions are the same in the non supersymmetric and in the supersymmetric case). The first diagram is

$$I(m_1, m_2) = \int_0^\Lambda d^4p \frac{1}{(p^2 + m_1^2)(p^2 + m_2^2)} = -\frac{m_2^2}{16\pi^2}\left(\log\frac{\Lambda^2}{m_2^2} - \frac{m_1^2}{m_1^2 - m_2^2}\log\frac{m_1^2}{m_2^2}\right) \quad (2.14)$$

while the second is

$$J(m_1) = \int_0^\Lambda d^4p \frac{1}{p^2 + m1^2} = \frac{1}{16\pi^2}\left(\Lambda^2 - 2m_1^2 \log\frac{\Lambda^2}{m_1^2}\right) \tag{2.15}$$

In this case the mass term in the Lagrangian is of the form

$$\mathcal{L} \sim \frac{1}{2} m_X^2 |X|^2 \tag{2.16}$$

and in this case, using (2.13) we have

$$\frac{1}{2} m_X^2 = \frac{1}{2} + (I(m_+, m) + I(m_-, m) - 2I(m, m)) + \frac{1}{2}(J(m_+) + J(m_-) - 2J(m)) \tag{2.17}$$

where $m_\pm = \sqrt{m^2 \pm 2f}$. Inserting (2.14) and (2.15) in (2.17) one find exactly (2.12).

This procedure is more involved in this case than the CW formula. Nevertheless it represents and useful strategy to compute the mass of the pseudomoduli in cases where it is not simple to find the eigenvalues of the mass matrices with the background field X switched on. Moreover this strategy will play a crucial role in the two loop calculations.

The Fayet-Iliopoulos Model

Another mechanism for supersymmetry breaking is the FI model. In that case a $U(1)$ gauge group is included in the theory, and the linear term is switched on. In the simplest case there are two chiral multiplet ϕ_1 and ϕ_2 with charges respectively $+1$ and -1 under the $U(1)$. The superpotential is $W = m\phi_1\phi_2$, and the Kahler potential is canonical. The scalar potential is

$$V = |m|^2(|\phi_1|^2 + |\phi_2|^2) + \frac{1}{2}\left(\xi + |\phi_1|^2 - |\phi_2|^2\right) \tag{2.18}$$

If $m \neq 0$ and $\xi \neq 0$ the scalar potential cannot vanish and supersymmetry is broken. The minimum always occurs for non zero D. If $|m_i|^2 > g\xi$ the minimum is placed at $\phi_1 = 0$. The *Gaugino* remain massless, and it is indeed the *Goldstino*.

Dynamical supersymmetry breaking

One of the most important motivation for supersymmetry is that it solves the hierarchy problem. It is true if the scale of supersymmetry breaking is well below the GUT scale, preferably of the order of EWSB, or not too much higher. This requires the scale of supersymmetry breaking to be small enough. In the spontaneous supersymmetry breaking models studied above a small scale of supersymmetry breaking is imposed at hand.

There is no explanation of the origin of the hierarchy.

A natural possibility is that the scale of supersymmetry breaking is determined by the strong dynamics. The dynamics of the model can naturally set this low scale. This possibility is named dynamical supersymmetry breaking.

In the past an important constraint on dynamical supersymmetry breaking was the Witten index. This is a topological index that is given by the difference between the bosonic and fermionic states in the vacuum of a supersymmetric theory

$$\mathrm{Tr}(-1)^F = n_B^0 - n_F^0 \tag{2.19}$$

If this index is non vanishing the zero energy vacuum is a state of the theory, and supersymmetry is unbroken. On the contrary theories with zero index can break supersymmetry. The index can be used as a guideline in the search of models with dynamical supersymmetry breaking, in virtue of its topological property. Indeed dynamical supersymmetry breaking is often related to strong dynamics effects, uncalculable in perturbation theory. Nevertheless one can compute the index in the weakly coupled regime and this should rule out some models, like SYM and many non-chiral theories.

The general strategy in the search of dynamical supersymmetry breaking is based on looking for models with no flat tree level directions. One choose models with zero energy at the origin and an increasing potential. Non-perturbative corrections usually generate a runaway potential, that is not zero at the origin. In these cases supersymmetry is broken by the competition of tree level and non perturbative effects. This argument is not rigorous, since many counterexamples are possible. For example the ITIY model has tree level, but it breaks supersymmetry dynamically. A general problem, common to most of these models, is that their spectra are often uncalculable, due to the strong dynamics.

Another characteristic of many of these models is that, when they are coupled with the MSSM supersymmetry is restored somewhere in the moduli space. The non supersymmetric vacua can be only metastable. At this point of the discussion one can ask what happens by requiring metastability from the beginning, i.e. in the supersymmetry breaking sector. The idea of metastable vacua already appeared in supersymmetric calculable models [26, 27], but it raised a great interest only after the ISS model. This model evades one important constraint of supersymmetry breaking: it has non zero Witten index. This is an attractive property since many non-chiral models, that where ruled out, come back as possible candidates of hidden supersymmetry breaking sectors.

In this section we discuss the ISS model of metastable vacua in $\mathcal{N} = 1$ SQCD. The model is the $SU(N_c)$ SQCD, with N_f flavours, and with a mass term for the quarks. This theory does not break supersymmetry, since the Witten index is N_c. Nevertheless, in the magnetic window, the dual description of this theory, does not possess zero energy ground states only. Indeed there are also non supersymmetric metastable vacua. Seiberg duality turns out to be fundamental for this mechanism and for its extensions in the rest of this chapter.

Differently from known models of dynamical supersymmetry breaking, here the perturbative theory is under control. The non perturbative effects from strong coupling are important in a region of the potential very far from the metastable state, and they drive the supersymmetry restoration. This large separation among the supersymmetric and the non supersymmetric sectors implies the long lifetime of the metastable state.

We know review the basic physical and technical aspect of the computation of [2].

The model

Consider $SU(N_c)$ SQCD with N_f quarks charged under a $SU(N_f) \times SU(N_F)$ flavour symmetry. This flavour symmetry is explicitly broken to the diagonal subgroup by the superpotential term

$$W = m_Q \mathrm{Tr} Q\tilde{Q} \tag{2.20}$$

The global symmetry is

$$SU(N_F) \times U(1)_B \times U(1)_R \tag{2.21}$$

where the R-symmetry is non-anomalous. The baryonic symmetry can be gauged, in this case one has to consider the $U(N_C)$ gauge symmetry

The charges of the fields under the global and local symmetries are

	$SU(N_c)$	$SU(N_F)$	$U(1)_B$	$U(1)_R$
Q	$\tilde{N}_c$	N_F	1	$1-\frac{N_F}{N_C}$
$\tilde{Q}$	N_c	$\tilde{N}_F$	-1	$1-\frac{N_F}{N_C}$

(2.22)

In the magnetic free window $N_c + 1 \leq N_f \leq \frac{3}{2}N_c$ the theory is UV free and the infrared theory is strongly coupled. These strong coupling effects drive the supersymmetric vacua at large vev. Indeed if one parameterizes the moduli space with the meson $M = Q \cdot \tilde{Q}$ the supersymmetric vacua are placed at

$$\langle M \rangle = \left(\Lambda^{3N_C - N_F} \det m_Q\right)^{\frac{1}{N_C}} \frac{1}{m_Q} \tag{2.23}$$

The mass matrix m can be diagonalized with positive eigenvalues. They are considered of the same magnitude and constrained by

$$m_{Q_i} \ll \Lambda \tag{2.24}$$

where Λ is the dynamical scale of the theory. In the limit of $m_i \to 0$ the supersymmetric vacua approach the origin of the field space. In this case one can use the Seiberg dual description.

Seiberg duality in this theory gives a magnetic theory which is IR free and perturbatively accessible at low energy. The gauge symmetry becomes $SU(\tilde{N}_C) \equiv SU(N_F - N_C)$, while the global symmetry is unchanged.

The dual superpotential becomes

$$W_{\text{magn}} = \frac{1}{\hat{\Lambda}} \mathrm{Tr} M q\tilde{q} + m \mathrm{Tr} M \tag{2.25}$$

where the q and $\tilde{q}$ fields are the magnetic quarks, and M is the electric meson. The Kahler potential around the origin is canonical, and higher correction are suppressed by the fact that the theory is IR free.

$$K_{\text{magn}} = \frac{1}{\beta} \mathrm{Tr}(qq^\dagger + \tilde{q}\tilde{q}^\dagger) + \frac{1}{\alpha|\Lambda|^2} \mathrm{Tr} M M^\dagger \tag{2.26}$$

The magnetic theory is characterized by two scales, $\hat{\Lambda}$ and a Landau pole $\tilde{\Lambda}$. They are not independent, but they are related to the dynamical electric scale Λ by the scale matching condition (1.11). By rescaling $\tilde{q}$ and q one can impose $\beta = 1$, but this cannot compute both $\tilde{\Lambda}$ and $\hat{\Lambda}$ in terms of the magnetic variables. The coefficient α cannot be imposed to be $\alpha = 1$ because there is no freedom on rescaling M.

As in [2] one can define new fields and couplings, such that all the fields have mass dimension 1 and no coefficient appears in the Kahler potential. These definition are

$$\Phi = \frac{M}{\sqrt{\alpha}\Lambda} \qquad h = \frac{\sqrt{\alpha}\Lambda}{\hat{\Lambda}} \qquad \mu^2 = -m\hat{\Lambda} \tag{2.27}$$

With these definitions the superpotential is

$$W = h\mathrm{Tr}q\Phi\tilde{q} - h\mu^2\mathrm{Tr}M \tag{2.28}$$

Supersymmetry breaking

The equation of motion arising from superpotential (2.28) are

$$\begin{aligned} F_\Phi &= q_i^\alpha q_\alpha^j - \mu^2\delta_i^j = 0 \\ F_\phi &= q_\alpha^i M_i^j = 0 \\ F_{\tilde{\phi}} &= M_i^j q_j^\alpha = 0 \end{aligned} \tag{2.29}$$

The first equation cannot be completely solved, since there is a rank condition. The second matrix has rank N_F, while the first matrix has rank $\tilde{N}_C = N_F - N_c$. There are N_c equations that cannot be solved, and supersymmetry is broken at the tree level.

The non-supersymmetric vacuum is

$$q\tilde{q} = \begin{pmatrix} \mu^2\mathbf{1}_{\tilde{N}_c} & 0 \\ 0 & 0 \end{pmatrix} \qquad M = \begin{pmatrix} 0 & 0 \\ 0 & X \end{pmatrix} \tag{2.30}$$

The D-term condition gives

$$q = \begin{pmatrix} \mu e^{i\theta} \\ 0 \end{pmatrix} \qquad \tilde{q}^T = \begin{pmatrix} \mu e^{-i\theta} \\ 0 \end{pmatrix} \tag{2.31}$$

This is a moduli space of non supersymmetric classical vacua parameterized by X. This is not a *Goldstone* flat direction, since it is not associated to the breakdown of the global symmetry. Indeed it is a pseudomodulus that can acquire a mass term at quantum level. The quantum analysis is necessary to decide if the tree level supersymmetry breaking minimum is a true minimum of the quantum theory.

The first step toward the analysis of the stability of the non-supersymmetric vacuum consists of calculating the squared mass matrices and look for potential tree level tachyons.

This is done by expanding the fields around the non-supersymmetric minimum. In terms of the fluctuations the common parameterization is

$$\Phi = \begin{pmatrix} \delta Y & \delta Z^T \\ \delta \tilde{Z} & X \end{pmatrix} \quad q = \begin{pmatrix} \mu + \frac{1}{\sqrt{2}}(\delta\chi_+ + \delta\chi_-) \\ \frac{1}{\sqrt{2}}(\delta\rho_+ + \delta\rho_-) \end{pmatrix} \quad \tilde{q}^T = \begin{pmatrix} \mu + \frac{1}{\sqrt{2}}(\delta\chi_+ - \delta\chi_-) \\ \frac{1}{\sqrt{2}}(\delta\rho_+ - \delta\rho_-) \end{pmatrix} \tag{2.32}$$

The combination of these fluctuations gives some massless scalars as

$$\frac{\mu^*}{|\mu|}\delta\chi_- - h.c., \qquad Re\left(\frac{\mu^*}{|\mu|}\delta\rho_+\right), \qquad Im\left(\frac{\mu^*}{|\mu|}\delta\rho_-\right) \tag{2.33}$$

that are Goldstone bosons of the broken global symmetries.
The traceless part of $Im\left(\frac{\mu^*}{|\mu|}\delta\chi_-\right)$ is associated to the breakdown of the gauge symmetry, and the Higgs mechanism is at work in this case. These Goldstone boson is eaten by the gauge fields, that acquire mass $g\mu^2$.

Moreover there are some massless fluctuations that are not associated to any Goldstone flat direction. They are

$$X, \qquad \frac{\mu^*}{|\mu|}\delta\chi_- + \text{h.c.} \tag{2.34}$$

The traceless part of the latter get a tree level mass from the D-term potential, of order $g\mu$. The classical pseudomoduli that remains are X and $Tr\left(\frac{\mu^*}{|\mu|}\delta\chi_- + \text{h.c.}\right)$. This last is identified with $(\theta + \theta^*)$ in the parameterization of the vacuum (2.31)

Since there are no tachyonic direction one can calculate the one loop corrections for the pseudomoduli. For simplicity we study the case with gauged baryonic symmetry, In this case only X remains as a pseudomodulus, since $(\theta + \theta^*)$ get a tree level mass from the D-term of $U(1)_B$.

The CW potential for X is calculated by expanding around the vacuum with $\theta = \theta^* = 0$ the fields as

$$q = \begin{pmatrix} \mu + \sigma_1 \\ \phi_1 \end{pmatrix} \quad \tilde{q}^T = \begin{pmatrix} \mu + \sigma_2 \\ \phi_2 \end{pmatrix} \quad M = \begin{pmatrix} \sigma_1 & \phi_3 \\ \phi_4 & X \end{pmatrix} \tag{2.35}$$

This parameterization divides the chiral fields in two sectors. The σ_i fields are "supersymmetric" fluctuations, which means that they do not enter in the one loop vacuum diagrams with the field X, and hence they are not necessary to the CW potential. On the contrary the fields ϕ_i participate to he one loop CW potential for X.

Note that with this distinction one can look at the mass matrices and see that only the matrices (A.44) and (A.45) contributes to the CW potential, and we can use (A.46). The O'Raifeartaigh models that contributes to the potential are N_C copies of

$$W = hX\phi_1\phi_2 + h\mu(\phi_1\phi_4 + \phi_2\phi_3) - h\mu^2 X \tag{2.36}$$

One finds that the X field is stabilized at the origin with mass

$$m^2_{|X|} = |h^4\mu^2|\frac{N_C(\log 4 - 1)}{8\pi^2} \tag{2.37}$$

The irrelevance of the marginal coupling h in this IR free theory ensures that higher loop corrections are suppressed. This is true in the case of masses of the same order, as here. If there are masses of different order it can happens that the pseudomoduli associated to the lower masses can become negative at the origin because of the contribution at the origin of the two loop potential, as in [28].

Supersymmetry restoration and lifetime

The non-supersymmetric minimum is stable at the origin, but in the large field region the non perturbative effects restore supersymmetry, as expected from Seiberg duality. One has to ensure that this restoration does not destabilizes the non-supersymmetric state at all, making its lifetime too short. For this reason it is necessary to know the position of the supersymmetric state and then calculate the bounce action between the two vacua. Once this action is known it is possible to calculate the lifetime of the vacuum.

The supersymmetric state has been given in the electric theory as a function of m and Λ in (2.23). In this magnetic theory one can map the variables. For future aims we calculate it in the dual theory. If the field Φ acquires a non zero vacuum expectation value, it massifies the flavours q and $\tilde{q}$. They have mass $\langle h\Phi\rangle$. Below this mass scale the flavours can be integrated out, at zero vev. The scale matching relation among the $\tilde{\Lambda}$ scale and the $\tilde{\Lambda}_L$ scale, at which the fields are integrated out, is

$$\tilde{\Lambda}_L^{3\tilde{N}_c} = h^{N_F} \det \Phi \tilde{\Lambda}^{3\tilde{N}_C - N_F} \tag{2.38}$$

The gauge theory is $SU(\tilde{N}_C)$ SYM, and gaugino condensation occurs. This implies that a dynamical superpotential $W_{dyn} = \tilde{N}_c \tilde{\Lambda}_L^3$ is generated. Using the scale matching relation the final superpotential is

$$W_{eff} = \tilde{N} \left(h^{N_f} \tilde{\Lambda}^{3N - N_F} \det \Phi \right)^{1/N} - h\mu^2 \mathrm{Tr}\Phi \tag{2.39}$$

This superpotential sets the supersymmetric vacua at

$$\langle h\Phi\rangle = \tilde{\Lambda}\varepsilon^{\frac{2N}{N_F - N}} = \frac{\mu}{\varepsilon^{\frac{N_F - 3N}{N_F - N}}} \tag{2.40}$$

where $\varepsilon = \frac{\mu}{\Lambda}$. The position of the minimum in the field space depends on the parameter ε. If $|\varepsilon| \ll 1$ then

$$|\mu| \ll |\langle h\Phi\rangle| \ll |\tilde{\Lambda}| \tag{2.41}$$

These bounds are fundamental for the stability of the metastable state. Indeed The relation $|\langle h\Phi\rangle| \ll |\tilde{\Lambda}|$ ensures that the non perturbative contribution does not ruin the perturbative analysis near the origin of the moduli space. Moreover, higher corrections to the Kahler potential are of order $1/|\tilde{\Lambda}|^2$. They do not invalidate neither the perturbative results neither the non perturbative ones.

The lifetime of the vacuum is calculated after the choice of the bounce action. This action has to connect the supersymmetric and the non supersymmetric vacua. The peak of the potential is at $\Phi = q = \tilde{q} = 0$, where the potential is $V_{max} = N_F|h\mu^2|^2$. From the non supersymmetric state to the supersymmetric one the motion takes place along the q and $\tilde{q}$ directions. After the maximum is reached the motion of the Φ field drives the theory to the supersymmetric minimum. This choice of the bounce action is well approximated by a triangular barrier. Indeed the two vacua are far in the field space and the gradient is nearly constant. Using the approximation of a triangular barrier, as discussed in the appendix A.7 one finds

$$S_B \sim \frac{\Delta\Phi^4}{V_+} \sim \frac{1}{\varepsilon^{4(N_F-3\tilde{N})/(N_F-\tilde{N})}} \tag{2.42}$$

The decay rate of the vacuum is $\Gamma \sim e^{-S_B}$ and more S_B is large more the lifetime is long, i.e. the vacuum is stable to decay. The action is parametrically large and we can act on the scale and modify the parameter ε such that it is $|\varepsilon| \ll 1$. This is important since the validity of the perturbative and non-perturbative analysis and the requirement of long lifetime fix the same bounds on the scales of the theory.

Validity of the computation

The ISS vacuum is a calculable model of supersymmetry breaking. The perturbative calculations have been performed without taking into account the dependence of the scale $\tilde{\Lambda}$, that is not under control. The validity of the approximation is guaranteed by the smallness of the parameter $\epsilon = \mu/\tilde{\Lambda}$. This choice is important for neglecting loop effects and non perturbative from the high energy theory. The former corrections are summarized in the correction of the Kahler potential. At quartic order it takes the form

$$\delta K = \frac{c}{|\tilde{\Lambda}|^2} X X^\dagger + \dots \tag{2.43}$$

If this correction to the Kahler potential is considered there is a correction of order $c|\epsilon\mu|^2$ to the mass of the pseudomodulus. This correction is negligible and the perturbative computation in not affected by the heavy modes. The dimensionless number c, whose sign is undetermined, can stabilize or destabilize the vacuum, but with the approximation $\epsilon \ll 1$ it is irrelevant. This discussion applies also on the non-perturbative irrelevant operator, which is suppressed by powers of $\tilde{\Lambda}$, and does not change the perturbative results for the non-supersymmetric vacuum.

In the calculation of the supersymmetric vacuum we can neglect the correction to the Kahler potential but we need to consider the non perturbative dynamical superpotential. Both are suppressed by the scale $\tilde{\Lambda}$, but in the first case this is an effect of the theory above the Landau pole, while in the second case the dynamical superpotential is generated by the dynamics of the gauge group at low energy. Indeed the expectation value of the field X is well below the scale $\tilde{\Lambda}$, and this guarantees the correctness of the approximation.

Gauging the flavour symmetry

In the main text we will look for ISS like vacua in quiver gauge theories. The main difference between SQCD and these theories is that in the latter the symmetries are all gauged, and hence the flavour groups are gauged as well. In the analysis of the moduli spaces the gauge contributions of these groups may become relevant.

Such groups may develops a strong dynamics that ruins the conclusions about the lifetime of the metastable vacua, since new supersymmetric vacua arise.

Another problem is that some fields charged under these groups could take non zero vev in the meta-stable vacua. This makes the one loop computation difficult, since we should take into account the D-term corrections to the effective potential. In fact the mass matrices which appear in the Coleman Weinberg potential are built using the F-terms of the superpotential, and the D-terms arising from the gauge groups. The D-term contributions to the mass matrix are irrelevant with respect to the F-term ones only if the corresponding gauge group is very weakly coupled.

The problems associated with the gauging of the flavour symmetries has already been handled in [10, 29, 30, 31] with different solutions. Basically one needs a scheme where the gauge contributions of such groups can be ignored. If these groups are IR free in the Seiberg dual description, the way out consists of tuning their Landau pole to be much higher than the Landau pole Λ_m of the dualized gauge group. In the opposite case, the gauged flavour groups are UV free. In this case we have to choose the opposite tuning, i.e. their strong coupling scale must be much lower than Λ_m and also lower than the supersymmetry breaking scale. Such tunings make the gauge contributions of the flavour groups negligible, and the problems mentioned above are avoided.

R-symmetry plays a crucial role in many aspects of supersymmetry breaking models. Indeed, under some hypothesis, the presence of this symmetry is a necessary condition for supersymmetry breaking. Moreover if it is spontaneously broken it is also a sufficient condition for supersymmetry to be broken.

Some general argument shows that spontaneous R-symmetry breaking is usually connected with the assignation of the R-charges in a specific model. This charge assignation prevents the pseudomodulus to acquire a vacuum expectation value, at one loop, in ISS model and in many generalizations.

Also phenomenology of supersymmetry breaking is constrained if R-symmetry is preserved, i.e. the absence of explicit or spontaneous R-symmetry breaking. Indeed if supersymmetry breaking is mediated to the MSSM through gauge interactions, the gauginos can only acquire a Dirac mass if R symmetry is a symmetry of the quantum theory. On the contrary, if R symmetry is broken, Majorana masses are allowed.

The assignation of R-charges is also important for the behaviour of the theory at large vev. It has been shown that the runaway behaviour of an O'Raifeartaigh model is connected with R-charges of the F-terms.

R-symmetry and supersymmetry breaking

The Nelson-Seiberg argument [3] connects R-symmetry and supersymmetry breaking. This results holds for models which are calculable, i.e. where the scale of supersymmetry breaking is much lower than the scale at which the gauge theory is strongly coupled. The superpotential has to be generic, all the terms admitted by the symmetries have to be present and the coefficients are not fine tuned. Finally the Kahler potential is finite and non-singular. In models in which these hypothesis hold, supersymmetry is unbroken if

$$\frac{\partial W_{eff}}{\partial \phi_i} \tag{2.44}$$

where the ϕ_i are gauge invariant functions of the parameters describing the D-flat directions.
In this class of models two result hold

1 Having an R-symmetry is a necessary condition for spontaneously supersymmetry breaking.

2 A spontaneously broken R-symmetry is a sufficient condition for spontaneous supersymmetry breaking

The first claim is proved in two steps. First one consider the case in which there are no global symmetries. In that case there are n F-term equations for n unknowns, and supersymmetry is unbroken (counterexamples are in the class of non generic models). The second possibility is that there are continuous global symmetries which commute with supersymmetry, which is not the case for R-symmetry. In this case the fact that supersymmetry is unbroken follows from the holomorphy of the superpotential. Indeed if there are l generators of global symmetry, than the superpotential is a function of $n-l$ variables. W_{eff} is independent of l variables but l equations are satisfied. In this case there are $n-l$ variables for $n-l$ unknowns, and supersymmetry is unbroken for generic models. This shows that the absence of R symmetry is a sufficient condition for the absence of supersymmetry breaking. In other words, R symmetry is necessary for supersymmetry breaking.

For the proof of the second claim one need a field ϕ_n whose R-charge is not vanishing. If this field has finite, but not zero, vev, then one can define a new class of variables for the fields ϕ_i $(i = 1, \ldots, n-1)$

$$X_i = \frac{\phi_i}{\phi_n^{q_i/q_n}} \tag{2.45}$$

If $R[W] = 2$, the superpotential written in the new X_i, ϕ_n variables become

$$W_{\text{eff}} = \phi_n^{2/q_n} f(X_i) \tag{2.46}$$

where f is an holomorphic function of the $n-1$ variables X_i. The F terms are

$$\begin{aligned} \frac{\partial W_{\text{eff}}}{\partial \phi_n} &= \frac{2}{q_n} \phi^{\frac{2-q_n}{q_n}} f(X_i) \sim f(X_i) \\ \frac{\partial W_{\text{eff}}}{\partial X_i} &\sim \frac{f(X_i)}{\partial X_i} \end{aligned} \tag{2.47}$$

We have n equations for $n-1$ unknowns, and generically there is no supersymmetric solution. This shows the second claim.

Although massive SQCD breaks R-symmetry to Z_2, the ISS model is not in conflict with these results on R-symmetry. Indeed around the origin of the moduli space of the magnetic theory the theory is described by an R-symmetric superpotential (2.36), and R symmetry breaking effects are negligible. These non-perturbative effects break R symmetry in the large field region, where supersymmetry is restored.

In many application the hypothesis of genericity is evaded. In this case one can write some models with no R-symmetry and with broken supersymmetry. For example the superpotential

$$W = fX + X\phi_1\phi_2 + \mu(\phi_1\phi_3 + \phi_2\phi_4) + m(\phi_1\phi_5 + \phi_2\phi_6) + \epsilon\phi_3\phi_4 \tag{2.48}$$

is not R-symmetric and it breaks supersymmetry at the classical level if $\phi_i = 0$ and X is arbitrary. We will discuss this model in detail in the case of metastable vacua with adjoint matter.

Spontaneous R symmetry breaking

The hypothesis of genericity constraints the landscape of models of supersymmetry breaking. Models that preserve R-symmetry at tree level are not phenomenologically appealing if R-symmetry is not broken in the quantum moduli space. This raised the interest on spontaneous R-symmetry breaking. Direct inspection is not promising, since the analysis can be involved, and general results are more attractive.

In the last two years some general result has been found in WZ models at one loop [4] and at tree level [32, 33]. They are based on the assignation of R-charges in an R-symmetric model.

Tree level spontaneous R symmetry breaking

R-symmetry is unbroken at tree level in a renormalizable WZ model, with canonical Kahler potential and superpotential

$$W_0 = \sum_i X_i f_i(\phi_J) \quad \text{where} \quad f(x) = a_0 + a_1 x + a_2 x^2 \tag{2.49}$$

if the field X has R-charge two and if the other ϕ_J fields have vanishing R-charge. This result was proved in ([32]). In another paper [33] the author modified the superpotential (2.49) by adding a term

$$W_0 \to W = W_0 + g(\phi_J, \tilde{\phi}_K) \tag{2.50}$$

The model with $g = 0$ has to break supersymmetry at tree level and it has to respects an ordinary $U(1)_{nonR}$ symmetry. Moreover the superpotential (2.50) with $g = 0$ has to stabilize the ϕ_J fields out of the origin, spontaneously breaking the $U(1)_{nonR}$. One then adds g such that it explicitly breaks the original R symmetry. The $U(1)_R$ and $U(1)_{nonR}$ are both broken, but a non trivial $U(1)_{R'}$ combination still remains. As long as $F_{\tilde{\phi}} = 0$, R' symmetry is broken at tree level, in fact $\langle\phi_J\rangle \neq 0$ and $R'[\phi_J] \neq 0$.

One loop spontaneous R symmetry breaking

Tree level spontaneous R-symmetry breaking is usually discarded in favor of one loop R-symmetry breaking for phenomenological reasons. At one loop an useful formula for the calculation of the first order mass term acquired from a classical flat direction was calculated in [4]. The superpotential studied in [4] is

$$W = fX + \frac{1}{2}\left(M^{ij} + XN^{ij}\right) \tag{2.51}$$

and the equation of motion fix $\phi_i = 0$ and X arbitrary. The mass matrices are defined as

$$m_B^2 = (\hat{M} + X\hat{N})^2 + f\hat{N} \quad m_F^2 = (\hat{M} + X\hat{N})^2 \tag{2.52}$$

where

$$\hat{M} = \left(\begin{array}{cc} 0 & M^\dagger \\ M & 0 \end{array}\right) \quad \hat{N} = \left(\begin{array}{cc} 0 & N^\dagger \\ N & 0 \end{array}\right) \tag{2.53}$$

In [4] it has been shown that the mass term for the pseudomodulus is

$$m_X^2 = \frac{1}{16\pi^2 f^2}\int_0^\infty dv v^3 \mathrm{Tr}\left(\frac{\mathcal{F}^4(v)}{1-\mathcal{F}^2(v)}v^2 - 2\frac{\mathcal{F}^2(v)}{1-\mathcal{F}^2(v)\hat{M}}\right) \equiv M_1^2 - M_2^2 \tag{2.54}$$

where $\mathcal{F}(v) = (v^2 + \hat{M}^2)^{-1} f\hat{N}$. Both M_1^2 and M_2^2 are positive, but M_2^2 is zero if all the fields in the theory have R-charges zero or two. R-symmetry is spontaneously broken at one loop only if some field with R-charge different from zero or two is present.

Some of the results of this section do not hold at two or higher loop. The behaviour of the pseudomoduli in the large field region has been studied in [34]. Although the analog of the CW potential does not exists at higher loop, the behaviour of a large class of models at the origin of the moduli space can be studied by explicit calculation. In section 2.5 we will give some example.

R symmetry and runaway directions

The assignment of R-charges is also related to the classical behaviour of the potential at large fields. The symmetry group cannot determine by itself the manifold of classical solution of the F-terms. The complexified of the symmetry group must be used. If there are fields with R charges higher than two and lower then two, than the theory usually has a runaway behaviour. This is runaway takes place if one is able to solve the equations $F_i = 0$ for all the fields with $R \leq 2$ or $R \geq 2$. In this case one can act on the solutions with a complexified transformation of the symmetry group and take the parameter to $\pm\infty$. In such a way a runaway direction is found. This runaway can be avoided if not all the F terms can be satisfied. There is a whole branch of the moduli space that breaks R-symmetry, but with no runaway. For an explicit realization of this mechanism see [35].

Since supersymmetry necessitate to be a broken symmetry, the MSSM requires to be completed by a supersymmetry breaking sector. One can ask if the supersymmetry breaking scale can directly belong to the MSSM but the answer is negative. Indeed a FI term for $U(1)_Y$ does not lead to an acceptable spectrum, and there are no gauge singlets with non zero F-terms.
This require a new sector that communicates the breakdown of supersymmetry to the MSSM partners. The STr formula [36] does not allow a tree level coupling between the supersymmetry breaking sector and the MSSM. Indeed some of the superpartners are too light and they should have been already detected at the colliders.
Another possibility is that the supersymmetry breaking sector is hidden, and couples to the MSSM through flavor blind gauge interactions. The MSSM soft masses, for *Gaugino* and sfermions raise up radiatively. This requirement evades the constraints of the STr theorem and the gauged group can be the SM gauge group or some GUT extension.

Minimal gauge mediation

This mechanism is based on the coupling of the MSSM sector and the Hidden supersymmetry breaking sector through new chiral fields, called messengers. These field feel supersymmetry breaking and are charged under the MSSM gauge group. They couple with the gauge bosons and *Gaugino* of the MSSM gauge group.
The superpotential for these messenger fields is

$$W_{mess} = \sum_i h_i X f_i \tilde{f}_i \tag{2.55}$$

where X is a field with non zero vev and F-term, $X = M + \theta^2 F$. This can be the field pseudomodulus of an O'Raifeartaigh sector, or it can belong to a dynamical supersymmetry breaking sector.
The *Gauginos* of the MSSM acquire a one loop mass, while the sfermion mass is obtained at two loop. The gauge bosons and the fermions do not acquire masses because of the gauge invariance of the MSSM. The masses are calculated once the classical spectrum of the messengers is known. One has

$$m_{\psi f_i \psi \tilde{f}_i} = |h_i M| \quad m^2_{f_i, \tilde{f}_i} = |h_i M|^2 \pm |h_i F| \tag{2.56}$$

The *Gaugino* and sfermion masses are [37, 38]

$$M_a = \frac{\alpha_a}{4\pi} \frac{F}{M} \sum_i n_a(i) g(x_i) \tag{2.57}$$

$$\tilde{M} = 2 \left| \frac{F}{M} \right|^2 \sum_a \left(\frac{\alpha_a}{4\pi} \right)^2 C_a \sum_i n_a(i) f(x_i)$$

where $x_i = |F/(h_i M)|$,$n_a(i)$ is the Dinkin Index of the pair $f_i + \tilde{f}_i$ (the fundamental-antifundamental pair is normalized by $n_a = 1$) and C_a is the Quadratic Casimir invariant.

The functions $g(x)$ and $f(x)$ are

$$f(x) = \frac{1}{x^2}\left((1+x)\log(1+x)\right) + (x \leftrightarrow -x) \tag{2.58}$$
$$g(x) = \frac{1+x}{x^2}\left(\log(1+x) - 2\ \mathrm{Li}_2\ \frac{x}{1+x} + +\frac{1}{2}\ \mathrm{Li}_2\ \frac{2x}{1+x}\right) + (x \leftrightarrow -x)$$

Direct Mediation

There is another mechanism of gauge mediation, in which the messenger sector disappears. A (sub)-group of the flavour group of the hidden sector is gauged, and it often coincides with the MSSM gauge group. The main problem is that the new gauge group usually implies a supersymmetric vacuum in the hidden sector.

One must add some interaction that lift this zero energy state, or alternatively one has to require that the non supersymmetric state is long lifetime. (Direct) gauge mediation requires metastability.

We now remark that many supersymmetry breaking models studied in the past where chiral models, since non chiral models have non zero Witten index. If we now accept metastability, we can consider hidden vector like sector with metastable vacua. The simplest example is the ISS model.

Direct mediation in ISS

Even if R-symmetry protects the *Gaugino* to be massive at every order in perturbation theory, we here gauge (a part of) the global symmetry of ISS and see explicitly what happens.

The global symmetry unbroken by the metastable ISS vacuum is $SU(N_f - \tilde{N}_c) \times SU(\tilde{N}_c)$. Gauging a subgroup $SU(3) \times SU(2) \times U(1)$ the gauginos of this subgroup interact with the fields ϕ_i and X of the superpotential (2.36).

This last can be written as

$$W = h \begin{pmatrix} \phi_1 & \phi_3 \end{pmatrix} \begin{pmatrix} X & \mu \\ \mu & 0 \end{pmatrix} \begin{pmatrix} \phi_2 \\ \phi_4 \end{pmatrix} - h\mu^2 X = h \begin{pmatrix} \phi_1 & \phi_3 \end{pmatrix} \mathcal{M} \begin{pmatrix} \phi_2 \\ \phi_4 \end{pmatrix} - h\mu^2 X \tag{2.59}$$

The mass of the gaugino at the lowest order in the supersymmetry breaking scale F_X [1] is given by the formula

$$m_\lambda = \frac{\alpha}{4\pi} N |F_X| \frac{\partial \log \det \mathcal{M}}{\partial X} \tag{2.60}$$

In this case the determinant of $\mathcal{M}$ is independent on X and the gaugino is massless.

In [39] it was shown that the addition of a R-symmetry breaking deformation $m\phi_3\phi_4$, changes the mass matrix and $\mathcal{M}$ and the gaugino acquires a mass

$$m_\lambda = \frac{\alpha}{4\pi} N \frac{|F_X|}{m} \tag{2.61}$$

[1] This formula gives the correct order for the gaugino mass also if $\mu^2 \sim F_X$.

Indeed in this case the R symmetry cannot protect the gaugino to be massless anymore.

In many cases the formula (2.60) cannot be used since the gaugino receives a leading order contribution at higher orders in the supersymmetry breaking scale $|F_X|$. This will be the case for the the metastable vacua with adjoint matter.

The Landau pole problem

The identification of a gauged global symmetry with the gauge symmetry of the MSSM presents a phenomenological problem, named the Landau pole problem. Indeed there are many particles that are charged under the GUT or $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry. They contribute to the beta function of the gauge group, and this can lead the Landau pole under the unification scale. This constraints the choice of the mass scale, of the dynamical scale and of the flavor symmetry that needs to be gauged in the hidden sector,

2.2 SQCD with adjoint matter

In this section we propose a generalization of the ISS mechanism to SQCD with adjoint matter. We study models with adjoint chiral fields with cubic superpotential à la KSS [21, 22, 23]. Such superpotentials generate a further meson in the dual magnetic theory: this might produce several pseudogoldstone excitations and jeopardize the 1-loop stability of the non supersymmetric vacua. There must be enough F and/or D equations to give tree level masses.

We consider a theory with one gauge group $SU(N_c)$ and two massive electric adjoint fields, where the most massive one gets integrated out. This amounts to add a massive mesonic deformation in the dual theory. This avoids dangerous extra flat directions which cannot be stabilized at 1-loop. A discussion of the possible interpretation via D-brane configurations can be found in [40, 41].

In the study of the magnetic dual theory we find a tree-level non supersymmetric vacuum which is stabilized by quantum corrections; we show that this is a metastable state that decays to a supersymmetric one after a parametrically long time. A landscape of non supersymmetric metastable vacua, present at classical level, disappears at quantum level. Differently from [2, 42, 43] in our model there is no $U(1)_R$ symmetry and our minimum will not be at the origin of the field space, making our computation much involved. We present most of our results graphically, giving analytic expressions in some sensible limits.

The explicit breaking of the R-symmetry is a starting point for phenomenological applications of the ISS mechanism. Indeed the ISS model and its generalizations can have phenomenological applications in connection with gauge mediation of dynamical supersymmetry breaking to the standard model sector [44]. R-symmetry plays here a relevant role since a $U(1)$ R-symmetry, even broken to Z_n, forbids a gaugino mass generation. To obtain a gaugino mass, deformations can be added to the superpotential making the R-symmetry trivial, and this might require a further careful analysis of its stability.

Metastable models have been analyzed in this direction [39, 45, 74, 46, 47]. In most cases extra terms, breaking R-symmetry, have been added to known models of dynamical supersymmetry breaking, leading to gaugino mass at 1 loop at the first or at the third order in the breaking scale. Our model which has meta-stable vacua, is rather non generic (in the sense of [3]), it has no R-symmetry and it is suitable for direct gauge mediation.

We first recall some basic elements of the KSS duality for the model that we consider. Then we solve the the D and F equations finding a local minimum where supersymmetry is broken by the rank condition. After that we compute the 1-loop effective potential around this vacuum and find that it is stabilized by the quantum corrections. Later we restore supersymmetry by non perturbative gauge dynamics and recover supersymmetric vacua. By using this result, we estimate the lifetime of the metastable state. Finally we conclude this section by showing that a gaugino mass gets generated at 1 loop at third order in the breaking parameter.

$\mathcal{N} = 1$ SQCD with adjoint matter

We consider $\mathcal{N} = 1$ supersymmetric $SU(N_c)$ Yang Mills theory coupled to N_f massive flavours $(Q^i_\alpha, \tilde{Q}^{j\beta})$ in the fundamental and antifundamental representations of the gauge group $(\alpha, \beta = 1, \dots N_c)$ and in the antifundamental and fundamental representations of the flavour group $(i, j = 1, \dots N_f)$, respectively. We also consider a charged chiral massive adjoint superfield X^α_β with superpotential[2]

$$W_{el} = \frac{g_X}{3}\mathrm{Tr}X^3 + \frac{m_X}{2}\mathrm{Tr}X^2 + \lambda_X \mathrm{Tr}X \tag{2.62}$$

where λ_X is a Lagrange multiplier enforcing the tracelessness condition $\mathrm{Tr}X = 0$. The Kahler potential for all the fields is taken to be canonical. This theory is asymptotically free in the range $N_f < 2N_c$ and it admits stable vacua for $N_f > \frac{N_c}{2}$ [22].

The dual theory [21, 22, 23] is $SU(2N_f - N_c \equiv \tilde{N})$ with N_f magnetic flavours $(q, \tilde{q})$, a magnetic adjoint field Y and two gauge singlets build from electric mesons ($M_1 = Q\tilde{Q}$, $M_2 = QX\tilde{Q}$), with magnetic superpotential

$$W_{magn} = \frac{\tilde{g}_Y}{3}\mathrm{Tr}Y^3 + \frac{\tilde{m}_Y}{2}\mathrm{Tr}Y^2 + \tilde{\lambda}_Y \mathrm{Tr}Y - \frac{1}{\mu^2}\mathrm{tr}\left(\frac{\tilde{m}_Y}{2}M_1 q\tilde{q} + \tilde{g}_Y M_2 q\tilde{q} + \tilde{g}_Y M_1 q Y \tilde{q}\right) \tag{2.63}$$

where the relations between the magnetic couplings and the electric ones are

$$\tilde{g}_Y = -g_X, \qquad \tilde{N}\tilde{m}_Y = N_c m_X. \tag{2.64}$$

The intermediate scale μ takes into account the mass dimension of the mesons in the dual description. The matching between the microscopic scale (Λ) and the macroscopic scale ($\tilde{\Lambda}$) is

$$\Lambda^{2N_c - N_f}\tilde{\Lambda}^{2\tilde{N} - N_f} = \left(\frac{\mu}{g_X}\right)^{2N_f}. \tag{2.65}$$

[2](Tr) means tracing on the color indices, while (tr) on the flavour ones.

We look for a magnetic infrared free regime in order to rely on perturbative computations at low energy. The b coefficient of the beta function is $b = (3\tilde{N} - N_f) - \tilde{N}$, negative for $N_f < \frac{2}{3}N_c$ and so we will consider the window for the number of flavours

$$\frac{N_c}{2} < N_f < \frac{2}{3}N_c \qquad \Rightarrow \qquad 0 < 2\tilde{N} < N_f \tag{2.66}$$

where the magnetic theory is IR free and it admits stable vacua.

Adding mesonic deformations

We now add to the electric potential (2.62) the gauge singlet deformations

$$W_{el} \to W_{el} + \Delta W_{el} \qquad \Delta W_{el} = \lambda_Q \,\mathrm{tr}\; QX\tilde{Q} + m_Q \,\mathrm{tr}\; Q\tilde{Q} + h \,\mathrm{tr}\; (Q\tilde{Q})^2 \tag{2.67}$$

The first two terms are standard deformations of the electric superpotential that don't spoil the duality relations (e.g. the scale matching condition (2.65)) [23]. The last term of (2.67) can be thought as originating from a second largely massive adjoint field Z in the electric theory with superpotential

$$W_Z = m_Z \mathrm{Tr} Z^2 + \mathrm{Tr}\, ZQ\tilde{Q} \tag{2.68}$$

and which has been integrated out [40, 41, 48]. The mass m_Z has to be considered larger than Λ_{2A}, the strong scale of the electric theory with two adjoint fields. This procedure leads to the scale matching relation

$$\Lambda_{2A}^{N_c - N_f} = \Lambda_{1A}^{2N_c - N_f} m_Z^{-N_c} \tag{2.69}$$

where Λ_{2A} and Λ_{1A} are the strong coupling scales before and after having integrated out the adjoint field Z, i.e. with two or one adjoint fields respectively.

The other masses in this theory have to be considered much smaller than the strong scale $\Lambda_{2A} \gg m_Q, m_X$. This forces, via (2.69), the scale Λ_{1A} and the masses to satisfy the relations

$$\frac{m_Q m_Z}{\Lambda_{1A}^2} \ll 1 \qquad\qquad \frac{m_X m_Z}{\Lambda_{1A}^2} \ll 1 \tag{2.70}$$

We will work in this range of parameters in the whole paper, translating these inequalities in the dual (magnetic) context.

We also observe that in (2.69) the coefficient b of the beta function for the starting electric theory with two adjoint fields is $b = N_c - N_f$ and the theory is asymptotically free for $N_f < N_c$. This range is still consistent with our magnetic IR free window (2.66). The dimensional coupling h in our effective theory (2.67) results $h = \frac{1}{m_Z}$ so it must be thought as a small deformation. In analogy with [48][3] we can suppose that when h is small the duality relations are still valid and obtain the full magnetic superpotential

$$\begin{aligned} W_{magn} \;\; &= \;\; \frac{\tilde{g}_Y}{3}\mathrm{Tr}Y^3 + \frac{\tilde{m}_Y}{2}\mathrm{Tr}Y^2 + \tilde{\lambda}_Y \mathrm{Tr}Y - \frac{1}{\mu^2}\,\mathrm{tr}\;\left(\frac{\tilde{m}_Y}{2}M_1 q\tilde{q} + \tilde{g}_Y M_2 q\tilde{q} + \tilde{g}_Y M_1 qY\tilde{q}\right) \\ &\quad + \lambda_Q \;\mathrm{tr}\; M_2 + m_Q \;\mathrm{tr}\; M_1 + h \;\mathrm{tr}\; (M_1)^2 \end{aligned} \tag{2.71}$$

[3]Where it was done in the context of Seiberg duality.

For this dual theory the scale matching relation is the same as (2.65) with $\Lambda \equiv \Lambda_{1A}$ defined in (2.69).

We consider the free magnetic range (2.66), where the metric on the moduli space is smooth around the origin. The Kahler potential is thus regular and has the canonical form

$$K = \frac{1}{\alpha_1^2\Lambda^2}\text{tr } M_1^\dagger M_1 + \frac{1}{\alpha_2^2\Lambda^4}\text{tr } M_2^\dagger M_2 + \frac{1}{\beta^2}\text{Tr}Y^\dagger Y + \frac{1}{\gamma^2}(\text{tr } q^\dagger q + \text{tr } \tilde{q}^\dagger \tilde{q}) \qquad (2.72)$$

where $(\alpha_i, \beta, \gamma)$ are unknown positive numerical coefficients.

Non supersymmetric metastable vacua

We solve the equations of motion for the chiral fields of the macroscopic description (2.71). We will find a non supersymmetric vacuum in the region of small fields where the $SU(\tilde{N})$ gauge dynamics is decoupled. The gauge dynamics becomes relevant in the large field region where it restores supersymmetry via non perturbative effects (see sec.5).

We rescale the magnetic fields appearing in (2.71) in order to work with elementary fields with mass dimension one. We then have a $\mathcal{N} = 1$ supersymmetric $SU(\tilde{N})$ gauge theory with N_f magnetic flavours $(q, \tilde{q})$, an adjoint field Y, and two gauge singlet mesons M_1,M_2, with canonical Kahler potential. The superpotential, with rescaled couplings, reads

$$\begin{aligned} W_{magn} \;&=\; \frac{g_Y}{3}\text{Tr}Y^3 + \frac{m_Y}{2}\text{Tr}Y^2 + \lambda_Y \text{Tr}Y + \text{tr }(h_1 M_1 q\tilde{q} + h_2 M_2 q\tilde{q} + h_3 M_1 q Y \tilde{q}) \\ &\quad -h_1 m_1^2 \text{ tr } M_1 - h_2 m_2^2 \text{ tr } M_2 + m_3 \text{ tr } M_1^2 \qquad (2.73) \end{aligned}$$

where the rescaled couplings in (2.73) are mapped to the original ones in (2.71) via

$$\begin{aligned} &h_1 = -\frac{\tilde{m}_Y}{2\mu^2}(\alpha_1\Lambda)\gamma^2 \qquad h_2 = -\frac{\tilde{g}_Y}{\mu^2}\left(\alpha_2\Lambda^2\right)\gamma^2 \qquad h_3 = -\frac{\tilde{g}_Y}{\mu^2}(\alpha_1\Lambda)\gamma^2\beta \\ &h_1 m_1^2 = -m_Q\,\alpha_1\,\Lambda \qquad h_2 m_2^2 = -\lambda_Q\,\alpha_2\,\Lambda^2 \qquad m_3 = h(\alpha_1\Lambda)^2 \qquad (2.74) \end{aligned}$$

We can choose the magnetic quarks $q, \tilde{q}^T$ (which are $N_f \times \tilde{N}$ matrices) to solve the D equations as

$$q = \begin{pmatrix} k \\ 0 \end{pmatrix} \qquad \tilde{q}^T = \begin{pmatrix} \tilde{k} \\ 0 \end{pmatrix} \qquad (2.75)$$

where $k, \tilde{k}$ are $\tilde{N} \times \tilde{N}$ diagonal matrices such that the diagonal entries satisfy $|k_i| = |\tilde{k}_i|$.

We impose the F equations of motion for the superpotential (2.73)

$$\begin{aligned} F_{\lambda_Y} &= \text{Tr}Y = 0 \\ F_Y &= g_Y Y^2 + m_Y Y + \lambda_Y + h_3 M_1 q\tilde{q} = 0 \\ F_q &= h_2 M_2 \tilde{q} + h_1 M_1 \tilde{q} + h_3 M_1 Y \tilde{q} = 0 \\ F_{\tilde{q}} &= h_2 M_2 q + h_1 M_1 q + h_3 M_1 q Y = 0 \qquad (2.76) \\ F_{M_1} &= h_1 q\tilde{q} + h_3 q Y \tilde{q} - h_1 m_1^2 \delta_{ij} + 2 m_3 M_1 = 0 \qquad i, j = 1, \ldots N_f \\ F_{M_2} &= h_2 q\tilde{q} - h_2 m_2^2 \delta_{ij} = 0 \qquad i, j = 1, \ldots N_f \qquad (2.77) \end{aligned}$$

Since we are in the range (2.66) where $N_f > \tilde{N}$ the equation (2.77) is the rank condition of [2]: supersymmetry is spontaneously broken at tree-level by these non trivial F-terms.

We can solve the first $\tilde{N}$ equations of (2.77) by fixing the product $k\tilde{k}$ to be $k\tilde{k} = m_2^2 \mathbf{1}_{\tilde{N}}$. We then parametrize the quarks vevs in the vacuum (2.75) with complex θ

$$q = \begin{pmatrix} m_2 e^{\theta}\, \mathbf{1}_{\tilde{N}} \\ 0 \end{pmatrix} \qquad \tilde{q}^T = \begin{pmatrix} m_2 e^{-\theta}\, \mathbf{1}_{\tilde{N}} \\ 0 \end{pmatrix} . \tag{2.78}$$

The other $N_f - \tilde{N}$ equations of (2.77) cannot be solved and so the corresponding F-terms don't vanish ($F_{M_2} \neq 0$). However we can find a vacuum configuration which satisfies all the other F-equations (2.76) and the D-ones. We solve the equations (2.76) for M_1, Y and λ_Y and we choose Y to be diagonal, finding

$$\lambda_Y = \frac{h_3 h_1 m_2^2}{2m_3}\left(m_2^2 - m_1^2\right) - \frac{m_Y^2}{g_Y}\left(1 - \frac{h_3^2 m_2^4}{2m_3 m_Y}\right)^2 \frac{n_1 n_2}{(n_1 - n_2)^2} \tag{2.79}$$

where the integers (n_1, n_2) count the eigenvalues degeneracy along the Y diagonal, with $(n_1 + n_2 = \tilde{N})$

$$\langle Y \rangle = \begin{pmatrix} y_1 \mathbf{1}_{n_1} & 0 \\ 0 & y_2 \mathbf{1}_{n_2} \end{pmatrix} \qquad y_1 = -\frac{m_Y - \frac{h_3^2 m_2^4}{2m_3}}{g_Y}\frac{n_2}{n_1 - n_2} \qquad y_2 = \frac{m_Y - \frac{h_3^2 m_2^4}{2m_3}}{g_Y}\frac{n_1}{n_1 - n_2}$$

We choose the vacuum in which the magnetic gauge group is not broken by the adjoint field choosing $n_1 = 0$, so y_2 vanishes and $\langle Y \rangle = 0$. We observe that other choices for $\langle Y \rangle$ with $n_1 \neq 0 \neq n_2$ wouldn't change the tree-level potential energy of the vacua which is given only by the non vanishing F_{M_2}. This classical landscape of equivalent vacua will be wiped out by 1-loop quantum corrections[4]. In our case ($n_1 = 0$) we have

$$\langle M_1 \rangle = \begin{pmatrix} \frac{h_1}{2m_3}(m_1^2 - m_2^2)\ \mathbf{1}_{\tilde{N}} & 0 \\ 0 & \frac{h_1 m_1^2}{2m_3}\ \mathbf{1}_{N_f - \tilde{N}} \end{pmatrix} = \begin{pmatrix} p_1^A & 0 \\ 0 & p_1^B \end{pmatrix} \tag{2.80}$$

The two non trivial blocks are respectively $\tilde{N}$ and $N_f - \tilde{N}$ diagonal squared matrices.

The $(q, \tilde{q})$ F equations fix the vev of the M_2 meson to be

$$\langle M_2 \rangle = \begin{pmatrix} -\frac{h_1^2}{2h_2 m_3}(m_1^2 - m_2^2)\ \mathbf{1}_{\tilde{N}} & 0 \\ 0 & \mathcal{X} \end{pmatrix} = \begin{pmatrix} p_2^A & 0 \\ 0 & \mathcal{X} \end{pmatrix} \tag{2.81}$$

where the blocks have the same dimensions of M_1, with $\mathcal{X}$ undetermined at the classical level.

Since supersymmetry is broken at tree level by the rank condition (2.77) the minimum of the scalar potential is

$$V_{MIN} = |F_{M_2}|^2 = (N_f - \tilde{N})|h_2 m_2^2|^2 = (N_f - \tilde{N})\, \alpha_2^2\, |\lambda_Q \Lambda^2|^2 \tag{2.82}$$

[4]This agrees with an observation in [43].

It depends on parameters that we can't compute from the electric theory (e.g. α_2); in any case we are only interested in the qualitative behaviour of the non supersymmetric state. The potential energy of the vacuum (2.82) doesn't depend on θ and $\mathcal{X}$; they are massless fields at tree level, not protected by any symmetry and hence are pseudomoduli. Their fate will be decided by the quantum corrections.

We don't expect the value of $\mathcal{X}$ in the quantum minimum to vanish because there isn't any $U(1)_R$ symmetry. Indeed, computing the 1-loop corrections, we will find that in the quantum minimum the value of θ is zero while $\mathcal{X}$ will get a nonzero vev. This makes our metastable minimum different from the one discovered in [2, 42, 43] where the quantum corrections didn't give the pseudomoduli a nonzero vev. Notice also that although we have many vevs different from zero in the non supersymmetric vacuum they are all smaller than the natural breaking mass scale $|F_{M_2}|^{\frac{1}{2}} = |h_2 m_2^2|^{\frac{1}{2}}$.

1-Loop effective potential

In this subsection we study the 1-loop quantum corrections to the effective potential for the fluctuations around the non supersymmetric vacuum selected in the previous subsection with $\langle Y \rangle = 0$. The aim is to establish the sign of the mass corrections for the pseudomoduli $\mathcal{X}, \theta$. The 1-loop corrections to the tree level potential energy depend on the choice of the adjoint vev $\langle Y \rangle$: as a matter of fact they are minimized by the choice $\langle Y \rangle = 0$.

The 1-loop contributions of the vector multiplet to the effective potential vanish since the D equations are satisfied by our non supersymmetric vacuum configuration.

The 1-loop corrections will be computed using the supertrace of the bosonic and fermionic squared mass matrices built up from the superpotential for the fluctuations of the fields around the vacuum. The standard expression of the 1-loop effective potential is

$$V_{1-loop} = \frac{1}{64\pi^2} S\mathrm{Tr}\mathcal{M}^4 \log \frac{\mathcal{M}^2}{\Lambda^2} = \frac{1}{64\pi^2} \sum \left(m_B^4 \log \frac{m_B^2}{\Lambda^2} - m_F^4 \log \frac{m_F^2}{\Lambda^2} \right) \tag{2.83}$$

where the F contributions to the mass matrices are read from the superpotential W

$$m_B^2 = \begin{pmatrix} W^{\dagger ac} W_{cb} & W^{\dagger abc} W_c \\ W_{abc} W^{\dagger c} & W_{ac} W^{\dagger cb} \end{pmatrix} \qquad m_f^2 = \begin{pmatrix} W^{\dagger ac} W_{cb} & 0 \\ 0 & W_{ac} W^{\dagger cb} \end{pmatrix} \tag{2.84}$$

We parametrize the fluctuations around the tree level vacuum as

$$q = \begin{pmatrix} k e^{\theta} + \xi_1 \\ \phi_1 \end{pmatrix} \qquad \tilde{q}^T = \begin{pmatrix} k e^{-\theta} + \xi_2 \\ \phi_2 \end{pmatrix} \qquad Y = \delta Y \tag{2.85}$$

$$M_1 = \begin{pmatrix} p_1^A + \xi_3 & \phi_3 \\ \phi_4 & p_1^B + \xi_4 \end{pmatrix} \qquad M_2 = \begin{pmatrix} p_2^A + \xi_5 & \phi_5 \\ \phi_6 & \mathcal{X} \end{pmatrix} \tag{2.86}$$

We expand the classical superpotential (2.73) up to three linear order in the fluctuations $\phi_i, \xi_i, \delta Y$. Most of these fields acquire tree level masses, but there are also massless fields.

Some of them are Goldstone bosons of the global symmetries considering $SU(\tilde{N})$ global, the others are pseudogoldstone.
In this set up, ξ_1 and ξ_2 combine to give the same Goldstone and pseudogoldstone bosons as in [2]. Gauging the $SU(\tilde{N})$ symmetry these goldstones are eaten by the vector fields, and the other massless fields, except $\theta+\theta^\star$, acquire positive masses from D-term potential as in [2]. Combinations of the ϕ_i fields give the Goldstone bosons related to the breaking of the flavour symmetry $SU(N_f) \to SU(\tilde{N}) \times SU(N_f - \tilde{N}) \times U(1)$. The off diagonal elements of the classically massless field $\mathcal{X}$ are Goldstone bosons of the $SU(N_f - \tilde{N})$ flavour symmetry as in [42]. We then end up with the pseudogoldstones $\theta + \theta^\star$ and the diagonal $\mathcal{X}$.

We now look for the fluctuations which give contributions to the mass matrices (2.84). They are only the ϕ_i fields, while the ξ_i and δY represent a decoupled supersymmetric sector. Indeed ξ_i and δY do not appear in bilinear terms coupled to the ϕ_i sector, so they do not contribute to the fermionic mass matrix (2.84). Even if they appear in three linear terms coupled to the ϕ_i, they do not have the corresponding linear term[5]: they do not contribute to the bosonic mass matrix (2.84). Since $(\xi_1, \xi_2, \delta Y)$ do not couple to the breaking sector at this order, also their D-term contributions to the mass matrices vanish and all of them can be neglected. We can then restrict ourselves to the chiral ϕ_i fields for computing the 1-loop quantum corrections to the effective scalar potential using (2.84). Without loss of generality we can set the pseudomoduli $\mathcal{X}$ proportional to the identity matrix.

The resulting superpotential for the sector affected by the supersymmetry breaking (the ϕ_i fields) is a sum of $\tilde{N} \times (N_f - \tilde{N})$ decoupled copies of a model of chiral fields which breaks supersymmetry at tree-level

$$\begin{aligned} W &= h_2 \left(\mathcal{X}\phi_1\phi_2 - m_2^2\mathcal{X}\right) + h_2 m_2 \left(e^{\theta}\phi_2\phi_5 + e^{-\theta}\phi_1\phi_6\right) + \\ &\quad + h_1 m_2 \left(e^{\theta}\phi_2\phi_3 + e^{-\theta}\phi_1\phi_4\right) + 2m_3\phi_3\phi_4 + \frac{h_1^2 m_1^2}{2m_3}\,\phi_1\phi_2 \end{aligned} \tag{2.87}$$

This superpotential doesn't have any $U(1)_R$ symmetry, differently from the ones studied in [2, 42, 43]. This may be read as an example of a non generic superpotential which breaks supersymmetry [3], without exact R symmetry.

The expressions for the eigenvalues, and then for the 1-loop scalar potential, are too complicated to be written here. We can plot our results numerically to give a pictorial representation.

The computation is carried out in this way: we first compute the eigenvalues of the bosonic and fermionic mass matrices (2.84) using the superpotential (2.87); we evaluate them where all the fluctuations ϕ_i are set to zero; finally we compute the 1-loop scalar potential using (2.83) as a function of the pseudomoduli $\mathcal{X}, \theta + \theta^\star$. The corrections will always be powers of $\theta + \theta^\star \equiv \tilde{\theta}$ so from now on we will treat only the $\tilde{\theta}$ dependence. We give graphical plots of the 1-loop effective potential treating fields and couplings as real. We have checked that our qualitative conclusions about the stability of the vacuum are not affected by using complex variables.

[5]The possible linear terms in ξ_i and δY factorize the F-equations (2.76) and so they all vanish.

We redefine the couplings in order to have the mass matrices as functions of three dimensionless parameters (ρ, η, ζ)

$$\rho = \frac{h_1}{h_2} \qquad \eta = \frac{2m_3}{h_2 m_2} \qquad \zeta = \frac{h_1^2 m_1^2}{2 h_2 m_2 m_3} \quad , \qquad\qquad \zeta < \rho < \eta \tag{2.88}$$

and we rescale the superpotential with an overall scale $h_2 m_2$ which becomes the fundamental unit of our plots. The inequality in (2.88) is a consequence of the range (2.70) and the redefinitions (2.74). We notice also that (ρ, η, ζ) have absolute values smaller than one.

In figure 2.1 we plot the 1-loop scalar potential as a function of $\mathcal{X}, \tilde{\theta}$ and for fixed values of the parameters ρ, η, ζ. We can see that there is a minimum, so the moduli space is lifted by the quantum corrections, the pseudomoduli get positive masses, and there is a stable non supersymmetric vacuum. Making a careful analysis we find that the quantum minimum in the 1-loop scalar potential is reached when $\langle\tilde{\theta}\rangle = 0$ but $\langle\mathcal{X}\rangle \neq 0$ and its vev in the minimum depends on the parameters (ρ, η, ζ). This agrees with what we observed in the previous subsection. It can be better seen in the second picture of figure 2.1 where we take a section of the first plot for $\tilde{\theta} = 0$.

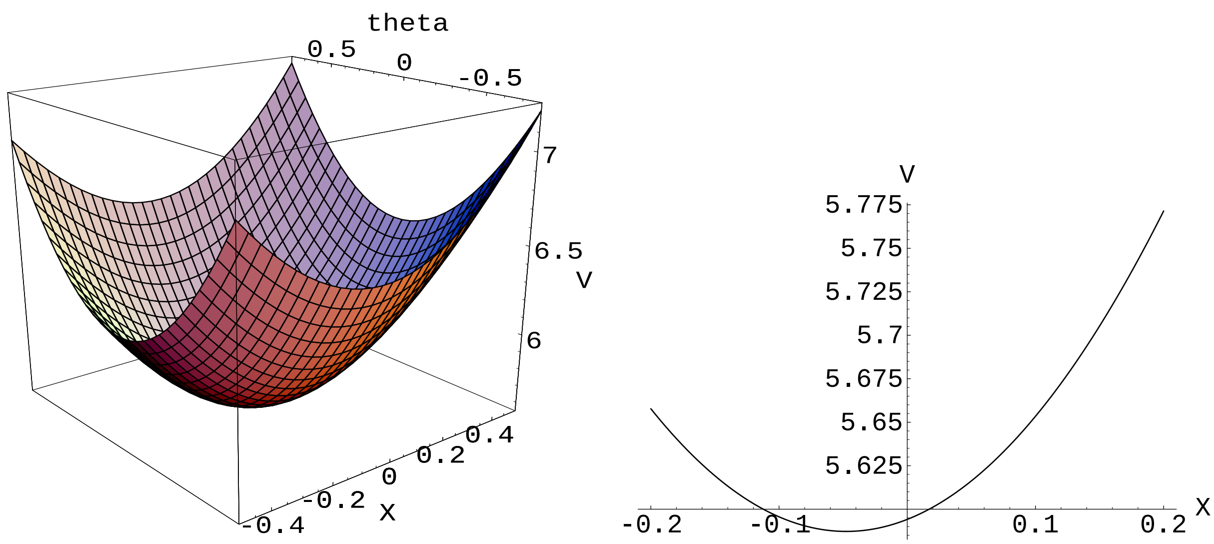


Figure 2.1: Scalar potential V^{1-loop} for $(\eta = 0.5, \rho = 0.1, \zeta = 0.05, \mathcal{X} = -0.5 \ldots 0.5, \tilde{\theta} = -0.8 \ldots 0.8)$, and its section for $\tilde{\theta} = 0$; $\mathcal{X}$ is in unit of m_2, while V is in unit of $|h_2^2 m_2^2|^2$.

In figure 2.2 we plot the 1-loop scalar potential for $\tilde{\theta} = 0$ as a function of $\mathcal{X}$ and of the parameter ρ, fixing η and ζ. For each value of ρ the curvature around the minimum gives a qualitative estimation of the generated mass for the pseudomoduli $\mathcal{X}$. We note that for large ρ the scalar potential become asymptotically flat, and so the 1-loop generated mass goes to zero, but this is outside our allowed range.

As already observed, there is a minimum for $\langle\mathcal{X}\rangle$ slightly different from zero due to quantum corrections, and we have found that it goes to zero in the limit $(\zeta \to 0, \rho \to 0)$.

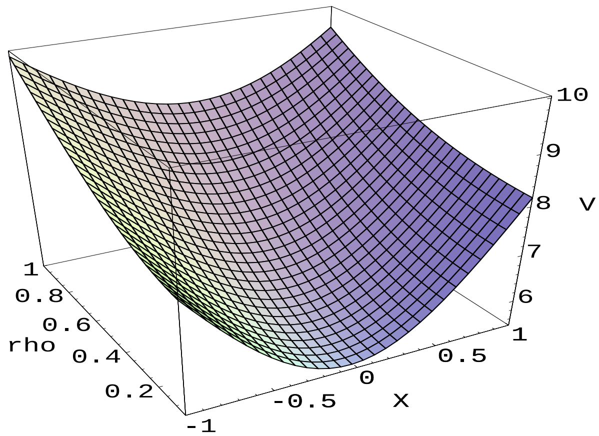


Figure 2.2: Scalar potential V^{1-loop} for $(\mathcal{X} = -1 \ldots 1, \rho = 0.05 \ldots 1, \eta = 0.5, \zeta = 0.05, \tilde{\theta} = 0)$; $\mathcal{X}$ is in unit of m_2, while V is in unit of $|h_2^2 m_2^2|^2$.

We can give analytic results in this limit[6]. We found at zero order in ρ and ζ, with arbitrary η, that the 1-loop generated masses for the pseudomoduli are

$$\begin{aligned} m_{\mathcal{X}}^2 &= \frac{\tilde{N}(N_f - \tilde{N})}{8\pi^2} |h_2^2 m_2|^2 (\log[4] - 1) + o(\rho) + o(\zeta) \\ m_{\tilde{\theta}}^2 &= \frac{\tilde{N}(N_f - \tilde{N})}{16\pi^2} |h_2^2 m_2^2|^2 (\log[4] - 1) + o(\rho) + o(\zeta) \end{aligned} \tag{2.89}$$

so in the limit of small ρ (and small ζ) quantum corrections don't depend on η. We can write the 1-loop scalar potential setting η to zero obtaining (for small ζ)

$$\begin{aligned} V^{(1)} = \frac{\tilde{N}(N_f - \tilde{N})}{64\pi^2} |h_2^2 m_2|^2 \Bigg\{ |m_2|^2 \Bigg(\log\Big(\frac{|h_2 m_2|^2}{\Lambda^2}\Big) + 2\rho^4 \log[\rho^2] - 4(1+\rho^2)^2 \log[1+\rho^2] + \\ + 2(2+\rho^2)^2 \log[2+\rho^2] \Bigg) + \Bigg(4(2+\rho^2)^2 \log[2+\rho^2] - 4\rho^4 \log[\rho^2] + \\ - 8(1+\rho^2)(1 + 2\log[1+\rho^2]) \Bigg) |\mathcal{X} + m_2\zeta|^2 + |m_2|^2 \Bigg(2(1+\rho^2) \Big[(2+\rho^2)^2 \log[2+\rho^2] + \\ - \rho^4 \log[\rho^2] - 2(1+\rho^2)(1 + 2\log[1+\rho^2]) \Big] + 4(\log[4] - \frac{5}{3})\zeta^2 \Bigg) (\theta + \theta^\star)^2 \Bigg\} (1 + o(\zeta)) \end{aligned} \tag{2.90}$$

where this expression is valid only in the regime of small ρ. In these approximations the vev for $\langle \mathcal{X} \rangle$ in the minimum is shifted linearly with ζ; however, in general, the complete behaviour for $\langle \mathcal{X} \rangle$ is more complicated and depends non trivially on η. We observe that, being ζ a simple shift for the vev of $\mathcal{X}$, it doesn't affect its mass, while it modifies $\tilde{\theta}$ mass.

[6]Considering η, ρ, ζ real.

From (2.90) we can read directly the masses expanding for small ρ

$$m^2_{\mathcal{X}} = \frac{\tilde{N}(N_f-\tilde{N})}{8\pi^2}|h_2 m_2|^2\Big(|h_2|^2(\log[4]-1)+|h_1|^2(\log[4]-2)\Big) \tag{2.91}$$

$$m^2_{\tilde{\theta}} = \frac{\tilde{N}(N_f-\tilde{N})}{16\pi^2}|h_2 m_2^2|^2\Big(|h_2|^2(\log[4]-1)+\left|\frac{h_1^2 m_1^2}{2m_2 m_3}\right|^2(\log[4]-\frac{5}{3})+ +|h_1|^2(2\log[4]-3)\Big). \tag{2.92}$$

These expressions are valid up to cubic order in ρ, ζ. The first term in (2.91,2.92), being independent of the deformations (ρ, η, ζ), agrees with [2]. The second term in (2.91) is the same as in [43].

Supersymmetric vacuum

Supersymmetry is restored via non perturbative effects [49], away from the metastable vacuum in the field space, when the $SU(\tilde{N})$ symmetry is gauged [2]. The non supersymmetric vacuum is a metastable state of the theory which decays to a supersymmetric one. We are interested in evaluating the lifetime of the metastable vacuum. We need an estimation of the vevs of the elementary magnetic fields in the supersymmetric state.

We first integrate out the massive fields in the superpotential (2.73) using their equations of motion. In (2.73) there are two massive fields (M_1, Y). We integrate out the meson M_1 and the adjoint field Y tuning λ_Y in such a way that the gauge group $SU(\tilde{N})$ is not broken by the adjoint[7], as in the metastable state, so $\langle Y\rangle = 0$. Using this last condition the equation of motion for the meson M_1 gives the simple relation $M_1 = \frac{h_1}{2m_3}(m_1^2 - q\tilde{q})$. Integrating out the charged field Y the scale matching condition reads

$$\tilde{\Lambda}^{2\tilde{N}-N_f} = \tilde{\Lambda}_{int}^{3\tilde{N}-N_f}\hat{m}_Y^{-\tilde{N}} \tag{2.93}$$

where we have indicated with $\hat{m}_Y$ the resulting mass for Y which is a combination of its tree-level mass m_Y and a term proportional to $\frac{h_3^2}{m_3}(q\tilde{q})^2$ which will be shown to be zero in the supersymmetric vacuum.

We obtain a superpotential for the meson M_2 and the flavours $(q, \tilde{q})$

$$W_{int} = \mathrm{tr}\,\left(\frac{h_1^2}{4m_3}\left(2m_1^2 q\tilde{q} - (q\tilde{q})^2\right) + h_2 M_2 q\tilde{q} - h_2 m_2^2 M_2\right) \tag{2.94}$$

We expect that the supersymmetric vacua lie in the large field region, where the $SU(\tilde{N})$ gauge dynamics becomes relevant [2]. We then consider large expectation value for the meson M_2. We can take as mass term for the flavours $(q, \tilde{q})$ only the vev $h_2\langle M_2\rangle$ neglecting the other contribution in (2.94) coming from the couplings of the magnetic theory.

[7]We are not interested in finding all the supersymmetric vacua.

We then integrate out the flavours $(q, \tilde{q})$ using their equations of motion $(q = 0, \tilde{q} = 0)$. The corresponding scale matching condition is

$$\Lambda_L^{3\tilde{N}} = \tilde{\Lambda}_{int}^{3\tilde{N}-N_f} \det(h_2 M_2) = \tilde{\Lambda}^{2\tilde{N}-N_f} \det(h_2 M_2)\, m_Y^{\tilde{N}} \,. \tag{2.95}$$

The low energy effective $SU(\tilde{N})$ superpotential gets a non-perturbative contribution from the gauge dynamics related to the gaugino condensation proportional to the low energy scale Λ_L

$$W = \tilde{N}\Lambda_L^3 \tag{2.96}$$

that can be written in terms of the macroscopical scale $\tilde{\Lambda}$ using (2.95). This contribution should be added to the M_2 linear term that survives in (2.94) after having integrated out the magnetic flavours $(q, \tilde{q})$. Via the scale matching relation (2.95) we can then express the low energy effective superpotential as a function of only the M_2 meson

$$W_{Low} = \tilde{N}\left(\tilde{\Lambda}^{2\tilde{N}-N_f} \det(h_2 M_2)\right)^{\frac{1}{\tilde{N}}} m_Y - m_2^2 h_2 \,\mathrm{tr}\; M_2 \tag{2.97}$$

Using this dynamically generated superpotential we can obtain the vev of the meson M_2 in the supersymmetric vacuum. Considering M_2 proportional to the identity $\mathbf{1}_{N_F}$ we minimize (2.97) and obtain

$$h_2\langle M_2\rangle = \tilde{\Lambda}\epsilon^{\frac{\tilde{N}}{N_f-\tilde{N}}}\xi^{\frac{\tilde{N}}{N_f-\tilde{N}}}\mathbf{1}_{N_f} = m_2\left(\frac{1}{\epsilon}\right)^{\frac{N_f-2\tilde{N}}{N_f-\tilde{N}}}\xi^{\frac{\tilde{N}}{N_f-\tilde{N}}}\mathbf{1}_{N_f} \tag{2.98}$$

where

$$\epsilon = \frac{m_2}{\tilde{\Lambda}} \qquad \xi = \frac{m_2}{m_Y}\,. \tag{2.99}$$

ϵ is a dimensionless parameter which can be made parametrically small sending the Landau pole $\tilde{\Lambda}$ to infinity. ξ is a dimensionless finite parameter which doesn't spoil our estimation of the supersymmetric vacuum in the sensible range $\epsilon < \frac{1}{\xi}$. All the exponents appearing in (2.98) are positive in our window (2.66).

We observe that in the small ϵ limit the vev $h_2\langle M_2\rangle$ is larger than the typically mass scale m_2 of the magnetic theory but much smaller than the scale $\tilde{\Lambda}$

$$m_2 \ll h_2\langle M_2\rangle \ll \tilde{\Lambda}. \tag{2.100}$$

This fact justifies our approximation in integrating out the massive flavours $(q, \tilde{q})$ neglecting the mass term in (2.94) except $h_2\langle M_2\rangle$. It also shows that the evaluation of the supersymmetric vacuum is reliable because the scale of $h_2\langle M_2\rangle$ is well below the Landau pole.

Lifetime of the metastable vacuum

We make a qualitative evaluation of the decay rate of the metastable vacuum. At semi classical level the decay probability is proportional to e^{-S_B} where S_B is the bounce action from the non supersymmetric vacuum to a supersymmetric one. We have to find a trajectory in the field space such that the potential energy barrier is minimized. We remind the non supersymmetric vacuum configuration (2.78,2.80,2.81) and the supersymmetric one

$$q = 0 \qquad \tilde{q} = 0 \qquad Y = 0 \qquad \langle M_1 \rangle = \frac{h_1 m_1^2}{2 m_3} \; \mathbf{1}_{N_f} \qquad \langle M_2 \rangle \neq 0 \tag{2.101}$$

where $\langle M_2 \rangle$ can be read from (2.98).

By inspection of the F-term contributions (2.76) to the potential energy it turns out that the most efficient path is to climb from the local non supersymmetric minimum to the local maximum where all the fields are set to zero but for M_1 which has the value $M_1 = \frac{h_1 m_1^2}{2m_3} \mathbf{1}_{N_f}$ as in the supersymmetric vacuum, and M_2, which is as in (2.81). This local maximum has potential energy

$$V_{MAX} = N_f |h_2 m_2^2|^2 \tag{2.102}$$

We can move from the local maximum to the supersymmetric minimum (2.101) along the M_2 meson direction. The two minima are not of the same order and so the thin wall approximation of [50] can't be used. We can approximate the potential barrier with a triangular one using the formula of [51]

$$S \simeq \frac{(\Delta\Phi)^4}{V_{MAX} - V_{MIN}} \tag{2.103}$$

We neglect the difference in the field space between all the vevs at the non supersymmetric vacuum and at the local maximum. We take as $\Delta\Phi$ the difference between the vevs of M_2 at the local maximum and at the supersymmetric vacuum. Disregarding the M_2 vev at the local maximum we can approximate $\Delta\Phi$ as (2.98). We then obtain as the decay rate

$$S \sim \left(\left(\frac{1}{\epsilon}\right)^{\frac{N_f - 2\tilde{N}}{N_f - \tilde{N}}} \xi^{\frac{\tilde{N}}{N_f - \tilde{N}}} \right)^4 \sim \left(\frac{1}{\epsilon}\right)^{4\frac{N_f - 2\tilde{N}}{N_f - \tilde{N}}} \tag{2.104}$$

This rate can be made parametrically large sending to zero the dimensionless ratio ϵ (i.e. sending $\tilde{\Lambda} \to \infty$) since the exponent $\left(4\frac{2\tilde{N} - N_f}{\tilde{N} - N_f}\right)$ is always positive in our window (2.66).

R-symmetry and gauge mediation

We are interested in direct gauge mediated supersymmetry breaking. In this framework the gauge group of the SM has to be embedded into a flavour group of the dynamical sector. The gauge sector of the SM directly couples to the supersymmetry breaking dynamics and a natural question for model building is whether the gauginos of the MSSM acquire masses.

We can embed the SM gauge group into the subgroups of the flavour symmetry $SU(2N_f - N_c)$ or $SU(N_c - N_f)$ provide $(2N_f - N_c > 5)$ or $(N_c - N_f > 5)$, respectively.

As in [39] we can compute the beta function coefficient $b_{SU(3)}$ at different renormalization scales and we conclude that in order to avoid Landau pole problems the embedding should be done in $SU(2N_f - N_c)$.

The full model has no R-symmetry, and, unlike [2, 43], no accidental R-symmetry arises at the non-supersymmetric meta-stable vacuum, and hence a gaugino mass generation is not forbidden [2]. Moreover the absence of R-symmetry implies that the non supersymmetric minimum is not at the origin of the moduli space, i.e. $\langle \mathcal{X} \rangle \neq 0$.

The R-breaking terms are the quadratic massive terms $\phi_1\phi_2$ and $\phi_3\phi_4$ in (2.87). The first one can be eliminated shifting the field $\mathcal{X}$. The second one cannot be eliminated rearranging the fields. If the mass m_3 is larger than the supersymmetry breaking scale, ϕ_3 and ϕ_4 could be integrated out, supersymmetrically, recovering an accidental R-symmetry: this, however, is not our range of parameters.

We analyze the dynamics at the meta-stable vacuum where the breaking of supersymmetry generates a gaugino mass proportional to the breaking scale $F_{\mathcal{X}}$. Contribution to this mass comes from the superpotential[8] of the messengers ϕ_i

$$W \supset h_2\left(\mathcal{X}\phi_1\phi_2\right) + h_2 m_2\left(\phi_2\phi_5 + \phi_1\phi_6\right) + h_1 m_2\left(\phi_2\phi_3 + \phi_1\phi_4\right) + 2m_3\phi_3\phi_4 + \frac{h_1^2 m_1^2}{2m_3}\phi_1\phi_2 \tag{2.105}$$

which, in a matrix notation, reads

$$\begin{pmatrix} \phi_1 & \phi_3 & \phi_5 \end{pmatrix} \mathcal{M} \begin{pmatrix} \phi_2 \\ \phi_4 \\ \phi_6 \end{pmatrix} \tag{2.106}$$

where $\mathcal{M}$ is a mass matrix for the messenger fields

$$\mathcal{M} = \begin{pmatrix} h_2\langle\mathcal{X}\rangle + \frac{h_1^2 m_1^2}{2m_3} & h_1 m_2 & h_2 m_2 \\ h_1 m_2 & 2m_3 & 0 \\ h_2 m_2 & 0 & 0 \end{pmatrix} \equiv h_2 m_2 \begin{pmatrix} \frac{\langle\mathcal{X}\rangle}{m_2} + \zeta & \rho & 1 \\ \rho & \eta & 0 \\ 1 & 0 & 0 \end{pmatrix} \tag{2.107}$$

This matrix does not generate a gaugino mass at one loop at first order in $F_{\mathcal{X}}$ as in [52]. However at the third order in $F_{\mathcal{X}}$, the gaugino mass arises as in [46, 52]. This contribution is not negligible when $\frac{F_{\mathcal{X}}}{h_2^2 m_2^2} \sim 1$, which is admitted in our range of parameters.

Diagonalization of (2.107) and use of the general formula in [37] for the computation of the 1 loop diagrams contributing to the gaugino mass m_λ lead to

$$m_\lambda \sim \frac{F_{\mathcal{X}}^3}{(h_2 m_2)^5}\left[\frac{1}{4}\left(\frac{\langle\mathcal{X}\rangle}{m_2} + \zeta\right) + \rho^2\eta\right] \tag{2.108}$$

The coefficient of $F_{\mathcal{X}}^3$ in (2.108) is evaluated at the third order in the adimensional small parameters (ρ, η, ζ): indeed by direct inspection we find that also the term $\left(\frac{\langle\mathcal{X}\rangle}{m_2} + \zeta\right)$ gives at least third order contributions in (η, ρ, ζ).

[8]For simplicity we consider only one copy of the chiral superpotentials.

In conclusion we have found that the $SU(N_c)$ SQCD with two adjoint chiral fields and mesonic deformations admits a metastable non supersymmetric vacuum with parametrically long life. It seems that particular care is needed in building models with adjoint matter exhibiting such vacua. The same can be said about the string geometrical construction realizing the gauge model we have studied [40, 41]. Furthermore we have embedded this model with adjoint matter in a scenario of direct mediation of supersymmetry breaking.

2.3 A_n gauge theories

Another interesting set of theories in which to study metastability à la ISS [2] is the ADE class of quiver gauge theories [48, 53, 54, 55]. These theories can be derived in type IIB string theory from $D5$-branes partially wrapping 2-cycles of non compact Calabi-Yau threefolds. These manifolds are ADE-fold geometries fibered over a plane, and the 2-cycles are blown up S_i^2 in one to one correspondence with the simple roots of ADE (see appendix 2.5 for further details).

In this section we investigate metastability in A_n $\mathcal{N} = 2$ (non affine) quiver gauge theories deformed to $\mathcal{N} = 1$ by superpotential terms in the adjoint fields. In the presence of many gauge groups we have, in principle, a large number of dualization choices.

In [39, 43, 56] A_2, A_3, A_4 quivers have been studied dualizing only one node in the quiver, where dynamical supersymmetry breaking occurs.

Here we consider A_n theories with arbitrary n, where several Seiberg dualities take place. In particular we will explore theories obtained by dualizing alternate nodes. This leads to a low energy description in terms of only magnetic fields.

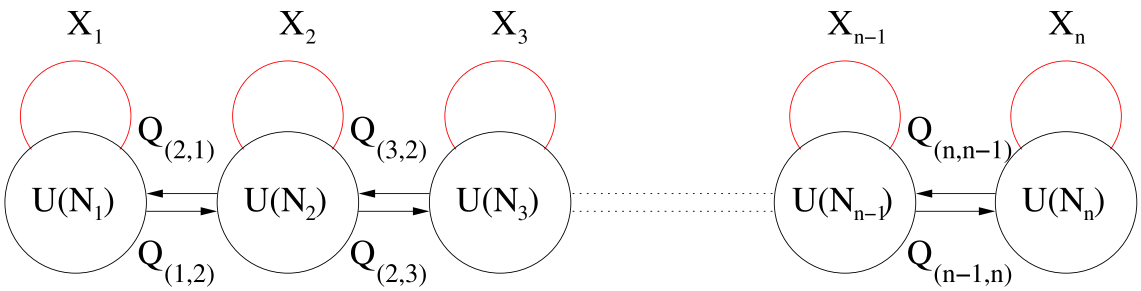
In the duality process the dualized groups are treated as genuine gauge groups whereas the other ones have to be weakly coupled at low energy, so that they act as flavour groups i.e. global symmetries. The procedure depends on the interplay of the RG flows of the dualized and of the non dualized gauge groups and is governed by the associated beta-functions. This translates into inequalities among the ranks of the gauge groups and in hierarchies among the strong coupling scales.

We first describe the $\mathcal{N} = 2$ quiver gauge theories, explicitly broken to $\mathcal{N} = 1$ by superpotential terms. After the integration of the massive adjoint fields, we give the general form of the superpotential. Than we investigate Seiberg duality on the alternate nodes of the quiver. The general theory obtained with this procedure on an A_n is expressed in terms of only magnetic fields. After that we consider the simplest case, i.e. A_3 quiver, showing that it possesses long living metastable vacua à la ISS. The analysis is done neglecting the gauge contributions of the odd nodes, which are treated as flavour symmetries. This last approximation is justified with an analysis of the running of the gauge coupling constants of the various groups. We conclude this section with some comments on the possible ways of enforcing gauge mediation of supersymmetry breaking in

A_n quivers.

A_n quiver gauge theories with massive adjoint fields

We consider a $\mathcal{N} = 2$ (non affine) A_n quiver gauge theory, deformed to $\mathcal{N} = 1$ by superpotential terms in the adjoint fields. The theory is associated with a Dynkin diagram where each node is a $U(N_i)$ gauge group.



The arrows connecting two nodes represent fields $Q_{i,i+1}, Q_{i+1,i}$ in the fundamental of the incoming node and anti fundamental of the out-coming node. The adjoint fields X_i refer to the i-th gauge group.

The gauge group of the whole theory is the product $\prod_{i=1}^{n} U(N_i)$. We call Λ_i the strong coupling scale of each gauge group.

The $\mathcal{N} = 1$ superpotential is

$$W = \sum_{i=1}^{n} W_i(X_i) + \sum_{i,j} s_{i,j}(Q_{i,j})^{\beta}_{\alpha}(X_j)^{\gamma}_{\beta}(Q_{j,i})^{\alpha}_{\gamma} \tag{2.109}$$

where $s_{i,j}$ is an antisymmetric matrix, with $|s_{i,j}| = 1$. The Latin labels run on the different nodes of the A_n quivers, the Greek labels runs on the ranks of the groups of each site. In the case of A_n theories the only non zero terms are $s_{i,i+1}$ and $s_{i,i-1}$. The superpotentials for the adjoint fields $W_i(X_i)$ break supersymmetry to $\mathcal{N} = 1$.

We choose these superpotentials to be

$$W_i(X_i) = \lambda_i Tr X_i + \frac{m_i}{2} Tr X_i^2 \tag{2.110}$$

As a consequence the adjoint fields are all massive. We consider the limit where the adjoint fields are so heavy that they can be integrated out, and we study the theory below the scale of their masses.

Integrating out these fields we obtain the effective superpotential describing the A_n theory (traces on the gauge groups are always implied).

$$\begin{aligned} W &= \sum_{i=1}^{n-1} \left(\left(\frac{\lambda_{i+1}}{m_{i+1}} - \frac{\lambda_i}{m_i} \right) Q_{i,i+1}Q_{i+1,i} - \frac{1}{2} \left(\frac{1}{m_i} + \frac{1}{m_{i+1}} \right) (Q_{i,i+1}Q_{i+1,i})^2 \right) \\ &+ \sum_{i=2}^{n-1} \frac{1}{m_i} Q_{i-1,i}Q_{i,i+1}Q_{i+1,i}Q_{i,i-1} \end{aligned} \tag{2.111}$$

A final important remark is that for the A_n theories the D-term equations of motion can be decoupled and simultaneously diagonalized [46].

Seiberg duality on the even nodes

We investigate the low energy dynamics of the gauge groups of the Dynkin diagram, governed by the ranks and by the hierarchy between the strong coupling scales of each node. We work in the regime where the even nodes develop strong dynamics and have to be Seiberg dualized.

We set all the strong coupling scales of the even nodes to be equal $\Lambda_{2i} \equiv \Lambda_G$ and we require the odd nodes to be less coupled at this scale. We impose the following window for the ranks of the nodes

$$N_{2i} + 1 \leq N_{2i-1} + N_{2i+1} < \frac{3}{2} N_{2i} \qquad i = 1, \ldots, \frac{n-1}{2} \tag{2.112}$$

We take n odd, the even case can be included setting to zero one of the ranks of the extremal nodes.

Along the flow toward the IR, we have to change the description at the scale Λ_G performing Seiberg duality on the even nodes. The even nodes are treated as gauge groups, whereas the odd nodes are treated as flavours. We will discuss the consistency of this description in section 2.3.

It is convenient to list the elementary fields of the dualized theory, i.e. the electric gauge singlets and the new magnetic quarks.

	$U(N_{2i-1})$	$U(\widetilde{N}_{2i})$	$U(N_{2i+1})$
$M_{2i+1,2i-1}$	N_{2i-1}	1	$\bar{N}_{2i+1}$
$M_{2i+1,2i+1}$	1	1	Bifund.
$M_{2i-1,2i-1}$	Bifund.	1	1
$M_{2i-1,2i+1}$	$\bar{N}_{2i-1}$	1	N_{2i+1}
$q_{2i-1,2i}$	N_{2i-1}	$\overline{\widetilde{N}}_{2i}$	1
$q_{2i,2i-1}$	$\bar{N}_{2i-1}$	$\widetilde{N}_{2i}$	1
$q_{2i,2i+1}$	1	$\widetilde{N}_{2i}$	$\bar{N}_{2i+1}$
$q_{2i+1,2i}$	1	$\overline{\widetilde{N}}_{2i}$	N_{2i+1}

The mesons are proportional to the original electric variables: $M_{2i+k,2i+j} \sim Q_{2i+k,2i} Q_{2i,2i+j}$. The even magnetic groups have ranks $\widetilde{N}_{2i} = N_{2i+1} + N_{2i-1} - N_{2i}$. The superpotential in the new magnetic variables results

$$\begin{aligned} W &= h M^{(2i)}_{2i+k,2i+j} q_{2i+j,2i} q_{2i,2i+k} + h \mu^2_{2i+k,(2i)} M^{(2i)}_{2i+k,2i+k} + \\ &+ hm M^{(2i)}_{2i+1,2i+1} M^{(2i+2)}_{2i+1,2i+1} + hm \left(M^{(2i)}_{2i+k,2i+k} \right)^2 + hm M^{(2i)}_{2i-1,2i+1} M^{(2i)}_{2i+1,2i-1} \end{aligned} \tag{2.113}$$

where the index i runs from 1 to $\frac{n-1}{2}$, and k and j are $+1$ or -1. The upper index $(2i)$ of the mesons indicates which site the meson refers to: it is necessary because some mesons have the same flavor indexes, but they are summed on different gauge groups, so they have to be labeled differently. We denote with hm_i the meson masses, related to the quartic terms in the electric superpotential, and with $h\mu_i^2$ the coefficients of the linear deformations, corresponding to the masses of the quarks in the electric description. In (2.113) we wrote a single coupling hm, for all the different mesons, considering all their masses of the same order.

The b coefficients of the beta functions before dualization are

$$b_i = 3N_i - N_{i-1} - N_{i+1} \qquad i = 1, \dots, n \tag{2.114}$$

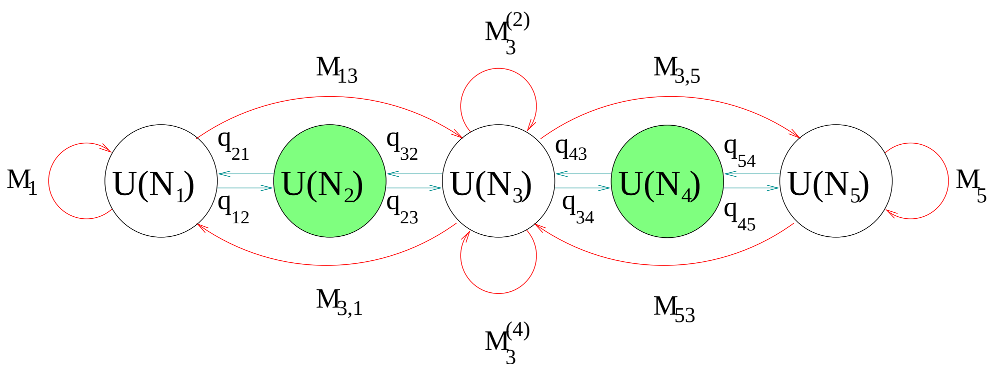
where $N_0 = N_{r+1} = 0$. After the dualization the coefficients $\widetilde{b}$ for the beta functions in the internal nodes result

$$\widetilde{b}_{2k} = 2N_{2k+1} + 2N_{2k-1} - 3N_{2k} \tag{2.115}$$
$$\widetilde{b}_{2k+1} = N_{2k} + N_{2k+2} - N_{2k+1} - 2N_{2k-1} - 2N_{2k+3} \tag{2.116}$$

where k runs from 1 to $\frac{n-1}{2}$, and $N_{n+1} = N_{n+2} = 0$. For the external nodes we have

$$\widetilde{b}_1 = N_1 + N_2 - 2N_3 \qquad \widetilde{b}_n = N_n + N_{n-1} - 2N_{n-2} \tag{2.117}$$

To visualize the resulting magnetic theory (2.113) we exhibit below the content of the magnetic dual theory for an A_5 quiver, which encodes the relevant features.



The superpotential is

$$\begin{aligned} W &= h\left(M_{11}q_{12}q_{21} + M_{13}q_{32}q_{21} + M_{31}q_{12}q_{23} + M_{33}^{(2)}q_{32}q_{23}\right) + \\ &+ h\left(M_{33}^{(4)}q_{34}q_{43} + M_{35}q_{54}q_{43} + M_{53}q_{34}q_{45} + M_{55}q_{54}q_{45}\right) + \\ &+ hm\left(M_{11}^2 + M_{13}M_{31} + {M_{33}^{(2)}}^2 + M_{33}^{(2)}M_{33}^{(4)} + {M_{33}^{(4)}}^2 + M_{35}M_{53} + M_{55}^2\right) + \\ &+ h\left(\mu_1^2 M_{11} + \mu_{3,(2)}^2 M_{33}^{(2)} + \mu_{3,(4)}^2 M_{33}^{(4)} + \mu_5^2 M_{55}\right) \end{aligned} \tag{2.118}$$

Metastable vacua in A_3 quivers

We start studying the existence and the slow decay of non supersymmetric meta-stable vacua in A_3 quiver gauge theory, the simplest example of an A_n theory. The A_3 gauge group is $U(N_1) \times U(N_2) \times U(N_3)$. As already mentioned in section 2.3 for a A_n theory, we integrate out the adjoint fields and we perform Seiberg duality on the central node under the constraint

$$N_2 + 1 \leq N_1 + N_3 < \frac{3}{2} N_2 \tag{2.119}$$

The superpotential reads

$$\begin{aligned} W &= h\left(M_{1,1} q_{1,2} q_{2,1} + M_{1,3} q_{3,2} q_{2,1} + M_{3,1} q_{1,2} q_{2,3} + M_{3,3} q_{3,2} q_{2,3}\right) + \\ &+ h\mu_1^2 M_{1,1} + h\mu_3^2 M_{3,3} \end{aligned} \tag{2.120}$$

where all the mass terms for the mesons have been neglected. Turning on these terms does not ruin the metastability analysis at least for very small masses compared to the supersymmetry breaking scale. Such deformations slightly shift the value of the pseudomoduli in the non supersymmetric minimum, breaking R-symmetry [45]. We neglect them in the following.

The central node yields the magnetic gauge group $U(N_1+N_3-N_2)$ whereas the groups at the two external nodes are considered as flavour groups, much less coupled. We discuss in section 2.3 the consistency of this assumption. Since the gauge group is IR free in the low energy description, and the flavours are less coupled, we are allowed to neglect Kahler corrections and take it as canonical [2]. Moreover the D-term corrections to the one loop effective potential due to the flavour nodes are negligible with respect to the F-term corrections.

Now, there are two different choices of ranks for the A_3 theories, which can give meta-stable vacua: the first possibility is that $N_1 < N_2 \leq N_3$, the second one is $N_1 < N_2 > N_3$. We study separately the two cases which show meta-stable vacua in a similar manner.

$N_1 < N_2 \leq N_3$

We analyze here the case $N_1 < N_2 < N_3$; the equal ranks limit can be easily included. After the dualization the ranks obey the following inequalities $N_1 < \widetilde{N}_2 = N_1 + N_3 - N_2 < N_3$.

We work in the regime where $|\mu_1| > |\mu_3|$, and we comment on what happens in the opposite limit in the in next subsection, where we shall discuss dangerous tachyonic directions in the quark fields.

We find that the following vacuum is a non supersymmetric tree level minimum

$$\begin{aligned} q_{1,2} = q_{2,1} &= \mu_1 \left(\mathbf{1}_{N_1} \quad 0\right) \qquad q_{2,3} = q_{3,2} = \begin{pmatrix} 0 & \mu_3 \mathbf{1}_{\widetilde{N}_2 - N_1} \\ 0 & 0 \end{pmatrix} \\ M_{1,1} &= 0 \qquad M_{1,3} = M_{3,1} = 0 \quad M_{3,3} = \begin{pmatrix} 0 & 0 \\ 0 & X \end{pmatrix} \end{aligned} \tag{2.121}$$

where the field X is the pseudomodulus, which is a massless field not associated with any broken global symmetries. This flat direction has to be stabilized by the one loop corrections. Westart the one loop analysis by rearranging the fields and expanding around the vevs

$$q=\left(\frac{q_{1,2}}{q_{3,2}}\right) = \left(\begin{array}{cc} \mu_1+\Sigma_1 & \Sigma 2 \\ \hline \Sigma_3 & \mu_3+\Sigma_4 \\ \Phi_1 & \Phi_2 \end{array}\right) \quad \widetilde{q} = \left(\, q_{2,1} \,\middle|\, q_{2,3} \, \right) = \left(\begin{array}{c|cc} \mu_1+\Sigma_5 & \Sigma_6 & \Phi_3 \\ \Sigma_7 & \mu_3+\Sigma_8 & \Phi_4 \end{array}\right)$$

$$M = \left(\begin{array}{c|c} M_{1,1} & M_{1,3} \\ \hline M_{3,1} & M_{3,3} \end{array}\right) = \left(\begin{array}{c|cc} \Sigma_9 & \Sigma_{10} & \Phi_5 \\ \hline \Sigma_{11} & \Sigma_{13} & \Phi_6 \\ \Phi_7 & \Phi_8 & X+\Sigma \end{array}\right) \tag{2.122}$$

We now compute the superpotential at the second order in the fluctuations. We find that the non supersymmetric sector is a set of decoupled O'Raifeartaigh like models with superpotential

$$W = h\mu_3^2 X + hX(\Phi_1\Phi_3+\Phi_2\Phi_4) + h\mu_3(\Phi_1\Phi_5+\Phi_2\Phi_6) + h\mu_1(\Phi_3\Phi_7+\Phi_4\Phi_8) \tag{2.123}$$

In this way all the pseudomoduli can get a mass. The quantum corrections behave exactly as in [2], which means that the pseudomoduli get positive squared mass around the origin of the field space.

The choice (2.121) guarantees that there are no tachyonic directions and have to be made coherently with the hierarchy of the couplings μ_i; see below for details.

The lifetime of the non supersymmetric vacuum is related to the value of the scalar potential in the minimum, and to the displacement of the vevs of the fields between the false and the true vacuum. The scalar potential in the non supersymmetric minimum is

$$V_{min} = (N_3+N_1-\widetilde{N}_2)|h\mu_3^2|^2 = N_2|h\mu_3^2|^2 \tag{2.124}$$

The vevs of the fields in the supersymmetric vacuum have to be studied considering the non perturbative contributions arising from gaugino condensation. When we take into account these non perturbative effects, we expect that the mesons get large vevs and this allows us to integrate out the quarks using their equation of motion, $q_{i,j}=0$. In the supersymmetric vacua also $M_{1,3}=0$ and $M_{3,1}=0$. If we define

$$M = \begin{pmatrix} M_{1,1} & 0 \\ 0 & M_{3,3} \end{pmatrix} \tag{2.125}$$

the effective superpotential is

$$W = (N_1+N_3-N_2)\left(\det(hM)\Lambda_{2i}^{2N_1+2N_3-3N_2}\right)^{\frac{1}{N_1+N_3-N_2}} - h\left(\mu_1^2 trM_{1,1}+\mu_3^2 trM_{3,3}\right) \tag{2.126}$$

We have now to solve the equation of motion for M_1 and M_3. The equations to be solved are

$$\begin{aligned} \left(h^M M_{1,1}^{(N_2-N_3)} M_{3,3}^{N_3} \Lambda_{2i}^{(2N_1+2N_3-3N_2)}\right)^{\frac{1}{N_1+N_3-N_2}} - \mu_1^2 &= 0 \\ \left(h^{N_2} M_{1,1}^{N_1} M_{3,3}^{(N_2-N_1)} \Lambda_{2i}^{(2N_1+2N_3-3N_2)}\right)^{\frac{1}{N_1+N_3-N_2}} - \mu_3^2 &= 0 \end{aligned} \tag{2.127}$$

The vevs of the mesons follow solving (2.127)

$$\langle hM_{1,1}\rangle = \mu_1^{2\frac{N_1-N_2}{N_2}}\mu_3^{2\frac{N_3}{N_2}}\Lambda_{2i}^{\frac{3N_2-2N_3-2N_1}{N_2}}\mathbf{1}_{N_1} \qquad \langle hM_{3,3}\rangle = \mu_1^{2\frac{N_1}{N_2}}\mu_3^{2\frac{N_3-N_2}{N_2}}\Lambda_{2i}^{\frac{3N_2-2N_3-2N_1}{N_2}}\mathbf{1}_{N_3} \tag{2.128}$$

Since $|\mu_1| > |\mu_3|$, it follows that $\langle hM_{3,3}\rangle > \langle hM_{1,1}\rangle$. This implies that in the evaluation of the bounce action, with the triangular barrier [51], we can consider only the displacement of M_3 in the field space. We obtain for the bounce action

$$S \sim \frac{(\Delta\Phi)^4}{\Delta V} = \left(\frac{\mu_1}{\mu_3}\right)^{\frac{3N_2-2N_3}{N_2}}\left(\frac{\Lambda_{2i}}{\mu_1}\right)^{4\frac{3N_2-2N_3-2N_1}{N_2}} \tag{2.129}$$

Both exponents are positive in the range (2.119). This implies that $S_B \gg 1$, and the vacuum is long living.

$N_1 < N_2 > N_3$

The ranks of the groups after the duality obey the relation $N_1 > \widetilde{N}_2 = N_1+N_3-N_2 < N_3$. We choose now $|\mu_1| > |\mu_3|$, but also the other choice is possible, leading to other vacua. In the meta-stable vacuum all the vevs of the fields have to be chosen to be zero except a block of the quarks $q_{1,2}$ and $q_{2,1}$ and the pseudomoduli. The vevs are

$$q_{1,2} = \mu_1 \begin{pmatrix} \mathbf{1}_{N_1} \\ \mathbf{0} \end{pmatrix} \qquad q_{2,1}^T = \mu_1 \begin{pmatrix} \mathbf{1}_{N_1} \\ \mathbf{0} \end{pmatrix} \tag{2.130}$$

The pseudomoduli come out from the meson $M_{3,3}$ and a $(\widetilde{N}_2 - N_1)\times(\widetilde{N}_2 - N_1)$ diagonal block of the other meson, $M_{1,1}$. The one loop analysis is the same as before and lifts all the flat directions.

In order to estimate the lifetime we need the vevs of the fields in the supersymmetric vacuum, which are again (2.128), and the value of the scalar potential in the non supersymmetric vacuum (2.130)

$$V_{min} = (N_2 - N_3)|h\mu_1|^2 + N_3|h\mu_3|^2 \tag{2.131}$$

Since $|\mu_1| > |\mu_3|$ we approximate the scalar potential by the term $\sim |\mu_1|^2$ and the field displacement by $\langle hM_3\rangle$, obtaining as bounce action

$$S \sim \left(\frac{\mu_1}{\mu_3}\right)^{2\frac{N_2-N_3}{N_2}}\left(\frac{\Lambda_{2i}}{\mu_1}\right)^{4\frac{3N_2-2N_1-2N_3}{N_2}} \gg 1 \tag{2.132}$$

Goldstone bosons

The analysis we made in the A_3 theories started from the limit $|\mu_1| > |\mu_3|$. Also the opposite limit can give meta-stable vacua. To understand the differences among the various choices, we have to study the classical masses acquired by the fields expanding them around their vevs.

We study the case with ranks $N_1 < \widetilde{N}_2 < N_3$. Since the flavor symmetry is $U(N_1) \times U(N_3)$, and not $U(N_1+N_3)$, the linear terms of the mesons are different. We are still free to choose the hierarchy between them. We here analyze the breaking of the global symmetries taking $|\mu_1| > |\mu_3|$. Treating the gauge symmetry as a global one, and rearranging the quarks in the form

$$\langle q \rangle = \begin{pmatrix} q_{1,2} \\ q_{3,2} \end{pmatrix} = \begin{pmatrix} \mu_1 \mathbf{1}_{N_1} & 0 \\ 0 & \mu_3 \mathbf{1}_{\widetilde{N}_2 - N_1} \\ 0 & 0 \end{pmatrix} \quad \langle \widetilde{q}^T \rangle = \begin{pmatrix} q_{2,1} \\ q_{2,3} \end{pmatrix} = \begin{pmatrix} \mu_1 \mathbf{1}_{N_1} & 0 \\ 0 & \mu_3 \mathbf{1}_{\widetilde{N}_2 - N_1} \\ 0 & 0 \end{pmatrix} \tag{2.133}$$

we see that the global symmetry breaks as

$$U(N_1) \times U(\widetilde{N}_2) \times U(N_3) \longrightarrow U(N_1)_D \times U(\widetilde{N}_2 - N_1)_D \times U(N_1 + N_2 - \widetilde{N}_2) \tag{2.134}$$

This implies that the Goldstone bosons are $\widetilde{N}_2^2 + 2(\widetilde{N}_2 - N_1)(N_1 + N_3 - \widetilde{N}_2)$. The first $\widetilde{N}_2^2$ Goldstone bosons come from the upper $\widetilde{N}_2 \times \widetilde{N}_2$ block matrices in the quark fields, exactly the same as in ISS. The second part is a bit different. In fact in ISS, with equal masses, the Goldstone bosons which come from the lower $(N_1+N_3-\widetilde{N}_2)\times \widetilde{N}_2$ sector in the quarks matrices, are $2\widetilde{N}_2(N_3 + N_1 - \widetilde{N}_2)$. In this case, since we started with lesser flavor symmetry, there are $2N_1(N_3+N_1-\widetilde{N}_2)$ massless Goldstone bosons fewer than in ISS. We have to control the other directions. From the scalar potential we have to compute the masses that the fields acquire expanding around the vacuum. The relevant expansions for the potentially tachyonic directions are the ones around the vevs of the quarks

$$\begin{aligned} q_{12} &= \begin{pmatrix} \mu_1 + \phi_1 & \phi_2 \end{pmatrix} \quad q_{21} = \begin{pmatrix} \mu_1 + \widetilde{\phi}_1 \\ \widetilde{\phi}_2 \end{pmatrix} \\ q_{23} &= \begin{pmatrix} \phi_3 & \mu_3 + \phi_4 \\ \phi_5 & \phi_6 \end{pmatrix} \quad q_{32} = \begin{pmatrix} \widetilde{\phi}_3 & \widetilde{\phi}_5 \\ \mu_3 + \widetilde{\phi}_4 & \widetilde{\phi}_6 \end{pmatrix} \end{aligned} \tag{2.135}$$

The relevant terms of the scalar potential come from the F-terms of the mesons

$$V = |F_{M_{11}}|^2 + |F_{M_{13}}|^2 + |F_{M_{31}}|^2 + |F_{M_{33}}|^2 \tag{2.136}$$

If we study the mass terms of the fields ϕ_5 and $\widetilde{\phi}_5$ we note that they are not zero, since $\mu_1 \neq \mu_3$. In fact their mass matrix is[9]

$$\begin{pmatrix} \phi_5 & \widetilde{\phi}_5^\dagger \end{pmatrix} \begin{pmatrix} \mu_1^2 & -\mu_3^2 \\ -\mu_3^2 & \mu_1^2 \end{pmatrix} \begin{pmatrix} \phi_5^\dagger \\ \widetilde{\phi}_5 \end{pmatrix} \tag{2.137}$$

with eigenvalues $\mu_1^2 \pm \mu_3^2$. A minimum of the scalar potential without tachyonic directions imposes a constraint on the masses, $\mu_1 > \mu_3$, consistent with the analysis of ISS.

[9]From now on we will consider all the mass terms as real.

We can ask now what happens if $\mu_1 < \mu_3$. The vacua we studied before are not true vacua any longer, but they have tachyonic directions in the quark fields. The meta-stable vacua are obtained choosing the vevs of $q_{1,2}$ and $q_{2,1}$ to be zero, and the vevs of the other quarks to be

$$q_{3,2} = q_{2,3}^T = \begin{pmatrix} \mu_3 \mathbf{1}_{\widetilde{N}_2} \\ 0 \end{pmatrix} \tag{2.138}$$

The differences in the two cases are the value of the scalar potential and the pseudo-moduli. In fact in the first limit $V_{vac} = (N_1 + N_3 - \widetilde{N}_2)|h\mu_3^2|^2$, and in the second limit the scalar potential is $V_{vac} = (N_3 - \widetilde{N}_2)|h\mu_3^2|^2 + N_1|h\mu_1^2|^2$. Since we choose the masses to be different, but of the same order, both cases have long lived meta-stable vacua. As far as the pseudo-moduli are concerned, in the case analyzed during the paper, they come out from a block of the $M_{3,3}$ meson, and in this case they come out from the whole M_1 meson and from a diagonal block $(N_3 - \widetilde{N}_2) \times (N_3 - \widetilde{N}_2)$ of the $M_{3,3}$ meson.

Renormalization group flow

The analysis of sections 2.3 and 2.3 relies on the fact that we neglect the contributions to the dynamics due to the odd nodes. It means that these groups have to be treated as flavours groups, i.e. global symmetries. However, in the A_n quiver theory each node represents a gauge group factor and we have to analyze how its coupling runs with the energy.

The magnetic window (2.112) constraints the even nodes to be UV free in the high energy description, i.e. $b_{2i} > 0$. The odd groups are not uniquely determined by (2.112) and can be both UV free or IR free in the electric description. In the first case we will choose their scale Λ_{2i+1} to be much lower than the even one

$$\Lambda_{2i+1} \ll \Lambda_{2i}. \tag{2.139}$$

In the second case, when $b_{2i+1} < 0$, Λ_{2i+1} is a Landau pole and we take

$$\Lambda_{2i+1} \gg \Lambda_{2i}. \tag{2.140}$$

In these regimes the even nodes become strongly coupled before the odd ones in the flow toward the infrared. This means that we need a new description provided by Seiberg dualities on the even nodes.

In order to trust the perturbative description at low energy, we have to impose that at the supersymmetry breaking scale (typically μ_i) the odd nodes (flavour), are less coupled than the even ones (gauge), which are always IR free. This requirement will give other constraints on the scales.

As already said there are two possible behaviors of the flavour groups above the scale Λ_{2i}: they can be IR free or UV free. For both cases there are three different possibilities about the beta coefficients in the low energy description.

We start discussing the case when the flavours group are UV free in the electric description. The following three possibilities arise for each flavour group $U(N_{2k+1})$ in the dual theory (Plots 1,2,3 in Figure 1).

1. The first one is characterized by

$$b_{2k+1} > 0 \qquad \widetilde{b}_{2k+1} < \widetilde{b}_{2i} < 0 \tag{2.141}$$

In this case the flavour groups $U(N_{2k+1})$ are more IR free than the even nodes after Seiberg duality. The couplings of the flavour groups become more and more smaller than the couplings of the gauge groups along the flow toward low energy. Hence we do not need other constraints on the scales except (2.139).

2. The second possibility is reported in Plot 2 in Figure 1

$$b_{2k+1} > 0 \qquad \widetilde{b}_{2i} < \widetilde{b}_{2k+1} < 0 \tag{2.142}$$

The flavour groups $U(N_{2k+1})$ are IR free in the dual theory, but less than the $U(\widetilde{N}_{2i})$ gauge groups (2.142). Below a certain energy scale the flavours become more coupled than the gauge groups. If this happens before the supersymmetry breaking scale we cannot trust our description anymore. To solve this problem we have to choose the correct hierarchy between the electric scales of the flavour and the gauge groups, and the supersymmetry breaking scale. We impose that the couplings of the flavours are smaller than the couplings of the gauge groups at the breaking scale, in the magnetic description. This condition can be rewritten in terms of electric scales only using the matching between the magnetic and the electric scales of the flavours. This procedure is explained in the Appendix A.3 and gives the following condition on Λ_{2k+1}

$$\Lambda_{2k+1} \ll \left(\frac{\mu}{\Lambda_{2i}}\right)^{\frac{\widetilde{b}_{2k+1}-\widetilde{b}_{2i}}{b_{2k+1}}} \Lambda_{2i} \ll \Lambda_{2i} \tag{2.143}$$

This imposes a constraint stronger than (2.139) on the strong coupling scale of the flavours.

3. The third possibility (Plot 3 Figure 1) is

$$b_{2k+1} > 0 \qquad \widetilde{b}_{2k+1} > 0 \tag{2.144}$$

In this case the flavour group $U(N_{2k+1})$ is asymptotically free in the low energy description. Once again we have to impose that at the breaking scale the flavours are less coupled than the gauge groups. The procedure is the same outlined above, and the condition is the same as (2.143). This case may become problematic in the far infrared. Indeed, since the flavour group is UV free, it develops strong dynamics at low energy. If we take into account the non perturbative contributions they could restore supersymmetry. Another interesting feature is the appearance of cascading gauge theories, flowing in the IR. We do not discuss these issues here.

If the flavour groups $U(N_{2k+1})$ are IR free in the electric description the same three possibilities discussed above arise (see Plots 4, 5, and 6 of Figure 1).

4. The plot 4 of Figure 1 is characterized by

$$b_{2k+1} < 0 \qquad\qquad \widetilde{b}_{2k+1} < \widetilde{b}_{2i} < 0 \tag{2.145}$$

Here we do not need any other constraint except (2.140).

5. The plot 5 in Figure 1 is

$$b_{2k+1} < 0 \qquad\qquad \widetilde{b}_{2i} < \widetilde{b}_{2k+1} < 0 \tag{2.146}$$

The requirement that the odd nodes are less coupled than the even ones at the supersymmetry breaking scale give once again non trivial constraints, with the same procedure outlined previously

$$\Lambda_{2k+1} \gg \left(\frac{\Lambda_{2i}}{\mu}\right)^{\frac{\widetilde{b}_{2i}-\widetilde{b}_{2k+1}}{b_{2k+1}}} \Lambda_{2i} \gg \Lambda_{2i} \tag{2.147}$$

where now the strong coupling scale of the flavour groups in the electric description is a Landau pole.

6. The last possibility (Plot 6 of Figure 1)

$$b_{2k+1} < 0 \qquad\qquad \widetilde{b}_{2k+1} > 0 \tag{2.148}$$

lead to the same constraint (2.147). In the far infrared the strong dynamics of the flavours node can lead to non perturbative phenomena, as in the case 3.

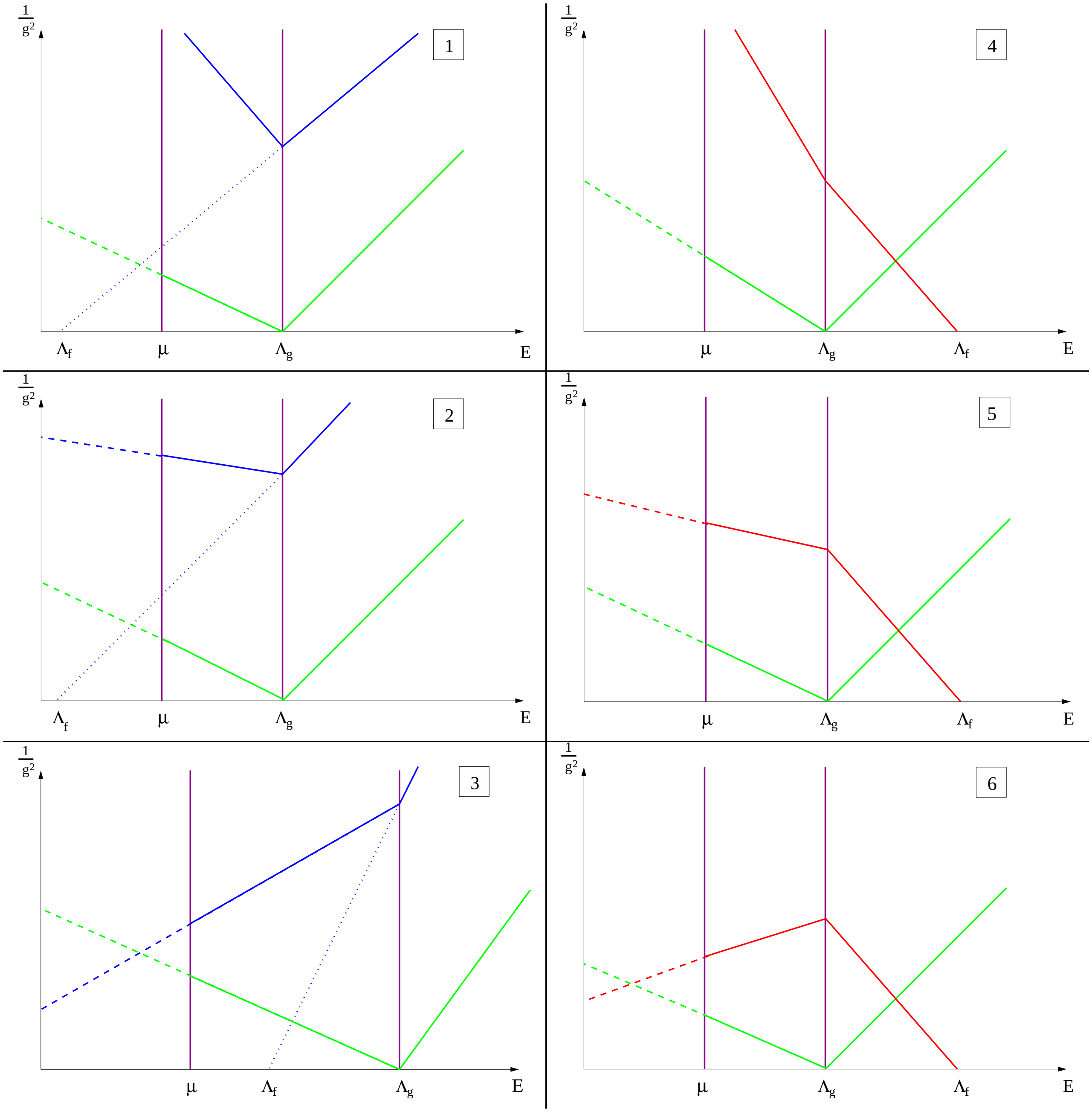


Figure 1: *The blue lines refer to flavour/odd groups which are UV free in the electric description, while the red ones are IR free. The green lines refer to the gauge/even group couplings. We denote with μ the supersymmetry breaking scale, and Λ_G and Λ_F are the strong coupling scales of the gauge and the flavour groups, respectively.*

Meta-stable A_n

We work in the regime where the ratio $\frac{\mu_i^2}{m}$ is larger than the strong scale of the even nodes Λ_{2i}. This requirement is satisfied if $\lambda_i \gg \Lambda_{2i}^2$ in the electric theory. This allows us to ignore in the dual superpotential (2.113) the presence of quadratic deformations in the mesonic fields.

In this approximation the superpotential of the A_n quiver (2.113) reduces to $\frac{n-1}{2}$ copies of A_3 superpotentials. Hence a generic A_n diagram results decomposable in copies of A_3 quivers, where every adjacent pair shares an odd node.

For each A_3 the even nodes provide the magnetic gauge groups, and each A_3 has long living metastable vacua, if the perturbative window is correct. It follows that the A_n quiver theory, which is a set of metastable A_3 quivers, possesses metastable vacua.

We still have to be sure of the perturbative regime. This means that we have to control the gauge contributions from the odd nodes of the A_n diagram. We have to proceed as in section 2.3, and study the beta coefficients of the groups. From (2.116) we can see that the magnetic beta coefficients of the internal odd nodes involve the ranks of the next to next neighbor groups, i.e. they depend on five integer numbers. This means that in order to know these beta coefficients it is enough to study the A_5 consistent with (2.112). Below we classify all the possible metastable A_5 diagrams and we give the corresponding electric and magnetic beta coefficients of the central flavour node. This classification describes the RG behaviour of all the internal odd nodes of the A_n.

The running of the first and of the n-th node of the A_n quiver is still undefined and it is discussed below.

This provides a classification of metastable A_n quiver gauge theories with alternate Seiberg dualities.

A5 classification

We study A_5 quiver gauge theories obtained gluing all the possible combinations of A_3 which present metastable vacua, i.e. the one of section (2.3)

We analyze the beta function coefficients for these A_5 quiver gauge theories, with gauge group $U(N_1) \times U(N_2) \times U(N_3) \times U(N_4) \times U(N_5)$. The even nodes are in the IR free window

$$N_2 < N_1 + N_3 < \frac{3}{2}N_2 \qquad N_4 < N_3 + N_5 < \frac{3}{2}N_4 \tag{2.149}$$

We write in the table the beta coefficients of the third node of the A_5, specifying the range, compatible with (2.149), when this node is UV free or IR free in the electric and in the magnetic descriptions, respectively. The table classifies the possible A_5 quiver gauge theories which present alternate Seiberg dualities and which have metastable vacua.

As explained in section 2.3 we can obtain an A_n quiver gauge theory by gluing the A_3 patches. For the renormalization group, the internal flavour nodes of the A_n chain behave as the third node of the A_5 patches.

The table does not say anything about the external nodes of the A_n. In the electric theory one has $b_1 = 3N_1 - N_2$ and $b_n = 3N_n - N_{n-1}$; after duality, in the low energy description we have $\widetilde{b}_1 = N_1 + N_2 - N_3$, and $\widetilde{b}_n = N_n + N_{n-1} - 2N_{n-2}$. The possible values for $\widetilde{b}_1$ and $\widetilde{b}_n$ have to be studied separately.

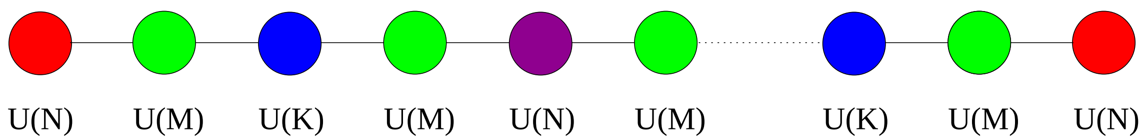
Ranks of A_5	Further condition(I)	Further condition(II)	electric $b-$factor	magnetic $b-$factor
$N_1 < N_2 \leq N_3 < N_4 \leq N_5$		$N_2 + N_4 < 3N_3$	$b_3 > 0$	$\widetilde{b}_3 < 0$
		$3N_3 < N_2 + N_4$	$b_3 < 0$	$\widetilde{b}_3 < 0$
$N_1 < N_2 > N_3 < N_4 \leq N_5$			$b_3 < 0$	$\widetilde{b}_3 < 0$
$N_1 \geq N_2 > N_3 < N_4 \leq N_5$			$b_3 < 0$	$\widetilde{b}_3 < 0$
$N_1 < N_2 > N_3 < N_4 > N_5$	$N_3 < N_1 + N_5$	$N_2 + N_4 < 3N_3$	$b_3 > 0$	$\widetilde{b}_3 < 0$
		$3N_3 < N_2 + N_4$	$b_3 < 0$	$\widetilde{b}_3 < 0$
	$N_3 > N_1 + N_5$	$N_2 + N_4 < N_3 + 2N_1 + 2N_5$	$b_3 > 0$	$\widetilde{b}_3 < 0$
		$N_3 + 2N_1 + 2N_5 < N_2 + N_4$	$b_3 > 0$	$\widetilde{b}_3 > 0$
$N_1 < N_2 \leq N_3 \geq N_4 > N_5$		$N_2 + N_4 < N_3 + 2N_1 + 2N_5$	$b_3 > 0$	$\widetilde{b}_3 < 0$
		$N_3 + 2N_1 + 2N_5 < N_2 + N_4$	$b_3 > 0$	$\widetilde{b}_3 > 0$
$N_1 < N_2 \leq N_3 < N_4 > N_5$	$N_3 < N_1 + N_5$	$N_2 + N_4 < 3N_3$	$b_3 > 0$	$\widetilde{b}_3 < 0$
		$3N_3 < N_2 + N_4$	$b_3 < 0$	$\widetilde{b}_3 < 0$
	$N_3 > N_1 + N_5$	$N_2 + N_4 < N_3 + 2N_1 + 2N_5$	$b_3 > 0$	$\widetilde{b}_3 < 0$
		$N_3 + 2N_1 + 2N_5 < N_2 + N_4$	$b_3 > 0$	$\widetilde{b}_3 > 0$

In the first column we report all the possible inequalities among the A_5 rank numbers consistent with (2.149). Moving from left to right the further condition fix the signs of b_3, $\widetilde{b}_3$.

Example

We show now a simple example of metastable A_n diagram. We choose the even nodes in the electric description to become strongly coupled at the same scale Λ_{2i}. We require that at such scale the flavours (odd nodes) are less coupled than the gauge ones. Moreover we will show that we can also require that in the low energy description all the nodes are IR free and also that the flavour groups (odd nodes) are less coupled than the gauge groups (even nodes) at any scale below the Λ_{2i}.

We study an A_n theory, where $n = 4k+1$, with k integer. The chain is built as follow



with $N < M < K$. This range allows for metastable vacuum in each A_3 piece as showed previously. We perform alternate Seiberg dualities, working in the in the window

$$M + 1 < N + K < \frac{3}{2}M$$

Thanks to the simple choice for the ranks we have four values for the b coefficients of the beta functions in the electric description, and four values for the coefficients $\widetilde{b}$. They are summarized in the following table

node	b	$\widetilde{b}$	
$1, n$ (red)	$3N - M$	$N - 2K + M$	
$2i$ (green)	$3M - N - K$	$2K + 2N - 3M$	
$4i - 1$ (blue)	$3K - 2M$	$2M - 4N - K$	$i = 1, \ldots, \frac{n-1}{4}$
$4j + 1$ (violet)	$3N - 2M$	$2M - 4K - N$	$j = 1, \ldots, \frac{n-5}{4}$

We require that in the magnetic description all the nodes are IR free. Moreover we require the beta coefficients of the odd groups to be lower than the even group ones, i.e. $\widetilde{b}_{\text{odd}} < \widetilde{b}_{2i}$. This restricts the window to

$$K > 2N \qquad\qquad 3N < 2M < 4N + K \tag{2.150}$$

In this regime all the nodes in the electric description are UV free except the $4j+1$-th ones. Seiberg duality is allowed on the even nodes, if we impose the following hierarchy of scales

$$\Lambda_1, \Lambda_n, \Lambda_{4i-1} \ll \Lambda_{2i} \ll \Lambda_{4j+1} \tag{2.151}$$

The running of the gauge couplings of the different nodes are depicted in Figure 2.

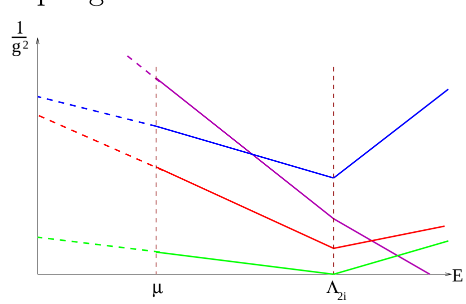


Figure 2: *The green line represents the running of the coupling of the even sites. The violet line is related to the $4j+1$-th sites, the blue one to the $4i-1$-th sites and the red to the first and the last nodes.*

At high energy the $4j+1$-th nodes are strongly coupled, while the other nodes are all UV free. At the scale Λ_{2i} the even nodes become strongly coupled and Seiberg dualities take place. All the runnings of the couplings are changed by these dualities, and all the coefficients of the beta functions $\widetilde{b}_i$ become negative. Hence at energy scale lower than Λ_{2i} the theory is weakly coupled. Furthermore the beta coefficients of the odd nodes are more negative than the even node ones. This guarantees that we can rely on perturbative computations, treating the odd nodes as flavours.

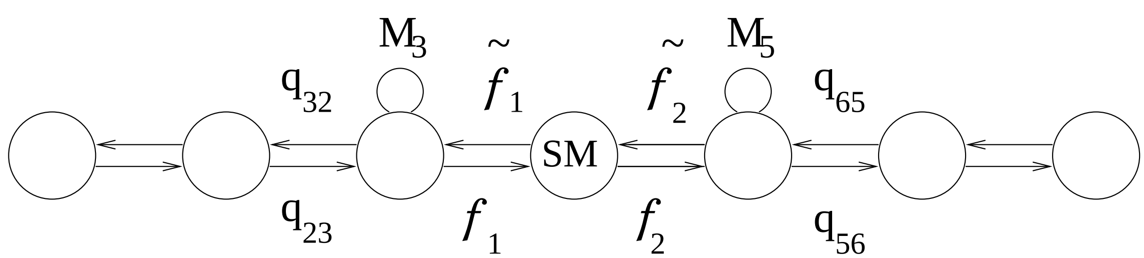
Gauge mediation

The models analyzed in this work can admit mechanisms of gauge mediation. This means that the breaking of supersymmetry can be transmitted to the Standard Model sector via a gauge interaction. This idea has already appeared in the literature of metastable vacua in A_n theories [39, 56].

Different realizations are possible here. A first one, of direct gauge mediation, identifies the SM gauge group with a subgroup of a flavour group in the quiver [39] and leads to a gaugino mass consistently with the bound of [45].

A second possibility [56] is to connect one of the extremal nodes of the A_n quiver with a new gauge group, which represents the Standard Model gauge group. The arrows connecting these nodes are associated with the messengers f and $\widetilde{f}$, which communicate the breaking of supersymmetry to the standard model. Neglecting all the quartic terms, except the term which couples the messengers $f, \widetilde{f}$ with the last meson, it is possible to show that also in this case gaugino masses arise at one loop.

In our models of metastable A_n quivers another possibility arises for gauge mediation. It consists in substituting an even node with the Standard Model gauge group.



The low energy description is constituted by two metastable A_n (A_3 in this case) which are connected through the SM sector. Both communicate the supersymmetry breaking to the standard model. The superpotential leads to two copies of messengers fields related to the two different hidden sectors

$$W = \left(m_1 + \theta^2 h_1 F_{M_3}\right) f_1 \widetilde{f}_1 + \left(m_2 + \theta^2 h_2 F_{M_5}\right) f_2 \widetilde{f}_2 \tag{2.152}$$

A gaugino mass arises at one loop proportional to $\left(h_1\frac{F_{M_3}}{m_1}+h_2\frac{F_{M_5}}{m_2}\right)$.

In this section we have studied metastability in models of A_n quiver gauge theories. The low energy description in terms of macroscopic fields can be achieved via Seiberg dualities at chosen nodes in the A_n diagram. This choice defines, to a certain extent, the models.

A strategy for building acceptable models unfolds from the request for a reliable perturbative analysis. This constrains the ranks of the gauge groups associated with the nodes and their strong coupling scales. We chose to dualize alternate nodes and we fixed two scales: a unique breaking scale μ and a common strong coupling scale Λ_G for each dualized node. The RG flows of the dualized and non dualized gauge groups must be such that at energy scale higher than μ the gauge groups of the dualized nodes are more coupled than the other ones.

The RG properties of the different nodes of an A_n quiver can be studied decomposing it in A_5 quivers and the decomposition of the A_n in A_3 patches gives the structure of the metastable vacuum. In this way we classify all the possible A_n quiver gauge theories which show metastable vacua with the technique of alternating Seiberg dualities.Finally we have discussed different patterns of gauge mediation.

2.4 Geometric deformations

A string approach to the ISS mechanism remains a problems and only partial results are at hand, either within the gauge/gravity correspondence or toward a more direct string origin or interpretation [6, 57, 58]

Some steps have emerged for the grounding of a geometrical interpretation of the features of metastability in simple quiver gauge theories on D-branes near a singularity inside a CY manifold [10, 59]. The aim is to phrase the metastable F-type susy breaking in a general geometrical language. A key point is that the non perturbative dynamics behind the existence of metastable vacua corresponds to deformations of a theory with unbroken supersymmetry [10]. The deformations regard the superpotential: in the D-brane setting of IIB string theory they are mapped into complex deformations in the local geometry.

In this section we develop this approach further. We study systems of branes at toric conical Calabi-Yau singularities of a special type, i.e. deformable singularities, in the sense of Altman's deformations [60], that are not isolated. These form a large subfamily of toric singularities and consist of a cone with a singularity at the tip and some set of lines of $\mathbb{C}^2/\mathbb{Z}_n$ singularities passing through it. Different combinations of fractional branes at these singularities give rise to different IR behaviors of the gauge theory: $\mathcal{N}=2$ dynamics, confinement, runaway supersymmetry breaking [61], and long living metastable vacua, as recently pointed out in [10]. Some of the different IR dynamics can be geometrically understood as motion in the moduli space of the CY singularities.

Our discussion is mainly focused on metastability in the quiver gauge theories living on deformed L^{aba} singularities. Such theories correspond to an infinite class of non isolated toric singularities, with a known metric. Beyond their role in model building and in the gauge/gravity duality, they form a fitting laboratory for the investigation of the field theory/geometry correspondence. In the analysis of general L^{aba} quivers we show that we can always extract subclasses where metastable vacua exist. The features of broken and restored supersymmetry find a systematic geometric counterpart in terms of appropriate deformations of the geometry of the unbroken susy phase.

We start this section by reviewing the case of the Suspended Pinch Point (SPP) singularity, its associated field theory and the relation between their corresponding deformations. This simple case will be the guideline for the whole section. Then we introduce the family of L^{aba} singularities and the corresponding quiver gauge theories. We analyze the metastable vacua in the L^{aba} gauge theories with $b \neq a$ and the L^{aaa} gauge theories. In all these cases we show that some deformation of the geometry leads to metastability and some other deformation restores supersymmetry. Metastability turns out to be a quite generic phenomenon in these deformed toric theories. Finally we try to extend this analysis to more complicated singularities.

Complex deformations and metastability: the SPP example

The SPP gauge theory [62] is obtained as the near horizon limit of a stack of D3 branes on the tip of the conical singularity

$$xy^2 = wz\,. \tag{2.153}$$

The holomorphic equation defining the singularity can be encoded in a graph called the toric diagram (see the Appendix A.1). In the paper we will use these diagrams to give an intuitive visual picture of the singularities.
The field theory has $U(N_1) \times U(N_2) \times U(N_3)$ gauge groups and chiral superfields that transform in the adjoint and bifundamental representations of the various gauge group factors. The fields X_{ii} are in the adjoint of the i-th gauge group and the fields q_{ij} transform in the fundamental representation of the $U(N_i)$ gauge group and in the anti-fundamental representation of the $U(N_j)$ gauge group. The symmetries and the matter content of a gauge theory related to branes at singularities can be encoded in a graph called the quiver diagram. The toric diagram and the quiver of the SPP singularity are shown in Figure 2.3.

Its superpotential is[10]

$$W = X_{11}(q_{13}q_{31} - q_{12}q_{21}) + q_{21}q_{12}q_{23}q_{32} - q_{32}q_{23}q_{31}q_{13}\,. \tag{2.154}$$

Taking into account the F-term equations for (2.154) we can choose

$$x = X_{11} = q_{23}q_{32}\ ,\ y = q_{12}q_{21} = q_{13}q_{31}\ ,\ w = q_{13}q_{32}q_{21}\ ,\ z = q_{12}q_{23}q_{31} \tag{2.155}$$

[10]The superpotential is a sum of gauge field monomials obtained contracting gauge indexes and taking the trace. Explicit index contractions and traces will be omitted in the paper.

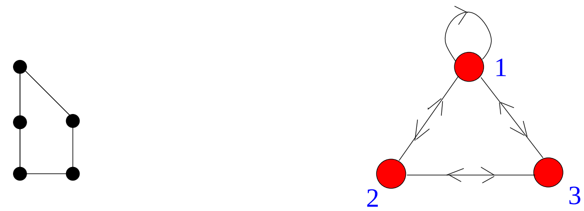


Figure 2.3: The toric diagram and the quiver of the SPP singularity

as generators of the mesonic chiral ring. The set of algebraic relations among these fields reproduces the geometric singularity (2.153). The presence of an adjoint chiral field is a signal for the presence of a non isolated singularity. In fact, giving a vev to X_{11} corresponds to motion in the geometry along the x direction, which is a line of non isolated $\mathbb{C}^2/\mathbb{Z}_2$ singularities: $y^2 = wz$. This line of singularities can be deformed to a smooth space

$$xy^2 = wz \to xy(y - \xi) = wz\,. \tag{2.156}$$

Moreover the conical singularity (2.153) has a complex deformation in which the tip of the cone is substituted by a three sphere S^3. In this case the SPP geometry is deformed as

$$xy^2 - y\epsilon - wz = 0\,. \tag{2.157}$$

This is the same process as the conifold transition in the KS solution [63].

Using toric geometry it is possible to visualize these two processes. First of all draw the toric diagram of the singularity. Then, if the dual graph has some parallel lines,

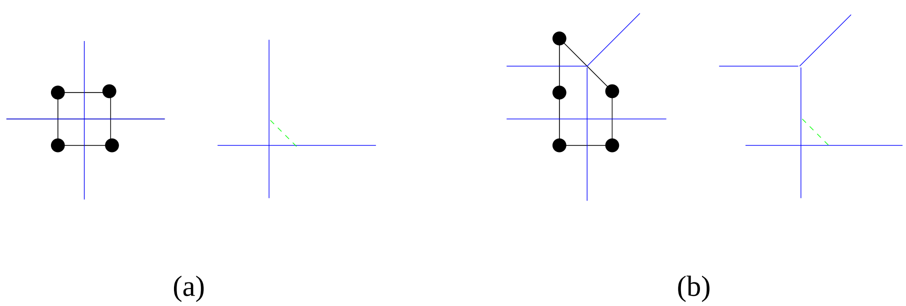


Figure 2.4: Toric diagram, dual diagram and complex deformation for (a) the conifold case $xy - wz = 0 \to xy - wz - \epsilon = 0$; (b) the SPP case $xy^2 - wz = 0 \to xy^2 - y\epsilon - wz = 0$. The broken line represents the S^3 due to the fluxes. The volume of S^3 is parameterized by ϵ.

this implies that there exist non isolated $\mathbb{C}^2/\mathbb{Z}_k$ lines of singularities (depending on the number of parallel lines). These singularities can be deformed by inserting two spheres S^2 parameterized by a set of complex ξ_i parameters. If the dual diagram admits splits in equilibrium (the edges of every sub-diagrams sum to zero), there exist deformations of the singularities on the tip of the cone. These deformations are obtained by inserting three spheres S^3, parameterized by some set of complex ϵ_j parameters (see Figure 2.4).

In this paper we argue that metastable supersymmetry breaking is geometrically realized by moving in the space of complex deformations. The motion in the ξ-parameter space breaks supersymmetry (in a metastable vacuum) while moving in the ϵ-parameter space restores the supersymmetry. We will provide several examples and show that this is a general phenomenon in an infinite class of quiver gauge theories.

We now review the possible IR behavior of the SPP gauge theory and their geometric interpretation. The SPP gauge theory has two kinds of fractional branes, because of the non anomalous distribution of ranks for the gauge group factors: $(1,0,0)$ and $(0,1,0)$. The different combinations of these set of branes and the possible geometric deformations of the singularity characterize different IR dynamics. We summarize the different possibilities.

The first fractional brane is called an $\mathcal{N}=2$ brane. The quiver in Figure 2.3 with $(N_1,0,0)$ fractional branes reduces to an $\mathcal{N}=2$ gauge theory. The vev of the adjoint field X_{11} is a modulus of the theory, corresponding to x in the geometry. Moving along x corresponds to the D-brane exploring the curve of A_1 singularities $y^2=wz$.

The second fractional brane is called deformation brane. Indeed the back reaction of $(0,N_2,0)$ $D5$ branes wrapped on the collapsed two cycle of the conifold inside the SPP induces a geometric transition which deforms the singularity to a smooth manifold: $xy^2+\epsilon y=wz$ (see Figure 2.4). In the gauge theory description, the deformation parameter ϵ is related to the gaugino condensate. The branes $(0,N_2,0)$ induce deformation in the geometry and confinement in the gauge theory [64].

The deformation brane and the $\mathcal{N}=2$ brane are incompatible. If we put $(N_1,N_2,0)$ branes in the SPP singularity the gauge theory has a runaway behavior, which is the most common behavior in non conformal quiver gauge theories [61, 65, 66]. Consider the case $N_2 \gg N_1=1$: the perturbative superpotential is

$$W_{pert}=X_{11}q_{12}q_{21}\,. \tag{2.158}$$

The node 2 is UV free and develops strong dynamics in the IR. The gauge invariant operators are the degrees of freedom that describe the IR dynamics of this node, i.e. the meson $M_{11}=q_{12}q_{21}$. The node 2 has $N_c>N_f$ and generates a non perturbative ADS superpotential. The complete IR superpotential is then

$$W_{IR}=X_{11}M_{11}+(N_2-1)\Big(\frac{\Lambda^{3N_2-1}}{M_{11}}\Big)^{\frac{1}{N_2-1}}\,. \tag{2.159}$$

The F term equations give the runaway.

Now we can include in the theory the deformation parameter ξ of the A_1 singularity and obtain the geometry (2.156). This corresponds to the superpotential term: $W_\xi = -\xi(X_{11}-q_{13}q_{31})$. Taking the same brane distribution as in the previous case, the IR superpotential is

$$W_{IR}=X_{11}M_{11}+(N_2-1)\Big(\frac{\Lambda^{3N_2-1}}{M_{11}}\Big)^{\frac{1}{N_2-1}}-\xi X_{11} \tag{2.160}$$

and hence the theory develops a supersymmetric vacuum.

Finally, as pointed out in [10], if we consider the theory deformed by ξ (2.156) in the regime $N_1 = N + M$ and $N_2 = N$ (Figure 2.5), the theory admits ISS like metastable vacua, provided $M > 2N$. In this case the node N_2 is the IR free gauge group and the

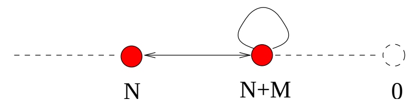


Figure 2.5: The fractional brane disposition to obtain the ISS theory from the SPP singularity.

node N_1 is treated as the flavour group (see the Appendix 2.1 for a discussion about this approximation). The superpotential is

$$W_{pert} = -\xi X_{11} + X_{11} q_{12} q_{21} \tag{2.161}$$

and supersymmetry is broken at tree level by the rank condition. Observe that from this construction we obtain directly the dual magnetic theory of the ISS model. This theory has also M supersymmetric vacua far away in the moduli space. As usual, these vacua are obtained by considering the non perturbative contribution to the superpotential due to the gaugino condensation

$$W_{IR} = -\xi X_{11} + N \left(\frac{\det X_{11}}{\Lambda_m^{2M-N}} \right)^{1/N} \quad \rightarrow \quad \langle X_{11} \rangle = \Lambda_m^{\frac{M-2N}{M}} \xi^{\frac{N}{M}} e^{\frac{2\pi i k}{M}} \mathbf{1}_{M+N} \tag{2.162}$$

The gauge theory dynamics that restore supersymmetry have a dual geometric interpretation. The geometry describing the IR gauge theory is the A_1 deformed conifold variety (2.157). Indeed, using the techniques of [10, 59], we can recover the complete IR non perturbative superpotential (2.162) from the geometry (2.157), performing a classical computation (see the Appendix A.4).

The SPP singularity can be considered the simplest representative of the family of deformable non isolated toric singularities. We will give a detailed analysis of an infinite sub-class of this family of singularities called the L^{aba} singularities [67, 68, 69, 70, 71],and we will then comment about their generalizations to more complicated examples.

The L^{aba} Singularities

L^{aba} with $b \geq a$ refers to an infinite class of deformable non isolated singularities that include the SPP as a special case: $L^{121} = SPP$ (see Figure 2.6). The L^{aba} singularities contain "a" conifold like singularities (hence "a" conifold like complex deformations) and two lines of non isolated singularities passing through the tip of the cone: $\mathbb{C}^2/\mathbb{Z}_a$ and $\mathbb{C}^2/\mathbb{Z}_b$. Indeed the L^{aba} singularities are described by a quadric in $\mathbb{C}^4$

$$x^a y^b = wz\,. \tag{2.163}$$

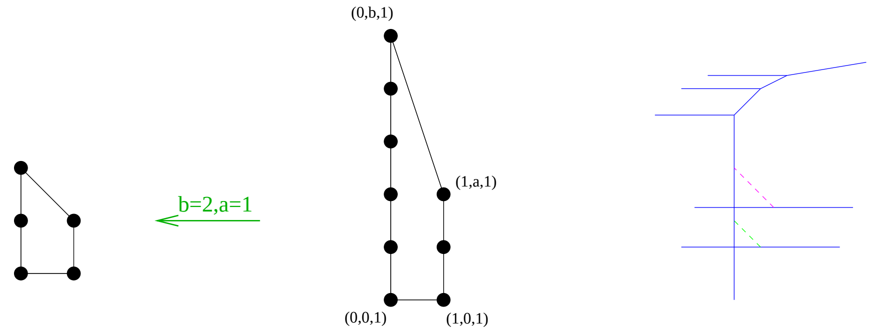


Figure 2.6: Toric diagram of L^{aba} singularity in the case $a = 2$, $b = 5$, its dual diagram with the complex deformations, and its reduction to the SPP toric diagram.

The lines parametrized by non zero values of x and y are the $\mathbb{C}^2/\mathbb{Z}_b$ and $\mathbb{C}^2/\mathbb{Z}_a$ non isolated singularities

$$x \neq 0 \ \rightarrow \ y^b = wz \ \ , \ y \neq 0 \ \rightarrow \ x^a = wz \, . \tag{2.164}$$

We can deform the singularities (2.164) by inserting two cycles at the singular point. A generic $\mathbb{C}^2/\mathbb{Z}_n$ contains, indeed, $n-1$ two spheres collapsed at the origin and can be deformed to a smooth space turning on $n-1$ generic complex deformation parameters ξ_i, $i = 1, ..., n-1$

$$x^n = yz \rightarrow x \prod_{i=1}^{n-1} (x - \xi_i) = yz \, . \tag{2.165}$$

On the other hand, from figure 2.6, we note that L^{aba} contain a conifolds that can be locally deformed as

$$xy - wz = 0 \ \rightarrow \ xy - wz - \epsilon_j = 0 \ , \quad \text{j=1,...a} \, . \tag{2.166}$$

We have thus identified two families of deformations: the ξ deformations and the ϵ deformations. As already mentioned, we argue that the motion in the ξ deformations breaks supersymmetry to a metastable vacuum, while the motion in the ϵ deformations restores it.

The gauge theories dual to these singularities [69, 70, 71] are non chiral and have the quiver representations[11] in Figure 2.7. The theory has gauge group $U(N_1) \times U(N_2) \times ... \times U(N_{a+b})$ and chiral fields transforming in the adjoint or in the bi-fundamental representations. The superpotentials are

$$W = \sum_{i=2a+1}^{b+a} X_{ii} (q_{i,i-1} q_{i-1,i} - q_{i,i+1} q_{i+1,i}) + \sum_{j=1}^{2a} (-1)^{j+1} q_{j,j-1} q_{j-1,j} q_{j,j+1} q_{j+1,j} \tag{2.167}$$

[11]This is just a possible toric phase. By Seiberg duality one can move to other toric phases with generically different content of matter and different superpotential but all flowing to the same IR fixed point and hence having the same singularity as mesonic moduli space.

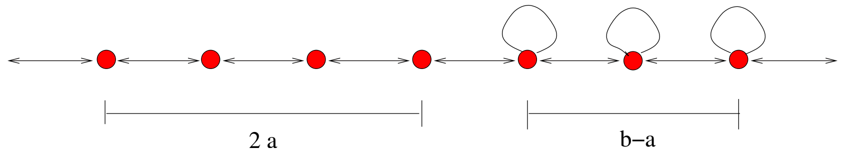


Figure 2.7: The quiver for the generic L^{aba} singularity.

where $a+b+1=1$ and the fields X_{ii} transform in the adjoint representation of the i-th gauge group, while $q_{i,i+1}$ transform in the fundamental representation of the i-th group and in the anti-fundamental of the i+1-th group.

The chiral ring constrains of the gauge theory can be related to the algebraic geometric description of the singularity. The complex deformations can be mapped into deformations of the superpotential, as well. Indeed the equation (2.163) can be reconstructed through the supersymmetric constraints on the mesonic chiral ring of the gauge theory. Define the following set of basic mesonic chiral operators

$$\begin{aligned} &x_1 = q_{12}q_{21} \quad , \quad x_2 = q_{34}q_{43} \quad , \quad \dots \quad , \quad x_a = q_{2a-1,2a}q_{2a,2a-1}; \\ &y_1 = q_{23}q_{32} \quad , \quad y_2 = q_{45}q_{54} \quad , \quad \dots \quad , \quad y_a = q_{2a,2a+1}q_{2a+1,2a}, \\ &y_{a+1} = q_{2a+1,2a+2}q_{2a+2,2a+1} \quad , \quad \dots \quad , \quad y_b = q_{b,1}q_{1,b}; \\ &X_{2a+1,2a+1} \quad , \quad X_{2a+2,2a+2} \quad , \quad \dots \quad , \quad X_{b,b}; \end{aligned} \tag{2.168}$$

$$w = q_{1,b}q_{b,b-1}\dots q_{3,2}q_{2,1} \quad , \quad z = q_{1,2}q_{2,3}\dots q_{b-1,b}q_{b,1}\,. \tag{2.169}$$

These operators satisfy

$$x_1 \dots x_a \; y_1 \dots y_b = wz\,. \tag{2.170}$$

From the F-term equations we get the relations

$$x_1 = \dots = x_a = X_{2a+1,2a+1} = \dots = X_{b,b} \quad , \quad y_1 = \dots = y_b\,. \tag{2.171}$$

The chiral ring constraints (2.170,2.171) reproduce the geometric singularity (2.163).

By this technique, using the F-term constraints, we can also map the complex deformations of the geometry to deformations of the superpotential.

A final, important, remark is that different UV gauge theories, flowing in the IR to the same conformal fixed point, correspond to the same toric singularity. These theories are related by Seiberg dualities and give equivalent physical descriptions at the conformal point. Here we choose the more convenient Seiberg phase for finding metastable vacua in the related non conformal case. This can be achieved by performing a set of Seiberg dualities on the quiver gauge theories with only regular branes, and then placing the right set of fractional branes that breaks conformal invariance.

Meta-stable vacua in L^{aba} theories

This paragraph is devoted to the analysis of metastability in the L^{aba} theories with $b > a$. The simplest example is the one studied in [10], where the ISS dynamics dynamics was found in the infrared of a deformed L^{121} theory. We now extend this analysis to more complicated cases, like L^{131} and then L^{1n1}. After that we show how to generate chains of theories that have supersymmetry breaking meta-stable vacua. Generally speaking, if we have a L^{aba} theory which shows metastability, we argue that the $L^{an,bn,an}$ theory behaves as a set of decoupled theories of this sort. At the end of this section, we give a general recipe for the existence of metastable vacua in a L^{aba} theory, by decomposing it into a set of shorter quivers.

In the analysis of the metastable vacua we consider some nodes of the quivers as gauge groups and other nodes as flavor groups, tuning the dynamical scales as explained in the section(2.1). This is implicit in all the cases that we treat.

Note that, in the notation of ISS, we are working in the magnetic description. This means that we deal with IR free gauge groups, without performing Seiberg duality on them. Another important remark is that, since we are dealing with the magnetic phase, if we want to realize metastable vacua, we need linear deformations in the mesons rather than massive quarks.

We present here several examples, as well as general results, to stress the fact that the ξ deformations lead to metastable non supersymmetric vacua whereas the ϵ deformations bring to supersymmetry restoration.

The L^{131} theory

The L^{131} theory is described by the quiver in figure 2.8, with superpotential

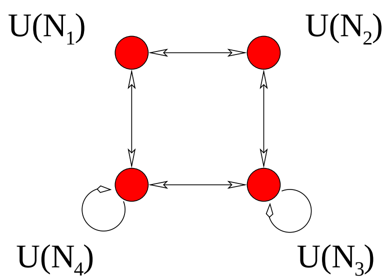


Figure 2.8: Quiver for the L^{131} theory.

$$W = \quad X_{33}(q_{32}q_{23} - q_{34}q_{43}) - hq_{21}q_{12}q_{23}q_{32} + hq_{12}q_{21}q_{14}q_{41} + \tag{2.172}$$
$$+X_{44}(q_{43}q_{34} - q_{41}q_{14}) \tag{2.173}$$

and it corresponds to the singular geometry

$$xy^3 = wz \tag{2.174}$$

which is correctly reproduced by the mesonic chiral ring as explained in section 2. We now add a superpotential deformation

$$W_{def} = -\xi_1(X_{33} - hq_{12}q_{21}) - \xi_2(X_{44} - hq_{12}q_{21})\,. \tag{2.175}$$

Imposing the constraints from the F-term equations we find the new relations on the mesonic chiral ring

$$y = q_{23}q_{32} = q_{34}q_{43} + \xi_1 = q_{41}q_{14} + \xi_1 + \xi_2\,. \qquad (2.176)$$

These constraints are translated into the deformed geometry

$$xy(y - \xi_1)(y - \xi_1 - \xi_2) = wz\,. \qquad (2.177)$$

Obviously, we are not obliged to add a linear term for each adjoint field but the case with only one deformation turns out to be unstable, as we show in the following.

We study this theory setting one node to zero. There are two different possibilities: we can set to zero a node with an adjoint field (N_3 or N_4) or a node without it (N_1 or N_2), obtaining a theory with one or two adjoint fields respectively. In the second case the scalar potential has dangerous flat directions and we cannot find metastable vacua. In the following we only analyze the first case and show the existence of long living non supersymmetric metastable vacua.

The theory under investigation is then obtained setting to zero the N_4 node (the case with $N_3 = 0$ is the same), described by the quiver in figure 2.9.

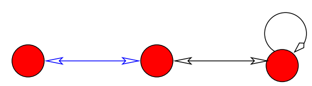


Figure 2.9: L^{131} theory with $N_4 = 0$. The blue lines indicate massive matter.

The superpotential is

$$W = X_{33}q_{32}q_{23} - hq_{21}q_{12}q_{23}q_{32} - \xi_1 X_{33} + h(\xi_1 + \xi_2)q_{21}q_{12}\,. \qquad (2.178)$$

For simplicity, in the analysis of the equations of motion we fix the ranks of the groups to be

$$N_1 = N_2 = N \qquad N_3 = N + M\,. \qquad (2.179)$$

First of all we have to impose the correct tuning on the scales of the gauge groups and on the rank numbers in order to treat the node N_2 as an infrared free gauge group and the other gauge groups as flavours. Calculating the beta functions we have

$$b_1 = 2N \quad b_2 = N - M \quad b_3 = N + 2M\,. \qquad (2.180)$$

Since we require the group $U(N_2)$ to be infrared free we impose the constraint $M > N$. Moreover, we require that this group is more coupled than the other groups at the supersymmetry breaking scale and at the scale of supersymmetry restoration[12]. This can

[12]With supersymmetry restoration we mean the supersymmetric vacua that arise due to the strong dynamics of $U(N_2)$. For what concern the other supersymmetric vacua, given by the strong dynamics of the other groups, the tuning on the scales put them far away in the field space.

be done by tuning the scales Λ_1 and Λ_3, which are the strong coupling scales of two UV free gauge groups. Their scales have to be chosen[13] much smaller than the scale of supersymmetry breaking (which is the deformation $h\xi_1$) and much smaller than the scale Λ_2 of $U(N_2)$.

Now that we have correctly set up the role played by each gauge group in the quiver we can proceed in finding the vacua. The F-term equation for the X_{33} field is the rank condition and breaks supersymmetry, fixing the vev of the fields q_{23} and q_{32}. The equation for the q_{12} quark is

$$F_{q_{12}} = h\left(-q_{23}q_{32} + (\xi_1 + \xi_2)\right) q_{21} = h\xi_2 q_{21} \tag{2.181}$$

and it is solved with $q_{12} = 0 = q_{21}$. This is related to the fact that we have added two deformation parameters (ξ_1, ξ_2), i.e. two linear contributions to the superpotential for the two adjoint fields. Otherwise (for $\xi_2 = 0$), the equation (2.181) would be automatically satisfied, leaving the fields q_{12} and q_{21} unfixed at tree level and leading to potentially dangerous flat directions.

The non supersymmetric vacuum at tree level is

$$q_{12} = q_{21}^T = 0 \qquad q_{32} = q_{23}^T = \begin{pmatrix} \sqrt{\xi_1}\mathbf{1}_N \\ 0 \end{pmatrix} \qquad X_{33} = \begin{pmatrix} 0 & 0 \\ 0 & \chi \end{pmatrix} \tag{2.182}$$

where χ is the pseudomodulus of dimension $M \times M$. This vacuum is stable under one loop correction, and the pseudomodulus is stabilized at $\chi = 0$.

The restoration of supersymmetry is obtained in the hypothesis that the group labeled by N_2 develops a strong dynamics, by adding to the low energy superpotential a non perturbative contribution

$$W_{IR} = -\xi_1 X_{33} + N_2\left(\Lambda^{3N_2 - N_3} \det X_{33}\right)^{\frac{1}{N_2}} \tag{2.183}$$

where we have integrated out all the massive fields. From the geometric point of view, supersymmetry restoration, governed by the dynamics of the $U(N_2)$ gauge group, can be described deforming the geometry with an S^3, i.e. an ϵ deformation,

$$(y - \xi_1)(y - \xi_1 - \xi_2)(xy - \epsilon) = wz\,. \tag{2.184}$$

The low energy field theory superpotential (2.183) can be recovered from the geometric data (2.184). Indeed, setting $y = x' - y'$ and $x = x' + y'$, equation (2.184) becomes:

$$(x' - y' - \xi_1)(x' - y' - \xi_1 - \xi_2)\left(x' + \sqrt{y'^2 + \epsilon}\right)\left(x' - \sqrt{y'^2 + \epsilon}\right) = wz\,. \tag{2.185}$$

The low energy superpotential can be written as a function of the glueball field S_2 (identified with $\epsilon/2$) and of the adjoint field X_{33}

$$W_{IR}(S_2, X_{33}) = W_{GVW}(S_2) + W_{adj}(S_2, X_{33}) = N_2 S_2\left(\log\frac{S_2}{\Lambda_2^3} - 1\right) + \frac{t}{g_2}S_2 + W_{adj}(S_2, X_{33})\,. \tag{2.186}$$

[13] See section 2.1 and [31] for a complete analysis.

Following the procedure explained in Appendix A.4 the last term is derived from the geometric data

$$\begin{aligned} W_{adj}(S_2, X_{33}) \quad &= \int (x'_2(y') - x'_3(y'))dy' = \\ &= \int \left(y' + \xi_1 - \sqrt{y'^2 + \epsilon}\right) \sim \xi_1 X_{33} - S_2 \log \frac{X_{33}}{\Lambda_m} \end{aligned} \tag{2.187}$$

where we have expanded the integral in the approximation $y' \gg \xi_1, \epsilon$. We can now solve the equation of motion for the glueball field S_2 and integrate it out, ignoring the multi-istanton contribution. In this way we recover from the geometry (2.184) the low energy superpotential (2.183).

As claimed in the introduction, we have shown, in this simple example, that the ξ_i deformations lead to metastable vacua whereas the ϵ deformation leads to supersymmetry restoration.

The L^{1n1} theories

The metastable L^{131} theory can be generalized to the more complicate L^{1n1} case. We find metastable supersymmetry breaking in the L^{141} and L^{151} theories and then we show how to extend this procedure to the L^{1n1} case. A relevant aspect in the analysis is the decoupling between the breaking sector and the supersymmetric one.

L^{141}

Here we study the quiver gauge theory of figure 2.10, with superpotential

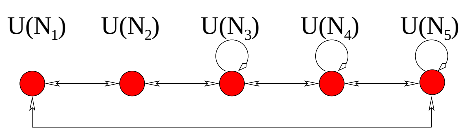


Figure 2.10: L^{141} theory.

$$\begin{aligned} W = \quad & hq_{12}q_{23}q_{32}q_{21} - X_{33}q_{32}q_{23} + X_{33}q_{34}q_{43} - X_{44}q_{43}q_{34} \\ & X_{44}q_{45}q_{54} - X_{55}q_{54}q_{45} + X_{55}q_{51}q_{15} - hq_{51}q_{12}q_{21}q_{15}\,. \end{aligned} \tag{2.188}$$

The geometry associated with this theory is described by the equation

$$xy^4 = wz\,. \tag{2.189}$$

Supersymmetry breaking is driven by linear terms for the adjoint fields. We add the deformation superpotential

$$W_{\rm def} = -\xi_3(X_{33} - hq_{32}q_{23}) + \xi_4(X_{44} - hq_{32}q_{23}) \ - \xi_5(X_{55} - hq_{32}q_{23})\,. \tag{2.190}$$

We have add a linear term for all the adjoint fields: this is crucial for the stability of the non supersymmetric vacuum. The q_{23} and q_{32} quarks become massive, since the F-terms constraints have to be compatible.

The corresponding geometry reads now

$$x(y-\xi_3)(y-\xi_3-\xi_4)(y-\xi_3-\xi_4-\xi_5)y = wz\,. \tag{2.191}$$

If we consider as gauge group the node $U(N_2)$ and choosing the ranks as[14]

$$N_1 = N_2 = N_5 = N \quad N_3 = N+M \quad N_4 = 0 \tag{2.192}$$

with $M > N$, this theory breaks supersymmetry through rank condition for the meson X_{33}. In the next paraghraph we give a detailed analysis that shows that this theory possesses metastable vacua without dangerous flat directions.

Two important remarks are in order. Without turning on the deformation ξ_4 (the one related to the node set to zero) we are not protected from instabilities of the scalar potential. Furthermore, as we did in the L^{131} case, we have decoupled an ISS like sector with supersymmetry breaking from a supersymmetric sector. These two facts hold in all the L^{1n1} cases.

The process of supersymmetry restoration works as in the L^{131}, when the dynamics of the gauge group $U(N_2)$ gives rise to non perturbative contributions to the superpotential.

L^{151}

Here we study metastability in the L^{151} quiver gauge theory. This is the basic example for the generalization of the analysis to the L^{1n1} case. The gauge theory, related to the

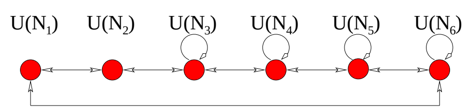


Figure 2.11: The L^{151} theory.

quiver in figure 2.11, has superpotential

$$\begin{aligned} W = \quad & hq_{12}q_{23}q_{32}q_{21} - X_{33}q_{32}q_{23} + X_{33}q_{34}q_{43} - X_{44}q_{43}q_{34} + X_{44}q_{45}q_{54} \\ & -X_{55}q_{54}q_{45} + X_{55}q_{51}q_{15} - X_{66}q_{65}q_{56} + X_{66}q_{61}q_{16} - hq_{61}q_{12}q_{21}q_{16} \end{aligned} \tag{2.193}$$

and it is associated to the geometry

$$xy^5 = wz\,. \tag{2.194}$$

[14]Note that also the situation with gauge group $U(N_1)$ and $N_3 = N$ and $N_5 = N + M$ leads to metastable vacua.

Once again we deform the superpotential with linear terms for the adjoint fields and masses for the quarks

$$W_{\rm def} = -\xi_3(X_{33} - hq_{32}q_{23}) - \xi_4(X_{44} - hq_{32}q_{23}) - \xi_5(X_{55} - hq_{32}q_{23}) - \xi_6(X_{66} - hq_{32}q_{23})\,. \tag{2.195}$$

The deformation (2.195) leads to the geometric deformation

$$x(y-\xi_3)(y-\xi_3-\xi_4)(y-\xi_3-\xi_4-\xi_5)(y-\xi_3-\xi_4-\xi_5-\xi_6)y = wz\,. \tag{2.196}$$

We choose the ranks of the groups as

$$N_1 = N_2 = N_5 = N_6 = N \qquad N_3 = N+M \qquad N_4 = 0 \tag{2.197}$$

with $M > N$. The equation of motion for the field X_{33} is the ISS rank condition, that breaks supersymmetry at the classical level. In the next paragraph we show that the supersymmetry breaking minimum is stable. Stability of the metastable vacuum requires $\xi_3, \xi_4, \xi_5 \neq 0$ and arbitrary ξ_6.

The supersymmetry restoration carries on exactly as in the L^{131}, with non perturbative contribution to the superpotential due to the dynamics of the gauge group $U(N_2)$.

L^{1n1}

We now extend the results about metastability to the general L^{1n1} theory. The superpotential is

$$W = \sum_{i=3}^{n} X_{i,i}(q_{i,i-1}q_{i-1,i} - q_{i,i+1}q_{i+1,i}) + hq_{21}q_{12}q_{23}q_{32} - hq_{12}q_{21}q_{1,n+1}q_{n+1,1} + \tag{2.198}$$

$$+X_{n+1,n+1}(q_{1,n+1}q_{n+1,1} - q_{n,n+1}q_{n+1,n}) \tag{2.199}$$

and the geometry

$$xy^n = wz\,. \tag{2.200}$$

The deformation of the superpotential is

$$\Delta W = \sum_{i=3}^{n+1} \xi_i(hq_{12}q_{21} - X_{i,i}) \tag{2.201}$$

which corresponds to the geometry

$$xy\prod_{i=3}^{n+1}(y - \sum_{j=1}^{i}\xi_i) = wz\,. \tag{2.202}$$

We choose the ranks of the nodes to be

$$N_4 = 0 \qquad N_3 = N+M \qquad N_j = N \quad (j \neq 3,4) \tag{2.203}$$

such that supersymmetry is broken at node 3. Moreover it should be $M > N$ for $U(N_2)$ to be IR free.

The deformations ξ_3, ξ_4 and ξ_5 have to be different from zero for the non supersymmetric vacuum to be stable. All the other deformations can be chosen arbitrarily The breaking sector is the same than all the other L^{1n1} cases analyzed before. The only difference is that the supersymmetric sector is larger here.

Supersymmetry is restored by the strong dynamics of the gauge group $U(N_2)$, and the metastable vacuum is long living. This concludes the analysis of the L^{1n1} theories.

Details on the non supersymmetric vacua

In this paragraph we discuss the stability of the non supersymmetric vacua studied in the rest of the section. The relevant aspects in the analysis of metastable vacua are related to the tree level flat directions that can arise in the scalar potential around the would be minimum. If these directions are not related to any broken global symmetry they are pseudomoduli, and they have to be lifted classically or quantum mechanically. Even if these directions arise in a sector which is supersymmetric up to the third order in the fluctuations around the vacuum, we have to check that all of them acquire positive squared masses. Otherwise these fields can acquire tachyonic masses due to their coupling to the non supersymmetric sector at higher order. In the analysis we treat all the gauge groups as $U(n)$. This implies that the D-term scalar potential for the fluctuations around the minimum receives contributions not only from the $SU(n)$ part of the gauge groups but also from the $U(1)$'s. These contributions could be relevant in some examples to lift flat directions. We comment on this when needed.

A last comment is necessary. In the text we called the complex deformations that lead to supersymmetry braking ξ_i. Here we use a different notation, denoting μ_i^2 these deformations. In this way we work with couplings of mass dimension one.

L^{131}

We analyze the quiver gauge theory of figure 2.12 with superpotential

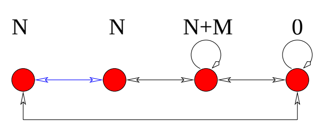


Figure 2.12: The L^{131} theory with $N_4 = 0$. The blue line indicate the massive fields

$$W = X_{33}q_{32}q_{23} - \mu_3^2 X_{33} - hq_{12}q_{23}q_{32}q_{21} + hm^2 q_{12}q_{21} \tag{2.204}$$

with $m^2 = \mu_3^2 + \mu_4^2$. The adjoint field has a linear term and the quarks have a mass generally different from the deformation of the adjoint field. We take the ranks of the gauge groups as

$$N_3 = N + M \qquad N_2 = N_3 = N \quad N_4 = 0 \tag{2.205}$$

with $M > N$. With this choice we are guaranteed that the second node is infrared free. We consider the other groups less coupled.

Solving the equation of motion and expanding around the tree level minimum we have

$$q_{32} = \begin{pmatrix} \mu_3 + \sigma_1 \\ \phi_1 \end{pmatrix} \quad q_{23} \begin{pmatrix} \mu_3 + \sigma_2 & \phi_2 \end{pmatrix} \quad X_{33} = \begin{pmatrix} \sigma_3 & \phi_3 \\ \phi_4 & \chi \end{pmatrix} \quad q_{21} = \sigma_4 \quad q_{12} = \sigma_5 \tag{2.206}$$

where χ is a classical flat direction not associated to any broken symmetry. The case with $\mu_4 = 0$ (and hence $m^2 = \mu_3^2$) is problematic since in this case the quarks q_{12} and q_{21} are potentially dangerous tree level flat directions.

Now, the non supersymmetric sector (the fields ϕ_i) gives the usual O'Raifeartaigh like model of ISS which gives positive squared mass through 1 loop corrections to the pseudomoduli[15] χ. The fields ϕ_i get tree level masses except the Goldstone bosons as in the ISS model.

In the supersymmetric sector, the $\sigma_1, \sigma_2, \sigma_3$ fields are stabilized as in ISS. The fields σ_4 and σ_5 get non trivial squared mass $\sim |hm^2 - h\mu^2|^2 = |h\mu_4^2|^2$.

L^{141}

We analyze here a more complicated example that arises setting to zero a node in the L^{141} quiver gauge theory. The resulting quiver is reported in figure 2.13 and the superpotential

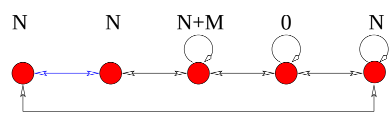


Figure 2.13: The L^{141} theory with $N_4 = 0$. The blue line indicate the massive fields

is the following

$$W = hq_{12}q_{23}q_{32}q_{21} - \mu_3^2 X_{33} - X_{33}q_{32}q_{23} + \mu_5^2 X_{55} + X_{55}q_{51}q_{15} - hq_{51}q_{12}q_{21}q_{15} + hm^2 q_{12}q_{21} \tag{2.207}$$

where all the adjoint fields receive a linear term. From the geometric description we know that

$$m^2 = \mu_3^2 + \mu_4^2 - \mu_5^2 \tag{2.208}$$

where μ_4^2 is related to the node we have set to zero. Having set the ranks of the gauge group to be

$$N_1 = N_2 = N_5 = N \qquad N_3 = N + M \quad N_4 = 0 \tag{2.209}$$

a rank condition mechanism is realized for the X_{33} meson.

[15]If the $U(1)$ factor of $U(N_2)$ decouples there is another pseudomodulus, $\theta + \theta^*$, stabilized by 1-loop corrections (see Section 2.1).

Solving the equation of motion and expanding around the tree level minimum we have

$$q_{23} = \begin{pmatrix} \mu_3 + \sigma_1 \\ \phi_1 \end{pmatrix} \quad q_{32} \begin{pmatrix} \mu_3 + \sigma_2 & \phi_2 \end{pmatrix} \quad X_{33} = \begin{pmatrix} \sigma_3 & \phi_3 \\ \phi_4 & \chi \end{pmatrix}$$
$$q_{12} = \sigma_4 \quad q_{21} = \sigma_5 \quad q_{51} = \mu_5 + \sigma_6 \quad q_{15} = \mu_5 + \sigma_7 \quad X_{55} = \sigma_8 \,. \tag{2.210}$$

The non supersymmetric sector (the ϕ_i fields) is like the ISS model, and give raise to an O'Raifeartaigh model which stabilize at one loop the pseudomodulus at $\chi = 0$.

The supersymmetric sector (the σ_i fields) has the following superpotential at the relevant order for the mass matrix

$$W = \mu_3\sigma_3(\sigma_1 + \sigma_2) - h\mu_4^2\sigma_4\sigma_5 - \mu_5\sigma_8(\sigma_6 + \sigma_7)\,. \tag{2.211}$$

The $\sigma_1, \sigma_2, \sigma_3$ fields behave exactly as in ISS: some of them acquire tree level positive mass. The massless ones are either Goldstone bosons either pseudomoduli. The latter are lifted by the D term potential for the $U(N_2)$ gauge group.

The σ_4, σ_5 fields have tree level masses and this is due to the fact that we have turned on all the possible deformation for the geometry, i.e. $\mu_4 \neq 0$. Otherwise they would be dangerous flat directions.

The $\sigma_6, \sigma_7, \sigma_8$ fields behave as the $\sigma_1, \sigma_2, \sigma_3$ sector. However we note that here the pseudomoduli arising in these fields are lifted by the D terms of the $U(N_5)$ gauge group, that we have considered less coupled than the gauge group $U(N_2)$.

L^{151}

We study here the quiver gauge theory associated to the L^{151} singularity. The aim is to find the relevant aspects for the generalization to the L^{1n1} theory. After setting to zero a node in the L^{151} theory we obtain the quiver in figure 2.14 with superpotential

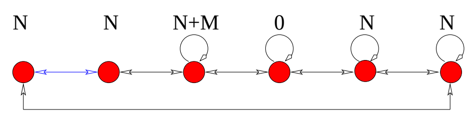


Figure 2.14: The L^{151} theory with $N_4 = 0$. The blue line indicate the massive fields

$$\begin{aligned} W \;\; = \;\; & X_{33}q_{23}q_{32} - hq_{12}q_{23}q_{32}q_{21} + hq_{61}q_{12}q_{21}q_{16} - X_{55}q_{56}q_{65} + X_{66}q_{65}q_{56} - X_{66}q_{61}q_{16} \\ & -\mu_3^2X_{33} + \mu_5^2X_{55} + \mu_6^2X_{66} + hm^2q_{12}q_{21}\,. \end{aligned} \tag{2.212}$$

The geometric description implies

$$m^2 = \mu_3^2 + \mu_4^2 - \mu_5^2 - \mu_6^2 \tag{2.213}$$

where the parameter μ_4 is related to the deformation for the node we have set to zero. The ranks of the groups are taken to be

$$N_3 = N + M \qquad N_1 = N_2 = N_5 = N_6 = N \quad N_4 = 0\,. \tag{2.214}$$

Solving the equation of motion and expanding around the tree level minimum we have

$$q_{23} = \begin{pmatrix} \mu_3 + \sigma_1 \\ \phi_1 \end{pmatrix} \quad q_{32} \begin{pmatrix} \mu_3 + \sigma_2 & \phi_2 \end{pmatrix} \quad X_{33} = \begin{pmatrix} \sigma_3 & \phi_3 \\ \phi_4 & \chi \end{pmatrix}$$

$$q_{12} = \sigma_4 \quad q_{21} = \sigma_5 \quad q_{16} = \sqrt{\mu_5^2 + \mu_6^2} + \sigma_6 \quad q_{61} = \sqrt{\mu_5^2 + \mu_6^2} + \sigma_7 \tag{2.215}$$

$$X_{66} = \sigma_8 \quad q_{65} = \mu_5 + \sigma_9 \quad q_{56} = \mu_5 + \sigma_{10} \quad X_{55} = \sigma_{11}\,. \tag{2.216}$$

The non supersymmetric sector works as in the previous examples and stabilize the pseudomodulus χ at $\chi = 0$. The supersymmetric sector (the σ_i) has, at the relevant order for the mass matrix, the following superpotential

$$W = \mu_6(\sigma_{11} - \sigma_8)(\sigma_9 + \sigma_{10}) + \sqrt{\mu_6^2 + \mu_5^2}\,\sigma_8(\sigma_6 + \sigma_7) - h\mu_4^2\sigma_4\sigma_5 + \mu_3\sigma_3(\sigma_1 + \sigma_2)\,. \tag{2.217}$$

It can be analyzed as three separated sectors.

The first one is made by the fields $\sigma_1, \sigma_2, \sigma_3$ and behave exactly as in ISS. The second one is made by the fields σ_4, σ_5. Here once again the parameter in the whole theory associated to the node set to zero (μ_4) is crucial for the stability of the vacuum. In fact if $\mu_4 = 0$ the directions σ_4 and σ_5 would result massless at tree level.

The third sector is made by the other fields and it is stabilized at tree level taking into account the D term contributions to the scalar potential for the gauge groups $U(N_5)$ and $U(N_6)$.

Another important fact to be stressed is that in this case we are not obliged to switch on the deformation μ_6.

L^{1n1}

The analysis made in the last example can be extended to the gauge theory obtained from the L^{1n1} quiver as explained in the text. The vacuum is chosen as a natural generalization of the previous examples, and the fluctuation superpotential has the same structure. The non supersymmetric sector is the same than in ISS. The supersymmetric sector is decoupled in three different parts as in the last subsection. The tree level flat directions are stabilized provided the deformation associated with the node set to zero and to the first and the last nodes are switched on.

Another requirement for stabilizing the flat directions in the L^{1n1} theories with $n > 3$ is to take into account the tree level D-term potential of some of the flavour groups. Note that for these nodes we need to consider also the $U(1)$ contribution to the D-term potential of the $U(n)$ groups. Otherwise, if the $U(1)$'s decouple, some flat directions due to the trace part of the fundamental fields can remain in the one loop spectrum. It would be interesting to explore their two loop behaviour.

Extension to longer quivers

We extend here the analysis of the L^{1n1} theories to more complicated L^{aba} cases. The strategy is to decouple an L^{aba} theory in a set of a metastable theories, adding $b - a$

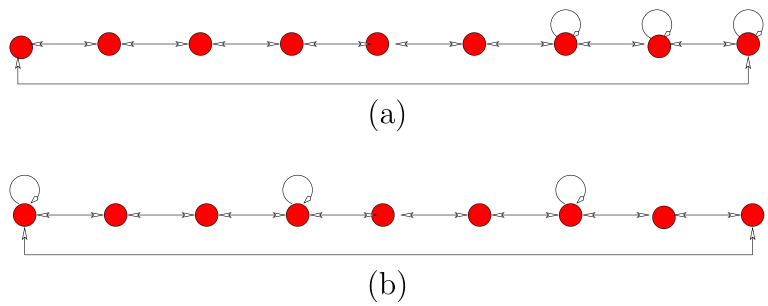


Figure 2.15: Two different Seiberg phases of the same L^{363} quiver gauge theory.

deformations, one for each adjoint field. In the case $b - a \geq a$,by using the results obtained for L^{1n1}, we are able to find metastable vacua in each L^{aba} theory.

Our general strategy will be to consider in each metastable subset only one group as a gauge group, since there are some difficulties in treating the dynamics of more than one gauge group simultaneously.

We study first the simplest cases, like the L^{n2nn} theory, which can be viewed as a set of decoupled ISS models. This is a pedagogical example, useful for the extension of the analysis to the general situations and for the proof that metastability is a generic phenomenon in the L^{aba} theories. At the end of this subsection, we furnish the general recipe to build metastable L^{aba} quivers.

L^{n2nn} as a set of decoupled ISS

The L^{121} gauge theory, in the ISS regime, has been shown to possess meta-stable vacua [10]. Starting from an L^{n2nn} quiver gauge theory we can perform a set of Seiberg dualities going from the first quiver in the Figure 2.15 to the second one. In fact, Seiberg duality on these theories has the effect of displacing the adjoint fields. Each duality moves one adjoint field two nodes farther.

We now deform the geometry, associating each ξ_i deformation with the i-th node, obtaining

$$\prod_{i=1}^{n} x(y - \xi_{3i-2})y = wz \,. \tag{2.218}$$

This deformation corresponds, on the gauge side, to the combined addition of linear terms for the adjoint fields and of masses for the appropriate bifundamentals (i.e. that ones not directly coupled to the adjoint fields). By setting to zero one node, without an adjoint field, every three nodes, we have a theory of decoupled metastable ISS models (see Figure 2.16). The analysis of metastable vacua is the same as in ISS for each sector. Supersymmetry is restored in the large field region in each ISS sector, where the gauge group gives rise to a non perturbative contribution in the effective theory. The non perturbative contributions

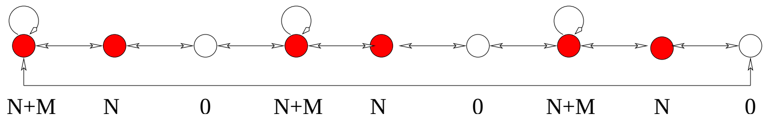
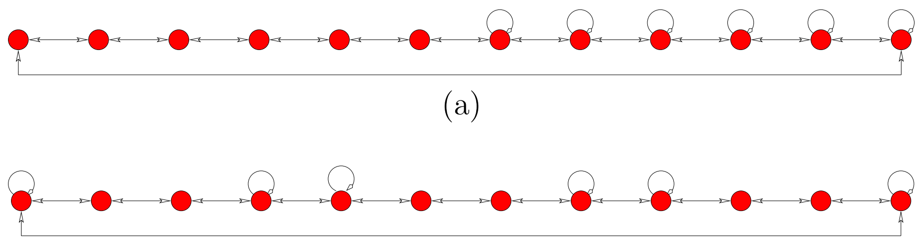


Figure 2.16: L^{363} as a set of three decoupled ISS models



(a)

(b)

Figure 2.17: Two different Seiberg phases of the same L^{393} quiver gauge theory.

modify the constraints on the mesonic moduli space, and hence the geometry, as

$$\prod_{i=1}^{n} y\left((y-\xi_{3i-2})x-\epsilon_i\right) = wz\,. \tag{2.219}$$

The technique of Appendix A.4 can be applied to the new singularities of the geometry to recover the correct low energy behavior of the field theory. The calculation proceeds exactly as in the L^{121} case

The L^{n3nn} theories

With this strategy we can build longer quivers with metastable vacua. For example the L^{131} case can be extended to metastable L^{n3nn} theories. Indeed we can perform a set of Seiberg dualities to obtain a new phase of the theory as shown in Figure 2.17. As we did in the L^{n2nn} case, we then deform the geometry

$$\prod_{i=1}^{2n}(y-\xi_{4(i-1)}-\xi_{4(i+1)+1})yx(y-\xi_{4i}) = wz \qquad \text{with} \quad \xi_0=\xi_{4n}\,. \tag{2.220}$$

The deformation brings in the superpotential a linear term for each adjoint field, and a mass term for the quarks stretched between two nodes without the adjoint fields.

We set then to zero the right nodes and breaks the L^{n3nn} into a set of metastable gauge theories. Indeed, setting the ranks number as in Figure 2.18, in each decoupled sector we have the same breaking patterns as in the L^{131} studied before. Each sector has the superpotential

$$W = hq_{i,i+1}q_{i+1,i+2}q_{i+2,i+1}q_{i+1,i} - q_{i+1,i}X_{i,i}q_{i,i+1} - \xi_i X_{i,i} + h(\xi_i+\xi_{i+3})q_{i+2,i+1}q_{i+1,i+2} \tag{2.221}$$

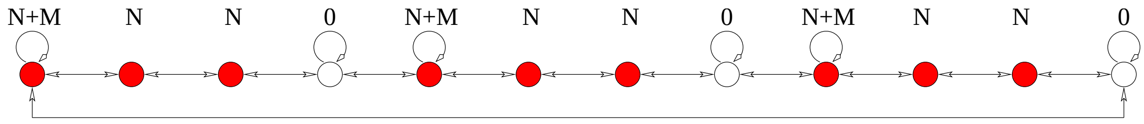


Figure 2.18: L^{393} as a set of three decoupled L^{131} models.

which leads to long living metastable vacua, as it has been explained for the L^{131} theory. Supersymmetry restoration can be obtained separately in each decoupled sector, through the strong dynamics of the gauge group. In the geometric description it can be read from the deformation of the variety

$$\prod_{i=1}^{2n} \left((y - \xi_{4(i-1)} - \xi_{4i-3})x - \epsilon_i\right) y(y - \xi_{4i}) = wz\,. \tag{2.222}$$

It is straightforward to show that it corresponds to adding a term proportional to $\det X_{ii}$ for each gauge group and restores supersymmetry.

Extension with an example

The procedure just outlined for the L^{121} and L^{131} can be applied also for the L^{1n1} case, extending it to $L^{m,nm,m}$ metastable theories.

More generally, we can consider an L^{aba} quiver that can be decomposed into subsets of different theories, each one metastable.

We show the technique in a clarifying example and then give a general recipe. For instance, we take the L^{252} theory and perform a Seiberg duality to obtain the phase of figure 2.19. By deforming all the adjoint fields with a linear term the chiral ring gets

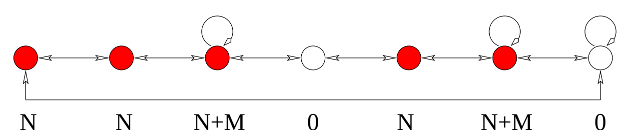


Figure 2.19: The Seiberg phase of L^{252} suitable for metastable vacua

modified to be

$$\begin{aligned} y &= q_{23}q_{32} = q_{34}q_{43} + \xi_3 = q_{56}q_{65} = q_{67}q_{76} + \xi_6 = q_{71}q_{17} + \xi_6 + \xi_7 \\ x &= q_{12}q_{21} = q_{45}q_{54} \end{aligned} \tag{2.223}$$

with the corresponding deformed geometry

$$x^2y^2(y - \xi_3)(y - \xi_6)(y - \xi_6 - \xi_7) = wz\,. \tag{2.224}$$

We choose then the sequence of the ranks of the groups as shown in figure 2.19, setting to zero the fourth and the last node. Now the first sector corresponds to the L^{131} theory and the second one to the L^{121} one. Each sector shows metastable supersymmetry breaking vacua. The superpotential is

$$W = hq_{12}q_{23}q_{32}q_{21} - q_{23}X_{33}q_{32} - q_{56}X_{66}q_{65} - \xi_3 X_{33} - \xi_6 X_{66} + (\xi_3 + \xi_7)q_{21}q_{12} \,. \quad (2.225)$$

Supersymmetry is restored by the strong dynamics of the nodes two and five that give rise to the non perturbative contribution

$$W_{\rm dyn} = N\left(\Lambda_2^{2N-M} \det X_{33}\right)^{1/N} + N\left(\Lambda_5^{2N-M} \det X_{66}\right)^{1/N} \quad (2.226)$$

which deforms the geometry to

$$(xy - \epsilon_1)(xy - \epsilon_2)(y - \xi_3)(y - \xi_6)(y - \xi_6 - \xi_7) = wz \,. \quad (2.227)$$

Indeed, from this geometry, with the technique discussed in the Appendix A.4, we can recover now the low energy superpotential of the field theory.

We start writing the general IR superpotential as a function of the mesons X_{33} and X_{66} and of the glueballs S_2 and S_5

$$W_{IR} = W_{GVW}(S_2) + W_{GVW}(S_5) + W_{adj}(S_2, X_{33}) + W_{adj}(S_5, X_{66}) \,. \quad (2.228)$$

Substituting $y = x' - y'$ and $x = x' + y'$ in (2.227) we can calculate the contributions $W(S, X)$ in the superpotential

$$\begin{aligned} W_{adj}(S_2, X_{33}) &= \int \left(y' + \xi_3 - \sqrt{y'^2 + \epsilon_1}\right) dy' \sim \xi_3 X_{33} - S_2 \log \frac{X_{33}}{\Lambda_2} \\ W_{adj}(S_5, X_{66}) &= \int \left(y' + \xi_6 - \sqrt{y'^2 + \epsilon_2}\right) dy' \sim \xi_6 X_{66} - S_5 \log \frac{X_{66}}{\Lambda_5} \end{aligned} \quad (2.229)$$

where we identify $(2S_2, 2S_5) = (\epsilon_1, \epsilon_2)$. We remark that the variable y', that parametrizes the position of the brane, can be interpreted as the vev of the field X_{ii} in each integral. Integrating out the glueball fields S_2 and S_5 we recover the low energy description of the field theory.
This example shows that we can obtain metastable L^{aba} theories by breaking them up into shorter quivers.

General analysis

Here we decompose an L^{aba} theory into a set of L^{1n_i1} theories, each one with metastable vacua.

We consider a distribution of gauge groups with ranks such that there are no consecutive nodes set to zero. Moreover we consider only Seiberg phases with $b - a$ adjoint fields to be distributed on the a gauge nodes. This implies that we can only describe theories with $b - a \geq a$. In the next paragraph we extend this result to theories with $b - a < a$, studying Seiberg phases with more adjoint fields.

With these assumptions, starting from a L^{aba} and setting a nodes to zero, we can obtain a metastable L^{1n_i1} theories. Each decoupled sector possesses long living metastable vacua like the ones studied in the L^{1n_i1} theories and hence the whole theory is metastable. The procedure is not unique: we can indeed decouple the L^{aba} theory in different sets of L^{1n_i1} quivers. This is related to the fact that we can distribute differently the $b-a$ adjoint fields on the a gauge nodes and set to zero nodes with or without adjoint fields.

This can be shown in a simple example. The L^{383} theory can be decoupled in three different sectors, where the number of adjoint fields totals up to five. There are two inequivalent possibilities to obtain metastable vacua as shown in figure 2.20. We set

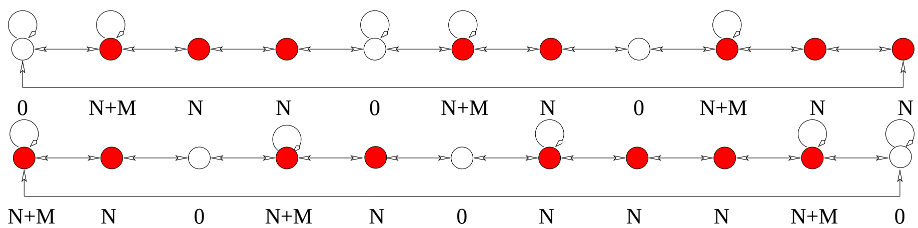


Figure 2.20: The two inequivalent possible L^{383} that give rise to three decoupled metastable sectors

three different nodes to zero (nodes $1, 5, 8$ in the first case and $3, 6, 11$ in the second one), obtaining three decoupled metastable theories. For the first case the analysis of metastability follows from L^{121} and L^{131}, while in the second case it follows from L^{121} and L^{141}. So we decouple L^{383} in two different ways: as $2L^{131} + L^{121}$ or as $2L^{121} + L^{141}$. By this technique, we can write L^{aba} as a sum of $\sum_{i=1}^{a} L^{1n_i1}$, with the constraint $\sum_{i=1}^{a} n_i = b$, $n_i \geq 2$. All these theories lead, with the right distribution of ranks, to metastable vacua.

Three nodes with two adjoint fields

We analyze the quiver gauge theory of figure 2.21 with superpotential

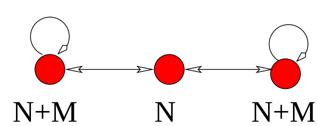


Figure 2.21: The quiver for the L^{222} theory with a node set to zero

$$W = X_{11}q_{12}q_{21} - \mu_1^2 X_{11} - hq_{12}q_{23}q_{32}q_{21} + hm_1^2 q_{12}q_{21} + hm_3^2 q_{32}q_{23} + X_{33}q_{32}q_{23} - \mu_3^2 X_{33}\,. \tag{2.230}$$

We keep the more general situation arising from the geometries analyzed here. That is the adjoint fields have linear terms and the quarks have masses generally different from the deformations of the adjoint field. The choice of the ranks for the gauge groups is

$$N_1 = N_3 = N + M \qquad N_2 = N \tag{2.231}$$

with $2M > N$ and so we are guaranteed that the second node is infrared free. We consider this infrared free group as the most strongly coupled.

Solving the equation of motion and expanding around the tree level minimum we have

$$
\begin{array}{lll}
q_{12} = \begin{pmatrix} \mu_1 + \sigma_1 \\ \phi_1 \end{pmatrix} & q_{21} \begin{pmatrix} \mu_1 + \sigma_2 & \phi_2 \end{pmatrix} & X_{11} = \begin{pmatrix} h(\mu_3^2 - m_1^2) + \sigma_3 & \phi_3 \\ \phi_4 & \chi_1 \end{pmatrix} \\
q_{32} = \begin{pmatrix} \mu_3 + \sigma_5 \\ \phi_5 \end{pmatrix} & q_{23} \begin{pmatrix} \mu_3 + \sigma_6 & \phi_6 \end{pmatrix} & X_{33} = \begin{pmatrix} h(\mu_1^2 - m_3^2) + \sigma_7 & \phi_7 \\ \phi_8 & \chi_2 \end{pmatrix}
\end{array}
$$

where χ_1 and χ_2 are the pseudomoduli. The superpotential for the supersymmetry breaking sector is

$$
\begin{aligned}
W &= \chi_1\phi_1\phi_2 - \mu_1^2\chi_1 + \mu_1(\phi_1\phi_4 + \phi_2\phi_3) - h(\mu_3^2 - m_1^2)\phi_1\phi_2 + \\
&+ \chi_2\phi_5\phi_6 - \mu_3^2\chi_2 + \mu_3(\phi_5\phi_8 + \phi_6\phi_7) - h(\mu_1^2 - m_3^2)\phi_5\phi_6
\end{aligned} \tag{2.232}
$$

and it consists in two O'Raifeartaigh like models after shifting the pseudomoduli as $\chi_1' = \chi_1 - h(\mu_3^2 - m_1^2)$ and $\chi_2' = \chi_2 - h(\mu_1^2 - m_3^2)$. Hence the pseudomoduli are stabilized at $\chi_1' = \chi_2' = 0$ such that the non supersymmetric vacuum at quantum level is where the mesons X_{11} and X_{33} are proportional to the identity.

Meta-stable vacua in L^{aaa} theories

In the case $a = b$, i.e. L^{aaa} the theory does not posses adjoint matter, since $b - a = 0$. Nevertheless, by performing Seiberg dualities, we can create the necessary adjoint fields. As explained above, this procedure does not affect the geometry, which is of the form

$$x^a y^a = wz\,. \tag{2.233}$$

We can then add the deformations for the adjoint fields and obtain theories suitable for metastable supersymmetry breaking.

Once again the strategy to analyze a long quiver consists in breaking it up in a set of shorter quivers, each one with metastable vacua.

We study in detail the simplest example, L^{222}, and then we comment on possible generalizations.

The L^{222} theory

We analyze here the L^{222} theory after a Seiberg duality. The quiver of the complete theory (see figure 2.22) is related to the double conifold. The superpotential is

$$W = -q_{21}X_{11}q_{12} + hq_{12}q_{23}q_{32}q_{21} - q_{23}X_{33}q_{32} + q_{41}X_{11}q_{14} - hq_{14}q_{43}q_{34}q_{41} + q_{43}X_{33}q_{34} \tag{2.234}$$

and the geometry is given by the equation

$$x^2 y^2 = wz\,. \tag{2.235}$$

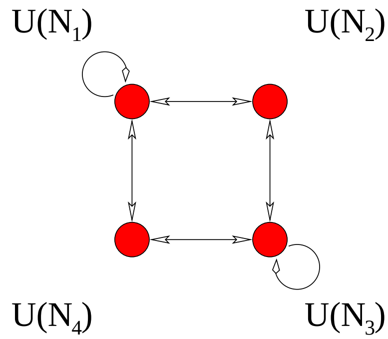


Figure 2.22: The L^{222} quiver without any node set to zero

We deform the geometry

$$x(y-\xi_1)y(x-\xi_3)=wz\,. \tag{2.236}$$

This deformation changes the constraints on the mesonic chiral ring. The new constraints can be satisfied by adding in W two linear terms of the form $\xi_1 X_{11}$ and $\xi_3 X_{33}$, and we have to switch on also two mass terms in the quarks fields. Setting to zero one node, we can have the three different cases, as shown in figure 2.23. They all have metastable

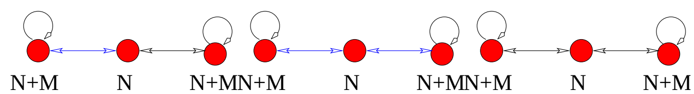


Figure 2.23: Three different quivers from the deformed L^{222}. The massive quarks are represented with blue lines, the massless quarks are represented with black lines.

vacua in the correct regime of couplings, ranks and scales.

These models are similar to ISS, but with two differences: the quartic term for the quarks and the mass term for some of the quarks.

We study here the case with only one group of massive quarks (the first case in the figure 2.23), and then we comment on the other at the end of this paragraph.

We choose the ranks of the groups to be

$$N_2=N \qquad N_1=N+M=N_3\,. \tag{2.237}$$

The second node is treated as the gauge group and the other two nodes as flavours. The superpotential is

$$W=-(\xi_1 X_{11}+\xi_3 X_{33})-q_{21}X_{11}q_{12}+hq_{12}q_{23}q_{32}q_{21}-q_{23}X_{33}q_{32}+h\xi_1 q_{32}q_{23}\,. \tag{2.238}$$

We then solve the equations of motion for the various fields, recognizing the ISS rank condition, responsible for breaking of supersymmetry. The F-terms fix the vacuum to be

$$q_{12}=q_{21}^T=\begin{pmatrix}\sqrt{\xi_1}\\ 0\end{pmatrix} \qquad q_{32}=q_{23}^T\begin{pmatrix}\sqrt{\xi_3}\\ 0\end{pmatrix} \qquad X_{11}=\begin{pmatrix}0 & 0\\ 0 & \chi_1\end{pmatrix} \qquad X_{33}=\begin{pmatrix}\xi_1 & 0\\ 0 & \chi_3\end{pmatrix}. \tag{2.239}$$

We show that this vacuum is stable up to one loop corrections, fixing the pseudomoduli to $\langle \chi_1 \rangle = 0$ and $\langle \chi_3 \rangle = \xi_1$. The two breaking sectors are separated at the one loop level, and their quantum corrections are as in ISS. The ξ_i deformations have thus lead to supersymmetry breaking vacua.

The strong dynamics of the gauge group restores supersymmetry, and is geometrically described by the ϵ deformation

$$(x-\xi_1)\,(x(y-\xi_3)-\epsilon)\,y = wz\,. \tag{2.240}$$

In the field theory analysis we explore the large field region for the mesons, by integrating out the massive fields, and by taking into account the non perturbative contributions due to gaugino condensation. The low energy superpotential results

$$W_{IR} = N\left(\Lambda^{-2M-N}\det X_{11}\det X_{33}\right)^{\frac{1}{N}} - (\xi_1 \mathrm{Tr} X_{11} + \xi_3 \mathrm{Tr} X_{33}) \tag{2.241}$$

which guarantees the long life of the vacuum[16].

On the other hand, we can use the geometric techniques of Appendix A.4 to recover the same low energy superpotential (2.241) from the geometry (2.240). Relabeling the variables in (2.240) by $y = (x'-y')$ and $x = (x'+y')$ we can rewrite

$$(x'-y'-\xi_1)\,((x'-y')((x'+y'-\xi_3)-\epsilon) = wz\,. \tag{2.242}$$

The geometric superpotential is

$$W_{IR}(S, X_{11}, X_{33}) = N_2 S\left(\log\frac{S}{\Lambda_m^3}-1\right) - \frac{t}{g}S + W_{adj}(S, X_{11}) + W_{adj}(S, X_{33})\,. \tag{2.243}$$

The two contributions W_{adj} derive from the singularities of the geometry. Repeating the computations as in Appendix A.4 we have

$$\begin{aligned} W_{adj}(S, X_{11}) = &\quad \int\left(y' + \xi_1 - \frac{\xi_3}{2} - \sqrt{\left(y'-\frac{\xi_3}{2}\right)^2 + \epsilon}\right)dy' \\ W_{adj}(S, X_{33}) = &\quad \int\left(\frac{\xi_3}{2} - \sqrt{\left(y'-\frac{\xi_3}{2}\right)^2 + \epsilon} + y'\right)dy'\,. \end{aligned} \tag{2.244}$$

In the previous integral we identify $2S$ with ϵ and y' with the vev of the adjoint fields X_{11} and X_{33} respectively. In the regime $y' \gg \epsilon, \xi_i$ we can compute the integrals expanding at first order in ϵ and ξ_i, obtaining the superpotential for the interaction between the glueball field and the adjoint fields

$$W_{adj}(S, X_{11}) + W_{adj}(S, X_{33}) = \xi_1 \mathrm{Tr} X_{11} + \xi_3 \mathrm{Tr} X_{33} - S\log\det\left(\frac{X_{11}}{\Lambda_m}\right) - S\log\det\left(\frac{X_{33}}{\Lambda_m}\right)\,. \tag{2.245}$$

[16]The restoration of supersymmetry in the other cases in figure 2.23 follows directly.

The equation for the glueball field S can be derived from (2.243) and (2.245). Solving for S and ignoring the multi-istanton contributions we have

$$S = \left(\Lambda_m^{3N_2-N_1-N_3} \det X_{11} \det X_{33}\right)^{\frac{1}{N_2}} = \left(\Lambda_m^{-N-2M} \det X_{11} \det X_{33}\right)^{\frac{1}{N}} . \tag{2.246}$$

Substitution of (2.246) in (2.243) gives the same low energy superpotential of field theory (2.241), up to an overall sign.

The ϵ deformation has lead to supersymmetry restoration.

The L^{333} theory

Here we search for metastable vacua in an L^{333} theory, after performing on it some Seiberg dualities. This theory has six nodes without adjoint fields, with superpotential

$$W = \sum_{i=1}^{4} (-1)^i h q_{i,i+1} q_{i+1,i+2} q_{i+2,i+1} q_{i+1,i} - h q_{56} q_{61} q_{16} q_{65} + h q_{61} q_{12} q_{21} q_{16} . \tag{2.247}$$

A Seiberg duality on the sixth node and integration out of the massive matter. leads to the superpotential

$$\begin{aligned} W = \quad & -q_{61} X_{11} q_{16} + q_{21} X_{11} q_{12} - h q_{12} q_{23} q_{32} q_{21} + h q_{23} q_{34} q_{43} q_{32} - h q_{34} q_{45} q_{54} q_{43} \\ & + q_{45} X_{55} q_{54} - q_{65} X_{55} q_{56} + h q_{56} q_{61} q_{16} q_{65} \end{aligned} \tag{2.248}$$

with the quiver given in figure 2.24. The geometry is then deformed by the ξ_i terms to

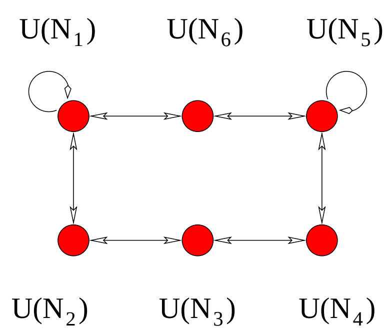


Figure 2.24: The L^{333} theory after a Seiberg duality on node 6.

$$x^2 y^2 (y - \xi_1)(x - \xi_5) = wz . \tag{2.249}$$

This deformation corresponds in the field theory to linear terms $\xi_1 X_{11}$ and $\xi_5 X_{55}$ in the superpotential. For consistency with the F-term constraints, we also add some mass term for the bifundamentals, i.e.

$$\Delta W = -\xi_1 X_{11} + \xi_5 X_{55} + h\xi_1 q_{23} q_{32} - h\xi_5 q_{43} q_{34} . \tag{2.250}$$

We set the ranks of the groups as follows

$$N_1 = N_5 = N + M \qquad N_2 = N_4 = N \qquad N_3 = N_6 = 0\,. \tag{2.251}$$

We then obtain two decoupled ISS like models that break supersymmetry through rank conditions for the mesons X_{11} and X_{55}.

The supersymmetric vacua can be recovered by adding the non perturbative contributions arising for each gauge group. From the geometry, restoration of supersymmetry can be described by the ϵ_i deformations

$$xy(x(y-\xi_1)+\epsilon_1)((x-\xi_5)y-\epsilon_2) = wz\,. \tag{2.252}$$

This deformed geometry gives, with the techniques of Appendix A.4, the right low energy superpotential that leads to the supersymmetric vacua.

Extension

We now briefly outline a procedure for finding metastable vacua in a generic L^{aaa} theory.

The strategy again consists in breaking the quiver into a set of shorter quivers, each one metastable.

We study a phase of the theory, derived by acting with Seiberg dualities, which has a number of a adjoint fields if a is even and $a-1$ if a is odd.

We then set to zero the right nodes[17] in order to obtain a set of decoupled theories that have the same structure of the deformed L^{222} and L^{333} studied above. This can be done choosing appropriately the Seiberg phases.

We now show how to proceed in a simple example, L^{555} in figure 2.25. We perform

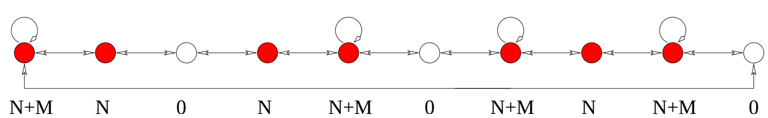


Figure 2.25: Quiver for the deformed L^{555} theory.

Seiberg dualities on the sixth and on the tenth node and obtain 4 adjoint fields. We then deform the geometry in such a way that, in the field theory description, all the adjoint fields get linear terms

$$x^3y^3(y-\xi_1)(x-\xi_5)(y-\xi_7)(x-\xi_9) = wz\,. \tag{2.253}$$

Indeed, this deformation give rise to linear terms for all the adjoint fields, and masses for some of the quarks. The new superpotential contribution is [18]

$$\Delta W = \xi_1 q_{23}q_{32} + \xi_5 q_{34}q_{43} + \xi_7 q_{56}q_{65} + \xi_9 q_{110}q_{101} - \xi_1 X_{11} - \xi_5 X_{55} - \xi_7 X_{77} - \xi_9 X_{99}\,. \tag{2.254}$$

[17] We set to zero only not consecutive nodes.

[18] Other choices for the masses of the quarks are possible, and all of them lead to metastability.

We now set to zero the third, the sixth and the tenth node. In this way we decompose the theory in three different metastable sectors. The first two sectors have the same structure of L^{333}, whereas the last sector is like the theory emerging from a L^{222}. In short we have decomposed the L^{555} as L^{222} and L^{333}.

Supersymmetry restoration is achieved in each sector separately. From the geometric point of view this transition is read as an ϵ deformation of (2.253) to

$$x^2y^2((y-\xi_1)x-\epsilon_1)(y(x-\xi_5)-\epsilon_2)((y-\xi_7)(x-\xi_9)-\epsilon_3)=wz \tag{2.255}$$

where the three ϵ_i take into account the deformations on the moduli space imposed by the strong dynamics of the three groups that we considered as gauge groups. Using the geometric techniques of Appendix A.4 it is possible also in this case to recover the correct low energy behavior in the supersymmetric vacua. The three different deformation parameters ϵ_i are interpreted as the three glueball fields of the three gauge groups.

Back to L^{aba}

Up to now we have found metastable vacua in all the L^{aaa} theories (with $a>1$) and in L^{aba} with the constraint $b-a>a$. The study of the L^{aaa} theories gives us a way out from the constraints imposed on L^{aba}. If we have an L^{aba} theory with $b-a<a$ we have indeed to look for a different Seiberg phase. Given an L^{aba} theory one can find a dual theory with at most $b+a-2$ adjoint fields, instead of $b-a$.

We proceed in a simple example: the L^{343} theory. A Seiberg duality on the fourth node gives the quiver in figure 2.26. By adding a linear deformation for each adjoint field,

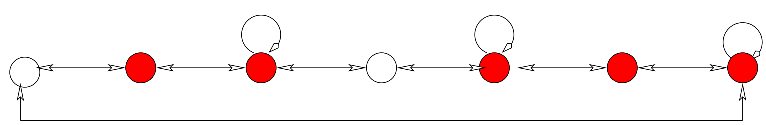


Figure 2.26: L^{343} theory with a Seiberg duality on the fourth node.

the geometry becomes

$$x^2y^2(y-\xi_1)(x-\xi_2)(y-\xi_3)=wz\,. \tag{2.256}$$

We can now set some node to zero and obtain a set of decoupled theories with a metastable IR behavior. A possible choice is shown in figure 2.26, where we set to zero the white nodes. We have broken up L^{343} theory in two sectors: the first one has the same property of metastability of L^{121}, and the second one of L^{222}, the double conifold.

Supersymmetry is restored by the geometric ϵ deformation [19]

$$xy((y-\xi_1)x-\epsilon_1)(x-\xi_2)(xy-\epsilon_2)(y-\xi_3)=wz \tag{2.257}$$

through the strong dynamics of the gauge group in each decoupled sector.

[19]Note that in this case we chose massless quarks in the last two lines of the quiver.

Beyond the L^{aba} cases

In the previous paragraphs we performed metastable supersymmetry breaking in the family of L^{aba} singularities. An immediate generalization is the embedding of L^{aba} in larger singularities and the recovering of metastable dynamics in the IR.

We need to start with a UV quiver gauge theory and flow by way of the renormalization group to a set of gauge theories with fewer gauge groups. These theories are decoupled at low energy, and they keep at least one L^{aba} singularity. These singularities trigger metastability in the IR.

In the RG flow to the IR two different decouplings are possible, the resolution and the deformation of the mother singularity.
Blowing up two spheres gives first the resolution of the mother singularity. The daughters singularities are geometrically separated by the volume of these two spheres. This corresponds to the motion in the Kahler moduli space of the singularities.
The second one, the deformation, is achieved by blowing-up three spheres. Here the singularities are separated by the volume of the three spheres.
In both cases the IR theories decouple at the level of massless states and the masses of the messenger fields are controlled by the volume of the two and three spheres respectively.
We now describe these two possibilities by proceeding with pictures and examples.

The graphical resolution of a singularity in the toric language corresponds to drawing a line in the toric diagram (the red line in our figures) and a perpendicular line in the dual diagram (the dashed line). This last line parametrizes the volume of the two sphere (see Figure 2.27).

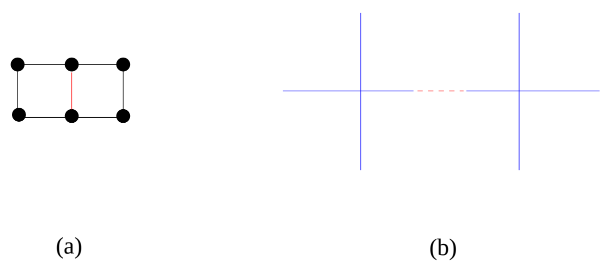


Figure 2.27: The toric resolution of the double conifold: L^{222}. (a) The toric diagram representation, (b) the dual diagram: the broken red arrow parametrize the volume of the blown up two sphere.

A natural laboratory for these constructions is the family of Pseudo del Pezzo singularities PdP_n. These are complex cones over $\mathbb{P}^2$, blown up at n non-generic points. This blowing up generates lines of singularities passing trough the tip of the cones (Figure 2.28).

In the PdP_4 and PdP_5 singularities it is possible to recover two of the singularities that show a metastable behavior, L^{121} (SPP) and L^{222} (double conifold), through the resolution of the singularities as shown in Figure 2.29. We first assign a set of fractional

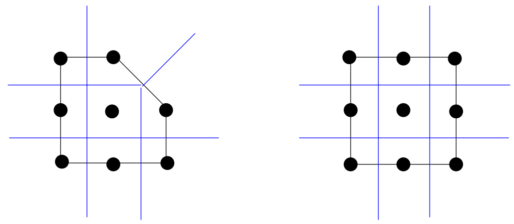


Figure 2.28: The toric diagrams and the dual diagrams for (a) PdP_4 and (b) PdP_5.

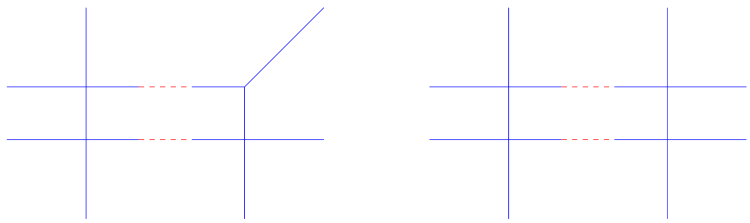


Figure 2.29: Resolutions of (a) PdP_4 and (b) PdP_5.

branes to the mother singularity such that it reproduces, at least for one of the daughter singularities, the set of fractional branes that has metastable non supersymmetric vacua. We turn then on Kahler moduli deformations, decoupling in the IR one L^{121} and one L^{222} singularities from the PdP_4. For the PdP_5 singularity we can decouple two L^{222} singularities. In each situation the two decoupled IR theories are separated at the level of massless states [20]. Finally, metastable supersymmetry breaking can be realized, since we can deform the A_1 singularities belonging to one or to both the IR theories.

The other possibility for the decoupling of a mother singularity is the deformation (see section 2.4 for a graphical description). It furnishes a second embedding of L^{121} and L^{222} into PdP_4 and PdP_5. These configurations are described in Figure 2.30.

We have to distribute the fractional branes at the mother singularity in such a way that they lead to the complex moduli deformation. Gaugino condensation is then induced by the strong dynamics of some gauge groups. This decoupling leads to the remaining daughter singularities in the IR, and, in this case, we are left with L^{121} and L^{222}. We can move in the complex moduli space deformations of the non local singularity, reproducing the supersymmetry breaking behaviour of the L^{aba} theories.
The advantage of this procedure is that the moduli associated with the volumes of the three spheres are automatically stabilized by the strong IR gauge dynamics. The draw-

[20]As discussed in [72] using Kahler moduli space deformations it is possible to compute the mass of the "messenger particles" but the Kahler moduli remain free parameters to be stabilized in some way.

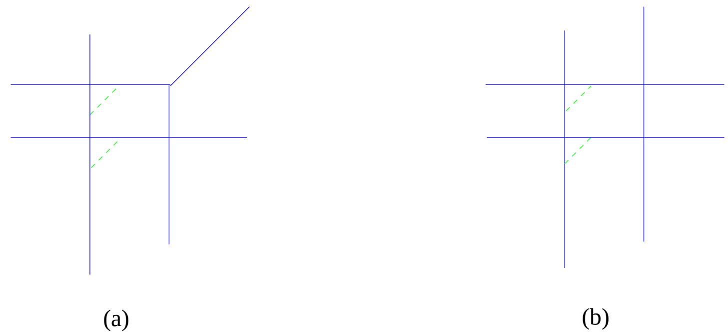


Figure 2.30: Deformations of (a) PdP_4 and (b) PdP_5.

back is that the computation of the masses of the messenger sector is not straightforward.

Following the two procedures explained in this section and the methods developed in [72] many examples, useful for model building, can be studied.

There exist conical singularities that provide extensions of MSSM as the IR limit of the dynamics of D3 branes put at the tip of the cone. The easiest example is given by $D3$ branes at dP_0 singularity.

Here, by using either Kahler moduli deformations or complex moduli deformations, it is possible to separate a singularity into a dP_0 sector and some L^{aba} sector. In the IR, dP_0 is an extension of the MSSM, L^{aba} is the hidden supersymmetry breaking sector, and the massive fields are the messengers. It is possible to find many examples of singularities that, after the resolution, decouple in a MSSM like sector and in a hidden supersymmetry breaking sector, also metastable. We show here two possibilities.

The first one, in Figure 2.31, admits a resolution that decouples in the IR a dP_0 and two SPP singularities. The dP_0 plays the role of phenomenological sector, while the two SPP singularities play the role of supersymmetry breaking hidden sectors. The second one, in figure 2.32, admits a complex deformation. It decouples a dP_0 sector and a single SPP sector. In this section we discussed the geometric interpretation of metastable vacua for systems of D3 branes at non isolated deformable toric CY singularities. We have generalized the analysis done in [10] to the infinite family of L^{aba} singularities and we have proposed the embedding of these theories in bigger singularities. The dynamical generation of the ξ deformation which sets the scale of the supersymmetry breaking is still an open problem. Since much is known about the metric of the L^{aba} spaces, another challenging question regards metastability in the gauge/gravity correspondence. The models here studied may play the role of hidden sector in mechanisms of gauge mediation of supersymmetry breaking [73] in metastable vacua [74].

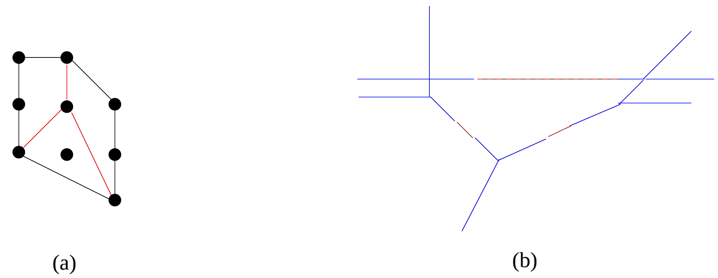


Figure 2.31: The resolved toric diagram (a) and the dual diagram (b). The triangle at the bottom is the dP_0 singularity that represents the "visible sector", the polygon on the top are two decoupled SPP singularities that represent the supersymmetry breaking sector.

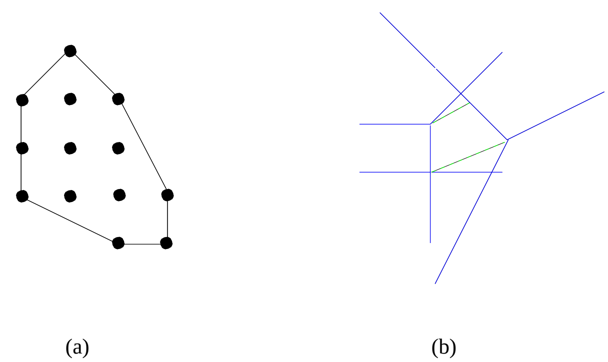


Figure 2.32: (a) The toric diagram of the mother singularity and (b) the deformed dual diagram that contain the dP_0 visible sector and the SPP supersymmetry breaking sector.

2.5 Metastable vacua at two loop

In section 2.1 we discussed the relation between supersymmetry breaking and spontaneous R-symmetry breaking. We reviewed the known results at tree level and at one loop. Here we propose a mechanism for spontaneous R-symmetry breaking in supersymmetry breaking vacua through two loop effects. In [28] two loop corrections were shown to destabilize an R-charged field at the origin of the pseudomoduli space. Then the addition of a small tree level effect stabilize this field in the large vev region, breaking R-symmetry.

Here we look for models with spontaneous R-symmetry breaking at two loop. This breaking occurs when an R-charged field gets a vev only from the two loop effective potential. We show that different couplings in the superpotential lead to different signs for the two loop mass. The strategy is to combine these contributions to give non zero vev to the R-charged field.

I	J	m_Z^2	
1	4	$h^6\mu^2(\log 4 - 1 - \frac{\pi^2}{6})$	< 0
1	5	$h^6\mu^2(\log 4 - 1)$	> 0
2	4	$h^6\mu^2(\log 2 - 1 + \frac{\pi^2}{12})$	> 0
2	5	$h^6\mu^2(\log 2 - 1)$	< 0

Table 2.1: Two loop squared mass for Z.

One loop flat directions

Here we present the class of models we consider through this section. They are theories of pure chiral fields with canonical Kahler potential and with a renormalizable superpotential. We study two loop corrections in models with spontaneous breaking of supersymmetry. The most natural way consists in coupling an O'Raifeartaigh sector to another bunch of fields through three-linear couplings. This implies that the one loop corrections lift the potential for the O'Raifeartaigh field but do not lift pseudomoduli space of the other sector. The superpotentials we consider are

$$W = h\left(-\frac{f}{2}X + X\phi_1^2 + \mu\phi_1\phi_2 + \phi_I\rho\xi_J + Z\xi_4^2 + \mu\xi_4\xi_5\right) \tag{2.258}$$

where $I = 1, 2$ and $J = 4, 5$.

The supersymmetry breaking vacuum is at the origin of the moduli space. The fields ϕ_2 and ξ_4 and ξ_5 have positive squared mass $h^2\mu^2$. The field ϕ_1 splits its mass in $h^2\mu^2 \pm h^2 f$, for its real and imaginary component. The other fields are pseudomoduli. The pseudomodulus X is stabilized at one loop at the origin. The pseudomodulus ρ is also stabilized at one loop at the origin, when $I = 1$, i.e. when it is directly coupled in the three-linear term with the field ϕ_1 which has a mass splitting. For the case with $I = 2$ we add a mass term for the field ρ, to avoid tachyons

$$\Delta W_{I=2} = m\rho^2 \tag{2.259}$$

with $m \ll \mu$.

The pseudomodulus Z is not lifted at one loop and a two loop analysis is required. In the Appendix A.6 we give the details of the calculation. We summarize in Table 1 the results for the two loop mass of the field Z, at order $o(m)$ for the cases $I = 2$. The model $(I, J) = (1, 4)$ gives the same result than [28]. In fact it is the same model of pure chiral fields. Then, the explicit calculation shows that the model with $(I, J) = (2, 5)$ has a runaway behaviour, while the two loop potential for the Z field in the models $(I, J) = (1, 5)$ and $(I, J) = (2, 4)$ has a stable minimum at the origin and the potential increases in the large field region.

The model $(I, J) = (1, 4)$ has the bad behaviour discussed in the surveying of [34]. Methods of [34] can be generalized for the other three models as well. For the case with $I = 4$ the beta function of the mass term $\mu\phi_1\phi_2$ has to be taken into account. Moreover,

	X	ϕ_1	ϕ_2	ϕ_4	ϕ_5	ϕ_6	ϕ_7	ϕ_8	Z
$U(1)_R$	2	0	2	3	-1	1	1	3	-4
Z_2	0	π	π	0	0	0	π	π	0
Z_2	0	π	π	π	π	π	0	0	0
Z_2	0	0	0	π	π	π	π	π	0

Table 2.2: $U(1)_R$ and Z_2 charges of the fields.

in the cases with $J = 5$, the field ξ_5 does not decouple at large Z, and the term $\phi_I \rho \xi_J$ affects the effective potential for long RG time (i.e. large Z). In all the cases this analysis gives the same qualitative behaviour than our explicit computation[21].

One can also notice that under the exchange $\xi_4 \leftrightarrow \xi_5$ the quadratic mass for the Z field changes sign. This change corresponds to an opposite R-symmetry charge for the field Z. Connecting the behaviour of the two loop potential for Z with its R-charge is an interesting question that we leave for future investigation.

Breaking R-symmetry at two loop

In Table 2.1 we observe that masses of different signs are related to different three-linear couplings between the ϕ_i and the ξ_i sector. Combining these contributions we can generate a two loop potential for the field Z which stabilizes it, but not at the origin. In the following we study a model with several three-linear couplings. The field Z acquires a non trivial vev $\langle Z \rangle \neq 0$ in the true quantum minimum at two loop. The model has a tree level R-symmetry, and the field Z has a non trivial R-charge, then the R-symmetry is spontaneously broken by the two loop corrections.

The basic example

In this section we present the model that breaks supersymmetry and perturbatively R-symmetry at two loop. The superpotential is

$$W = -h\frac{f}{2}X + hX\phi_1^2 + h\mu\phi_1\phi_2 + h\alpha\phi_1\phi_7\phi_6 + h\beta\phi_1\phi_8\phi_5 + h\gamma\phi_2\phi_7\phi_5 + h\, Z\phi_4^2 + h\mu\phi_4\phi_5 + h\mu\phi_6^2 \quad (2.260)$$

where h is a marginal coupling, and α, β and γ are numerical constants. All the couplings can be made real with a phase shift of the fields.

We give in Table 2.2 the R charges and the Z_2 discrete symmetries. These global symmetries and renormalizability constraints the theory to the form (2.260), except for three terms $Z\phi_8^2$, $\mu\phi_7^2$, $X\phi_5\phi_6$. In the limit $f \to 0$ the theory admits a $U(1)$ global symmetry which forbids the terms $Z\phi_8^2$, $\mu\phi_7^2$. The term $X\phi_5\phi_6$ has to be tuned to zero. It cannot be forbidden even introducing global symmetries involving the couplings, to be

[21] We are grateful to Ken Intriligator for explaining us how to analyze these cases with the techniques of [34].

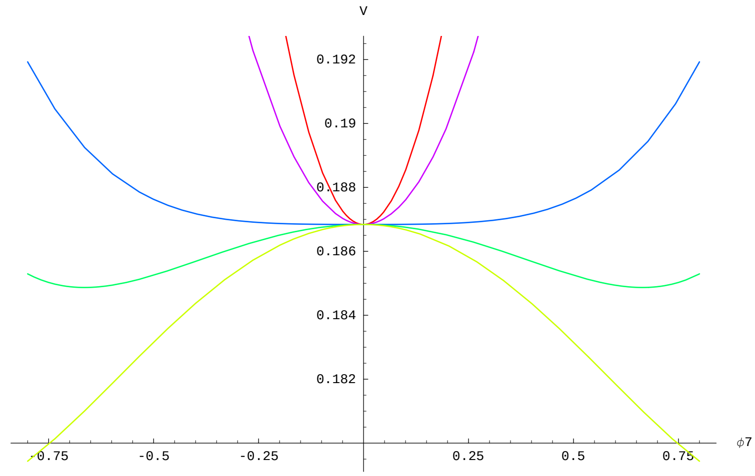


Figure 2.33: Scalar potential for ϕ_7 at the origin. The ratio α/γ is respectively 2, 1, 0.9, 0.8 and 0.7 from the red to the yellow curve.

thought as spurion fields. A possible solution to this tuning is discussed at the end of this section.

There is a supersymmetry breaking vacuum at the origin of the moduli space. Around this vacuum the fields ϕ_2, ϕ_4, ϕ_5 and ϕ_6 have positive squared mass $h^2\mu^2$. The field ϕ_1 splits its mass in $h^2\mu^2 \pm h^2 f$, which are both positive for $y = |f/\mu^2| < 1$. The other fields are pseudomoduli, and their squared mass spectrum has to be analyzed by looking at the loop expansion of the scalar potential.

One loop corrections

The one loop corrections lift the X and ϕ_8 directions and set $\langle X\rangle = 0$ and $\langle\phi_8\rangle = 0$, with positive squared masses

$$m_X^2 = \frac{16h^4}{f}\left(-2\mu^2 f - 8\mu^2 f\log[h\mu] - (\mu^2 - f)^2\log[h^2(\mu^2 - f)] + (\mu^2 + f)^2\log[h^2(\mu^2 + f)]\right)$$
$$m_{\phi_8}^2 = \frac{|\beta|^2}{4}m_X^2 \tag{2.261}$$

The fields ϕ_7 is also stabilized at the origin but this direction can develop a runaway behaviour to be analyzed. First note that this pseudomoduli space is stable for

$$|\langle\phi_7\rangle| < \frac{\mu}{\gamma}\sqrt{\frac{1-y}{y}} \tag{2.262}$$

Figure 2.33 then shows for which values of the ratio α/γ the one loop mass of ϕ_7 is positive, after fixing $f/\mu^2 = 0.5$ (all the other choices with $y < 1$ are possible). We choose the ratio α/γ to stabilize the field ϕ_7 at the origin. For ϕ_7 larger than (2.262) the theory has a runaway behaviour, parametrized by $\phi_7 \to \infty$,

$$\phi_1 \sim \sqrt{\frac{f}{2}} \qquad \phi_2 \sim \sqrt{\frac{f}{2}}\frac{\alpha^2}{2\gamma\mu^2}\phi_7^2 \qquad \phi_4 \sim 0 \qquad \phi_5 \sim \sqrt{\frac{f}{2}}\frac{\mu}{\gamma}\frac{1}{\phi_7} \qquad \phi_6 \sim \sqrt{\frac{f}{2}}\frac{\alpha}{2\mu}\phi_7$$
$$\phi_8 \sim \frac{\alpha^2}{2\beta\mu^2}\phi_7^3 \qquad X \sim \frac{\alpha^2}{2\gamma\mu}\phi_7^2 \qquad Z \sim \sqrt{\frac{f}{2}}\frac{\mu^2}{\gamma}\frac{1}{\phi_4\phi_7} \tag{2.263}$$

Two loop corrections

The potential for the field Z is not lifted at one loop, and a two loop analysis is necessary. Considering Z as a background field, the masses of ϕ_4 and ϕ_5 mix. We diagonalize the fermionic mass matrix for these two fields. The rotation is

$$\begin{aligned} \phi_4 &= -s\theta\rho_4 + c\theta\rho_5 \\ \phi_5 &= c\theta\rho_4 + s\theta\rho_5 \end{aligned} \tag{2.264}$$

where

$$s\theta^2 = \frac{h^2\mu^2 - \lambda_-^2}{\lambda_+^2 - \lambda_-^2} \tag{2.265}$$

and

$$\lambda_\mp^2 = \frac{h^2}{2}\left(Z^2 + 2\mu^2 \mp Z\sqrt{Z^2 + 4\mu^2}\right) \tag{2.266}$$

The contributions to the two loop effective potential for Z are computed with the same strategy of [28], which is reviewed in Appendix A.6. The three contributions are given in Figure A.6 and are called V_{SS}, V_{SSS} and V_{FFS}. We found

$$\begin{aligned} V_{SS} &= h^2\beta^2 c\theta^2\left(f_{SS}(h^2(\mu^2 - f), \lambda_-^2) + f_{SS}(h^2(\mu^2 + f), \lambda_-^2) - 2f_{SS}(h^2\mu^2, \lambda_-^2)\right) \\ &\quad + \left(c\theta^2, \lambda_- \leftrightarrow s\theta^2, \lambda_+\right) \\ \\ V_{SSS} &= h^4\mu^2(s\theta^2\beta^2 + c\theta^2\gamma^2)(f_{SSS}(0, h^2(\mu^2 - f), \lambda_-^2) + f_{SSS}(0, h^2(\mu^2 + f), \lambda_-^2) \\ &\quad -2f_{FSS}(0, h^2\mu^2, \lambda_-^2)) + \left(s\theta^2, c\theta^2, \lambda_- \leftrightarrow c\theta^2, s\theta^2, \lambda_+\right) \\ \\ V_{FFS} &= h^2(\beta^2 c\theta^2)(f_{FSS}(0, \lambda_-^2, h^2(\mu^2 - f)) + f_{FSS}(0, \lambda_-^2, h^2(\mu^2 + f)) \\ &\quad -2f_{FSS}(0, \lambda_-^2, h^2\mu^2)) + \left(c\theta^2, \lambda_- \leftrightarrow s\theta^2, \lambda_+\right) \end{aligned} \tag{2.267}$$

Expanding the two loop effective potential for small Z, the mass term at the origin is

$$m_Z^2 = h^6\mu^2(\beta^2 f(\tau^2) - \gamma^2 g(\tau^2)) \tag{2.268}$$

where

$$\tau^2 = f/\mu^2 \tag{2.269}$$

$$\begin{aligned} f(x) &= -2 - \frac{(1-x)^2}{x}\log(1-x) + \frac{(1+x)^2}{x}\log(1+x) \\ g(x) &= 2 + \frac{(1-x)}{x}\log(1-x) - \frac{(1+x)}{x}\log(1+x) \end{aligned} \tag{2.270}$$

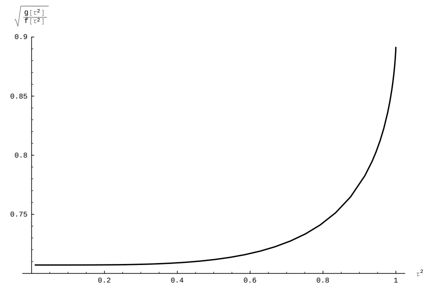


Figure 2.34: Function $\sqrt{\frac{g(\tau^2)}{f(\tau^2)}}$. The region of interest is the one below the curve.

and the functions $f(x)$ and $g(x)$ are positive for $x < 1$. There is a regime of the parameters for which this mass term is negative. This happens in the region where

$$\frac{\beta}{\gamma} < \sqrt{\frac{g(\tau^2)}{f(\tau^2)}}$$

as in Figure 2.34. In such a regime of the parameter we look for a minimum of the two loop scalar potential. We indeed observe in Figure 2.35, by plotting the scalar potential, that there is a choice of the ratio β/γ where the scalar potential has a minimum at $\langle Z \rangle \neq 0$.

We can then conclude that the model (2.260) spontaneously breaks R-symmetry at two loop in a non supersymmetric (metastable) vacuum. All the tree level flat directions of the scalar potential are lifted by quantum effects. The vacuum is metastable because the field ϕ_7, which acquires positive squared mass around the origin through one loop corrections, develops a runaway in the large vev region. The effective potential for ϕ_7 has to be analyzed to estimate the lifetime of the vacuum. We found a supersymmetry breaking model in which R-symmetry is spontaneously broken at two loop in the scalar potential. It is a model of pure chiral fields without any gauge symmetry. There is a tuning in the superpotential, since we did not consider all the terms invariant under the global symmetries of the theory. Adding the allowed term should spoil some of the infrared properties, i.e. supersymmetry breaking.

The tuning problem can be solved by embedding the superpotential in a quiver gauge theory. In this case the pure chiral fields model has to be considered as the effective theory around the non supersymmetric vacuum found at tree level in the gauge theory, as in [2]. This embedding might also stabilize the runaway behavior in the large field region, where strong dynamics effects of the gauge groups add non perturbative terms to the superpotential.

Moreover, this two loop analysis can be applied to many models with metastable vacua. In most of them an approximate R-symmetry exists at such vacua. Two loop effects can offer a solution for this problem. Indeed, as in the model we studied, we can couple the theory to an R-charged pseudomodulus that receives two loop corrections from the supersymmetry breaking sector. This field can acquire a quantum scalar potential that breaks spontaneously R-symmetry.

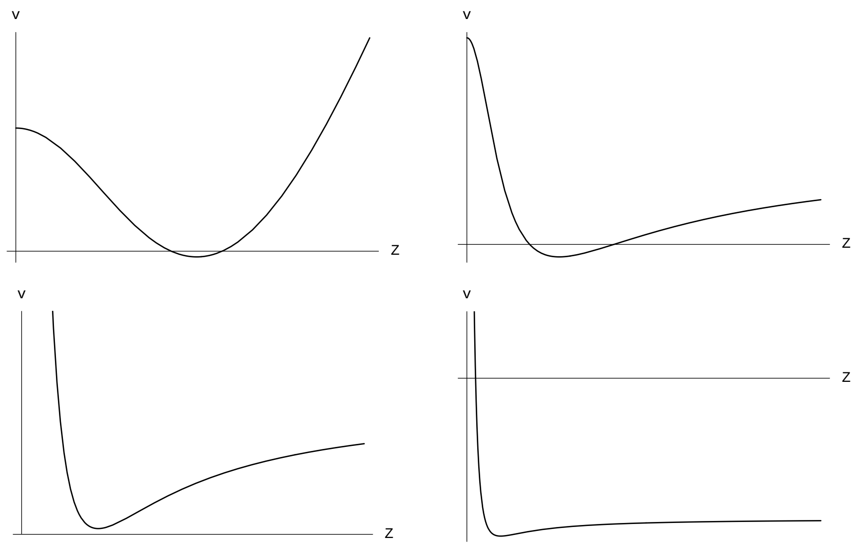


Figure 2.35: Scalar potential for Z, plotted for different values of the ratio β/γ, respectively 0.7, 0.65, 0.6 and 0.55, from left to right. The ration f/μ^2 has been chosen to be 0.5

Another possibility is to build a model with a "tension" between the one loop and the two loop contributions for some pseudomoduli. This competition could shift the minimum from the origin, breaking R-symmetry. In [28] the one and two loop corrections in the ISS model with a mass hierarchy among the fundamental fields have been studied. However in that case one can check that the quantum corrections lead to a runaway, without any local minimum, and then restore supersymmetry. It would be interesting to find models where the combination of one loop and two loop quantum corrections lead to metastable vacua.

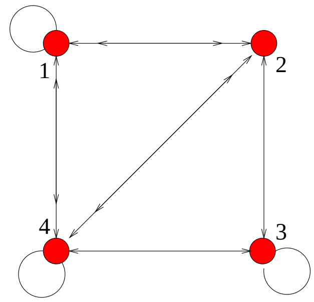


Figure 2.36: Quiver for the superpotential (2.271).

Embedding in quiver gauge theories

A possible embedding in quiver gauge theory is a four $U(1)$ nodes theory (note that also non-abelian groups are admitted) with superpotential

$$\begin{aligned} W &= q_{12}^{(1)}q_{21}^{(1)}X_{11} - fX_{11} + \mu\left(q_{12}^{(1)}q_{21}^{(2)} + q_{12}^{(2)}q_{21}^{(1)}\right) + q_{21}^{(1)}q_{14}^{(7)}q_{42}^{(6)} \\ + & \quad q_{12}^{(1)}q_{24}^{(6)}q_{41}^{(7)} + q_{21}^{(2)}q_{14}^{(7)}q_{42}^{(5)} + q_{12}^{(2)}q_{24}^{(5)}q_{41}^{(7)} + q_{21}^{(1)}q_{14}^{(8)}q_{42}^{(5)} \\ + & \quad q_{12}^{(1)}q_{24}^{(5)}q_{41}^{(8)} + q_{23}^{(4)}Z_{33}q_{32}^{(4)} + q_{43}q_{32}^{(4)}q_{24}^{(5)} + q_{42}^{(5)}q_{23}^{(4)}q_{34} + q_{34}Y_{44}q_{43} - \mu^2 Y_{44} \end{aligned} \quad (2.271)$$

The quiver is shown in Figure 2.36. The upper scripts map the fields in (2.271) with the corresponding fields in (2.260). The fields q_{34} and q_{43} get a vev μ from the equation of motion of the field Y_{44}. This gives a mass term for the fields $q_{32}^{(4)}q_{24}^{(5)}$ and $q_{42}^{(5)}q_{23}^{(4)}$, as in (2.260).

In this model the requirement of gauge invariance forbids the dangerous term $X\phi_5\phi_6$ that we discussed in section 2.5.

Chapter 3

Stringy instantons

Instantons are responsible for non-perturbative phenomena in $4D$ gauge theory and have, by now, found meaningful roles in string theory as well. Relevant work [15] has been done in different branches. It has been shown that instantons in string theory generate non perturbative contributions to the superpotential [16, 18, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87] and higher F-term contributions [88, 89, 90, 91, 92]. Moreover they have a role in model building, since they can be responsible for moduli stabilization [93, 94, 95] and for other phenomenological aspects like neutrino masses, supersymmetry breaking and gauge mediation [30, 58, 96, 97, 98, 99, 100, 101, 102]. Other results are found by adding fluxes [103, 104, 105, 106] and more generally by looking at the string compactification scenarios [107, 108, 109, 110, 111, 112, 113, 114, 115].

One distinguishes between ordinary D-brane instantons and stringy D-brane instantons. Ordinary D-brane instantons are euclidean D-branes wrapping cycles in the geometry occupied by other space time filling D-branes. They reproduce ordinary instantons effects for the gauge theory living on the space time filling D-branes. Stringy D-brane instantons are euclidean branes wrapped over cycles in the geometry which are not occupied by any space-time filling brane.

In this chapter we explain the role of Seiberg duality in connecting the contribution of a class of stingy instanton to $\mathcal{N} = 1$ quiver gauge theories. These contribution were called exotic, since they did not seem to have an explanation in term of gauge theory. Indeed, looking at the four dimensional quiver gauge theories as theories of regular and fractional D3 branes at CY singularity, the stringy instanton are $D(-1)$ branes wrapped on cycles with no D3. Since a D(-1) is believed to be an instanton for a D3, there is no reason to expect a contribution in the superpotential from this construction. Nevertheless, by explicit calculation, it has been shown that this contribution exists. For this reason this contribution has been called exotic.

Here we explain that the this contribution corresponds to the classical constraint on the moduli space of a gauge theory with $N_f = N_c + 1$. This is the limiting case of Seiberg duality and we have already discussed it in section 1. In this way we can connect the low energy theory with the theory one step below in the cascade, connecting the instanton contributions with the strong dynamics effects.

In the following we first review the ADHM construction of $\mathcal{N} = 4$ instantons using branes. In this picture instantons are D(−1) branes on D3 branes, which represents the $U(N)$ gauge theory. This brane construction is useful in the calculation of the instanton action of four dimensional theories with less amount of supersymmetry. Indeed, by applying orbifold and higgsing procedure, one can calculate the action of many theory at CY singularity. Here we concentrate on toric quiver gauge theories. The knowledge of the action is enough to calculate the general form of the contribution of the stringy instantons to these quiver gauge theories. We see that these theories, that admit a cascading behavior, have some regime without regular branes. Nodes with 1 fractional brane (in the unitary case) or 0 branes (in the symplectic case) give rise to exotic contributions in the superpotential. We show that they are not exotic, if they are thought at higher steps in the cascade.

3.1 D-instantons

D-instantons are D(−1) branes located on D3 branes. Their contribution is determined by the collective coordinate measure. The collective coordinate are given by the massless states in the open string spectrum. For a generic Dp brane the collective coordinates come from the reduction of a $U(1)$ vector multiplet from ten to $p + 1$ dimensions. After the dimensional reduction the ten dimensional gauge field A_μ, $\mu = 1, \ldots, 9$, gives raise to a $p + 1$ dimensional gauge field and $9 - p$ real scalars. Moreover the ten dimensional multiplet contains a Majorana-Weyl fermion that has to be opportunely reduced.

In the case of k separated parallel D-branes instantons there are k copies of collective coordinates, and the gauge group is $U(1)^k$. If some of the D-branes are coincident a part (or at least all) of the gauge group becomes non abelian and the low energy theory is obtained from dimensional reduction from the ten dimensional action. It is

$$S_k^{(10)} = \frac{1}{g_{10}^2} \int d^{10}x \mathrm{tr}\ _k \left(\frac{1}{2} F_{\mu\nu}^2 + i\lambda \Gamma_\mu \mathcal{D}_\mu \Psi \right) \tag{3.1}$$

with $\mathcal{D}_\mu \Psi = \partial_\mu \Psi - i[A_\mu, \Psi]$. The Γ matrices compose the ten dimensional Euclidean Clifford algebra (see appendix B.1) The instanton contribution to string theory is calculated by integrating over the collective coordinates. This is similar to the quantum field theory calculation. Indeed the instantons contribute to correlation functions through their saddle-point contribution to the (Euclidean) path integral, but in the semiclassical limit the path integral reduces to an ordinary integral over the instanton moduli. Analogously here, a charge-k instanton contributes via the partition function

$$Z_k = \frac{1}{VolU(k)} \int_{U(k)} d^{10}Ad^{16}\Psi exp(-S_k) \tag{3.2}$$

and the contribution of the integral to the correlators of the low energy supergravity fields can be interpreted as instanton induced vertex in the low energy effective action. The position x_μ is the location of the vertex itself.

The contribution of D-instantons to four dimensional gauge theory are calculated after determining how the D-instanton measure is modified by the presence of D3 branes. The dynamics of N D3-branes in flat spacetime in Type IIB give raise to $\mathcal{N}=4$ gauge theory, with gauge group $U(N)$. Yang-Mills instanton in the gauge theory is equivalent to a D-instantons placed on the D3-branes. In general a Dp brane placed on a D$(p+4)$ brane is an instanton p-brane for the gauge theory living on the D$(p+4)$ brane. Here we review the case of D5 instantons placed on D9 brane in type I, and then obtain the D3/D(-1) case via dimensional reduction

In six dimensions it has $\mathcal{N}=(1,1)$ supersymmetry (two Weyl supercharges, with opposite chirality). The Lorentz group is now $SO(10)\to SO(1,5)\times SO(4)$, i.e. the six dimensional Lorentz and R-symmetry. The ten dimensional gauge field A_M and the Majorana-Weyl fermion are decomposed as

$$\begin{aligned} A_M &= i\left(\chi_a, \frac{1}{2\pi\alpha'}{a'}_n\right) \qquad a=1,\dots,6 \quad n=1,\dots,4 \\ \Psi &= \frac{1}{2\pi\alpha'}\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}{\mathcal{M}'}^A_\alpha+\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix}\lambda^{\dot\alpha}_a \qquad A=1,\dots,4 \quad \alpha,\dot\alpha=1,2 \end{aligned} \tag{3.3}$$

where both χ_a, adjoint scalars, and a'_μ, a six dimensional gauge fields, are Hermitian. The fermion Ψ has been decomposed under the covering group of $SO(1,5)\times SO(4)$, which is $SU(4)\times SU(2)_L\times SU(2)_R$. Using $A=1,\dots,4$ as a spinor index for $SO(4)$ and $\alpha,\dot\alpha$ as spinor indexes for the two $SU(2)$'s, the Majorana-Weyl spinor Ψ is decomposed as in the $(4,2,1)$ and in the $(\bar 4,1,2)$ representations. This shows that the two fermions $\mathcal{M}'$ and λ have opposite chirality. The fields $(\chi_a, {a'}_n, {\mathcal{M}'}^A_\alpha, \lambda^{\dot\alpha}_a)$ form a vector multiplet of $\mathcal{N}=(1,1)$ in six dimensions.The action of the dimensional reduced theory is

$$S=\frac{1}{g_6^2}\left(S_{\text{gauge}}+\frac{1}{4\pi^2\alpha'^2}S^{(a)}_{\text{matter}}\right)\equiv\frac{4\pi^2}{g^2_{p+5}}(4\pi^2\alpha'^2S_{\text{gauge}}+S^{(a)}_{\text{matter}}) \tag{3.4}$$

where the last relation holds since $g^2_{p+5}=(2\pi)^4\alpha'^2g^2_{p+1}$.

$$S_{gauge}=\int d^6\xi \text{tr}_{\ k}\left(\frac{1}{2}F^2_{ab}-i\Sigma^{aAB}\lambda_AD_a\lambda_B+\frac{1}{2}D^2_{mn}\right) \tag{3.5}$$

and

$$S^{(a)}_{\text{matter}}=\int d^6\xi \text{ tr}_{\ k}\Big\{\mathcal{D}^a a'_n\mathcal{D}_a a'_n-\frac{i}{4}\bar\Sigma^a_{AB}\mathcal{M}^A\mathcal{D}_a\mathcal{M}'^B-i[\mathcal{M}'^{\alpha A},a'_{\alpha\dot\alpha}]\lambda^{\dot\alpha}_A+i\vec{D}\cdot\vec\tau^{\dot\alpha}_{\dot\beta}\bar a'^{\dot\beta\alpha}a'_{\alpha\dot\alpha}\Big\} \tag{3.6}$$

where $\mathcal{D}$ is a covariant derivative, D transforms in the adjoint representation of $SU(2)_R$ and we have defined $D_{mn}=-D^c\bar\eta^c_{mn}$. The Σ matrices have been defined in appendix B.1.

After that, one introduces the N D9 branes, whose world volume theory is $U(N)$ supersymmetric Yang-Mills in ten dimensions. In the effective action there is a coupling [116] between the two-form F (the field strength of the gauge field) and a six-form RR field $C^{(6)}$. This coupling is given by the ten dimensional integral

$$\int C^{(6)}\wedge F\wedge F \tag{3.7}$$

The six form $C^{(6)}$ couples also with the RR charge carried by the D5 branes, i.e. $U(N)$ gauge fields act as a source for $D5$-brane charge. The D5 is interpreted as an instanton on the D9.

The system of D5/D9 branes breaks the $\mathcal{N} = (1,1)$ supersymmetry down to the $\mathcal{N} = (0,1)$ sub-algebra, with a vector multiplet $(\chi_a, \lambda_a^{\dot{\alpha}})$ and a scalar adjoint, $(a'_n, \mathcal{M}'^A_\alpha)$. There are new hyper-multiplets of $\mathcal{N} = (0,1)$, in the bifundamental representation of $U(k) \times U(N)$. There are two complex scalars $w_{ui\dot{\alpha}}$, with $\dot{\alpha} = 1,2$, and i and u are respectively fundamental of $U(k)$ and $U(N)$. Each hyper-multiplet contains a pair of complex Weyl spinors, μ^A_{ui} and $\bar{\mu}^A_{iu}$. The action is

$$S^{(f)}_{\text{matter}} = \int d^6\xi \,\text{tr}\,_k\Big\{ -\mathcal{D}^a \bar{w}^{\dot{\alpha}} \mathcal{D}_a w_{\dot{\alpha}} - \frac{i}{2}\bar{\Sigma}^a_{AB}\bar{\mu}^A \mathcal{D}_a \mu^B - i\big(\bar{\mu}^A w_{\dot{\alpha}} + \bar{w}_{\dot{\alpha}}\mu^A\big)\lambda^{\dot{\alpha}}_A + i\vec{D}\cdot\vec{\tau}^{\dot{\alpha}}_{\dot{\beta}}\bar{w}^{\dot{\beta}}w_{\dot{\alpha}}\Big\} \tag{3.8}$$

The complete action is

$$S = \frac{4\pi^2}{g^2_{p+5}}\left(4\pi^2\alpha'^2 S_{gauge} + S^{(a)}_{\text{matter}} + s^{(f)}_{\text{matter}}\right) \tag{3.9}$$

One conclude that the six-dimensional fields are the collective coordinates of the D5-branes. The vacuum moduli space of the $U(k)$ gauge theory on the D5 branes represent the ADHM construction of the supersymmetric k-instanton moduli space of $\mathcal{N} = 4$ SYM.

The ADHM constraints are also recovered in this formalism. Indeed the D term equations are the bosonic ADHM constraints

$$\alpha'^2\vec{D} = \frac{i}{16\pi^2}\vec{\tau}^{\dot{\alpha}}_{\dot{\beta}}\left(\bar{w}^{\dot{\beta}}w_{\dot{\alpha}} + \bar{a}'^{\dot{\beta}\alpha}a'_{\alpha\dot{\alpha}}\right) = 0 \tag{3.10}$$

The ADHM fermionic constraint are found after an integration on the fermionic zero mode λ_A. Indeed this integration leaves a delta function that imposes

$$\bar{\mu}w_{\dot{\alpha}} + \bar{w}_{\dot{\alpha}}\mu + \left[\mathcal{M}'^{\alpha}, a'_{\alpha\dot{\alpha}}\right] = 0 \tag{3.11}$$

This last is exactly the ADHM fermionic constraint.

The D3/D(-1) system is recovered from this D9/D5 system by a dimensional reduction. The contribution of k D-instantons to the correlation functions of the low energy fields in presence of N D3 branes is given by the partition function

$$Z_k = \frac{1}{Vol(U(k))}\int d^6\xi d^8\lambda d^3 D d^4 a' d^8\mathcal{M} d^2 w d^2\bar{w}\delta^4\mu d^4\bar{\mu} s^{-S} \tag{3.12}$$

The action is calculated by dimensional reducing (3.9) to zero dimensions. One find

$$\begin{aligned}
S_G &= \text{tr}\,_k\Big\{\frac{1}{2}[\chi_a,\chi_b]^2 - \Sigma^{AB}_a\lambda_A[\chi_a,\lambda_B] - \frac{1}{2}D^2_{mn}\Big\} \\
S_K &= \text{tr}\,_k\Big\{ -[\chi_a, a'_n]^2 + \chi_a\bar{w}^{\dot{\alpha}}w_{\dot{\alpha}}\chi_a - \frac{1}{4}\bar{\Sigma}_{aAB}\mathcal{M}'^{\alpha A}[\chi_a,\mathcal{M}'^B_\alpha] + \frac{1}{2}\bar{\Sigma}_{aAB}\bar{\mu}^A\mu^B\chi_a\Big\} \\
S_D &= \text{tr}\,_k\Big\{ -i\vec{D}\cdot\vec{\tau}^{\dot{\alpha}}_{\dot{\beta}}\big(\bar{w}^{\dot{\beta}}w_{\dot{\alpha}} + \bar{a}'^{\dot{\beta}\alpha}a'_{\alpha\dot{\alpha}}\big) + i\big(\bar{\mu}^A w_{\dot{\alpha}} + \bar{w}_{\dot{\alpha}}\mu^A + [\mathcal{M}'^{A\alpha}, a'_{\alpha\dot{\alpha}}]\big)\lambda^{\dot{\alpha}_A}\Big\}
\end{aligned}$$

This construction ends once we consider the $\mathcal{N} = 4$ theory on its Coulomb branch, i.e. the N D3 branes separate in the six dimensional transverse space. The lengths of the strings stretched between the D-instantons and the D3-branes changes and this process introduces a mass term for the fundamental hypermultiplets. This displacement gives raise to an interaction of the sector charged under the gauge group and the scalars of the gauge theory. This part of the action is

$$S_C = \text{tr} \left(\bar{w}_{\dot{\alpha}} X^a X_a w^{\dot{\alpha}} + \frac{i}{2} (\bar{\Sigma}^a)_{AB} \bar{\mu}^A X_a \mu^B \right) \tag{3.13}$$

For the $\mathcal{N} = 1$ application is more useful a notation in term of explicit $SU(4)$ indexes,then broken into $SU(3)$ representations. The six scalars in the antisymmetric representation of $SU(4)$ are

$$X_{AB} = -X_{BA} = (\bar{\Sigma}^a)_{AB} \tag{3.14}$$

and with the action becomes

$$S_C = \text{tr} \left(\frac{1}{8} \epsilon_{ABCD} \bar{w}_{\dot{\alpha}} X_{AB} X_{CD} w^{\dot{\alpha}} + \frac{i}{2} \bar{\mu}^A X_{AB} \mu^B \right) \tag{3.15}$$

The breaking into $SU(3)$ splits the index A as $A = (i, 4)$, and one can identify the scalars of $\mathcal{N} = 4$ as $\Phi_i^\dagger = X_{i4}$ in the $\bar{3}$ and $\Phi_i = \frac{1}{2}\epsilon_{ijk} X_{jk}$ in the 3. In term of these field the action for this part of the charged sector becomes

$$S_c = \text{tr} \left(\frac{1}{2} \bar{w}_{\dot{\alpha}} \Phi^i, \Phi_i^\dagger w^{\dot{\alpha}} + \frac{i}{2} \bar{\mu}^i \Phi_i^\dagger \mu^4 - \frac{i}{2} \bar{\mu}^4 \Phi_i^\dagger \mu^i + \frac{i}{2} \epsilon_{ijk} \bar{\mu}^i \Phi^j \mu^k \right) \tag{3.16}$$

where we have distinguish the fourth component of the fields μ and $\bar{\mu}$ as $\mu^A = (\mu^i, \mu^4)$ and $\bar{\mu}_A = (\bar{\mu}^i, \bar{\mu}^4)$. The same distinction in term of $SU(3)$ representations can be done for the other fields with the index A. Finally also the six real fields χ_a can be written in term of the fields s_i $i = 1, 2, 3$, which are their complexification

The ADHM limit

In order to recover the same results of ADHM construction in gauge theory we still have to set $\alpha' \to 0$. Indeed the collective integral depends explicitly from α' via the kinetic term of the vector multiplet. Correlation functions will have a non trivial expression in terms of the α'. This is not possible in the pure gauge theory construction. This implies that the decoupling of gauge theory from gravity is necessary, and it is done by taking the limit $\alpha' \to 0$.

3.2 A general action for instanton in toric quiver gauge theories

Once the action for the $\mathcal{N} = 4$ theory is known, one can extend the calculation to theories with a lower amount of supersymmetry. In this section we explain a procedure studied in [17], that calculates the multi-instanton action for a generic $\mathcal{N}=1$ toric quiver gauge theory. Through many examples it is possible to extrapolate the general behavior of the action of the bosonic and fermionic zero modes in the neutral and charged instantonic sector. For the charged sector, a general rule, in term of the superpotential of the associated quiver gauge theory is given by generalizing the rules of [17, 18].

Before going on we reorganize the $\mathcal{N}=4$ instanton action in three sectors as in [17]

$$\begin{aligned} S_1 \;=\; \mathrm{tr}\,\Big\{ & -[a_m, s_i^\dagger][a^m, s^i] - \frac{i}{2}\Big(M^{\alpha i}[s_i^\dagger, M_\alpha^4] - \frac{1}{2}\epsilon_{ijk}M^{\alpha i}[s^j, M_\alpha^k]\Big) \\ & + i\left(\bar{\mu}^i\omega_{\dot\alpha} + \bar{\omega}_{\dot\alpha}\mu^i + \sigma^m_{\beta\dot\alpha}[M^{\beta i}, a_m]\right)\lambda_i^{\dot\alpha} + i\left(\bar{\mu}^4\omega_{\dot\alpha} + \bar{\omega}_{\dot\alpha}\mu^4 + \sigma^m_{\beta\dot\alpha}[M^{\beta 4}, a_m]\right)\lambda_4^{\dot\alpha} \\ & - iD^c\Big(\bar{\omega}^{\dot\alpha}(\tau^c)^{\dot\beta}_{\dot\alpha}\omega_{\dot\beta} + i\bar{\eta}^c_{mn}[a^m, a^n]\Big)\Big\} \end{aligned} \tag{3.17}$$

A second part of the action, involving the Φ_i fields is

$$\begin{aligned} S_2 \;=\; \mathrm{tr}\,\Big\{ & \frac{1}{2}\Big(\bar{\omega}_{\dot\alpha}\Phi^i + s^i\bar{\omega}_{\dot\alpha}\Big)\Big(\Phi_i^\dagger\omega^{\dot\alpha} + \omega^{\dot\alpha}s_i^\dagger\Big) + \frac{1}{2}\Big(\bar{\omega}_{\dot\alpha}\Phi_i^\dagger + s_i^\dagger\bar{\omega}_{\dot\alpha}\Big)\Big(\Phi^i\omega^{\dot\alpha} + \omega^{\dot\alpha}s^i\Big) \\ & + \frac{i}{2}\bar{\mu}^i\left(\Phi_i^\dagger\mu^4 + \mu^4 s_i^\dagger\right) - \frac{i}{2}\bar{\mu}^4\left(\Phi_i^\dagger\mu^i + \mu^i s_i^\dagger\right) - \frac{i}{2}\epsilon_{ijk}\bar{\mu}^i\left(\Phi^j\mu^k - \mu^j s^k\right)\Big\} \end{aligned} \tag{3.18}$$

The last term is

$$S_3 = \mathrm{tr}\,\Big\{\frac{1}{2}D_c^2 - \frac{i}{2}\Big(\lambda_{\dot\alpha i}[s^i, \lambda_4^{\dot\alpha}] - \frac{1}{2}\epsilon^{ijk}\lambda_{\dot\alpha i}[s_j^\dagger, \lambda_k^{\dot\alpha}]\Big) + [s^i, s^j][s_j^\dagger, s_i^\dagger] + \frac{1}{2}[s^i, s_i^\dagger][s^j, s_j^\dagger]\Big\} \tag{3.19}$$

The last sectors is usually suppressed in the limit $\alpha' \to 0$, but here it plays a crucial role.

The techniques necessary for the calculation of the instanton action in this framework are the action of orbifold group and the higgsing procedure. The orbifold that acts on the $\mathcal{N}=4$ gauge theory is mapped to the whole instanton sector, giving the action for all these theories (e.g. $\mathbb{C}^2/Z_2$, $\mathbb{C}^2/(Z_2 \times Z_2)$ etc ...). Then one can come through the orbifold limit and obtain the action for all the toric singularities, via the higgsing.

In this supersymmetric scenario the higgsing takes place through the partial resolution of the singularity, which means that some FI is switched on in the GLSM associated to the singularity. Some of the fields of the theory acquire a vev, and some fields become dynamically massive, because of this vev. The massive fields are integrated out and the effective action is the one describing the non-orbifold toric singularity.

This is a standard procedure in the analysis of toric quiver gauge theory. In [17] the authors has shown how this procedure applies to give raise to the correct action for the instanton zero modes. By inspection one notes that some care is necessary in the mapping of the partial resolution to the neutral instanton sector. Indeed some consistency check show that a FI term ξ for a chiral field Φ of the gauge theory has to be reproduced by a FI term $-\xi$ for a field χ in the associated neutral instanton sector. Otherwise the integration out procedure cannot take into account all the orders in the Gaussian integral, and for consistency an opportune scaling limit is considered.

Without describing the details of the calculation we here review the general behaviour of the instantonic action for toric quivers.

A first observation is based on the behaviour of the fields after the orbifold projection. The same structure of the fields Φ_i is acquired by the fields $s^i, M^i, \lambda_i, \mu^i$ and $\bar{\mu}^i$. Otherwise the fields $a_m, D^c, w, \bar{w}, M^4, \lambda^4, \mu^4, \bar{\mu}^4$ are block diagonal and their behaviour does not change after the projection.

Another important remark is the role of the FI in the S_G action. The commutator $[s^i, s_j^\dagger]$ is changed in $[s^i, s_j^\dagger] - \xi$ and this change in the S_G action is important (although this action vanishes in the ADHM limit, this term is important in the higgsing procedure).

We can now discuss explicitly what happens in (3.17) and (3.18). The first term changes only via the orbifold projection, but the equation of motion and the new interactions do not alter it. This is constituted by a neutral sector that is totally unchanged and by a charged sector. For every pair of nodes a and b for which the relevant field exist, there are the couplings

$$\bar{\omega}_{aa}\Phi_{ab}\Phi^\dagger_{ba}\omega_{aa}, \quad \bar{\omega}_{aa}\Phi^\dagger_{ab}\Phi_{ba}\omega_{aa}, \quad \bar{\mu}_{aa}\Phi^\dagger_{ab}\mu_{ba}, \quad \bar{\mu}_{ab}\Phi^\dagger_{ba}\mu_{aa} \tag{3.20}$$

Differently the second term, S_2 is strongly constrained from the interactions. Indeed one sees that, if one has a term tr $\Phi_{12}\Phi_{23}\Phi_{34}\Phi_{41}$ in the superpotential, then in the charged sector some new term arise

$$\text{tr}\left(\bar{\mu}_{12}\Phi_{23}\Phi_{34}\mu_{41} + \bar{\mu}_{23}\Phi_{34}\Phi_{41}\mu_{12} + \bar{\mu}_{34}\Phi_{41}\Phi_{12}\mu_{23} + \bar{\mu}_{41}\Phi_{12}\Phi_{23}\mu_{34}\right) \tag{3.21}$$

An analog result is obtained for s. For future aims we will ignore the s_i, because we are interested only in the one instanton action, without multi instanton contributions.

One instanton action

For the convenience of the reader we briefly review the basic instanton framework of relevance here. We describe the most general configuration with a $SU(1)$ node in a toric quiver gauge theory and we place a (stringy) instanton on that node. We consider only rigid instantons, without adjoint fields charged under the $SU(1)$ gauge group. We use the instantonic action for toric quiver gauge theories as discussed above.

The system consists of N $D3$ branes and k $D(-1)$ brane in type II B. The strings with endpoints attached to the $D3$ branes lead to $SU(N)$ $\mathcal{N}=4$ SYM. The strings with endpoints attached to the $D(-1)$ branes lead to the neutral sector, uncharged under the gauge group. It includes bosonic moduli a^μ and fermionic zero modes $M^{\alpha\mathcal{A}}$ and $\lambda_{\dot{\alpha}\mathcal{A}}$ where α and $\dot{\alpha}$ denote the positive and negative chirality in four dimension and $\mathcal{A}$ is an $SU(4)$ index (fundamental or anti fundamental) denoting the chirality in the transverse six dimensions. The equations of motion for the zero modes $\lambda_{\dot{\alpha}\mathcal{A}}$ implement the fermionic ADHM constraint. There is also a triplet of auxiliary bosonic fields D^c whose equations of motion implement the bosonic ADHM constraint. The charged sector is associated with strings stretching between $D3$ branes and the $D(-1)$ branes. It includes bosonic spinors $\omega_{\dot{\alpha}}$ and $\bar{\omega}_{\dot{\alpha}}$ and fermions $\mu^{\mathcal{A}}$ and $\bar{\mu}^{\mathcal{A}}$. These fields are matrices of dimension $N \times k$.

In order to obtain the toric quiver gauge theory together with the instanton sector the whole field content has to be projected with the orbifold and then higgsed. Notice that instanton moduli scale with the same Chan-Paton structure of ordinary gauge theory fields.

The resulting gauge theory is a toric quiver gauge theory with many gauge groups, where we can change the ranks of the groups by adding fractional D-branes. The instanton sector works in a similar way. There are k instantons placed on each node, and we can add instantonic fractional branes (not to be confused with fractional instantons) to obtain a different numbers k_i of instantons on the various nodes. Here we are interested in one instanton corrections without multi-instantons effects.

From now on we consider one instanton placed on a $SU(1)$ node in a generic toric quiver gauge theory (see Figure 3.1). We denote with A the index associated with that

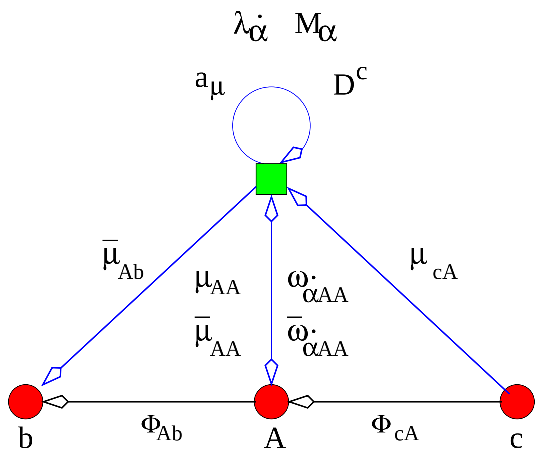


Figure 3.1: Stringy instanton on a $SU(1)$ node in a generic quiver. This is only the sector directly connected to the node A of an anomaly free quiver.

node. The auxiliary instanton group is $U(1)$. The node A could be connected to the neighbor nodes with fields Φ_{Ab}, for outgoing arrows in the quiver, or with fields Φ_{cA}, for incoming arrows. In general, there could be more fields with the same gauge groups indexes. To simplify the notation we suppose here that every neighbor node is connected to the node A with a single field. The general case is treated in the appendix B.2.

The spectrum is reported in Figure 3.1 and in Table 3.1. The toric quiver represents the gauge sector. The neutral sector includes the bosonic zero mode a_μ and D^c, and the fermionic zero modes M^α and $\lambda^{\dot\alpha}$ (only the 4 component survive the orbifold projection in the one instanton case). There is a charged sector connecting the node A and the instanton, given by $\omega_{\dot\alpha AA}, \bar\omega_{\dot\alpha AA}, \mu_{AA}, \bar\mu_{AA}$, and a charged sector connecting the instanton with the neighbor nodes, in a way similar to the field content of the gauge theory. For each existing outgoing arrow Φ_{Ab} there is a fermionic zero mode $\bar\mu_{Ab}$, and for each incoming arrow Φ_{cA} there is μ_{cA}.

Sector	ADHM	Statistic	Chan-Paton
Charged	μ_{Ab}	Fermion	$k \times N_b$
Charged	$\bar{\mu}_{cA}$	Fermion	$N_c \times k$
Charged	μ_{AA}	Fermion	$k \times N_A$
Charged	$\bar{\mu}_{AA}$	Fermion	$N_A \times k$
Charged	$\omega_{\dot{\alpha}\ AA}$	Boson	$k \times N_A$
Charged	$\bar{\omega}_{\dot{\alpha}\ AA}$	Boson	$N_A \times k$
Neutral	a_μ	Boson	$k \times k$
Neutral	M_α	Fermion	$k \times k$
Neutral	$\lambda_{\dot{\alpha}}$	Fermion	$k \times k$
Neutral	D^c	Boson	$k \times k$

Table 3.1: Spectrum of the ADHM moduli in the charged and in the neutral sector

The instantonic action reads

$$S_{inst} = S_1 + S_2 + S_W \tag{3.22}$$

where

$$S_1 = i(\bar{\mu}_{AA}\omega_{\dot{\alpha}AA} + \bar{\omega}_{\dot{\alpha}AA}\mu_{AA})\lambda^{\dot{\alpha}} - iD^c(\bar{\omega}_{\dot{\alpha}AA}\tau^c\omega_{\dot{\alpha}AA}) \tag{3.23}$$

$$S_2 = \frac{1}{2}\sum_b \left[\bar{\omega}_{\dot{\alpha}AA}\Phi_{Ab}(\Phi_{Ab})^\dagger\omega_{\dot{\alpha}AA} + i\,\bar{\mu}_{Ab}(\Phi_{Ab})^\dagger\mu_{AA}\right] + \tag{3.24}$$

$$\frac{1}{2}\sum_c \left[\bar{\omega}_{\dot{\alpha}AA}(\Phi_{cA})^\dagger\Phi_{cA}\omega_{\dot{\alpha}AA} - i\,\bar{\mu}_{AA}(\Phi_{cA})^\dagger\mu_{cA}\right] \tag{3.25}$$

$$S_W = -\frac{i}{2}\sum_{b,c}\bar{\mu}_{Ab}\frac{\partial W}{\partial(\Phi_{cA}\Phi_{Ab})}\mu_{cA}, \tag{3.26}$$

Observe that the S_W action involves derivatives of the superpotential with respect to bilinears of fields contracted on the A index[1].

3.3 Stringy Instanton and Seiberg Duality

In this section we investigate the relations between stringy instantons and strong dynamics effects in type IIB toric quiver gauge theories. Stringy instanton contributions to the superpotential in quiver gauge theories have been shown to exist for $SP(0)$, $SU(1)$ and $SO(3)$ nodes. The second and third cases are named stringy since the low energy dynamics associated to those groups is trivial and no instanton contribution is expected. The results,

[1]This is necessary in order to take into account the contribution to this expression for non abelian superpotential and for superpotentials with terms involving more than 3 fields.

up to now, show that the stringy instanton contributions reproduce the non perturbative part of the superpotential of the gauge theory, i.e. part of the classical constraint on the moduli space. Here we present a clear-cut argument based on the involutive nature of the Seiberg duality, which explains the retrieval of the exact instanton contribution as a strong dynamical effect. We shall speak of equivalence or correspondence between the instantonic and gauge schemes.

In the following we first we give a general overview of the correspondence between stringy instantons and dynamical effects. Then we review the one-instanton action for a general quiver gauge theory and compute the contribution to the superpotential. We argue that the correspondence is implied by the involutive property of Seiberg duality, and we give two examples, the L^{121} and the dP_1 quiver gauge theories. After that we discuss the correspondence for stringy instantons on $SP(0)$ and $SO(3)$ gauge groups in orientifolded quiver gauge theories, with clarifying examples.

Overview

Consider a quiver gauge theory with an $SU(1)$ node and a tree level superpotential W_{tree}. A stringy instanton on a $SU(1)$ node gives rise to a superpotential term [18, 82, 84]. The cycle wrapped by the euclidean D-brane is occupied also by one D-brane and the non trivial interaction lifts the fermionic zero modes. The resulting superpotential is

$$W = W_{tree} + W_{inst} \tag{3.27}$$

Gauge theories with a $SU(1)$ gauge group are obtained as low energy (magnetic) descriptions of a strongly coupled $SU(N_c)$ gauge theory with $N_c + 1$ flavours. The low energy description of this strongly coupled $SU(N_c)$ gauge theory is a *limiting case* of Seiberg duality. Indeed, it can be described by a magnetic gauge group $SU(1)$, where the elementary degrees of freedom are mesons and baryons. The baryons are the dual magnetic quarks. The classical moduli space of such a theory is not modified at quantum level. The classical constraint is imposed in the dual description by the addition of a non trivial superpotential for the mesons and the baryons, of the form

$$W_{eff} \sim B\mathcal{M}\tilde{B} - \det \mathcal{M} \tag{3.28}$$

We shall show that the second term in (3.28) is exactly reproduced by the stringy instanton contribution in (3.27). Here and in the rest of the paper we set to unity the dimension-full coefficients.

Relations between non perturbative dynamics and stringy instantons has been already observed in [81, 82] for cascading gauge theories. The correspondence we ascertain holds at every step of a cascade when the Seiberg duality is in the limiting case. The non perturbative contribution to the superpotential is then continuously mapped at every step until the bottom of the cascade [82].

Stringy instanton contribution

The stringy instanton contribution is obtained by integrating over all the zero modes

$$Z = \mathcal{C} \int d\{a_\mu, M, \lambda^{\dot\alpha}, D, \omega_{AA}, \bar\omega_{AA}, \mu_{AA}, \bar\mu_{AA}, \bar\mu_{Ab}, \mu_{cA}\}\, e^{-S_{inst}} \tag{3.29}$$

where $\mathcal{C}$ is a dimension-full parameter which is discussed in appendix B.3. The integration over the a_μ and the M^α zero mode is interpreted as the superspace integration. Hence the stringy instanton contribution to the superpotential is given by

$$W_{inst} \sim \int d\{\lambda^{\dot\alpha}, D, \omega_{AA}, \bar\omega_{AA}, \mu_{AA}, \bar\mu_{AA}, \bar\mu_{Ab}, \mu_{cA}\}\, e^{-S_{inst}} \tag{3.30}$$

The bosonic integration is discussed in appendix B.3. As for the fermionic integration

$$W_{inst} \sim \int d\lambda^{\dot\alpha} d\bar\mu_{AA}\, d\mu_{AA} \prod_{b,c} (d\bar\mu_{Ab})^{N_b}\, (d\mu_{cA})^{N_c} e^{-S_{inst}} \tag{3.31}$$

the integral on $\lambda^{\dot\alpha}$ can be performed as in [84] using the S_1 part of the instanton action and it gives the ADHM fermionic constraints. It also saturates the fermionic integration on $\bar\mu_{AA}$ and μ_{AA}. We end up with the integral

$$W_{inst} \sim \int \prod_{b,c} (d\bar\mu_{Ab})^{N_b}\, (d\mu_{cA})^{N_c} e^{-S_W} \tag{3.32}$$

and this fermionic integration gives

$$W_{inst} \sim \det\big(\frac{\partial W}{\partial(\Phi_{cA}\Phi_{Ab})}\big) \equiv \det\big(\mathcal{M}\big) \tag{3.33}$$

Notice that $\mathcal{M}$ is a a square matrix from the anomaly free condition for the node A, which is $\sum_b N_b = \sum_c N_c$.

Discussion on the equivalence

The contribution generated by a stringy instanton on an $SU(1)$ gauge node is here obtained from the strong dynamics of the gauge theory. This equivalence follows from the involutive property of the (limiting case) of Seiberg duality (i.e. the case with $N_f = N_c+1$ for unitary gauge groups).

We consider the previous toric quiver gauge theory with a $SU(1)$ gauge group labeled by A, with N_f flavours spread on the nodes connected to the $SU(1)$ one. The part of the superpotential involving the fields charged under the gauge group A is a generic holomorphic function

$$W = W_0(\Phi_{cA}\Phi_{Ab}, X_{bc}^{(p)}) \tag{3.34}$$

where the Φ fields are bifundamentals charged under the $SU(1)$. The $X_{bc}^{(p)}$ are fields or products of fields charged under the gauge groups connected to A in the quiver. In section 3.2 we showed that a stringy instanton on node A gives a contribution to the superpotential of the form

$$W_{inst} \sim \det \frac{\partial W_0}{\partial(\Phi_{cA}\Phi_{Ab})} \tag{3.35}$$

We now perform two consecutive Seiberg dualities on the node A and compare the resulting theory with the original one. The first step is a formal Seiberg duality for the gauge group $SU(1)$. This gives a $SU(\tilde{N} = N_f - 1)$ gauge theory with N_f flavours, and superpotential

$$W_{dual} = W_0(M_{cb}, X^{(p)}_{bc}) + M_{cb}q_{bA}q_{Ac} \tag{3.36}$$

where $M_{cb} = \Phi_{cA}\Phi_{Ab}$ and q_{bA} and q_{Ac} are the dual quarks.

The next step is another duality on the node A. Since $N_f = \tilde{N} + 1$, the dual gauge group is $SU(1)$, and the superpotential is

$$W_{eff} = W_0(M_{cb}, X^{(p)}_{bc}) + M_{cb}N_{bc} - N_{bc}b_{cA}b_{Ab} + \det N_{bc} \tag{3.37}$$

where $N_{bc} = q_{bA}q_{Ac}$, the b are baryons, and we have changed the sign of the interaction term as in [117]. The last two terms implement the classical constraint on the moduli space [1].

For the involution to hold, this theory should coincide with the original one, after integrating out the massive mesons M_{cb}, N_{bc}. The equations of motions of the fields N_{bc} give

$$b_{cA}b_{Ab} = M_{cb} = \Phi_{cA}\Phi_{Ab} \tag{3.38}$$

Hence we identify the baryons b with the original fields Φ. The equation of motion of the meson M_{cb} implies that

$$N_{bc} \sim \frac{\partial W_0}{\partial M_{cb}} = \frac{\partial W_0}{\partial(\Phi_{cA}\Phi_{Ab})} \tag{3.39}$$

so we recover in (3.37) the original theory (3.34) and also the stringy instanton contribution (3.35), i.e. the determinant term. This proves the correspondence. We conclude that the involution of the Seiberg duality in the limiting case provides a gauge theory explanation of the stringy instanton contribution.

Examples

In this section we exhibit two examples of the correspondence: a non chiral theory, the L^{121} quiver gauge theory, and the dP_1 chiral theory.

We begin with a theory where there is a node with $N_f = N_c + 1$, we then consider strong dynamics for that node and we study the low energy theory, performing a Seiberg duality in the limiting case, obtaining a non trivial contribution to the superpotential. The same contribution is obtained analyzing directly the low energy theory and taking into account the stringy instanton effect on the dualized node, an $SU(1)$ node.

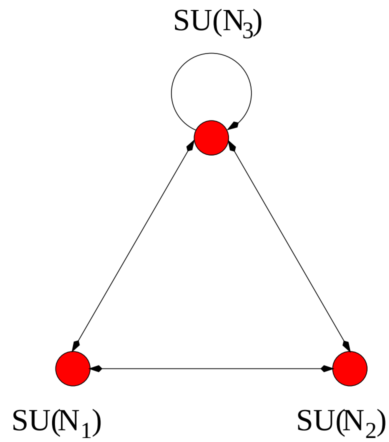


Figure 3.2: L^{121} quiver gauge theory.

Non chiral example: L^{121}

The superpotential is

$$W = -X_{33}Q_{31}Q_{13} + X_{33}Q_{32}Q_{23} + Q_{21}Q_{13}Q_{31}Q_{12} - Q_{32}Q_{21}Q_{12}Q_{23} \tag{3.40}$$

We choose the assignment of ranks for the gauge groups such that

$$N_2 + N_3 = N_1 + 1 \tag{3.41}$$

We now consider strong dynamics for the node 1. This node has $N_f = N_c + 1$. The low energy can be analyzed performing a limiting case of Seiberg duality. The magnetic gauge group is $SU(1)$, and the magnetic quarks are identified with the baryons of the electric description. The resulting theory has superpotential

$$\begin{aligned} W &= -X_{33}M_{33} + X_{33}Q_{32}Q_{23} + M_{23}M_{32} - M_{22}Q_{23}Q_{32} \\ &\quad + M_{33}q_{31}q_{13} + M_{22}q_{21}q_{12} - M_{23}q_{31}q_{12} - M_{32}q_{21}q_{13} + \det \begin{pmatrix} M_{22} & -M_{23} \\ -M_{32} & M_{33} \end{pmatrix} \end{aligned} \tag{3.42}$$

We have added the determinant contribution in order to correctly implement the classical constraint on the moduli space. Integrating out the massive fields, we obtain the quiver in figure 3.3 with the following superpotential

$$W = -q_{21}q_{13}q_{31}q_{12} + Q_{23}q_{31}q_{13}Q_{32} + q_{12}M_{22}q_{21} - Q_{32}M_{22}Q_{23} + \det \begin{pmatrix} M_{22} & -q_{21}q_{13} \\ -q_{31}q_{12} & Q_{32}Q_{23} \end{pmatrix} \tag{3.43}$$

where there is an extra determinant term with respect to the usual SPP superpotential. The theory of figure 3.3 has an $SU(1)$ gauge group. Strong dynamics effects from the theory one step backward in Seiberg duality have produced a non trivial superpotential contribution, i.e. the determinant term in (3.43). We show here that the same term is generated by a stringy instanton in the theory of figure 3.3.

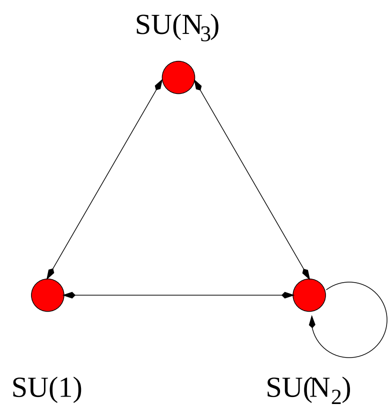


Figure 3.3: L^{121} after dualizing node 1.

The instantonic action for the D-instantons in the SPP has been constructed in [17]:

$$
\begin{aligned}
S_{inst} = \quad & i(\bar{\mu}_{11}\omega_{\dot{\alpha}11} + \bar{\omega}_{\dot{\alpha}11}\mu_{11})\lambda^{\dot{\alpha}} - iD^c(\bar{\omega}_{\dot{\alpha}11}\,(\tau^c)^{\dot{\alpha}}_{\dot{\beta}}\,\omega^{\dot{\beta}}_{11}) \\
& + \frac{1}{2}\bar{\omega}_{\dot{\alpha}11}\left(q_{12}q^{\dagger}_{12} + q^{\dagger}_{21}q_{21} + q_{13}q^{\dagger}_{13} + q^{\dagger}_{13}q_{31}\right)\omega_{\dot{\alpha}11} \\
& + \frac{i}{2}\left(\bar{\mu}_{12}q^{\dagger}_{12}\mu_{11} + \bar{\mu}_{13}q^{\dagger}_{13}\mu_{11} - \bar{\mu}_{11}q^{\dagger}_{21}\mu_{21} - \bar{\mu}_{11}q^{\dagger}_{31}\mu_{31}\right) \\
& - \frac{i}{2}\left(\bar{\mu}_{12}M_{22}\mu_{21} - \bar{\mu}_{12}q_{21}q_{13}\mu_{31} - \bar{\mu}_{13}q_{31}q_{12}\mu_{21} + \bar{\mu}_{13}Q_{32}Q_{23}\mu_{31}\right) \qquad (3.44)
\end{aligned}
$$

The corresponding quiver is given in figure 3.4. The solid lines are the chiral superfields. The dashed lines are the fermionic zero modes connecting the instanton and the other D-branes in the theory. In order to compute the instanton contribution to the effective superpotential we have to integrate over the fermionic and bosonic zero mode that couple among them and with the chiral superfields. The a_μ and M^α zero modes are the superspace coordinates. The integral over the $\lambda^{\dot{\alpha}}$ and D^c can be done using the first line in (3.44), and they give the two fermionic and the three fermionic ADHM constraints. The other bosonic integration has been shown in appendix B.3 to give only a constant, via a general dimensional argument. Here we explicitly show this result via an explicit calculation [83, 84] The fermionic ADHM constraints are

$$
\begin{aligned}
& \delta\left(\bar{\mu}_{11}\omega_{\dot{1}11} + \bar{\omega}_{\dot{1}11}\mu_{11}\right)\delta\left(\bar{\mu}_{11}\omega_{\dot{2}11} + \bar{\omega}_{\dot{2}11}\mu_{11}\right) = \left(\bar{\mu}_{11}\omega_{\dot{1}11} + \bar{\omega}_{\dot{1}11}\mu_{11}\right)\left(\bar{\mu}_{11}\omega_{\dot{2}11} + \bar{\omega}_{\dot{2}11}\mu_{11}\right) = \\
& = \bar{\mu}_{11}\left(\omega_{\dot{1}11}\bar{\omega}_{\dot{2}11} - \omega_{\dot{2}11}\bar{\omega}_{\dot{1}11}\right)\mu_{11} \qquad (3.45)
\end{aligned}
$$

This term saturates also the integrations over the zero modes μ_{11} and $\bar{\mu}_{11}$. A non vanishing result can be found only expanding the action at zero order in μ_{11} and $\bar{\mu}_{11}$ The bosonic integral factorizes and it is

$$
I_{bos} = \int_0^\infty d^3D^c d^2w_{\dot{\alpha}11}d^2w_{\alpha11}\left(\omega_{\dot{1}11}\bar{\omega}_{\dot{2}11} - \omega_{\dot{2}11}\bar{\omega}_{\dot{1}11}\right)e^{-iD^c(\bar{\omega}_{\dot{\alpha}11}(\tau^c)^{\dot{\alpha}}_{\dot{\beta}}\omega^{\dot{\beta}}_{11})\omega_{\dot{2}11}\bar{\omega}_{\dot{1}11} + \frac{1}{2}\bar{\omega}_{\dot{\alpha}11}g(\{q\})\omega_{\dot{\alpha}11}} \qquad (3.46)
$$

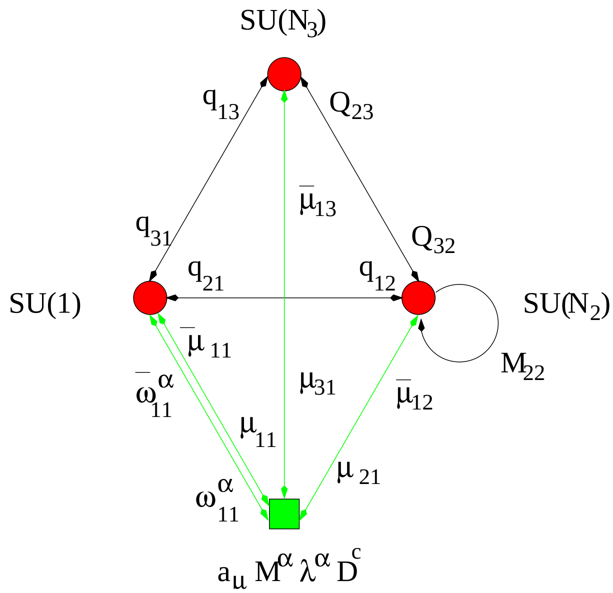


Figure 3.4: L^{121} with instanton on node $N_1 = 1$.

where $g(\{q\}) = \left(q_{12}q_{12}^{\dagger} + q_{21}^{\dagger}q_{21} + q_{13}q_{13}^{\dagger} + q_{13}^{\dagger}q_{31}\right)$. The integral I_{bos} can be calculated by expliciting the Pauli matrices τ^c. One arrives at the expression

$$I_{bos} = \int_0^{\infty} d^3D^c d^2 w_{\dot{\alpha}11} d^2 w_{\alpha 11} \left(-\frac{\partial}{\partial M_1} - \frac{\partial}{\partial M_4}\right) e^{-\left(\begin{array}{cc} \bar{w}_{\dot{1}11} \bar{w}_{\dot{2}11} \end{array}\right)\left(\begin{array}{cc} M_1 & M_2 \\ M_3 & M_4 \end{array}\right)\left(\begin{array}{c} w_{\dot{1}11} \\ w_{\dot{2}11} \end{array}\right)} \tag{3.47}$$

where $M_1 = -iD^3 + \frac{1}{2}g(\{q\})$, $M_2 = -iD^1 - D^2$, $M_3 = -iD^1 + D^2$ and $M_4 = iD^3\frac{1}{2}g(\{q\})$. The integration over the zero modes w and $\bar{w}$ gives

$$I_{bos} = \int_0^{\infty} d^3D \left(-\frac{\partial}{\partial M_1} - \frac{\partial}{\partial M_4}\right) \frac{1}{M_1M_4 - M_2M_3} \tag{3.48}$$

If we define $D^2 = \sum (D^c)^2$ the bosonic integral can be written as

$$I_{bos} = \int_0^{\infty} d^3D \frac{g(\{q\})}{(D^2 + \frac{1}{4}g(\{q\}))^2} \tag{3.49}$$

In spherical coordinates, $d^3D = D^2 dD d\Omega$, the angular measure gives a 4π factor and the linear variable can be rescaled as $D' = 2D/g(\{q\})$. The integral is

$$I_{bos} = 4\pi \int_0^{\infty} dD' \frac{D'}{(D'^2 + 1)^2} = \pi^2 \tag{3.50}$$

This shows that the contribution of the bosonic integral is only a non vanishing numerical factor.

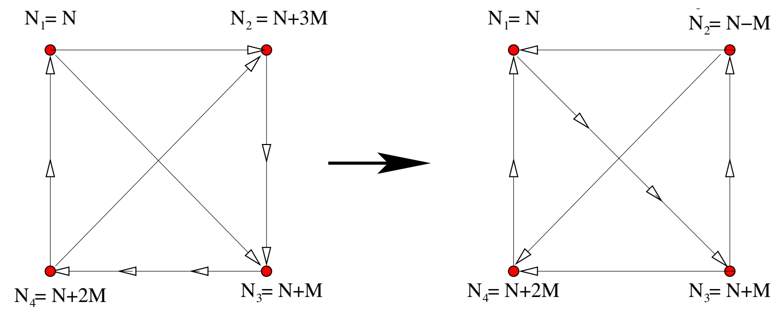


Figure 3.5: Quivers representing the two dual phases studied for dP_1

We are left with the following fermionic integral

$$W_{inst} \sim \int d^{N_2}\bar{\mu}_{12} d^{N_2}\mu_{21} d^{N_3}\bar{\mu}_{13} d^{N_3}\mu_{31} e^{-S_{inst}} \tag{3.51}$$

The relevant part of the action for this integral is the last line in (3.44). It can be rearranged as

$$S_{inst} = \cdots - \frac{i}{2} \begin{pmatrix} \bar{\mu}_{12} & \bar{\mu}_{13} \end{pmatrix} \begin{pmatrix} M_{22} & -q_{21}q_{13} \\ -q_{31}q_{12} & Q_{32}Q_{23} \end{pmatrix} \begin{pmatrix} \mu_{21} \\ \mu_{31} \end{pmatrix} \tag{3.52}$$

and the fermionic integration (3.51) gives the contribution

$$W_{inst} \sim \det \begin{pmatrix} M_{22} & -q_{21}q_{13} \\ -q_{31}q_{12} & Q_{32}Q_{23} \end{pmatrix} \tag{3.53}$$

This is exactly the same determinant contribution we have obtained in (3.43). The correspondence between the superpotential terms holds. Indeed, adding (3.53) to the tree level superpotential for the quiver in figure 3.3, we exactly recover (3.43).

dP_1

Here we study the the chiral dP_1 toric quiver gauge theory. The quiver of the theory is in figure 3.5.a . The superpotential is

$$W = \epsilon_{\alpha\beta} X_{23}^{\alpha} X_{34}^{\beta} X_{42} + \epsilon_{\alpha\beta} X_{34}^{\alpha} X_{41}^{\beta} X_{13} - \epsilon_{\alpha\beta} X_{12} X_{23}^{\alpha} X_{34}^{3} X_{41}^{\beta} \tag{3.54}$$

We choose the ranks to be

$$N_1 = N \qquad N_2 = N + 3M \qquad N_3 = N + M \qquad N_4 = N + 2M$$

and consider strong dynamics for the node 2. The dual degrees of freedom are the dual quarks $(b_{32}^{\alpha}, b_{21}, b_{24})$ and the mesons $(M_{13}^{\alpha}, M_{43}^{\alpha})$. The resulting quiver is in figure 3.5.b and the superpotential, after integrating out the massive matter, is

$$W = \epsilon_{\alpha\beta} b_{32}^{\alpha} b_{24} X_{41}^{\beta} X_{13} + \epsilon_{\alpha\beta} M_{13}^{\alpha} b_{32}^{\beta} b_{21} - \epsilon_{\alpha\beta} M_{13}^{\alpha} X_{34}^{3} X_{41}^{\beta} \tag{3.55}$$

We used the equations of motion of the massive fields $(M_{43}^{\alpha}, X_{34}^{\alpha})$ that fix

$$M_{43}^1 = X_{41}^1 X_{13} \qquad M_{43}^2 = X_{41}^2 X_{13} \tag{3.56}$$

Choosing $N = M + 1$ we are in the limiting case of Seiberg duality, where the dualized magnetic gauge group is $SU(1)$. The classical constraint on the moduli space in this case is implemented adding to the superpotential (3.55) a determinant term

$$\Delta W = \det \mathcal{M} = \det \begin{pmatrix} M_{13}^1 & -M_{13}^2 \\ -M_{43}^1 & M_{43}^2 \end{pmatrix} = \det \begin{pmatrix} M_{13}^1 & -M_{13}^2 \\ -X_{41}^1 X_{13} & X_{41}^2 X_{13} \end{pmatrix} \tag{3.57}$$

where we used the equation of motions to express it as a function of the fields of the effective theory.

We now recover the same contribution as a stringy instanton effect in the magnetic theory, the one in figure 3.5.b. We place a stringy instanton on the $SU(1)$ node. The saturation of the zero modes proceed as usual and we are left with the following integral

$$W_{inst} \sim \int (d\bar{\mu}_{24})^{N_4} \, (d\bar{\mu}_{21})^{N_1} \, (d\bar{\mu}_{32}^1)^{N_2} \, (d\bar{\mu}_{32}^2)^{N_2} \, e^{-S_{inst}} \tag{3.58}$$

The relevant part of the instantonic action is (3.26) and can be deduced from the superpotential (3.55) to be

$$S_{inst} \supset \epsilon_{\alpha\beta} \bar{\mu}_{24} X_{41}^{\beta} X_{13} \mu_{32}^{\alpha} + \epsilon_{\alpha\beta} \bar{\mu}_{21} M_{13}^{\alpha} \mu_{32}^{\beta} = \begin{pmatrix} \bar{\mu}_{21} & \bar{\mu}_{24} \end{pmatrix} \begin{pmatrix} M_{13}^1 & -M_{13}^2 \\ -X_{41}^1 X_{13} & X_{41}^2 X_{13} \end{pmatrix} \begin{pmatrix} \mu_{32}^2 \\ \mu_{32}^1 \end{pmatrix} \tag{3.59}$$

Performing the fermionic integrals we then find that

$$W_{inst} \sim \det \begin{pmatrix} M_{13}^1 & -M_{13}^2 \\ -X_{41}^1 X_{13} & X_{41}^2 X_{13} \end{pmatrix} \tag{3.60}$$

that is $W_{inst} = \Delta W$ as claimed. The stringy instanton contribution has been exactly mapped to the strong dynamics effect.

Orthogonal and symplectic gauge groups

In this section we generalize the correspondence to orthogonal and symplectic gauge groups. Quiver gauge theories with these groups can be obtained from unitary gauge groups applying orientifold projections. Since we consider toric quiver gauge theories, we use the technology developed in [118] to perform orientifold projections on dimer models [70, 71, 119, 120, 121, 122]. We review this procedure in appendix A.2

The O-plane projects out some degrees of freedom both in the gauge sector and in the instanton sector. As a consequence the number of bosonic and fermionic zero modes and the corresponding ADHM constraints are different [125].

The stringy instanton contribution to the superpotential for symplectic and orthogonal gauge groups has been studied in [16, 81, 82, 87]. Non trivial contributions, in analogy

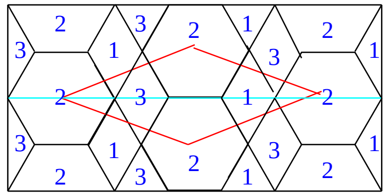


Figure 3.6: Dimer model for the fixed line orientifold of the SPP.

with the $SU(1)$ case, arise for stringy instantons of $SP(0)$ and $SO(3)$ gauge groups. The auxiliary instantonic groups are in these cases $O(1)$ and $SP(2)$, respectively[2].

The relation between stringy instantons and strong dynamics effects of the gauge theory holds also in these cases. Electric magnetic dualities have been studied in [1, 123, 124] for symplectic and orthogonal gauge groups. For these groups there exist limiting cases of the duality, as the $N_f = N_c + 1$ case for $SU(N_c)$. They are respectively the $N_f = N_c + 4$ for $SP(N_c)$ and $N_f = N_c - 1$ for $SO(N_c)$ gauge groups. For unitary groups the dual description is a $SU(1)$ gauge theory. For symplectic and orthogonal gauge groups the dual descriptions are $SP(0)$ and $SO(3)$, respectively. Indeed they are the configuration where stringy instanton effects add to the superpotential.

We now point out the agreement between stringy instanton and gauge theory analysis with some example.

The orthogonal case

In this subsection we study an orientifold projection of the SPP. We choose an orientifold from the dimer with a fixed line, where the unit cell of the dimer has a rhombus geometry (see Figure 3.6). The orientifold charge for the fixed line is chosen positive. In this case all the unitary groups $SU(N_i)$ become orthogonal $SO(N_i)$ groups. Half of the bifundamentals survive the projection, and they become

$$q_{i,j} = (\square_i, \square_j) \tag{3.61}$$

The adjoint field M_{22} is projected to a symmetric representation. We then choose the number of fractional branes for each group such that

$$N_1 = N \quad N_2 = 3 \quad N_3 = 0 \tag{3.62}$$

This theory is described by the superpotential

$$W = q_{12}^T q_{12} M_{22} \tag{3.63}$$

We add D-brane instantons on the $SO(3)$ node. In the ADHM construction of $SO(N)$ $\mathcal{N} = 4$ SYM the instantonic auxiliary group [125] is $SP(k)$. In this case the counting

[2] In our convention $SP(2) \simeq SU(2)$.

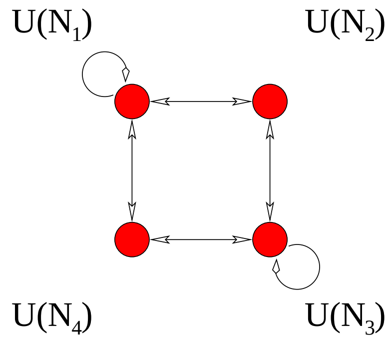


Figure 3.7: Zero modes (in green) for the $\tilde{SP}(2)$ instanton placed on an $SO(3)$ gauge group.

of the zero modes tells that stringy instanton contributes to the superpotential if the auxiliary group is $SP(2)$. The orientifolded quiver gauge theory with the instanton and the relative zero modes are shown in the figure 3.7. The action for the zero modes is

$$\begin{aligned} S_{inst} &= i\lambda^i_{\dot\alpha}\left(w_1^{\dot\alpha a}\sigma^i_{ab}\mu_1^b\right) + i\lambda'^i_{\dot\alpha}\left(\bar w_1^{\dot\alpha a}\sigma^i_{ab}\mu_1^b\right) - iD^k_i\left(w_1^{\dot\alpha a}\sigma^k_{\dot\alpha\dot\beta}\sigma^i_{ab}\bar w_1^{\beta b}\right) \\ &+ \frac{1}{2}\left(w_1^{\dot\alpha a}\left(q_{12}q^\dagger_{12}\right)\bar w_1^{\dot\alpha a} + i\mu_2^a q^\dagger_{12}\mu_1^b\epsilon_{ab} - i\mu_2^a\epsilon_{ab}\mu_2^{T^b}M_{22}\right) \end{aligned} \tag{3.64}$$

where a and b are $SP(2)$ indexes. Imposing the reality conditions we find six independent $\lambda^i_{\dot\alpha}, \lambda'^i_{\dot\alpha}$ and four M_α zero modes. The a_μ, in the adjoint of $SP(2)$, are symplectic antisymmetric matrices. This representation has dimension 1, which implies that there are four zero modes from a_μ. The D_c are nine while there are twelve independent $w_1^{\dot\alpha a}, \bar w_1^{\dot\alpha a}$ bosonic spinors. There are six fermionic μ_1^a fields connecting the gauge group $SO(3)$ with the auxiliary instantonic group $SP(2)$. The sector connecting the $SP(2)$ instanton with the flavor group gives $2N_f$ fermionic zero modes μ_2^a. We can now perform the integration over the fermionic and bosonic zero modes to obtain the instanton contribution. The (a_μ, M_α) zero modes are as usual interpreted as superspace coordinates, giving the superpotential contribution

$$W_{inst} = \mathcal{C}\int d\{\lambda, \lambda', D, \omega_1, \bar\omega_1, \mu_1, \mu_2\}\, e^{-S_{inst}} \tag{3.65}$$

We discuss in the appendix B.3 the bosonic integration and the dimension-full constant $\mathcal{C}$. We only quote here the nine ADHM bosonic constraints obtained integrating over the D^k_i

$$\delta^{(9)}\left(w_c^{\dot\alpha a}\sigma^k_{\dot\alpha\dot\beta}\sigma^i_{ab}\bar w_c^{\beta b}\right) \tag{3.66}$$

Now we focus on the fermionic integration. The integration over the $\lambda^i_{\dot\alpha}, \lambda'^i_{\dot\alpha}$ fermionic zero modes can be done using the first two terms in (3.64) and it gives the six ADHM fermionic constraints

$$\delta^{(3)}(\omega_c^a\sigma^i_{ab}\mu_c^b)\delta^{(3)}(\bar\omega_c^a\sigma^i_{ab}\mu_c^b) \tag{3.67}$$

This saturate also the fermionic integration on μ_1^a in (3.65). We are left with the fermionic integral

$$W_{inst} \sim \int [d\mu_2^a]\, e^{-S_{inst}} \tag{3.68}$$

The integration is done expanding the relevant part of the action in the exponent

$$S_{inst} \supset \mu_2^1 {\mu_2^T}^2 M_{22} - \mu_2^2 {\mu_2^T}^1 M_{22} = \begin{pmatrix} \mu_2^1 & \mu_2^{T2} \end{pmatrix} \begin{pmatrix} 0 & M_{22} \\ -M_{22} & 0 \end{pmatrix} \begin{pmatrix} \mu_2^1 \\ \mu_2^{T2} \end{pmatrix} \tag{3.69}$$

The gaussian integration gives the contribution

$$W_{inst} \sim \mathrm{Pf} \begin{pmatrix} 0 & M_{22} \\ -M_{22} & 0 \end{pmatrix} = \det M_{22} \tag{3.70}$$

This last equality holds since M is a symmetric matrix. In appendix B.3 we show, using dimensional analysis, that the bosonic integral is adimensional. This means that it is independent from the physical fields and it gives only a constant contribution. We conclude that (3.70) is the $SP(2)$ stringy instanton contribution on the $SO(3)$ node.

We now argue that the same relationship between stringy instanton and strong dynamics that holds in the case of unitary groups is valid also in this situation.

Once again we exploit the involutive property of Seiberg duality. We thus perform two consecutive Seiberg duality, recovering in the end the starting theory. The first one is a formal Seiberg duality on the $SO(3)$ node with N flavours, and we obtain the theory one step backwards. This gives an $SO(\tilde{N} = N_f - N_c + 4 = N + 1)$ gauge group with N flavor. The superpotential of this theory is

$$W = Q_{12}^T Q_{12} N_{22} + N_{22} M_{22} \tag{3.71}$$

Integrating out the massive field this superpotential vanishes. We perform then another Seiberg duality. Since for this theory $N_f = N_c - 1$ we are in the limiting case of Seiberg duality for orthogonal gauge groups. The dual gauge group is $SO(N_f - N_c + 4 = 3)$ and the superpotential

$$W = q_{12}^T q_{12} M_{22} + \det M_{22} \tag{3.72}$$

where we have added the determinant to take into account the classical constraint on the moduli space.

We have thus recovered in (3.72) the starting superpotential (3.63) and the stringy instanton contribution (3.70). So also for orthogonal gauge group we have mapped the stringy instanton contribution in strong dynamics effects.

The symplectic case

We first consider symplectic SQCD. We take $SP(0)$ as the gauge group, with $SP(N)$ flavours. There is a meson M in the antisymmetric representation and there is no superpotential.

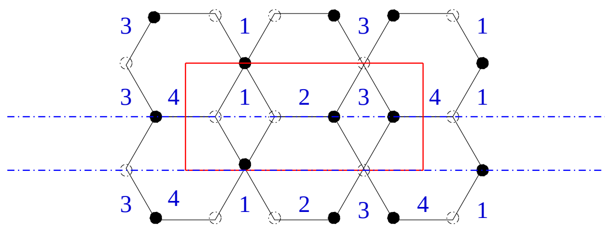


Figure 3.8: Dimer for the orientifold of the double conifold. The dashed blue lines represents the orientifold fixed lines.

It has been shown that a stringy instanton on the $SP(0)$ gauge group gives a non trivial contribution to the superpotential. In the ADHM construction the instantonic auxiliary group for a symplectic gauge group is $O(k)$. The non perturbative contribution is obtained if the instantonic number is $k = 1$, with auxiliary group $O(1)$. There are no fermionic and bosonic ADHM constraint, no w, D and λ fields. There are two M_α and four a_μ which are interpreted as the superspace coordinates. The instantonic action is given by the interaction of the meson with the fermionic zero modes μ connecting the $O(1)$ instanton and the flavor group

$$S = -\frac{i}{2}\mu M \mu^T \tag{3.73}$$

The superpotential contribution is obtained integrating over the μ fermionic zero modes

$$W_{inst} \sim \int d[\mu] e^{-S} = \text{Pf} M \tag{3.74}$$

Also for symplectic gauge groups we relate this contribution to strong dynamics effects. Through a formal electric magnetic duality on the $SP(0)$ node we obtain the dual theory. It is an $SP(\tilde{N} = N_f - N_c - 4 = N - 4)$ gauge group with N flavours and no mesons. We then perform another duality obtaining a $SP(N_f - N_c - 4 = N - \tilde{N} - 4 = 0)$ gauge group, where the only degree of freedom is the meson M. This is the starting theory. However, since we are in the limiting case of Seiberg duality for symplectic gauge group, we also obtain the following contribution to the superpotential

$$W_{eff} = \text{Pf} M \tag{3.75}$$

which implement the classical constraints on the moduli space. The equivalence between (3.74) and (3.75) shows that, also for symplectic gauge groups, the strong dynamic effect coincides with the stringy instanton contribution to the superpotential.

Example: Orientifold of the double conifold

In this subsection we give an example of the correspondence using an orientifold of the double conifolds. The dimer model is represented in figure 3.8. The unit cell is delimited

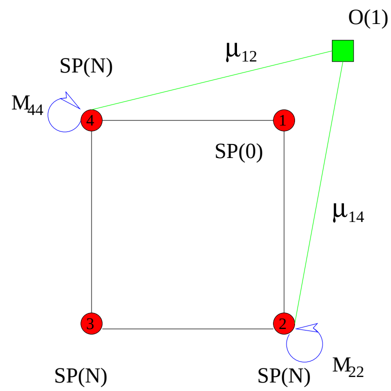


Figure 3.9: Stringy instanton on the orientifolded double conifold. The green lines represent the fermionic zero modes in the instantonic action.

by the red lines. The projection is done by the two independent dashed blue fixed lines. We choose their orientifold charge to be negative. This implies that all the groups are symplectic. The bifundamentals fields are in the $(\square_i, \square_j)$ representation of the $SP(2N_i) \times SP(2N_j)$ gauge groups. The fields in the adjoint representation of the $SU(N_i)$ gauge groups are now in the antisymmetric representation of the $SP(2N_i)$ groups. The rank of the first group $SP(2N_1)$ is chosen to be zero. The choice of the others ranks is free. Here, for simplicity, we choose the same rank N for all of them. The superpotential for this theory is

$$W = M_{22} \cdot Q_{23} \cdot Q_{23} - Q_{23} \cdot Q_{23} \cdot Q_{34} \cdot Q_{34} + M_{44} \cdot Q_{34} \cdot Q_{34} \tag{3.76}$$

where the $\cdot$ represent the symplectic products.

We add a stringy instanton on the $SP(0)$ node and we study its contribution to the superpotential. The zero modes are shown in figure 3.9. The instantonic action is

$$S_{\text{inst}} = -\frac{i}{2}\mu_{12} M_{22} \mu_{12}^T - \frac{i}{2}\mu_{14} M_{44} \mu_{14}^T \tag{3.77}$$

The integration over the fermionic zero modes μ_{12} and μ_{14} gives a non perturbative contribution

$$W_{inst} \sim \int d[\mu_{12}] d[\mu_{14}] e^{-S_{\text{inst}}} = \text{Pf} M_{22} \text{Pf} M_{44} \tag{3.78}$$

to the superpotential (3.76).

The same result can be found from the gauge theory analysis. The theory one step backwards in Seiberg duality is obtained by a formal duality on the $SP(0)$ node. We get a $SP(2N-4)$ gauge group with $2N$ flavours. The superpotential of this dual theory is

$$\begin{aligned} W &= Q_{12} \cdot Q_{12} \cdot Q_{23} \cdot Q_{23} - Q_{23} \cdot Q_{23} \cdot Q_{34} \cdot Q_{34} \\ &+ Q_{34} \cdot Q_{34} \cdot Q_{41} \cdot Q_{41} - Q_{41} \cdot Q_{41} \cdot Q_{12} \cdot Q_{12} \end{aligned} \tag{3.79}$$

We perform then another Seiberg duality to go back to the starting theory. We obtain a $SP(0)$ gauge group and superpotential

$$W = -M_{24} \cdot M_{42} + M_{22} \cdot Q_{23} \cdot Q_{23} - Q_{23} \cdot Q_{23} \cdot Q_{34} \cdot Q_{34} + M_{44} \cdot Q_{34} \cdot Q_{34} + \mathrm{Pf} \begin{pmatrix} M_{22} & M_{24} \\ M_{42} & M_{44} \end{pmatrix} \tag{3.80}$$

The mesons are defined as $M_{ij} = q_{i\lambda_i} J_{\lambda_1\lambda_2} q_{j,\lambda_2}$. Since we are in the case $N_f = N_c + 4$ we have added the non perturbative term to implement the classical constraint on the moduli space. Note that the antisymmetry of J_{λ_1,λ_2} implies that $M_{42} = -M_{24}^T$. Integrating out the massive fields (M_{24}, M_{42}) we recover the starting theory (3.76) plus the non perturbative contribution

$$W_{np} = \mathrm{Pf} M_{22} \mathrm{Pf} M_{44} \tag{3.81}$$

This is exactly the same contribution obtained from the stringy instanton computation (3.78).

In this section we have considered stringy instantons in toric quiver gauge theories deriving from $D3/D(-1)$ systems. We have provided an interpretation for the stringy instanton contribution as a strong dynamics effect by analyzing the theory one step backwards in Seiberg duality, for the node where the instanton is located. Our result is valid for stringy instantons on $SU(1)$, $SP(0)$ and $SO(3)$ nodes[3].

There are interesting aspects we have not discussed here. The results we presented could be extended to non toric quiver gauge theories. Our analysis might also be useful in understanding the role played by stringy instantons in dynamical supersymmetry breaking in quiver gauge theories. Another issue would be the study of non-rigid instantons and multi-instantons effects in toric quiver gauge theories, and their relation to strong dynamics of the gauge theory. Finally[4] a similar correspondence should exist for instantonic higher F-term contributions in relation with strong dynamics leading to magnetic $SU(0)$ gauge group, i.e. the $N_f = N_c$ case.

[3]See [126, 127] for related discussion in the context of matrix models.

[4]We thank A. M. Uranga for suggesting this to us.

Chapter 4

Three dimensions

Recently a breakthrough toward the explicit realization of the AdS_4/CFT_3 correspondence was done in [128] where the authors propose $U(N)_k \times U(N)_{-k}$ CS matter theories, with $\mathcal{N} = 6$ supersymmetry, to be the low energy theories of N M2-branes at the $\mathbb{C}^4/\mathbb{Z}_k$ singularities. Afterward, this construction has been extended to many others CS matter theories with a lower amount of supersymmetries [129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141].

A subclass of $\mathcal{N} = 2$ quiver gauge theories have then been conjectured to be dual to M theory on $AdS_4 \times SE_7$, where SE_7 is a seven dimensional Sasaki Einstein manifold. The theories are Chern-Simons matter theories associated with CY_4 toric singularity.

In this chapter we propose a three dimensional Seiberg duality by studying the classical moduli space. In this case the computation simplifies because of the powerful of the toric geometry. Indeed the moduli space is encoded in the toric diagram, and we propose that the identification, up to an $SL(3, Z)$ transformation, of the toric diagrams (Toric Duality) is Seiberg duality.

Since this duality is of the strong/weak type one can provide a perturbative study of supersymmetry breaking in the dual theory. However higher orders in the perturbative expansions are usually important in three dimensions. It the last part of this chapter we analyze some method to make the perturbative expansion valid.

4.1 The ABJM model

In this section we review the basic properties of the ABJM model. It is a CS matter theory with gauge group $U(N) \times U(N)$ and with an $\mathcal{N} = 6$ superconformal symmetry. It is a special case of a $\mathcal{N} = 3$ theory, where supersymmetry is enhanced for a specific choice of the gauge groups and field content to $\mathcal{N} = 6$. We start the review of CS gauge theories starting from the $\mathcal{N} = 2$ case. A pure CS theory in 2+1 dimensions is a topological theory. After been coupled to matter fields the theory is not topological anymore, but it can be conformal invariant. The $\mathcal{N} = 2$ CS matter theory with no superpotential is an example of an exactly conformal theory of this class. This theory is obtained from dimensional reduction from four dimensions. Dimensional reduction gives a vector multiplet V in the adjoint of the gauge group and chiral multiplets Φ charged under the gauge group. The chiral multiplet has canonical kinetic term. The vector multiplet has a CS supersymmetric kinetic term. When it is written in components, in the WZ gauge the kinetic CS term takes the form

$$S_{\mathcal{N}=\in} = \frac{k}{4\pi}\int \mathrm{Tr}\left(A\wedge A + \frac{2}{3}A^3 - \chi\bar{\chi} + 2\sigma D\right) \tag{4.1}$$

In this superpotential χ is the *Gaugino* field, D is the auxiliary field of the vector multiplet, and σ is another scalar in the vector multiplet, that arises from dimensional reduction). If the gauge symmetry is non abelian the level k is quantized. Moreover it is an integer number in the case of unitary group if the trace is in the fundamental representation.
The fields of the vector multiplet have no kinetic terms, and they are all auxiliary. Integrating out the field D and χ the action is

$$\begin{aligned} S = &\int \frac{k}{4\pi}\mathrm{Tr}\left(A\wedge A + \frac{2}{3}A^3\right) + D_\mu\bar{\phi}_1 D^\mu\phi_i + i\bar{\psi}_i\gamma^\mu D_\mu\psi_i \\ &-\frac{16\pi^2}{k^2}(\bar{\phi}_i T^a_{R_i}\phi_i)(\bar{\phi}_j T^b_{R_j}\phi_j)\bar{\phi}_k T^a_{R_k} T^b_{R_k}\phi_(k) \\ &-\frac{4\pi}{k}(\bar{\phi}_i T^a_{R_i}\phi_i)(\bar{\psi}_j T^a_{R_j}\psi_j) - \frac{8\pi}{k}(\bar{\psi}_i T^a_{R_i}\phi_i)(\bar{\phi}_j T^a_{R_j}\psi_j) \end{aligned} \tag{4.2}$$

This action preserves conformal invariance also at quantum level.
The $\mathcal{N}=3$ generalization of this action is obtained by considering the field content of an $\mathcal{N}=4$ theory. An additional auxiliary field φ in the adjoint is added to the vector multiplet. The action includes a term $\tilde{\Phi}_i\varphi\Phi_i$, and the CS term has an additional term $-\frac{k}{8\pi}\mathrm{Tr}(\varphi^2)$. ϕ is an auxiliary field that can be integrated out. It gives

$$W = \frac{4\pi}{k}(\tilde{\Phi}_i T^a_{R_i}\Phi_i)(\tilde{\Phi}_j T^a_{R_j}\Phi_j) \tag{4.3}$$

This action is the same of the $\mathcal{N}=3$ at which the term (4.3) has been added. The R-symmetry group in three dimensions is $SO(\mathcal{N})$. This theory has also an $U(N_F)$ global symmetry. In a special case the $\mathcal{N}=3$ theory has gauge group $U(N)\times U(N)$ with two pairs of hypermultiplets (A_1, A_2) and (B_1, B_2). The CS levels are equal and opposite, k and $-k$, After integration of massive matter the superpotential becomes

$$W = \frac{4\pi}{k}Tr(A_1B_1A_2B_2 - A_2B_1A_1B_2) \tag{4.4}$$

After a counting of the global symmetries associated to the superconformal group one shows that in this case supersymmetry is enhanced to $\mathcal{N}=6$. The coupling constant is $1/k$ and at large k the theory is weakly coupled. In the large N limit, at fixed N/k one can expand in $1/N^2$ and consider only the planar contribution. The effective coupling constant in the planar diagrams is the 't Hooft coupling $\lambda = N/k$. The theory is weakly coupled for $N \ll k$ and strongly coupled for $N \gg k$.

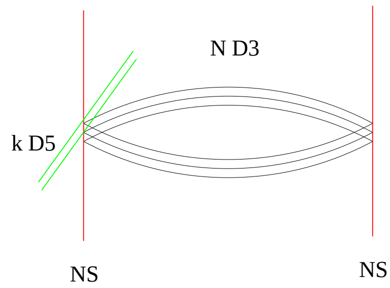


Figure 4.1: The $\mathcal{N} = 2$ brane configuration in IIB with the$k$ D5 branes

Figure 4.2: Web deformation of the system of NS5 and D5 branes, and creation of the fivebrane.

This field theory can be also described as a system of brane in type II B string theory. The ABJM model is realized from branes as an $\mathcal{N} = 3$ supersymmetric theory that flows in the IR to the $\mathcal{N} = 6$ theory. Consider a system of two parallels NS5 branes along (012345) and separated along 6, which is a compact direction, and N D3 branes along 0126. The common (012) directions are the coordinates of the three dimensional field theory. The N D3 branes can break on the NS5 branes, and this implies that there are two $U(N)$ gauge groups. The bifundamental fields are the open strings that ends on the D3 and cross the NS, the A_i and B_i fields. This is an $\mathcal{N} = 4$ theory $U(N) \times U(N)$ gauge theory.

Then one adds k D5 branes along (012349) as in figure 4.1, that intersect the D3 along (012) and one of the NS5 along (01234). Supersymmetry is broken to $\mathcal{N} = 2$ and $2k$ massless chiral multiplet (a two component Majorana fermion and a complex scalar), k in the fundamental and k in the antifundamental are added.

The CS term is obtained from this brane configuration via mass deformation. The mass deformation of interest is a web deformation, in which the k D5 and the NS break along the 1234 directions and generate a $(1, k)$ or $(1, -k)$ fivebrane. The angle in the 59 placed is chosen to be $\tan\theta = k$. This process is represented in figure 4.2

The CS terms are produced from parity anomaly after integrating out the fermions in the chiral and anti-chiral multiplet. Each Majorana fermion with positive mass contributes with a $+1/2$ coefficient to the CS level and each negative mass term gives a $-1/2$ contribution. At the end of this process one group has a level k and the other has level $-k$.

The final configuration is an $\mathcal{N}=2$ theory with one NS5 brane along (012345) and a $(1,k)$ fivebrane along $(1234[59]_\theta)$, where $[59]_\theta = x_5 \cos\theta + X_9 \sin\theta$.

The $\mathcal{N}=3$ theory is found by rotating the $(1,k)$5-brane in the 37 and 78 planes. If the two angles are equal $\mathcal{N}=2$ supersymmetry is preserved. If the angles coincide with θ, the angle in the 9 plane, supersymmetry is enhanced to $\mathcal{N}=3$ and this is the brane picture that realizes the high energy ABJM theory (see figure 4.3). The $\mathcal{N}=6$ theory is then recovered integrating out all the massive fields. This IIB brane construction turns

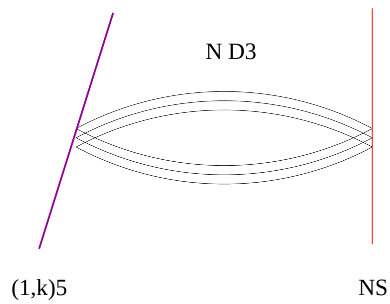


Figure 4.3: The $\mathcal{N}=3$ configuration reproducing ABJM in IIB.

into a IIA description through a T-duality along the x^6 compact direction. Note the D3 branes transform in D2 branes. Finally one lifts the configuration to M theory, and the D2 lift to M2 branes. In this M-theory picture the IR limit becomes the near horizon limit of N M2 branes probing a $\mathbb{C}^4/\mathbb{Z}_k$ singularity.

The ABJ model: fractional branes

As we explain in chapter 1 Seiberg duality in four dimensions has a direct interpretation in systems of intersecting branes, as an exchange of NS5 branes. In three dimensions there is an analog argument, as explained in [142]. The exchange of a $(1,k)$ and a NS5 brane has been interpreted in [19] as a Seiberg duality in SQCD CS gauge theories in three dimensions. In the ABJM model this exchange of branes produce D3 fractional branes in the compact direction. This imply that Seiberg duality in this three dimensional scenario is intimately connected with fractional branes. Fractional branes in the ABJM model were studied in [143]. Here we review the relevant aspect of their result for our analysis. Adding l fractional D3 branes as in figure 4.4 changes the gauge groups from $U(N)\times U(N)$ to $U(N+l)\times U(N)$, while the field content and the interactions remain the same. The classical moduli space is unchanged. Indeed the l fractional D3 do not have a moduli space because they branes are not free to move, they are constrained by NS5 brane and the $(1,k)$ fivebrane.

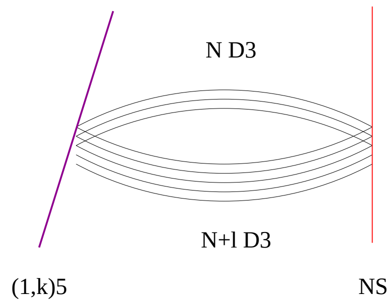


Figure 4.4: Addition of k D3 branes to the $\mathcal{N} = 3$ theory

There is a constraint on the number of fractional brane that we can add. There cannot be more than one D3 brane connecting a D5/NS5 pair. This condition is the s-rule [142]. In this case this rule imposes that a supersymmetric configuration is found only if $l \leq k$.

If one moves one of the fivebrane along the compact direction another effect of [142] becomes important. When one NS5 crosses a $(1, k)$ fivebrane k D3 branes are created. In this case if the NS5 brane crosses the $(1, k)$ fivebrane as in figure 4.5 we are left with $k - l$ branes.

The equivalence stated in [143] is between the $U(N+l)_k \times U(N)_{-k}$ theory and the new theory whose gauge group content is $U(N)_{-k} \times U(N + k - l)_k$. The superpotential and the field content of the two theories does not change, and in terms of Seiberg duality this is due to the self similarity of the superpotential. The fivebranes can crosses more than once, but the new configuration are not supersymmetric, and they have been ignored. We

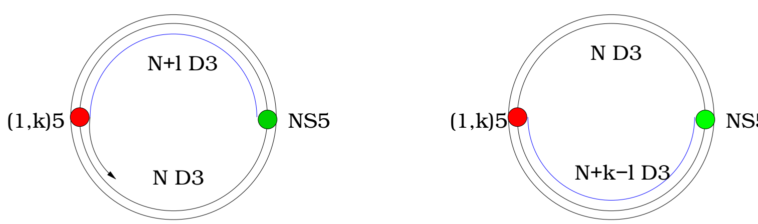


Figure 4.5: Displacing the $(1, k)$ and the NS

are interested to the $k = l$ case, that relates the original ABJM theory and a theory with fractional branes.

$$U(N + k)_k \times U(N)_{-k} \leftrightarrow U(N)_{-k} \times U(N)_k \tag{4.5}$$

The two theories have the same classical moduli space, and are equivalent. In three dimensional CS gauge theories a theory with no fractional branes is connected with a theory containing fractional branes. This relation will be important in the derivation of the rules for three dimensional Seiberg like duality.

4.2 $N = 2$ toric CS matter theory

A large and interesting, but still very peculiar, class of AdS$_4$/CFT$_3$ pairs is realized by M2 branes at Calabi Yau four-fold toric singularities [134, 135, 136, 140, 141, 144, 145]. The low energy theories are proposed to be a special kind of $\mathcal{N} = 2$ Chern-Simons matter theories. It was soon realized [136] that, as in the AdS$_5$/CFT$_4$ case, different $\mathcal{N} = 2$ Chern-Simons matter theories can be associated with the same Calabi Yau fourfold geometry.

$\mathcal{N} = 2$ Chern-Simons matter theories for M2 branes at singularities are typically described by a quiver with an assignment of Chern-Simons levels k_i and a superpotential in a way similar to the gauge theories for D3 branes at Calabi Yau three-fold singularities. Indeed a class of $\mathcal{N} = 2$ three dimensional theories can be simply obtained from four dimensional $\mathcal{N} = 1$ quivers with superpotential in the following way: rewrite the theory in three dimensions, change the $SU(N)$ gauge factors to $U(N)$ factors, disregard the super Yang-Mills actions and add a super Chern-Simons term for every factors. We will say that these three dimensional theories have a four dimensional parent. Viceversa we will call theories without four dimensional parents the three dimensional theories that cannot be obtained in the way just explained [141].

M2 branes and $\mathcal{N} = 2$ Chern Simons theories

As discussed in the introduction, supersymmetric Chern Simons theories coupled to matter fields are good candidates to describe the low energy dynamics of M2 branes [128, 146]. We are interested in M2 branes at Calabi Yau four fold toric conical singularities. The authors of [135] proposed that the field theories living on these M2 branes are $(2 + 1)$ dimensional $\mathcal{N} = 2$ Chern Simons theories with gauge group $\prod_{i=1}^{G} U_i(N)$ with bifundamental and adjoint matter fields. The Lagrangian in $\mathcal{N} = 2$ superspace notation is:

$$\mathrm{Tr}\left(-i\sum_a k_a \int_0^1 dt V_a \bar{D}^{\alpha}(e^{tV_a} D_{\alpha} e^{-tV_a}) - \int d^4\theta \sum_{X_{ab}} X_{ab}^{\dagger} e^{-V_a} X_{ab} e^{V_b} + \int d^2\theta W(X_{ab}) + c.c.\right) \tag{4.6}$$

where V_a are the vector superfields and X_{ab} are bifundamental chiral superfields. The superpotential $W(X_{ab})$ satisfies the toricity conditions: every field appears just two times: one time with plus sign and the other time with minus sign. Since these theories are conjectured to be dual to M theory on $AdS_4 \times SE_7$, where SE_7 is a seven dimensional Sasaki Einstein manifold, the moduli space of these theories must contain a branch isomorphic to the four-fold Calabi Yau real cone over SE_7: $\mathcal{M}_4 = C(SE_7)$. To study the moduli space we need to find the vanishing conditions for the scalar potential. The scalar potential is:

$$\mathrm{Tr}\left(-4\sum_a k_a \sigma_a D_a + \sum_a D_a \mu_a(X) - \sum_{X_{ab}} |\sigma_a X_{ab} - X_{ab}\sigma_b|^2 - \sum_{X_{ab}} |\partial_{X_{ab}} W|^2\right)$$

where $\mu_a(X) = \sum_b X_{ab}X_{ab}^{\dagger} - \sum_c X_{ca}^{\dagger} X_{ca} + [X_{aa}, X_{aa}^{\dagger}]$, σ_a and D_a are scalar components of the vector superfield V_a, and with abuse of notation X_{ab} is the lowest scalar component of the chiral superfield X_{ab}. The moduli space is the zero locus of the scalar potential and it is given by the equations:

$$\partial_{X_{ab}} W = 0, \qquad \sigma_a X_{ab} - X_{ab}\sigma_b = 0, \qquad \mu_a(X) = 4k_a\sigma_a. \tag{4.7}$$

In [135] it was shown that if

$$\sum_a k_a = 0 \tag{4.8}$$

then the moduli space contains a branch isomorphic to a four-fold Calabi Yau singularity. This branch is interpreted as the space transverse to the M2 branes. Let us start with the abelian case in which the gauge group is $U(1)^G$. We are interested in the branch in which all the bifundamental fields are generically different from zero. In this case the solution to the first equation in (4.7) gives the irreducible component of the master space ${}^{\text{Irr}}\mathcal{F}^\flat$ [147, 148]. The second equation in (4.7) imposes $\sigma_{a_1} = ... = \sigma_{a_G} = \sigma$. The last equation in (4.7) are G equations; the sum of all the equations gives zero and there are just $G-1$ linearly independent equations. The remaining $G-1$ equations can be divided in one along the direction of the Chern Simons levels, and $G-2$ perpendicular to the direction of the Chern Simons levels. The first equation fixes the value of the field σ while the other $G-2$ equations looks like $\mu_i(X) = 0$ and can be imposed, together with their corresponding $U(1)$ gauge transformations, modding ${}^{\text{Irr}}\mathcal{F}^\flat$ by the complexified gauge group action $(\mathbb{C}^*)^{G-2}$. The equation fixing the field σ leaves a $\mathbb{Z}_k$ action with $gcd(\{k_\alpha\}) = k$ by which we need to quotient to obtain the moduli space. In the following we will take $gcd(\{k_\alpha\}) = 1$. Summarizing, the branch of the moduli space we just analyzed is:

$$\mathcal{M}_4 = {}^{\text{Irr}}\mathcal{F}^\flat \diagup H \tag{4.9}$$

where H is the $(\mathbb{C}^*)^{G-2}$ kernel of

$$C = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ k_1 & k_2 & \dots & \dots & k_{G-1} & k_G \end{pmatrix} \tag{4.10}$$

${}^{\text{Irr}}\mathcal{F}^\flat$ is a $G+2$ dimensional toric Calabi Yau cone [147, 148] and the vectors of charges in H are traceless by construction; it implies that $\mathcal{M}_4$ is a four dimensional Calabi Yau cone and it is understood as the transverse space to the $M2$ branes. Following the same procedure in the non abelian case it is possible to see that the moduli space contains the N-times symmetric product of $\mathcal{M}_4$, and it is interpreted as the the transverse space to a set of N BPS M2 branes.

It is quite generic that a specific Calabi Yau four-fold is a branch of the moduli space of apparently completely different $\mathcal{N} = 2$ Chern-Simons theories. This fact it is called toric duality. We want to systematically study $\mathcal{M}_4$ for some set of Chern Simons theories and see if it is possible to find examples of toric dual pairs. To do this we will use the algorithm proposed in [140].

An Algorithm to compute $\mathcal{M}_4$

Let us review the algorithm proposed in [140] to compute $\mathcal{M}_4$. We consider an $\mathcal{N}=2$ Chern-Simons theory described in the previous section with gauge group $U(1)^G$, and with the following constraints on the Chern-Simons levels:

$$\sum k_a = 0 \qquad gcd(\{k_a\}) = 1 \tag{4.11}$$

To compute $\mathcal{M}_4$ we need three matrices: the incidence matrix d, the perfect matching matrix P, and the Chern-Simons levels matrix C. d contains the charges of the chiral fields under the gauge group $U(1)$ factors of the theory, and can be easily obtained from the quiver. P is a map between the gauge linear sigma model variables and the chiral fields in the Chern-Simons theory. It can be obtained from the superpotential of the theory and we refer the reader to [140, 141] for explanations. Summarizing, the determination of the field theory contains the three matrices d, P, C. They are defined respectively by the gauge group representations of the chiral fields, the chiral fields interactions and the Chern Simons levels. Once we get these three matrices we can obtain the toric diagram of $\mathcal{M}_4$. From P and d we compute the matrix Q. It is the matrix of charges of the gauge linear sigma model variables under the $U(1)^G$ gauge group: $d = Q \cdot P^T$. From Q and C we construct the charge matrix $Q_D = \ker(C) \cdot Q$. We denote with $K \equiv \ker(C)$. From P we get the charge matrix Q_F: $Q_F = \ker(P^T)$. Once we have Q_D and Q_F we combine them in the total charge matrix Q_t:

$$Q_t = \begin{pmatrix} Q_D \\ Q_F \end{pmatrix} \tag{4.12}$$

The toric diagram of $\mathcal{M}_4$ is given by the kernel of Q_t:

$$G_t = (\ker^*(Q_t))^T \tag{4.13}$$

where the columns of G_t are the vectors defining the toric diagram of $\mathcal{M}_4$. Note that, as pointed out in [140], we have to find the integer kernel, that we denote $\ker^*$, and not the null-space of the charge matrix. Each row of G_t is reduced to a basis over the integer for every choice of the CS levels. We will see this algorithm at work in the following sections.

4.3 M2 branes and Seiberg duality

In the AdS_5/CFT_4 it happens that to a single geometry correspond different UV field theory descriptions. This phenomenon was called Toric Duality in [149], analyzed in [150, 151] and identified as a Seiberg duality in [152, 153]. Due to the difficulties to understand the field theory living on M2 branes the AdS_4/CFT_3 correspondence was less mastered.

The phenomenon of toric duality reappears in the AdS_4/CFT_3 correspondence, but in a much general context. Indeed, contrary to the four dimensional case, in three dimensions one find models with different numbers of gauge group factors to describe the same IR physics (for example mirror symmetry pairs [154]). In the literature some very specific pairs of dual field theories were constructed. A step was done in [19, 143] where a sort of Seiberg like duality for three dimensional Chern-Simons matter theory was proposed[1]. In the context of M2 branes at singularities, we can divide the set of dualities in the ones that change and in the ones that do not change the number of gauge group factors. In this section we will call the second type of duality Seiberg-like toric duality.

In this section we investigate Seiberg-like toric dualities for (2+1) dimensional $\mathcal{N} = 2$ Chern-Simons matter theories associated with M2 branes at Calabi Yau four-fold toric singularities. Using a generalization of the forward algorithm for D3 branes [140] we analyze a particular branch of the moduli space that is supposed to reproduce the transverse four-fold Calabi Yau singularity. We identify a set of Seiberg-like toric dualities for three dimensional Chern-Simons quiver theories.

For theories with four dimensional parents one could try to simply extend the four dimensional Seiberg duality to the three dimensional case. In fact, three dimensional theories share the same Master Spaces [147, 148] of their four dimensional parents. In the map between four and three dimensions a direction of the Master Space become a direction of the physical Calabi Yau four-fold. Unfortunately, it turns out that an arbitrary assignment of 2+1 dimensional Chern-Simons levels does not in general commute with 3+1 dimensional Seiberg duality [136]. In fact it was shown in [157] that the Master Space for four dimensional Seiberg dual theories are not in general isomorphic. Actually it seems that three dimensional CS theories with chiral four dimensional parents do not admit a simple generalization of the three dimensional SQCD Seiberg duality as it happens in the four dimensional case.

Here, we first analyze non chiral three dimensional CS theories with (3+1)d parents. Using a type IIB brane realization, we propose a Seiberg like duality, with a precise prescription for the transformation of the CS levels and the gauge groups factors. We then check that this proposed Seiberg like duality is indeed a toric duality, namely that the two dual theories are associated with M2 branes probing the same Calabi Yau four-fold singularity.

We try to simply extend to chiral CS theories with (3+1)d parents the rules that we have found for the non chiral theories. For chiral four dimensional theories the Master Space is not isomorphic among Seiberg dual phases [157]. This fact presumably puts constraints on the duality transformations for the three dimensional case. We find indeed difficulties for a straightforward realization of Seiberg like toric dualities for 2+1 dimensional Chern-Simons matter theories with four dimensional chiral parents.

However, by analyzing several examples, we find a rule for the assignments of the CS levels such that toric duality still holds among Seiberg like dual phases.

We finally give some examples of Chern-Simons theories without four dimensional parents. In this case there is no immediate insight from the four dimensions, but we show that the duality proposed for the chiral theories works also for theories without a 3+1

[1]Seiberg duality for 3D gauge theories were previously studied in [155, 156].

parents.

Our analysis is a first step to the study of Seiberg-like toric dualities in the context of M2 branes. We tried to use the intuition from the non-chiral case and to leave as arbitrary as possible the values of the Chern-Simons levels. It is reasonable that more general transformation rules exist. Moreover it would be nice to investigate more general families of toric dualities, like the ones changing the number of gauge group factors and the large limit for Chern-Simons levels. We leave these topics for future investigations.

Non-Chiral Theories

We consider non-chiral 3D $\mathcal{N}=2$ CS matter quiver gauge theories which are the three dimensional analog of the four dimensional L^{aba} theories [69, 70, 71]. and will be denoted as $\widetilde{L^{aba}}_{\{k_i\}}$. We say that a theory is non chiral if for every pair of gauge group factors $U(N)_i$ and $U(N)_{i+1}$ the number of bifundamental fields in the representation $(N_i, \bar{N}_{i+1})$ is the same as the number of bifundamental fields in the conjugate representation $(\bar{N}_i, N_{i+1})$. Otherwise the theory is chiral. The quiver for the $\widetilde{L^{aba}}_{\{k_i\}}$ is in figure 4.6, with gauge groups $\prod_i U(N)_{k_i}$. We label the nodes from left to right. The action is

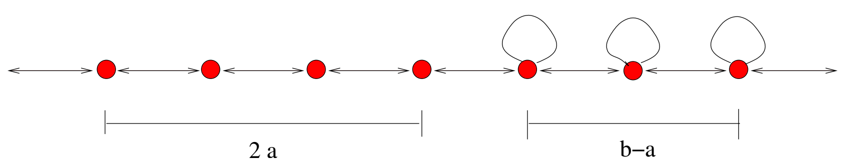


Figure 4.6: The quiver for the generic $\widetilde{L^{aba}}_{\{k_i\}}$.

$$
\begin{aligned}
S \;=\; & \sum_i S_{CS}(k_i, V_i) \qquad (4.14)\\
+ & \int d^4\theta \mathrm{Tr} \sum_i (e^{-V_i} Q^\dagger_{i,i+1} e^{V_{i+1}} Q_{i,i+1} + e^{V_i} Q_{i+1,i} e^{-V_{i+1}} Q^\dagger_{i+1,i}) + \sum_j X^\dagger_{j,j} e^{-2V_j} X_{j,j}\\
+ & \int d^2\theta \sum_l (-1)^l \mathrm{Tr} Q_{l-1,l} Q_{l,l+1} Q_{l+1,l} Q_{l,l-1} + \sum_j \mathrm{Tr} Q_{j-1,j} X_{j,j} Q_{j,j-1} - Q_{j+1,j} X_{j,j} Q_{j,j+1}
\end{aligned}
$$

where

$$i = 1, \dots a+b, \qquad j = 2a+1 \dots a+b, \qquad l = 1 \dots 2a, \qquad (4.15)$$

and $S_{CS}(k_i, V_i)$ is the first term in (4.6).

Brane construction

3D gauge theories can be engineered in type IIB string theory as $D3$ branes suspended among five branes [142]. For 3d CS theories the setup includes $(p,q)5$ branes [158, 159].

#	brane	directions
N	D3	012 6
1	NS_1	012 3 45
1	NS_2	012 3 89
1	NS_3	012 3 89
p_1	$D5_1$	012 45 7
p_2	$D5_2$	012 7 89
p_3	$D5_3$	012 7 89

Table 4.1: Brane content for the $\widetilde{L^{121}}_{\{k_1,k_2,k_3\}}$ theory.

Here we construct $\mathcal{N}=2$ three dimensional $\widetilde{L^{aba}}_{\{k_i\}}$ CS theories, in analogy with the 4D construction [160].

As an example we show in figure 4.7 the realization of the $\widetilde{L^{121}}_{\{k_1,k_2,k_3\}}$ theory. The generalization to the $\widetilde{L^{aba}}_{\{k_i\}}$ is straightforward. We have the brane content resumed in

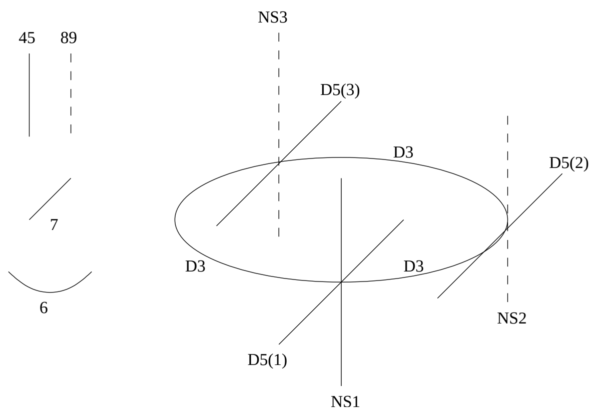


Figure 4.7: Brane construction for $\widetilde{L^{121}}_{\{k_1,k_2,k_3\}}$. The D5 branes fill also the vertical directions of the corresponding NS5.

Table 4.1. The NS branes and the corresponding D5 branes get deformed in $(1,p_i)$ five branes at angles $\tan\theta_i \simeq p_i$, obtaining

- N D3 brane along 012 6
- $(1,p_1)$ brane along 012 $[3,7]_{\theta_1}$ 45
- $(1,p_2)$ brane along 012 $[3,7]_{\theta_2}$ 89
- $(1,p_3)$ brane along 012 $[3,7]_{\theta_3}$ 89

Since the $(1, p_2)$ and the $(1, p_3)$ branes are parallel in the 89 direction there is a massless adjoint field on the node 2. This brane system gives the three dimensional $\widetilde{L^{121}}_{\{k_1,k_2,k_3\}}$ CS theory. The Chern-Simons levels are associated with the relative angle of the branes in the $[3, 7]$ directions, i.e.

$$k_i = p_i - p_{i+1} \qquad i = 1, \ldots, a + b \tag{4.16}$$

automatically satisfying (4.8). The gauge groups are all $U(N)$. Similar brane configurations have been studied in [137, 138, 139] for $\mathcal{N} = 3$ and/or non toric theories.

Seiberg-like duality

As in the four dimensional case [161], we argue that electric magnetic duality corresponds to the exchange of two orthogonal $(1, p_i)$ and $(1, p_{i+1})$ branes. In appendix C.1 we briefly discuss the process of Seiberg duality in three dimensional CS SQCD,and we see that it reduces to the exchange of fivebranes. During this process, $|p_i - p_{i+1}| = |k_i|$ D3 branes are created [158]. Observe that, since the p_i and p_{i+1} of the dualized gauge group are exchanged, this gives non trivial transformations also for the CS level of the neighbor nodes.

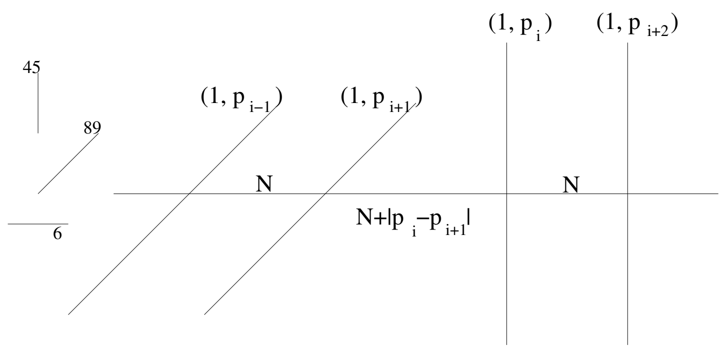


Figure 4.8: Configuration after exchanging the position of the $(1, p_i)$ and $(1, p_{i+1})$ branes. The movement implies that $|p_i - p_{i+1}|$ $D3$ are created in the middle interval. The rank of the dualized group is $N + |p_i - p_{i+1}| = N + |k_i|$. The CS levels change as $(p_{i-1} - p_i, p_i - p_{i+1}, p_{i+1} - p_{i+2}) \to (p_{i-1} - p_{i+1}, p_{i+1} - p_i, p_i - p_{i+2})$.

From the brane picture (see figure 4.8) we obtain the rules for a Seiberg like duality on a node without adjoint fields in the $\widetilde{L^{aba}}_{\{k_i\}}$ quiver gauge theories. Duality on the i-th node gives

$$\begin{array}{rcl} U(N)_{k_i} & \to & U(N + |k_i|)_{-k_i} \\ U(N)_{k_{i-1}} & \to & U(N)_{k_{i-1}+k_i} \\ U(N)_{k_{i+1}} & \to & U(N)_{k_{i+1}+k_i} \end{array} \tag{4.17}$$

and the field content and the superpotential changes as in 4D Seiberg duality.

In the following we will verify that this is indeed a toric duality by computing and comparing the branch $\mathcal{M}_4$ of the moduli space (i.e. the toric diagram) of the two dual descriptions.

We observe that the Seiberg like duality (4.17) modifies the rank of the dualized gauge group, introducing fractional branes. This is a novelty of this $3d$ duality with respect to the $4d$ case (see also [19, 143, 162]).

The k fractional $D3$ branes are stuck between the five branes, so there is no moduli space associated with their motion. This is as discussed in [143], and the same field theory argument can be repeated here. The moduli space of the magnetic description is then the N symmetric product of the abelian moduli space.

The fractional branes can break supersymmetry as a consequence of the s-rule [142]. Indeed it was suggested in [159, 163, 164] that for $U(l)_k$ YM-CS theories supersymmetry is broken if $l > |k|$. We notice from (4.17) that in the moduli space of the magnetic description, there is a pure $U(k)_{-k}$ YM-CS theory. Thus the bound is satisfied and supersymmetry is unbroken. However, if we perform multiple dualities we can realize configurations with several fractional branes on different nodes. At every duality we have to control via s-rule that supersymmetry is not broken. We leave a more thorough study of these issues related to fractional branes for future investigation.

Finally, the duality proposed maps a theory with a weak coupling limit to a strongly coupled theory. Indeed if we define the i-th 't Hooft coupling as $\lambda_i = N/k_i$, the original theory is weakly coupled for $k_i >> N$. In this limit the dual theory is strongly coupled since the i-th dual 't Hooft coupling is $\tilde{\lambda}_i = 1 + O(N/k_i)$.

$\widetilde{L^{121}}_{\{k_1,k_2,k_3\}}$

The $\widetilde{L^{121}}_{\{k_1,k_2,k_3\}}$ is the first example that we study. The quiver is given in Figure 4.9. The

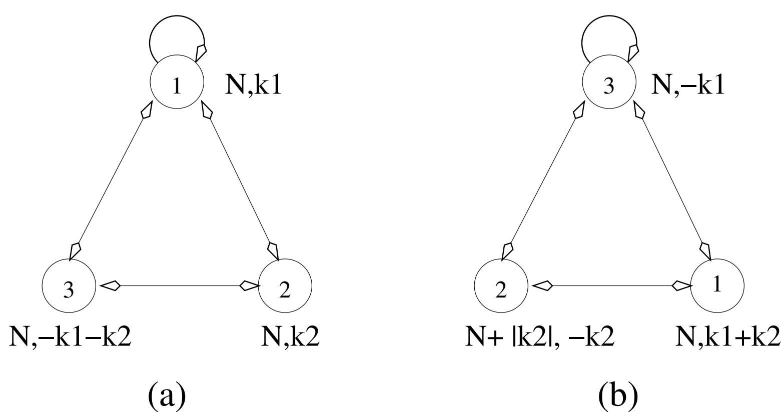


Figure 4.9: Quiver, ranks and CS levels for the $\widetilde{L^{121}}_{\{k_1,k_2,k_3\}}$ in the two phases related by Seiberg like duality on node 2.

toric diagram that encodes the information about the classical mesonic moduli space is computed with the techniques explained in section 4.2. We extract the incidence matrix $d_{G,i}$, where $G = 1, \ldots, N_g$ runs over the labels of the gauge groups, and i runs over the fields ($i = 1, \ldots, 7$ for the $\widetilde{L^{121}}_{\{k_1,k_2,k_3\}}$, six bifundamental and one adjoint)

$$d = \begin{pmatrix} 1 & -1 & 0 & 0 & -1 & 1 & 0 \\ -1 & 1 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & -1 & 0 \end{pmatrix} \tag{4.18}$$

The matrix of the perfect matchings $P_{\alpha,i}$ is computed from the determinant of the Kastelein matrix

$$Kas = \begin{pmatrix} Q_{23} + Q_{32} & Q_{12} + Q_{21} \\ Q_{13} + Q_{31} & X_{11} \end{pmatrix} \tag{4.19}$$

If we order the fields in the determinant of (4.19) we can build the matrix $P_{i,\alpha}$ where $\alpha = 1, \ldots, c$ is the number of perfect matchings, that corresponds to the number of monomials of the $\det Kas$. In this matrix we have 1 if the i-th field appears in the α-th element of the determinant, 0 otherwise

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \tag{4.20}$$

The matrix Q that represents the charge matrix for the GLSM fields is obtained from the relation $d_{G,i} = Q_{G,\alpha} \cdot P^T_{i,\alpha}$

$$Q = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 & -1 & 0 \\ -1 & 0 & 1 & -1 & 1 & 0 \end{pmatrix} \tag{4.21}$$

The contribution of the D-terms to the moduli space is given by quotienting by the $G-2$ FI parameters induced by the CS couplings. These FI parameters are in the integer kernel of the matrix of the CS level

$$K = Ker \begin{pmatrix} 1 & 1 & 1 \\ k_1 & k_2 & -k_1 - k_2 \end{pmatrix} \tag{4.22}$$

The F-term equations are encoded in the matrix $Q_F = Ker(P)$. The Toric diagram is the kernel of the matrix obtained by combining $Q_D = K \cdot Q$ and Q_F. Acting with an $SL(4, Z)$ transformation the toric diagram reads

$$G_t = Ker^* \left[K \cdot Q, Ker[P^T]\right] = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ k_2 & k_1 + k_2 & 0 & 0 & k_1 + 2k_2 & 0 \\ 1 & 1 & 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \tag{4.23}$$

This system of vectors is co-spatial. This is a CY condition that guarantees that the toric diagram lives on a three dimensional hypersurface in Z_4.

The last three rows of (4.23) defines the toric diagram for the three dimensional Chern-Simons $\widetilde{L^{121}}_{\{k_1,k_2,k_3\}}$ toric quiver gauge theory. Note that the toric diagram of the (3+1)d parent is recovered by setting to zero the row with the CS levels.

We perform the Seiberg-like duality (4.17) on node 2. The resulting theory is shown in figure 4.9(b). The L^{121} four dimensional parent theory has only one toric phase. The dual theory is in the same phase, thanks to the mapping among the nodes $(1 \to 3, 2 \to 1, 3 \to 2)$, see Figure 4.9. In the $\widetilde{L^{121}}_{\{k_1,k_2,k_3\}}$ we should also properly map the CS levels in the two dual descriptions. The transformation rules (4.17) change the CS level as in figure 4.9. Then we apply the same mapping we used for the gauge groups. After these steps the resulting K matrix is

$$\begin{aligned} K_{dual} = \quad & Ker \begin{pmatrix} 1 & 1 & 1 \\ \tilde{k}_3 & \tilde{k}_1 & \tilde{k}_2 \end{pmatrix} = Ker \begin{pmatrix} 1 & 1 & 1 \\ k_3 + k_2 & k_1 + k_2 & -k_2 \end{pmatrix} = \\ = \quad & Ker \begin{pmatrix} 1 & 1 & 1 \\ -k_1 & k_1 + k_2 & -k_2 \end{pmatrix} \end{aligned} \tag{4.24}$$

where with $\tilde{k}_i$ we denote the CS level of the i-th node in the dual phase. Concerning the field content and the superpotential, the dual theory is in the same phase than the starting theory. Thus we use the same matrices P, d, Q for the computation of the moduli space. The toric diagram is then computed with the usual algorithm. Up to an $SL(4, Z)$ transformation it coincides with the same as the one computed in the original theory (4.23).

In this example we have shown that the Seiberg like duality (4.17) is a toric duality. Observe that the non trivial transformation on the CS levels of (4.17) are necessary for the equivalence of the moduli spaces of the two phases.

$\widetilde{L^{222}}_{\{k_i\}}$

The second example is the $\widetilde{L^{222}}_{\{k_i\}}$ theory. The main difference is that this theory has two phases with a different matter content and superpotential (see Figure 4.10), obtained by dualizing node 2. These phases are dual for the (3+1)d parents theory. Here we show that the same holds in three dimensions with the Seiberg like duality (4.17). The P, d, Q

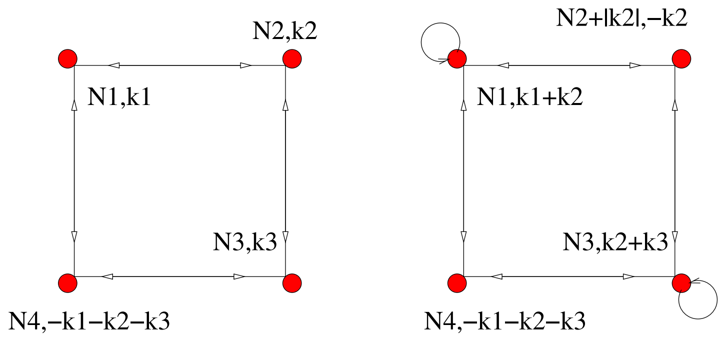


Figure 4.10: The quivers for the two phases of L^{222}.

and K matrices for the first phase are

$$d = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 & -1 & 1 \\ -1 & 1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 1 & -1 \end{pmatrix} \quad P = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{4.25}$$

$$Q = \begin{pmatrix} -1 & 0 & 1 & 0 & 1 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 & -1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 & 0 & -1 & 0 & 1 \end{pmatrix} \quad K = Ker \begin{pmatrix} 1 & 1 & 1 & 1 \\ k_1 & k_2 & k_3 & -k_1 - k_2 - k_3 \end{pmatrix} \tag{4.26}$$

The resulting toric diagram is

$$G_t = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 0 & 2 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ -k1 - k_2 & -k_1 & k_2 & 0 & k_2 + k_3 & k_2 + k_3 & 0 & 0 \end{pmatrix} \tag{4.27}$$

By duality on node 2 we obtain the inequivalent phase of $\widetilde{L^{222}}_{\{k_i\}}$, see figure 4.10. The toric diagram is computed with new d, P, Q and K matrices

$$d = \begin{pmatrix} 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & -1 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 & -1 \end{pmatrix} \quad P = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{4.28}$$

$$Q = \begin{pmatrix} 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 & 1 \end{pmatrix} \quad K = Ker \begin{pmatrix} 1 & 1 & 1 & 1 \\ k_1+k_2 & -k_2 & k_3+k_2 & -k_1-k_2-k_3 \end{pmatrix} \tag{4.29}$$

It results

$$G_t = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 0 & 2 & 1 & 1 & 0 \\ -k1-k_2 & -k_1 & k_2 & 0 & k_2+k_3 & k_2+k_3 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{4.30}$$

which is equivalent to (4.27).

The general $\widetilde{L^{aba}}_{\{k_i\}}$

In the previous section we have seen two simple examples. In this section we consider the generic case of non-chiral $\mathcal{N} = 2$ toric three dimensional CS quiver gauge theories $\widetilde{L^{aba}}_{\{k_i\}}$ singularities. Four dimensional theories based on these singularities share the same toric diagram among the different Seiberg dual phases. Here we show that two theories that are related by Seiberg like duality in three dimension (4.17) share the same toric diagram. Our argument is based on the algorithm [144] that extracts toric data of the CY four-fold by using brane tiling.

Toric diagrams from bipartite graphs

Let us remind the reader that to every quiver describing a four dimensional conformal field theory on D3 branes at Calabi Yau three-fold singularities it is possible to associate a bipartite diagram drawn on a torus. It is called tiling or dimer [120, 121], and it encodes all the informations in the quiver and in the superpotential. To every face in the dimer we can associate a gauge group factor, to every edge a bifundamental field and to every node a term in the superpotential. A similar tiling can be associated with three dimensional Chern-Simons matter theories living on M2 branes probing a Calabi Yau four-fold singularity simply adding a flux of Chern-Simons charge. The CS levels are described as a conserved flow on the quiver, or equivalently on the dimer. To every edge we associate a flux of Chern-Simons charge and the CS level of the gauge group is the sum of these contributions taken with sign depending on the orientation of the arrow.

In this section we review the proposal of [144] for the computation of the moduli space of three dimensional CS toric quiver gauge theories. This method furnishes the toric data from the bipartite graphs associated with the quiver model and with the CS levels.

One has to choose a set of paths $(p_1, \ldots, p_4)$on the dimer. The paths p_1 and p_2 correspond to the α and β cycles of the torus described by the dimer. The path p_4 is a paths encircling one of the vertexes. One can also associate mesonic operators to these paths. These operators correspond to the product of the corresponding bifundamentals along the paths. For example the operator associated on the p_4 path is a term of the superpotential. The moduli space of the three dimensional theory requires also the definition of the path p_3. This is a product of paths corresponding to a closed flow of CS charges along the quiver. We choose p_3 in the tiling of the L^{aba} singularity by taking a minimal closed path connecting the bifundamentals from the first node to the $a + b$-th node of the quiver. Then one associates the CS charge

$$\sum_{i=1}^{n} k_i \tag{4.31}$$

to each bifundamental in this closed loop, with $n = 1, \ldots, a + b$. The last charge is zero since it corresponds to sum of all the CS levels in the theory. This conserved CS charges flow is represented on the dimer by a set of oriented arrows connecting the faces of the dimer, the gauge groups.

In [144] it has been shown that the toric polytope of CY_4 is given by the convex hull of all lattice point $v^\alpha = (v_1^\alpha, \ldots, v_4^\alpha)$, with

$$v_i^\alpha = \langle p_i, D_\alpha \rangle \tag{4.32}$$

where D_α are the perfect matchings. The operation $\langle ., . \rangle$ in (4.32) is the signed intersection number of the perfect matching D_α with the path p_i. Note that v_4 is always 1, since there is always only one perfect matching connected with the node encircled by p_4.

Seiberg-like duality on $\widetilde{L^{aba}}_{\{k_i\}}$ and toric duality

In subsection (4.3) we argued that the action of Seiberg duality on the field content and on the superpotential is the same as in four dimensions. The only difference is the change of the CS levels associated with the groups involved in the duality (4.17).

In an L^{aba} theory, if duality is performed on node N_2, the levels become

$$\begin{aligned} k_1 &\to k_1 + k_2 \\ k_2 &\to -k_2 \\ k_3 &\to k_3 + k_2 \end{aligned} \tag{4.33}$$

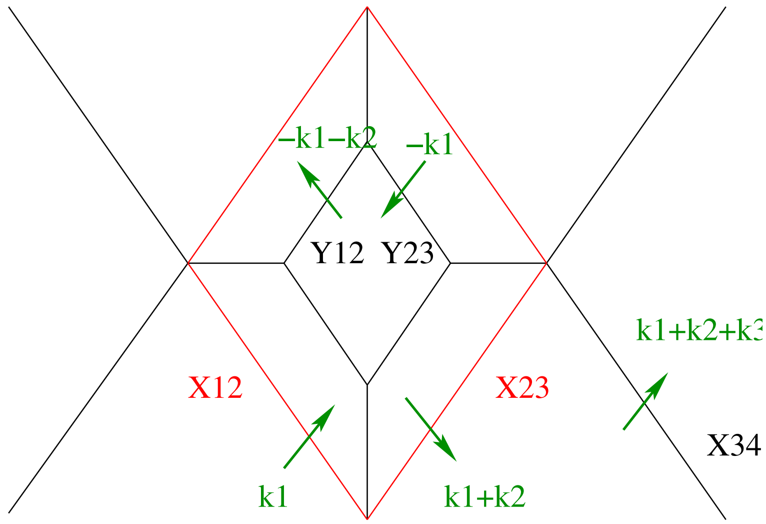


Figure 4.11: Action of Seiberg duality on the dimer and modification of the CS flow.

The action of Seiberg duality (4.17) not only modifies the dimer as in the 4d, but also the CS levels of the gauge groups. This changes the path p_3 in the dual description. Many $SL(4, Z)$ equivalent choices are possible. Among them we select the p_3 path as in figure 4.11. We associate a charge l_1 to Y_{12} and l_2 to Y_{23}, but with the opposite arrows that before. The values of the charges l_1 and l_2 are derived from (4.33) and are

$$l_1 = -k_1 - k_2 \qquad l_2 = -k_1 \tag{4.34}$$

The field X_{34} is not involved in this duality and it contributes to the CS flow with the same charge $k_1 + k_2 + k_3$ in both phases.

We claim that the two theories share the same toric diagram. For the proof of this relation it is useful to distinguish two sector of fields from which all the perfect matching are built. In Figure 4.12 we separated these two sectors for the electric and magnetic phase of an L^{aaa} theory (the same distinction is possible in a generic L^{aba} theory). Every perfect matching is built by choosing in these sets only one field associated with each vertex. For example in Figure 4.12(a) every perfect matching is a set of blue lines chosen such that every vertex is involved only once.

The paths p_1 and p_2 are shown in figure 4.13 for the electric and the magnetic phase. The intersection numbers $\langle p_1, D^\alpha \rangle$ and $\langle p_2, D^\alpha \rangle$ give the same bi-dimensional toric diagram of the associated four dimensional L^{aba} theories, up to an overall $SL(3, Z)$ translation. This is shown by mapping the perfect matching in the two description. This mapping is done with a prescription on the choice of the fields in the perfect matching of the dual description. If duality is performed on node N_i, the field $X_{i-1,i}$, $X_{i,i-1}$, $X_{i,i+1}$ and $X_{i+1,i}$ are respectively mapped in the fields $Y_{i,i+1}$, $Y_{i+1,i}$, $Y_{i-1,i}$ and $Y_{i.i-1}$ of the dual theory. This prescription gives a $1-1$ map of each point of the 2d toric diagrams of the two dual theories.

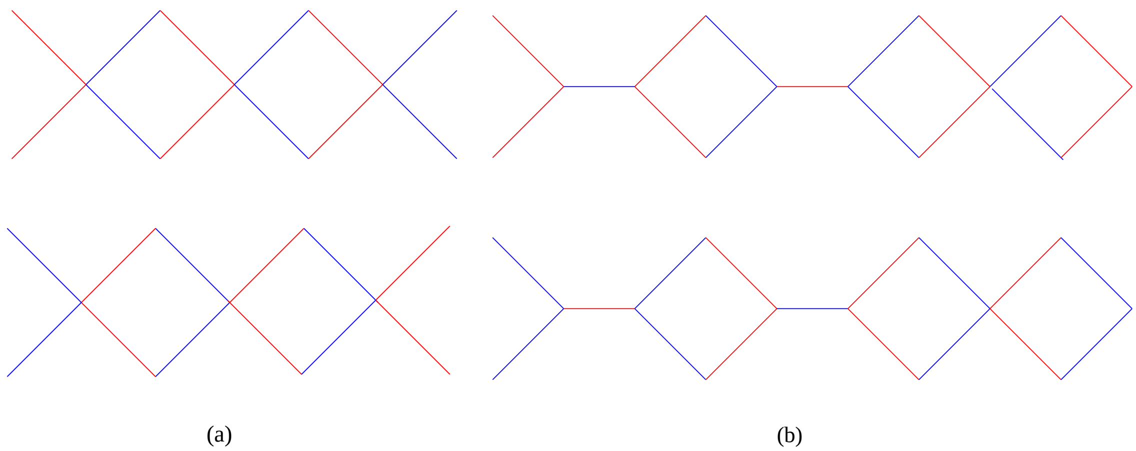


Figure 4.12: Different sectors of fields that generate all perfect matching in the (a) electric and (b) magnetic theory

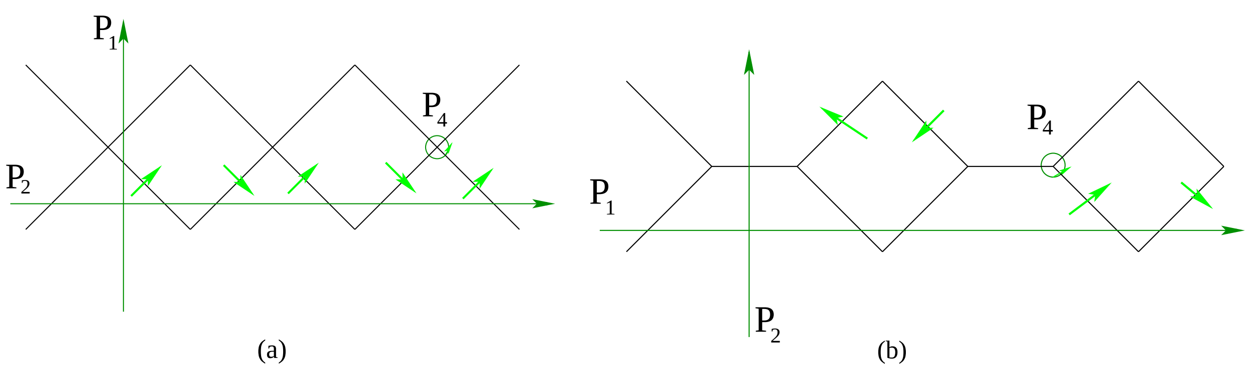


Figure 4.13: Paths p_i in the two dual versions of the theory

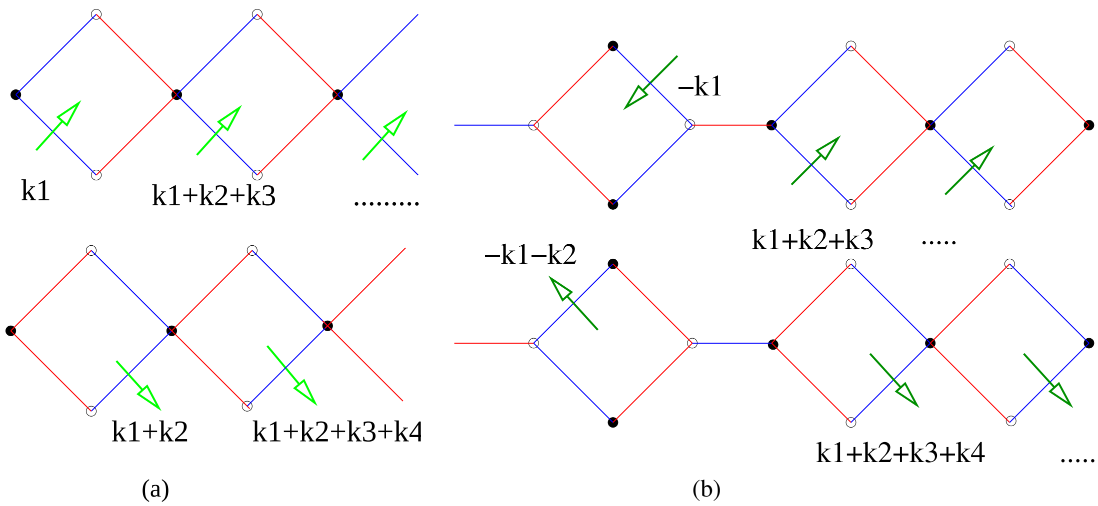


Figure 4.14: Decomposition of the path p_3 on the different perfect matchings

The whole diagram for the three dimensional $\widetilde{L^{aba}}$ theory is obtained by considering the intersection numbers $\langle p_3, D^\alpha \rangle$, which give the third component of the vectors v^α. The path p_3 corresponds to the flux of CS charges from the group N_1 to the group N_g, where the last arrow is omitted since it carries zero charge. In the dual theory the path p_3 changes as explained above, and as we show in Figure 4.13 for duality on a node labeled by 2. Note that only the arrows connected with the dualized gauge group change.

With this choice of p_3 and using the basis of perfect matching we prescribed, the intersection numbers $\langle p_3, D^\alpha \rangle$ coincide in the two phases for every point of the 2d toric diagram. This is shown by associating the relevant part of the path p_3 to each sector of perfect matching as in Figure 4.14. The arrow that carries charge k_1 in the electric theory corresponds to the arrow with charge $-k_1$ in the magnetic theory. Its contribution to the moduli space remains the same, since also the orientation of this arrow is the opposite. The same happens for the arrow carrying charge $k_1 + k_2$.

Thus the 3d toric diagrams of the two dual theories are the same. We conclude that the action of three dimensional Seiberg like duality (4.17) in the $\widetilde{L^{aba}}_{\{k_i\}}$ theories implies toric duality.

Chiral theories

In this section we study Seiberg like duality for $\mathcal{N} = 2$ three dimensional Chern-Simons theories with four dimensional parent chiral theories, which as such suffer from anomalies. The anomaly free condition imposes constraints on the rank distribution. In 3d there are no local gauge anomalies. Nevertheless we work with all $U(N)$ gauge groups such that the moduli space is the N symmetric product of the abelian one. Moreover for three dimensional Chern-Simons chiral theories we do not have a brane construction as simple as for the non-chiral case, and were not able to deduce the duality from the brane picture.

However, using what we learned from the non-chiral case, we infer that at least a subset of the possible three dimensional Seiberg-like toric dualities acts on the field content and on the superpotential as it does in 4d and moreover recombines the Chern-Simons levels in a similar way as in (4.17). As a matter of fact, a straightforward extension of the rule (4.17) does not seem to work in the chiral case. This could be related to the fact that the Master Spaces of the four dimensional dual Seiberg parents are not isomorphic [157]. For this reason we restrict ourself to the case where the CS level of the group which undergoes duality is set to zero. We assume that the other CS levels are unchanged, and no fractional branes are created. This could be suggested by the parity anomaly matching argument (see appendix C.2). We also set to zero the CS level of those gauge groups that after duality have the same interactions with the rest of the quiver as the dualized gauge group.

By direct inspection we find that under these assumptions on the CS levels also for chiral 3D CS theories Seiberg like duality leads to toric duality. For $\widetilde{\mathbb{F}_0}$ we can take milder assumptions. Indeed we find that a generalization of the rule (4.17) to the chiral case still gives toric duality for this theory.

$\widetilde{\mathbb{F}_{0\{k_i\}}}$

Here we study the (2+1)d CS chiral theory whose (3+1)d parent is $\mathbb{F}_0$. In (3+1)d there are two dual toric phases of $\mathbb{F}_0$, denoted as $\mathbb{F}_0^I$ and $\mathbb{F}_0^{II}$. In (2+1)d the two phases for arbitrary choices of the CS levels do not have the same moduli space. Nevertheless it is possible to find assignments for the Chern-Simons levels such that the two phases have the same toric diagram.

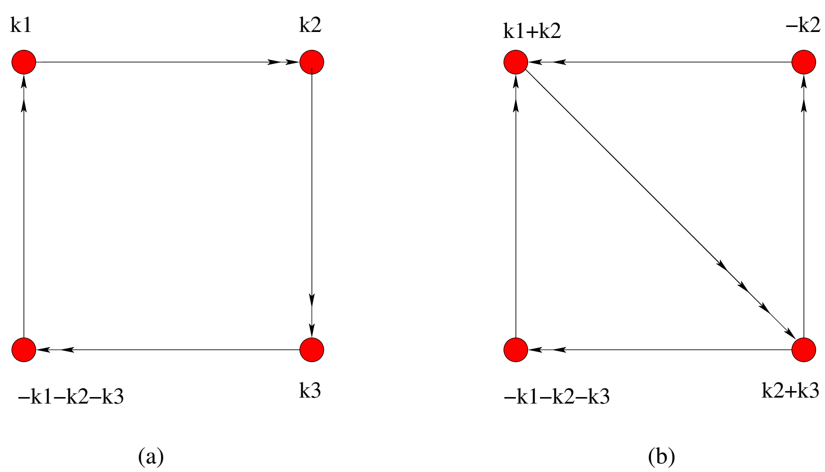


Figure 4.15: (a) quiver for $\widetilde{\mathbb{F}_0^I}$ and (b) quiver for $\widetilde{\mathbb{F}_0^{II}}$ for one of the possible choice of CS levels.

The quiver representing the $\widetilde{\mathbb{F}_0^I}$ phase is in figure 4.15(a). The superpotential is

$$W = \varepsilon_{ij}\varepsilon_{kl}X_{12}^i X_{23}^k X_{34}^j X_{41}^l \tag{4.35}$$

The incidence matrix and the matrix of perfect matchings are

$$d = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 & -1 & 1 \\ -1 & 1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 1 & -1 \end{pmatrix} \quad P = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \tag{4.36}$$

The charges of the GLSM fields determine the matrix Q, the can be chosen as

$$Q = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{4.37}$$

The second phase $\widetilde{\mathbb{F}_0^{II}}$ is obtained by dualizing node 2. The dual superpotential is

$$W = \varepsilon_{ij}\varepsilon_{kl} X_{13}^{ik} X_{32}^{l} X_{21}^{j} - \varepsilon_{ij}\varepsilon_{kl} X_{13}^{ik} X_{34}^{l} X_{41}^{j} \tag{4.38}$$

The matrices d, P and Q are determined from the quiver and the superpotential

$$d = \begin{pmatrix} -1 & -1 & 0 & 0 & 0 & 0 & -1 & -1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & -1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & -1 & -1 & 1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \quad P = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{4.39}$$

$$Q = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \end{pmatrix} \tag{4.40}$$

Two families

The $\widetilde{\mathbb{F}_{0\{k_i\}}}$ theories turn out to be a special case. Indeed one can single out two different possibilities: in the first one we put to zero just the CS level associated with the group 2; while in the second case we can fix to zero just the Chern-Simons level of the group 4 and transform the CS levels as in the non-chiral case. In the first case we choose the CS levels as $(k_1, k_2, k_3, k_4) = (k, 0, p, -k-p)$. The CS level matrix for both phases is:

$$C = \begin{pmatrix} 1 & 1 & 1 & 1 \\ k & 0 & p & -k-p \end{pmatrix} \tag{4.41}$$

and the toric diagram is given by:

$$G_t^{(I)} = \left(\begin{array}{cccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ \text{p} & 0 & \text{p} & 0 & \text{p} & \text{p} & \text{k+p} & \text{k+p} \\ 0 & 1 & \text{-1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & \text{-1} & 0 \end{array}\right) \tag{4.42}$$

The CS levels for the dual phase, obtained by duality on N_2, are unchanged and the toric diagram

$$G_t^{(II)} = \left(\begin{array}{ccccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ \text{p} & \text{k+p} & \text{p} & \text{p} & \text{k+p} & \text{k+p} & 0 & 0 & 0 \\ 0 & 0 & \text{-1} & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & \text{-1} & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \tag{4.43}$$

is equivalent to the one above.

In the second case we choose the CS levels as $(k_1, k_2, k_3, k_4) = (k, p, -k-p, 0)$. The phase $\widetilde{\mathbb{F}_0^{II}}$ is computed by dualizing the node 2. We observe that by applying the rules (4.17) the CS levels of $\widetilde{\mathbb{F}_0^{II}}$ are $(k+p, -p, -k, 0)$. The C matrices for the two phases are:

$$C_I = \left(\begin{array}{cccc} 1 & 1 & 1 & 1 \\ k & p & -k-p & 0 \end{array}\right) \qquad C_{II} = \left(\begin{array}{cccc} 1 & 1 & 1 & 1 \\ k+p & -p & -k & 0 \end{array}\right) \tag{4.44}$$

The toric diagram for the first phase is:

$$G_t^{(I)} = \left(\begin{array}{cccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ \text{k} & \text{k} & 0 & 0 & \text{k+p} & 0 & \text{k+p} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & \text{-1} & 0 \\ 0 & \text{-1} & 1 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \tag{4.45}$$

while the toric diagram for the second phase is:

$$G_t^{(II)} = \left(\begin{array}{ccccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & \text{k} & \text{k} & \text{k+p} & \text{k} & \text{k+p} & \text{k+p} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & \text{-1} & 0 \\ 0 & 1 & 0 & \text{-1} & 0 & 0 & 0 & 0 & 0 \end{array}\right) \tag{4.46}$$

And they are equivalent.

The $\mathbb{F}_0$ theory seems to be the only case where the assumptions we gave at the beginning of this section can be relaxed. In the following examples we will just apply those basic rules.

$\widetilde{dP}_{1\{k_i\}}$

Here we study the (2+1)d CS chiral theory whose (3+1)d parent is dP_1. In (3+1)d dP_1 has only one phase. After Seiberg duality on node 2 the theory has a self similar structure and it is described by the same quiver. The only difference is that we have to change

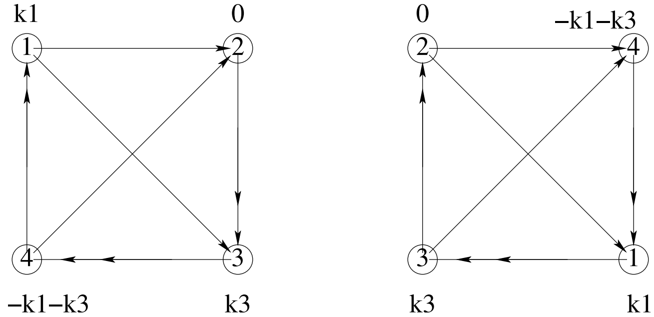


Figure 4.16: Quiver and CS level for the $\widetilde{dP}_{1\{k_i\}}$ in the two phases related by duality on node 2.

the labels of the groups as $(1 \to 2, 2 \to 4, 3 \to 1, 4 \to 3)$. The matrix d, P and Q are unchanged by duality. The CS levels in the C matrix change as the labels of the gauge groups do. We give in figure 4.16 the two phases.

We take the assumption described in the introduction of section 4.3. Hence we choose the CS level of the group that undergoes duality to be 0. With this choice the two phases of the $2+1$ dimensional theory (see figure 4.16) have the same toric diagram.

The superpotential is

$$W = \varepsilon_{ab} X_{13} X_{34}^a X_{41}^b + \varepsilon_{ab} X_{42} X_{23}^a X_{34}^b + \varepsilon_{ab} X_{34} X_{41}^a X_{12} X_{23}^b \tag{4.47}$$

The d, P, Q matrices are

$$d = \begin{pmatrix} 1 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 1 & 0 & -1 \\ -1 & 1 & 0 & 1 & 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & -1 & 1 & 0 & 1 & 0 & -1 & 0 \end{pmatrix} \quad P = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \tag{4.48}$$

$$Q = \begin{pmatrix} 0 & 0 & 1 & 0 & 1 & -1 & -1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 & -1 & 1 & 1 & 0 \end{pmatrix} \tag{4.49}$$

Following the relabeling of the gauge groups, the C matrix in the two phases are

$$C_1 = \begin{pmatrix} 1 & 1 & 1 & 1 \\ k_1 & 0 & k_3 & -k_1 - k_3 \end{pmatrix} \quad C_2 = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & -k_1 - k_3 & k_1 & k_3 \end{pmatrix} \tag{4.50}$$

The toric diagram for the first theory is given by the matrix $G_t^{(1)}$

$$G_t^{(1)} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ k_1 + k_3 & k_1 & k_1 & k_1 + k_3 & k_1 & k_1 & 0 & 0 \end{pmatrix} \tag{4.51}$$

The toric diagram of the dual theory, up to $SL(4,Z)$ transformation, is

$$G_t^{(2)} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ k_1+k_3 & k_1+k_3 & 0 & k_1 & k_1 & 0 & k_1 & k_1+k_3 \end{pmatrix} \tag{4.52}$$

This shows that the two systems of vectors give the same toric diagram and that the two theories have the same abelian moduli space also in the 2+1 dimensions, provided $k_2 = 0$.

$\widetilde{dP}_{2\{k_i\}}$

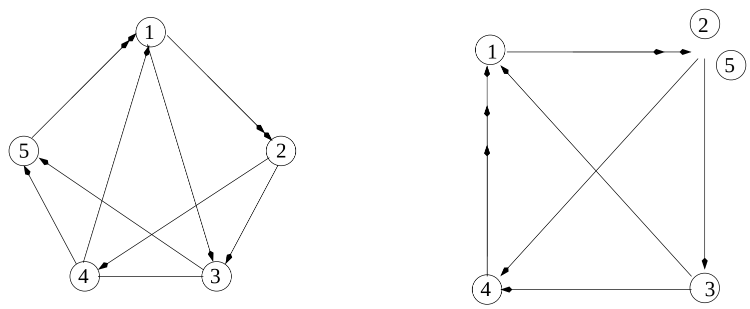


Figure 4.17: The quivers representing the dual phases of $dP2$

We analyze the (2+1)d CS chiral theory with dP_2 as (3+1)d parents. The $4D$ theory has two inequivalent phases. The two phases are connected by duality on node 5 and are reported in figure 4.17.

The constraint on the CS levels explained in the introduction of this section imposes $k_2 = 0$ and $k_5 = 0$. Under this assumption the two phases have the same toric diagram also for the (2+1)d CS theory.

The superpotential for the two phases are

$$\begin{aligned} W_I &= X_{13}X_{34}X_{41} - Y_{12}X_{24}X_{41} + X_{12}X_{24}X_{45}Y_{51} - X_{13}X_{35}Y_{51} \\ &+ Y_{12}X_{23}X_{35}X_{51} - X_{12}X_{23}X_{34}X_{45}X_{51} \\ \\ W_{II} &= Y_{41}X_{15}X_{54} - X_{31}X_{15}X_{53} + Y_{12}X_{23}X_{31} - Y_{12}X_{24}X_{41} + Y_{15}X_{53}X_{34}X_{41} \\ &- Z_{41}Y_{15}X_{54} + X_{12}X_{24}Z_{41} - X_{12}X_{23}X_{34}Y_{41} \end{aligned} \tag{4.53}$$

We compute the toric diagrams $G_t^{(I)}$ and $G_t^{(II)}$ for the two phases

$$\begin{aligned} G_t^{(I)} &= \left(\begin{array}{cccccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & -1 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \\ k_1 & k_1 & k_1 & k_1 & 0 & 0 & 0 & 0 & -k_3 & -k_3 \end{array} \right) \\ G_t^{(II)} &= \left(\begin{array}{ccccccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & -1 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -k_3 & k_1 & 0 & k_1 & -k_3 & k_1 & 0 & 0 & 0 & 0 \end{array} \right) \end{aligned} \tag{4.54}$$

They result the same.

$\widetilde{dP}_{3\{k_i\}}$

Here we study $\widetilde{dP}_{3\{k_i\}}$. This theory has four phases in four dimensions, with superpotentials

$$\begin{aligned} W_I &= X_{13}X_{34}X_{46}X_{61} - X_{24}X_{46}X_{62} + X_{12}X_{24}X_{45}X_{51} - X_{13}X_{35}X_{51} \\ &+ X_{23}X_{35}X_{56}X_{62} - X_{12}X_{23}X_{34}X_{45}X_{56}X_{61} \end{aligned}$$

$$\begin{aligned} W_{II} &= X_{13}X_{34}X_{41} - X_{13}X_{35}X_{51} + X_{23}X_{35}X_{52} - X_{26}X_{65}X_{52} + X_{16}X_{65}Y_{51} \\ &- X_{16}X_{64}X_{41} + X_{12}X_{26}X_{64}X_{45}X_{51} - X_{12}X_{23}X_{34}X_{45}Y_{51} \end{aligned}$$

$$\begin{aligned} W_{III} &= X_{23}X_{35}X_{52} - X_{26}X_{65}X_{52} + X_{14}X_{46}X_{65}Y_{51} - X_{12}X_{23}Y_{35}Y_{51} + X_{43}Y_{35}X_{54} \\ &- Y_{65}X_{54}X_{46} + X_{12}X_{26}Y_{65}X_{51} - X_{14}X_{43}X_{35}X_{51} \end{aligned}$$

$$\begin{aligned} W_{IV} &= X_{23}X_{35}X_{52} - X_{52}X_{26}X_{65} + X_{65}Z_{54}X_{46} - Z_{54}X_{41}Y_{15} + Y_{15}Z_{52}X_{21} - Z_{52}X_{23}Y_{35} \\ &+ Y_{35}X_{54}X_{43} - X_{54}X_{46}Y_{65} + Y_{65}Y_{52}X_{26} - Y_{52}X_{21}X_{15} + X_{15}Y_{54}X_{41} - Y_{54}X_{43}X_{35} \end{aligned}$$

Phases (II, III, IV) are computed from phases (I, II, III) by dualizing nodes (6, 4, 1) respectively. The quivers associated with each phase are given in Figure 4.18. We now show the equivalence of phases (I,II), (II,III) and (III,IV) by choosing ($k_3 = k_6 = 0$), ($k_2 = k_4 = 0$) and ($k_1 = k_3 = k_6 = 0$) respectively. For phases (I, II) we have

$$G_t^{(I)} = \left(\begin{array}{cccccccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 1 & 0 \\ -1 & 0 & -1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ k_1+k_2 & 0 & k_1 & k_1+k_2+k_4 & k_1+k_2 & 0 & k_1 & k_1+k_2+k_4 & k_1+k_2 & k_1+k_2 & 0 & 0 \end{array} \right)$$

$$G_t^{(II)} = \left(\begin{array}{ccccccccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 1 \\ k_1 & k_1+k_2+k_4 & k_1+k_2 & k_1+k_2 & k_1+k_2 & k_1 & k_1+k_2+k_4 & k_1+k_2 & k_1+k_2 & k_1+k_2 & 0 & 0 & 0 \end{array} \right)$$

For phases (II, III) we have

$$G_t^{(II)} = \left(\begin{array}{ccccccccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & -1 \\ -k_1 & k_3+k_5 & k_3+k_5 & k_3+k_5 & -k_1 & -k_1-k_3 & k_5 & -k_1-k_3 & k_5 & -k_1-k_3 & 0 & 0 & 0 \end{array} \right)$$

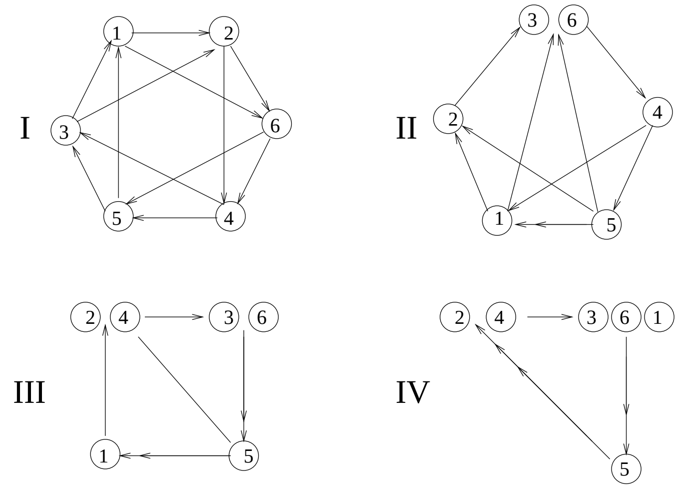


Figure 4.18: The quiver of dP_3

$$G_t^{(III)} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & k_3+k_5 & 0 & -k_1 & k_3+k_5 & -k_1 & 0 & k_3+k_5 & k_5 & -k_1-k_3 & -k_1 & -k_1 & -k_1-k_3 & -k_1-k_3 \end{pmatrix}$$

For phases (III, IV) we have

$$G_t^{(III)} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & -1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ k_2+k_4 & k_2 & k_2+k_4 & k_2 & k_2+k_4 & k_2+k_4 & k_4 & 0 & 0 & 0 & 0 & k_4 & 0 & 0 \end{pmatrix}$$

$$G_t^{(IV)} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 1 & 0 & 0 \\ k_2+k_4 & k_2+k_4 & k_2+k_4 & k_2+k_4 & k_2+k_4 & k_2+k_4 & k_2+k_4 & k_2+k_4 & k_2+k_4 & k_2+k_4 & k_4 & k_4 & k_2 & k_2 & 0 & 0 & 0 \end{pmatrix}$$

$\widetilde{Y^{32}}_{\{k_i\}}$

This is the last chiral theory we analyze. In this case, after duality, there is not an identification between the gauge group that undergoes duality with other groups. This implies that the assumptions of section 4.3 impose only $k_g = 0$, where k_g is the CS level of the dualized gauge group. As for the case of dP_1 and all the $Y^{p,p-1}$ theories, the $Y^{3,2}$ theory is self similar under four dimensional Seiberg duality. We can evaluate $\mathcal{M}_4$ for one phase and then the toric diagram associated with a dual phase is given by an appropriate change of the D-term modding matrix.

We fix the conventions on the groups by giving the tiling of the two dual phases, see Figure 4.19. The two phases are connected by duality on node 5, so we set $k_5 = 0$.

The CS matrices for the two dual phases are:

$$C_1 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ k_1 & k_2 & k_3 & k_4 & 0 & -k_1-k_2-k_3-k_4 \end{pmatrix} \quad C_2 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ k_4 & -k_1-k_2-k_3-k_4 & k_1 & k_2 & k_3 & 0 \end{pmatrix}$$

The toric diagrams are encoded in the G_t matrices.

$$G_t^{(1)} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \\ 2 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & -1 & 1 & 0 & 0 & 0 \\ k_3+k_4 & 0 & -k_6 & k_2+k_3+k_4 & k_2 & k_3+k_4 & 0 & -k_6 & k_4 & -k_3 & k_1+k_2+k_4 & k_2+k_4 & k_1+2k_2+k_4 & k_2-k_3 & 0 & k_2 & 0 & 0 \end{pmatrix}$$

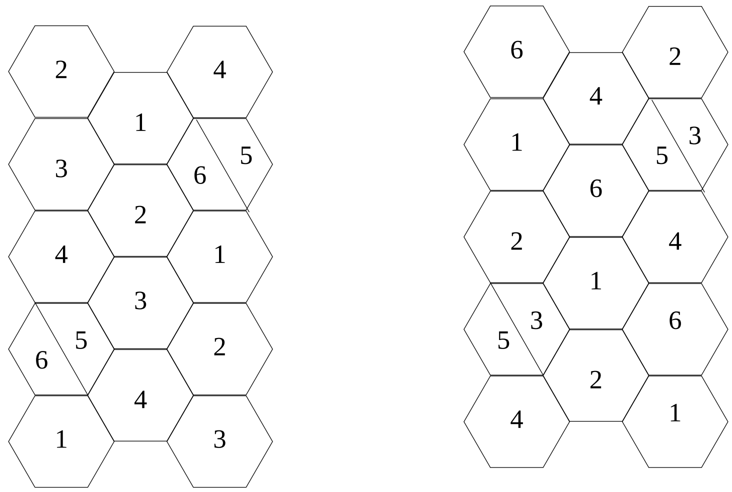


Figure 4.19: The tiling for the two dual phases of Y^{32}. Seiberg duality has been performed on groups 5.

$$
G_t^{(2)} = \begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \\
2 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & -1 & 1 & 0 & 0 & 0 \\
k_3+k_4 & k_4 & k_4 & 0 & -k_3 & -k_6 & k_1+k_2+k_4 & k_1+k_2+k_3 & k_2+k_3+k_4 & k_2+k_4 & k_2+k_4 & k_2 & k_1+2k_2+k_4 & k_2-k_3 & k_3+k_4 & 0 & -k_6 & 0
\end{pmatrix}
$$

and they coincide for arbitrary k_1, k_2, k_3, k_4, remind $k_6 = -k_1 - k_2 - k_3 - k_4$.

Dualities for CS theories without 4d parents

Three dimensional CS theories with four dimensional parents are a subset of all the possible 3d CS theories [141]. For CS theories without four dimensional parents we miss in principle the intuition from the 4d Seiberg duality. In this short section we see that we can still describe a subset of 3d CS theories with the same mesonic moduli space if we just apply the rules we learn in the previous sections.

We study a case associated with Q^{111}. We show that by performing a Seiberg-like duality and by setting the CS level of the dualized gauge group to zero, the toric diagrams of the two models coincide.

Example

The theory is described by the quiver given in Figure 4.20a. It is a generalization of the $C(Q_{1,1,1})$ with arbitrary CS levels [141]. The superpotential is

$$
W = X_{41}X_{13}X_{34}^1X_{42}X_{23}X_{34}^2 - X_{41}X_{13}X_{34}^2X_{42}X_{23}X_{34}^1 \tag{4.55}
$$

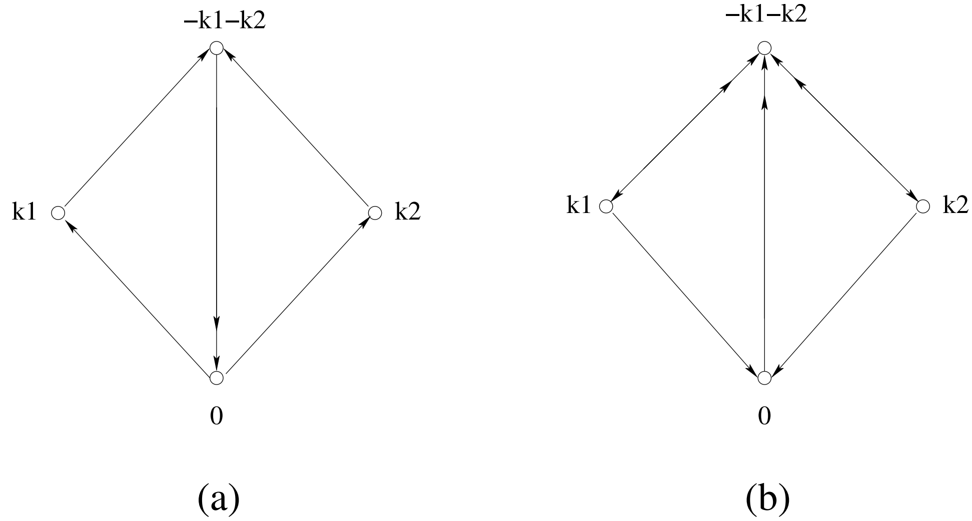


Figure 4.20: The quiver for Q^{111} in the two dual phases.

The toric diagram for $k_4 = 0$ is given by

$$G_t = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ k_1 & k_2 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{4.56}$$

Seiberg duality on node N_4 gives the superpotential

$$\begin{aligned} W &= X_{13}X_{32}^{(1)}X_{23}X_{31}^{(2)} - X_{13}X_{32}^{(2)}X_{23}X_{31}^{(1)} + X_{24}X_{43}^{(2)}X_{32}^{(2)} - X_{24}X_{43}^{(1)}X_{32}^{(1)} \\ &+ X_{14}X_{43}^{(1)}X_{31}^{(1)} - X_{14}X_{43}^{(2)}X_{31}^{(2)} \end{aligned} \tag{4.57}$$

and the theory is described by the quiver given in Figure 4.20b. The toric diagram in this case is

$$G_t = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ k_1 & k_2 & k_1 & k_2 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{4.58}$$

and it is equivalent to the toric diagram for the first phase.

In this section we made some advances towards the understanding of toric duality for M2 branes. Generalizing the work of [19, 143], we proposed a Seiberg-like duality for non-chiral three dimensional CS matter theories and we verified that the mesonic moduli space of dual theories is indeed the same four-fold Calabi Yau probed by the M2 branes. In the chiral case and in the case in which the three dimensional theories do not have a four dimensional parent, the situation is more complicated. However, fixing to zero the value of some of the Chern-Simons levels, we were able to realize toric dual pairs.

We have just analyzed the mesonic moduli space, it would be important to study the complete moduli space, including baryonic operators.

For the non-chiral case the two main limitations are the lack of understanding of the transformation rule for the rank of the gauge groups and the fact that we forced to zero some of the k_i. It is reasonable that there exist some more general and precise transformation rules and we would like to investigate them.

We concentrated on Seiberg-like transformations, but it is well known that in three dimension there exist duality maps that change the number of gauge group factors. It would be interesting to systematically study these more general transformations.

A lot of possible directions and generalizations are opening up and after these first steps we hope to step up.

4.4 Supersymmetry breaking

In three dimensions the mechanisms of supersymmetry breaking have been still rather unexplored. In a recent paper [20] the authors have shown that a mechanism analog to the ISS takes place in three dimensional massive SQCD with CS or YM gauge theories. The low energy dynamics is controlled by a Wess-Zumino (WZ) model. In four dimensions WZ models have been useful laboratories for supersymmetry breaking, playing a crucial role in the ISS mechanism.

In this section we analyze supersymmetry breaking in three dimensional WZ models. The WZ models studied in [20] had relevant couplings and the quantum corrections could be computed only after the addiction of an explicit R-symmetry breaking deformation. On the contrary, a different solution to the problem of the computation of quantum corrections in three-dimensional WZ models is given by preserving an $SO(2)_R \simeq U(1)_R$ R-symmetry and by adding only marginal deformations to the superpotential. The non-supersymmetric vacua turn out to be only metastable, since the marginal couplings induce a runaway behavior in the scalar potential. A property of these models is that R-symmetry is spontaneously broken in the non supersymmetric vacua. As a general result it seems that in three dimensions R-symmetry needs to be broken (explicitly or spontaneously) for the validity of the perturbative expansion.

We first review the model of [20] and the problems of the perturbative approach. Then we present a model with marginal couplings and long lifetime metastable vacua, and we study the general behavior of WZ models with marginal couplings The regime of validity of the perturbative approximation in models with relevant coupling is also discussed.

Effective potential in $3D$ WZ models

While a systematic study of supersymmetry breaking mechanisms in $3+1$ dimensions has been done, in $2+1$ dimension such an analysis still lacks. A recent step towards the

	q	$\tilde{q}$	M
$U(N)$	N	$\overline{N}$	1
$U(N_F)$	$\overline{N}_F$	N_F	N_F^2

Table 4.2: Representation of the fields in the model of [20]

comprehension of supersymmetry breaking in $2+1$ dimensions has been done in [20]. In this section we briefly review their model and results.
The theory is a WZ model with canonical Kähler potential

$$K = \mathrm{Tr}\left(M^\dagger M + q_i^\dagger q^i + \tilde{q}_i^\dagger \tilde{q}^i\right) \tag{4.59}$$

and superpotential

$$W = hqM\tilde{q} + h\mathrm{Tr}\left(\frac{1}{2}\epsilon\mu M^2 - \mu^2 M\right) \tag{4.60}$$

with an $U(N)\times U(N_F)$ global symmetry. The representations of the matrix valued chiral superfields q, $\tilde{q}$ and M are given in Table 4.2. All the three dimensional couplings and fields in (4.60) have mass dimension $1/2$, except ϵ which is adimensional. The model (4.60) has supersymmetric vacua labeled by $k = 0,\ldots,N$. At given k the expectation values of the chiral fields in the supersymmetric vacuum is

$$M = \begin{pmatrix} 0 & 0 \\ 0 & \frac{\mu}{\epsilon}\mathbf{1}_{N_F-k} \end{pmatrix} \quad q\tilde{q} = \begin{pmatrix} \mu^2\mathbf{1}_k & 0 \\ 0 & 0 \end{pmatrix} \tag{4.61}$$

Moreover this model also has metastable vacua, in which the combination of the tree level and one loop scalar potential stabilizes the fields. In the analysis of [20] the authors studied the case of different values of k. Here we only refer to the simplified case $k = N$.
The vacuum is

$$M = \begin{pmatrix} 0 & 0 \\ 0 & X\mathbf{1}_{N_F-N} \end{pmatrix} \quad q\tilde{q} = \begin{pmatrix} \mu^2\mathbf{1}_N & 0 \\ 0 & 0 \end{pmatrix} \tag{4.62}$$

where X is a pseudomodulus, stabilized, in this case, by the one loop effective potential. This potential is given by the Coleman-Weinberg formula, that in three dimensions is [20]

$$V_{eff}^{(1)} = -\frac{1}{12\pi^2}\mathrm{STr}|\mathcal{M}|^3 \equiv -\frac{1}{12\pi^2}\mathrm{Tr}\left(|\mathcal{M}_B|^3 - |\mathcal{M}_F|^3\right) \tag{4.63}$$

The cubic dependence on the bosonic and fermionic mass matrices $\mathcal{M}_B$ and $\mathcal{M}_F$ can be eliminated by expressing (4.63) as

$$V_{eff}^{(1)} = -\frac{1}{6\pi^2}\mathrm{STr}\int_0^\infty \frac{v^4}{v^2+\mathcal{M}^2}\mathrm{d}v \tag{4.64}$$

In appendix C.4 we observe that (4.64) can be generalized to every dimension.
The superpotential that is necessary to calculate the one loop corrections for the WZ

model (4.60) simplifies by expanding the fields around (4.62). The fluctuations of the fields can be organized in two sectors, respectively called ϕ_i and σ_i. The former represents the fluctuation necessary for the one loop corrections of the field X, while the latter parameterizes the supersymmetric fields that do not contribute to the one loop effective potential. We have

$$q = \begin{pmatrix} \mu + \sigma_1 \\ \phi_1 \end{pmatrix} \quad \tilde{q}^T = \begin{pmatrix} \mu + \sigma_2 \\ \phi_2 \end{pmatrix} \quad M = \begin{pmatrix} \sigma_3 & \phi_3 \\ \phi_4 & X \end{pmatrix} \tag{4.65}$$

The one loop CW is calculated by inserting (4.65) in the superpotential (4.60). There are $N_F(N_F - N)$ copies of WZ models with superpotential

$$W = \frac{1}{2} h\mu\epsilon X^2 - h\mu^2 X + h\mu\epsilon\phi_3\phi_4 + hX\phi_1\phi_2 + h\mu(\phi_1\phi_3 + \phi_2\phi_4) \tag{4.66}$$

The tree level potential and the one loop corrections calculated from (4.66) give raise to a non-supersymmetric vacuum at

$$\phi_i = 0 \qquad X \simeq \frac{\epsilon\mu}{b} \tag{4.67}$$

where $b = \frac{(3-2\sqrt{2})h}{4\pi\mu}$ is a dimensionless parameter. Thereafter we use a notation that makes clear the relevancy of the cubic three-dimensional couplings, by rewriting (4.66) as

$$W = \frac{1}{2}\epsilon\, m\, X^2 - fX + \epsilon\, m\, \phi_3\phi_4 + \frac{m^2}{f} X\phi_1\phi_2 + m(\phi_1\phi_3 + \phi_2\phi_4) \tag{4.68}$$

where we have defined

$$f \equiv h\mu^2 \qquad\qquad m \equiv h\mu \tag{4.69}$$

The new parameters of the theory,, f and m,respectively have mass dimension 3/2 and 1. The X field vacuum expectation value is proportional to $\epsilon f/(bm)$ and the expansion of the one-loop potential near the origin is possible if the R-symmetry breaking parameter ϵ satisfies

$$\epsilon \ll b \tag{4.70}$$

which expresses the condition $X \ll \mu$ of [20].

Moreover, for the perturbative expansion to be valid, higher orders in the loop expansion must be negligible. This last condition is satisfied when the relevant coupling is small at the mass scale of the chiral fields

$$h^2 \ll hX \iff \frac{m^4}{f^2} \ll \frac{m^2}{f} X \tag{4.71}$$

This requirement imposes a lower bound on the R-breaking parameter ϵ

$$b \gg \epsilon \gg b\frac{m^3}{f^2} \tag{4.72}$$

and by using the definition of $b = \frac{3-2\sqrt{2}}{4\pi}\frac{m^3}{f^2}$

$$\frac{3-2\sqrt{2}}{4\pi}\frac{m^3}{f^2} \gg \epsilon \gg \frac{3-2\sqrt{2}}{4\pi}\frac{m^6}{f^4} \tag{4.73}$$

The parameter ϵ cannot approach zero. In fact, in this case the theory becomes strongly coupled and the effective potential cannot be evaluated perturbatively.

Three dimensional WZ models with marginal couplings

Relevant couplings do not complete the renormalizable interactions of a three dimensional superpotential. In fact quartic marginal terms can be also added to a WZ model. Here we study supersymmetry breaking in a renormalizable WZ model with quartic marginal couplings and no trialling interactions. We show that supersymmetry is broken at tree level and the perturbative approximation is valid without any explicit R-symmetry breaking. The three dimensional $\mathcal{N} = 2$ superpotential is

$$W = -fX + hX^2\phi_1^2 + \mu\phi_1\phi_2 \tag{4.74}$$

and the classical scalar potential is

$$V_{\text{tree}} = |2hX\phi_1^2 - f|^2 + |2hX^2\phi_1 + \mu\phi_2|^2 + |\mu\phi_1|^2 \tag{4.75}$$

The chiral superfields have R-charges

$$R(X) = 2, \quad R(\phi_1) = -1, \quad R(\phi_2) = 3 \tag{4.76}$$

The F-terms of the fields X, ϕ_1 and ϕ_2 cannot be solved simultaneously and supersymmetry is broken at tree level. We study the theory around the classical vacuum $\langle\phi_1\rangle = \langle\phi_2\rangle = 0$ and arbitrary $\langle X\rangle$. Stability of supersymmetry breaking requires the computation of the one loop effective potential for the X field. The squared masses of the scalar components of the fields ϕ_1 and ϕ_2 read

$$m_{1,2}^2 = \mu^2 + 2h\langle|X|\rangle\left(h\langle|X|\rangle^3 + \eta f + \sigma\sqrt{f^2 + 2\eta f h\langle|X|\rangle^3 + h^2\langle|X|\rangle^6 + \langle|X|\rangle^2\mu^2}\right) \tag{4.77}$$

where $\langle|X|\rangle$ is the vacuum expectation value of the field X and η and σ are ± 1. These masses are positive for

$$\langle|X|\rangle < \frac{\mu^2}{4fh} \tag{4.78}$$

In this regime the pseudomoduli space is tachyon free and classically stable. Outside this region there is a runaway in the scalar potential. The squared masses of the fermions in the superfields ϕ_1 and ϕ_2 are

$$m_{1,2}^2 = \mu^2 + 2h\langle|X|\rangle\left(h\langle|X|\rangle^3 + \sigma\sqrt{h^2\langle|X|\rangle^6 + \langle|X|\rangle^2\mu^2}\right) \tag{4.79}$$

The two real combinations of the fermions of X and $X^\dagger$ are the two goldstinos of the $\mathcal{N}=2 \to \mathcal{N}=0$ supersymmetry breaking.

The one loop effective potential is computed with the CW formula (4.64). At small $\frac{X}{\sqrt{\mu}}$ the field X has a negative squared mass

$$m^2_{X=0} \sim -\frac{f^2 h^2}{\mu} \tag{4.80}$$

and the origin is unstable.

A (meta)stable vacuum is found if there is a minimum such that (4.78) is satisfied. As long as the adimensional parameter $\frac{f^2}{\mu^3}$ is small the effective potential has a minimum at

$$\langle X \rangle \simeq \sqrt{\frac{\sqrt{2}\mu}{h}} \tag{4.81}$$

This imposes a bound on the coupling constant

$$h < \frac{\mu^3}{16\sqrt{2} f^2} \tag{4.82}$$

since the scalar potential has to be tachyon free.

When (4.78) or (4.82) are saturated the classical scalar potential (4.75) has a runaway behavior. Indeed, if we parameterize the fields by their R-charges (4.76), we have

$$X = \frac{f}{2h\mu} e^{2\alpha}, \qquad \phi_1 = \sqrt{\mu} e^{-\alpha}, \qquad \phi_2 = -\frac{f^2}{2h\sqrt{\mu^5}} e^{3\alpha} \tag{4.83}$$

and we get $F_X = F_{\phi_1} = 0$ and $F_{\phi_2} \to 0$ as $\alpha \to \infty$.

Lifetime

The decay rate of the non-supersymmetric state is proportional to the semi-classical decay probability. This probability is proportional to e^{-S_B}, where S_B is the bounce action. Here the lifetime of the metastable vacuum is estimated from the bounce action of a triangular potential barrier, since the two vacua are well separated in field space and the maximum is approximately in the middle of them.

The computation is similar to [51], but in this case we have to deal with a three dimensional theory. In the appendix C.4 we compute the bounce action for a triangular potential barrier in three dimensions. We found that it is

$$S_B \sim \sqrt{\frac{(\Delta\Phi)^6}{\Delta V}} \tag{4.84}$$

In our case we estimate the behavior of the potential barrier by using the evolution of the scalar potential along the field X. The non supersymmetric minimum has been found in

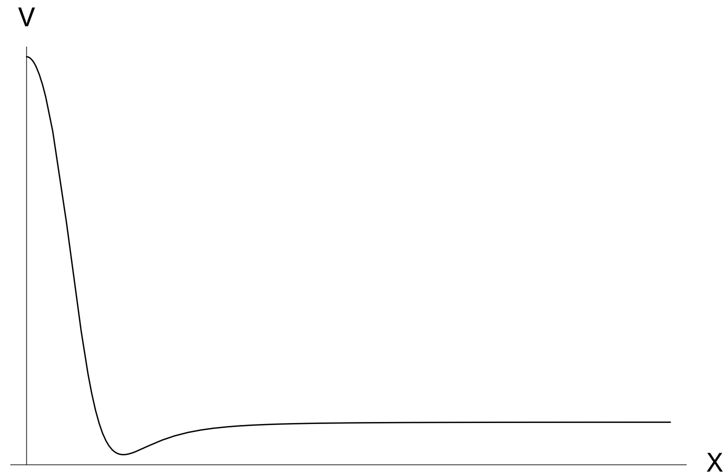


Figure 4.21: One loop scalar potential for the one-loop validity region $X < \frac{\mu^2}{4hf}$. Over this value, we find a classical runaway. At the origin the pseudomodulus has negative squared mass. The potential is plotted for $\mu = 1$, $f = h = 0.1$

(4.81) and the potential is $V_{min} = |f|^2$.
The one loop scalar potential plotted in Figure 4.21 is always increasing between the metastable minimum and (4.78). When (4.78) is saturated there is a classical runaway direction, and the local maximum of the potential can be estimated to be at $\langle X \rangle = \frac{\mu^2}{4hf}$, where the potential is $V_{max} \sim 2|f|^2$. After this maximum the potential starts to decrease and the field X acquires large values. There is not a local minimum, nevertheless the lifetime of the non-supersymmetric state can be estimated as in [42, 4]. Indeed, by using the parametrization (4.83) of the fields along the runaway the scalar potential has the same value as V_{min} for $\langle X_{\rm R} \rangle \sim \frac{\mu^2}{2hf}$.

In the regime $\frac{f^2}{\mu^3} \ll h \ll 1$ the barrier is approximated to be triangular, and the gradient of the potential is constant. The non-supersymmetric state is near the origin of the moduli space, and the bounce action is

$$S_B \sim \sqrt{\frac{\langle X_{\rm R} \rangle^6}{V_{\rm min}}} \sim \sqrt{\frac{1}{h}\left(\frac{\mu^3}{f^2}\right)^4} \gg \left(\frac{\mu^3}{f^2}\right)^2 \gg 1 \tag{4.85}$$

The general case

In this section we consider the class of models with a single pseudo-modulus X which marginally couples to n chiral superfields ϕ_i. As shown in [4] for the class of four-dimensional renormalizable and R-symmetric models, many general features are worked out by R-symmetry considerations. For the three-dimensional case, we find some interesting features concerning such models. The perturbative expansion is reliable under the weak condition that the coupling constants are small numbers, i.e. one can use the one-loop approximation and made higher loop corrections suppressed. The origin of the moduli space is a local maximum of the one-loop potential, and the pseudomodulus acquires a negative squared mass. Finally the scalar tree-level potential exhibits runaway directions for every choice of the couplings.

To deal with renormalizable three-dimensional WZ models, we consider only canonical Kähler potential, and superpotential of the type

$$W = -fX + \frac{1}{2}(M^{ij} + X^2\, N^{ij})\, \phi_i\, \phi_j \tag{4.86}$$

in which R-symmetry imposes the conditions

$$M^{ij} \neq 0 \Rightarrow R(\phi_i) + R(\phi_j) = 2 \qquad\qquad N^{ij} \neq 0 \Rightarrow R(\phi_i) + R(\phi_j) = -2 \tag{4.87}$$

The conditions (4.87) could not be sufficient to uniquely fix the R–charges. However, it is clear that a basis always exists in which there are both charges greater than two and charges lower than two.

In a basis where the fields with the same R-charge are grouped together, the M matrix is written in the form

$$M = \begin{pmatrix} & & & & M_1 \\ & & & M_2 & \\ & & \cdot & & \\ & \cdot & & & \\ M_2^T & & & & \\ M_1^T & & & & \end{pmatrix} \tag{4.88}$$

and similar for the N matrix. The scalar potential of this model can be written as

$$V_S = \left|-f + XN^{ij}\phi_i\phi_j\right|^2 + \left|M^{ij}\phi_j + X^2 N^{ij}\phi_j\right|^2 \tag{4.89}$$

which we assume to have a one-dimensional space of extrema given by

$$\phi_i = 0 \qquad\qquad X \,\text{arbitrary} \qquad\qquad V_S = |f|^2 \tag{4.90}$$

For general couplings, there can be other extrema and in particular some lower local minima away from of the origin of ϕ's. Furthermore for some choices of the coupling constants at least some of (4.90) are saddle points. Here we work under the hypothesis that this is not the case.

We show now that the effective potential always has a local maximum at the origin of the pseudo-moduli space:

$$V_{eff}(X) = V_0 + m_X^2|X|^2 + \mathcal{O}(X^3) \tag{4.91}$$

and the X field acquires a negative squared mass. We derive a general formula for m_X^2 in the one-loop approximation by using the Coleman-Weinberg formula

$$\begin{aligned} V_{eff}^{(1)} &= -\frac{1}{12\pi}\mathrm{STr}|\mathcal{M}|^3 \\ &= -\frac{1}{6\pi^2}\mathrm{Tr}\int_0^\Lambda dv\, v^4\left(\frac{1}{v^2+\mathcal{M}_B^2} - \frac{1}{v^2+\mathcal{M}_F^2}\right) \end{aligned} \tag{4.92}$$

where $\mathcal{M}_B^2$ and $\mathcal{M}_F^2$ are, respectively, the squared mass matrices of the bosonic and fermionic components of the superfields of the theory

$$\begin{aligned}\mathcal{M}_B^2 &= (\hat{M} + X^2\hat{N})^2 - 2fX\hat{N} \\ \mathcal{M}_F^2 &= (\hat{M} + X^2\hat{N})^2\end{aligned} \tag{4.93}$$

where we have defined

$$\hat{M} \equiv \begin{pmatrix} 0 & M^\dagger \\ M & 0 \end{pmatrix} \qquad\qquad \hat{N} \equiv \begin{pmatrix} 0 & N^\dagger \\ N & 0 \end{pmatrix} \tag{4.94}$$

which take a form analogous to (4.88).

Substituting the mass formulas into the Coleman-Weinberg potential (4.92) and expanding up to second order in the field X we find

$$\begin{aligned}V_{eff}^{(1)} &= -\frac{1}{6\pi^2}\mathrm{Tr}\int_0^\Lambda dv\, v^4\, \frac{1}{v^2+\hat{M}^2}\left(\frac{1}{1+\frac{X^2\{\hat{M},\hat{N}\}-2fX\hat{N}}{v^2+\hat{M}^2}} - \frac{1}{1+\frac{X^2\{\hat{M},\hat{N}\}}{v^2+\hat{M}^2}}\right) + \dots \\ &= -\frac{2f^2X^2}{3\pi^2}\mathrm{Tr}\int_0^\Lambda dv\, v^4\, \frac{1}{v^2+\hat{M}^2}\,\frac{1}{v^2+\hat{M}^2}\,\hat{N}\,\frac{1}{v^2+\hat{M}^2}\,\hat{N} + \mathcal{O}(X^3) \\ &= -\frac{f^2X^2}{2\pi^2}\mathrm{Tr}\int_0^\Lambda dv\, v^2\, \frac{1}{v^2+\hat{M}^2}\,\hat{N}\,\frac{1}{v^2+\hat{M}^2}\,\hat{N} + \mathcal{O}(X^3)\end{aligned} \tag{4.95}$$

where the last step follows after an integration by parts. From the previous formula we note that the origin of moduli space is always a local maximum, i.e. the pseudo-modulus always acquires a negative squared mass at one-loop level. The vacuum cannot be at the origin, and we have to find it at $X \neq 0$, where R-symmetry is spontaneously broken. The existence of this vacuum is not guaranteed by (4.95), but it depends on the couplings in (4.94)

We show now that the models in (4.86) have a runaway direction. In four-dimensional theories if the R-charges of the superfields are both greater than two and lower than two then the potential exhibits runaway [35]. In three-dimensional renormalizable theories (4.86), conditions (4.87) state there are always both superfields with R-charge greater than two and superfields with R-charge lower than two. We parametrize the fields by their R-charge $R(\phi_i) \equiv R_i$ as

$$\begin{aligned}\phi_i &= c_i\, e^{R_i\alpha} \\ X &= c_X\, e^{2\alpha}\end{aligned} \tag{4.96}$$

The runaway behavior of the potential is analyzed by looking at the R-charges of F-terms. The F-terms with R charges lower or equal to zero can be solved. All the non vanishing F terms have charge greater than zero. They vanish only if $\alpha \to \infty$. By (4.87), the F-terms are

$$F_X = -f + XN^{ij}\phi_i\phi_j = -f + N^{ij}c_X c_i c_j \tag{4.97}$$

$$F_{\phi_i} = M^{ij}\phi_j + X^2 N^{ik}\phi_k = \left(\sum_j M^{ij}c_j + \sum_k N^{ik}c_X^2 c_k\right) e^{(2-R_i)\alpha} \tag{4.98}$$

We distinguish three possibilities. The first is that the field ϕ_i couples both to the matrices M and N. In this case, the equation (4.98) fixes the relative coefficients of the fields. The second possibility is that the field ϕ_i does not couple with the matrix N but it has more than one entry in the matrix M. In this case we fix the relative coefficients.The last possibility is that ϕ_i only couples once to M. In this case we set $c_j = 0$.

The discussion above does not solve all the $F_{\phi_i} = 0$ in (4.98). Some of the F-terms with R-charge greater than zero have not been set to zero yet. They vanish when $\alpha \to \infty$, which implies a runaway behavior in the directions parameterized by some fields in (4.96)

Relevant couplings

In three dimensions there exist WZ models with relevant deformations that can be perturbatively studied without the addition of explicit R-symmetry breaking deformations. Even if R-symmetry is not explicitly broken in three dimensions, quantum non-supersymmetric vacua can appear out of the origin of the moduli space. The vevs at which the vacua are found set the spontaneous R-symmetry breaking scale which plays the same role as the ϵ deformation in [20].

The models which exhibit explicit or spontaneous R-symmetry breaking can be perturbatively studied in three dimensions. The former case has been analyzed in [20]. We treat here the latter case. Assuming the R-symmetry breaking vacuum is near the origin, we require that the pseudomodulus acquires a negative squared mass at the origin of moduli space, i.e.there is a vacuum of the quantum theory which spontaneously breaks the R-symmetry. This happens if not all the R-charges take the values $R = 0$ and $R = 2$. This result was shown in four dimensions in [4] and can be analogously demonstrated in three dimensions. We classify these models in two subclasses.

In the first class we identify the models without runaway behavior, i.e. all the charges are lower or equal to two. There can be a regime of couplings in which supersymmetry is broken in non R-symmetric vacua. The vev of the field that breaks R-symmetry introduce a scale which bounds the perturbative window for the relevant couplings.

In the second class, that we consider in the following, there are runaway models. They have R-charges lower and higher than two. Under the assumption of a hierarchy on the mass scales, we distinguish two possibilities. Some of these models flow in the infrared to models with only marginal couplings, that have been treated in sections 4.4 and 4.4. The other possibility is that the effective descriptions of these theories share marginal and relevant terms in the superpotential. In both cases a perturbative regime is allowed.

A model with relevant couplings

Consider the superpotential

$$\begin{aligned} W &= \lambda X\phi_1\phi_2 - fX + \mu\phi_1\phi_3 + \mu\phi_2\phi_4 \\ &+ m\phi_1\phi_5 + m\phi_3\phi_5 \end{aligned} \tag{4.99}$$

The first line corresponds to the ISS low energy superpotential, while the second line of (4.99) asymmetrizes the behavior of ϕ_1 and ϕ_2. If the mass term m is higher than the other scales of the theory ($m^2 \gg \mu^2$ and $m^2 \gg f^{4/3}$) we can integrate out the second line of (4.99), and obtain

$$W = hX^2\phi_2^2 - fX + \mu\phi_2\phi_4 \tag{4.100}$$

where $h = \frac{\lambda}{4\mu}$ is a marginal coupling. The perturbative analysis of this model is now possible, with the only requirement that $h \ll 1$.

The superpotential (4.100) is identical to (4.74). This example shows that in three dimensions models with cubic couplings in the UV can flow to theories with quartic terms in the IR, which are perturbatively accessible.

Models with relevant and marginal couplings

A model with relevant couplings can flow to a model with both marginal and relevant couplings. For example if we take the superpotential

$$W = -fX + \lambda_5 X\phi_1\phi_5 - \frac{m}{2}\phi_5^2 + \mu\phi_1\phi_2 + \lambda X\phi_3^2 + \mu\phi_3\phi_4 \tag{4.101}$$

and we study this model in the regime $m^2 \gg \mu^2$, $m^2 \gg f^{4/3}$, we can integrate the field ϕ_5 out. The effective theory becomes

$$W = -fX + hX^2\phi_1^2 + \mu\phi_1\phi_2 + \lambda X\phi_3^2 + \mu\phi_3\phi_4 \tag{4.102}$$

where $h = \frac{\lambda_5^2}{m}$. This model preserves R-symmetry and the charges of the fields are

$$R(X) = 2 \quad R(\phi_1) = -1 \quad R(\phi_2) = 3 \quad R(\phi_3) = 0 \quad R(\phi_4) = 2 \tag{4.103}$$

As before this theory has a runaway behavior in the large field region, and the fields are parametrized as

$$X = \frac{f}{2h\mu}e^{2\alpha}, \qquad \phi_2 = \sqrt{\mu}e^{-\alpha}, \qquad \phi_4 = -\frac{f^2}{2h\sqrt{\mu^5}}e^{3\alpha} \qquad \phi_3 = 0 \quad \phi_4 = 0 \tag{4.104}$$

Near the origin the classical equations of motion break supersymmetry at tree level at $\phi_i = 0$. The field X is a classical pseudomodulus whose stability has to be studied perturbatively. The pseudomoduli space is stable if $|\langle X\rangle| < \frac{\mu^2}{4fh}$ and $\frac{\lambda}{\sqrt{\mu}} < \frac{\mu^{3/2}}{2f}$.
We study the effective potential by expanding it in the adimensional parameter $\frac{f^2}{\mu^3}$, finding

$$V_{eff}^{(1)}(X) = -\frac{3f^2\lambda^2(\lambda^2X^2 + 2\mu^2)}{2(\lambda^2X^2+\mu^2)^{3/2}} - \frac{6f^2h^2X^2(h^2X^4+2\mu^2)}{(h^2X^4+\mu^2)^{3/2}} \tag{4.105}$$

This perturbative analysis holds if the coupling λ is small at the mass scale of the chiral field ϕ_3

$$\lambda^2 \ll \lambda X \tag{4.106}$$

This requirement imposes that the field X cannot be fixed at the origin, and R-symmetry has to be broken in the non supersymmetric vacuum. The coupling λ has to be small, and we can expand the potential in the adimensional parameter $\frac{\lambda}{\sqrt{\mu}}$. At the lowest order we found that a minimum exists and it is

$$X \sim 2^{1/4} \left(\frac{\mu^2}{h^2} - \frac{9\sqrt{3}\lambda^2\mu^2}{15\sqrt{3}h^2\lambda^4 + 4h^4\mu^2} \right)^{1/4} \tag{4.107}$$

Inserting (4.107) in (4.106) we find the condition under which the one loop approximation is valid. In this range we found a (meta)stable vacuum at non zero vev for the pseudo-modulus.

We have shown that metastable supersymmetry breaking vacua in three dimensional WZ models are generic. Relevant couplings potentially invalidate the perturbative approximation. Nevertheless, as we have seen, this problem is removed by the addiction of marginal couplings.

Our study may be useful for the analysis of spontaneous supersymmetry breaking in $3D$ gauge theories. This issue has been investigated in [159, 163, 165] as a consequence of brane dynamics. A preliminary step towards the study of supersymmetry breaking in the dual field theory living on the branes appeared in [20], where the three dimensional ISS mechanism has been discovered for Yang-Mills and Chern-Simons gauge theories. The first class has been deeply studied in [155]. The second class has become more important in the last years, because of its relation with the AdS_4/CFT_3 correspondence. It would be interesting to generalize the three dimensional ISS mechanism of [20] in theories which admit a Seiberg-like dual description [19, 162, 166, 167].

Another interesting aspect, that needs a further analysis, is the role of R-symmetry. In fact in three dimensions supersymmetry breaking seems always paired with R-symmetry breaking, spontaneous or explicit. A similar result holds in four dimensions [3]. Here the condition seems stronger, since known models without R-symmetry breaking are not perturbatively accessible.

Conclusions

In this thesis we have discussed various application of Seiberg duality in supersymmetric gauge theories. Many other direction can be investigated, both in four and three dimensions.

For example the discover of the existence of metastable vacua receives a great interest both for the theoretics developments both for the phenomenological application. Much of the theoretical interest is devoted to the application of metastability to the gauge/gravity duality. For example quiver gauge theories have given many extensions of the ISS mechanism. Moreover these theories, representing wrapped branes at CY singularity, can in principle relate the metastable supersymmetry breaking vacua to the dS vacua of [93]. A similar idea appeared in [30], but a general comprehension of this phenomenon still lacks. It is not clear how to match the dS vacua in the CY throat with the ISS vacua. The phenomenological zoo of supersymmetry breaking models has been enlarged by ISS like models. Indeed many deformations of ISS model have been investigated as hidden sectors of supersymmetry breaking. Gauge mediation is the natural mechanism used to propagate the supersymmetry breaking to the MSSM superpartners, sfermions and *gauginos*. Recently [168, 169] have applicated the idea of pseudomoduli to DM. It seems a promising possibility, since it fits with the requirements on relic abundance and pseudomoduli DM can be rather light, i.e. at the TeV scale.

Another application that we discussed is the connection between exotic stringy instanton contribution and gauge theory. Duality furnishes an interpretation in terms of gauge instanton contribution on one side that becomes classical constraints on the moduli space in the dual model. This behaviour is typical of cascading theories and is general. It is possible that a similar result holds in the case of multi F-term contributions.

The last application of duality that we have shown is the connection among different theories describing the motion of M2 branes on CY_4 toric singularities. These theories have a CFT_3 interpretation in terms of CS matter $\mathcal{N}=2$ gauge theories. Seiberg duality in three dimensional CS theories has similar rules than in the four dimensional case. Nevertheless the introduction of the CS levels change the rules of transformation of the gauge groups. The intuition given by the knowledge of the four dimensional case is not enough in three dimensions. Indeed a larger duality symmetry, called mirror symmetry, should connect theories with different number of gauge groups in three dimensions. This implies that in three dimensions duality acts in a more complicated way than in four dimensions, and a complete comprehension and classification of dual theories in here still lacks. It is also interesting to study the ISS mechanism in these three dimensional quiver gauge theories. From our analysis it is evident the gauge theories must break R-symmetry in proximity of the metastable state. From four dimensions we learned that it is happens only explicitly. Indeed no four dimensional gauge theories that spontaneously break R-symmetry at the metastable vacua are known. The ISS mechanism in quiver gauge theories has to be realized by deforming the singularity with supersymmetry and R-symmetry breaking terms.

Appendix A

Analysis of $\mathcal{N} = 1$ supersymmetric theories

A.1 Quiver gauge theories

In this appendix we review the general properties of two classes of quiver gauge theories. The first class is the quiver gauge theory associated to a ALE space with a A-D-E singularity. The second class is associated to toric singularities.

Quiver gauge theory arises in AdS/CFT in the study of D3 branes at CY singularity. In four dimensions quiver gauge theories are supersymmetric models with a product of $U(N)$ or $SU(N)$ gauge groups and bifundamental or adjoint two index tensors matter fields. We focus only on $\mathcal{N} = 1$ theories. The structure of the gauge groups and of the matter are encoded in a graph composed of directed arrows and vertexes. The arrows represents the matter fields and the vertexes the gauge group. Every arrow that connects two vertexes represents a bifundamental field. The tail of the arrow tells us that the field is in the fundamental representation of the gauge group associated to the vertex, while the head entails the antifundamental representation. If the two endpoints of an arrow coincide the matter field is in the adjoint representation of the associated vertex.

One can define the incidence matrix Q_i^a of the quiver, that encodes the $U(1)$ factors of $U(N)$ under which the fields are charged. In this matrix a $+1$ factor is associated to the fundamental representation, a -1 factor to the antifundamental and 0 for the adjoint.

The superpotential of a quiver is a function of the gauge invariant operators (closed loops in the quiver), but there is no algorithm to work it out for a given quiver, and other information are necessary.

The other information are given by the AdS/CFT origin of quiver theories. The structure of the internal CY_3 geometry, and of its singularity determine the superpotential of the theory. In some simple case, like ADE and toric singularities there are straightforward algorithm to compute the structure of the superpotential.

A_n quiver gauge theories

We consider a class of $\mathcal{N} = 1$ supersymmetric gauge theories that takes origin from wrapping D-branes on ADE fibered CY_3. These theories are $\mathcal{N} = 2$ quiver gauge theories associated to the ADE Dinkin diagrams deformed to $\mathcal{N} = 1$ by polynomial superpotential terms in the adjoint fields.

The undeformed theory

The $\mathcal{N} = 2$ theory arises from ALE spaces with ADE singularities at the origin. These spaces are obtained by quotienting $\mathbb{C}^2$ by a discrete subgroup Γ of $SU(2)$. We study only the cyclic A_n geometry. The geometry is an hypersurface $f(x, y, z) = 0$ on $\mathbb{C}^3$, $f = x^2 + y^2 + z^{r+1}$ and the origin is singular. These spaces can be desingularized by proper deformations. There are two possibility, one can add deformation to the equation $f(0, 0, 0) = 0$, or blow up the singularity.

In the first case one can deform the equation for an A_{r-1} space as

$$x^2 + y^2 + \prod_{i=1}^{r}(z + t_i) \tag{A.1}$$

with the constraint $\sum_{1=1}^{r} t_i = 0$. The space $f = 0$ admits r non-vanishing S^2. There exists a basis for which the intersection of these S_i^2 is the same of the A_{r-1} Dynkin diagram. These S^2 are related to the volume of the corresponding two-cycles in the geometry. The integral of the holomorphic two forms over the corresponding S^2 is

$$\alpha_i = \int_{S_i^2} dw = \int_{S_i^2} \frac{dxdy}{z} \tag{A.2}$$

The α_i are identified with the simple roots of the A_{r-1}. One can alternatively use these α_i instead of the t_i as deformation parameter, since the two quantities are related by $\alpha_i = t_i - t_{i+1}$.

The other possibility to obtain an S^2 is to blow up the singularity. The ALE space has a three dimensional space of deformations (complex deformation and blow up) for each S_i^2. Moreover in IIB string theory one can turn on B fields, B^{NS} and B^R. This gives a 5 parameter family of deformation. The volume of each S_i^2 is

$$V_i = \left((B_i^{NS})^2 + r_I^2 + |\alpha_i|^2\right)^2 \tag{A.3}$$

where $r_i = \int_{S_i^2} k$, and k is the Kahler form.

Then D_5 spacetime filling branes are wrapped on the deformed space (N_i branes at each S_i^2). The gauge theory living on the branes is a $\mathcal{N} = 2$ quiver gauge theory with gauge group G

$$G = \prod_{i=1}^{r} U(N_i) \tag{A.4}$$

and each group has coupling constant given by

$$\frac{1}{g_{\rm YM}^2} = \frac{V_i}{g_s} \tag{A.5}$$

For each intersecting pair of S^2_i, S^2_j there is an $\mathcal{N}=2$ hypermultiplet. In $\mathcal{N}=1$ notations this is a pair of bifundamental superfields Q_{ij} and Q_{ji}, that transforms in the fundamental of the first index and in the antifundamental of the second. The superpotential is

$$W = \sum_{i,j} s_{ij} \mathrm{Tr}\left(Q_{ij}\Phi_j Q_{ji}\right) \tag{A.6}$$

where Φ_j is the $\mathcal{N}=1$ adjoint field charged under $U(N_j)$. The matrix s_{ij} is antisymmetric and $|s_{ij}|=1$.

Before fibering the geometry an observation is necessary. The space $\mathcal{C}^2/\Gamma$ gives $r+1$ choices for the basis of wrapped D-branes in the case of A_r. Here we have added r D5 branes wrapped over two cycles in the H_2 homology class of the ALE space. The extra charge arises from the H_0 class of homology of the ALE space, and it corresponds to add a spacetime filling D3, which is point like in the ALE space. This new brane has the effect of connecting the first and the r-th node of the quiver with the fields Q_{1r} and Q_{r1}. From now on we concentrate on the non affine case only, where the D3 branes do not appear.

Fibration of the space and deformation to $\mathcal{N}=1$

Supersymmetry breaking to a $\mathcal{N}=1$ theory is obtained after wrapping the branes on the fibered geometry. The fibration takes place over the complex plane t, transverse to the D5. The Kahler class is fixed on the 2-fold geometry and the complex moduli of the ALE space are varied. One has $t_i = t_i(t)$ and $\alpha_i = \alpha_i(t)$. If α_i is not a single value function of t one has a monodromic fibration, on the contrary if α_i is single valued the fibration is non monodromic.

In the simplest case, A_1 the equation for the 3-fold is

$$f = x^2 + y^2 + z^2 + \alpha(t)^2 = 0 \tag{A.7}$$

and over each point in the complex plane t there is a S^2 with holomorphic volume $\alpha(t)$.

The D5 branes are wrapped around the S^2 fiber as before, and the modulus t parameterizes the vev of the adjoint field Φ. The superpotential is [170]

$$W(\Phi) = \int_{S^2(t)\times I}^{t=\Phi} w \wedge dt = \int_I^{\Phi} \alpha(t) dt \tag{A.8}$$

and I is an interval on t. Modulo a constant factor one has

$$\frac{dW}{d\Phi} = \alpha(\Phi) \tag{A.9}$$

In the general case of an A_n Dynkin diagram one considers non monodromic fibrations, such that the branes can be wrapped over non trivial 2-cycles in the fibered geometry. Each α_i can be taken to be a polynomial in t. Defining $\alpha_i(t) = dP_i(t)/dt$ the complete superpotential is

$$W = \sum_{i,j} s_{ij} \left(\mathrm{Tr} Q_{ij} \Phi_j Q_{ji} \right) + \sum_i \mathrm{Tr} P_i(t) \tag{A.10}$$

Toric quiver gauge theories

The other class of quiver gauge theory that we need to review is the class of toric quiver gauge theories. In the study of branes at singularities we have a stack of N D3 branes placed at a toric CY singularity. The toric condition asks a $U(1)^3$ isometry group in the internal geometry. From the field theory side this means that there is a $U(1)_R$ symmetry and two additional $U(1)$ global flavour symmetries. Moreover the theory may have additional baryonic $U(1)$ symmetries (corresponding to gauged symmetries in the dual AdS string theory). Differently from non-toric theories the requirement of having three $U(1)^3$ fixes the superpotential of the gauge theories.

From the geometry side the D3 brane are stable far from the singularity, but they become unstable and decay into fractional branes at the singularities. Fractional branes break the conformal symmetry and give raise to a non trivial dynamics in the flow through the IR. For example many theories have a cascading behaviour after fractional branes are added. The singular structure determine the structure of the gauge theory. The gauge theory is a product of $U(N)$ gauge group. In the IR the $U(1)$ factors become global symmetries and we are left with a $\prod SU(N_i)$ gauge theory. These symmetries were in origin non anomalous. In the original theory there could be also anomalous $U(1)$ gauge symmetries, that are canceled in string theory, with the associated gauge fields getting a mass. Examples of toric quiver gauge theories are the $\mathcal{N} = 4$, the conifold and their orbifold. In this thesis we studied also other examples of toric theories as the Y^{pq}, the L^{pqr} and the del Pezzo singularities. An additional property typical of toric gauge theories is the F terms structure. Each field appears linearly twice in the superpotential, with opposite sign and F terms have a structure "monomial"="monomial". Moreover these equation constraints the mesonic operators to form a chiral ring. In many applications the moduli space of these toric theories is constructed from a GLSM [171].

Dimer models

The analysis of quiver toric theories is further simplified by another geometric structure, the dimer model. This simplify the analysis of the moduli space of the toric model, and the identification of the GLSM fields is straightforward in this formalism.

The first step in the building of a dimer model is the association of each superpotential term with a polygon. Each edge of the polygon is associated to a field. Each pairs of polygons are glued together with only one of their edge (every field in a toric theory appear only twice, and with opposite sign in the superpotential). This construction represents a polygonal tiling of a orientable Riemann surface, called the planar quiver. This Riemann surface has genus $g = 1$, because of the conditions of toricity and conformity, and it is topologically a torus T^2.

	Planar Quiver	Brane Tiling
Vertex	Gauge group	Superpotential Term
Edge	Matter Field	Matter Field
Face	Superpotential Term	Gauge group

The dual graph, called the brane tiling, is built by inverting the role of faces and vertexes. The dual vertexes represents the superpotential interactions and the dual faces represents the gauge groups. The role of the edges in unchanged, and they still represents the matter fields. This graph is bipartite, and the sign of the superpotential term reflected in the black or with color of the vertexes (e.g. "black=+1" and "white=−1").

A dimer is defined as a marked edge connecting the black and the white in a bipartite tiling. A collection of dimers that involve all the edges only once is called a perfect matching. A dimer model is the collection of the bipartite graph and the perfect matchings. In four dimensions the knowledge of the perfect matchings is fundamental for the building of the GLSM that describe the classical moduli space (see [172] for reviews).

The Newton polygon is a convex polygon in Z^2 generated by the integer exponent of the monomial in P. It represents the toric diagram associated to the moduli space. The perfect matchings are in $1-1$ correspondence with the fields of the GLSM that describes the toric geometry. As explained in [149] there is an algorithm for the calculation of this diagram. Since it is the moduli space one has to solve the F terms and the D terms. The contribution of the F terms is contained in the matrix that maps the bifundamental field in the GLSM fields, e.g. the perfect matching. This charge matrix Q has dimensions $(c - N_G - 2) \times c$, where c is the number of perfect matchings. Another charge matrix Q_D determines the action of the D-terms on the perfect matchings, and Q_D is a $N_G - 1 \times c$ matrix. The toric diagrams corresponds to the column of the matrix $G_t = Ker(Q_t)^T$, where Q_t is given by joining Q and Q_D.

Toric diagrams

From the algebraic-geometric point of view the data of a conical toric Calabi-Yau are encoded in a rational polyhedral cone $\mathcal{C}$ in $\mathbb{Z}^3$ defined by a set of vectors V_α $\alpha = 1, ..., d$. For a CY cone, using an $SL(3, \mathbb{Z})$ transformation, it is always possible to carry these vectors to the form $V_\alpha = (x_\alpha, y_\alpha, 1)$. In this way the toric diagram can be drawn in the x, y plane (see for example Figure 2.3). The CY equations can be reconstructed from this set of combinatorial data using the dual cone $\mathcal{C}^*$.
The two cones are related as follow. The geometric generators for the cone $\mathcal{C}^*$, which are vectors aligned along the edges of $\mathcal{C}^*$, are the perpendicular vectors to the facets of $\mathcal{C}$. To give an algebraic-geometric description of the CY, we consider the cone $\mathcal{C}^*$ as a semigroup and find its generators over the integer numbers. The primitive vectors pointing along the edges generate the cone over the real numbers but we generically need to add

other vectors to obtain a basis over the integers. Denote by W_j with $j = 1, ..., k$ a set of generators of $\mathcal{C}^*$ over the integers. To every vector W_j one can associate a coordinate x_j in some ambient space. k vectors in $\mathbb{Z}^3$ are linearly dependent for $k > 3$, and the additive relations satisfied by the generators W_j translate into a set of multiplicative relations among the coordinates x_j. These are the algebraic equations defining the six-dimensional CY cone.
All the relations between points in the dual cone become relations among mesons in the field theory. In fact, there exists a one to one correspondence among the integer points inside $\mathcal{C}^*$ and the mesonic operators in the dual field theory, modulo F-term constraints [1]. To every integer point m_j in $\mathcal{C}^*$ we indeed associate a meson M_{m_j} in the gauge theory with $U(1)^3$ charge m_j, which uniquely determine them. The first two coordinates $Q^{m_j} = (m_j^1, m_j^2)$ of the vector m_j are the charges of the meson under the two flavour $U(1)$ symmetries. Since the cone $\mathcal{C}^*$ is generated as a semi-group by the vectors W_j the generic meson will be obtained as a product of basic mesons M_{W_j}, and we can restrict to these generators for all our purposes. The multiplicative relations satisfied by the coordinates x_j become a set of multiplicative relations among the mesonic operators M_{W_j} inside the chiral ring of the gauge theory. It is possible to prove that these relations are a consequence of the F-term constraints of the gauge theory. The abelian version of this set of relations is just the set of algebraic equations defining the CY variety as embedded in $\mathbb{C}^k$. In the example of SPP from the four mesons x, y, z, w we associate the quadric $xy^2 = zw$ in $\mathbb{C}^4$.

Example: the conifold

We study the moduli space of the conifold by using the rules of toric geometry. This theory represents the CY$_3$ singularity whose affine coordinates are given by the hypersurface in $\mathbb{C}^4$

$$xy - wz = 0 \tag{A.11}$$

The worldvolume gauge theory of a stack of N D3 branes on this singularity has two gauge groups $SU(N) \times SU(N)$ with four bifundamental fields A_i and B_i. The superpotential is

$$W = A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1 \tag{A.12}$$

We start by studying the case $N = 1$ where the superpotential is $W = 0$. The gauge invariant mesonic operators $x = A_1 B_1$, $y = A_2 B_2$, $w = A_1 B_2$ and $z = A_2 B_1$ are subject to (A.11). For generic N one has the N-symmetrized products of N copies of the conifold.

The quiver and the brane tiling of this theory are given in figure A.1. The Kastelein matrix is a 1×1 matrix, the sum of the four bifundamental fields. The perfect matchings

[1]For the relations between the chiral ring of toric CFT and the geometry of the singularities see [173, 174, 175, 176, 177].

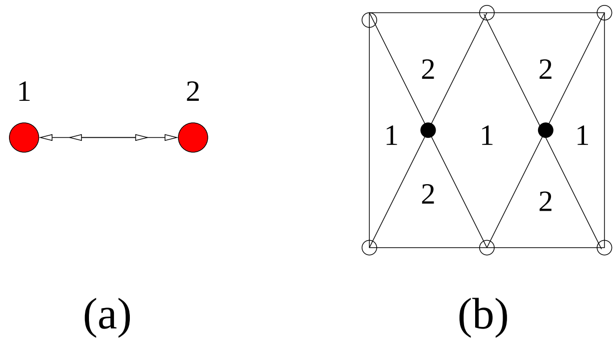


Figure A.1: (a) Quiver for the conifold, (b) brane tiling of the conifold

are four and they correspond to the fields A_i and B_i. The d and P matrices are

$$d = \begin{pmatrix} 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \end{pmatrix} \qquad P = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{A.13}$$

The matrix of the GLSM fields Q corresponds to the matrix d in this case. The D term

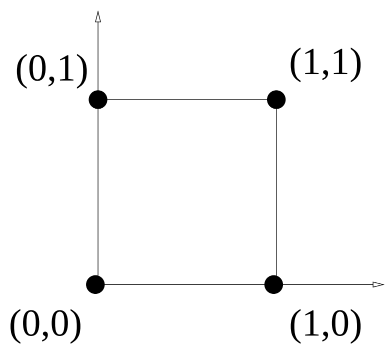


Figure A.2: Toric diagram for the conifold

contribution is obtained by quotienting the matrix Q by the $G-1$ FI parameters of the gauge theory, that are in the integer kernel of $K = (1,1)$. The toric diagram, drawn in figure A.2 is

$$G = \begin{pmatrix} Q_D \\ Q_F \end{pmatrix} = \begin{pmatrix} Ker(K) \cdot Q \\ Ker(P^T) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix} \tag{A.14}$$

ant it encodes the classical moduli space for the GLSM model.

A.2 Orientifold projection from dimers

In this appendix we discuss the procedure to follow for orientifolding a toric quiver gauge theory using dimers. This operation has been worked out in [118], and we needed it to built the instantonic action in chapter 3.

There are two possibility, orientifold from dimers with fixed points and orientifold from dimers with fixed lines.

Orientifolds from dimers with fixed points

We consider systems of D3 branes on toric CY_3 singularity. If the orientifold action commute with the $U(1)^3$ mesonic symmetry group the orientifold act as a Z_2 symmetry on the dimer corresponding to reflections of the two coordinated of the T^2 torus.

There are four fixed points under this Z_2 in the dimer diagram, these fixed points correspond to the orientifold planes, O^+ and O^-. This sign assignment determines the projection on the chiral and vector multiplets. If we name with an a the index of a face and a' the index of the image of a under the projection, the rules that relate the un-orientifolded theory and the orientifolded theory are

* Every face $a \neq a'$ gives a gauge factor $U(N_a)$;

* Every face $a = a'$ on a $O^+(O^-)$ plane gives a factor $SO(N_a)$ $(Sp(N_a/2))$;

• A chiral multiplet in the bi-fundamental $(\square_a, \overline{\square}_b)$ with $b \neq a'$ gives a bi-fundamental in the orientifolded theory $(\square_a, \overline{\square}_b)$ (the image is $(\square_{b'}, \overline{\square}_{b'})$);

* A bi-fundamental $(\square_a, \overline{\square}_{b'})$ gives a bi-fundamental $(\square_a, \square_b)$;

* An edge with $a = a'$ that represents a bifundamental $(\square_a, \overline{\square}_{a'})$ on the top of a plane O^+ (O^-) is projected on $\square\!\square_a$ $\left(\begin{smallmatrix}\square\\ \square\end{smallmatrix}_a\right)$

* The orientifolded theories are in general chiral theories (also if the parents were vector like). This introduces gauge anomalies, that are usually canceled by the Green-Schwartz mechanism. If it is not the case some new D7 branes are required;

* The superpotential of the orientifolded theory is obtained from the parent theory by projecting out an half of the terms and substituting the other terms with their images under orientifold.

The sign of the orientifold is determined by the number of terms in the superpotential. The number of orientifold planes with the same sign is even (odd) if the number of therm in the superpotential is even (odd).

Orientifolds from dimers with fixed lines

In this case the orientifold planes do not preserve the mesonic symmetry but only a combination. In the dimer these are symmetries with a fixed line. Orientifold with fixed lines are obtained from dimers whose fundamental cell is invariant under a line reflection (and the constraint of mapping black nodes with black nodes and white nodes with white nodes). There are two possible fundamental cells with this characteristic, rectangles and rhombus, see figure A.3 and A.4. Since the brane tiling is interpreted a set suspended

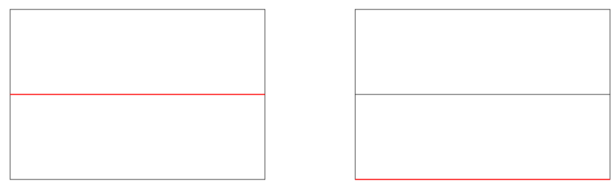


Figure A.3: Orientifold with fixed lines: rectangular fundamental cell

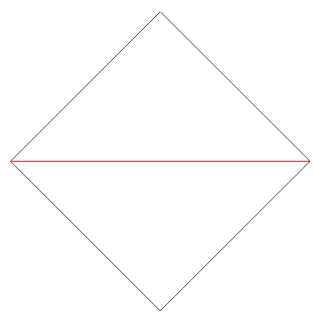


Figure A.4: Orientifold with fixed lines: rhombus fundamental cell

D5 on NS5 branes, the fixed lines are interpreted as physical orientifold planes. The rules of the projection are

* The faces that are mapped into themselves from the orientifold action have the gauge factors projected on SO or Sp, depending on the orientifold charge. The different faces that are identified become unitary groups.

* The edges mapped in themselves become chiral multiplets in the symmetric (anti-symmetric) representation in the case of O^+ (O^-). The edges that are coupled by a line are identified as bifundamental matter.

* The nodes that are mapped in themselves are the interaction terms in the superpotential.

A.3 Hierarchy of scales

One of the main approximation we used to find metastable vacua has been to neglect the fact that the odd nodes are gauge nodes. In order to treat them as flavours groups in the region of interest, it is necessary that their gauge couplings are lower than the couplings of the even nodes. We can treat the odd groups as flavour groups only if this relation holds.

In order to substantiate this idea we have to relate the electric scale of the flavour group to the other scales of the theory. The latter ones are the strong coupling scale of the gauge theories, Λ_{2i}, and the supersymmetry breaking scale μ, which is the value of the linear term in the dual version of the theory.

We must impose the groups related to the flavour/odd nodes to be less coupled than the gauge/even groups in the magnetic region. A similar analysis was performed in [29].

There are six possibilities, shown in Figure 1 in section 2.3. We have already discussed what happens in all these different cases. We will now show how to derive the formulas (2.143) and (2.147).

Let's denote by f all the objects related to the flavour group, and by g all the objects related to the gauge group. We have to distinguish four different cases, all with $\widetilde{b}_f > \widetilde{b}_g$ [2]. In fact the flavours can be IR free or UV free in the electric description (i.e. above the scale Λ_{2i}) and also UV free or IR free in the magnetic description.

We start studying a single case, and then we will comment about the others. Let's study the case (2) in Figure 1, where the flavours are UV free in the electric and IR free in the magnetic description, i.e. $b_f > 0$ and $\widetilde{b}_f < 0$.

We require that after Seiberg duality the gauge coupling g_g is larger than the flavour coupling g_f. More precisely we require that this happens at the supersymmetry breaking scale μ

$$\frac{1}{g_f^2(\mu)} > \frac{1}{g_g^2(\mu)} \qquad \Rightarrow \qquad \widetilde{b}_f \log\left(\frac{\widetilde{\Lambda}_f}{\mu}\right) < \widetilde{b}_g \log\left(\frac{\widetilde{\Lambda}_g}{\mu}\right) \tag{A.15}$$

from which follows

$$\widetilde{\Lambda}_f > \left(\frac{\widetilde{\Lambda}_g}{\mu}\right)^{\frac{\widetilde{b}_g - \widetilde{b}_f}{\widetilde{b}_f}} \widetilde{\Lambda}_g > \widetilde{\Lambda}_g \tag{A.16}$$

The scale matching relation coming from Seiberg duality

$$\Lambda_g^{3n_g - n_f} \widetilde{\Lambda}_g^{2n_f - 3n_g} = \hat{\Lambda}_g^{n_f} \tag{A.17}$$

fixes $\Lambda_g = \widetilde{\Lambda}_g$, if we choose the intermediate scale to be $\hat{\Lambda}_g = \Lambda_g$.

For the flavour scale we observe that, at the scale Λ_g, where we perform Seiberg duality, the coupling in the electric description for the odd node is the same that the coupling of

[2] The opposite inequality do not require this analysis, since at low energy the flavours are always less coupled than the gauge.

the magnetic description, and this implies

$$g_f = \widetilde{g}_f \quad \rightarrow \quad \left(\frac{\Lambda_f}{\Lambda_g}\right)^{b_f} = \left(\frac{\widetilde{\Lambda}_f}{\Lambda_g}\right)^{\widetilde{b}_f} \tag{A.18}$$

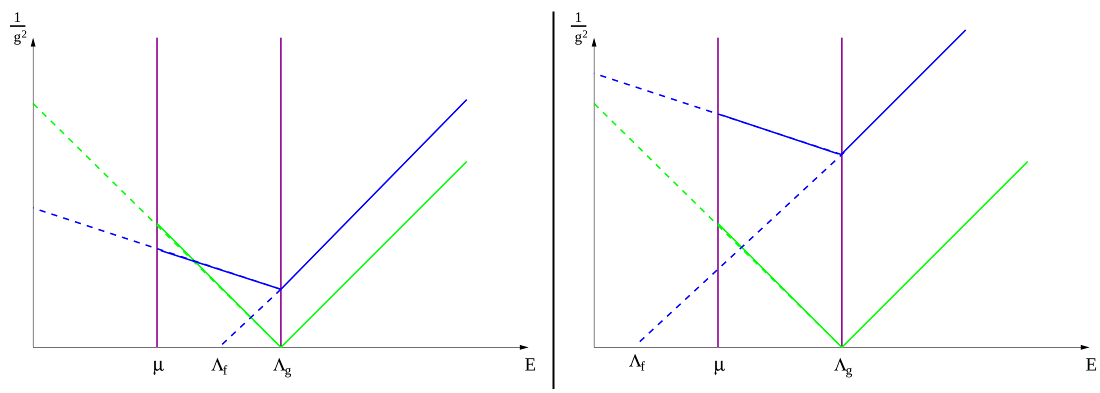
We can now write (A.16) in term of the electric scales (Λ_f and Λ_g) using (A.18), and we obtain

$$\Lambda_f < \mu^{\frac{\widetilde{b}_f - \widetilde{b}_g}{b_f}} \Lambda_g^{\frac{\widetilde{b}_g - \widetilde{b}_f + b_f}{b_f}} \tag{A.19}$$

Since the exponent of μ is positive we have

$$\frac{\widetilde{b}_f - \widetilde{b}_g}{b_f} > 0 \quad \rightarrow \quad \Lambda_f < \left(\frac{\mu}{\Lambda_g}\right)^{\frac{\widetilde{b}_f - \widetilde{b}_g}{b_f}} \Lambda_g \ll \Lambda_g \tag{A.20}$$

This imposes a stronger constraint on the scale of the flavour group Λ_f. In fact it is not enough to choose it lower than the gauge strong coupling scale Λ_g. It is also constrained by (A.20). The next figure explains what happens



In the first picture the scale Λ_f is lower than Λ_g but not enough: at the breaking scale it is not possible to neglect the contribution coming from $\widetilde{g}_f$. Instead, if we constrain the scale Λ_f using (A.20), we obtain the runnings depicted in the second picture: here the flavour groups are less coupled than the gauge groups at the supersymmetry breaking scale.

As explained above there are four different possibilities. The second possibility is that the flavours are UV free both in the electric description and in the magnetic description, with $\widetilde{b}_f > 0$. The analysis is the same as before, and we obtain the same inequality as (A.20). However this situation requires a more careful analysis, since in the infrared the gauge coupling associated to the flavour group develops a strong dynamics which has to be taken under control.

For the other two possibilities, where $b_f < 0$, one finds

$$\Lambda_f > \left(\frac{\Lambda_g}{\mu}\right)^{\frac{\widetilde{b}_g - \widetilde{b}_f}{b_f}} \Lambda_g \gg \Lambda_g \tag{A.21}$$

The general recipe we learn from this analysis can be summarized in three different cases

- If the inequality $\widetilde{b}_f < \widetilde{b}_g$ holds one has simply to choose $\Lambda_f \ll \Lambda_g$ or $\Lambda_f \gg \Lambda_g$ if $b_f > 0$ or $b_f < 0$ respectively as in (2.139,2.140).
- If $\widetilde{b}_f > \widetilde{b}_g$ we can still distinguish two cases
  - In the first case $b_f > 0$, and we have to constraint Λ_f with (A.20).
  - In the second case $b_f < 0$, and we have to constraint Λ_f with (A.21).

A.4 Geometric transition and the superpotential

In this Appendix we review the geometric transition techniques of [59] for computing the low energy superpotential from the geometrical data. The computation is illustrated here for the ϵ-deformed geometries. These deformations are due to the strong dynamics developed by the gauge groups that lead to the supersymmetric vacua.

With this technique it is possible to write the superpotential for the gaugino condensate and its interaction with the adjoint fields, which are the mesons describing the low energy theory. The dynamical deformation ϵ of the geometry is related to the gaugino condensate, while the adjoint field is interpreted as the location of the $D5$-branes relative to the dynamically deformed conifold.

In the SPP example, the deformed geometry is

$$(x(y-\xi)-\epsilon)\, y = wz \tag{A.22}$$

and the glueball field is given by $\epsilon = 2S$.

The low energy superpotential W_{IR} is composed by two contributions

$$W_{IR} = W_{GVW}(S) + W_{adj}(S, X) \tag{A.23}$$

the first one involves the glueball field S whereas the second one is the contribution of the adjoint field X.

The superpotential for the glueball field is the GVW flux superpotential

$$W_{GVW}(S) = \int H \wedge \Omega = NS \left(\log \frac{S}{\Lambda_m^3} - 1 \right) + \frac{t}{g_s} S \,. \tag{A.24}$$

This perturbative superpotential is a function of the glueball field S and of a parameter t. The t parameter takes into account the multistanton contribution to the low energy superpotential. In fact since we have D5-branes wrapping rigid $\mathbb{P}^1$ in a Calabi-Yau, D1-brane istantons wrapping the $\mathbb{P}^1$ generate a superpotential proportional to $\exp^{-\frac{t}{g_s N}}$ with $t = \int_{S^2} B^{NS} + i g_s B^{RR}$. Expanding with respect of t in the low energy theory we can take into account the multistanton contribution.

In [59] it has been shown how to compute from geometrical data the adjoint contribution $W_{adj}(S, X)$ to the low energy superpotential. It is given by the integral over holomorphic 3-form

$$W_{adj}(S, X) = \int_\Gamma \Omega \tag{A.25}$$

where Γ is a 3-chain bounded by the 2-cycle that the D5 brane wraps. This can be computed writing the geometry (A.22) in terms of new variables $x = x' - y'$ and $y = x' + y'$

$$\prod_{i=1}^{3}(x' - {x'}_i(y')) = wz \tag{A.26}$$

and evaluating

$$W_{adj} = \int ({x'}_3(y') - {x'}_1(y'))dy' \,. \tag{A.27}$$

More generally, [59] if we have a geometry of the form

$$\prod_{i=1}^{n}(x' - {x'}_i(y')) = wz \tag{A.28}$$

the contribution of the j-th node to this superpotential is of the form

$$W_{j,adj} = \int ({x'}_j(y') - {x'}_{j+1}(y'))dy' \,. \tag{A.29}$$

In the SPP case the only node in the quiver with the adjoint field is N_1, and indeed the contribution to the superpotential is (A.27). In the regime where all the deformations are lower than y' ($y' \gg \epsilon, \xi_i$), we can expand the integral (A.27) at first order in ϵ, and obtain

$$W_{adj} = \xi X_{11} - S \log\left(\frac{X_{11}}{\Lambda_m}\right) \tag{A.30}$$

where we have identified $\epsilon = 2S$. From the full low energy superpotential W_{IR} (A.23) we can now obtain a description in terms of the adjoint field only. This is achieved by integrating out the glueball field, using $(N + M)$ copies of (A.30)

$$S = \left(\Lambda_m^{2N-M} \det X_{11}\right)^{1/N} \mathrm{e}^{-t/g_s} \sim \left(\Lambda_m^{2N-M} \det X_{11}\right)^{1/N} \tag{A.31}$$

without considering multi-istanton contributions. With this procedure we recover the expected result

$$W_{IR} = \xi_1 X_{11} - N\left(\Lambda_m^{2N-M} \det X_{11}\right)^{1/N} \tag{A.32}$$

which is understood in field theory as the low energy contribution to the superpotential due to the gaugino condensation of the node N_1.

A.5 Stability and UV completion

In this Appendix we discuss the issue of UV completion. A related problem concerns the unstable directions that can arise when we set some node to zero. The most natural UV completion to the IR theories analyzed in this paper seems to describe them as the last step of a duality cascade. If this is the case there could be potentially dangerous baryonic flat directions, due to the breaking of the baryonic symmetry. It occurs if we choose the baryonic branch after the confinement of some of the gauge groups. For supersymmetry, the Goldstone boson associated to the breaking of baryonic symmetry fits in a chiral supermultiplet containing another scalar particle that is not protected by any symmetry. This particle is a pseudogoldstone and signals a dangerous flat direction.

This scalar mode is decoupled at one loop and studying the stability of this direction remains an open problem. This was the case in [30, 42, 178]. A possible solution is the gauging of the baryonic symmetry. The resulting D-term potential lift these dangerous directions. Another possible way out, as noticed in [30], is to consider non canonical terms in the Kahler potential. We comment on this problem and discuss it in a simple example, the L^{444} theory.

We consider the quiver in figure A.5 and we study its low energy dynamics. Tuning the

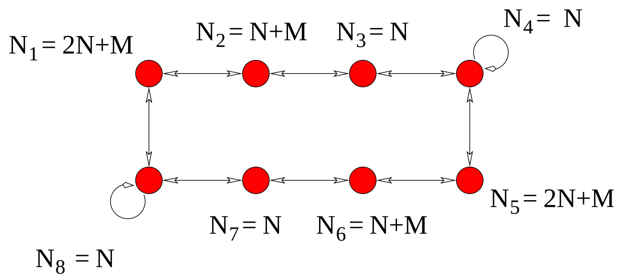


Figure A.5: The L^{444} theory which gives metastable vacua after the confinement of the nodes N_1 and N_5

scales such that the first and the fifth node are the more strongly coupled gauge groups, we can describe the low energy with gauge singlets for these groups as

$$\begin{aligned} W = \quad & q_{23}q_{34}q_{43}q_{32} - q_{34}X_{44}q_{43} + M_{44}X_{44} - M_{46}M_{64} + q_{76}M_{66}q_{67} - \xi_1 M_{66} + \xi_1 X_{44} \\ - \quad & q_{67}q_{78}q_{87}q_{76} + q_{78}X_{88}q_{87} - M_{88}X_{88} + M_{82}M_{28} - q_{32}M_{22}q_{23} + \xi_2 M_{22} - \xi_2 X_{88} \,. \end{aligned}$$

We observe that for the first and the fifth nodes the number of flavour coincides with the number of colors. Hence we have to impose the following quantum constraint on the moduli space

$$\begin{aligned} \det \begin{pmatrix} M_{44} & M_{46} \\ M_{64} & M_{66} \end{pmatrix} - b_1\tilde{b}_1 - \Lambda_1^{4N+2M} = 0 \\ \det \begin{pmatrix} M_{22} & M_{28} \\ M_{82} & M_{88} \end{pmatrix} - b_2\tilde{b}_2 - \Lambda_2^{4N+2M} = 0 \,. \end{aligned} \tag{A.33}$$

Choosing the baryonic branch, we have $b_i\tilde{b}_i = \Lambda_i^{4N+2M}$, which breaks the baryonic symmetries. If we integrate out the massive mesons we obtain the low energy theory corresponding to set the nodes N_1 and N_5 to zero

$$\begin{aligned} W = \quad & q_{76}M_{66}q_{67} - \xi_1 M_{66} - q_{67}q_{78}q_{87}q_{76} \\ - \quad & q_{32}M_{22}q_{23} + \xi_2 M_{22} + q_{23}q_{34}q_{43}q_{32} \,. \end{aligned} \tag{A.34}$$

This superpotential corresponds to two decoupled copies of theories obtained from L^{131} setting to zero a node with an adjoint field, and where we set the two deformations to have the same value but opposite sign. This implies that there is not a mass term for the quarks. The two theories have metastable vacua, as shown in section 2.4.

As mentioned, the problem here is that the breaking of the global baryonic symmetry gives rise to a Goldstone boson and to a pseudoflat direction, which is not protected by any global symmetry. This direction does not receive any one loop contribution by the CW effective potential, and can get tachyonic at higher loops. The possible way out to this source of instability is that we are dealing with a compactified theory. This implies that the baryonic symmetry is gauged, and this gauging gives origin to a positive squared mass term for the pseudoflat direction.

A.6 Quantum analysis

In this appendix we review some formulas, useful for the calculation of the spectrum of supersymmetry breaking models. First we give the explicit expressions for the squared mass matrices. Than we review the basic aspects for the one loop and two loop calculation used in the text.

Mass Matrices

In the analysis of supersymmetry breaking we made use of the squared bosonic and fermionic mass matrices. These can be calculated not only in the supersymmetric cases, where they coincide, but also in non supersymmetric case, i.e. where some F or D terms are not vanishing.

In this section we review the general formulation for these mass matrices in the general case of $F \neq 0$ and $D \neq 0$, for the vector, fermion and spinor field.

Vector Fields

In the general case, where some of the chiral fields get a vev, $\langle \phi_i \rangle \neq 0$, the standard Higgs mechanism is at work and some vector fields get a mass. In the Lagrangian it is due to the interaction

$$g^2 \phi^\dagger T^a T^b \phi v_\mu^a v^{a\mu} \tag{A.35}$$

If one defines the derivatives of the D terms as

$$D_i^a = \frac{\partial D^a}{\partial \phi^i} = -g(\phi^\dagger T^a)_i \quad D^{i\,a} = \frac{\partial D^a}{\partial \phi_i^\dagger} = -g(T^a \phi)^i \quad D_j^{ai} = -g T_j^{ai} \tag{A.36}$$

the mass matrix takes the form

$$m_v^2 = 2 D_i^a D^{bi} \tag{A.37}$$

where the ϕ_i's fields are substituted by their vevs.

Fermionic Fields

In the Lagrangian there are two terms that can participate to the mass matrix for the fermions. One comes from the D terms and relates the gauginos with the fermions of the chiral multiplet. the second one is only due to the superpotential.

$$i\sqrt{2}g^a\phi_j^\dagger(T^a)_i^j\lambda^a\psi^i - \frac{1}{2}W_{ij}\psi^j\psi^i + \text{h.c.} \tag{A.38}$$

where

$$W^{ij} = \frac{\partial^2 W}{\partial\phi^j\partial\phi^i} \tag{A.39}$$

If one organizes (A.38) in a matrix form the expression is

$$-\frac{1}{2}\begin{pmatrix} \psi^i & \lambda^a \end{pmatrix}\begin{pmatrix} W_{ij} & \sqrt{2}iD_i^b \\ \sqrt{2}iD_j^a & 0 \end{pmatrix}\begin{pmatrix} \psi^j \\ \lambda^b \end{pmatrix} \tag{A.40}$$

The squared mass matrix is given by the product of the matrix above with the conjugated one. One has

$$m_f^2 = \begin{pmatrix} W_{il}W^{jl} + 2D_i^cD^{cj} & -\sqrt{2}iW_{il}D^{bl} \\ \sqrt{2}iD_l^aW^{jl} & 2D_l^aD^{bl} \end{pmatrix} \tag{A.41}$$

Scalar Fields

The squared mass matrix is given by the second derivatives of the scalar potential. In term of the superpotential we have [3]

$$m_B^2 = \begin{pmatrix} W_{ip}W^{kp} + D^{ak}D_i^a + D^aD_i^{ak} & W^pW_{ilp} + D_i^aD_l^a \\ W_pW^{jkp} + D^{aj}D^{ak} & W_{lp}W^{jp} + D^{aj}D_l^a + D^aD_l^{aj} \end{pmatrix} \tag{A.42}$$

Once we have written the mass matrices we can write a general relation, that holds also in the case of spontaneous supersymmetry breaking. It is

$$\text{STr}\mathcal{M}^2 = 3\text{Tr}m_v^2 - 2\text{Tr}m_f^2 + \text{Tr}m_B^2 = -2gD^a\text{Tr}T^a \tag{A.43}$$

The 3 and 2 factors count the physical degrees of freedom of massive vectors and fermions. The + and − signs represent the fermionic and bosonic number. The $\text{STr}\mathcal{M}^2$ is zero if there are no $U(1)$ or FI terms. This strongly constraints the spectrum of a supersymmetry breaking theory.

[3] Note that we use the terminology m_B for this matrix since in the cases that we have analyzed the only bosons necessary for the analysis were the scalars.

Note that we can use a different form of the mass matrices, in order to have the same coefficients in (A.43) in front of the different traces. Here we discuss only the case of interest, i.e. the case where susy is spontaneously broken by F terms, and the D terms are all zero. If also the gauge symmetry is unbroken we have that m_f can be written as

$$m_f^2 = \begin{pmatrix} W_{il}W^{jl} & 0 \\ 0 & W^{il}W_{jl} \end{pmatrix} \tag{A.44}$$

We have doubled the mass matrix for m_f^2. In this way we will have only a (-1) factor in the supertrace formula, since the correct number of degrees of freedom is counted already in the mass matrix. For the m_b^2 we have

$$m_B^2 = \begin{pmatrix} W_{ip}W^{kp} & W^p W_{ilp} \\ W_p W^{jkp} & W_{lp}W^{jp} \end{pmatrix} \tag{A.45}$$

There it is manifest that supersymmetry is broken by the out diagonal blocks in (A.45). Note that now the supertrace formula assumes the simple form $\text{Tr}m_B^2 - \text{Tr}m_F^2 = 0$.

The one loop effective potential

In a non supersymmetric theory the vacuum energy is corrected at quantum level. The one loop correction is given by summing the one loop diagrams vacuum and the Coleman-Weinberg formula (CW) is the result of this summation. If we consider a model without gauge interactions, the CW formula takes the form

$$V_{\text{eff}}^{(1)} = \frac{1}{64\pi^2}\text{STr}\mathcal{M}^4 \log\frac{\mathcal{M}^2}{\Lambda^2} = \frac{1}{64\pi^2}\left(\text{Tr}\, m_B^4 \log\frac{m_B^2}{\Lambda^2} - \text{Tr}\, m_F^4 \log\frac{m_F^2}{\Lambda^2}\right) \tag{A.46}$$

This formula is commonly used to calculate the one loop scalar potential as a function of a tree level massless field. Indeed if the mass matrices are calculated as functions of this fields, than (A.46) gives a correction to V_0 and in addiction the effective potential, at one loop, for the pseudomodulus.

The ultraviolet cut-off Λ is absorbed in the renormalization of the coupling constant in V_0. Moreover the term $\text{Str}\mathcal{M}^4$ is independent of the pseudomodulus. This implies that the effective potential for the pseudomodulus does not depend on the cut-off Λ.

Two loop effective potential

The calculation of the two loop effective potential for a pseudomodulus is more difficult, since a general formula like the CW potential does not exists. One can sum all the vacuum diagrams and calculate in each model the effective potential. This procedure involve a lot of graphs that potentially can contribute.

Here we used the trick of [28], which makes the calculation simpler. One has to switch off the supersymmetry breaking scale and compute the supersymmetric masses for all the fields. The pseudomoduli are massless also in this supersymmetric version of the model,

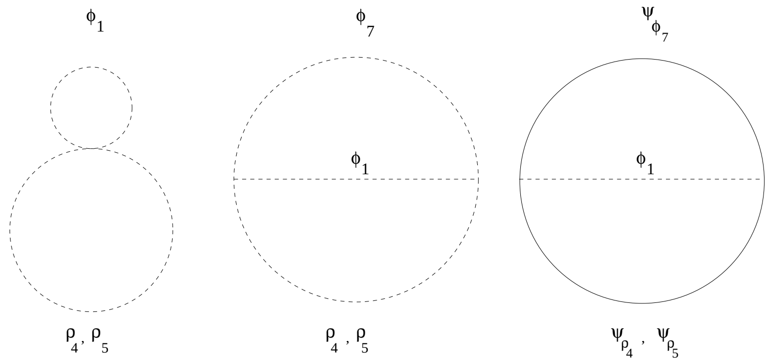


Figure A.6: Relevant Feynman graphs to the two loop potential

but in this case they cannot be lifted by quantum corrections. The two loop potential for these fields can be calculated by subtracting the supersymmetric part to the non supersymmetric one. In formulas, calling $V^{(2)}(Z)$ the two loop potential, it is given by

$$V^{(2)}(Z) = V^{(2)}_{nonSUSY}(Z) - V^{(2)}_{SUSY}(Z) \tag{A.47}$$

This formula means that the effective potential for Z is due to the diagrams that depends both on the fields whose masses split in the non supersymmetric case (to respect to the supersymmetric one) and on the fields whose masses depend on Z. This trick generically reduces the number of diagrams that contribute to the two loop potential.

Example

Here we see the trick at work by studying the model (2.260) There are only few diagrams of this form and they are computed using the formulas of [179, 180]. The model (2.260) gives rise to three different diagrams, V_{SS}, V_{SSS} and V_{FFS}, and they are given in Figure A.6.

Details on the calculation

Here we explain the details on the computation of the mass for the pseudomodulus Z around the origin. The potential is made of three different pieces

$$V^{(2)}(Z) = V_{SS} + V_{SSS} + V_{FFS} \tag{A.48}$$

They come from three different Feynman graphs, and they have been explicitly derived in [179, 180] They are

$$\begin{aligned} V_{SS}(x,y) &= J(x)J(y) \\ V_{SSS}(x,y,x) &= -I(x,y,z) \\ V_{FFS}(x,y,z) &= J(x)J(y) - J(x)J(z) - J(y)J(z) + (x+y-z)I(x,y,z) \end{aligned} \tag{A.49}$$

where

$$J(x) = x\left(\log\frac{x}{Q^2} - 1\right) \tag{A.50}$$

In our calculation one argument of the function $I(x,y,z)$ is always zero. We give the expression for this simplified case

$$\begin{aligned} I(0,x,y) = &\quad (x-y)\left(\mathrm{Li}_2(y/x) - \log(x/y)\log\frac{x-y}{Q^2} + \frac{1}{2}\log^2\frac{x}{Q^2} - \frac{\pi^2}{6}\right) \\ - &\quad \frac{5}{2}(x+y) + 2x\log\frac{x}{Q^2} + 2y\log\frac{y}{Q^2} - x\log\frac{x}{Q^2}\log\frac{y}{Q^2} \end{aligned} \tag{A.51}$$

Using these formulas we found in (2.268) that the mass term for Z is $m_Z^2 = m_{Z_\beta}^2 + m_{Z_\gamma}^2$, with $\tau^2 = \frac{f}{\mu^2}$,

$$\begin{aligned} m_{Z_\beta}^2 &= \frac{h^6\beta^2\mu^2}{\tau^2}\Bigg(-2\tau^2 - (1-\tau^2)^2\log(1-\tau^2) + (1+\tau^2)^2\log(1+\tau^2) \\ + &\quad \frac{1}{2}\log^2(1+\tau^2) + Li_2(-\tau^2) + Li_2\left(\frac{\tau^2}{1+\tau^2}\right)\Bigg) \end{aligned} \tag{A.52}$$

and

$$m_{Z_\gamma}^2 = -\frac{h^6\gamma^2\mu^2}{\tau^2}(2\tau^2 + (1-\tau^2)\log(1-\tau^2) - (1+\tau^2)\log(1+\tau^2))) \tag{A.53}$$

The last line in (A.52) vanishes for $\tau^2 < 1$ because of an identity of dilogarithms.

A.7 Bounce action for a triangular barrier in 4 D

Usually a supersymmetric gauge theory does not have a single vacuum state. In some case there is a set of discrete separated vacua, counted by the Witten index. In other case there is a moduli space of states at zero energy. In the case of metastable vacua there are also minima of the scalar potential with non zero energy. These metastable states are unstable and decay into the supersymmetric states. The lifetime of a false vacuum state depends on it decay rate to the true vacuum.

In the case of SQCD and its extensions the decay rate can be estimated by approximating the potential with a triangular barrier, represented in Figure A.7. The tunneling rate is estimated by using the trajectory of minimal energy connecting the two vacuum states. This is the bounce action, and it is defined as the difference between the tunneling configuration and the metastable vacuum in the Euclidean action.

In the ISS model this action corresponds to a motion in the space parameterized by the scalar components of the chiral multiplets. Usually it reduces to a mono-dimensional motion.in this field space. From the vacuum state to the local maximum the motion evolves along the direction of the pseudomodulus. Then after passing the maximum the motion takes place in the quark directions.

It is necessary to estimate the bounce action for a single scalar field in four dimensions, from one false vacuum, say ϕ_F to a true vacuum, namely ϕ_T. This action is

$$S_E[\phi] = 2\pi^2 \int_0^\infty r^2 dr \left(\frac{1}{2} \frac{d\dot{\phi}^2}{dr} + V(\phi) \right) \tag{A.54}$$

The field $\phi(r)$ is the solution that minimizes the action. It equation of motion is

$$\frac{d^2\phi}{d^2r} + \frac{3}{r}\frac{d\phi}{dr} = \frac{dV(\phi)}{d\phi} \tag{A.55}$$

This equation is solved after imposing the boundary conditions on the field at large radius and on its derivative at the false vacuum. The first requirement is that the field at large radius approaches to the configuration of the false vacuum. The second condition is that the theory makes sense at the false vacuum. The conditions are

$$\lim_{r\to\infty} \phi(r) = \phi_F, \qquad \dot{\phi}(R_F) = 0 \tag{A.56}$$

The bounce action is the difference of the action calculated on the solution of (A.55). and the action of for the case in which the field remains at the false vacuum. This bounce action is given by

$$B = S_E[\phi(r)] - S_E[\phi_F] \tag{A.57}$$

where we have called the action for the field sitting at the false vacuum ϕ_F.

It is then helpful to define the gradient of the potential $V'(\phi)$ in terms of the parameters at the extremal points,

$$\begin{aligned} \lambda_F = & \quad \tfrac{V_{max}-V_F}{\phi_{max}-\phi_F} \equiv \frac{\Delta V_F}{\Delta \phi_F} \\ \lambda_T = & \quad -\tfrac{V_{max}-V_T}{\phi_T-\phi_{max}} \equiv -\frac{\Delta V_T}{\Delta \phi_T} \end{aligned} \tag{A.58}$$

where the first has positive sign and the second has negative one.

The solution of the equation of motion at the two side of the triangular barrier are solved by imposing the boundary conditions, and a matching condition at some radius $r + R_{max}$, that has to be determinate.

The choice of the boundary condition proceeds as follows. Firstly the field ϕ reaches the false vacuum ϕ_F at a finite radius R_F and stays there. This imposes

$$\begin{aligned} \phi(R_F) &= \phi_F \\ \dot{\phi}(R_F) &= 0 \end{aligned} \tag{A.59}$$

For the second boundary condition we work in the simplified situation such that the field has initial value $\phi_0 < \phi_T$ at radius $r = 0$. In this way we imposes the conditions

$$\begin{aligned} \phi(0) &= \phi_0 \\ \dot{\phi}(0) &= 0 \end{aligned} \tag{A.60}$$

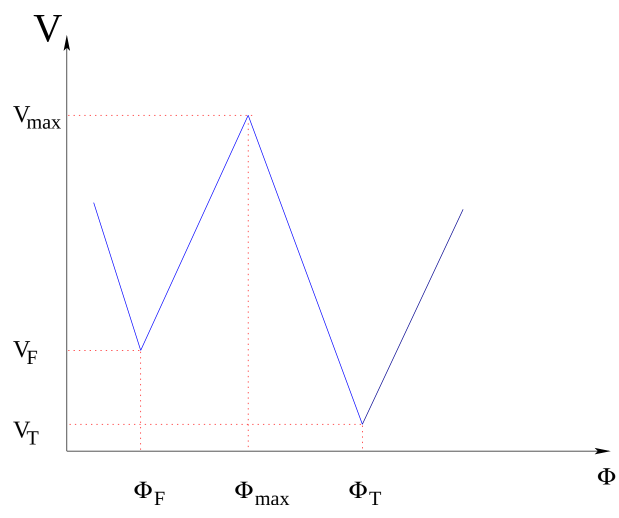


Figure A.7: Triangular potential barrier

Solving the equations of motion we have two solution at the two sides of the barrier

$$\phi_R(r) = \phi_0 - \frac{\lambda_T r^2}{8} \qquad \text{for} \qquad 0 < r < R_{max} \quad \text{(A.61)}$$

$$\phi_L(r) = \phi^F_{max} + \frac{\lambda_F}{8r^2}(r^2 - R_F^2)^2 \qquad \text{for} \quad R_{max} < r < R_F \quad \text{(A.62)}$$

By matching the derivatives at R_{max} we are able to express R_F as a function of R_{max}

$$R_F^4 = (1+c)R_{max}^4 \quad \text{(A.63)}$$

where $c = -\lambda_T/\lambda_F$. Out of the value of the field at R_{max} we get two useful relations

$$\begin{aligned} \phi_0 &= \phi_{max} + \frac{\lambda_T}{8}R_{max}^2 \\ \Delta\phi_F &= \frac{\lambda_F(\sqrt{1+c}-1)}{8}R_{max}^2 \end{aligned} \quad \text{(A.64)}$$

The bounce action B can be evaluated by integrating the equation (A.57) from $r = 0$ to $r = R_F$. We found

$$B = \frac{32\pi^2}{3}\frac{1+c}{\sqrt{1+c}-1}\frac{\Delta\phi_F^4}{\Delta V_F} \quad \text{(A.65)}$$

There is a second possibility that holds if $\phi_0 > \phi_T$. In this case the field is closed to ϕ_0 from $R = 0$ until a radius R_0, and then it evolves outside. In this case the boundary conditions (A.60) become

$$\begin{aligned} \phi(r) &= \phi_T \qquad 0 < r < R_T \\ \phi(R_T) &= \phi_T \\ \dot{\phi}(R_T) &= 0 \end{aligned} \quad \text{(A.66)}$$

In this case the equations of motion are solved by

$$\begin{aligned}
\phi_R(r) &= \phi_T & \text{for} \quad & 0 < r < R_T \\
\phi_R(r) &= \phi_T - \frac{\lambda_T}{8r^2}(r^2 - R_T)^2 & \text{for} \quad & R_T < r < R_{max} \\
\phi_L(r) &= \phi_F - \frac{\lambda_F}{8r^2}(r^2 - R_F)^2 & \text{for} \quad & R_{max} < r < R_F
\end{aligned} \tag{A.67}$$

There are three unknowns to determine, R_T, R_{max} and R_F. Matching the derivatives at the top of the barrier one relation is found

$$R_F^4 - R_{max}^4 = c(R_{max}^4 - R_F^4) \tag{A.68}$$

The matching of the fields at R_{max} gives

$$\Delta\phi_T = \frac{\lambda_T}{8R_{max}^2}(R_{max}^2 - R_T^2)^2 \quad \Delta\phi_F = \frac{\lambda_F}{8R_{max}^2}(R_{max}^2 - R_T^2)^2 \tag{A.69}$$

Then one expresses R_T and R_{max} in terms of R_F, and then determine $R_T$as a function of the parameters of the potential. This is done by defining

$$\beta_F = \sqrt{\left(\frac{8\delta\phi_F}{\lambda_F}\right)} \quad \beta_T = \sqrt{\left(\frac{8\delta\phi_T}{\lambda_T}\right)} \tag{A.70}$$

Combining (A.69) with (A.70) we have

$$R_T^2 = R_{max}^2 - \beta_T R_{max} \quad R_F^2 = R_{max}^2 + \beta_F R_{max} \tag{A.71}$$

and (A.68) becomes

$$R_{max} = \frac{1}{2}\left(\frac{\beta_F^2 + c\beta_+^2}{c\beta_- - \beta_+}\right) \tag{A.72}$$

By integrating the solution we can write the bounce action as

$$B = \frac{1}{96\pi^2}\pi^2\lambda_F^2 R_{max}^3\left(-\beta_F^3 + 3c\beta_F^2\beta_T + 3\beta_F\beta_T^2 - c^2\beta_T^3\right) \tag{A.73}$$

If

$$\left(\frac{\Delta V_F}{\Delta V_T}\right)^{1/2} = \frac{2\Delta\phi_T}{\Delta\phi_T - \Delta\phi_F} \tag{A.74}$$

then (A.65) coincides with (A.73). and the bounce action reduces to

$$B = \frac{2\pi^2}{3}\frac{(\Delta\phi_F^2 - \Delta\phi_T^2)^2}{\Delta V_F} \tag{A.75}$$

Appendix B

Details of the instantonic calculations

B.1 Clifford Algebra and Spinors

In this appendix we review some useful formulas necessary for the calculation of D-instanton action. It is necessary to connect fermions and Clifford algebra in various dimensions.

In $d = 4$ the gamma matrices have been taken in the representation

$$\gamma_n = \begin{pmatrix} 0 & \sigma_n \\ \bar{\sigma}_n & 0 \end{pmatrix} \quad \gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{B.1}$$

this representation holds in Euclidean space. In the minkoskian case $(\sigma_0, \sigma_i) \rightarrow (\sigma_0, i\sigma_i)$ In $d = 6$ a similar representation holds. For $a = 1, \ldots, 6$ it is

$$\gamma_a = \begin{pmatrix} 0 & \Sigma_a \\ \bar{\Sigma}_a & 0 \end{pmatrix} \quad \gamma_7 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \tag{B.2}$$

The four dimensional matrices Σ_a and $\bar{\Sigma}_a$ are usually expressed in term of the 't Hooft eta symbols

$$\Sigma^a_{AB} = (\eta^c_{AB}, i\bar{\eta}^c_{AB} \qquad \bar{\Sigma}^{AB}_a = (-\eta^c_{AB}, i\bar{\eta}^c_{AB} \tag{B.3}$$

For $c = 1, 2, 3$ the following relations holds

$$\begin{aligned} &\bar{\eta}^c_{AB} = \eta^c_{AB} = \epsilon_{cAB} \quad A, B = 1, 2, 3 \\ &\bar{\eta}^c_{4A} = \eta^c_{A4} = \delta_{cA} \\ &\eta^c_{AB} = -\eta^c_{BA} \quad \bar{\eta}^c_{AB} = -\bar{\eta}^c_{BA} \end{aligned} \tag{B.4}$$

Then in both $d = 4$ and $d = 6$ it is useful to define the matrices

$$\gamma_{mn} = \frac{i}{4}[\gamma_n, \gamma_m] = i \begin{pmatrix} \sigma_{nm} & 0 \\ 0 & \bar{\sigma}_{nm} \end{pmatrix} \qquad \gamma_{ab} = \frac{i}{4}[\gamma_a, \gamma_b] = i \begin{pmatrix} \Sigma_{ab} & 0 \\ 0 & \bar{\Sigma}_{ab} \end{pmatrix} \tag{B.5}$$

We can now built the representations of the Clifford Algebra in ten Euclidean space. Suppose that Clifford algebra in p and q euclidean dimensions are known and the generators are $\Gamma_n^{(p)}$ and $\Gamma_a^{(q)}$ with $n = 1, \ldots, p$ and $a = 1, \ldots, q$. A representation of the Clifford algebra in $p + q$ dimensions is

$$\Gamma_N = \left\{\Gamma_n^{(p)} \otimes 1, \Gamma_{p+1}^{(p)} \otimes \Gamma_a^{(q)}\right\} \qquad N = 1, \ldots, p+q \tag{B.6}$$

B.2 General result for $U(1)$ instanton

In this appendix we compute the general contribution to the superpotential for a rigid $U(1)$ instanton placed on a $SU(1)$ node (denoted with A) in a toric quiver gauge theory, generalizing the result of section 3.2. The more general configuration includes the possibility of having more than one field with the same gauge group indexes, connected to the node A. We label these fields with an extra index α for outgoing arrow and β for incoming arrow. Hence the fields connecting the node A to the other gauge nodes are referred as Φ_{Ab}^{α} or Φ_{cA}^{β}. These extra indexes have to be inserted, and summed over, in all the formula of section 3.2, e.g. for the instantonic action. An important remark is that now the anomaly free condition for the node A is

$$\sum_{b,\alpha} N_b = \sum_{c,\beta} N_c \tag{B.7}$$

The procedure for getting the superpotential contribution is as in section 3.2. The integration of the bosonic zero modes and of the fermionic zero mode $\lambda^{\dot{\alpha}}$ and $\mu_{AA}, \bar{\mu}_{AA}$, give the same result. We have to perform the following integral

$$W_{inst} \sim \int \prod_{b,\alpha,c,\beta} (d\bar{\mu}_{Ab}^{\alpha})^{N_b} \, (d\mu_{cA}^{\beta})^{N_c} e^{-S_W} \tag{B.8}$$

where now

$$S_W = -\frac{i}{2} \sum_{b,\alpha,c,\beta} \bar{\mu}_{Ab}^{\alpha} \frac{\partial W}{\partial(\Phi_{cA}^{\beta}\Phi_{Ab}^{\alpha})} \mu_{cA}^{\beta} \tag{B.9}$$

In order to compute this integral we can arrange the fermionic variable in vectors

$$\bar{\mu}_{AB} = (\mu_{Ab}^{\alpha}) \qquad \mu_{CA} = (\mu_{cA}^{\beta})^T \tag{B.10}$$

of dimension

$$B = 1, \ldots, \sum_{b,\alpha} N_b \qquad C = 1, \ldots, \sum_{c,\beta} N_c \tag{B.11}$$

and rewrite the instantonic action as

$$S_W = -\frac{i}{2} \bar{\mu}_{AB} \mathcal{M}_{BC} \mu_{CA} \tag{B.12}$$

where $\mathcal{M}$ is a matrix of dimension $\sum_{b,\alpha} N_b \times \sum_{c,\beta} N_c$, built taking derivatives of the superpotential

$$\mathcal{M} = \frac{\partial W}{\partial(\Phi^{\beta}_{cA}\Phi^{\alpha}_{Ab})} \tag{B.13}$$

$\mathcal{M}$ is a square matrix because of the anomaly free condition (B.7). The ordering of the fields in building $\mathcal{M}$ is irrelevant for the final contribution to the superpotential, which is a determinant. Indeed we can perform the fermionic integration and obtain the stringy instanton contribution

$$W_{inst} \sim \det \frac{\partial W}{\partial(\Phi^{\alpha}_{cA}\Phi^{\beta}_{Ab})} \tag{B.14}$$

B.3 Bosonic integration

In this appendix we show, via dimensional arguments similar to [84], that the integration over the bosonic zero modes of the stringy instantons change the results of the fermionic integration only by a constant factor. We analyze the general bosonic integration for the $U(1)$ and the $SP(2)$ instanton. The $O(1)$ case is trivial since there are no bosonic zero modes to integrate over.

The $U(1)$-instanton

In section 3.2 we have considered an $SU(1)$ node A in the quiver and label with index b all the outgoing arrows, and with c all the incoming arrows. We have then the collection of fields Φ_{Ab} and Φ_{cA}.

We have seen that the contribution to the superpotential after fermionic integration, due to an instanton on node A is given by the determinant of the squared matrix $\mathcal{M}$. The determinant of this matrix has mass dimension

$$[\det \mathcal{M}] = M_s^{(\sum_c N_c)} = M_s^{(\sum_b N_b)} \tag{B.15}$$

We can now compute the dimension of the measure factor for the general instanton computation of section 3.2

$$Z = \mathcal{C} \int d\{a_\mu, M, \lambda, D, \omega_{AA}, \bar{\omega}_{AA}, \mu_{AA}, \bar{\mu}_{AA}, \bar{\mu}_{Ab}, \mu_{cA}\}\, e^{-S_{inst}} \tag{B.16}$$

The dimension-full coefficient $\mathcal{C}$ is for the moment unknown. Using the usual standard dimensions we arrive at

$$[d\{a_\mu, M, \lambda, D, \omega_{AA}, \bar{\omega}_{AA}, \mu_{AA}, \bar{\mu}_{AA}, \bar{\mu}_{Ab}, \mu_{cA}\}] = M_s^{-n_a + \frac{1}{2}n_M - \frac{3}{2}n_\lambda + 2n_D - n_{\omega,\bar{\omega}} + \frac{1}{2}n_{\mu,\bar{\mu}}} \tag{B.17}$$

Since

$$n_a = 4 \quad n_M = n_\lambda = 2 \quad n_D = 3 \quad n_{\omega,\bar{\omega}} = 4N_A \quad n_{\mu\bar{\mu}} = 2N_A + \sum_b N_b + \sum_c N_c \tag{B.18}$$

we obtain

$$[d\{a_\mu, M, \lambda, D, \omega_{AA}, \bar{\omega}_{AA}, \mu_{AA}, \bar{\mu}_{AA}, \bar{\mu}_{Ab}, \mu_{cA}\}] = M_s^{-(3N_A - \frac{1}{2}(\sum_b N_b + \sum_c N_c))} = M_s^{-\beta_A} \quad \text{(B.19)}$$

where we have recognized the 1 loop beta function of the node A.

Now, since Z in (B.16) should be adimensional we obtain that

$$\mathcal{C} = \Lambda^{\beta_A} \quad \text{(B.20)}$$

Hence we have

$$Z = \Lambda^{\beta_A} \int d\{a_\mu, M, \lambda, D, \omega_{AA}, \bar{\omega}_{AA}, \mu_{AA}, \bar{\mu}_{AA}, \bar{\mu}_{Ab}, \mu_{cA}\}\, e^{-S_{inst}} \quad \text{(B.21)}$$

Now, we expect that

$$Z = \int d^4x d^2\theta \, W_{inst} \quad \text{(B.22)}$$

and then

$$W_{inst} = \Lambda^{\beta_A} \int d\{\lambda, D, \omega_{AA}, \bar{\omega}_{AA}, \mu_{AA}, \bar{\mu}_{AA}, \bar{\mu}_{Ab}, \mu_{cA}\}\, e^{-S_{inst}} \quad \text{(B.23)}$$

Now, we have seen that the fermionic (plus the D) integrations give, when $N_A = 1$, the following

$$W_{inst} = \Lambda^{\beta_A} I_{boson} \det \mathcal{M} \quad \text{(B.24)}$$

where $\mathcal{M}$ is the meson built before and we still have to perform the bosonic integration I_{boson}, and show that it gives a numerical coefficient. Indeed the dimensional analysis gives

$$\begin{aligned} [W] &= [\Lambda^{\beta_A}] + [I_{boson}] + [\det \mathcal{M}] = \beta_A + [I_{boson}] + (\sum_b N_b) = \quad \text{(B.25)} \\ &= 3 - \frac{1}{2}(\sum_b N_b + \sum_c N_c) + [I_{boson}] + (\sum_b N_b) = 3 + [I_{boson}] \end{aligned}$$

where we have used the anomaly free condition. In order to have a superpotential of dimension 3 we have to set $[I_{boson}] = 0$, i.e. a number.

The $SP(2)$-instanton

We can easily repeat the analysis done in the previous section for the $SP(2)$ instanton on the $SO(3)$ gauge node. We denote with A the $SO(3)$ gauge group where we place the instantons and label with index b all the outgoing arrows, and with c all the incoming arrows. In general the contribution to the superpotential after fermionic integration, due to $SP(2)$ instantons on node A is given by a pfaffian of dimension

$$[\text{Pf}\mathcal{M}] = M_s^{(\sum_c N_c)} = M_s^{(\sum_b N_b)} \quad \text{(B.26)}$$

We can now compute the dimension of the instanton measure factor

$$Z = \mathcal{C} \int d\{a_\mu, M, \lambda, D, \omega_{AA}, \mu_{AA}, \bar{\mu}_{Ab}, \mu_{cA}\}\, e^{-S_{inst}} \tag{B.27}$$

The dimension-full coefficient $\mathcal{C}$ is up to now unknown. Using the usual dimensions we arrive at

$$[d\{a_\mu, M, \lambda, D, \omega_{AA}, \mu_{AA}, \bar{\mu}_{Ab}, \mu_{cA}\}] = M_s^{-n_a+\frac{1}{2}n_M-\frac{3}{2}n_\lambda+2n_D-n_{\omega,\bar{\omega}}+\frac{1}{2}n_{\mu,\bar{\mu}}} \tag{B.28}$$

Now we have to remind that the auxiliary group for the instanton is $SP(2)$ and this gives different numbers of components respect to the $U(1)$ case, that is

$$n_a = 4 \quad n_M = 2, \quad n_\lambda = 6 \quad n_D = 9 \quad n_\omega = 4N_A \quad n_{\mu\bar{\mu}} = 2N_A + \sum_b N_b + \sum_c N_c \tag{B.29}$$

we obtain

$$[d\{a_\mu, M, \lambda, D, \omega_{AA}, \mu_{AA}, \bar{\mu}_{Ab}, \mu_{cA}\}] = M_s^{-(3N_A-6-\frac{1}{2}(\sum_b N_b+\sum_c N_c))} = M_s^{-\beta_A} \tag{B.30}$$

where we have recognized the 1 loop beta function of the $SO(N_A)$ node.

Now, since Z in (B.27) should be adimensional we obtain that

$$\mathcal{C} = \Lambda^{\beta_A} \tag{B.31}$$

Hence we have

$$Z = \Lambda^{\beta_A} \int d\{a_\mu, M, \lambda, D, \omega_{AA}, \mu_{AA}, \bar{\mu}_{Ab}, \mu_{cA}\}\, e^{-S_{inst}} \tag{B.32}$$

Now, we expect that

$$Z = \int d^4x d^2\theta\, W_{inst} \tag{B.33}$$

and then

$$W_{inst} = \Lambda^{\beta_A} \int d\{\lambda, D, \omega_{AA}, \mu_{AA}, \bar{\mu}_{Ab}, \mu_{cA}\}\, e^{-S_{inst}} \tag{B.34}$$

Now, we have seen that the fermionic (plus the D) integrations give, when $N_A = 3$, the following

$$W_{inst} = \Lambda^{\beta_A} I_{boson} \mathrm{Pf}\mathcal{M} \tag{B.35}$$

where I_{boson} is the bosonic integration. The dimensional analysis told us that

$$\begin{aligned}[W] &= [\Lambda^{\beta_A}] + [I_{boson}] + [\mathrm{Pf}\mathcal{M}] = \beta_A + [I_{boson}] + (\sum_b N_b) = \\ &= 3N_A - 6 - \frac{1}{2}(\sum_b N_b + \sum_c N_c) + [I_{boson}] + (\sum_b N_b) = 3N_A - 6 + [I_{boson}]\end{aligned} \tag{B.36}$$

Since we have $N_A = 3$, in order to have a superpotential of dimension 3 we have to set $[I_{boson}] = 0$, i.e. a number.

B.4 Relation between our results and known models

In this appendix we show that there is no disagreement between the stringy instanton contributions of [16, 84] and our results.

The $SU(1)$ theory

The theory studied in [84] is the $\mathbb{C}^3/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orbifold. This is a quiver gauge theory with four gauge groups, described by the superpotential

$$\begin{aligned} W \quad &= \Phi_{12}\Phi_{23}\Phi_{31} - \Phi_{13}\Phi_{32}\Phi_{21} + \Phi_{13}\Phi_{34}\Phi_{41} - \Phi_{14}\Phi_{43}\Phi_{31} \\ &+\Phi_{23}\Phi_{34}\Phi_{42} - \Phi_{24}\Phi_{43}\Phi_{32} + \Phi_{12}\Phi_{24}\Phi_{41} - \Phi_{14}\Phi_{42}\Phi_{21} \end{aligned} \tag{B.37}$$

The ranks of the groups are $(N_1, N_2, N_3, N_4) = (N_1, N_2, 1, 0)$. A stringy instanton placed on node N_3 contributes to the superpotential only if $N_1 = N_2$. In this case it has be shown that its contribution is

$$W_{inst} = \det \Phi_{12} \det \Phi_{21} \tag{B.38}$$

We now find the same result from gauge theory analysis. Dualizing the node 3 we find a theory with gauge group $SU(\tilde{N}_3 = N_1 + N_2 - 1)$ and superpotential

$$W = M_{11}Q_{13}Q_{31} - M_{22}Q_{23}Q_{32} \tag{B.39}$$

We then dualize again node 3. Since it is in the case $N_f = N_c + 1$, the dual theory has $SU(N_3) = SU(1)$ gauge group, and the superpotential is

$$W = M_{11}\Phi_{11} - M_{22}\Phi_{22} + \Phi_{11}\Phi_{13}\Phi_{31} - \Phi_{22}\Phi_{23}\Phi_{32} + \Phi_{12}\Phi_{23}\Phi_{31} - \Phi_{13}\Phi_{32}\Phi_{21} + \det \begin{pmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{pmatrix} \tag{B.40}$$

After the integration of the massive field M_{11},M_{22},Φ_{11},Φ_{22}, the superpotential is

$$W = \Phi_{12}\Phi_{23}\Phi_{31} - \Phi_{13}\Phi_{32}\Phi_{21} + \det \begin{pmatrix} 0 & \Phi_{12} \\ \Phi_{21} & 0 \end{pmatrix} \tag{B.41}$$

The first two terms are the same than (B.37). The det piece coincide with (B.38), as expected. Note that it vanishes if $N_1 \neq N_2$ as in the stringy instanton computation.

The $SP(0)$ theory

It is also possible to make an orientifold projection of the $\mathbb{C}^3/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orbifold. A possible orientifold is described by the dimer in Figure B.1. It is a fixed point projection. Since $N[W] = 8$, the total orientifold charge is positive. This condition can be imposed choosing all the single charge to be negative. All the groups are identified with themselves and they are all symplectic. All the fields are bifundamental in the $(\square_i, \square_j)$ of the $SP(N_i) \times SP(N_j)$

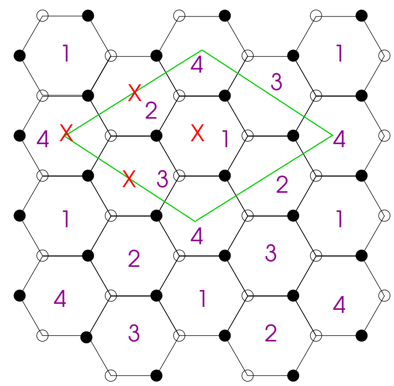


Figure B.1: Unit cell and fixed point for $\mathbb{C}^3/(\mathbb{Z}_2 \times \mathbb{Z}_2)$

gauge groups. There is no superpotential for these fields. This is the same projection described in [16]. In that paper the ranks were $(N_1, N_2, N_3, N_4) = (N, N, 0, 0)$, and the stringy instanton was located on the third node. The stringy instanton contribution to the superpotential is given by

$$W_{inst} = \det \Phi_{12} \tag{B.42}$$

The same result can be found by the gauge theory analysis. The dual theory has rank $2N - 4$ for the third node, and superpotential

$$W = M_{11} \cdot Q_{13} \cdot Q_{13} - M_{22} \cdot Q_{23} \cdot Q_{32} \tag{B.43}$$

We then perform again electric magnetic duality on the third node. The gauge group becomes $SP(N_1 + N_2 - N_3 - 4) = SP(0)$, and the superpotential is

$$W = M_{11} \cdot \Phi_{11} - M_{22} \cdot \Phi_{22} + \text{Pf} \begin{pmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{pmatrix} \tag{B.44}$$

where all the blocks of the meson are in an antisymmetric representation of the flavor, and $\Phi_{21} = -\Phi_{12}^T$. Integrating out the massive fields the only non vanishing term of the superpotential is the non perturbative one

$$W = \text{Pf} \begin{pmatrix} 0 & \Phi_{12} \\ -\Phi_{12}^T & 0 \end{pmatrix} = \det \Phi_{12} \tag{B.45}$$

It is exactly the same than the stringy instanton contribution (B.42).

Appendix C

Aspects of field theories in 3D

C.1 Seiberg duality in three dimensional CS SQCD from branes

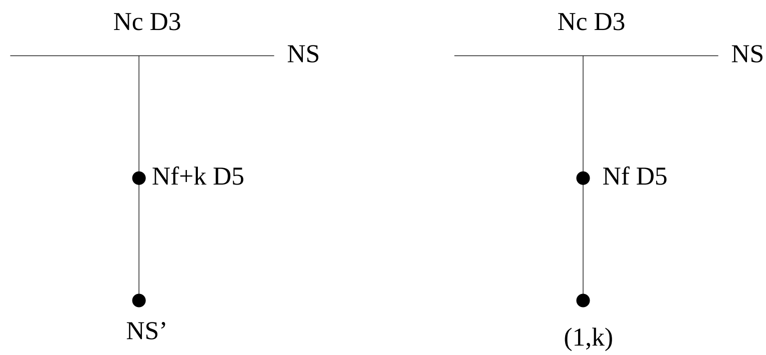


Figure C.1: The electric theory

The rules of Seiberg duality in three dimensional CS matter theory are worked out from systems of intersecting branes. The simplest example is a three dimensional $\mathcal{N} = 2$ CS gauge theory with gauge group $U(N_c)$, CS level k and matter fields charged under a global $U(N_f)$ flavor symmetry. Here we review the behaviour of this theory under Seiberg duality, by engineering it in a system of intersecting NS5, D5 and D3 branes in IIB, as in [19]. The branes are displaced as in the table.

	0	1	2	3	4	5	6	7	8	9
NS	x	x	x	x	x	x				
NS'	x	x	x	x					x	x
N_C D3	x	x	x				x			
$N_f + k$ D5	x	x	x					x	x	x

This system preserves an $\mathcal{N} = 2$ supersymmetry in the (012) space time dimensions. The NS' brane intersect the stack of k D5branes in the (37) plane and there is a recombination of branes that gives a $(1, k)$ fivebrane at angle θ such that $\tan\theta = g_s k$ (see figure C.1). This system of branes represents a $U(N_C)$ field theory with $N_f + k$ flavors Q and $\tilde{Q}$. Then one moves k of the D5 branes to the NS' brane, replacing these branes with the $(1, k)$ fivebrane. This is the level k CS $U(N_c)$ gauge theory with N_f fundamental fields.

The dual theory is obtained by exchanging the NS and the $(1, k)$ fivebranes. The D5 branes and the $(1, k)$ fivebrane move along the x^6 direction and they pass the NS brane. This process produces new D3 branes in the 0126 directions, and the final configuration is represented in figure C.2. This system of branes is identified with the Seiberg dual theory. It is a $U(N_f + k - N_c)$ level $(-k)$ CS gauge theory, with N_f fundamentals q and $\tilde{q}$ and the bifundamental M, a $N_f \times N_f$ matrix. The fundamentals couple with M through a superpotential term

$$W_{\text{dual}} = \text{Tr} M q \tilde{q} \tag{C.1}$$

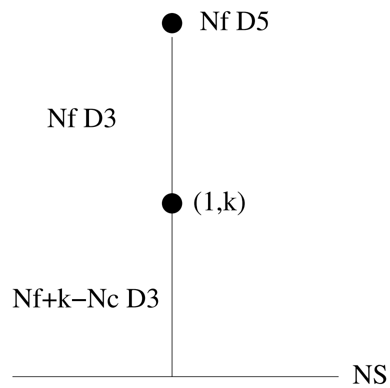


Figure C.2: The magnetic theory

The constraint

$$N_f + k - N_c > 0 \tag{C.2}$$

has to be imposed on both the theories. It is the condition that the theories have supersymmetric vacua and it coincides with the constraint imposed by the s-rule, i.e. no more than one D3 brane can end on each fivebrane. In this case the costraint C.2 coincides with the window in which duality makes sense.

C.2 Parity anomaly

We briefly review parity anomaly for 3D gauge theories [181, 182] and the parity anomaly matching argument [155]. In three dimensions there are no local gauge anomalies. However gauge invariance can require the introduction of a classical Chern-Simons term, which breaks parity. This is referred to as parity anomaly.

For abelian theories with multiple $U(1)$'s, there is a parity anomaly if

$$\mathcal{A}_{ij} = \frac{1}{2} \sum_{\text{fermion}} (q_f)_i (q_f)_j \quad \in \quad \mathbb{Z} + \frac{1}{2} \tag{C.3}$$

Here $(q_f)_i$ is the charge of the fermion f under the $U(1)_i$. We work in a basis where all the charges are integers.

Parity anomaly matching

In the context of dualities in 4D gauge theory, a relevant tool have been the 't Hooft anomaly matching between the electric and the magnetic description. Having some global symmetries, we suppose that they are gauged and we compute their anomaly. The result of this computation should be equal in the two dual descriptions. The same technique can be used here for the parity anomaly. We suppose we gauge the global $U(1)$'s of the theory, and we compute their parity anomaly both in the electric and in the magnetic description. The two computations should match.

The parity anomaly matching is much weaker then the 't Hooft one. Indeed in 4D the precise anomalies associated with gauging global symmetries must match. In 3D there is a weaker $\mathbb{Z}_2$ type condition.

For the $\widetilde{L^{aba}}_{\{k_i\}}$ theories, parity anomaly matching is obeyed by the Seiberg like duality we propose.

In chiral theories, parity anomaly matching can be non trivial between dual phases if fractional branes are introduced by the duality.

As an example we analyze the toric chiral (2+1)d CS theory which has as (3+1)d parents dP_2. We consider the parity anomaly associated to the two $U(1)$ flavour symmetries F_1 and F_2. The charges of the chiral fields of the theory under these symmetries can be derived from [157]. The electric theory has equal ranks n. In the magnetic description we set rank $n+k$ for the dualized gauge group (number 5) and n for the others. The integer k counts the number of fractional branes introduced in the duality. The parity anomaly matrices are

$$\mathcal{A}_{ele} \in \begin{pmatrix} \mathbb{Z} & \mathbb{Z} \\ \mathbb{Z} & \mathbb{Z} \end{pmatrix} \tag{C.4}$$

and

$$\mathcal{A}_{mag} \in \begin{pmatrix} \mathbb{Z} & \mathbb{Z} + \frac{nk}{2} \\ \mathbb{Z} + \frac{nk}{2} & \mathbb{Z} \end{pmatrix} \tag{C.5}$$

One can see that in the electric description there are no parity anomalies. In the magnetic description the off diagonal components of $\mathcal{A}_{mag}$ can instead lead to parity anomaly if kn is odd. If we set $k = 0$ the electric and magnetic theories satisfies the parity anomaly matching for the two flavour symmetries.

C.3 Bounce action for a triangular barrier in 3 D

In this appendix we calculate the bounce action B for a triangular barrier in three dimensions (Figure A.7). The bounce action is the difference between the tunneling configuration and the metastable vacuum in the euclidean action. The tunneling rate of the metastable state is then given by $\Gamma = e^{-B}$

Following [51] we reduce to the case of a single scalar field with only a false vacuum ϕ_F decaying to the true vacuum, ϕ_T. The tunneling action is

$$S_E[\phi] = 4\pi \int_0^\infty r^2 dr \left(\frac{1}{2}\dot{\phi}^2 + V(\phi) \right) \tag{C.6}$$

where $\phi(r)$ is the tunneling solution, function of the Euclidean radius r. Solving the equation of motion for the ϕ field and imposing the boundary conditions

$$\begin{aligned} &\lim_{r\to\infty} \phi(r) = \phi_F \\ &\dot{\phi}(R_F) = 0 \end{aligned} \tag{C.7}$$

the bounce action is given by

$$B = S_E[\phi(r)] - S_E[\phi_F] \tag{C.8}$$

where we have subtracted the action for the field sitting at the false vacuum ϕ_F.

It is then helpful to define the gradient of the potential $V'(\phi)$ in terms of the parameters at the extremal points,

$$\begin{aligned} \lambda_F = &\quad \tfrac{V_{max}-V_F}{\phi_{max}-\phi_F} &\equiv \frac{\Delta V_F}{\Delta \phi_F} \\ \lambda_T = &\; -\tfrac{V_{max}-V_T}{\phi_T-\phi_{max}} &\equiv -\frac{\Delta V_T}{\Delta \phi_T} \end{aligned} \tag{C.9}$$

where the first has positive sign and the second has negative one.

The solution of the equation of motion at the two side of the triangular barrier are solved by imposing the boundary conditions, and a matching condition at some radius $r + R_{max}$, that has to be determinate.

The choice of the boundary condition proceeds as follows. Firstly the field ϕ reaches the false vacuum ϕ_F at a finite radius R_F and stays there. This imposes

$$\begin{aligned} \phi(R_F) &= \phi_F \\ \dot{\phi}(R_F) &= 0 \end{aligned} \tag{C.10}$$

For the second boundary condition we work in the simplified situation such that the field has initial value $\phi_0 < \phi_T$ at radius $r = 0$. In this way we imposes the conditions

$$\begin{aligned} \phi(0) &= \phi_0 \\ \dot{\phi}(0) &= 0 \end{aligned} \tag{C.11}$$

Solving the equations of motion we have two solution at the two sides of the barrier

$$\phi_R(r) = \phi_0 - \frac{\lambda_T r^2}{6} \qquad \text{for} \qquad 0 < r < R_{max} \tag{C.12}$$

$$\phi_L(r) = \phi^F_{max} - \frac{\lambda_F R_F^2}{2} + \frac{\lambda_F R_F^3}{3r} + \frac{\lambda_F}{6} r^2 \qquad \text{for} \quad R_{max} < r < R_F \tag{C.13}$$

By matching the derivatives at R_{max} we are able to express R_F as a function of R_{max}

$$R_F^3 = (1+c)R_{max}^3 \tag{C.14}$$

where $c = -\lambda_T/\lambda_F$. Out of the value of the field at R_{max} we get two useful relations

$$\begin{aligned} \phi_0 &= \phi_{max} + \frac{\lambda_T}{6} R_{max}^2 \\ \Delta\phi_F &= \frac{\lambda_F (3 + 2c - 3(1+c)^{\frac{2}{3}})}{6} R_{max}^2 \end{aligned} \tag{C.15}$$

The bounce action B can be evaluated by integrating the equation (C.8) from $r = 0$ to $r = R_F$. We found

$$B = \frac{16\sqrt{6}\pi}{5} \frac{1+c}{(3 + 2c - 3(1+c)^{2/3})^{3/2}} \sqrt{\frac{\Delta\phi_F^6}{\Delta V_F}} \tag{C.16}$$

There is a second possibility that holds if $\phi_0 > \phi_T$. In this case the field is closed to ϕ_0 from $R = 0$ until a radius R_0, and then it evolves outside. In this case the boundary conditions (C.11) become

$$\begin{aligned} \phi(r) &= \phi_T \qquad 0 < r < R_T \\ \phi(R_T) &= \phi_T \\ \dot{\phi}(R_T) &= 0 \end{aligned} \tag{C.17}$$

In this case the equations of motion are solved by

$$\begin{aligned} \phi_R(r) &= \phi_T && \text{for} \quad 0 < r < R_T \\ \phi_R(r) &= \phi_T + \frac{\lambda_T R_T^2}{2} - \frac{\lambda_T R_T^3}{3r} - \frac{\lambda_T r^2}{6} && \text{for} \quad R_T < r < R_{max} \\ \phi_L(r) &= \phi_F - \frac{\lambda_F R_F^2}{2} + \frac{\lambda_F R_F^3}{3r} + \frac{\lambda_F r^2}{6} && \text{for} \quad R_{max} < r < R_F \end{aligned} \tag{C.18}$$

By integrating these solution we can write the bounce action as

$$B = \frac{8\pi}{15} \lambda_F (R_F^3 \Delta\phi_T - c R_T^3 \Delta\phi_F) \tag{C.19}$$

where the relations among the unknowns and the parameters of the potential are

$$
\begin{aligned}
&R_{max}^3(1+c) = R_F^3 + cR_T^3 \\
&\Delta\phi_T = \frac{(R_T - R_{max})^2(2R_T + R_{max})\lambda_T}{6R_{max}} \qquad (C.20) \\
&\Delta\phi_F = \frac{(R_F - R_{max})^2(2R_F + R_{max})\lambda_F}{6R_{max}} \qquad (C.21)
\end{aligned}
$$

We conclude by observing that in the limit $R_T = 0$ (C.19) coincides with (C.16).

C.4 Coleman-Weinberg formula in various dimensions

The CW formula for the one-loop superpotential

$$V_{eff}^{(1)} = \frac{1}{2}\,\mathrm{STr}\int \frac{d^dp}{(2\pi)^d}\ln(p^2+m^2) \qquad (C.22)$$

is not always straightforward to compute, since the theory can contain many fields, and one has to diagonalize the squared mass matrices. Some property of the models with metastable vacua can be analyzed without evaluating the eigenvalues of the squared mass matrices of component fields of the theory. To this purpose, we generalize a formula previously given for four-dimensional theories [4] to work in any dimension. Indeed, writing (C.22) in spherical coordinates and integrating by parts, we have

$$
\begin{aligned}
V_{eff}^{(1)} &= \frac{\pi^{d/2}}{\Gamma(d/2)}\,\mathrm{STr}\int \frac{dp}{(2\pi^d)}\,p^{d-1}\,\ln(p^2+m^2) \\
&= -\frac{1}{d}\,\frac{1}{2^{d-1}\,\pi^{d/2}\Gamma(d/2)}\,\mathrm{STr}\int dp\,\frac{p^{d+1}}{p^2+m^2} \qquad (C.23)
\end{aligned}
$$

where $A_d = 2\pi^{d/2}/\Gamma(d/2)$ is the d-dimensional spherical surface. By substituting $d=3$ we recover (4.64).

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