Decompositions of Zenga's Inequality Measure by Subgroups [(⋆)]

Scomposizioni per gruppi dell'indice di disuguaglianza di Zenga

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1. Introduction

Given a distribution frequency $\{(x_i, n_i) : \ i = 1, \ldots, r; \ 0 \leqslant x_1 < \ldots < x_r; \ \sum n_i = N\}$ of a non negative variable X, Zenga (2007) proposed to measure the uniformity between the lower group (including the N_i values $\leqslant x_i$) and the upper group (including the remaining $N - N_i$ observations) by the ratio of the correspondent arithmetic means:

$$U_i = \overset{-}{M}_i / \overset{+}{M}_i \qquad i = 1, \ldots, r. \tag{1}$$

The inequality index between the two groups is obtained as:

$$I_i = 1 - U_i \qquad i = 1, \ldots, r. \tag{2}$$

Both I_i and U_i lie in the interval $[0; 1]$; in particular $I_i = 0$ means no inequality between lower and upper groups and $I_i = 1$ means maximum inequality (i.e. lower group mean is null). The author also proposes an inequality diagram in the unit square and derives a synthetic inequality measure as the weighted arithmetic mean of the point measures I_i with weights $\frac{n_i}{N}$:

$$I = \sum_{i=1}^{r} I_i \cdot \frac{n_i}{N}. \tag{3}$$

2. Decomposition of the overall uniformity by subgroups

When the population is partitioned, according to some criterion, in c subgroups the common aim is to evaluate the extent to which the overall uniformity (inequality) depends on the uniformities (inequalities) within and between subgroups.

Reporting the whole distribution as in table 1, with n_{ij} denoting the frequency of the value x_i in subgroup j, Radaelli (2006) showed that the i-th overall point uniformity index U_i can be decomposed into the sum of two terms measuring, respectively, the within and the between subgroups contributions. The key point of the proposed decomposition is the definition of the cross point uniformity index:

$$_{j,h}U_i = {}_j\overset{-}{M}_i / {}_h\overset{+}{M}_i \qquad j, h = 1, \ldots, c; \ i = 1, \ldots, r \tag{4}$$

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which measures the uniformity between the lower and the upper group within the same subgroup for $j = h$, and between two different subgroups for $j \neq h$. The resultant decomposition for the synthetic uniformity index is:

$$U = \sum_{i=1}^{r} U_i \frac{n_{i\cdot}}{N} = \sum_i \sum_j {}_{j,j}U_i \frac{{}_jN_i}{N_i} {}_jw_i \frac{n_{i\cdot}}{N} + \sum_i \sum_j \sum_{h \neq j} {}_{j,h}U_i \frac{{}_jN_i}{N_i} {}_hw_i \frac{n_{i\cdot}}{N} \qquad (5)$$

where the weight ${}_hw_i$ is the upper group share of the h-th subgroup. The between term

Table 1: *Table of the whole distribution of the* c *subgroups*

	subgroup					
X	1	$\dots$	j	$\dots$	c	tot
x_1	n_{11}	$\dots$	n_{1j}	$\dots$	n_{1c}	$n_{1\cdot}$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
x_i	n_{i1}	$\dots$	n_{ij}	$\dots$	n_{ic}	$n_{i\cdot}$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
x_r	n_{r1}	$\dots$	n_{rj}	$\dots$	n_{rc}	$n_{r\cdot}$
tot	$n_{\cdot 1}$	$\dots$	$n_{\cdot j}$	$\dots$	$n_{\cdot c}$	N

computation, in decomposition (5) requires the evaluation of the $c(c-1)$ cross point uniformity indexes ${}_{j,h}U_i$ and this should become time consuming as the number c of subgroups rises. In this paper we show that the between term can also be obtained by comparing the lower mean of each subgroup with the upper mean of the remaining $(c-1)$ subgroups considered as a whole. This approach reduces calculus and allow us to evaluate the uniformity between the lower group of the j-th subgroup (values $x \leqslant x_i$ in subgroup j) and the upper group formed by grouping together the units (with values $x > x_i$) of the other $(c-1)$ subgroups regardless of the subgroups they belong to. Denoting by the subscript $\bar{j}$ the subgroup composed by the c subgroups except the j-th one, we get:

$$U = \sum_{i=1}^{r} U_i \frac{n_{i\cdot}}{N} = \sum_i \sum_j {}_{j,j}U_i \frac{{}_jN_i}{N_i} {}_jw_i \frac{n_{i\cdot}}{N} + \sum_i \sum_j {}_{j,\bar{j}}U_i \frac{{}_jN_i}{N_i} \left(1 - {}_jw_i\right) \frac{n_{i\cdot}}{N}. \qquad (6)$$

Moreover we are interested in comparing the decomposition obtained with others proposed in the literature with particular reference to the decomposition of the Gini concentration ratio proposed by Dagum (1997) which appears to follow an approach close to the one here proposed.

References

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