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Integrating stochastic programming and decision tree techniques in land conversion problems

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Abstract This paper is concerned with gradual land conversion problems, placing the main focus on the interaction between time and uncertainty. This aspect is extremely relevant since most decisions made in the field of natural resources and sustainable development are irreversible decisions. In particular, we discuss and develop a scenario-based multi-stage stochastic programming model in order to determine the optimal land portfolio in time, given uncertainty affecting the market. The approach is then integrated in a decision tree framework in order to account for domain specific (environmental) uncertainty that, diversely from market uncertainty, may depend on the decision taken. Although, the designed methodology has many general applications, in the present work we focus on a particular case study, concerning a semi-degraded natural park located in northern Italy.

Keywords Stochastic programming · Decision analysis · Land management

Introduction

We examine the effects of uncertainty and irreversibility in valuing and timing conversion and development projects involving land areas or natural resources. This topic has been addressed first by Arrow and Fisher (1974) and Henry (1974), who treat the complete conversion problem as an optimal stopping problem, while the gradual conversion problem was first introduced by Clarke and Reed (1990). Our analysis is focused on a more general problem, that is finding an optimal land/resources portfolio composition through time, in the presence of future market uncertainty. In fact, it is often more realistic to assume that an optimal land management program will involve a gradual sequence of conversion decisions through time, evolving as each land allocation/resource value becomes known more accurately.

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In this paper we formulate a scenario-based multi-stage stochastic programming model, which takes into account the uncertainty related to the market value of revenues accruing from the land in different states. As most of the real options literature assumes, whenever the riskiness of a project is diversifiable, as it is in the case of market uncertainty, it is possible to compute the value of the project applying the "risk-neutral" probability distribution and using the risk-free interest rate. In order to take into account the non-constant incremental benefits accruing from different land allocations, we consider piece-wise linear land-use value functions. In fact, in realistic settings, where quantity-dependence is admitted, the incremental value of developed land is contingent on the size of the conversion, see Spencer (2000).

The second issue investigated is environmental uncertainty. In fact, when we allow for the option of converting an area into a natural park, or, more generally, we include in the model preservation investments, we have to deal, not only with market uncertainty, but also with the more complex and domain specific uncertainty about environmental quality, see Conrad (2000) and Bosetti and Messina (2002). The latter issue is a consequence of the fact that policies are usually concerned with long time horizon decisions, their effects on the environment may be partly unknown and also that environmental processes are stochastic. For example, we typically lack information on the amount of ultimately recoverable resources, characteristics of future technologies and their arrival dates, tastes of people in the future, and so on. Historical data series on fluctuations of natural land value shows discontinuities and frequently a greater volatility when compared to other commercial development opportunities. This often implies that markets are incomplete, thence the riskiness of the environmental investment cannot be hedged by a replicating portfolio and the "risk-neutral" approach cannot be applied.

Nevertheless, option pricing can be profitably integrated to decision analysis methods in order to deal with this domain specific risk. In particular, option pricing techniques can be used to simplify decision analysis when some risks can be hedged by trading and, conversely, decision analysis techniques can be used to extend option pricing techniques to problems with incomplete securities markets, as shown in Smith and Nau (1995). Supposing that the decision maker can either choose to sharpen knowledge about the initial value accruing from the preserved land, or take decisions without any further inquiry, we model the gradual conversion problem using a decision tree framework, Birge and Louveaux (1997).

This approach has been applied to a real case concerned with the remediation of a semi-degraded area located in northern Italy, the Appiano Gentile and Tradate park.[1] This area embodies a class of problems typical of environmental management in Europe, where wilderness areas are scarce and investments in land remediation are often required.

The paper is organized as follows. Section 1 and 2 present the deterministic and stochastic models, respectively. Section 3 presents the application concerning the remediation of Appiano Gentile and Tradate park. In Section 4 we address the issue of environmental quality uncertainty. Section 5 concludes.

1. Problem formulation

Consider a relatively undeveloped area which supports a minor economic activity and that exhibits some environmental degradation as a result (e.g., resource extraction). We refer to the land portion left to this initial state as Status Quo (SQ). The portion of the area where the environmental damage is remediated and that is returned to a natural state will be referred to as Natural Park (NP). Finally, the portion of the initial area that is more intensively transformed

[1] The results of the study have been fundamental input for the 2003 territorial plan, see Piano di Settore (2003).

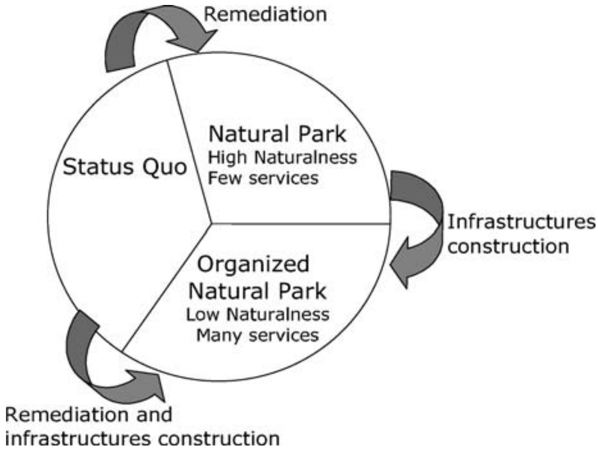


Fig. 1 Conversion directions in the Appiano Gentile and Tradate Park problem

in order to offer a number of services, loosing some of the original naturalness, is denoted as Organized Natural Park (NPO). The conversion directions can be as follows: land can be remediated from the SQ and turned to the NP state (see Figure 1), while land can be converted to the organized natural park both from SQ and from NP.

For each time period $t, t = 1, \ldots, T$, state and decision variables are:

$x(t) \in [0, 1]$, the portion of land in the Status Quo state (Strategy #1);
$y(t) \in [0, 1]$, the portion of land in the Natural Park state (Strategy #2);
$z(t) \in [0, 1]$, the portion of land in the Organized Natural Park state (Strategy #3);
$u_\alpha(t) \in [0, 1]$, the area converted to Strategy #2 from the Status Quo;
$u_\beta(t) \in [0, 1]$, the area converted to Strategy #3 from Natural Park;
$u_\gamma(t) \in [0, 1]$, the area converted to Strategy #3 from the Status Quo;

We suppose that, at time $t = 0$, the whole lot of land is in the SQ state, formally:

$$x(0) = 1 \quad \text{and} \quad y(0) = z(0) = u_\alpha(0) = u_\beta(0) = u_\gamma(0) = 0. \tag{1}$$

For the formulation to be consistent, the land cannot be generated or disappear, this leads to the conservation constraints, for $t \in \{0, \ldots, T\}$:

$$x(t) + y(t) + z(t) = 1. \tag{2}$$

We suppose that, once developed, the land is irreversibly compromised, while when remediated, the land can still be successively converted to commercial development (once again see Figure 1). The conversion conditions, for $t \in \{1, \ldots, T\}$, are:

$$\begin{aligned}
x(t) &= x(t-1) - u_\alpha(t) - u_\gamma(t), \\
y(t) &= y(t-1) + u_\alpha(t) - u_\beta(t), \\
z(t) &= z(t-1) + u_\beta(t) + u_\gamma(t), \\
u_\beta(t) &\le y(t-1).
\end{aligned} \tag{3}$$

Now we define the benefits and costs accruing from alternative land allocation: $\pi(t, x(t)), \kappa(t, y(t)), \nu(t, z(t))$ are the discounted values of revenues arising at time t from the land in Strategy #1, #2, #3 respectively. In particular, we assume that the value of the land in each state depends not only on the time period, t, but also on the decision concerning the size of the land devoted to that particular state. This allows us to capture the non constant marginal benefit accruing from the converted land, depending on the size. The first units of land converted in each state may have higher/lower value than successively converted units. The details concerning how to model the size dependency effect are discussed in Section 2. $\alpha(t)$, $\beta(t)$, $\gamma(t)$ represent the discounted values of variable costs deriving from converting the area from SQ to NP, from NP to NPO and from SQ to NPO, respectively. $I_\alpha(t)$, $I_\beta(t)$, $I_\gamma(t)$ are the initial sunk costs necessary to start the conversion of the area from SQ to NP, from NP to NPO and from the SQ to NPO, respectively.

The decision maker objective is to maximize the Net Present Value (*NPV*) deriving from the managed area, considered both revenues and variable and fixed conversion costs:

$$\max \sum_{t=1}^{T}[\pi(t, x(t))x(t) + \kappa(t, y(t))y(t) + \nu(t, z(t))z(t) +$$
$$- \sum_{t=1}^{T}[\alpha(t)u_\alpha(t) + \beta(t)u_\beta(t) + \gamma(t)u_\gamma(t)] +$$
$$- \sum_{t=1}^{T}[I_\alpha(t)\lambda_\alpha(t) + I_\beta(t)\lambda_\beta(t) + I_\gamma(t)\lambda_\gamma(t)] \ . \tag{4}$$

To include fixed initial investment costs in the model, we have introduced three binary variables each assuming the value one (zero) when the correspondent conversion activity has (has not) been undertaken. $\lambda_\alpha(t) \in \{0, 1\}$, $\lambda_\alpha(t) = 1$ if conversion from SQ into NP starts at time t; $\lambda_\alpha(t) = 0$, otherwise; analogously we introduce $\lambda_\beta(t) \in \{0, 1\}$, relating to conversion from NP into NPO and $\lambda_\gamma(t) \in \{0, 1\}$, relating to conversion from SQ into NPO.

The maximization problem is constrained to investment constraints, formalizing the presence of fixed costs when conversion starts, for $i = \alpha, \beta, \gamma$, with $t \in \{1, \dots, T\}$:

$$u_i(t) \le \sum_{t'=1}^{t} \lambda_i(t').$$
$$\sum_{t=1}^{T} \lambda_i(t) \le 1. \tag{5}$$

2. The stochastic programming problem

When uncertainty concerning benefit flows is included in the model, the decision made at each period should take into account all future uncertainties and future decisions. As in most of the land allocation literature, see Clarke and Reed (1990); Scheinkman and Zariphopulou (2001) and Coggins and Ramezani (1998), we assume that revenues accruing from the three states can be approximated by geometric Brownian motions with drift as

$$d\pi = \mu_\pi \pi(x(t))\, dt + \sigma_\pi \pi(x(t))\, d\xi_\pi,$$
$$d\kappa = \mu_\kappa \kappa(z(t))\, dt + \sigma_\kappa \kappa(z(t))\, d\xi_\kappa,$$
$$d\nu = \mu_\nu \nu(y(t))\, dt + \sigma_\nu \nu(y(t))\, d\xi_\nu, \tag{6}$$

where $d\xi_\pi, d\xi_\nu, d\xi_\kappa$ are independent Wiener processes. For the sake of simplicity we denote by ξ the random vector defined as $\xi = (\xi_\pi, \xi_\nu, \xi_\kappa)$.

We can now set out the stochastic linear programming problem as follows:

$$\max E_\xi \sum_{t=1}^{T}[\pi(t, x(t); \xi)x(t, \xi) + \kappa(t, y(t); \xi)y(t, \xi) + \nu(t, z(t); \xi)z(t, \xi)] +$$

$$- E_\xi \sum_{t=1}^{T}[\alpha(t)u_\alpha(t, \xi) + \beta(t)u_\beta(t, \xi) + \gamma(t)u_\gamma(t, \xi)] +$$

$$- E_\xi \sum_{t=1}^{T}[I_\alpha(t)\lambda_\alpha(t, \xi) + I_\beta\lambda_\beta(t, \xi) + I_\gamma(t)\lambda_\gamma(t, \xi)] \tag{7}$$

where E_ξ represents the expectation operator relative to the random vector ξ. Again the maximization problem is subject to the following constraints:

1. Initial conditions, for $t = 0$:

$$x(t, \xi) = 1 \text{ and } y(t, \xi) = z(t, \xi) = u_\alpha(t, \xi) = u_\beta(t, \xi) = u_\gamma(t, \xi) = 0\ . \tag{8}$$

2. Investment constraint, for $i = \alpha, \beta, \gamma$, and $t \in \{1, \dots, T\}$,

$$u_i(t, \xi) \leq \sum_{t'=1}^{t} \lambda_i(t', \xi), \text{ a.s.}$$

$$\sum_{t=1}^{T} \lambda_i(t, \xi) \leq 1. \tag{9}$$

3. Development constraints, for $t \in \{1, \dots, T\}$,

$$\begin{aligned} x(t, \xi) &= x(t-1, \xi) - u_\alpha(t, \xi) - u_\gamma(t, \xi), \\ y(t, \xi) &= y(t-1, \xi) + u_\alpha(t, \xi) - u_\beta(t, \xi), \\ z(t, \xi) &= z(t-1, \xi) + u_\beta(t, \xi) + u_\gamma(t, \xi), \\ u_\beta(t, \xi) &\leq y(t-1, \xi), \text{ a.s.} \end{aligned} \tag{10}$$

4. Conservation constraints, for $t \in \{1, \dots, T\}$,

$$x(t, \xi) + y(t, \xi) + z(t, \xi) = 1\ . \tag{11}$$

5. Information constraints, i.e., for $t \in \{1, \dots, T\}$, u_i is the $\Im_t$-measurable, where $\Im_t$ is the σ-field generated by the observations, i.e.,

$$\Im_t = \sigma\{\xi_\tau | \tau \leq t\}, \tag{12}$$

where ξ_t, represents the realization of the random vector at time t, for $i = \alpha, \beta, \gamma$.

An important prerequisite, in order to solve the maximization problem, is the discretization of the stochastic processes representing the evolution of the random data, ξ. The aggregation of the discrete processes can be represented by a scenario (event) tree, that defines the possible

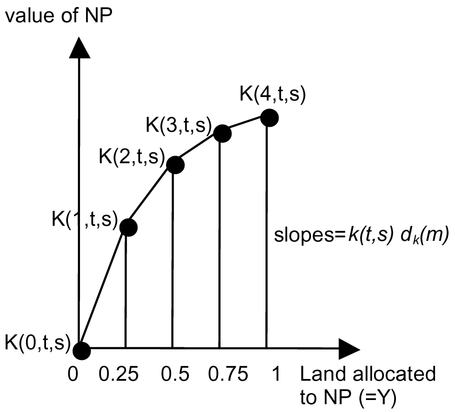


Fig. 2 Piecewise linear value of NP. Where $K(m, t, s)$ is the value of linear approximation of NP value at point $A(m)$ and $d_k(m)$ is the decrement of the marginal value of NP

sequences of realizations over the whole planning horizon. Note that nodes in the event tree are associated with decision points while arcs represent realizations of random variables. In particular, the root is associated with the first stage decision variables while leaves are related to all the possible last stage ones. If we denote with N_t the set of nodes at the t-th level, then each node $n \in N_t$ represents a particular realization sequence $\{\xi_\tau\}_{\tau=1}^{t}$ of the data process and it can be thought as a particular state of the system at a given time. A probability p_n can be associated with each node n at level t such that $p_n = p\{\xi_t|\xi_{t-1}, \dots, \xi_1\}$.

Scenario generation can be based on statistical approximations, see for example Hoyland and Wallace (2001), (Hoyland, Wallace, and Kaut, 2003) and also Koivu and Pennanen (2002), and on approximation theory, see, for example, Pflug (2000). Although many efforts have been made to find the most appropriate methodology, still one of the chief challenges in the field of stochastic programming is in finding the best way to generate scenarios, evaluate their importance and to trim unimportant information in order to solve smaller optimization problems. In Section 3, we discuss how we deal with the issue of generating the scenario tree for the case study.

First, it is important to mention that we introduce piecewise linear value functions in order to approximate the idea of non-constant marginal value of the land converted to the NP and NPO states. We introduce an index $m = 1, \dots, |M|$ where M is the set of linear approximation points. An example of piecewise linear function for the value of NP for $M = 4$ (four segments) is shown in Figure 2.

Hence, the deterministic equivalent problem can be stated as:

$$\max \sum_{s\in S} p(s) \sum_{t>0} [\pi(t,s)x(t,s)] +$$
$$+ \sum_{s\in S} p(s) \sum_{t>0} \sum_{m=1}^{M} [K(m,t,s)dY(m,t,s)] +$$
$$+ \sum_{s\in S} p(s) \sum_{m=1}^{M} m[V(m,t,s)dZ(m,t,s)] +$$
$$- \sum_{s\in S} p(s) \sum_{t>0} [\alpha(t)u_\alpha(t,s) + \beta(t)u_\beta(t,s) + \gamma(t)u_\gamma(t,s)] +$$
$$- \sum_{s\in S} p(s) \sum_{t>0} [I_\alpha(t)\lambda_\alpha(t,s) + I_\beta(t)\lambda_\beta(t,s) + I_\gamma(t)\lambda_\gamma(t,s)] \qquad (13)$$

The objective function is constrained to:

1. Initial conditions, for each $s \in S$ and for $t = 0$,

$$x(t, x) = 1 \text{ and } y(t, s) = z(t, s) = u_\alpha(t, s) = u_\beta(t, s) = u_\gamma(t, s) = 0. \quad (14)$$

2. Investment constraints, for each $s \in S$, for $i = \alpha, \beta, \gamma$, and $t \in \{1, \ldots, T\}$,

$$u_i(t, s) \leq \sum_{t' \leq t} \lambda_i(t', s),$$
$$\sum_t \lambda_i(t, s) \leq 1. \quad (15)$$

3. Development constraints, for each $s \in S$ and $t \in \{1, \ldots, T\}$,

$$\begin{aligned} x(t, s) &= x(t-1, s) - u_\alpha(t, s) - u_\gamma(t, s), \\ y(t, s) &= y(t-1, s) + u_\alpha(t, s) - u_\beta(t, s), \\ z(t, s) &= z(t-1, s) + u_\beta(t, s) - u_\gamma(t, s), \\ u_\beta(t, s) &\leq y(t-1, s). \end{aligned} \quad (16)$$

4. Conservation constraint, for any $s \in S$ and for $t \in \{1, \ldots, T\}$,

$$z(t, s) + y(t, s) + z(t, s) = 1. \quad (17)$$

5. Non-anticipativity constraints, defined in such a way that the dependencies implied by the scenario tree are satisfied (see Figure 3 as an example). Defining B_n the bundle of scenarios passing through node n, then

$$\begin{aligned} x(t, s_i) &= x(t, s_j), i \neq j, \forall s_i, s_j \in B_n, \text{ with } n \in N_t, t = 1, \ldots, T-1, \\ y(t, s_i) &= y(t, s_j), i \neq j, \forall s_i, s_j \in B_n, \text{ with } n \in N_t, t = 1, \ldots, T-1, \\ z(t, s_i) &= z(t, s_j), i \neq j, \forall s_i, s_j \in B_n, \text{ with } n \in N_t, t = 1, \ldots, T-1. \end{aligned} \quad (18)$$

6. Piece-wise linear value of land constraints, defined for each $s \in S$, with $t \in \{0, \ldots, T\}$.

3. The case of Appiano Gentile and Tradate park

Appiano Gentile and Tradate Park, located in Lombardia (region of northern Italy), is one of the few parks in an area that is extremely populated and urbanized, see Figure 4. The extreme fragmentation of the park and of its ownership (partly private, partly publicly owned) has led to a degradation of the naturalness of the site and to the loss of several ecosystems. Hence, citizens, paying through an annual tax for the maintenance of the park, are nowadays complaining about the low amenity value offered by the park to the public. For this reason, managers of the park have decided that more resources should be allocated in a rational management and in the sustainable development of the area.

We have modelled the problem as a three-stage stochastic programming problem, assuming that the managers can gradually transform the area. The conversion possibilities are once again staying in the status quo, developing a more organized natural park or, finally, investing

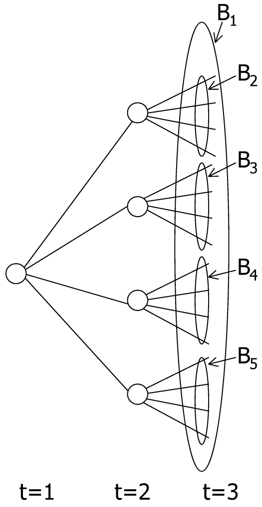


Fig. 3 An example of non-anticipativity constraints in the event tree

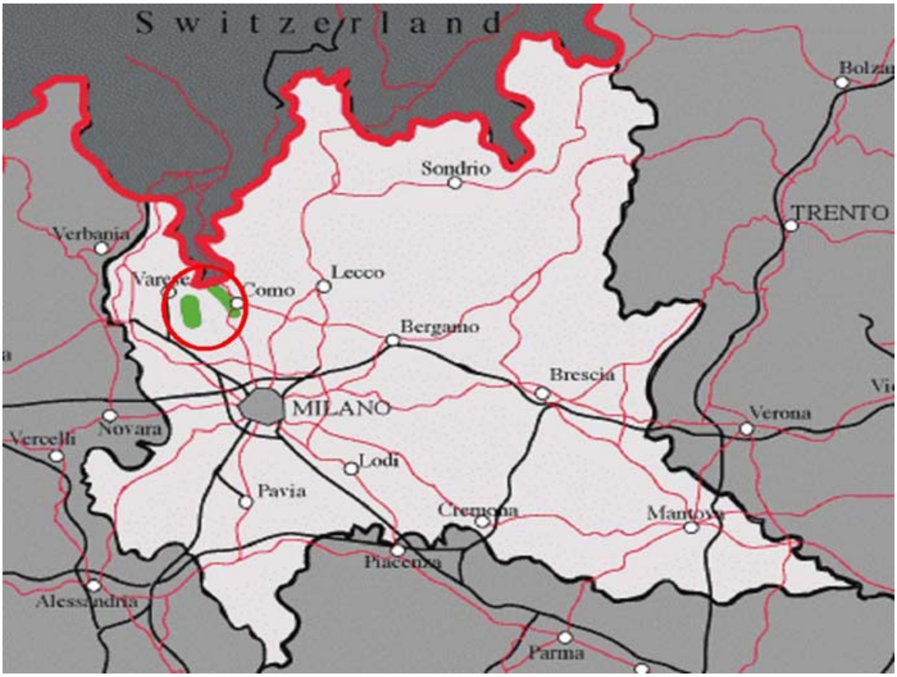


Fig. 4 Appiano Gentile and Tradate Park

more and developing a more organized and less natural version of the natural park, see Figure 1, p. 6. In order to solve the problem we use a scenario approach, thus assuming that the distribution of the three land values is concentrated on a finite number of scenarios generated using the Hoyland and Wallace approach. Hoyland and Wallace (2001). Thus an additional set of constraints is added to the model in order to account for non-anticipativity, that is each decision taken at a given stage cannot include information that will be revealed at subsequent stages.

Fig. 5 Values are expressed in Euro

Values	SQ	NP	NPO
Gross Annual Revenues	16,550	69,590	117,600
Costs	**SQ to NP**	**SQ to NPO**	**NP to NPO**
Conversion Costs	10,000	20,000	10,000
Investment Costs	100,000	200,000	100,000

Data required from the local manager belong mainly to two categories. The first set of data concerns the value of annual benefit flows deriving from each of the three possible allocations. Given that there are not dataset on revenues accruing from actual and potential allocations of the land, data were collected through a contingent valuation survey[2]. The contingent valuation study was conducted from July to October 2001; 250 users were surveyed in order to capture their willingness to pay for the area in each of the three possible states, using both a Multi-Attribute Choice[3] (MAC) and an Open-Ended[4] (OE) format.

Successively, through econometric analysis of the collected data, the likely willingness to pay per person to enter the park in its different hypothetical forms has been computed. Once the distribution of willingness to pay per person per day (in Euros) related to each of the land allocations was inferred, it was aggregated over the actual number of annual visitors (data are summarized in Figure 5). Moreover, unit conversion costs and initial conversion investment costs are approximated values estimated on the basis of similar projects.

The second data set comprises the historical data concerning annual visits to the parks, required to proceed with scenario generation. We assumed that the net benefits accruing from the three strategies would be proportional to the number of visitors and used time-series data to estimate the mean drift, μ, and standard deviation, σ, of the visits rate. With estimates of μ and σ for each option, it is possible to generate the scenario tree.

Accurate time series data for the Appiano Gentile and Tradate Park are not available, as, of course, they are not available for the hypothetical allocations NP and NPO. We have therefore used data concerning similar parks (parks that may be reasonable substitutes of the three states considered in the model[5]). Figure 6 shows time-series data for Parco delle Groane (the comparable to Appiano Gentile and Tradate Park in the SQ state), for Parco della Maremma (the comparable to Appiano Gentile and Tradate Park in the NP state) and Parco Nazionale dello Stelvio (the comparable to Appiano Gentile and Tradate Park in the NPO state).

[2] The Contingent Valuation method is a direct method applied in order to elicit the consumers' willingness to pay for a good that has no market and therefore has no price. The methodology implies the use of questionnaires that create a hypothetical market with which the respondents are confronted. Questions may have different formats and questionnaires should be conducted in person.

[3] Multiple attribute method is also known as the Stated Preference Approach or Choice Experiments. Rather than asking people to choose between a base situation and a specific alternative, the multi attribute choice presents respondents with a range of policy options and their costs and benefits, and constructs total values for an option based on marginal choices between options Adamowicz (1998).

[4] The open-ended question format (e.g., what is your maximum willingness to pay for... ?) was first applied by Davis in 1963 in one of the first CV studies Davis (1963). It is a continuous method, which generates continuous data.

[5] As far as organisation is concerned, Parco delle Groane is extremely similar to the actual situation of Parco Appiano Gentile (Activities are free, except for a course in environmental education).
Parco della Maremma is more organised than the actual Parco di Appiano Gentile (guided tours, canoe experiences, night visits, horse riding).
Parco Nazionale dello Stelvio is extremely innovative and organised (several museums, theme paths, picnic areas, etc).

Year	SQ	NP	NPO
1992	9225	521842	40000
1993	9375	447961	36000
1994	11750	522201	50873
1995	12575	547018	40542
1996	13275	560691	37189
1997	13700	603533	45628
1998	16075	595675	11688
1999	13075	583890	71403
2000	15850	676485	102965
2001	16800	857864	98478

Fig. 6 Values are expressed in annual number of visitors

Fig. 7 The geometric Brownian motion estimated parameters

Parameter	SQ	NP	NPO
μ	0,07946	0,06585	0,03416
σ	0,16032	0,14575	0,10077

For each option, we sought the values μ and σ which would maximize the log likelihood function:

$$\ln(L) = -\frac{T}{2}\ln(2\pi) - \left(\frac{T}{2}\right)\ln(\sigma) - \left[\frac{1}{2\sigma^2}\right]\sum_{t=0}^{T-1}\left[\ln\left(\frac{x_{t+1}}{x_t}\right) - \left(\mu - \frac{\sigma^2}{2}\right)\right]^2, \tag{19}$$

where T is the number of years we have data for, x_t, is the visiting rate to the park in year t and the time step is assumed to be $\Delta t = 1$. The estimated parameters are given in Figure 7.

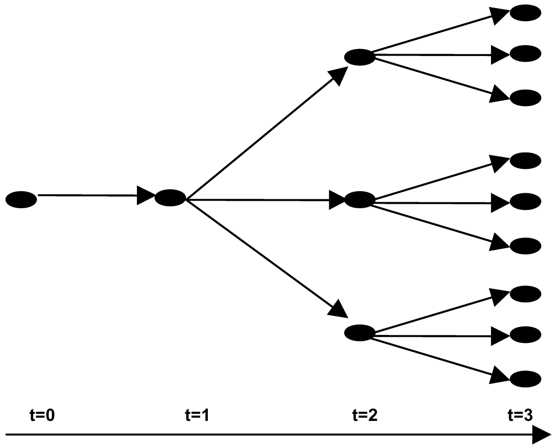
Generation of the Scenario Tree

In order to generate the scenario tree we have followed a methodology developed in Hoyland and Wallace (2001). The method is based on non linear programming, used to generate a limited number of scenarios that satisfy specified statistical properties. Following Hoyland and Wallace (2001), we know that in order to obtain a perfect match there must be a relationship between the characteristics of the statistical specifications and the number of outcomes; in particular, degrees of freedom provide a guess about the size of the tree. In our case, the problem dimension and the deriving number of specifications (three variables, two moments and requirements on the correlations) lead us to construct a tree with three outcomes deriving from each node at time period $t = 2, 3$. Moreover, for the sake of simplicity, we assume the tree is symmetric, see Figure 8.

Given the statistical properties of the random variables, the objective is to construct a tree such that the statistical properties of the approximating distribution match the specified statistical properties. Let Ξ be the set of all statistical properties and ϑ_i be the specified value of statistical property $i \in \Xi$. Moreover, let $f_i(x, p)$ be the mathematical expression for statistical property $i \in \Xi$ expressed as a function of the outcome value, x, and of the outcome probability, p. We use the square norm as a measure of the distance between the statistical properties and the constructed distribution and we want to construct x and p such that:

$$\min_{x,p}\sum_{i\in\Xi} w_i(f_i(x, p) - \vartheta_i)^2 \tag{20}$$

Fig. 8 The Scenario Tree for the Appiano Gentile and Tradate Park



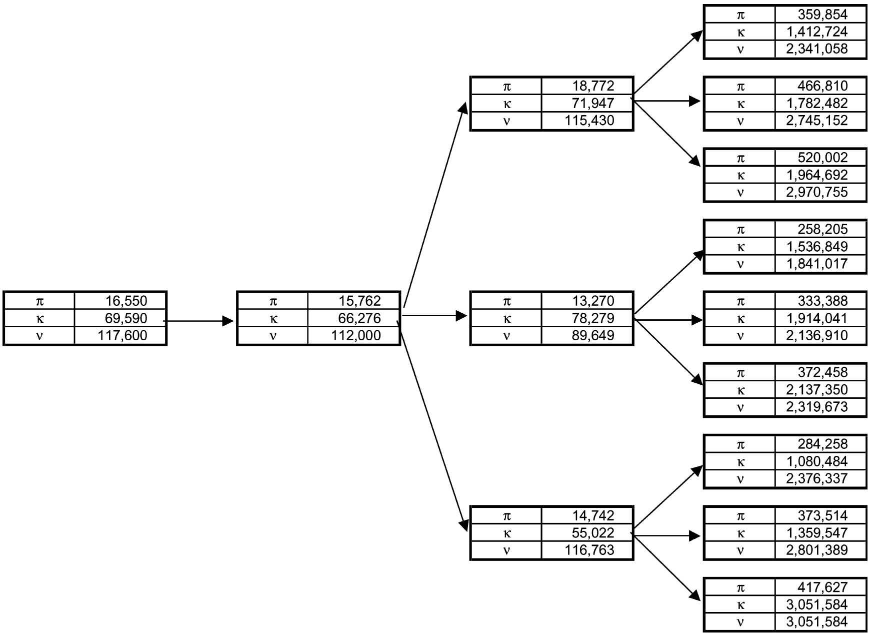


Fig. 9 The scenario tree generated for the Appiano Gentile and Tradate Park. Note that in period $t = 3$ scenarios include the stream of all remaining future cash flows calculated assuming that revenues have reached a steady state

where w_i is the weight associated to statistical property $i \in \Xi$ and the minimization problem is subject to constraints defining the probabilities to be nonnegative and to sum up to one. Given the statistical properties of our processes, defined in Figure 7, the deriving scenario tree is reported in Figure 9.

By solving the scenario based stochastic programming problem we obtain an optimal value for the objective function equal to 1,717,780 Euros, while the optimal first stage solution vector is $\{x(1) = 0.63, y(1) = 0.17, z(1) = 0.2\}$.

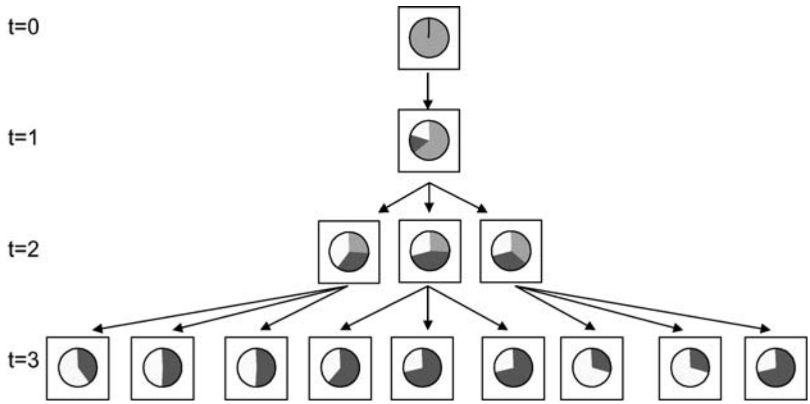


Fig. 10 The solution vectors represented in the form of a tree, where grey represents the Status Quo, black the NP and white the NPO portions, respectively

The suggested policy for the first period is to convert 17% of the area to the NP state, 20% to the NPO state, while the remaining land is left to the Status Quo state, see Figure 10. This decision provides the flexibility to convert greater amount of land to the NP and NPO states in successive periods, depending on the actual scenario.

Solution analysis

We analyze different solutions to the conversion problem by fixing their first stage decision and simulating their behavior for 10,000 trials of random generated scenarios. In particular, we analyze the solution obtained solving the stochastic programming model and compare it with other possible solutions, as the solutions obtained solving the deterministic problem for each of the nine significative scenarios as well as the solution to the mean value problem. In addition, we analyze solutions that are considered significative from the park managers (as, for example, immediate complete conversion to NP or NPO states).

When compared with the solutions to the deterministic problems solved for each of the nine significative scenarios, the stochastic programming first stage solution has a distribution that is more right-skewed and, in several cases, a mean value shifted to the right.

We also report the frequency chart of the objective value obtained by simulating future random scenarios given the first stage solution to the stochastic programming problem (the column graph in Figure 11) and by overlaying frequency charts obtained for the two immediate conversion solutions ($x(0) = 0$, $y(0) = 1$, $z(0) = 0$ and $x(0) = 0$, $y(0) = 0$, $z(0) = 1$) and the solution to the mean value problem ($x(0) = 0$, $y(0) = 0.5$, $z(0) = 0.5$) (the three outline graphs in Figure 11).

It is noticeable how, all the solutions considered are dominated by the solution obtained through the stochastic programming model, whose mean value is shifted to the right.

4. Integrating the stochastic programming procedure and the decision tree framework

Further, we investigated the possibility of including in the model the uncertainty about environmental quality, which typically affects the initial value of projects implying costly

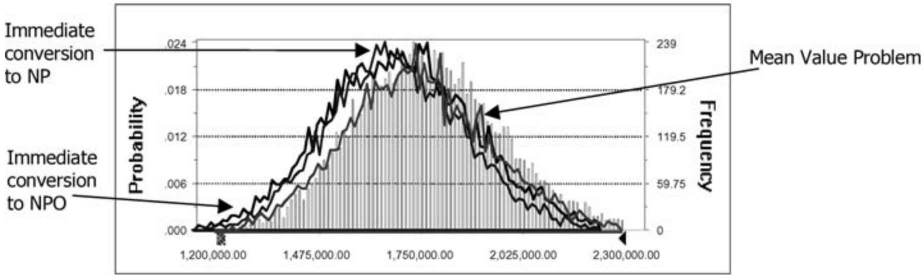


Fig. 11 Frequency chart representing the simulated behavior of the solution to the stochastic problem (column graph). The outline graphs represent the immediate NP and NPO conversion solutions and the solution to the mean value problem (respectively pointed by the indicators)

conversion decisions aiming at nature preservation. We assume that it is not possible to know *a priori* the environmental quality of a site, the efficiency of the remediation process and the resulting initial value of revenues deriving from consumers' response to the amenity value associated to the area. Moreover, we assume that the initial value of revenues deriving from Strategy #2 and #3 would be proportional to the environmental quality of the site, which depends on a series of uncertain attributes of the area, such as biodiversity, rareness, size of the area, naturalness, level of representation of the inhabitant species (i.e., their uniqueness,etc.). These attributes may be combined in an overall evaluation index of the site. In this work we follow a criteria-based evaluation method based on [Gaston et al. (1998)] and we consider an environmental quality index assuming three possible values[6], each characterized by a prior probability measure.

1. When either size or naturalness of the site are insufficient due to previous damage, the initial value of the *NP* and *NPO* allocations, say, κ_L and ν_L, are low (with probability $q_L, 0 \leq q_L \leq 1$).
2. When the site attributes jointly give a high value index, the initial value of the *NP* and *NPO* allocations, say, κ_M and ν_M, are high (with probability $q_H, 0 \leq q_H \leq 1 - q_L$).
3. When the site attributes jointly give a middle range index, the initial value of the *NP* and *NPO* allocations, say, κ_M and ν_M, are medium, such that $\kappa_L < \kappa_M < \kappa_H$ and $\nu_L < \nu_M < \nu_H$ (with probability $1 - q_H - q_L$).

Prior probabilities associated with the three environmental quality states are provided by experts or may be deduced from similar case studies. Hence we model uncertainty about environmental quality as uncertainty on the initial value of the price process, while we assume uncertainty affecting successive evolution of the value can be modelled as a diffusion process. Information on the actual realization of the initial value may become available depending on the decision maker's deliberation (e.g., he may choose to allocate money to investigate environmental quality characteristics of the site). The possibility of enclosing updated information concerning the initial value is framed as a decision step, preceding the conversion problem and concerning whether to invest in research or proceed in the decision process exclusively with the information known at time $t = 0$. Decision analysis dramatically emphasizes the need to acquire information about outcomes, thus proving to be the optimal

[6] This is clearly an attempt to simplify the complexity of possible environmental quality scenarios, however the assumptions are useful for explanation purposes and may be easily modified to provide a more realistic representation of environmental quality.

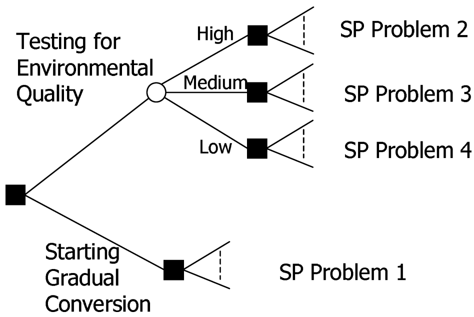


Fig. 12 The integrated decision environment

tool to deal with this type of problems (that is, when the set of possible actions is limited). However, once the decision whether to investigate the environmental quality of the site has been made, we have to deal with the gradual conversion decision problem, which implies a set of actions that cannot be enumerated easily. For this reason we install on each leaf of the decision tree a separate stochastic program, each with the same formulation but solving a different event tree. The value of each different event tree depends on the decision taken at $t = 0$ that is represented through a decision tree.

In particular, the decision framework models the possibility of investing in research, for example, acquiring more information about the overall environmental quality of the site: such data will influence our knowledge of the distribution and initial value of consumers' demand and reservation price, thus affecting the magnitude of revenues accruing from the preservation decision. If research is undertaken it will be possible to consider the actual initial discounted value of revenues arising from the Natural Park state, instead of its expected value. Moreover, acquiring new information may change the characteristics of the stochastic process describing the value of revenues in time. Alternatively, the decision maker could decide to immediately convert the area, without any further inquiry.

Stochastic programming problems are inserted on such a decision tree framework (see Figure 12). thus providing the decision maker with an optimal strategy accounting for both environmental and market uncertainty.

We return now to the case study. Let us start the analysis considering that each possible state (high, medium, low) of the environmental quality of the site is considered equally likely, provided we do not have any more specific information, i.e. $\{q_H = q_L = q_M = 0.\bar{3}\}$. The three deriving stochastic programming problems are inserted on the leaves of the "perform a test" portion of the decision tree, see Figure 13. By computing the discounted expected value, it is possible to evaluate the strategy of conducting a preliminary test. Indeed, testing the environmental quality of the site yields an expected return of 1,927,801 Euros, if we assume that conversion will be thereafter performed gradually, given information on the state of nature. The testing decision, when compared to the immediate gradual conversion decision, is worth an extra 210,020 Euros, which represents the maximum amount the decision maker should be willing to pay in order to gather new information. Clearly, it is useful for management purposes to gain an insight into the nature of the computed values and to consider how these values are affected by changes in the probability measures associated with the high, medium and low initial value of the NP and NPO states. As the probability of a high state value of the NP and NPO strategies grows, so does the value of information. For example, for a probability vector equal to $\{q_H = 0.5, q_L = q_M = 0.25\}$ then we would obtain a value of information equal to 370,537 Euros.

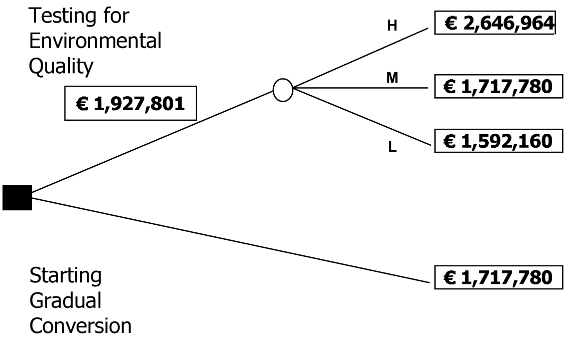


Fig. 13 The problem decision tree. Each final leaf represents the value of the optimal objective function, solution of the relative stochastic programming proble

5. Conclusions

Typically, in a dynamic setting more conservative strategies are preferred to irreversible ones; in other words, if we apply dynamic optimization tools, the returns required from irreversible decisions must be higher in order to undertake them. This makes the stochastic programming approach a tool of critical importance in the rationale management of renewable and non-renewable resources. Moreover, our approach, which combines both stochastic programming and decision tree techniques, is a way to successfully deal with the issue of enclosing environmental uncertainty in the decision process. We have shown how our approach has been applied in order to produce guidelines on the first stage optimal conversion vector and in order to give insights on how much would be economically rationale to invest in further research on the actual environmental quality of the site.

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