# **ELEMENTARY CRYPTOGRAPHY GAMES FOR PRIMARY SCHOOL STUDENTS THROUGH WIMS**

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## **Abstract**

The importance of an active engagement of students in the learning process is nowadays recognised as a central aspect for a successful teaching practice. Many researchers highlight that teaching through games is a very effective way to involve pupils, especially in primary school. Indeed, the methodology of Game Based Learning not only stimulates students' interest, but increases their motivation and satisfaction, making it easier to understand and remember new concepts. This is particularly relevant for Mathematics, where learning a new rule through experimentation leads to a better understanding.

Such a methodology becomes even more powerful when combined with the use of the so-called "new technologies". We developed a proposal for a web-based mathematical "turnkey" game environment for primary school pupils, using the WIMS system.

The WIMS system (WWW Interactive Multipurpose Server, e.g. see [\[1\]](#page-7-0)), conceived by Xiao Gang and made public in 1998, is now in widespread use throughout France and is slowly spreading to other countries. It is available in various languages, and it is recognized for its vitality and effectiveness as a server for publishing interactive learning objects. One of its advantages is that its activities can be offered to the user with no need to install any additional software on the local pc (or on any mobile device) other than a standard web browser. Also, teachers can create virtual classes to propose activities to students and actively monitor their works. Obviously, WIMS can be a very useful tool for distance learning.

The aim of this work is to present an activity on elementary cryptography through games. Our proposal consists of essentially two parts. The first activity challenges the students with the problem of encrypting and decrypting secret messages, taking inspiration from Caesar code. The second part involves a game inspired by modern cryptography techniques, such as Hamming codes for error correction. It is presented as a magic trick: one volunteer is asked to think about a number from 0 to 15 and to answer 7 yes/no questions regarding it. The interesting part is that the volunteer is allowed to lie once. The "mathe-magician" will then guess not only the number, but also the possible lie. It is a very intriguing game (for adults as much as for kids), that stimulates pupils in asking the question "How does it work?". This is enough to capture the students' attention, enabling the teacher to introduce them to the inner mathematical motivations of the game, such as the binary code.

The applets we will present are publicly available on any WIMS server (e.g. the document [\[2\]](#page-7-1) and the module [\[3\]](#page-7-2), the latter available in Italian and French, and soon in English as well).

In our presentation we will also share the outcomes of a trial testing of such activity with four classes of Italian students, for a total of 74 pupils of 8 and 9 years old, focusing on a qualitative analysis of the reactions of the pupils. Also, we will outline how the WIMS contents were designed to support the explanation of how the mathematics behind the magic works.

Keywords: mathematics, interactive games, educational games, game-based learning, educational software.

## **1. ENGAGING PUPILS THROUGH GAMES**

We start from the assumption that problem solving *à la* Polya (e.g. see [\[4\]](#page-7-3)) is a crucial step for a real understanding of mathematics. Effective learning takes place only if pupils have "a reasonable share of the work" ([\[5\]](#page-7-4)). In this view, active methodologies such as Problem-Based Learning (PBL, e.g. see [\[6\]](#page-7-5)) are particularly suitable for developing a deeper understanding of mathematical facts and methodologies, as well as for leading the learners to acquire fundamental skills such as critical thinking and self-regulating. When actually designing PBL activities, a game can be the starting point for a PBL session, especially with younger students, and in this sense, the use of games in teaching is a key tool for engaging students

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in maths activities. The "game" becomes the problem to be solved, and as such can improve attitude towards mathematics, can motivate and engage the students and can involve higher-level thinking. In spite of research documenting the importance of these goals in education and the importance of active, student-centered instruction based on games, we have to regret that such methodologies are not widespread in real teaching practice at school and the sentence "you do not come to school to play!" is unfortunately not uncommon. Teachers too often rely on self-perpetuating "traditional" methods, and even the most willing teachers on numerous occasions express regret on not being able to design engaging activities for their pupils. We thus think that research in mathematics education should focus also on helping teachers to plan the classroom activities by producing and discussing ready to use examples. The proposal we describe goes exactly in this direction.

# **2. WIMS**

WIMS (WWW Interactive Multipurpose Server) is a very flexible system that allows for the creation of a large variety of learning objects with automatic marking. It was conceived by Xiao Gang and made public in 1998 (e.g. see [\[7\]](#page-7-6)). We already introduced WIMS in [\[8\]](#page-7-7) and refer to [\[7\]](#page-7-6) and [\[1\]](#page-7-0) for a detailed description of the system. We also refer to  $[9]$ ,  $[10]$ ,  $[11]$ ,  $[12]$ ,  $[13]$ ,  $[14]$ ,  $[15]$ ,  $[16]$ ,  $[17]$ , and  $[18]$  for accounts of experiences of its use in teaching. For the purposes of this work, you can think of WIMS as "a network of servers sharing interactive resources at many levels in various subjects" ([\[1\]](#page-7-0)).

For some years now WIMS has included more and more modules suitable for primary school children, but sometimes the modular structure of WIMS makes it difficult to find suitable material. Indeed, documents within WIMS can be used to build a path into the vastness of available activities. The proposal we will describe starts from the WIMS document [\[2\]](#page-7-1), which is freely available on any WIMS server and can be visited for example starting from the url [https://wims.matapp.unimib.it/wims/wims.cgi?module=](https://wims.matapp.unimib.it/wims/wims.cgi?module=E4/number/doccrypto.it) [E4/number/doccrypto.it](https://wims.matapp.unimib.it/wims/wims.cgi?module=E4/number/doccrypto.it). The document links the various activities available in WIMS, thus constituting a base on which the teacher can build the class activities. The document can also be browsed autonomously by the pupils. The WIMS modules linked to this document are freely available in WIMS and can be used independently one from the other at ones' own will. Some of these modules were already available in the international distribution of WIMS, some other were created for this project. The WIMS system allows for teachers to build their own documents so to adapt the path to the needs of their own classes (e.g. teachers can translate [\[2\]](#page-7-1) into their own language).

# **3. A GAME IN ELEMENTARY CRYPTOGRAPHY**

# **3.1 Why cryptography?**

Cryptography is not a typical topic in primary school mathematics, but we believe it can be a powerful ally for teachers. First of all, the topics involved are challenging and useful to capture pupils' attention, as they are connected to everyday problems. Secondly, cryptography allows for connections to other subjects, such as history (from the ancient attempts at encrypting a secret message to its use in World War II), literature (using known texts as examples for encryption and decryption) and civic education (e.g. to become aware of computer scams such as the stealing of personal data and so on).

Moreover, many cryptography questions can be presented in the form of games and are suitable for setting up PBL activities. Finally, cryptography can be used as a bridge to introduce deep mathematical topics, helping the students to recognize the topics themselves as interesting, useful and meaningful. All these reasons led us into designing the activity described in the next section.

# <span id="page-1-1"></span>**3.2 Description of the activity**

Our proposal involves the problem of encrypting and decrypting secret messages and a game inspired by the modern cryptography technique of Hamming code for error correction (e.g. see [\[19,](#page-8-10) p.271-273]).

## <span id="page-1-0"></span>**3.2.1 Encrypting and decrypting secret messages**

We will briefly outline the first part of our proposal.

**Step 1: Introducing the problem.** The ability to send and receive secret messages is a subject that always inspires students' enthusiasm. We start our activity recalling that such a practice has a



<span id="page-2-0"></span>Fig. 1: To use Caesar's wheel to encrypt a message with key  $k = 2$ 



Fig. 2: To use Caesar's wheel to decrypt a message encrypted with key  $k = 2$ 

long history and was used even in ancient Rome to communicate during battles; this approach not only captures the pupils' attention, but also allows for multidisciplinary connections.

**Step 2: Encrypting.** We are ready to present Caesar's code. The idea is to fix a key *k* and then to move every letter of the used alphabet of *k* steps. For example, if we use the English alphabet (consisting of 26 letters) and we choose  $k = 2$ , then the letter "A" becomes "C", the letter "B" becomes "D" and so on. So, the word "HELLO" encrypted with key  $k = 2$  becomes "JGNNQ". A powerful tool to present visually this mechanism is the use of two concentric wheels with the letters of the alphabet written on both, fixed in such a way that the outer wheel can rotate.

Using the double wheel, we can see that the outer letter "H" is now on top of the inner letter "J", the outer letter "E" now corresponds to the inner letter "G" and so on.

The WIMS document [\[2\]](#page-7-1) contains a detailed explanation and various games of encrypting using the double wheel. These games are all available in the WIMS module [\[3\]](#page-7-2) and are of increasing difficulty, configurable by the teachers: firstly, it's asked to encrypt one letter, then a word and finally an entire sentence. Also, for each task there are two levels: using the double wheel already shifted of the right number of steps (as in Figure [1\)](#page-2-0) or visualizing the inner wheel only and having to imagine the shifting.

It is also useful to ask the students to create a paper-made double wheel that they can use in combination with the virtual one. In this way manipulation techniques are involved and the pupils are even more engaged in the task.

**Step 3: Decrypting.** The obvious next step in this path is to build the ability to decrypt an encrypted message. If one knows the key *k* used to encrypt the message, it is enough to take the double wheel and rotate the outer one of *k* steps anticlockwise.

Considering the example introduced above, it is now easy to see that the encrypted word "JGNNQ" corresponds to the original word "HELLO".

As for the previous task, the WIMS document  $[2]$  contains the explanation of the decrypting techniques and different games of increasing difficulty, all available in the same module [\[3\]](#page-7-2).

The activity is more effective if the pupils are asked to discover the rule of decrypting by themself, based on what they learnt about the encrypting process.

**Step 4: Guessing the key.** At this point we should highlight the importance of knowing the key: without its knowledge, the task of decrypting secret messages becomes harder (and that's a good thing

for real life necessities!). This task therefore focuses on the ability of guessing the key. The WIMS module [\[20\]](#page-8-11) presents a long encrypted text (in the Italian version, the texts used are extracts of Carlo Collodi's "Le Avventure di Pinocchio" and Lewis Carroll's "Alice's Adventures in Wonderland') and the students are asked to discover the key.

An example in the Italian version:

*"Paadgp vgpcsx gxhpit sprrped: bp a'dbxcd, xcktrt sx gxstgt, hx htcix egthd sp ipcid pbdgt etg fjtaa'xggtfjxtid phxctaad rwt, rdc jc qprxd, vax edgid kxp sx ctiid ap btip sx fjtaa'paigd dgtrrwxd. Edx sxhht pa qjgpiixcd: - Gxbdcip ejgt p rpkpaad, t cdc pktg epjgp. Fjta rxjrwxcd pktkp fjparwt vgxaad etg xa rped: bp xd vax wd stiid sjt epgdaxct ctvax dgtrrwx, t hetgd sx pktgad gthd bpchjtid t gpvxdctkdat. - "*

After a bit of work, one finds that here the key is  $k = 11$  and the decrypted text is

"Allora grandi risate daccapo: ma l'omino, invece di ridere, si sentì preso da tanto amore *per quell'irrequieto asinello che, con un bacio, gli porto via di netto la met ` a di quell'altro ` orecchio. Poi disse al burattino: - Rimonta pure a cavallo, e non aver paura. Quel ciuchino aveva qualche grillo per il capo: ma io gli ho detto due paroline negli orecchi, e spero di averlo reso mansueto e ragionevole. -"*

There are various strategies to find the key, like looking for short words (the letter "t" appears various time isolated and there is a good chance that it corresponds to "e", that means "and" in Italian) or considering how often a letter is repeated (that is carry out a frequency analysis) or its position in the words (in Italian most words end by vowel).

In our experience, students find this problem very intriguing and offer a plethora of strategies to guess the right key. As in the previous tasks, the WIMS module  $[20]$  contains many texts to play with, with two levels: "long and easy" or "short and hard".

**Step 5: conclusions.** Thanks to this activity, the students should have realized that in order to exchange secret messages with a friend/ally it is necessary to know how to encrypt and decrypt them and that there is a key involved. Such a key is crucial in the process and the harder it is to guess it, the safer the secret message will be. Indeed, it's important to remark that Caesar's code is not the only encrypting method! In the WIMS module [\[20\]](#page-8-11) one can find other games involving different types of encrypting techniques. For instance, given a encrypted text, the players are required to discover what letters need to be swapped to get the original message.

#### <span id="page-3-0"></span>**3.2.2 The game: "7 questions and 1 lie"**

The second activity we propose is presented as a magic game; this is said explicitly to the pupils in order to stimulate their curiosity. One volunteer is asked to think of a number from 0 to 15 and to not reveal it. Then he/she should answer 7 questions about this number. To make things even more interesting, the volunteer is given the chance to lie once. These are the 7 questions:

- 1. Is your number greater than 7 (and not equal to 7)?
- 2. Is it among the numbers <sup>4</sup>,5,6,7,12,13,14,15?
- 3. Is it among the numbers <sup>2</sup>,3,6,7,10,11,14,15?
- 4. Is it odd?
- 5. Is it among the numbers <sup>2</sup>,3,4,5,8,9,14,15?
- 6. Is it among the numbers <sup>1</sup>,2,4,7,9,10,12,15?
- 7. Is it among the numbers <sup>1</sup>,3,4,6,8,10,13,15?

After receiving the 7 answers, the mathe-magician is immediately able to guess the volunteer's number and also to reveal which answer was a lie (if there was one). The typical pupils' reaction is awe and, more importantly, the formulation of the question "How does it work?". The explanation is given in steps, that require the students' contribution, that is, their *reasonable share of the work*.

**Step 1: introducing the main tool.** First of all, it's revealed that the mathe-magician uses a special scheme to solve the problem, as shown in Figure 3. Each numbered circle corresponds to a question and the first step consists of coloring the dots corresponding to the positive answers and leaving blank the ones corresponding to the negative answers.

In order to actively involve the pupils, we designed the following activity. A number from 0 to 15 was assigned to each student and then pupils were asked to color the scheme according to their



Fig. 3: Scheme used in the game *7 questions and 1 lie*



Fig. 4: Scheme corresponding to the number 9

answers to the 7 questions (avoiding lies for the moment). This task can also be completed in couples, where first one student answers the question and the other fills the scheme and then they swap roles. At the end of the activity each student will have a coloured scheme corresponding to a precise number.

**Step 2: collecting observations.** Each student is asked to describe how he/she coloured its scheme and to what number it corresponds to. The idea is that all students can visualize the various filled schemes and their corresponding numbers (for example the teacher can reproduce the schemes on the blackboard or on the interactive screen). Then the most important part of the activity takes place: students are asked to share their observations and describe the properties of the coloured diagrams.

There is not a unique correct answer, making it an open problem. Eventually helped by the teacher, the pupils should discover in particular two fundamental rules:

- 1. Each number from 0 to 15 corresponds to exactly one and only one way of filling the scheme. This means that if we have the list of all possible outcomes (see Figure 5), then we can solve the game of 7 questions (without lies).
- 2. There are 4 possible behaviours in the filled schemes:
	- all circles are blank (corresponding to the number 0);
	- all circles are coloured (corresponding to the number 15);
	- there are 4 coloured circles and 3 blank circles aligned or positioned around the inner circumference;
	- there are 4 blank circles and 3 coloured circles aligned or positioned around the inner circumference;

For instance, in Figure 4 we see that the number 9 corresponds to the scheme with 4 coloured circles (1, 4, 5 and 6) and 3 blank circles aligned (2, 3, 7).

**Step 3: recognizing the lie.** The second rule discovered in Step 2 gives a hint on how to detect the lie. Indeed, if the filled scheme does not satisfy one of the 4 options listed, then there is a unique circle that breaks the pattern, and it is enough to identify it and change its status (colour the circle if it was blank and vice versa). With this procedure we kill two birds with a stone: we recognize the lie (corresponding to the wrong circle), and we recover the correctly filled scheme, enabling us to identify the mysterious number.

The WIMS module [\[3\]](#page-7-2) contains the game in two versions: letting the software play the role of the magician (without explanations of the trick) and in the "Now you guess" mode ("Ora indovina tu" in the Italian version), where a step-by-step solution is offered and starting from a filled scheme (as in Figure 6) the player is asked to detect the lie and to guess the mysterious number.

**Step 4: making the bridge to deep mathematical concepts.** The activity could be considered concluded at Step 3, but one of our already mentioned goals is to exploit the game to introduce some



Fig. 5: All possible outcomes without lies



Fig. 6: Scheme corresponding to the number 9 with a lie in the answer to question 7

deep mathematics. Therefore, we can give the students a different method to guess the mysterious number (instead of reading the list of all possible correctly filled schemes). Once the lie is detected and the scheme is correctly filled, the strategy consists of focusing the attention on circles numbered 1 to 4 and writing 0 if the circle is blank and 1 if it is coloured. For instance, the scheme associated to number 9 will give the sequence

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One may immediately recognize that this is the way of writing the number 9 in base 2. Therefore, this game can be a good expedient to introduce the topic of number bases. Once again, the WIMS document [\[2\]](#page-7-1) represents an invaluable help, offering various interactive games on the subject. The strategy is to use small cubes to represent units and to ask the player to count the total amount of cubes. Such cubes are grouped in "blocks" of different sizes (the powers of 2) that can be put together in order to form the wanted number of small cubes (e.g. see Figure [7\)](#page-6-0).

**Step 5: conclusions.** Now that the trick to solve the game is no longer a secret, we can analyze what we did. We discovered a special code (the filled scheme) that may contain an error (the circle corresponding to the lie). Once we fix the error and we recover the correct scheme, we can translate it into the binary code (i.e. the way of writing numbers in base 2). Now we can decrypt the code and find the solution to our game. Also, we may notice that the answers to questions 5 to 7 are used to detect the lie only: if we do not allow any lie, then it is enough to ask the first 4

<span id="page-6-0"></span>

Fig. 7: Blocks in base 2

questions. This is an example of Hamming code for error correction and can be related to everyday life problems.

## <span id="page-6-1"></span>**4. OUR EXPERIENCE**

We first presented this activity in February 2021 to two groups of Italian 4th grade primary school students (9 years old). In such an occasion, pupils were in the classroom while the tutors were connected via web conference. The activity "Encrypting and decrypting secret messages" has been conducted following the steps described in section [3.2.1.](#page-1-0) The game "7 questions and 1 lie" was proposed and pupils got totally involved by its magic. We realized that the question "how does it works?" naturally arose, but we did not have the tools to explain to primary school children the inner mechanism behind the trick. This first experience made us realize the importance of increasing the involvement of the students in the discovery of the rules associated to the coloured scheme. That is the reason why we added "step 1: introducing the main tool" as described in section [3.2.2](#page-3-0) and designed the learning object "Ora indovina tu" ("Now you guess") you can find in the module [\[3\]](#page-7-2). We also developed the section on binary code in the document [\[2\]](#page-7-1), linking the activities available in the WIMS module [\[21\]](#page-8-12).

We then proposed the activity in April 2021 to four classes of Italian primary school pupils, for a total of 74 pupils in 3rd and 4th grade. As in the previous experience, the tutor was connected via web conference while the students were in their classrooms, supported by their teachers. Such a setup allowed the tutor to share the interactive pages of WIMS on screen and the pupils to work in pairs and to use physical tools: a paper-made double wheel (as described in section [3.2.1,](#page-1-0) Step 2) and a form with the blank scheme introduced in section [3.2.2,](#page-3-0) Step 1. The activity was proposed following the steps outlined in section [3.2](#page-1-1) and took place in two distinct meetings, one dedicated to the tasks of encrypting and decrypting secret messages (section [3.2.1\)](#page-1-0) and one to the game "7 questions and 1 lie" (section [3.2.2\)](#page-3-0). At the end of the project, the students were asked to give written feedback answering the questions "What did you learn today?" and "Do you have doubts or curiosities after today's meeting?".

We built our analysis based on direct observations during the activities and a detailed analysis of these feedback.

The pupils showed great enthusiasm toward the various activities, as testified by the frequent use of expressions like *fun*, *very interesting*, *beautiful* in their feedback. They were engaged in the tasks proposed and eager to share their opinions and discoveries. This was particularly true for the task "Step 2: collecting observations" of the activity "7 questions and 1 lie": the students discovered the fundamental rules by themselves, and they also shared many more. For instance, they noticed that the number 0 corresponded to a negative answer to questions 1 and 4 and did not appear in any of the other questions: this was the reason why its scheme was completely blank. The opposite worked for the number 15. It's important to highlight the students' ability to not only describe a behaviour, but also to explain the reasons behind it. Our proposal therefore helped in the stimulation of the skill of argumentation, an example of mathematical higher-level thinking.

The activity of encrypting and decrypting secret messages produced similar outcomes, especially concerning Step 4: students were competing with their peers to discover the secret key first and suggested clever strategies to do that, without any prompt from the tutor. They showed in this sense their ability in recognizing patterns in long texts and in using them to solve the problem of decrypting. It is relevant that they came up with the idea of considering how often certain letters appear, informally applying the cryptography technique of frequency analysis.

Another benefit of using a game environment for our activities is the fact that the students felt free to try their solutions, without the fear of committing mistakes, and to cooperate as a group to find the right answers together. This attitude led to interesting exchanges of ideas. For instance, in Step 2 of the game "7 questions and 1 lie", one student made a mistake in describing his way of colouring the scheme

corresponding to the number 5 (indicating as coloured a circle that was supposed to remain blank). Immediately another student raised her hand to attract the attention of tutor and teachers. She firmly declared that her friend must have made a mistake because she also had the number 5 assigned, but her coloured scheme was different. In saying this, the pupil showed that she had already understood that each number corresponded to a unique filled scheme, observation not obvious a priori. That's an example of how a playful, and so relaxed, environment encourages higher-level thinking.

Analyzing the students' feedback, we can also deduce that our activity had an impact on their view of mathematics. Some students wrote "Today I learnt that you can play with numbers" or "Today I discovered that numbers can be fun". Also, one student recognized the usefulness of mathematics, stating that she was very happy to have learnt a way of exchanging secret messages with her friends. Finally, pupils shared their desire to know more about the subject. For example, referring to the game "7 questions and 1 lie", one student asked if it was possible to play the same game with more questions or more lies. This apparently simple requirement reveals an attempt of generalization and is actually connected to a deep mathematics question.

## **5. CONCLUSIONS**

The experience we described allows us to conclude that the activities we have designed can form a solid foundation for building student-centered activities that are engaging and meaningful for primary school children. As outlined in section [4.,](#page-6-1) our proposal achieved the goals of engaging students, increasing their motivation, improving their attitude towards mathematics and stimulating higher-level thinking. Also, our activity offers further evidence of the benefits of introducing cryptography in primary school, especially for designing PBL laboratory sessions. The strength of the virtual proposal through WIMS is the possibility to integrate computer-based activities with pen and paper ones, thus involving multiple skills.

Our aim is to support teachers with ready to use examples, and we hope that the materials available in the WIMS distribution (the module [\[3\]](#page-7-2), currently available in Italian and French, the modules [\[20\]](#page-8-11), and [\[21\]](#page-8-12) currently available in Italian, English and French, and the document [\[2\]](#page-7-1), currently available in Italian) can prove to be useful tools for them. Also, we hope we have shown how game-based student centered activities can indeed improve mathematics education. Our next step will be to organize training courses for teachers, in order to make them capable of fully exploiting our digital proposal, e.g. by setting up their own virtual space, organizing virtual experiences for their pupils and monitor their progresses.

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