ACC 92

Identification of a Distributed Parameter (Conductivity) in a System Governed by a Parabolic Equation: Uniqueness and Stability Results

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INTRODUCTION

The *distributed parameter system*

conductivity ↓   ↓ *potential*   ↓ *source term*

$$\frac{\partial}{\partial x}\left[a(x)\ \frac{\partial}{\partial x}u(x,\ t)\right] = \frac{\partial}{\partial t}u(x,\ t) + f(x,\ t) \text{ in } D \times T$$

IC, BC

An *inverse problem*

$$\{\ u,\ f\ \} \overset{?}{\longmapsto} a$$

Why inverse problems ?

accessibility, ease of measurement, ...

Difficulties

The ODE for the unknown conductivity:

$$a'(x)\frac{\partial}{\partial x}u(x,\ t) + a(x)\ \frac{\partial^2}{\partial x^2}u(x,\ t) = g(x,\ t) \text{ in } D \times T$$

Under– *vs.* overspecification => (non–)existence, (non–)uniqueness

Key issues:

existence, uniqueness, stability

How to attain *uniqueness*, given existence ?

Supply a Cauchy datum to the ODE

What is *stability* (of the unique solution) ?

Given $a(u,\ f)$, $b(v,\ f)$,

i) find suitable Banach spaces $\mathbb{X}$ (data), $\mathbb{Z}$ (parameters),

ii) try to relate $\| v - u \|_{\mathbb{X}}$ to $\| b - a \|_{\mathbb{Z}}$ by

$$\| b - a \|_{\mathbb{Z}} \leq const.\ \| v - u \|_{\mathbb{X}}$$

DEFINITIONS AND PROBLEM STATEMENT

State equation:

$$Q := (x_0, x_1) \times (t_0, t_1) = D \times T$$

$$(au_x)_x =_{\text{d.w.}} u_t + f \qquad \text{(d.w. = distribution – wise)}$$

$$u(x_i, t) = u_i(t),\ i = 0,\ 1;\ \text{IC}$$

The inverse problem:

given

$$f \in C^0(\bar{T}; H^{-1}(D)) \ , \qquad \textit{(source term)}$$

$$u \in \mathbb{X} \qquad \textit{(potential)}$$

find

$$a \in \mathbb{A}_{ad} \qquad \textit{(conductivity)}$$

s.t.,

$$(au_x)_x =_{\text{d.w.}} u_t + f \ \text{ in } Q$$

where

$$\mathbb{X} := C^0(\bar{T}; H^2(D)) \cap C^1(\bar{T}; L^2(D))$$

$$\mathbb{A}_{ad} := \{\, a \mid a \in L^\infty(D),\ 0 < a_L = \operatorname*{ess\,inf}_D a\ ;\ \operatorname*{ess\,sup}_D a = a_H \,\}$$

Hp. $\{ u, f \} \Rightarrow \exists\ \hat{a} \in \mathbb{A}_{ad}$ *(reference solution)*

Defect equation:

$$(Bv_x)_x =_{\text{d.w.}} V_t - (aV_x)_x\ ,\ \forall\ t \in \bar{T}$$

where

$$V := v - u\ \ ;\ \ u, v \in \mathbb{X}$$

$$B := b(\, v, f\,) - a(\, u, f\,)$$

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ASIDE:

LESS REGULAR DATA PAIRS (*UNIQUENESS* CONDITIONS ONLY)

$f \in L^2(T; H^{-1}(D))$, (*source term*)

$u \in \mathbb{U}_{ad}$ (*potential*)

where

$$\mathbb{U}_{ad} := \{u \mid u \in L^2(T; H^1(D));\ u_t \in L^2(T; H^{-1}(D));\ u_x \in C^0(\bar{T}; C^0(\bar{D} \setminus S_{u(t)}))\}$$

the set of points where u_x discontinuous is

$$S_u(t) := \{\ x(t) \mid x(t) \in D,\ x(.) \in C^0(\bar{T})\ \};$$

$$\text{meas}[S_u(t)] \overset{Hp}{=} 0,\ \forall t \in T$$

UNIQUENESS CONDITIONS:
1 – GENERAL

General property:

Uniqueness of a is due to a Cauchy datum for the defect equation (an ODE w.r. to a)

Classification

	Cauchy datum @ regular point	*
EXTRINSIC	*independent* $\{ u, f \}$ data pairs	
	zoning i.e., $\frac{da}{dx} = 0$ somewhere	*
	set of pts., where $\frac{\partial u}{\partial x}(\, . \, , t \,) = 0$	*
INTRINSIC	*self–identifiability*	*
	identity principle	