



Intrinsic charm in a matched general-mass scheme



Richard D. Ball^a, Valerio Bertone^{b,c}, Marco Bonvini^c, Stefano Forte^{d,*},
Patrick Groth Merrild^a, Juan Rojo^c, Luca Rottoli^c

^a The Higgs Centre for Theoretical Physics, University of Edinburgh, JCMB, KB, Mayfield Rd., Edinburgh EH9 3JZ, Scotland, United Kingdom

^b PH Department, TH Unit, CERN, CH-1211 Geneva 23, Switzerland

^c Rudolf Peierls Centre for Theoretical Physics, 1 Keble Road, University of Oxford, OX1 3NP Oxford, United Kingdom

^d TIF Lab, Dipartimento di Fisica, Università di Milano and INFN, Sezione di Milano, Via Celoria 16, I-20133 Milano, Italy

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ABSTRACT

The FONLL general-mass variable-flavour number scheme provides a framework for the matching of a calculation in which a heavy quark is treated as a massless parton to one in which the mass dependence is retained throughout. We describe how the usual formulation of FONLL can be extended in such a way that the heavy quark parton distribution functions are freely parameterized at some initial scale, rather than being generated entirely perturbatively. We specifically consider the case of deep-inelastic scattering, in view of applications to PDF determination, and the possible impact of a fitted charm quark distribution on F_2^c is assessed.

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In the perturbative computation of hard processes involving heavy quarks, it is usually assumed that the heavy quark content of colliding hadrons is generated perturbatively, namely, heavy quarks are generated from radiation by light partons. This assumption may be unsatisfactory both for reasons of principle and of practice. As a matter of principle, an “intrinsic” heavy quark component [1] may well be non-zero, especially in the case of charm (see Ref. [2] for a recent review). Such intrinsic charm component might have observable consequences at the LHC in processes like $\gamma + c$ [3,4] or open charm production [5]. Also, in practice, if the heavy quark is assigned a parton distribution (PDF), as required for accurate collider phenomenology [6,7], and this PDF is generated perturbatively, it will in general depend on the choice of scale at which the perturbative boundary condition is imposed. In a matched calculation this dependence will disappear at high enough perturbative orders, but at low orders it might be non-negligible in practice.

Both problems are solved by introducing a fitted heavy quark PDF, which can describe a possible non-perturbative intrinsic component, and also, reabsorb in the initial condition the dependence on the choice of starting scale of the perturbative component. It is the purpose of the present paper to explain how the so-called FONLL approach of Ref. [8], for the treatment of heavy quarks with

inclusion both of mass dependence, and resummation of collinear logs, can be generalized to include such a fitted heavy quark PDF.

The FONLL approach, originally proposed in Ref. [9], specifically applied there to heavy quark production in hadronic collisions, and generalized to deep-inelastic scattering in Ref. [8] (and more recently to Higgs production in bottom quark fusion [10]), is a general-mass, variable-flavour number (GM-VFN) scheme. Such schemes are designed to deal with the fact that hard processes involving heavy quarks can be computed in perturbative QCD using different renormalization and factorization schemes: a massive, or decoupling scheme, in which the heavy quark does not contribute to the running of α_s or the DGLAP evolution equation, and it appears as a massive field in the computation of hard cross-sections; and a massless scheme, in which the heavy quark is treated as a massless parton. In the former scheme, the mass dependence of the hard cross-section is kept, but logs of the hard scale over the heavy quark mass are only included to finite order, while in the latter scheme these logs are resummed to all orders to some logarithmic accuracy through perturbative evolution, but heavy quark mass effects are neglected. For simplicity, we will henceforth refer to the former as a three-flavour scheme (3FS), and to the latter as a four-flavour scheme (4FS), which we will respectively take as synonyms of massive and massless scheme (though all we say would apply equally to four and five, or five and six flavours).

In GM-VFN schemes, the information contained in the three- and four-flavour schemes is combined through a suitable matching

* Corresponding author.

E-mail address: forte@mi.infn.it (S. Forte).

procedure. As the problem arises each time a heavy-quark threshold is crossed, the matching is performed each time this happens. The FONLL scheme has the dual advantage that it can be generally applied to any hard electro- or hadro-production process, and also, that it allows for the combination of a three- and four-flavour computations each performed at any desired perturbative order (fixed, in the former case, and logarithmically resummed, in the latter). Further GM-VFN schemes include ACOT [11–13] (and its variants S-ACOT [14] and S-ACOT- χ [15]) and TR [16,17] (and its variant TR' [18,6]), both of which have been mostly developed in the context of deep-inelastic scattering (see Ref. [19] for detailed comparisons), though applications of GM-VFN schemes to LHC processes have been presented very recently [20,21].

The possibility of including an intrinsic charm component in a global PDF fit has been considered previously by the CT Collaboration [22,23] in the context of the ACOT scheme. In these references, however, the fitted PDF was only introduced in massless contributions. This may bias phenomenological conclusions, and, perhaps more importantly, it does not allow for a fully consistent treatment of the interplay between the fitted charm contribution to the fixed-order and resummed computations. A determination of intrinsic charm which instead only uses the fixed-flavour number scheme (i.e. a 3FS throughout) has been presented recently in Ref. [24].

The basic idea of the FONLL method consists of expanding out the massless-scheme computation in powers of the strong coupling α_s , and replacing a finite number of terms with their massive-scheme counterparts. The result then has at the massive level the fixed-order accuracy which corresponds to the number of massive orders which have been included (“FO”, standing for fixed order), and at the massless level the same logarithmic accuracy as the starting 4FS computation (“NLL”, standing for next^k-to-leading logarithmic¹). The only technical complication of the method is that the starting three- and four-flavour scheme computations are performed in different renormalization and factorization schemes. The difficulty is overcome by re-expressing both α_s and the PDFs of the 3FS (which thus have $n_f = 3$ in the running of α_s and the evolution of PDFs) in terms of those of the 4FS. This must be done order by order in perturbation theory, to the desired accuracy of the computation.

The generic form of deep-inelastic structure functions in the FONLL approach is

$$F(x, Q^2) = F^{(3)}(x, Q^2) + F^{(4)}(x, Q^2) - F^{(3,0)}(x, Q^2), \quad (1)$$

where the three- and four-flavour scheme structure functions are respectively given by

$$\begin{aligned} F^{(3)}(x, Q^2) &= x \int_x^1 \frac{dy}{y} \sum_{i=g,q,\bar{q}} C_i^{(3)} \left(\frac{x}{y}, \frac{Q^2}{m_h^2}, \alpha_s^{(3)}(Q^2) \right) f_i^{(3)}(y, Q^2) \\ &= \sum_{i=g,q,\bar{q}} C_i^{(3)} \left(\frac{Q^2}{m_h^2}, \alpha_s^{(3)}(Q^2) \right) \otimes f_i^{(3)}(Q^2), \end{aligned} \quad (2)$$

$$\begin{aligned} F^{(4)}(x, Q^2) &= x \int_x^1 \frac{dy}{y} \sum_{i=g,q,\bar{q},h,\bar{h}} C_i^{(4)} \left(\frac{x}{y}, \alpha_s^{(4)}(Q^2) \right) f_i^{(4)}(y, Q^2) \\ &= \sum_{i=g,q,\bar{q},h,\bar{h}} C_i^{(4)} \left(\alpha_s^{(4)}(Q^2) \right) \otimes f_i^{(4)}(Q^2), \end{aligned} \quad (3)$$

¹ The name ‘FONLL’ is perhaps a misnomer, as it suggests that the resummed calculation is necessarily NLL, while in actual fact it can be performed at any logarithmic order, but we stick to it for historical reasons.

in terms of hard coefficient functions C_i and PDFs f_i , and it is understood that, in Eq. (1), in the 3FS contributions, PDFs and α_s are re-expressed perturbatively in terms of their 4FS counterparts. The structure function $F^{(3,0)}(x, Q^2)$ is the sum of all the contributions to the massless-scheme computation which are already contained in the massive-scheme one. Its subtraction thus avoid double counting of these contributions, which are contained both in the four-flavor expression $F^{(4)}(x, Q^2)$, from which they can be extracted by expanding in powers of the strong coupling, and in the three-flavor expression $F^{(3)}(x, Q^2)$, where they correspond to the sum of all contributions which do not vanish as the heavy quark mass tends to zero, namely constants, and collinear logarithmic terms of the form $\ln Q^2/m_h^2$, which in $F^{(4)}(x, Q^2)$ are present as a consequence of perturbative evolution of the PDFs.

The decomposition Eq. (1) holds generically for all structure functions: F_2 , F_1 , F_3 , neutral current and charged current. It is useful to decompose the structure function Eq. (1) in a “heavy” and “light” component

$$F(x, Q^2) = F_h(x, Q^2) + F_l(x, Q^2), \quad (4)$$

defined respectively as the contribution to $F(x, Q^2)$ which survives if only the electric (or weak) charge of the heavy quark is non-zero, or that which survives if the electric and weak charge of the heavy quark vanishes. The expressions Eq. (1) then separately apply to F_h and F_l .

In the remainder of this paper we will consider the structure function Eq. (1) as our basic hard observable. The generalization to other observables, and specifically to hadronic processes will in general require a relabelling of perturbative orders. Indeed, in general, the perturbative order at which the 3FS and 4FS cross-sections start being non-zero is process dependent, and thus so is what one calls leading, next-to-leading, and so on.

As mentioned, an advantage of the FONLL method is that the perturbative order at which heavy quark terms are included in $F^{(3)}(x, Q^2)$ and $F^{(4)}(x, Q^2)$ can be chosen freely. In Ref. [8] (to which we refer for more details) in particular, three cases were considered explicitly: FONLL-A, in which $F^{(3)}(x, Q^2)$ is computed up to order α_s , while $F^{(4)}(x, Q^2)$ is determined up to the next-to-leading log (NLL) level; FONLL-B in which $F^{(3)}(x, Q^2)$ is to order α_s^2 and $F^{(4)}(x, Q^2)$ is up to NLL; and FONLL-C in which $F^{(3)}(x, Q^2)$ is to order α_s^2 and $F^{(4)}(x, Q^2)$ is up to NNLL.

If we wish to include a fitted heavy quark PDF, both $F^{(3)}(x, Q^2)$ and $F^{(4)}(x, Q^2)$ must be modified. The modification of the 4FS expression is straightforward. In the absence of a fitted heavy quark PDFs, the 4FS scheme heavy-quark (and antiquark) PDFs are completely determined by perturbative evolution from a vanishing boundary condition at a scale of the order of the quark mass. For simplicity, and in analogy to Ref. [8], in this work we set this scale equal to the quark mass itself. Then, for all $Q^2 > m_h^2$, $f_h^{(4)}(x, Q^2)$ and $f_{\bar{h}}^{(4)}(x, Q^2)$ satisfy perturbative evolution with $n_f = 4$, with the boundary condition at $Q^2 = m_h^2$ determined by standard VFN matching to $f_h^{(3)}(x, m_h^2) = f_{\bar{h}}^{(3)}(x, m_h^2) = 0$. With a fitted heavy quark PDF the vanishing condition is relaxed, and $f_h^{(4)}(x, Q_0^2)$ and $f_{\bar{h}}^{(4)}(x, Q_0^2)$, with $Q_0 \sim m_h$, are just given by some parameterization, with parameters to be determined by comparing to experimental data.

In the presence of a fitted heavy quark component, the 3FS heavy PDFs $f_h^{(3)}(x, m_h^2)$ and $f_{\bar{h}}^{(3)}(x, m_h^2)$ are thus non-zero, and heavy quark PDFs must then be introduced for consistency, at all scales. Because in this scheme the heavy quark is treated as a massive object which decouples from renormalization-group equations, these PDFs are scale independent.

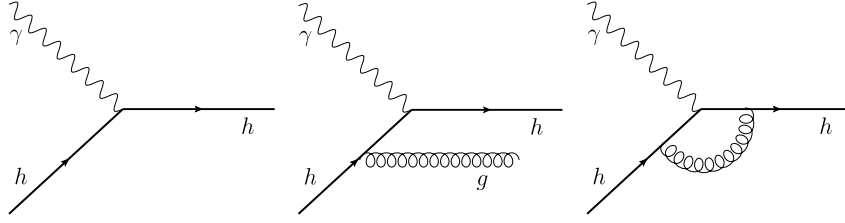


Fig. 1. Representative Feynman diagrams for the contributions to the heavy $F_h(x, Q^2)$ structure function induced by a heavy quark PDF. The fermion line represents the heavy quark. From left to right, the LO diagram and the NLO real and virtual diagrams are shown.

When a fitted heavy quark PDF is introduced, the expression of the 4FS structure functions Eq. (3) is thus unchanged: the only change is in the boundary condition satisfied by the heavy quark PDFs when solving the evolution equations. The expression of the 3FS structure functions Eq. (2) instead does change, because new contributions to the structure function arise, namely those with a heavy quark in the initial state.

Specifically, decomposing the structure functions into a heavy and light contribution according to Eq. (4), the heavy structure functions F_h now receive a contribution from $f_h^{(3)}$ and $f_{\bar{h}}^{(3)}$ which starts at $\mathcal{O}(\alpha_s^0)$ (i.e. at the parton-model level). The corresponding coefficient functions have been computed, both in the neutral- and charged-current sector, in Refs. [25,26], up to $\mathcal{O}(\alpha_s)$. The corresponding Feynman diagrams are shown in Fig. 1. The light structure functions F_l instead receive a contribution which starts at $\mathcal{O}(\alpha_s^2)$. Because heavy-quark initiated massive contributions are only known up to $\mathcal{O}(\alpha_s^2)$ corrections, the highest accuracy which can be achieved at present in the inclusion of a fitted heavy quark is FONLL-A.

We thus define a correction term $\Delta F(x, Q^2)$, which must be added to the standard FONLL structure functions $F^{\text{FLNR}}(x, Q^2)$ of Ref. [8] in order to account for the inclusion of a fitted heavy quark PDF: the structure function Eq. (1) is thus now given by

$$F(x, Q^2) = F^{\text{FLNR}}(x, Q^2) + \Delta F(x, Q^2). \quad (5)$$

Because the 4FS expressions are unaffected by the correction, only $F^{(3)}(x, Q^2)$ and $F^{(3,0)}(x, Q^2)$ contribute to $\Delta F(x, Q^2)$, and thus up to $\mathcal{O}(\alpha_s)$ we find

$$\begin{aligned} \Delta F_h(x, Q^2) = & \sum_{i=h, \bar{h}} \left[C_i^{(3)} \left(\frac{Q^2}{m_h^2}, \alpha_s^{(3)}(Q^2) \right) \right. \\ & \left. - C_i^{(3,0)} \left(\frac{Q^2}{m_h^2}, \alpha_s^{(3)}(Q^2) \right) \right] \otimes f_i^{(3)} \end{aligned} \quad (6)$$

(at higher orders, further terms due to operator mixing would contribute to the difference). Note that $f_h^{(3)}$ and $f_{\bar{h}}^{(3)}$ are scale-independent.

The actual FONLL expression is obtained by re-expressing the coupling and PDFs in the 3FS contribution to Eq. (1) in terms of their 4FS counterparts. This is done by first, matching the coupling and PDFs at some fixed scale, and then, evolving them in the respective schemes. Matching at the heavy quark mass we have

$$\begin{aligned} \alpha_s^{(4)}(m_h^2) &= \alpha_s^{(3)}(m_h^2) + \mathcal{O}(\alpha_s^3), \\ f_i^{(4)}(m_h^2) &= \sum_j K_{ij}(m_h^2) \otimes f_j^{(3)}(m_h^2), \quad i, j = q, \bar{q}, g, h, \bar{h}. \end{aligned} \quad (7)$$

The matching functions $K_{ij}(m_h^2) = \sum_n \alpha_s^n K_{ij}^{(n)}(m_h^2)$ at zeroth order are of course $K_{ij}^{(0)} = \delta_{ij}$. For $i, j = q, \bar{q}, g$, they start receiving non-trivial contributions at $\mathcal{O}(\alpha_s^2)$, accounting for a two-loop normalization mismatch between quark and gluon operators in the

three- and four-flavour schemes (due to the different number of quark flavours circulating in loops) first determined in Ref. [27]. The $K_{hi}(m_h^2)$ functions, with $i = q, \bar{q}, g$, start at $\mathcal{O}(\alpha_s^2)$: in the absence of a fitted quark contribution one may actually express the 4FS heavy quark PDF in terms of massless partons, thus avoiding their explicit use [8]. The $K_{ih}(m_h^2)$ functions are irrelevant in the absence of intrinsic charm and are discussed here in the context of FONLL for the first time: $K_{hh}(m_h^2)$ already receives non-trivial corrections at $\mathcal{O}(\alpha_s)$, while $K_{gh}(m_h^2)$ starts at $\mathcal{O}(\alpha_s)$, and $K_{qh}(m_h^2)$ starts at higher orders. It follows that in the absence of a fitted charm component, all matching conditions coincide with the trivial zeroth-order ones up to and including $\mathcal{O}(\alpha_s)$ (FONLL-A), while to $\mathcal{O}(\alpha_s^2)$ (FONLL-B or FONLL-C) knowledge of the $\mathcal{O}(\alpha_s^2)$ contribution to $K_{qq}(m_h^2)$ is sufficient [8]. In the presence of intrinsic charm, they are already non-trivial at $\mathcal{O}(\alpha_s)$ (FONLL-A), where knowledge of the $\mathcal{O}(\alpha_s)$ correction to $K_{hh}(m_h^2)$ is required. Its explicit expression can be extracted from the known $\mathcal{O}(\alpha_s)$ massive coefficient functions of Refs. [25,26], and is given in the Appendix (see Eqs. (20) below). In order to upgrade to $\mathcal{O}(\alpha_s^2)$ (FONLL-B or FONLL-C), the yet unknown $\mathcal{O}(\alpha_s^2)$ correction to $K_{hh}(m_h^2)$ as well as the (known) $\mathcal{O}(\alpha_s)$ contribution to $K_{gh}(m_h^2)$ would also be required.

Evolving both the three-flavour and four-flavour quantities in the respective schemes one can turn Eq. (7) into matching conditions at any scale Q^2 : this then defines a matching matrix $K_{ij}(Q^2)$; of course this will then generate logarithmic contributions to all matching functions starting at $\mathcal{O}(\alpha_s)$.

In particular, the matching condition satisfied by the heavy quark PDF at a generic scale Q^2 is found recalling that in the 3FS the heavy quark distribution does not evolve: up to $\mathcal{O}(\alpha_s)$ one then gets

$$\begin{aligned} f_h^{(3)} &= f_h^{(4)}(Q^2) - \alpha_s^{(4)}(Q^2) \left(K_{hh}^{(1)}(m_h^2) + P_{qq}^{(0)}L \right) \otimes f_h^{(4)}(Q^2) \\ &\quad - \alpha_s^{(4)}(Q^2) L P_{qg}^{(0)} \otimes g^{(4)}(Q^2) + \mathcal{O}(\alpha_s^2), \end{aligned} \quad (8)$$

where (following Ref. [8]) we have defined $L \equiv \ln \frac{Q^2}{m_h^2}$ and $P_{ij}^{(0)}(z)$ are the usual leading-order splitting functions. Note that, whereas the 3FS heavy PDF $f_h^{(3)}$ is scale independent, in practice in Eq. (8) $K_{hh}(Q^2)$ is expanded out perturbatively and only terms up to $\mathcal{O}(\alpha_s)$ are kept, thereby inducing a subleading dependence on the scale Q^2 when $f_h^{(3)}$ is expressed in terms of the 4FS PDFs.

We can finally get a simple, explicit expression for $\Delta F(x, Q^2)$ up to $\mathcal{O}(\alpha_s)$ by noting that, because of standard collinear factorization together with the matching conditions Eq. (7), the 3FS coefficient functions in the massless limit are simply related to the 4FS mass-independent coefficient functions:

$$\begin{aligned} C_i^{(3,0),0} \left(\frac{Q^2}{m_h^2} \right) &= C_i^{(4),0}, \\ C_i^{(3,0),1} \left(\frac{Q^2}{m_h^2} \right) &= C_i^{(4),1} + C_i^{(4),0} \otimes \left(K_{hh}^{(1)}(m_h^2) + P_{qq}^{(0)}L \right), \\ &\quad (i = h, \bar{h}), \end{aligned} \quad (9)$$

$$\begin{aligned} C_i^{(3,0),1} \left(\frac{Q^2}{m_h^2} \right) &= C_i^{(4),1} + C_i^{(4),0} \otimes \left(K_{hh}^{(1)}(m_h^2) + P_{qq}^{(0)}L \right), \\ &\quad (i = h, \bar{h}), \end{aligned} \quad (10)$$

where we have defined $C_i = \sum_n C_i^n \alpha_s^n$. Eq. (9) holds for any $\mathcal{O}(\alpha_s^0)$ coefficient function and Eq. (10) holds for $\mathcal{O}(\alpha_s^1)$ heavy quark coefficient functions. Explicit expression for the heavy-quark initiated massive 3FS coefficient functions are collected in the Appendix, while the remaining ones can be found in Ref. [8].

Substituting Eqs. (8)–(10) into Eq. (6) we obtain

$$\begin{aligned} \Delta F_h(x, Q^2) = & \sum_{i=h, \bar{h}} \left\{ \left[\left(C_i^{(3),0} \left(\frac{Q^2}{m_h^2} \right) - C_i^{(4),0} \right) \right. \right. \\ & + \alpha_s^{(4)}(Q^2) \left(C_i^{(3),1} \left(\frac{Q^2}{m_h^2} \right) - C_i^{(4),1} \right) \left. \right] \\ & - \alpha_s^{(4)}(Q^2) C_i^{(3),0} \left(\frac{Q^2}{m_h^2} \right) \otimes \left(K_{hh}^{(1)}(m_h^2) + P_{qq}^{(0)} L \right) \left. \right\} \\ & \otimes f_i^{(4)}(Q^2) \\ & - \alpha_s^{(4)}(Q^2) \sum_{i=h, \bar{h}} \left(C_i^{(3),0} \left(\frac{Q^2}{m_h^2} \right) - C_i^{(4),0} \right) \\ & \otimes P_{qg}^{(0)} L \otimes f_g^{(4)}(Q^2) + \mathcal{O}(\alpha_s^2), \end{aligned} \quad (11)$$

where by $\mathcal{O}(\alpha_s^2)$ we really mean up to subleading terms with respect to FONLL-A (i.e. $\mathcal{O}(\alpha_s^2)$ in the 3FS, and NNLL in the 4FS).

By definition, $\Delta F_h(x, Q^2)$ Eq. (11) is a contribution to the “heavy” component F_h Eq. (4) of the structure function. We list for completeness the remaining contributions to F_h in the FONLL-A scheme, as given in Ref. [8]:

$$\begin{aligned} F_h^{\text{FLNR}}(x, Q^2) = & \sum_{i=h, \bar{h}} \left(C_i^{(4),0} + \alpha_s^{(4)}(Q^2) C_i^{(4),1} \right) \otimes f_i^{(4)}(Q^2) \\ & + \alpha_s^{(4)}(Q^2) C_g^{(4),1} \otimes f_g^{(4)}(Q^2) \\ & + \alpha_s^{(4)}(Q^2) \left(C_g^{(3),1} \left(\frac{Q^2}{m_h^2} \right) - C_g^{(4),1} - \sum_{i=h, \bar{h}} C_i^{(4),0} \otimes P_{qg}^{(0)} L \right) \\ & \otimes f_g^{(4)}(Q^2) + \mathcal{O}(\alpha_s^2) \\ = & \sum_{i=h, \bar{h}} \left(C_i^{(4),0} + \alpha_s^{(4)}(Q^2) C_i^{(4),1} \right) \otimes f_i^{(4)}(Q^2) \\ & + \alpha_s^{(4)}(Q^2) \left(C_g^{(3),1} \left(\frac{Q^2}{m_h^2} \right) - \sum_{i=h, \bar{h}} C_i^{(4),0} \otimes P_{qg}^{(0)} L \right) \\ & \otimes f_g^{(4)}(Q^2) + \mathcal{O}(\alpha_s^2). \end{aligned} \quad (12)$$

Substituting Eq. (11) and Eq. (12) in Eq. (5) provides the final expression of the heavy structure functions in the FONLL-A scheme, with the latter providing the result in the absence of a fitted heavy quark PDF as given in Ref. [8], and the former the correction due to a non-vanishing heavy quark PDF. Note that even if the fitted heavy quark PDFs vanishes (i.e. $f_h^{(4)}(x, m_h^2) = f_{\bar{h}}^{(4)}(x, m_h^2) = 0$), the new contribution $\Delta F_h(x, Q^2)$ Eq. (11), though subleading, does not vanish when $Q^2 > m_h^2$: it only vanishes when $Q^2 = m_h^2$. This is due to the fact that, when re-expressing $f_i^{(3)}(Q^2)$ in terms of $f_i^{(4)}(Q^2)$, only terms up to $\mathcal{O}(\alpha_s)$ were kept, as can be seen from Eq. (8).

This means that, even in the absence of a fitted component, our expressions are not identical to those of Ref. [8], from which they differ by subleading terms. However, we will show below that in

the absence of fitted charm this difference is completely negligible, so that it makes no difference in practice whether one uses Eq. (5), or Eq. (12) as in Ref. [8].

As well known [19], the FONLL-A expression, as given by Eq. (12), coincides with the NLO S-ACOT [14] GM-VFN scheme result. It is easy to show that FONLL-A as given by Eq. (5) coincides with the original NLO ACOT [11,12] scheme. Indeed, note that once Eq. (11) and Eq. (12) are combined into Eq. (5) there is a certain amount of cancellation, and one ends up with the relatively simpler expression for the heavy structure function

$$\begin{aligned} F_h(x, Q^2) = & \sum_{i=h, \bar{h}} \left\{ C_i^{(3),0} \left(\frac{Q^2}{m_h^2} \right) \right. \\ & + \alpha_s^{(4)}(Q^2) \left[C_i^{(3),1} \left(\frac{Q^2}{m_h^2} \right) - C_i^{(3),0} \left(\frac{Q^2}{m_h^2} \right) \right. \\ & \left. \left. \otimes \left(K_{hh}^{(1)}(m_h^2) + P_{qq}^{(0)} L \right) \right] \right\} \otimes f_i^{(4)}(Q^2) \\ & + \alpha_s^{(4)}(Q^2) \left[C_g^{(3),1} \left(\frac{Q^2}{m_h^2} \right) - \sum_{i=h, \bar{h}} C_i^{(3),0} \left(\frac{Q^2}{m_h^2} \right) \otimes P_{qg}^{(0)} L \right] \\ & \otimes f_g^{(4)}(Q^2) + \mathcal{O}(\alpha_s^2). \end{aligned} \quad (13)$$

In plain words, the result reduces to the expression obtained by combining the PDFs $f_i^{(4)}$, evolved in the 4FS, with the massive 3FS coefficient functions $C_i^{(3)}$, and subtracting from the latter the unsummed collinear logarithms. This coincides with the ACOT result.

An interesting feature of our result Eq. (13) is the following. The FONLL expression Eq. (1) can be viewed as the sum of the 3FS expression, and a “difference” contribution

$$F^{(d)}(x, Q^2) = F^{(4)}(x, Q^2) - F^{(3,0)}(x, Q^2), \quad (14)$$

which is in fact subleading with respect to the accuracy of the massive computation: it only contains logarithmic terms beyond the order of the 3FS result. If the new FONLL expression Eq. (5) is adopted the difference term $F^{(d)}(x, Q^2)$ vanishes identically. This is not accidental: it is due to the fact that, when re-expressing the 3FS PDFs in terms of the 4FS ones, Eq. (8), the difference in evolution is only compensated up to $\mathcal{O}(\alpha_s)$. The higher-order collinear logs which would normally contribute to the difference terms are thus also included in the 3FS contribution, and subtracted off.

A consequence of this is that the phenomenologically motivated modification of the FONLL expression by subleading terms (akin to the ACOT- χ [15] modification of ACOT) which was considered in Ref. [8] is no longer possible. This modification had the purpose of leading to $\mathcal{O}(\alpha_s)$ (FONLL-A) results which approximate the full $\mathcal{O}(\alpha_s^2)$ (FONLL-B) result [28]. It consisted of multiplying $F^{(d)}(x, Q^2)$ Eq. (14) by a kinematically motivated function of m_h^2/Q^2 which tends to one in the large Q^2 limit: but this term now vanishes, and thus this modification would have no effect. However, this does not appear to be a limitation, as in the presence of a fitted charm PDF, subleading terms can now be reabsorbed in the initial PDF.

While we have presented so far results only up to $\mathcal{O}(\alpha_s)$ (FONLL-A) accuracy, our discussion is easily generalized to higher orders. Indeed, quite in general, the structure function Eq. (1) has the form

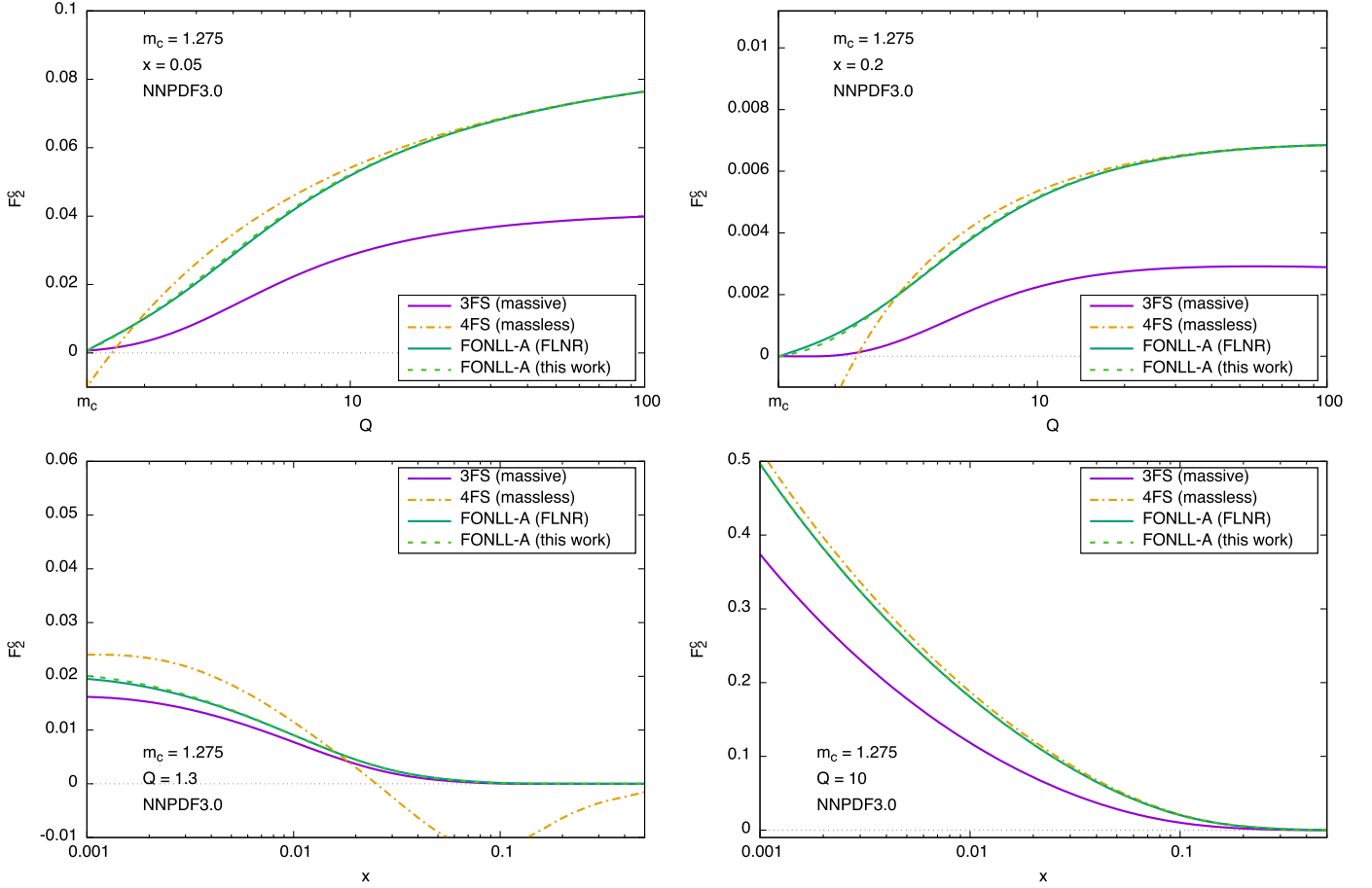


Fig. 2. The charm structure function $F_2^c(x, Q^2)$ in the 3FS to $\mathcal{O}(\alpha_s^3)$, in the 4FS scheme to NLL, and using the FONLL-A matched scheme, in the absence of a fitted charm component. The FONLL implementation of Ref. [8] (labelled FLNR) and the implementation of this paper, which differs from it by subleading terms in the absence of fitted charm, are both shown. Results are shown as a function of Q for $x = 0.05$ and $x = 0.2$, and as a function of x for $Q = 1.3$ GeV and $Q = 10$ GeV.

$$\begin{aligned}
 F(x, Q^2) = & \sum_{i,j=g,q,\bar{q},h,\bar{h}} \left[C_i^{(3)} \left(\frac{Q^2}{m_h^2} \right) - C_i^{(3,0)} \left(\frac{Q^2}{m_h^2} \right) \right] \\
 & \otimes K_{ij}^{-1}(Q^2) \otimes f_j^{(4)}(Q^2) \\
 & + \sum_{i,j=g,q,\bar{q},h,\bar{h}} C_i^{(4)} \otimes f_i^{(4)}(Q^2), \quad (15)
 \end{aligned}$$

where $K_{ij}^{-1}(Q^2)$ is the inverse of the matching matrix Eq. (7), and it is understood that all quantities are re-expressed in terms of $\alpha_s^{(4)}$ and then expanded out to the desired accuracy, with the 4FS PDFs written in terms of a set of PDFs at a reference scale through perturbative evolution in the usual way.

The compact form of Eq. (15) reveals an interesting feature: using the matching conditions Eq. (7) evolved up to a generic scale Q^2 , the second and third terms in Eq. (15) cancel, and one ends up with the very simple expression

$$F(x, Q^2) = \sum_{i,j=g,q,\bar{q},h,\bar{h}} C_i^{(3)} \left(\frac{Q^2}{m_h^2} \right) \otimes K_{ij}^{-1}(Q^2) \otimes f_j^{(4)}(Q^2). \quad (16)$$

This shows explicitly the vanishing of the difference contribution Eq. (14), which thus appears to be an all-order feature of this approach. Higher-order generalizations then simply require the determination of the matching matrix $K_{ij}^{-1}(Q^2)$, its perturbative inversion to the desired order, and the re-expansion of $C_i^{(3)}$ in terms of $\alpha_s^{(4)}$. An all-order proof of Eq. (16) is given in Ref. [29], where

its implications are discussed in detail (see in particular Sect. 3.2 of this reference).

We finally turn to a first assessment of the phenomenological impact of a possible non-vanishing fitted charm component. We consider specifically $F_2^c(x, Q^2)$, the heavy contribution Eq. (4) to the neutral-current DIS structure function $F_2(x, Q^2)$. In the following, results have been obtained using the NNPDF3.0 sets [30], with the corresponding value of the charm mass $m_c = 1.275$ GeV (see Sect. 2.3.4 of Ref. [30]). We have generated the results for FONLL-A and FONLL-B structure functions determined according to the expressions of Refs. [8,19] using APFEL [31,32]. We have then supplemented them with the extra fitted-charm contributions Eq. (11), which was implemented in a new stand-alone public code [33].

For the sake of a first qualitative assessment, we have generated a “fitted” charm component by assuming two different models for the charm PDF $f_c^{(3)}(x) = f_c^{(3)}(x)$, and then matching it to the 4FS expressions for $Q^2 \geq m_c^2$. We specifically model $f_c^{(3)}(x) = f_c^{(3)}(x)$ by using the “intrinsic charm” model of Ref. [1], which we will refer to as the BHPS model:

$$f_c^{(3)}(x) = f_c^{(3)}(x) = A x^2 [6x(1+x) \ln x + (1-x)(1+10x+x^2)]. \quad (17)$$

In this model $f_c^{(3)}(x) = f_c^{(3)}(x)$ is peaked strongly around $x \simeq 0.2$. Alternatively, we consider a scenario, which we refer to as SEA model, in which the shape of the fitted charm is similar to that of all light quark sea PDFs, as one would expect if charm was gener-

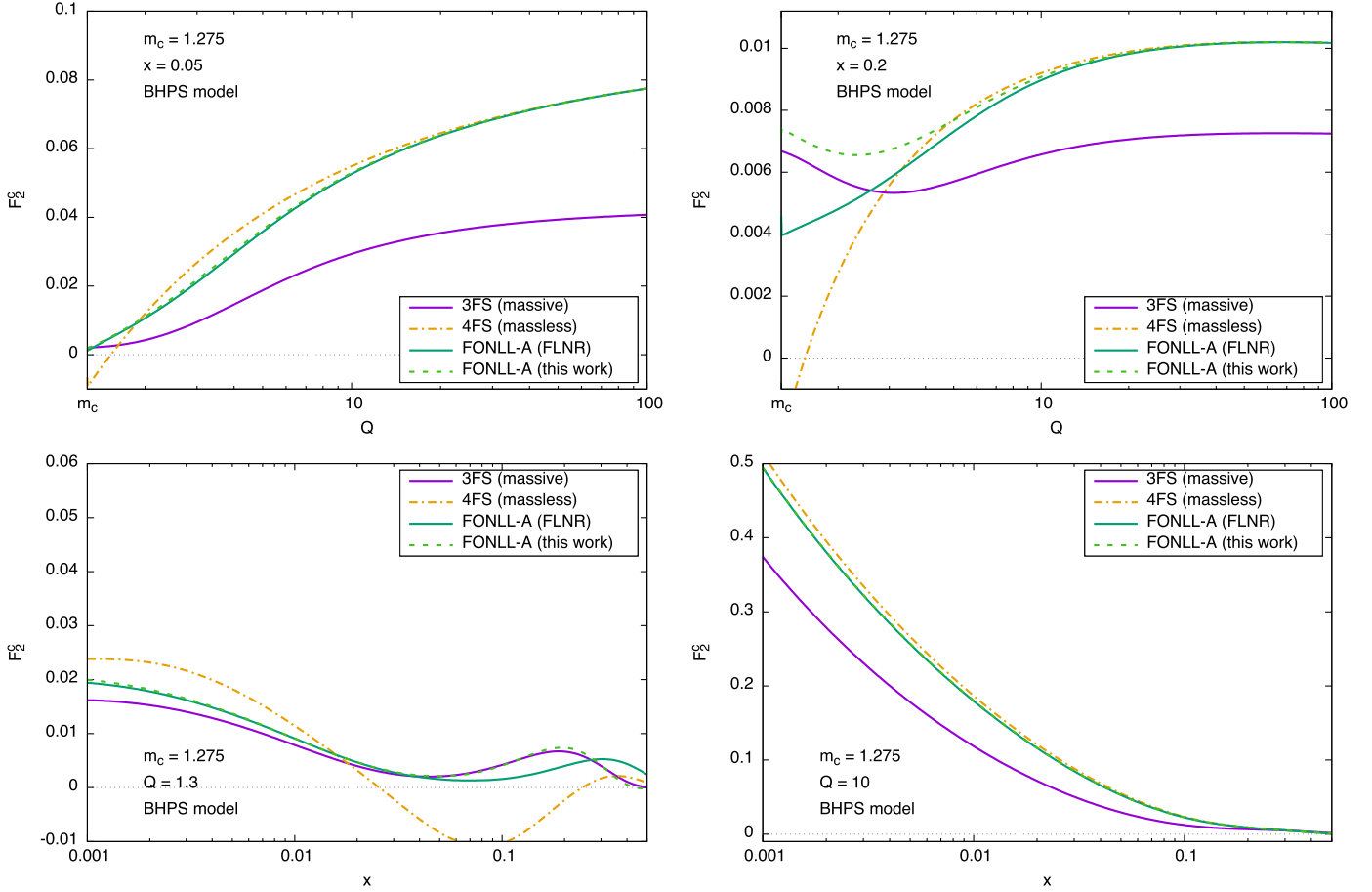


Fig. 3. Same as Fig. 2, but now including a charm component, modelled using the BHPS scenario Eq. (17). The FLNR curve, same as in Fig. 2, shows the result in the absence of charm.

ated by evolution. For illustrative purposes, we thus take

$$f_c^{(3)}(x) = f_{\bar{c}}^{(3)}(x) = A x^{-1.25} (1-x)^3, \quad (18)$$

which has been verified to provide a reasonably good approximation to the sea PDFs of the NNPDF3.0 NLO set.

For both scenarios, BHPS, Eq. (17), and SEA, Eq. (18), we determine the value of the normalization constant A , i.e. the overall size of the fitted charm contribution, by imposing that the momentum fraction carried by it is equal to a fixed amount, which we take to be

$$\langle x \rangle_{c+\bar{c}}(Q_0) \equiv \int_0^1 dx x (f_c^{(3)}(x) + f_{\bar{c}}^{(3)}(x)) = 5 \cdot 10^{-3}, \quad (19)$$

roughly in line with the phenomenological estimate of the CT10IC study [23]. These starting PDFs have then been combined with the gluon and the light quark PDFs from the NNPDF3.0, adjusting the gluon in order to ensure that the momentum sum rule still holds after accounting for Eq. (19), and evolved to all scales using APFEL. Note that in an actual PDF fit, it would be more advantageous to directly parameterize the heavy quark PDF above threshold, in the 4FS.

Before turning to these models, we first check that the modification of the FONLL scheme of Ref. [8], Eq. (11), which is sub-leading in the absence of a fitted charm component, is indeed negligible for all practical purposes, as mentioned above. In Fig. 2, we show, for two different values of Bjorken x as a function of Q and for two different values of Q as a function of x , the charm

structure function $F_2^c(x, Q^2)$ computed to $\mathcal{O}(\alpha_s)$ in the 3FS, Eq. (2) (with PDFs and α_s also in the 3FS), in the 4FS NLL Eq. (3), and using the FONLL-A matched scheme. In the latter case, we both show the original FONLL result Eq. (12) and the new form of FONLL presented here, which includes the extra term $\Delta F_h(x, Q^2)$ Eq. (11). It is clear that the correction is indeed negligible: this means that when the fitted charm component vanishes the FONLL result of Ref. [8] is reproduced.

We now include a fitted charm component. Results are shown in Fig. 3 for the BHPS scenario Eq. (17), and in Fig. 4 for the SEA scenario Eq. (18). For the BHPS scenario, the charm contribution is only significant for large $x \gtrsim 0.08$, and peaks around $x \sim 0.2$, while for the SEA model it provides a non-negligible correction at small x . In both cases, the “fitted” charm component is significant just above threshold, but already for $Q \sim 10$ GeV it is overwhelmed by the perturbatively generated component, and the results of Ref. [8] are reproduced. This suggests that a possible intrinsic component would be most easily revealed in precise measurements close to the charm threshold, with only relatively minor effects on LHC processes.

The FONLL-A scheme is the only one for which a fitted charm PDF can be consistently included, until the $\mathcal{O}(\alpha_s^2)$ charm-initiated massive coefficient functions are computed. However, FONLL-B and FONLL-C provide a rather more accurate description of the low- Q^2 charm structure functions, thanks to the inclusion of $\mathcal{O}(\alpha_s^2)$ gluon and light-quark initiated terms. Therefore, a practical compromise could be to assume that the unknown $\mathcal{O}(\alpha_s^2)$ charm-initiated massive coefficient functions is in practice negligible, and simply add the same correction, Eq. (11), to the standard FONLL-B result. This

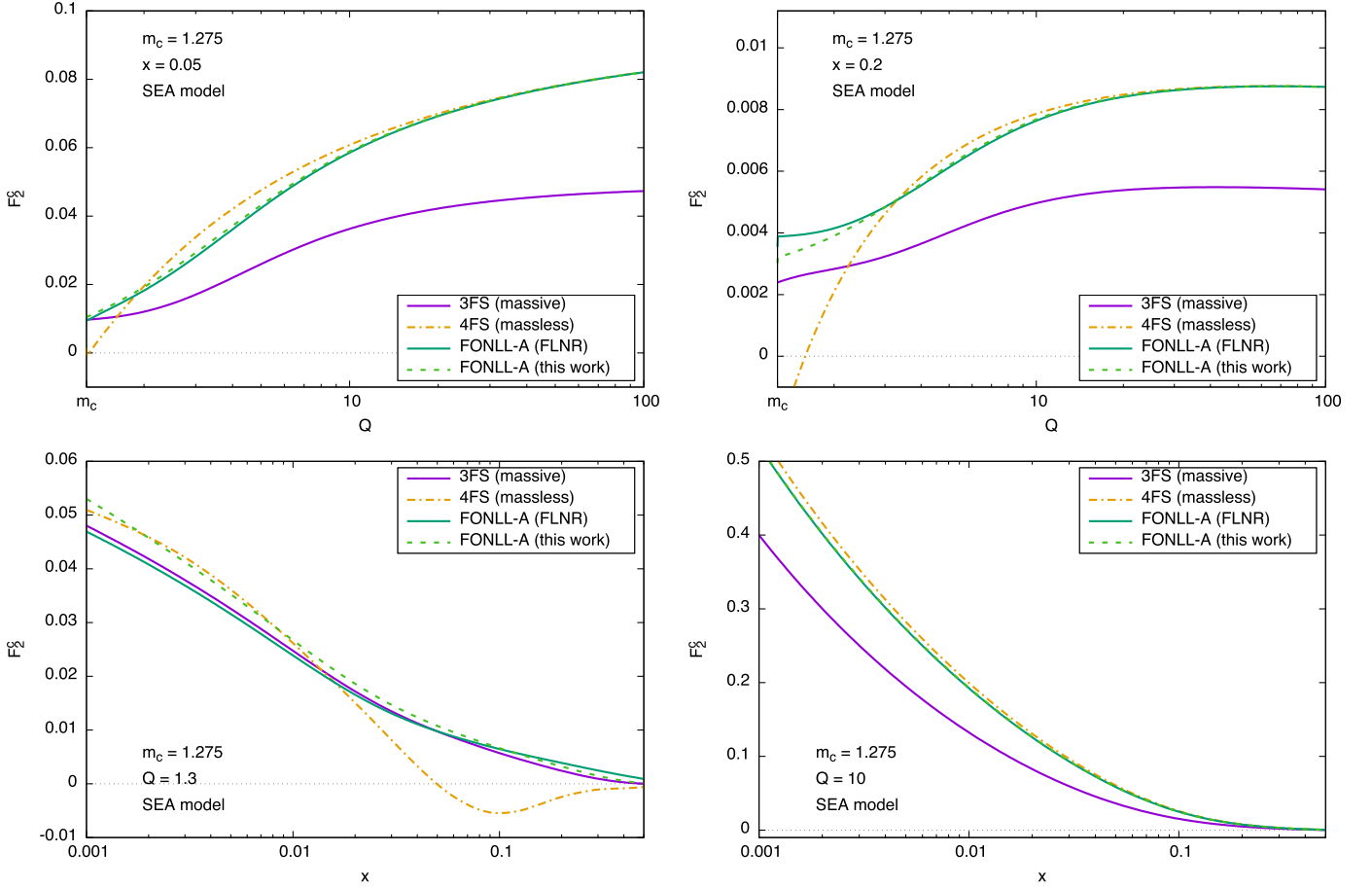


Fig. 4. Same as Fig. 3, but now with the charm component modelled using the SEA scenario Eq. (18).

is likely to be a rather good approximation, given that, as Figs. 3–4 show, the contribution of the “fitted” charm component is actually quite small in all reasonable scenarios. As a final check, we have thus recomputed predictions in the various scenarios in this approximate FONLL-B scheme. Results with the BHPS model are shown in Fig. 5: it is seen that the previous conclusions are qualitatively unchanged.

In summary, in this work we have generalized the FONLL GM-VFN scheme to account for the possibility that the heavy quark PDF can be fitted from the data, rather than being generated perturbatively. The next step will be to use the calculations presented here in a global analysis in the NNPDF framework [34–38], with the goal of determining the charm PDF from the data. This, also thanks to the unbiased NNPDF methodology, will remove the need to resort to ad-hoc modelling, and it will allow us to settle quantitatively a question that has been left open for more than 30 years: is it possible to unambiguously determine the charm content of the proton? This will also allow us to remove any possible bias induced by the hypothesis that charm vanishes at some more or less arbitrary scale, and explore the possible implications of this assumption for precision phenomenology at the LHC, such as for example to the determination of the heavy quark masses.

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Appendix A

We collect the explicit expressions for the additional matching conditions and coefficient functions that are required to generalize the FONLL scheme to the case in which a fitted charm PDF is included, up to FONLL-A accuracy. All other matching and coefficient functions were given in the Appendix to Ref. [8].

The new matching conditions involve the K_{ih} entries of the matching matrix Eq. (7). Up to $\mathcal{O}(\alpha_s)$, only K_{qh} and K_{hh} receive non-trivial contributions, both of which we give for completeness, even though to FONLL-A accuracy only K_{hh} is needed:

$$\begin{aligned}
 K_{hh}(Q^2) &= K_{h\bar{h}}(Q^2) \\
 &= 1 + \alpha_s \left[\bar{P}_{qq}^{(0)}(z) \left(\ln \frac{Q^2}{m_h^2(1-z)^2} - 1 \right) \right]_+ + \mathcal{O}(\alpha_s^2),
 \end{aligned}
 \tag{20a}$$

$$\begin{aligned}
 K_{gh}(Q^2) &= K_{g\bar{h}}(Q^2) = \alpha_s P_{gq}^{(0)}(z) \left(\ln \frac{Q^2}{m_h^2 z^2} - 1 \right) + \mathcal{O}(\alpha_s^2).
 \end{aligned}
 \tag{20b}$$

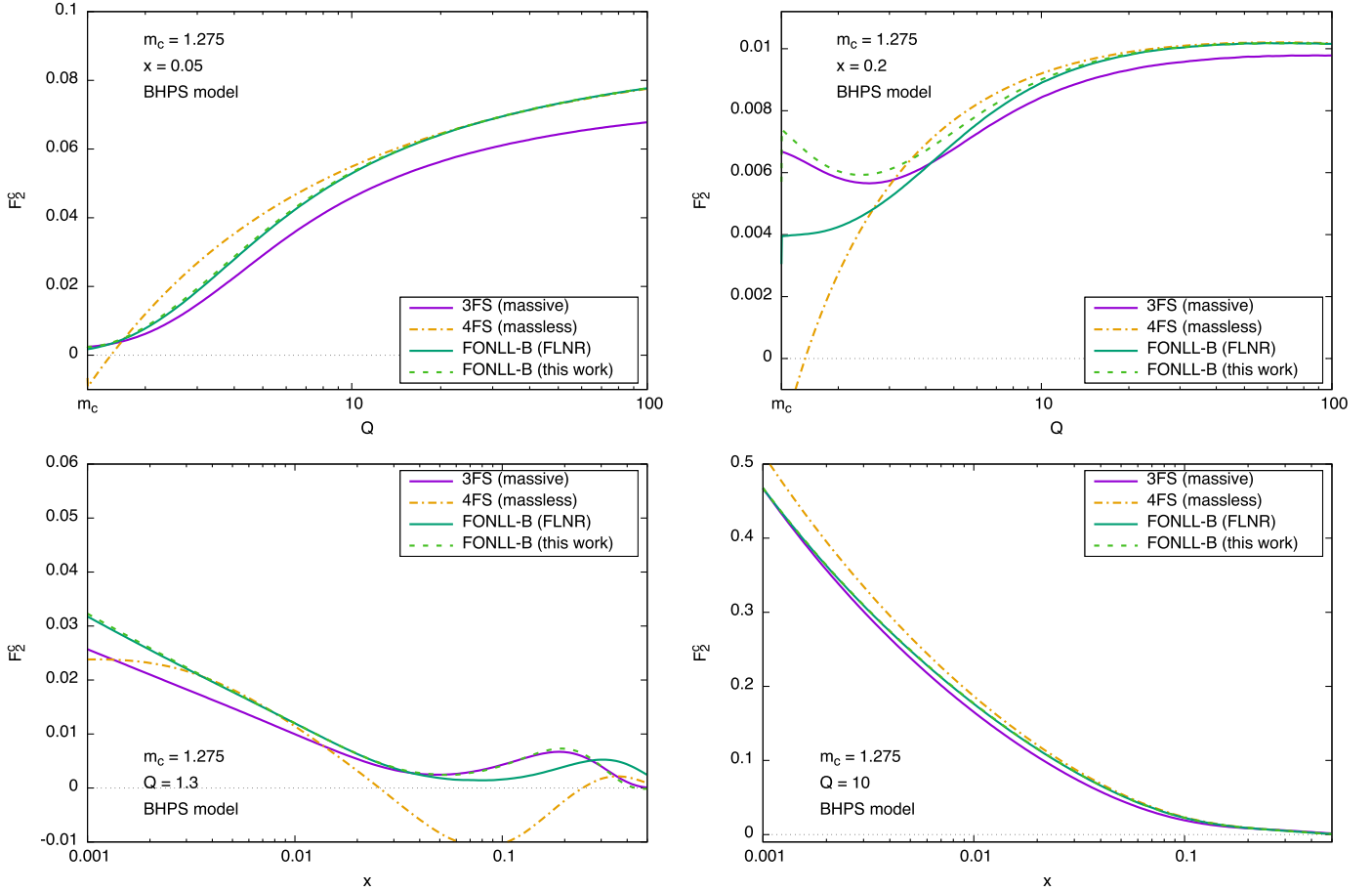


Fig. 5. Same as Fig. 3, but now using with an approximate FONLL-B matched scheme (see text).

In Eq. (20)

$$\bar{P}_{qq}^{(0)}(z) = \frac{C_F}{2\pi} \frac{1+z^2}{1-z}, \quad P_{gq}^{(0)}(z) = \frac{C_F}{2\pi} \frac{1+(1-z)^2}{z}, \quad (21)$$

where $P_{qq}^{(0)}(z) = [\bar{P}_{qq}^{(0)}(z)]_+$. The expressions Eq. (20) can be obtained by combining the matching functions Eq. (7) at $Q = m_h$ with standard perturbative evolution:

$$K_{ij}^{(1)}(Q^2) = K_{ij}^{(1)}(m_h^2) + LP_{ij}^{(0)}, \quad (22)$$

where $P_{ij}^{(0)}$ denote the usual leading-order splitting functions, with $P_{hh}^{(0)} = P_{qq}^{(0)}$ and $P_{gh}^{(0)} = P_{gq}^{(0)}$.

In order to write the charm-initiated massive coefficient functions up to $\mathcal{O}(\alpha_s)$, we introduce a number of useful definitions:

$$\lambda = m_c^2/Q^2, \quad \chi = \frac{x(1 + \sqrt{1+4\lambda})}{2}. \quad (23)$$

The contribution from the subprocess $\gamma^*c \rightarrow c$ to the charm structure function $F_{2,c}^{(3)}(x, Q^2)$ in the massive calculation can be written as

$$F_{2,c}^{(3)}|_{f_c} = x \int_{\chi}^1 \frac{d\xi}{\xi} C_{2,c}^{(3)}\left(\xi, \frac{Q^2}{m_c^2}\right) \left(f_c^{(3)}\left(\frac{\chi}{\xi}, Q^2\right) + f_{\bar{c}}^{(3)}\left(\frac{\chi}{\xi}, Q^2\right) \right), \quad (24)$$

where the $\mathcal{O}(\alpha_s^0)$ and $\mathcal{O}(\alpha_s)$ coefficient functions

$$C_{2,c}^{(3)}\left(\xi, \frac{Q^2}{m_c^2}\right) = C_{2,c}^{(3),0}\left(\xi, \frac{Q^2}{m_c^2}\right) + \alpha_s C_{2,c}^{(3),1}\left(\xi, \frac{Q^2}{m_c^2}\right) + \mathcal{O}(\alpha_s^2), \quad (25)$$

have been computed in Refs. [25,26]. Note that the lower limit of the convolution integral in Eq. (24) is χ , hence a change of variable is needed in order to recover the form Eq. (2) of the convolution. The complete structure function $F_{2,c}^{(3)}(x, Q^2)$ in the massive scheme is constructed by adding Eq. (24) to the corresponding gluon- and light-quark initiated contributions.

At $\mathcal{O}(\alpha_s^0)$ the massive coefficient function for the charm-initiated contribution reads

$$C_{2,c}^{(3),0}\left(\xi, \frac{Q^2}{m_c^2}\right) = e_c^2 \sqrt{1+4\lambda} \delta(1-\xi), \quad (26)$$

whose massless limit, $C_{2,c}^{(3,0),0}$, is given by

$$C_{2,c}^{(3,0),0}\left(\xi, \frac{Q^2}{m_c^2}\right) = e_c^2 \delta(1-\xi), \quad (27)$$

which of course coincides with the leading-order massless quark coefficient function.

At the next order, $\mathcal{O}(\alpha_s)$, it is possible to express the massive coefficient function for the charm-initiated contribution from Ref. [26] as follows:

$$C_{2,c}^{(3),1}\left(\xi, \frac{Q^2}{m_c^2}\right) = \frac{2}{3} \frac{e_c^2}{\pi} \delta(1-\xi) \sqrt{1+4\lambda} \left\{ 4 \ln \lambda - 2 + \sqrt{1+4\lambda} \tilde{L} - 2 \ln(1+4\lambda) \right\}$$

$$\begin{aligned}
& + \frac{1+2\lambda}{\sqrt{1+4\lambda}} [3\tilde{L}^2 + 4\tilde{L} + 4\text{Li}_2(-d/a) + 2\tilde{L} \ln \lambda \\
& - 2\tilde{L} \ln(1+4\lambda) + 2\text{Li}_2(d^2/a^2)] \\
& \times \frac{1}{3} \frac{e_c^2}{\pi} \frac{1}{(1-\xi)_+} \frac{(1-\xi)}{\Delta'^2 \xi^4 \hat{s}_1} \left\{ \frac{1}{2\xi^2} \right. \\
& \times \left[(1-\xi)^2 (1-2\xi-6\xi^2-6\xi^6-2\xi^7+\xi^8) + \lambda(1-\xi)^2 \right. \\
& \times (5+12\xi-115\xi^2-20\xi^3-4\xi^4-20\xi^5-115\xi^6 \\
& + 12\xi^7+5\xi^8) + \lambda^2 (5\xi^{10}+64\xi^9-361\xi^8-8\xi^7 \\
& + 828\xi^6-1072\xi^5+828\xi^4-8\xi^3-361\xi^2+64\xi+5) \\
& + 4\xi\lambda^3 (9+24\xi-202\xi^2-112\xi^3+530\xi^4-112\xi^5 \\
& - 202\xi^6+24\xi^7+9\xi^8) + 32\lambda^4 (\xi^7-10\xi^5+\xi^3) \\
& + \sqrt{4\lambda+1} \left[(1-\xi)^3 (1+\xi)(1-2\xi-5\xi^2-2\xi^3-5\xi^4 \right. \\
& - 2\xi^5+\xi^6) + \lambda (-3\xi^{10}-10\xi^9+132\xi^8-202\xi^7+79\xi^6 \\
& - 79\xi^4+202\xi^3-132\xi^2+10\xi+3) \\
& + \lambda^2 (1-\xi)(1+\xi) (1+36\xi-98\xi^2 \\
& - 376\xi^3+730\xi^4-376\xi^5-98\xi^6+36\xi^7+\xi^8) \\
& + 16\lambda^3 (1-\xi)\xi^2 (1+\xi) (2+\xi-26\xi^2+\xi^3+2\xi^4) \left. \right] \\
& + \frac{\hat{L}}{\Delta'} \left[-(1+\xi^2)^2 (1-\xi^2+\xi^4) \right. \\
& - 4\lambda (1-\xi+6\xi^2-2\xi^4+6\xi^6-\xi^7+\xi^8) \\
& - 2\lambda^2 (1-6\xi+18\xi^2+46\xi^3-54\xi^4+46\xi^5+18\xi^6 \\
& - 6\xi^7+\xi^8) + 16\lambda^3 (\xi^2-10\xi^4+\xi^6) \\
& + \sqrt{4\lambda+1} (\xi^8+\xi^6-\xi^2-1 \\
& + 2\lambda (-1+2\xi-11\xi^2+11\xi^6-2\xi^7+\xi^8) \\
& \left. + 4\lambda^2 (1-\xi)\xi(1+\xi) (1+2\xi-22\xi^2+2\xi^3+\xi^4) \right] \left. \right\}, \quad (28)
\end{aligned}$$

where we have introduced the following definitions

$$d = \frac{\sqrt{1+4\lambda}-1}{2}, \quad a = d+1 \quad (29)$$

$$\hat{s}_1 = \frac{(1-\xi)}{\xi} (a+d\xi), \quad \hat{s} = \hat{s}_1 - \lambda, \quad (30)$$

$$\Delta' = \frac{1}{\xi} \sqrt{\lambda(1+\xi^2)^2 + a + d\xi^4} \quad (31)$$

$$\tilde{L} = \ln\left(\frac{a}{d}\right), \quad \hat{L} = \ln\left(\frac{1+2\lambda+\hat{s}_1-\Delta'}{1+2\lambda+\hat{s}_1+\Delta'}\right) \quad (32)$$

and

$$\text{Li}_2(x) = - \int_0^x dz \frac{\ln(1-z)}{z}. \quad (33)$$

The massless limit of this coefficient function is

$$\begin{aligned}
& C_{2,c}^{(3,0),1} \left(\xi, \frac{Q^2}{m_c^2} \right) \\
& = \frac{e_c^2}{3\pi} \left\{ \frac{(1-2\xi-6\xi^2)}{(1-\xi)_+} - \frac{2(1+\xi^2) \ln \xi}{(1-\xi)} - \frac{2(1+\xi^2) \ln \lambda}{(1-\xi)_+} \right. \\
& \quad - 2(1+\xi^2) \left[\frac{\ln(1-\xi)}{1-\xi} \right]_+ \\
& \quad \left. - \delta(1-\xi) [3 \ln \lambda + 5 + 2\pi^2/3] \right\}. \quad (34)
\end{aligned}$$

Note that, as pointed out in Ref. [26], the previous expressions presented in Ref. [25] are affected by a typo, and also differ by terms which vanish in the massless limit.

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