Global Games of Policy Change

The role of sociopolitical variables in affecting the equilibrium and its uniqueness

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Table of Contents

List of Figures

SECTION 1

Introduction

1.1 General Framework

Global games in literature are commonly defined as games of incomplete information whose type space is determined by the players each observing a noisy signal of the underlying state [8] [25]. Over the past few years, they have proved useful in modelling scenarios and problems related to various fields of application. There are examples where global games have been used to model regime change scenarios [6] [7] [3], bank runs phenomena [19], currency crises [23], pricing debt [27].

The classical definition of global games traces the identification of a single equilibrium as their central characteristic, obviously this restricts the class only to games characterized by two-sided limit dominance. Taking advantage of the recent literature [7] the definition can be generalized and extended to also include games characterized by one-sided limit dominance and thus with multiple equilibria, if the class of global games is defined by information technology. This definition also emphasises how the presence of two-sided limit dominance is not sufficient to guarantee the uniqueness of the equilibrium, which is also linked to the property of thick tails.

Our discussion focuses on modelling the phenomena of mass uprisings through the tool represented by global games. In this context, players use cutpoint strategies, i.e. they participate in the protest if their information level is sufficiently high. What is interesting is to outline how the equilibrium and outcomes of the game are influenced both by the actions of the strategic actors and the characteristics of the society.

As argued in [7] the phenomena of mass uprisings were originally treated as complete information coordination game [35] [37]. This type of game typically involves the presence of two equilibria, one with total participation and one with zero participation, so that any type of structural or strategic variation either does not influence the outcome of the game or overturns it.
The global games approach to the phenomena of mass uprisings is realized through the class of models known as global game of regime change [13] [14] [31] and contrary to the previous case induce not only an unique finite equilibrium, but structural or strategic variations cause continuous variations in the equilibrium and consequently in both the mass of protesting citizens and the probabilities of the outcomes of the game. These kind of games consider the use of cutpoint strategies therefore the players participate if their private signal level is greater than a certain threshold representing the cutoff equilibrium. If this threshold assumes a finite value, equilibrium is finite, whereas if it assumes an infinite value, equilibrium is infinite. The latter involves total or zero participation, while the former a participation percentage between 0 and 1.

This apparent contrast can be solved through the approach described in [6] [7] where in a context of an incomplete information game by eliminating the conditions that induce uniqueness of the equilibrium in global games it is possible to construct a game that provides both a finite equilibrium with the desirable smoothness property of the global games and infinite equilibria of total or null mobilization typical of complete information coordination game retaining the advantages of the two approaches.
The innovative approach seems reasonable as it assumes that when certain conditions are met, mobilization cannot be observed, while when these vary, a discontinuous jump in the level of mobilization is observed, which can lead to a failed or successful protest.

First the aim of our work is to enrich this approach considering its goodness in terms of interpretation. This translates into both a more in-depth description of the composition of the population in strategic terms and an analytical definition of the equilibrium point and the functions that contribute to defining the level of mobilization. This makes it possible not only to derive the analytical form of the probabilities associated with the outcomes of the game, but also to carry out a comprehensive study of comparative statics by observing how structural and strategic variations influence the composition of the population in strategic terms and the game at each level.

The second objective is to better characterize the role of the revolutionary entrepreneurs through the study of the game without the presence of a revolutionary vanguard and thus no public signal comparing it to the original game. This also offers the possibility of redesigning the game by evaluating distributions for the parameters of the game by recovering the randomness generated earlier by the public signal.

The third task is to structurally redesign the game so that the multiplicity of equilibria is linked to the possibility of observing null or positive mobilization, thus eliminating the possibility that the finite equilibria defining the level of mobilization are multiple and above all that they behave logically in relation to structural or strategic variations. This mainly results in the conception of new payoff matrices or alternative cost functions that maintain a good level of intrepretability.

Finally, given the complexity of the analytical treatment, the last part is devoted to the study of the cumulative distribution function of the standard normal distribution with the aim of both identifying additional properties useful in the study of comparative statics and defining a simple but reliable approximation for estimating the equilibrium point.

1.2 Reference Model

We consider as reference model for mass uprisings the one designed in [6] and initially describe its structure and main features. Following chapters will highlight modifications and general extensions of the model with the aim of characterizing the problem, i.e. the occurrence and the outcomes of riots, more effectively in a variety of contexts.

1.2.1 Players

The game considers a continuum of population members of mass 1.

1.2.2 Set of Actions

Each player i takes a dichotomous choice $a_i \in \{0,1\}$, where $a_i = 1$ is the decision to participate and $a_i = 0$ the opposite.

1.2.3 Conflict Technology and Outcomes

The conflict technology is such that there is a regime change if and only if enough players attack. Let denote by $R = 0$ the outcome of no change, and $R = 1$ the one of regime change. Thus, formally the conflict tecnology is discontinuous in this way:

$$\mathbb{P}\{R|N\} = \begin{cases} 0 & \text{if } N < T \\ 1 & \text{if } N \geq T \end{cases}$$

where N is the mass of attacking players and T is the minimum participation threshold (commonly known) for observing regime change.

1.2.4 Payoff Matrix

To conclude the description of the game structure, it is necessary to define the players' payoffs:

	$R=0$	$R=1$
$a_i=0$	0	θ_i
$a_i=1$	$-k$	$(1+\gamma)\theta_i - k$

where k is the private cost for a citizen to protests, while γ is the advantage of being part of the protests when the protest is successful. The original payoff matrix in [6] is:

	$R=0$	$R=1$
$a_i=0$	0	$(1-\gamma)\theta_i$
$a_i=1$	$-k$	$\theta_i - k$

however we prefer our notation, which is strategically equivalent, [1] because it emphasizes that the model's results require an intrinsic utility from participating to a successful protest. Actually, γ can be interpreted as a selective incentive to overcome collective action problems [4][22].

Assumption 1.1 $\gamma \in (0,1]$, $k > 0$

Remark 1.1 *$\gamma > 0$ is crucial, otherwise $a_i = 1$ is always a strictly dominated action*

1.2.5 Information Structure

The country is characterized by a common sentiment towards the government policy, $\theta \in \mathbb{R}$, where $\theta < 0$ means a support for the policy, vice-versa $\theta > 0$. The true θ is unknown by the citizens and each citizen i receives a private signal:

$$\theta_i = \theta + \varepsilon_i,$$

while all the citizens receive a public signal:

$$v = \theta + t + \eta$$

where t is a parameter not observed by the citizens representing the level of effort exerted by the revolutionary vanguard, while η and ϵ_i are random noises, the former is common to all citizens and the latter is idiosyncratic.

Remark 1.2 *The relationship between v, t, θ and η means that the higher the country's sentiment towards the government θ and the higher the parameter t, the higher the value of the observed v, however these variables cumulate and have a noise η so that θ and t can't be extrapolated from the realization of the random variable v : t and θ are confounded in the observed public signal v. This is a classic example of what [17] call "Signal-Jamming", in the sense that t is a variable that interferes with the citizens' inference on θ, given the private and the public signal (θ_i, v). Examples of signal-jamming by nature is [17], while in [21], the variable is strategically used by a player.*

Consider the following assumptions[2] on these signals[3],

Assumption 1.2

$$\theta \sim N\left(0,\sigma_\theta^2\right),\ \varepsilon_i \sim N\left(0,\sigma_\varepsilon^2\right),\ \eta \sim N\left(0,\sigma_\eta^2\right),\ \ t \in [0,\infty)$$

Assumption 1.3 *θ, ε_i and η are independent.*

These assumptions imply:

$$\theta_i \sim N\left(0,\sigma_\varepsilon^2+\sigma_\theta^2\right),\ \theta_i|\theta \sim N\left(\theta,\sigma_\varepsilon^2\right)$$

[1] It is immediate to see that the best reply correspondences coincide for the two payoff structures.

[2] $N\left(\mu,\sigma^2\right)$ is the Normal distribution with expected value μ and variance σ^2.

[3] In the original model [6], $t \in \left[\underline{t},\infty\right)$ and $E(\theta) = m$, however to simplify the discussion we assume $\underline{t} = 0$ and $m = 0$.

Remark 1.3 *Since θ_i does not provide any information on t, we can denote by t^* the common expectation of t by any citizen i. Then*

$$v - t^* = \theta + \left(t - t^*\right) + \eta$$

is the unexpected component of the public signal, which ex ante is expected to be 0, since $E[t - t^*] = 0$, however ex post its realization can be greater or smaller than expected, depending on the realization of the random variables, so that citizen i can not distinguish whether a big public signal is due to exceptional political activism or to big anti-government sentiment. For this reason, the following analysis will be in terms of $v - t^*$, which is distributed as follows :

$$v - t^* \sim N\left(0, \sigma_\eta^2 + \sigma_\theta^2\right), \;\; v - t^* | \theta \sim N\left(\theta, \sigma_\eta^2\right)$$

1.2.6 Timing and Choices

The timing of players' strategic interaction is as follows:

- **Nature choices:** nature choose the random variables $\theta, \eta, \varepsilon_i$ and the revolutionary vanguard chooses a level of t
- **Citizens' information:** each citizen i observes a private signal $\theta_i = \theta + \varepsilon_i$ and a public signal $v = t + \theta + \eta$
- **Protest stage:** each citizen i decides whether to join the protest, $a_i = 1$, or not, $a_i = 0$
- **Final Outcome:** the protest succeed, $R = 1$, if the number of citizens joining the protest, N, is greater or equal to T, otherwise it fails.

Consider the game design a citizen i strategy is a map of the following type:

$$s_i\left(\theta_i, v\right) : \mathbb{R} \times \mathbb{R} \to \{0, 1\},$$

while the conflict technology is :

$$\mathbb{P}\left(R = 1; a_i\right) = \begin{cases} 1 & if \quad N \geq T \\ 0 & if \quad N < T. \end{cases}$$

This situation of strategic interaction is called protest game.

1.2.7 Equilibrium concept

The solution concept of the game is pure strategy perfect Bayesian equilibrium (PBE), where the set of equilibria is restricted in two ways:

- In the revolution stage the players use cutoff strategies such that:

$$s_i\left(\theta_i, v - t^*; p, s\right) = \begin{cases} 1 & if \quad \theta_i \geq \widehat{\theta}\left(v - t^*; p, s\right) \\ 0 & if \quad \theta_i < \widehat{\theta}\left(v - t^*; p, \mathbf{s}\right) \end{cases}$$

- For some values of $v - t^*$ the revolution stage is characterized by multiple finite equilibria in cutoff strategies (also infinite cutoff rules are allowed), in this scenario the players play the same selection in terms of equilibrium

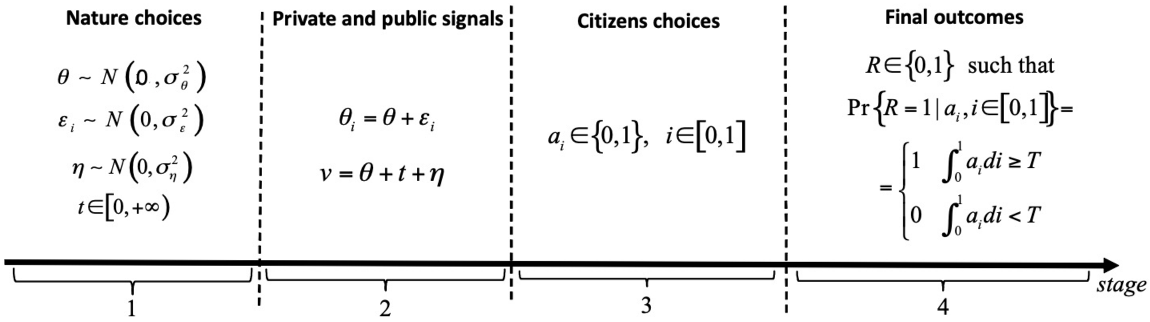

Figure 1.1: Games's stages

1.2.8 Posterior beliefs

The public opinion,i.e. the private signal component common to all citizens, affects citizens' behavior and thus represents an important driver of public policy changes. Of course, political outcomes will depend not only on public opinion, but also on how this reflects on citizens' behavior depending on the political regime and the political society, and in turn citizens' behavior will induce specific political outcomes depending on government responsiveness. Therefore, it is necessary to report the posterior beliefs induced by the previous assumptions:

- Citizen's i posterior beliefs on country unknown level of antigovernment sentiment given $i's$ private information:

$$\theta|\theta_i \sim N\left(\lambda\theta_i, \lambda\sigma_\varepsilon^2\right)$$

  where
$$\lambda = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\varepsilon^2}.$$

- Citizen's i posterior beliefs on country unknown level of antigovernment sentiment given $i's$ private and public information:

$$\theta|\theta_i, v - t^* \sim N\left(\psi\left(v - t^*\right) + (1 - \psi)\lambda\theta_i, \psi\sigma_\eta^2\right)$$

  where
$$\psi = \frac{\lambda\sigma_\varepsilon^2}{\lambda\sigma_\varepsilon^2 + \sigma_\eta^2} = \frac{\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}\sigma_\varepsilon^2}{\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}\sigma_\varepsilon^2 + \sigma_\eta^2} = \frac{\sigma_\theta^2\sigma_\varepsilon^2}{\sigma_\theta^2\sigma_\varepsilon^2 + \sigma_\theta^2\sigma_\eta^2 + \sigma_\varepsilon^2\sigma_\eta^2}.$$

1.3 Model's Interpretation

Although the model is highly stylized, it admits a variety of interpretations and possible applications. As argued in [34], the interpretation of a model is an essential ingredient of a model. We interpret the model as one of political change, in which each citizen observes both a private and a public signal, where the public one is related to political vanguard's activism: given the signals each citizen decides whether or not to take actions, mobilize or not, to induce a change in the established policy.
In the original model the interpretation of only a few variables involved in the game is provided, the purpose of this paragraph is to compose a descriptive framework of all the quantities involved inside the game by developing the original work in qualitative terms.

1.3.1 Random Variables

The stochastic structure of the model contemplate five random variables. Let consider these variables and their interpretation within this model of citizens' political behavior:

- θ is a value drawn from a normal distribution with mean 0 and variance σ_θ^2. It is the country average sentiment towards the government's policy supposed unknown by the agents. Note that $\theta > 0$ means opposition to the government's policy, while $\theta < 0$ means support for the government's policy, hence to assume $E(\theta) = 0$ means that ex ante the agents expect that the country sentiment towards the government is neutral. From now on we will write of θ as the country antigovernment sentiment since a positive realization of this variable means a positive return from a change in government policies

- $\theta_i = \theta + \varepsilon_i \sim N\left(\theta, \sigma_\varepsilon^2\right)$ is the individual $i's$ sentiment towards the government's policy, i.e., i's private signal on the country common sentiment towards the government's policy. From now on we will write of θ_i as the individual antigovernment sentiment since a positive realization of this variable means a positive return to i from a change in government policies

- $\varepsilon_i \sim N\left(0, \sigma_\varepsilon^2\right)$ is what differentiate i's individual antigovernment sentiment from the average country antigovernment sentiment

- $v = \theta + t + \eta \sim N\left(t^* + \theta, +\sigma_\eta^2\right)$ is the public signal on the country average sentiment towards the government's policy, which depends on θ and on the parameter t, which is interpreted below, as an exogenous sociopolitical variable

- $\eta \sim N\left(0, \sigma_\eta^2\right)$ is the stochastic noise in the public signal that meddle with the public signal, so that agents are not able, given the expectation of t, to infer the true value of the country antigovernment sentiment

1.3.2 Exogenous Sociopolitical Variables

The set of exogenous sociopolitical variables is denoted by:

$$S = \{(t^*, \sigma_\theta^2, \sigma_\varepsilon^2) \in \mathbb{R}_+^3\}$$

These are the variables related to social and political aspects of a country in a given historical period. A specific vector of realization is denoted by $s \in S$. Let consider these variables and their interpretation within this model of citizens' political behavior:

- $t^* \in (0, +\infty)$ is the common expectation of t, a parameter representing the vanguard's antigovernment activism, which is unknown by the citizens, but imperfectly observed through the public random signal v. It is a way activists use to send a public signal to the citizens about the unknown common antigovernment sentiment of the country. Since in this paper we are interested in understanding the causes and the consequences of citizens' political behavior, we take the vanguard's activism t as exogenous as well as t^*. In [6], t is the vanguard's effort chosen by the revolutionary entrepreneurs to induce the citizens' revolt, and v is the violence observed by the citizens. We think that this interpretation is not fully convincing since v does not impact directly on players' payoff as it should in violent conflicts: even the extension at the end of the paper where v affects negatively T does not seem to us to catch the effects of the violence on citizens' and government's payoffs

- $\sigma_\theta^2 \in (0, +\infty)$ is the variance of the country's common antigovernment sentiment, thus a bigger σ_θ^2 implies a greater probability of extreme values for θ, positive and negative, hence we can interpret σ_θ^2 as a political radicalization parameter

- $\sigma_\varepsilon^2 \in (0, +\infty)$ is the variance of the idiosyncratic component of individual antigovernment sentiment θ_i, which has a twofold role in the model, since it is both a private signal and $i's$ sentiment towards the government's policy. Thus a bigger σ_ε^2 implies a reduction in the informativeness of the private signal, but also a greater dispersion in the possible realizations of the individual antigovernment sentiment θ_i : from this second point of view, we can interpret σ_ε^2 as the country political diversity

1.3.3 Exogenous Policy Variables

The set of exogenous policy variables is denoted by:

$$P = \left\{ \left(T, \gamma, k, \sigma_\eta^2 \right) \in (0,1]^2 \times \mathbb{R}_+^2 \right\}$$

These are the variables that characterize a specific political regime. A specific vector of realization is denoted by $p \in P$. Let consider these policy variables and their interpretation within this model of citizens' political behavior.

- $T \in (0,1)$ is the threshold such that, once the mass of protesting citizens exceeds this level, then the government would change its policy; thus the greater T, the more difficult is to induce a change through a public protest. According to the political and economic literature[32], a government is responsive if it adopts policies that are signaled as preferred by citizens, where signals include various form of direct political actions, such as demonstrations, letter campaigns, and the like. Thus, $(1 - T)$ can be interpreted as the government responsiveness

- $\gamma \in (0,1]$ is the intrinsic utility the citizens get from participating to successful protests, i.e. it measures the selective incentives to overcome collective action problems, thus we can interpret γ as a measure of the regime's inclusiveness. According to the Global State of Democracy Indices[4] that measures democratic performance for 165 countries around the world across 29 aspects of democracy, to ensure inclusive and participatory decision-making at all levels is a crucial aspect to evaluate a political regime: the greater the inclusiveness, the greater the measure of democracy

- $k \in (0, +\infty)$ is the cost of protesting for the citizens, thus part of a measure of the repression by the regime. As we will see, a key parameter in the analysis is the cost of protesting with respect to the benefit from successful protest,

$$\frac{k}{\gamma},$$

  that we interpret as an index of government repression

- $\sigma_\eta^2 > 0$ is the variance of the noise in the public signal, thus a bigger σ_η^2 implies a looser connection between vanguard's activism, the common antigovernment sentiment and the public signal, hence we interpret σ_η^2 as the country opacity in public information: as well known, the effectivity of citizens' checks on the policies of the government depends on the informativeness of public information, and the greater the noise, the smaller the information provided by the public signal[5]

Finally, for each function $f(x)$ involved in the following developments I will use the notation $f\left(x; p, s\right)$ to emphasize its dependence from the exogenous vector $\left(p, s\right) \in P \times S$ of sociopolitical and policy variables.
The following table sum up the variables of the model and their interpretation

[4] International Institute for Democracy and Electoral Assistance (IDEA), 2021.

Variables	Interpretation
Endogenous Variables	
$a_i \in \{1,0\}$	citizen i protests or not
$R \in \{1,0\}$	success or not of protests
Random Variables	
$\theta \sim N\left(0,\sigma_\theta^2\right)$	country unknown level of antigovernment sentiment
$\theta_i \sim N\left(0,\sigma_\theta^2+\sigma_\varepsilon^2\right)$	$i's$ level of antigovernment sentiment
$\varepsilon_i \sim N\left(0,\sigma_\varepsilon^2\right)$	idiosyncratic component of $i's$ antigovernment sentiment
$\upsilon \sim N\left(t^*,\sigma_\eta^2\right)$	public signal on country antigovernment sentiment
$\eta \sim N\left(0,\sigma_\eta^2\right)$	noise in the public signal
Exogenous sociopolitical variables: $S=\left\{\left(t^*,\sigma_\theta^2,\sigma_\varepsilon^2\right)\in\mathbb{R}_+^3\right\}$	
(t^*) $t \in [0,\infty)$	(expected) vanguard's antigovernment activism
σ_θ^2	country political radicalization
σ_ε^2	country political diversity
Exogenous policy variables: $P=\left\{\left(T,\gamma,k,\sigma_\eta^2\right)\in(0,1]^2\times\mathbb{R}_+^2\right\}$	
$1-T$	government responsiveness
γ	political inclusiveness
k	cost of protesting
k/γ	government political repression
σ_η^2	opacity in the public information

Table 1: game variables and their meanings

1.4 Taxonomy of the Political Regimes and Societies

Given the previous interpretations of the exogenous variables, we propose an intuitive qualitative classification of the possible different political regimes and of the different political societies. The combination of the policy variables identifies eight different political regimes:

	σ_η^2 *small*	σ_η^2 *big*
$(1-T)$ *big*, $\frac{k}{\gamma}$ *small*	**RRT** responsive, tolerant, transparent	**RTO** responsive, tolerant, opaque
$(1-T)$ *small*, $\frac{k}{\gamma}$ *small*	**UTT** unresponsive, tolerant, transparent	**UTO** unresponsive, tolerant, opaque
$(1-T)$ *big*, $\frac{k}{\gamma}$ *big*	**RIT** responsive, intolerant, transparent	**RIO** responsive, intolerant, opaque
$(1-T)$ *small*, $\frac{k}{\gamma}$ *big*	**UIT** unresponsive, intolerant, transparent	**UIO** unresponsive, intolerant, opaque

Table 2: different political regimes P

These eight possible political regimes are defined on the basis of three dimensions. Leaving aside vanguard's activism that will play a specific role later, a political society can be identified on the basis of the remaining sociopolitical characteristics as follows :

	σ_θ^2 *small*	σ_θ^2 *big*
t^* *small*, σ_ε^2 *small*	**QHM** quiet, homogeneous, moderate	**QHR** quiet, homogeneous, radicalized
t^* *big*, σ_ε^2 *small*	**THM** turbulent, homogeneous, moderate	**THR** turbulent, homogeneous, radicalized
t^* *small*, σ_ε^2 *big*	**QDM** quiet, diverse, moderate	**QDR** quiet, diverse, radicalized
t^* *big*, σ_ε^2 *big*	**TDM** turbulent, diverse, moderate	**TDR** turbulent, diverse, radicalized

Table 3: different political societies S

In the following analysis I will consider how the causes and the consequences of citizens' political behavior change in different political regimes and different societies.

SECTION

Citizens' Protests: causes and consequences

2.1 Notes

This paper was presented in its preliminary form during the 5th International Conference on the political economy of democracy and dictatorship (University of Münster, Germany, 24 February 2022).

It is part of the collection of the working papers of the CefES research group[5] and the FEEM [6].

[5]available at https://cefes-dems.unimib.it/publications/wp-latest/

[6]available at https://www.feem.it/en/publications/citizens-protests-causes-and-consequences-a-research-on-regime-change-and-revolutionary-entrepreneurs-by-bueno-de-mesquita/

2.2 Introduction

Why is the world protesting so much? The last fifteen years have witnessed a significant increase in mass protest: the number of large protests and demonstrations globally has risen by 36% since the global financial crisis in 2008-2009, from an average of 355 per year in the decade to 2009 to 482 per year in the decade following the Global Financial Crisis.[7] In particular, increases in large protests[8] have been notable across all over the world, from Europe to the Middle East, from Africa to South-America, from Asia to North-America. For example, the number of large protests in Europe increased by 71%, averaging 92 annually in the period 2000-2009 and 157 in 2010-2019. Average annual figures for the Middle East and North Africa region increased by 229 % (22 to 72 per annum), while those for sub-Saharan Africa increased by 48% (59 to 88 per annum).[9] In general, mass protests have characterized countries as different as Bolivia, Brazil, Chile, Ecuador, Guinea, Haiti, Honduras, Hong Kong, India, Iraq, Kazakhstan, Lebanon, Pakistan, United States, often but not always leading to a reverse of government policies.
An important aspect of this stylized facts is that mass protest involves all kind of political regimes, from the most democratic to highest authoritarian. Are we in a historic age of protest? Probably not since the wave of "people power" movements swept Asian and east European countries in the late 1980s and early 1990s has the world experienced such a simultaneous outpouring of popular anger on the streets. Before that, only the global unrest of the late 1960s bears comparison in terms of the number of countries and of the number of people mobilized. However, those two waves of global unrest seemed more joined-up than the present spate of apparently unconnected and spontaneous movements. Protesters in many different countries had similar grievances and aims. This time, some themes inevitably crop up in country after country. A study[10] by a team of researchers of the Friedrich-Ebert-Stiftung (FES) and the Initiative for Policy Dialogue, a nonprofit organization based at Columbia University, looking at more than 900 protest movements or episodes across 101 countries and territories, concludes that we are living through a period of history like the years around 1848, 1917 or 1968 when large numbers of people rebelled against the way things were, demanding change.
But why? Why do protests spread globally at particular points in history? What accounts for the important differences that we find between similar protests movements in different political and social contexts? What explains the shift in strategies, aims, and organizational forms that we regularly observe over the course of protest waves? Is the search for a unifying theory pointless? After all, when you look more closely at the earlier waves, the impression of coherence might seem illusory. They too were more variegated than is often assumed. The global upheavals of the late 1960s ranged from Red Guards in China, to affluent Western youths who had stumbled on the joys of life without strict old traditional rules. In between were protesters against the Vietnam war, the Soviet domination of eastern Europe and the tedious traditional lectures at universities. Even

[7] See the study by (re)insurance group Chaucer https://www.chaucergroup.com/.

[8] Large protest are defined as a protest with at least 100 participants.

[9] Data by Binghampton University, NY.

[10] [30]

the people-power revolutions of 20 years later were as marked by their differences as their similarities. Right-wing strongmen such as the Philippines' Ferdinand Marcos or South Korea's Chun Doo-Hwan were a far cry from east European thugs such as Nicolae Ceausescu and Wojciech Jaruzelski.

The difficulty in discerning a pattern has not stopped scholars from trying. In general, on one hand, multiple empirical studies have disproved the common assumption that the greater the grievances, the more likely people will engage in political protests, on the other hand, different theoretical school have emphasized the role of different set of factors such as: the perception and interpretation of grievances and their causes, the expected impact of protest, i.e. the government responsiveness, the commonalities of the protesters, thus the country political diversity and radicalization, the expected benefit from success with respect to the cost from protesting, the context structure including political, economic, cultural opportunities and restrictions on protest, and, more generally, the social, political and cultural characteristics of a country.[11] We believe the answer is to go back to first principles and analyze what makes people take their grievances to the streets. Three reasons seem basic. First, for all its legal and physical dangers, successful protest can be more exciting than the drudgery of daily life. Second, when everybody else is doing it, solidarity becomes the fashion and makes protests effective. The third reason for demonstrating is that, in many situations, using conventional political channels may seem useless. In the protests of the late 1980s, the targets were usually autocratic governments that allowed at best sham elections. In the absence of the ballot box, the street was the only way to demonstrate "people power". Some of these years protests - for example against Abdelaziz Bouteflika in Algeria or Omar al-Bashir in Sudan - have been analogous. But also apparently well-functioning liberal democracies have been affected by large protests. The point is that, for a number of reasons, people may be feeling unusually powerless these days, believing that their votes do not matter and that the government is not accountable.

Whatever the reasons, citizens' political participation to protests is a crucial issue for any political system, whether democratic or autocratic. Of course, the ways, the costs and the effects differ in different political regimes, and actually the government answer to citizens' protests define a crucial characteristic of a polity. But, democratic or autocratic, all polities have some form of public involvement in the political process, if only to accept public policies. And all political systems have different ways of dealing with citizens' participation to political behavior, determining cost and benefit of public dissent, responsiveness to public requests and transparency in public information.

In this work, we are interested in citizens' political protests, i.e. in the deliberate and public expression of dissent towards a government policy with the intent of influencing a political decision that a group of citizens perceive as having negative consequences for themselves or for their vision of the public good. Protests can refer to any political and social issue that regards the citizens as a collectivity, whether it is a specific policy or a political regime as a whole. The aim of the protest can be narrow or broad, re-

[11] See e.g. [12], [15], [28], [36]

formist or revolutionary. The forms of protests, too, include a broad range of activities, from writing a petition, to attending a march, from blocking traffic to injuring or even killing people. The aim of this paper is to analyze causes and consequences of citizens' protests. In particular, we want to analyze when and why unsatisfied citizens are able to overcome the collective action problem, protesting against a government and possibly inducing changes in government policy. Specifically, this analysis will focus on how private and public information affect citizens' opinion and political behavior, and how these effects depend on sociopolitical factors as well on the political governance system. In [6] Bueno de Mesquita proposed an important model to study why revolutionary vanguards might use violence to mobilize a mass of citizens against a regime. His analysis in particular focused on how vanguard's violence, affecting the population sentiment on a regime, may help its overthrown. In the paper Bueno de Mesquita proposes a particular global game with one-sided limit dominance that, differently from the usual global games with two-sided-limit dominance, have multiple equilibria, arguing for selecting one of these equilibria. The selected equilibrium has three possible probabilistic outcomes relative to citizens' protests: one where there is no mobilization, one with insufficient mobilization, and one with successful mobilization and thus protest. We claim that the Bueno de Mesquita model can be used more generally to investigate citizens' behavior within different political regimes and different countries.[12] In particular, the aim of this work is to use and revise the Bueno de Mesquita model to deepen the understanding of causes and consequences of citizens' political behavior, studying the relationship between the model's structural parameters and causes and consequences of citizens' protests, to better understanding the basic principles of how citizens function within the political process across different political systems. Thus, in this paper, we review and revise the model, adopting a new approach to derive some of the paper's main results, discussing the interpretation of the exogenous variables of the model, extending and correcting some of Bueno de Mesquita results.
The paper is organized as follows: initially we present and discuss the model and the interpretation of the exogenous variables, while in the following sections: we review the properties of the citizens' beliefs, i.e. of a country public opinion, while section 4 derives citizens' behavior and we derives the possible equilibrium outcomes and discuss the relationships between the sociopolitical variables and the outcome probabilities.

2.3 The public opinion

Definition 2.1 *A citizen i is an **extremist** if $\theta_i > E(\theta)$, i.e. if his/her antigovernment sentiment is greater than the average country's antigovernment sentiment, otherwise he/she is a **moderate**.*

[12]As argued in [34], an economic model differs from a purely mathematical model in that it is a combination of mathematical structures and interpretation. Thus, interpretation is a substantial part of an economic model.

Notation 1 *Let denote by* $\searrow$ *a decreasing behavior, by* $\nearrow$ *an incresing behavior, by* $\nearrow\searrow$ *and* $\searrow\nearrow$ *a non monotonic behavior.*

Result 2.1 *The expected country's level of antigovernment sentiment given* $i's$ *private signal is:*

- *increasing in* $i's$ *level of antigovernment sentiment*
- *when* i *is a moderate, decreasing in country radicalization and increasing in country diversity*
- *when* i *is an extremist, increasing in country radicalization and decreasing in country diversity*
- *almost coinciding with* $i's$ *antigovernment sentiment, when country radicalization is increasing without limit with a finite amount of country diversity*
- *almost degenerated in* 0*, when country diversity is increasing without limit with a finite amount of country radicalization*

Socio-pol. variables	**Private signal**	
	Moderate	Extremist
i's antigovernment sentiment	$\nearrow$	$\nearrow$
radicalization	$\searrow$	$\nearrow$
diversity	$\nearrow$	$\searrow$

Table 4: behavior of $E(\theta|\theta_i)$

This result is simple but interesting because it explains why people think their political position on average is shared by other citizens, and it shows the difference in perceptions between moderates and extremists in a society, in particular on country radicalization and diversity.

The public signal v changes this result and has different consequences for extremists and for moderates; before stating the result, consider the following intuitive terminology.

Definition 2.2 *The unexpected component of the public signal* $v-t^*$ *and the private signal* θ_i

- *for a moderate are:*
  - *strongly incendiary if* $(v-t^*) > -\frac{\sigma_\eta^2}{\sigma_\varepsilon^2}\theta_i$
  - *incendiary if* $(v-t^*) \in \left[E(\theta;\theta_i;p,s) = \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}\theta_i, -\frac{\sigma_\eta^2}{\sigma_\varepsilon^2}\theta_i\right]$
  - *moderating if* $(v-t^*) \in \left[\frac{\sigma_\theta^2+\sigma_\eta^2}{\sigma_\theta^2}\theta_i, E(\theta;\theta_i;p,s) = \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}\theta_i\right]$
  - *strongly moderating if* $(v-t^*) < \frac{\sigma_\theta^2+\sigma_\eta^2}{\sigma_\theta^2}\theta_i$

- *for an extremist are:*
  - *strongly incendiary if* $(v-t^*) > \frac{\sigma_\theta^2+\sigma_\eta^2}{\sigma_\theta^2}\theta_i$
  - *incendiary if* $(v-t^*) \in \left[E\left(\theta;\theta_i;p,s\right) = \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}\theta_i, \frac{\sigma_\theta^2+\sigma_\eta^2}{\sigma_\theta^2}\theta_i\right]$
  - *moderating if* $(v-t^*) \in \left[-\frac{\sigma_\eta^2}{\sigma_\varepsilon^2}\theta_i, E\left(\theta;\theta_i;p,s\right) = \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}\theta_i\right]$
  - *strongly moderating if* $(v-t^*) < -\frac{\sigma_\eta^2}{\sigma_\varepsilon^2}\theta_i$

This definition is illustrated in the following picture:

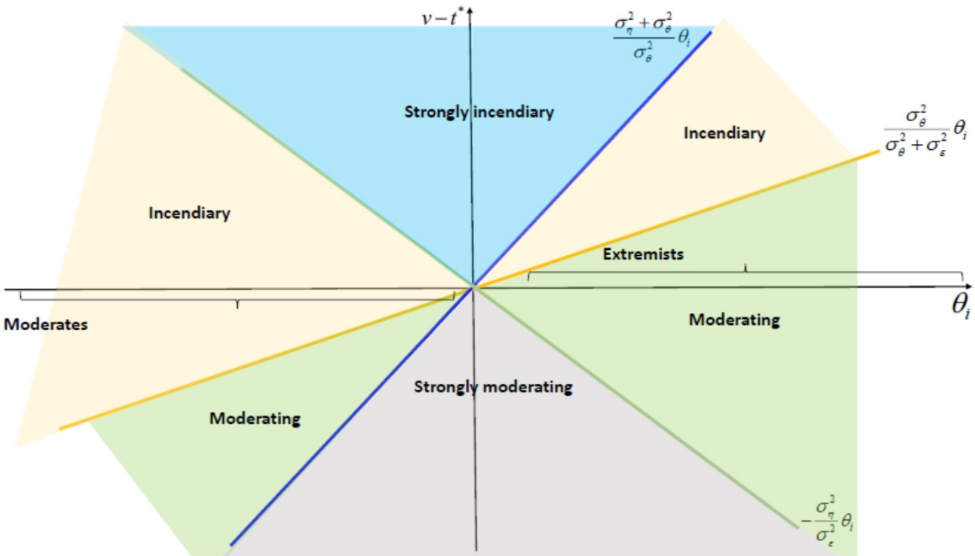

Figure 2.1: The role of public signal on citizens' beliefs

Then, the following result is immediate:

Result 2.2 *The expected country antigovernment sentiment given* $i's$ *private and public signals is:*

- *increasing in* $i's$ *level of antigovernment sentiment and in the unexpected component of the public signal*
- *increasing in the opacity of public information if and only if the unexpected component of the public signal is smaller than* $\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}\theta_i$
- *increasing in country radicalization if and only if the unexpected component of the public signal is greater than* $-\frac{\sigma_\eta^2}{\sigma_\varepsilon^2}\theta_i$
- *increasing in country political diversity if and only if the unexpected component of the public signal is greater than* $\frac{\sigma_\eta^2+\sigma_\theta^2}{\sigma_\theta^2}\theta_i$

Socio-pol. var.	Public and private signals					
	Strong inc	Ext&Inc	Extr&Moder	Strongly moder	Moder&Moder	Moder&Inc
i's antigov sent	↗	↗	↗	↗	↗	↗
unexpect activism	↗	↗	↗	↗	↗	↗
radicalization	↗	↗	↗	↘	↘	↘
diversity	↗	↘	↘	↘	↗	↗
opacity	↘	↘	↗	↗	↗	↘

Table 5: behavior of $E(\theta|\theta_i, v - t^*)$

These results show the different properties of the expected country antigovernment sentiment depending on whether i is moderate or extremist, and the different effects of private and public signal, with their subtle interaction, on the expected country antigovernment sentiment. In particular, it is immediate to derive the following result.

Result 2.3 *The public signal changes the behavior of citizens' expectations with respect to the case of private signal only when the signals are strongly incendiary or strongly moderating. Moreover, the space of strongly incendiary/moderating signals is shrinking when:*

- *the opacity in public information is increasing*
- *country radicalization is decreasing*
- *country diversity is decreasing*

Socio-pol.var.	Public and private signals					
	Strong inc	Ext&Inc	Extr&Moder	Strong.moder	Moder&Moder	Moder&Inc
radicalization	↗ also for Moder	↗	↗	↘ also for Extr	↘	↘
diversity	↗ also for Extr	↘	↘	↘ also for Moder	↗	↗

Table 6: behavior of $E(\theta|\theta_i, v - t^*)$ **and** $E(\theta|\theta_i, v - t^*)$

The following figure synthesizes the two previous results:

2.4 Citizens' behavior

From the payoff table presented in the introductory chapter the following result is immediate:

Result 2.4 *Any citizen with type* $\theta_i \in \left(-\infty, \frac{k}{\gamma}\right)$ *has a dominant strategy not to participate whatever the private and public signals*

This result justify the following terminology:

Definition 2.3 *The group of citizens that will never protest notwithstanding the public and private signals is called silent group*

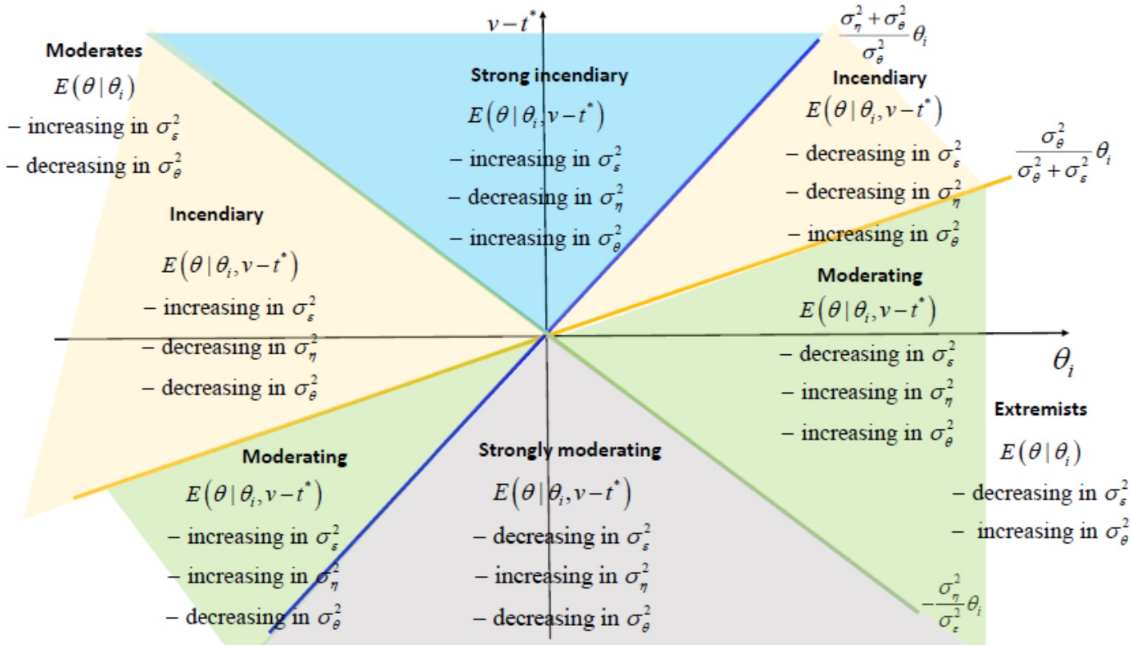

Figure 2.2: Expected country antigovernment sentiment end the socio-political variables

Note that the silent group is different from the moderates, because it also depends on the variables related to the polity, besides the sociopolitical variables as for the moderates. In particular, considerate the following definition:

Definition 2.4 *A political regime is repressive if* $\frac{k}{\gamma} > \theta$, *tolerant otherwise.*

Then, we can say that the moderates are a subset of the silent citizens in repressive political regimes, and viceversa in tolerant political regimes. The following figure illustrates the situation for the two case of repressive and tolerant political regimes.

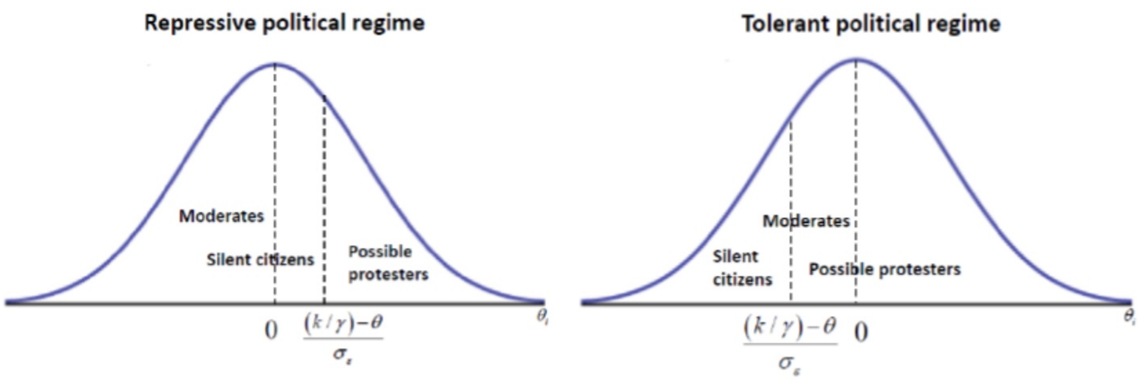

Figure 2.3: Moderates and silent group as determined by the political regime

Result 2.5 *The measure of the silent group is* $\Phi\left(\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon}; p, s\right) > 0$, *which is:*

- *increasing in the government repression*
- *decreasing in the country antigovernment sentiment*
- *decreasing in the country diversity for repressive political regimes*
- *increasing in the country diversity for tolerant political regimes*

These results are summed up in the following table:

Socio-pol. Var	**Socio political situation**	
	any	
repression	$\searrow$	
	any	
country antigovernment sentiment	$\searrow$	
	Tolerant	Repressive
diversity	$\nearrow$	$\searrow$

Table 7: behavior of the measure of the silent group

Remark 2.1 *An interesting aspect of this results is that political diversity reduces the measure of the silent group only in repressive polities, which means that in more democratic countries, political participation is increasing for more heterogenous societies.*

A further interesting result is the following:

Result 2.6 *Regarding the protests, it is possible to state that:*

- *Protests are impossible if and only if*

$$\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon} \to \infty,$$

  hence protests are always possible for tolerant political regimes

- *Protests can be successful if and only if*

$$\Phi\left(\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon}; p, s\right) \leq 1-T \Leftrightarrow \frac{k}{\gamma} \leq \theta + \sigma_\varepsilon \Phi^{-1}(1-T)$$

  restricting the set of political regims and societies where protests can succeed. In particular

  - *responsiveness should be greater than $\Phi\left(\frac{\theta}{\sigma_\varepsilon}\right)$, otherwise protests can't succeed;*
  - *when the regime is repressive, responsiveness should be greater than $\frac{1}{2}$, otherwise protests can't succeed*

The following figure illustrates the situation:

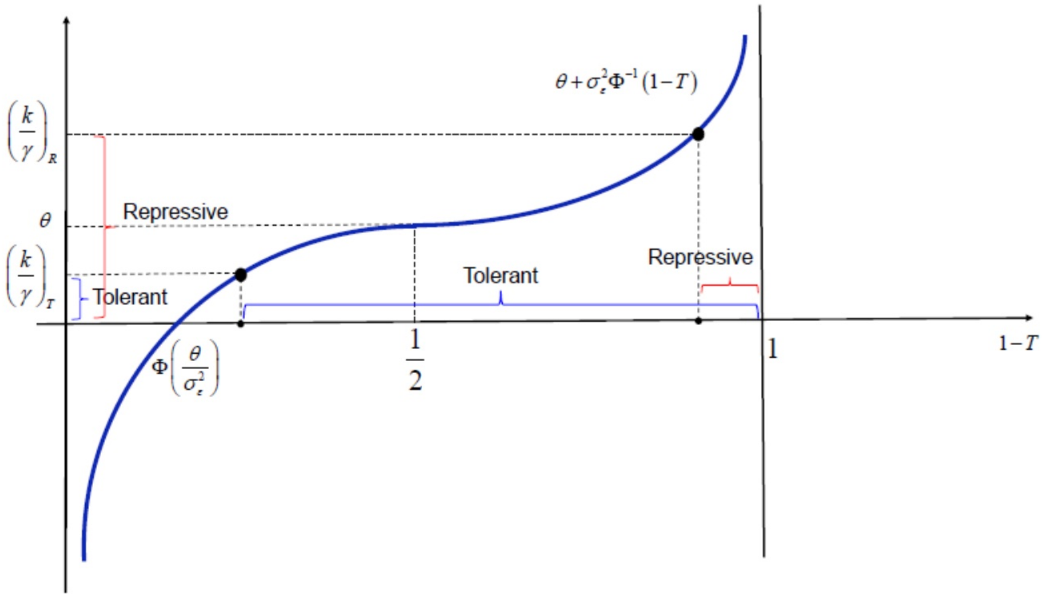

Figure 2.4: Repression and responsiveness such that protest can succeed

2.5 Citizens' Equilibrium Behavior

2.5.1 The Equilibrium Concept

We consider cutoff equilibria, i.e. Perfect Bayesian Equilibria (PBE) in pure strategies, with two restrictions as in [6]:

- cutoff strategies:

$$s_i\left(\theta_i, v-t^*; p, s\right) = \begin{cases} 1 & if \quad \theta_i \geq \widehat{\theta}\left(v-t^*; p, s\right) \\ 0 & if \quad \theta_i < \widehat{\theta}\left(v-t^*; p, s\right) \end{cases}$$

- common cutoff selection: when there are multiple equilibria with possible multiple cutoff points, the citizens will choose the same threshold $\widehat{\theta}\left(v-t^*; p, s\right)$.

Remark 2.2 *The common cutoff selection is requested by the possibility of multiple equilibria in the model (see [1] for a discussion about the reasons for uniqueness and multiplicity in global games), while cutoff strategies are commonly used in global games (see [25] for a comprehensive review of global games).*

2.5.2 Deriving the Cutoff Equilibria of the Protest Game.

Following [6], the analysis is organized in four steps.

Step 1: The Cutoff Rule

Let conjecture there exists an increasing map:

$$\widehat{\theta} : \left(v-t^*\right) \mapsto \left[\frac{k}{\gamma}, +\infty\right]$$

such that

$$s_i\left(\theta_i, v-t^*; p, s\right) = \begin{cases} 1 & if \quad \theta_i \geq \widehat{\theta}\left(v-t^*; p, s\right) \\ 0 & if \quad \theta_i < \widehat{\theta}\left(v-t^*; p, s\right) \end{cases} \Leftrightarrow$$

$$\Leftrightarrow s_i\left(\theta_i, v-t^*; p, s\right) = \begin{cases} 1 & if \quad \varepsilon_i \geq \widehat{\theta}\left(v-t^*; p, s\right) - \theta \\ 0 & if \quad \varepsilon_i < \widehat{\theta}\left(v-t^*; p, s\right) - \theta. \end{cases}$$

Step 2: ***i*'s Beliefs about the Probability of Policy Change**

From player i's perspective, if all other players use the cutoff rule $\widehat{\theta}\left(v-t^*; p, s\right)$, then j protests if $\varepsilon_j \geq \widehat{\theta}\left(v-t^*; p, s\right) - \theta$. Thus, the mass of citizens protesting is

$$\mathcal{N}\left(\theta, \widehat{\theta}\left(v-t^*; p, s\right)\right) = 1 - \Phi\left(\frac{\widehat{\theta}\left(v-t^*; p, s\right) - \theta}{\sigma_\varepsilon}\right),$$

so that in equilibrium the protest succeeds if

$$\mathcal{N}\left(\theta, \widehat{\theta}\left(v-t^*; p, s\right)\right) = 1 - \Phi\left(\frac{\widehat{\theta}\left(v-t^*; p, s\right) - \theta}{\sigma_\varepsilon}\right) \geq T.$$

Since $\mathcal{N}\left(\theta, \widehat{\theta}\left(v-t^*; p, s\right)\right)$ is increasing in θ, for any cutoff rule $\widehat{\theta}\left(v-t^*; p, s\right)$ there exists a minimal level of θ, such that the protest is successful, which is the solution of the following equation in θ^*

$$\mathcal{N}\left(\theta^*, \widehat{\theta}\left(v-t^*; p, s\right)\right) = 1 - \Phi\left(\frac{\widehat{\theta}\left(v-t^*; p, s\right) - \theta^*}{\sigma_\varepsilon}\right) = T \Rightarrow$$

$$\Rightarrow \theta^*\left(\widehat{\theta}\left(v-t^*; p, s\right); T, \sigma_\varepsilon^2\right) = \widehat{\theta}\left(v-t^*; p, s\right) - \sigma_\varepsilon \Phi^{-1}\left(1-T\right) \in \mathbb{R}.$$

Then, we might conclude as follows.

Conclusion 2.1 *The minimal level of the country antigovernment sentiment, such that the protest is successful when the common cutoff rule is* $\widehat{\theta}\left(v-t^*; p, s\right)$, *is*

$$\theta^*\left(\widehat{\theta}\left(v-t^*; p, s\right); T, \sigma_\varepsilon^2\right) = \widehat{\theta}\left(v-t^*; p, s\right) - \sigma_\varepsilon \Phi^{-1}\left(1-T\right),$$

which is

- *decreasing in the responsiveness of the political regime, such that*
  - $\lim_{1-T \to 1} \theta^*\left(\widehat{\theta}\left(v-t^*; p, s\right); T, \sigma_\varepsilon^2\right) = -\infty$;
  - $\lim_{1-T \to 0} \theta^*\left(\widehat{\theta}\left(v-t^*; p, s\right); T, \sigma_\varepsilon^2\right) = \infty$;
  - $\theta^*\left(\widehat{\theta}\left(v-t^*; p, s\right); 1-T = \frac{1}{2}, \right) = \widehat{\theta}\left(v-t^*; p, s\right) \geq \frac{k}{\gamma}$;
  - $\theta^*\left(\widehat{\theta}\left(v-t^*; p, s\right); 1-T = \Phi^{-1}\left(\frac{\widehat{\theta}\left(v-t^*; p, s\right)}{\sigma_\varepsilon}\right), \sigma_\varepsilon^2\right) = 0$;
- *linearly increasing or decreasing in the diversity of the country depending whether the political regime is unresponsive or responsive;*
- *linearly increasing in the common cutoff* $\widehat{\theta}\left(v-t^*; p, s\right)$, *such that*
  - *the minimum is* $\frac{k}{\gamma} - \sigma_\varepsilon \Phi^{-1}\left(1-T\right)$;
  - $\lim_{\widehat{\theta}(v-t^*) \to \infty} \theta^*\left(\widehat{\theta}\left(v-t^*; p, s\right); T, \sigma_\varepsilon^2\right) = \infty$;
  - $\theta^*\left(\widehat{\theta}\left(v-t^*; p, s\right); T, \sigma_\varepsilon^2\right) = 0$ *if and only if* $\widehat{\theta}\left(v-t^*; p, s\right) = \sigma_\varepsilon \Phi^{-1}\left(1-T\right)$.

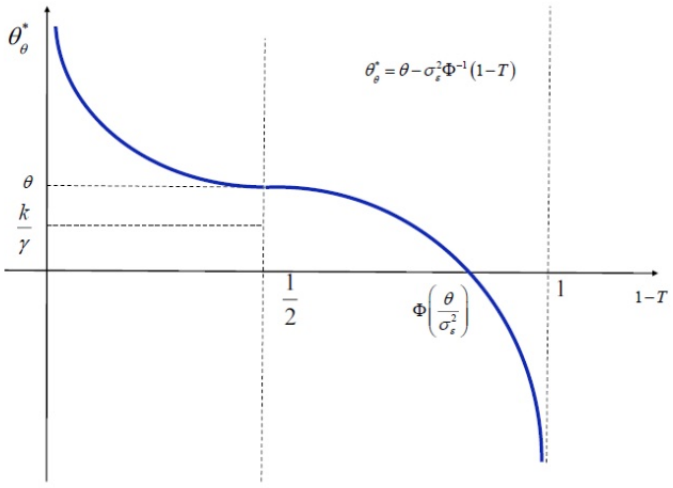

Figure 2.5: $\theta^*\left(\widehat{\theta}\left(v-t^*;p,s\right);T,\sigma^2_\varepsilon\right)$ as a function of T

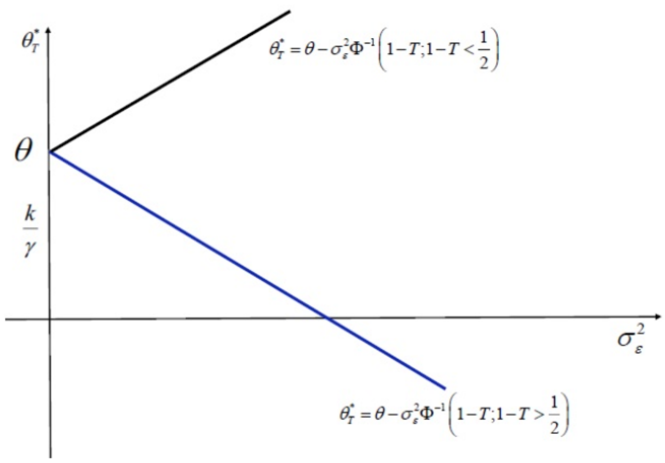

Figure 2.6: $\theta^*\left(\widehat{\theta}\left(v-t^*;p,s\right);T,\sigma^2_\varepsilon\right)$ as a function of σ_ε

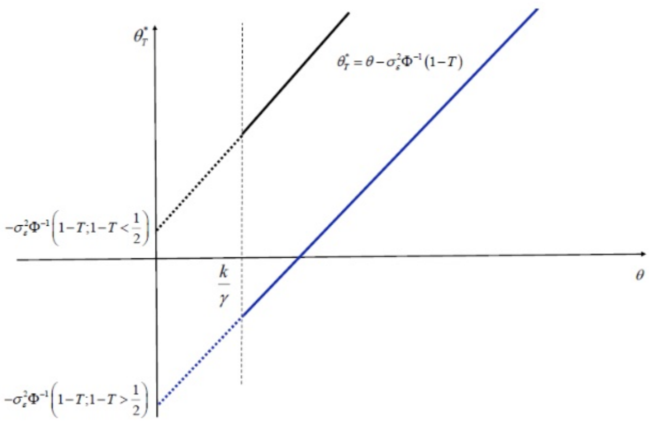

Figure 2.7: $\theta^*\left(\widehat{\theta}\left(v-t^*;p,s\right);T,\sigma^2_\varepsilon\right)$ as a function of $\widehat{\theta}$

Remark 2.3 *The previous results show that a more heterogenous country is more (less) likely to have a successful protest if the political regime is responsive (unresponsive). Hence, to evaluate the likelihood of a successful protest, it is important to consider the combination of political and social variables that characterize a country.*

Now, it is possible to evaluate i's subjective belief about the probability of policy change, $\mathbb{P}\left(\mathcal{N} \geq T|\theta_i, v-t^*, \widehat{\theta}\left(v-t^*;p,s\right);p,s\right)$, given the private and the public signals and the belief that all other players j participate if and only if $\theta_j \geq \widehat{\theta}\left(v-t^*;p,s\right)$:

$$\mathbb{P}\left(\mathcal{N} \geq T|\theta_i, v-t^*, \widehat{\theta}\left(v-t^*;p,s\right);p,s\right) = \mathbb{P}\left(\theta \geq \theta^*\left(\widehat{\theta}\left(v-t^*;p,s\right)\right)|\theta_i, v-t^*;p,s\right) =$$

$$= 1-\Phi\left(\frac{\widehat{\theta}\left(v-t^*;p,s\right) - \sigma_\varepsilon \Phi^{-1}\left(1-T\right) - \psi\left(v-t^*\right) - \left(1-\psi\right)\lambda\theta_i}{\sqrt{\psi\sigma_\eta^2}};p,s\right),$$

since

$$\theta|\theta_i, v-t^* \sim N\left(\psi\left(v-t^*\right) + \left(1-\psi\right)\lambda\theta_i, \psi\sigma_\eta^2\right).$$

Result 2.7 *i's subjective belief about the probability of policy change, given the private and the public signals and the belief that all other players j participate if and only if $\theta_j \geq \widehat{\theta}\left(v-t^*;p,s\right)$, is*

- *increasing in the responsiveness of the political regime*
- *increasing in the public signal and in its unexpected component*
- *increasing in the private signal*
- *uncertain in radicalization, diversity and opacity*

When the formal analysis does not lead to well defined relationships, we investigate the relationships through simulations that can be found in Appendix B. From these simulations we are able to derive the following results.

Result 2.8 *i's subjective belief about the probability of policy change, given the private and the public signals and the belief that all other players j participate if and only if $\theta_j \geq \widehat{\theta}\left(v-t^*;p,s\right)$, is*

- *increasing in opacity unless the political regime is responsive and the society is radicalized and heterogenous;*
- *increasing in diversity unless the political regime is unresponsive;*
- *increasing in radicalization unless the political regime is responsive.*

The result is summed up in the following table:

Socio-pol. Var	Socio political situation	
	any	
repression	↘	
	any	
responsiveness	↗	
	DR&R	other
opacity	↘	↗
	R	other
radicalization	↘	↗
	R	other
diversity	↗	↘
	any	
unexp activism	↗	

Table 8: ***i*'s belief about the probability of policy change**

Step 3: Citizens that Will Participate to Protests A player i who believes that everyone else is using the cutoff rule $\widehat{\theta}\left(v-t^{*};p,s\right)$ will participate to protests if and only if

$$E\left[U_i\left(1,R\right);p,s\right]\geq E\left[U_i\left(0,R\right);p,s\right]\Leftrightarrow\mathbb{P}\left(\mathcal{N}\geq T|\theta_i,v-t^{*},\widehat{\theta}\left(v-t^{*};p,s\right)\right)\gamma\theta_i\geq k.$$

$$\Leftrightarrow\mathbb{P}\left(\mathcal{N}\geq T|\theta_i,v-t^{*},\widehat{\theta}\left(v-t^{*};p,s\right)\right)\gamma\theta_i\geq k\Leftrightarrow s_i\left(\theta_i,v;p,s\right)=1\Leftrightarrow$$

$$\Leftrightarrow\left[1-\Phi\left(\frac{\widehat{\theta}\left(v-t^{*};p,s\right)-\sigma_{\varepsilon}\Phi^{-1}\left(1-T\right)-\psi\left(v-t^{*}\right)-\left(1-\psi\right)\lambda\theta_i}{\sqrt{\psi\sigma_{\eta}^{2}}};p,s\right)\right]\gamma\theta_i\geq k.$$

Step 4: Citizens' Equilibrium Behavior Let define $i's$ expected incremental benefit (IB) from protesting when i is expecting the same cutoff behavior from the other citizens as

$$E\left[U_i\left(1,R;p,s\right)\right]=:IB\left(\theta_i,\widehat{\theta}\left(v-t^{*};p,s\right),v-t^{*};p,s\right)=$$

$$=\left[1-\Phi\left(\frac{\widehat{\theta}\left(v-t^{*};p,s\right)-\sigma_{\varepsilon}\Phi^{-1}\left(1-T\right)-\left[\psi\left(v-t^{*}\right)+\left(1-\psi\right)\lambda\theta_i\right]}{\sqrt{\psi\sigma_{\eta}^{2}}}\right)\right]\gamma\theta_i.$$

Equilibrium requires $\widehat{\theta}$ to be the solution of the following equation

$$IB\left(\widehat{\theta}\left(v-t^{*};p,s\right),\widehat{\theta}\left(v-t^{*};p,s\right),v-t^{*};p,s\right)=k.$$

Let define

$$\widehat{IB}\left(\widehat{\theta},v-t^{*};p,s\right):=IB\left(\widehat{\theta}\left(v-t^{*}\right),\widehat{\theta}\left(v-t^{*}\right),v-t^{*};p,s\right).$$

Then a generic $\widehat{\theta}$ is the equilibrium cutoff if and only if

$$\widehat{IB}\left(\widehat{\theta},v-t^{*};p,s\right)=k\Leftrightarrow$$

$$\Leftrightarrow\left[1-\Phi\left(\frac{\widehat{\theta}-\sigma_{\varepsilon}\Phi^{-1}\left(1-T\right)-\left[\psi\left(v-t^{*}\right)+\left(1-\psi\right)\lambda\widehat{\theta}\right]}{\sqrt{\psi\sigma_{\eta}^{2}}}\right)\right]\gamma\widehat{\theta}=k\Leftrightarrow$$

$$\Leftrightarrow \left[1-\Phi\left(\frac{[1-(1-\psi)\lambda]\widehat{\theta}-\sigma_\varepsilon\Phi^{-1}(1-T)-\psi(v-t^*)}{\sqrt{\psi\sigma_\eta^2}}\right)\right]\gamma\widehat{\theta}=k \Leftrightarrow$$

$$\Leftrightarrow \widehat{\theta}=\frac{k}{\gamma}\frac{1}{1-\Phi\left(\frac{[1-(1-\psi)\lambda]\widehat{\theta}-\sigma_\varepsilon\Phi^{-1}(1-T)-\psi(v-t^*)}{\sqrt{\psi\sigma_\eta^2}}\right)} \Leftrightarrow$$

$$\Leftrightarrow \widehat{\theta}=\frac{k}{\gamma}F\left(\widehat{\theta};p,s\right), \text{ where } F\left(\widehat{\theta};p,s\right):=\frac{1}{1-\Phi\left(\frac{[1-(1-\psi)\lambda]\widehat{\theta}-\sigma_\varepsilon\Phi^{-1}(1-T)-\psi(v-t^*)}{\sqrt{\psi\sigma_\eta^2}}\right)}.$$

Note that

$$F(0;p,s)=\frac{1}{1-\Phi\left(-\frac{\sigma_\varepsilon\Phi^{-1}(1-T)+\psi(v-t^*)}{\sqrt{\psi\sigma_\eta^2}}\right)}>1$$

$$\lim_{\widehat{\theta}\to\infty}F\left(\widehat{\theta};p,s\right)=\infty,\quad \lim_{\widehat{\theta}\to-\infty}F\left(\widehat{\theta};p,s\right)=1$$

$$\frac{\partial F\left(\widehat{\theta};p,s\right)}{\partial\widehat{\theta}}=\frac{\phi\left(\frac{[1-(1-\psi)\lambda]\widehat{\theta}-\sigma_\varepsilon\Phi^{-1}(1-T)-\psi(v-t^*)}{\sqrt{\psi\sigma_\eta^2}}\right)\frac{[1-(1-\psi)\lambda]}{\sqrt{\psi\sigma_\eta^2}}}{\left[1-\Phi\left(\frac{[1-(1-\psi)\lambda]\widehat{\theta}-\sigma_\varepsilon\Phi^{-1}(1-T)-\psi(v-t^*)}{\sqrt{\psi\sigma_\eta^2}}\right)\right]^2}>0.$$

Thus, the situation is represented in the following figure that shows the existence of 0 or 2 cutoff equilibria, depending on the values of p,s.

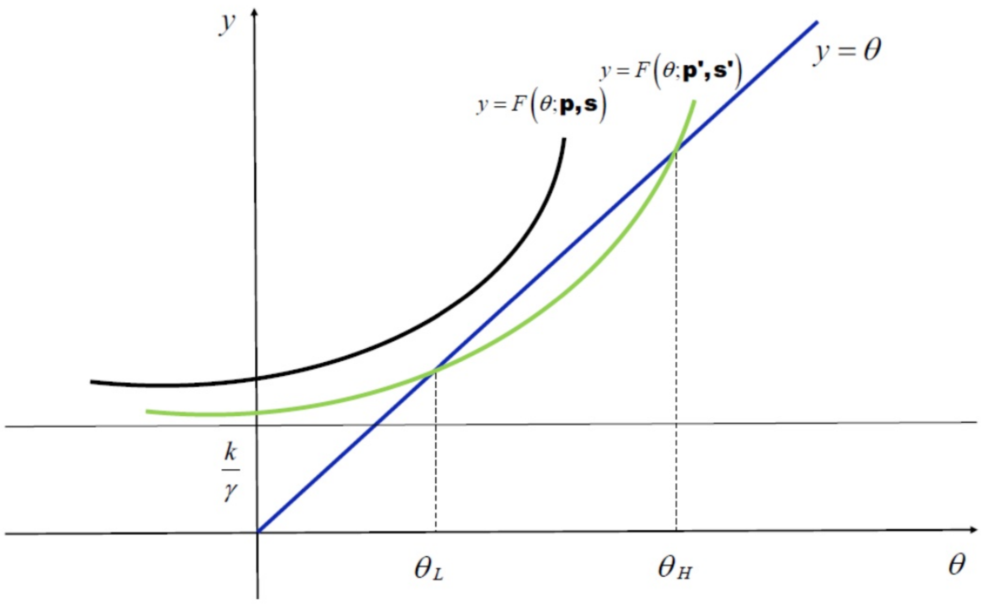

Figure 2.8: The equilibrium cutoff

Thus, the following result is quite immediate.

Result 2.9 *Any possible equilibrium cutoff satisfies the following restriction*

$$\widehat{\theta}\left(v-t^*;p,s\right)\in\left[\frac{k}{\gamma},\infty\right].$$

According to the previous result, any equilibrium cutoff satisfies the condition of individual rationality, i.e. $\widehat{\theta} \geq \frac{k}{\gamma}$. Moreover, any $\widehat{\theta} \in \left[\frac{k}{\gamma}, \infty\right]$ can be an equilibrium cutoff for certain combinations of the exogenous variables.

2.5.3 Vanguard Activism, Public Signal and Equilibrium Cutoff

In this subsection, which is original with respect to [6], we study the relationship between the level of $v - t^*$ and the citizens' equilibrium behavior such that a generic $\widehat{\theta} \in \left[\frac{k}{\gamma}, \infty\right]$ is an equilibrium cutoff, in order to characterize such equilibrium cutoff $\widehat{\theta}$. A generic $\widehat{\theta} \in \left[\frac{k}{\gamma}, \infty\right]$ is an equilibrium cutoff if and only if

$$\widehat{IB}\left(\widehat{\theta}, v - t^*; p, s\right) = k \Leftrightarrow$$

$$\Leftrightarrow \Phi\left(\frac{\widehat{\theta} - \sigma_\varepsilon \Phi^{-1}(1-T) - \left[\psi(v - t^*) + (1-\psi)\lambda\widehat{\theta}\right]}{\sqrt{\psi\sigma_\eta^2}}\right) = 1 - \frac{k}{\gamma\widehat{\theta}} \Leftrightarrow$$

$$\Leftrightarrow \frac{\widehat{\theta} - \sigma_\varepsilon \Phi^{-1}(1-T) - \left[\psi(v - t^*) + (1-\psi)\lambda\widehat{\theta}\right]}{\sqrt{\psi\sigma_\eta^2}} = -\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right) \Leftrightarrow$$

$$\Leftrightarrow v - t^* = \left(\sqrt{\frac{\sigma_\eta^2}{\psi}}\right)\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right) + \frac{[1-(1-\psi)\lambda]}{\psi}\widehat{\theta} - \frac{\sigma_\varepsilon}{\psi}\Phi^{-1}(1-T)$$

Definition 2.5 *Let define*

$$f\left(\widehat{\theta}; p, s\right) := \left(\sqrt{\frac{\sigma_\eta^2}{\psi}}\right)\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right) + \frac{[1-(1-\psi)\lambda]}{\psi}\widehat{\theta} - \frac{\sigma_\varepsilon}{\psi}\Phi^{-1}(1-T)$$

so that

$$v - t^* = f\left(\widehat{\theta}; p, s\right)$$

is the level of public signal (and of its unexpected component) such that a generic $\widehat{\theta} \in \left[\frac{k}{\gamma}, \infty\right]$ is an equilibrium cutoff.

Let we state some properties of $f\left(\widehat{\theta}; p, s\right)$.

Lemma 2.1 $f\left(\widehat{\theta}; p, s\right)$ *is*

- *U shaped in $\widehat{\theta} \in \left(\frac{k}{\gamma}, \infty\right)$, reaching a global minimum denoted by*

$$\widehat{\theta}^*(p, s) = \arg\min_{\widehat{\theta}} f\left(\widehat{\theta}; p, s\right);$$

- *decreasing in the responsiveness of the political regime;*
- *increasing in the repression of the political regime;*
- *uncertain in country diversity, radicalization and in information opacity.*

Using simulations, we are also able to derive the following result.

Result 2.10 *$f\left(\widehat{\theta};p,s\right)$ is*

- *increasing in opacity unless the political regime is responsive and tolerant;*
- *increasing in diversity unless the political regime is responsive;*
- *decreasing in radicalization unless the political regime is responsive but opaque and the society heterogenous.*

The results are summed up in the following table

Socio-pol. Var	**Socio political situation**	
	any	
repression	↗	
	any	
responsiveness	↘	
	RT	other
opacity	↘	↗
	D&RO	other
radicalization	↗	↘
	R	other
diversity	↘	↗
	any	
unexp activism	constant	

Table 9: $f\left(\widehat{\theta};\mathbf{p},s\right)$ and the sociopolitical variables

In conclusion, we can say that there exist and it is unique a $\widehat{\theta}\left(p,s\right)=\widehat{\theta}^{*}\left(p,s\right)$ for which the level of unexpected activism $v-t^{*}$ required for $\widehat{\theta}\left(p,s\right)=\widehat{\theta}^{*}\left(p,s\right)$ to be an equilibrium cutoff is minimal. Let we stress that while $\widehat{\theta}^{*}\left(p,s\right)$ does not depend on T, however $f\left(\widehat{\theta}^{*}\left(p,s\right);p,s\right)$ does depend on T.
The following figure shows the relationship between $v-t^{*}$ and $\widehat{\theta}$ in $v-t^{*}=f\left(\widehat{\theta};p,s\right)$:

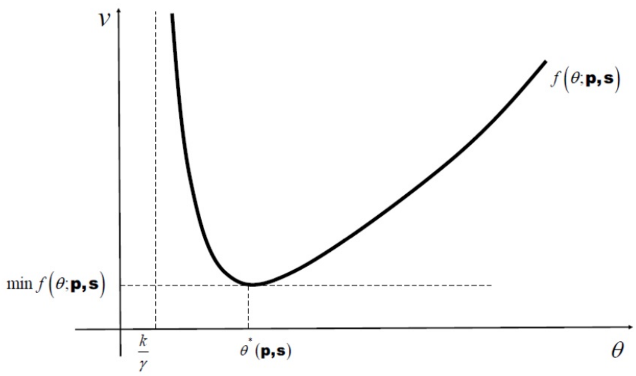

Figure 2.9: The function $v-t^{*}=f\left(\widehat{\theta}|p,s\right)$

Consider the properties of

$$\widehat{\theta}^*(p,s) = \arg\min_{\widehat{\theta}} f\left(\widehat{\theta}; p, s\right):$$

Lemma 2.2 *$\widehat{\theta}^*(p,s)$ is*

- *independent from responsiveness;*
- *increasing in the level of government repression;*
- *decreasing in the country diversity;*
- *increasing in the country radicalization;*
- *increasing in opacity.*

The results are summed up in the following table

Socio-pol. Var	**Socio political situation**
	any
repression	$\nearrow$
	any
responsiveness	constant
	any
opacity	constant
	any
radicalization	$\nearrow$
	any
diversity	$\searrow$
	any
unexp activism	constant

Table 10: $\widehat{\theta}^*(p,s)$ and the sociopolitical variables

The above results have the following implications for the possible equilibrium cutoff $\widehat{\theta}(v-t^*;p,s)$.

Result 2.11 *Consider the possible equilibrium payoff $\widehat{\theta}(v-t^*;p,s)$:*

- *If $(v-t^*)$ is small enough, i.e. if $(v-t^*) < f\left(\widehat{\theta}^*(p,s);p,s\right)$, then there exists no finite equilibrium cutoff, thus $\widehat{\theta}(v-t^*;p,s) = \infty$;*
- *If $(v-t^*) = f\left(\widehat{\theta}^*(p,s);p,s\right)$, then there exists one finite equilibrium cutoff and $\widehat{\theta}(v-t^*;p,s) = \widehat{\theta}^*(p,s)$;*
- *If $(v-t^*)$ is big enough, i.e. if $(v-t^*) > f\left(\widehat{\theta}^*(p,s);p,s\right)$, then there exists two finite equilibrium cutoff, thus $\widehat{\theta}(v-t^*;p,s) \in \left\{\widehat{\theta}_L(v-t^*;p,s), \widehat{\theta}_H(v-t^*;p,s)\right\}$ with*

$$\frac{k}{\gamma} < \widehat{\theta}_L(v-t^*;p,s) < \widehat{\theta}^*(p,s) < \widehat{\theta}_H(v-t^*;p,s).$$

The following figure illustrate the result:

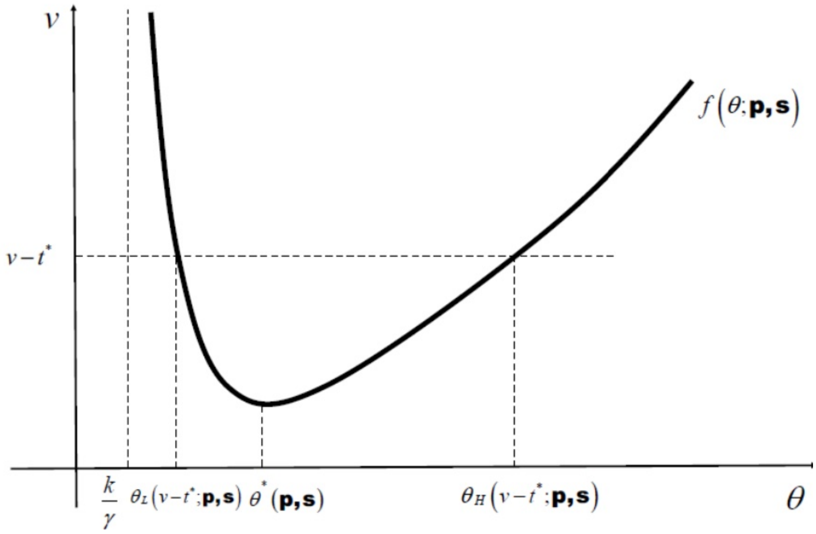

Figure 2.10: The equilibrium cutoffs

Note that the necessary and sufficient condition to get two finite equilibrium cutoff $\widehat{\theta}_L \leq \widehat{\theta}_H$ is

$$\left(v - t^*\right) > f\left(\widehat{\theta}^*\left(p, s\right); p, s\right),$$

so that for given $\left(v - t^*\right)$,this inequality restricts the set of possible sociopolitical and policy variables $\left(p, s\right) \in P \times S$.

Definition 2.6 *Let define*

$$\mathscr{PS}$$

the set of $\left(p, s\right)$ *such that for a given* $\left(v - t^*\right)$, $\left(v - t^*\right) > f\left(\widehat{\theta}^*\left(p, s\right); p, s\right)$. *Formally*

$$\mathscr{PS} := \left\{\left(p, s\right): \ \left(v - t^*\right) > min_{\widehat{\theta}} f\left(\widehat{\theta}; p, s\right)\right\}.$$

Therefore, we get the following result.

Proposition 2.1 *A strategy profile, where all citizens use the same strategy*

$$s : \mathbb{R} \times \mathbb{R} \to \{0, 1\}$$

which is not identically 0, *i.e. never participate to the protest, is consistent with a cutoff equilibrium if and only if:*

$$s\left(\theta_i, v - t^*; p, s\right) = \begin{cases} 1 & if \quad \theta_i \geq \widehat{\theta}\left(v - t^*; p, s\right) \\ 0 & \qquad otherwise \end{cases}$$

with $\widehat{\theta}\left(v - t^*; p, s\right)$ *satisfying the following conditions*

- $v - t^* = f\left(\widehat{\theta}\left(v - t^*; p, s\right); p, s\right)$;
- $\left(p, s\right) \in \mathscr{PS}$;

- *continuity in* (p,s).

To guarantee positive participation for a given realization of $v-t^*$, we make the following assumptions.

Assumption 2.1 $(p,s)\in\mathscr{PS}$.

It is now possible to characterize the citizens' equilibrium behavior in the protest stage.

Proposition 2.2 *There are three strategies for the citizens that are consistent with a cutoff equilibrium of the full game:*

- $$s^{\infty}\left(\theta_i, v-t^*; p,s\right)=0 \quad \textit{for all } \theta_i \textit{ and } v-t^*;$$

- $$s^{M}\left(\theta_i, v-t^*; p,s\right)=\begin{cases} 1 & if \quad \theta_i \geq \widehat{\theta}_H\left(v-t^*;p,s\right) \\ 0 & if \quad otherwise \end{cases}$$

- $$s^{L}\left(\theta_i, v-t^*; p,s\right)=\begin{cases} 1 & if \quad \theta_i \geq \widehat{\theta}_L\left(v-t^*;p,s\right) \\ 0 & if \quad otherwise. \end{cases}$$

This means that, for any level of activism above the minimum $\widehat{\theta}^*(p,s)$, there are two compatible equilibrium cutoffs $\widehat{\theta}_L$ and $\widehat{\theta}_H$.

Remark 2.4 *Note that* $\widehat{\theta}_L$ *decreases towards* $\frac{k}{\gamma}$ *as the level of unexpected activism increases, while* $\widehat{\theta}_H$ *grows towards infinity as activism increases. Intuitively, the growth in unexpected activism, net of the other variables in the game, should prompt citizens characterized by more restrained anti-government sentiment to join the protest. In this perspective it is implausible that as unexpected activism increases the level of anti-government sentiment for which one is indifferent to participate increases, as it happens with* $\widehat{\theta}_H$. *For this reason Bueno de Mesquita 2010 considers only the lower cutoff* $\widehat{\theta}_L$.

Assumption 2.2 *The space of possible equilibrium cutoff* $\widehat{\theta}$ *is restricted to*

$$\widehat{\theta}\in\left(\frac{k}{\gamma},\widehat{\theta}^*(p,s)\right)$$

so that citizens do no play the equilibrium strategy $s^M\left(\theta_i, v-t^*;p,s\right)$.

As a result of these assumptions, for each level of activism we will have a single finite equilibrium cutoff $\widehat{\theta}_L\left(v-t^*;p,s\right)$ with the following properties, that derives immediately from the previous characterization.

Result 2.12 *The finite equilibrium cutoff* $\widehat{\theta}_L\left(v-t^*;p,s\right)$ *is*

- *decreasing in the unexpected component of the public signal* $v-t^*$*;*
- *decreasing in the responsiveness of the political regime;*

- *increasing in the repression of the political regime;;*
- *uncertain in country diversity, radicalization and in information opacity.*

Using simulations, we are able to derive the following result.

Result 2.13 *The finite equilibrium cutoff $\widehat{\theta}_L\left(v - t^*; p, s\right)$ is*

- *increasing in opacity unless the political regime is responsive and tolerant;*
- *increasing in diversity unless the political regime is responsive;*
- *decreasing in radicalization unless the political regime is responsive but opaque and the society diverse.*

The following table sum up this results

Socio-pol. Var	**Socio political situation**	
	any	
repression	$\nearrow$	
	any	
responsiveness	$\searrow$	
	RT	other
opacity	$\searrow$	$\nearrow$
	D&RO	other
radicalization	$\nearrow$	$\searrow$
	R	other
diversity	$\searrow$	$\nearrow$
	any	
unexp activism	constant	

Table 9: cutoff and the sociopolitical variables

Definition 2.7 *The citizens that are not part of the silent group, but that do not demonstrate given the threshold $\widehat{\theta}_L\left(v - t^*; p, s\right)$ are called swing citizens,*[13] *because they can swing to protest if there are changes in the exogenous variables*

Remark 2.5 *The following figure shows how the citizens distribute among different categories depending on the political regime*

[13]Of course, the name is taken from the literature on swing voters.

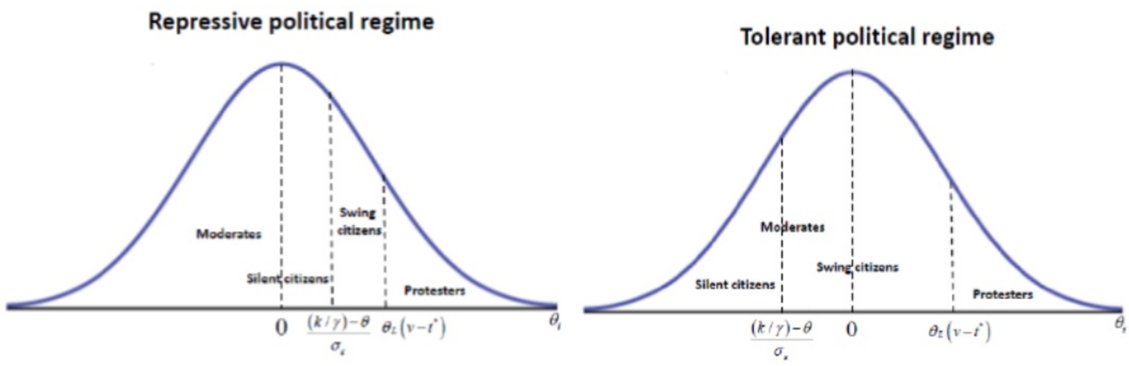

Figure 2.11: Moderates, Silent, Swing an Protesting Citizens

The previous result on $\widehat{\theta}_L\left(v-t^*;p,s\right)$ and the previous definitions imply the following result.

Result 2.14 *The percentage of protesting citizens is*

$$1-\Phi\left(\frac{\widehat{\theta}_L\left(v-t^*;p,s\right)-\theta}{\sigma_\varepsilon}\right),$$

which is

- *increasing in the antigovernment sentiment;*
- *increasing in the unexpected component of the public signal $v-t^*$;*
- *increasing in the responsiveness of the political regime;*
- *decreasing in the repression of the political regime;*
- *decreasing in the opacity of the political regime unless the political regime is responsive and tolerant;*
- *decreasing in diversity unless the political regime is responsive when $\widehat{\theta}_L\left(v-t^*;p,s\right)<\theta$ and after this threshold is increasing with upper limit $\frac{1}{2}$;*
- *increasing in radicalization unless the political regime is responsive but opaque and the society diverse.*

The following table sum up these results

Socio-pol. Var	Socio political situation	
	any	
repression	↘	
	any	
responsiveness	↗	
	RT	other
opacity	↗	↘
	D&RO	other
radicalization	↘	↗
	R	other
diversity	↗	↘
	any	
unexp activism	↗	
	any	
antigovernment sentiment	↗	

Table 12: protesting citizens and sociopolitical variables

This result is interesting because it states that the more democratic a political regime is, the greater is the percentage of protesting citizens, with an interesting exception: an increase in transparence would reduce participation when the political regime is highly democratic (D1 and D2). Hence in authoritarian regimes, the control of public information complements repression and unresponsiveness. As usual, the interaction between political and social variables is more complex: homogenous society are more likely to witness highly participated protests unless the political regime is partially democratic, i.e. responsive (regimes D1, D2, D4, D6 in table 2).

Result 2.15 *The percentage of swing citizens is:*

$$\Phi\left(\frac{\widehat{\theta}_L\left(v-t^*;p,s\right)-\theta}{\sigma_\varepsilon}\right)-\Phi\left(\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon}\right),$$

which is

- *first increasing, then decreasing in the antigovernment sentiment: the maximum is reached when*
$$\theta=\frac{1}{2}\left[\widehat{\theta}_L\left(v-t^*;p,s\right)+\frac{k}{\gamma}\right];$$
- *decreasing in the unexpected component of the public signal $v-t^*$;*
- *decreasing in the responsiveness of the political regime;*
- *first increasing and then decreasing in the level of government repression;*
- *increasing in opacity unless the political regime is responsive and the society radicalized;*

- *decreasing in diversity unless the political regime is tolerant and the society radicalized or the political regime is repressive and the society moderate, when it is first increasing and then decreasing;*
- *decreasing in radicalization unless the political regime is responsive but opaque and the society diverse.*

The following table sum up these results

Socio-pol. Var	**Socio political situation**		
	any		
repression	$\nearrow\searrow$		
	any		
responsiveness	$\searrow$		
	R&R		other
opacity	$\searrow$		$\nearrow$
	D&RO		other
radicalization	$\nearrow$		$\searrow$
	R&T	M&R	other
diversity	$\nearrow\searrow$		$\searrow$
	any		
unexp activism	$\searrow$		
	any		
antigovernment sentiment	$\nearrow\searrow$		

Table 13: swing citizens and sociopolitical variables

2.5.4 Vanguard Activism, Public Signal and Citizens' Protests

The aim of this subsection is to specify the situations characterized by point (η^*, θ) such that given vanguard's activism $(v - t^*)$, and the exogenous variables (p, s), some citizens will protest. These situations will be delimited by a curve representing the locus of points (η^*, θ) for which the participation to the protest showed by the citizens is strictly positive.

If the citizens play the strategy $s^{\infty}(\theta_i, v - t^*; p, s)$ so that no one ever participates, then vanguard activism has no effect on population members' behavior. But if population members play $s^L(\theta_i, v - t^*; p, s)$, activism can affect their behavior. The higher $v - t^*$, the more antigovernment sentiment each population member believes there is in society, since the expected average anti-government sentiment

$$E(\theta|\theta_i, v - t^*; p, s)$$

is increasing in $v - t^*$. Moreover, for a given cutoff rule, the higher θ, the more people will participate. Hence, higher levels of $v - t^*$ make population members believe that protest is more likely to succeed. This increases the incremental benefit of participation.

Consider the minimal level $v-t^*$ for which it is possible the existence of an equilibrium cutoff, i.e.

$$v-t^* \geq f\left(\widehat{\theta}^*\left(p,s\right);p,s\right):$$

since

$$v-t^* = \theta + \underbrace{t-t^*+\eta}_{\eta^*} = \theta + \eta^*$$

then

$$\begin{aligned} v-t^* &\geq f\left(\widehat{\theta}^*\left(p,s\right);p,s\right) \Leftrightarrow \theta+\eta^* \geq f\left(\widehat{\theta}^*\left(p,s\right);p,s\right) \Leftrightarrow \\ &\Leftrightarrow \theta \geq f\left(\widehat{\theta}^*\left(p,s\right);p,s\right) - \eta^*. \end{aligned}$$

Thus

Definition 2.8 *The function*

$$\mathring{\theta} := f\left(\widehat{\theta}^*\left(p,s\right);p,s\right) - \eta^* =: \mathring{\theta}\left(\eta^*;\left(\widehat{\theta}^*\left(p,s\right)\right);p,s\right)$$

represents the locus of points $\left(\eta^*,\theta\right)$ *such that activism is equal to the minimum level requested to have a finite equilibrium cutoff* $\widehat{\theta}_L\left(v-t^*;p,s\right)$.

From this definition and previous results, it is immediate to derive the following properties of this function

Lemma 2.3 *The function*

$$\mathring{\theta}\left(\eta^* \mid \left(\widehat{\theta}^*\left(p,s\right)\right),p,s\right)$$

has the following properties:

- *it is a straight line with domain* $\mathbb{R}$, *codomain* $\mathbb{R}$, *slope -1 and vertical intercept*

  $$f\left(\widehat{\theta}^*\left(p,s\right);p,s\right)$$

  defined for $\widehat{\theta}^*\left(p,s\right) \geq \frac{k}{\gamma}$;

- *all the points* (η^*,θ) *of the curve* $\mathring{\theta}\left(\eta^* \mid \left(\widehat{\theta}^*\left(p,s\right)\right),p,s\right)$ *are characterized by the same equilibrium cutoff* $\widehat{\theta}^*\left(p,s\right)$ *which is the cutoff that requires the lowest level of activism in order to be an equilibrium;*

- *as repression increases, the points on the line will be characterized by a higher level of unexpected activism* $v-t^*$ *and a higher cutoff* $\widehat{\theta}^*\left(p,s\right)$;

- *as responsiveness decreases, the points on the line will be characterized by a higher level of activism, but with the same cutoff* $\widehat{\theta}^*\left(p,s\right)$;

- *the vertical intercept*

  $$f\left(\widehat{\theta}^*\left(p,s\right);p,s\right)$$

  is

- *decreasing in the responsiveness of the political regime;*
- *increasing in the repression of the political regime;*
- *increasing in opacity unless the political regime is responsive and tolerant;*
- *increasing in diversity unless the political regime is responsive;*
- *decreasing in radicalization unless the political regime is responsive but opaque and the society heterogenous.*

The function $\mathring{\theta}\left(\eta^*;\left(\widehat{\theta}^*\left(p,s\right)\right);p,s\right)$is linear because, given values of $\left(p,s\right)$, there is a minimum level of activism so that the cutoff associated with it is $\widehat{\theta}^*\left(p,s\right)$. Given this equilibrium cutoff any real value of θ is compatible with the equilibrium, provided that η^* varies along with it while keeping the minimal level of activism constant. This leads to conclude that in this context θ is unrelated to the equilibrium cutoff $\widehat{\theta}^*\left(p,s\right)$ for which the level of activism is minimal; therefore despite the presence of the term $\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right)$ in the formulation of $f(\widehat{\theta};p,s)$ the function $\mathring{\theta}\left(\eta^*;\left(\widehat{\theta}^*\left(p,s\right)\right),p,s\right)$ is linear..

2.5.5 Vanguard Activism, Public Signal and Successful Protests

The aim of this subsection is to specify the situations characterized by point $\left(\eta^*,\theta\right)$ such that given vanguard's activism $(v-t^*)$, and the exogenous variables $\left(p,s\right)$, enough citizens will protest so that the protest is successful. These situations will be delimited by a curve representing the locus of points $\left(\eta^*,\theta\right)$ such that the participation to the protest showed by the citizens is equal to T. In the previous subsections, it has been shown that a generic $\widehat{\theta}\geq\frac{k}{\gamma}$ is the equilibrium cutoff if and only if

$$v-t^*=\theta+\eta=\frac{\left[1-(1-\psi)\lambda\right]\widehat{\theta}+\sqrt{\psi\sigma_\eta^2}\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right)-\sigma_\varepsilon\Phi^{-1}(1-T)}{\psi}.$$

Moreover, the participation is exactly equal to T if and only if

$$\mathbb{P}(\theta_i\geq\widehat{\theta})=T\Rightarrow 1-\Phi\left(\frac{\widehat{\theta}-\theta}{\sigma_\varepsilon}\right)=T\Rightarrow\widehat{\theta}=\theta+\sigma_\varepsilon\Phi^{-1}(1-T)_\varepsilon.$$

From these equations, it follows that

$$\theta+\eta=\frac{\left[1-(1-\psi)\lambda\right]\left[\theta+\sigma_\varepsilon\Phi^{-1}(1-T)\right]+\sqrt{\psi\sigma_\eta^2}\Phi^{-1}\left(\frac{k}{\gamma[\theta+\sigma_\varepsilon\Phi^{-1}(1-T)]}\right)-\sigma_\varepsilon\Phi^{-1}(1-T)}{\psi}\Rightarrow$$

$$\Rightarrow\eta^*=-\frac{\sigma_\varepsilon(1-\psi)\lambda\Phi^{-1}(1-T)}{\psi}+\frac{\left[\left(1-\psi\right)(1-\lambda)\right]\theta+\sqrt{\psi\sigma_\eta^2}\Phi^{-1}\left(\frac{k}{\gamma[\theta+\sigma_\varepsilon\Phi^{-1}(1-T)]}\right)}{\psi}.$$

Then we get the following definition.

Definition 2.9 *The function*

$$\eta^*\left(\theta;p,s\right)=\frac{\left[\left(1-\psi\right)(1-\lambda)\right]\theta+\sqrt{\psi\sigma_\eta^2}\Phi^{-1}\left(\frac{k}{\gamma[\theta+\sigma_\varepsilon\Phi^{-1}(1-T)]}\right)}{\psi}-\frac{\sigma_\varepsilon(1-\psi)\lambda\Phi^{-1}(1-T)}{\psi}$$

represents the locus of points $\left(\eta^*,\theta\right)$ such that the citizens' participation to the protest is equal to T.

The following result describes the properties of $\eta^*\left(\theta; p, s\right)$.

Lemma 2.4 *The curve*

$$\eta^*\left(\theta; p, s\right)$$

has the following properties:

- $\theta \in \left(\frac{k}{\gamma} - \sigma_\varepsilon \Phi^{-1}(1-T), \widehat{\theta}^*\left(p, s\right) - \sigma_\varepsilon \Phi^{-1}(1-T)\right]$*, while* $\eta^* \in (-\infty, \infty)$. *Note that* $\frac{k}{\gamma} - \sigma_\varepsilon \Phi^{-1}(1-T)$ *is positive if and only if* $1 - T \leq \Phi(\frac{k}{\gamma \sigma_\varepsilon})$;
- *it has a minimum in* $\widetilde{\theta}$;
- *it has an asymptote for* $\theta = \frac{k}{\gamma} - \sigma_\varepsilon \Phi^{-1}(1-T)$;
- *it is convex;*[14]
- *all points* $\left(\eta^*, \theta\right)$ *of the curve manifest a level of activism composed by a fixed part that varies in* $\left(p, s\right)$*, and a random part that varies in* $\left(\theta, p, s\right)$;
- *if the government repression grows, all the points* $\left(\eta^*, \theta\right)$ *of the curve will be characterized by a higher level of activism and a higher cutoff;*
- *if responsiveness decreases, all the points* $\left(\eta^*, \theta\right)$ *of the curve will be characterized by a higher level of activism and the same cutoff;*
- *the relationship between all the points* $\left(\eta^*, \theta\right)$ *of the curve and the country radicalization, diversity and opacity in public information is uncertain: it can be increasing, decreasing or non monotonic, depending on the values of the other parameters. Using simulations, we are able to derive the following results:* $\left(\eta^*, \theta\right)$ *is*
  - *increasing in opacity unless the political regime is responsive and the society is radicalized and homogeneous;*
  - *increasing in diversity;*
  - *decreasing in radicalization.*

2.6 The Equilibrium Outcomes

Which outcome will prevail depends on the citizen's behavior, which in turn will depend on the combinations of the functions

$$\mathring{\theta}\left(\eta^0; \widehat{\theta}^*\left(p, s\right); p, s\right) := f\left(\widehat{\theta}^*\left(p, s\right); p, s\right) - \eta^0 =$$

$$= \frac{\left[1 - (1-\psi)\lambda\right]\widehat{\theta}^* + \sqrt{\psi \sigma_\eta^2}\Phi^{-1}\left(\frac{k}{\gamma \widehat{\theta}^*}\right) - \sigma_\varepsilon \Phi^{-1}(1-T)}{\psi} - \eta^0 \Leftrightarrow$$

[14]This result implies that Figure 4 in De Mesquita 2020 is wrong because it depicts a function with an inflection point. Actually, $\eta^*\left(\theta; p, s\right)$ is a function as a function of θ, but it is not invertible, because for some values of η, ther are two values of θ.

$$\Leftrightarrow \eta^0 = -\mathring{\theta} + \left[\frac{1-(1-\psi)\lambda}{\psi}\right]\widehat{\theta}^* + \sqrt{\frac{\sigma_\eta^2}{\psi}}\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}^*}\right) + \frac{\sigma_\varepsilon}{\psi}\Phi^{-1}(1-T)$$

and

$$\eta^*\left(\theta; p, s\right) = \frac{\left[\left(1-\psi\right)(1-\lambda)\right]\theta + \sqrt{\psi\sigma_\eta^2}\Phi^{-1}\left(\frac{k}{\gamma(\theta-\Phi^{-1}(T)\sigma_\varepsilon)}\right)}{\psi} + \frac{(1-\psi)\lambda\Phi^{-1}(T)\sigma_\varepsilon}{\psi}$$

in the space

$$\left(\theta, \eta\right).$$

The following result describes the relationship between the two curves:

Result 2.16

- $\eta^*\left(\theta; p, s\right) \geq \eta^0\left(\theta; p, s\right)$ *for any* θ;
- $\eta^*\left(\theta; p, s\right) = \eta^0\left(\theta; p, s\right)$ *when* $\theta = \widehat{\theta}^* - \sigma_\varepsilon\Phi^{-1}(1-T)$.

For any level of vanguard's activism $(v - t^*)$, there are three possible equilibrium outcomes:

- **No protest**, when no citizens joins the protests against the government;
- **Failed protest,** when some citizens protest, but they are not enough to change the policy;
- **Successful protest**, when enough citizens join the protest and thus are able to change the policy.

Which outcome will prevail depends on the citizen's behavior, which in tun will depend on the realization of vanguard's activism, i.e. on $(v - t^*)$: since $v - t^* = \theta + t + \eta - t^*$, for given vanguard's effort and expected effort (t, t^*), then the citizens' behavior depend on the combinations of the behavioral functions described before

$$\mathring{\theta}\left(\eta^* | \widehat{\theta}^*\left(p, s\right); p, s\right) \text{ and } \eta^*\left(\theta; p, s\right)$$

in the space

$$\left(\eta, \theta\right).$$

To characterize these three possible outcome, consider the properties of these two functions.

Result 2.17 *There exist a unique*

$$\eta_0^* = f\left(\widehat{\theta}^*; p, s\right) - \widehat{\theta}^*\left(p, s\right) + \sigma_\varepsilon\Phi^{-1}(1-T)$$

such that

$$\mathring{\theta}\left(\eta_0^* | \widehat{\theta}^*\left(p, s\right), p, s\right) = \widehat{\theta}^*\left(p, s\right) - \sigma_\varepsilon\Phi^{-1}(1-T) = \theta_0.$$

Moreover the point (η_0^*, θ_0)

- *belongs to the curve* $\mathring{\theta}\left(\eta^* | \widehat{\theta}^*(p,s), p, s\right)$*;*

- *it is the unique point of the curve characterized by participation exactly equal to* T*;*

- *all the points of the curve characterized by* $\eta^* > \eta_0^*$ *will exhibit participation strictly less than* T*;*

- *all the points of the curve characterized by* $\eta^* < \eta_0^*$ *will exhibit participation strictly greater than* T*;*

- *it is the unique intersection with the curve* $\eta^*(\theta; p, s)$*.*

Result 2.18 *The function* $\eta^*(\theta; p, s)$ *has the following properties*

- *for a given* $\theta \in \left(\frac{k}{\gamma} - \sigma_\varepsilon \Phi^{-1}(1-T), \widehat{\theta}^*(p,s) - \sigma_\varepsilon \Phi^{-1}(1-T)\right]$*, the points of* $\eta^*(\theta; p, s)$ *are always on the east of* $\mathring{\theta}\left(\eta^* | \widehat{\theta}^*(p,s), p, s\right)$*, therefore* $\mathring{\theta}\left(\eta^* | \widehat{\theta}^*(p,s), p, s\right)$ *is dominated by* $\eta^*(\theta; p, s)$ *for all* $\theta \in \left(\frac{k}{\gamma} - \sigma_\varepsilon \Phi^{-1}(1-T), \widehat{\theta}^*(p,s) - \sigma_\varepsilon \Phi^{-1}(1-T)\right]$*;*

- *For descending values of* $\theta \in \left(\frac{k}{\gamma} - \sigma_\varepsilon \Phi^{-1}(1-T), \widehat{\theta}^*(p,s) - \sigma_\varepsilon \Phi^{-1}(1-T)\right]$ *the points of* $\eta^*(\theta; p, s)$ *exhibit ascending values of activism.*

These results justify the following picture, where we combine them.

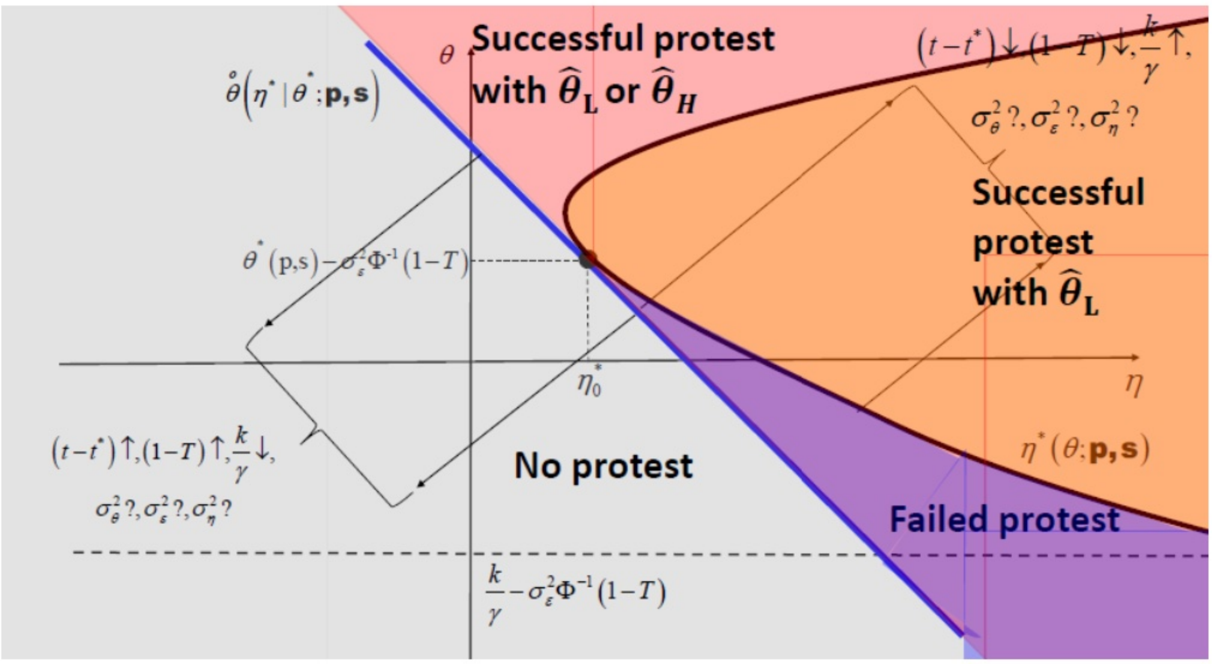

Figure 2.12: The possible equilibrium outcomes

2.7 The Probabilities of No Protest, of Failed Protest, and of Successful Protest

Our aim in this subsection is to evaluate the probabilities of the three possible equilibrium outcomes:

- no protest, the grey area;
- unsuccessful protest, the viola area;
- successful protest, the orange and the pink areas: orange, when the protest is successful only with the equilibrium cutoff $\widehat{\theta}_L\left(v-t^*;p,s\right)$, pink when the protest is successful also with the equilibrium cutoff $\widehat{\theta}_H\left(v-t^*;p,s\right)$.

Because of the results of previous subsection, there exists a unique $\eta^*=\eta_0^*$ such that

- $\eta_0^*=f\left(\widehat{\theta}^*\left(p,s\right);p,s\right)-\widehat{\theta}^*\left(p,s\right)-\Phi^{-1}(T)\sigma_\varepsilon$
- $\mathring{\theta}\left(\eta_0^*;\widehat{\theta}^*\left(p,s\right),p,s\right)=\theta_0$
- $\eta^*\left(\theta_0;p,s\right)=\eta_0^*$
- $\eta^*\left(\theta;p,s\right)$ is defined for $\eta^*\geq\eta_0^*$
- $\eta^*\geq\eta_0^*\Rightarrow\eta^*\left(\theta;p,s\right)\geq\mathring{\theta}\left(\eta^*;\widehat{\theta}^*\left(p,s\right),p,s\right)$.

A scenario where $\eta^*\left(\theta;p,s\right)\leq\mathring{\theta}\left(\eta^*;\widehat{\theta}^*\left(p,s\right),p,s\right)$ for $\eta^*\geq\eta_0^*$ is impossible, because if this were the case along the line $\eta^*\left(\theta;p,s\right)\geq\mathring{\theta}\left(\eta^*;\widehat{\theta}^*\left(p,s\right),p,s\right)$ for descending θ we would observe increasing participation at the same cutoff and activism level.

2.7.1 The Probability of Citizens' Protest

Result 2.19 *The probability of citizens' protest in equilibrium is*

- *increasing in the responsiveness of the political regime*;
- *decreasing in the repression of the political regime;*
- *uncertain in country radicalization, diversity and opacity.*

Using simulations, we are able to derive the following result:

Result 2.20 *The probability of citizens' protest in equilibrium is*

- *has no clear trend in opacity, even if responsiveness seems to induce an increasing trend;*
- *increasing in diversity unless the country is radicalized and the political regime is responsive but intolerant and opaque or unresponsive but tolerant and opaque;*

- *increasing in radicalization unless the political regime is responsive and tolerant and the society heterogenous.*

The following table sum up these results:

Socio-pol. Var	Socio political situation		
	any		
repression	$\searrow$		
	any		
responsiveness	$\nearrow$		
	D&RT	R&R	other
opacity	$\searrow$		$\nearrow$
	D&RT		other
radicalization	$\searrow$		$\nearrow$
	R&RRO	UTO	other
diversity	$\searrow$	$\searrow$	$\nearrow$
	any		
unexp activism	$\nearrow$		
	any		
antigovernment sentiment	$\nearrow$		

Table 14: probability of protest and sociopolitical variables

2.7.2 The Probability of No Protest

This result is the mirror image of previous one, however it has been proved independently and it interesting in itself.

Result 2.21 *The probability of no protest in equilibrium is*

- *decreasing in the responsiveness of the political regime;*
- *increasing in the repression of the political regime;*
- *uncertain in country radicalization, diversity and public information opacity.*

Using simulations, we are able to derive the following result:

Result 2.22 *The probability of no protest in equilibrium*

- *has no clear trend in opacity, even if responsiveness seems to induce a decreasing trend;*
- *is decreasing in diversity unless the country is radicalized and the political regime is responsive but intolerant and opaque or unresponsive but tolerant and opaque;*
- *is decreasing in radicalization unless the political regime is responsive and tolerant and the society heterogenous.*

2.7.3 The Probability of Successful Protest

Result 2.23 *The probability of successful protest is*

- *tending to increase in the responsiveness of the political regime;*
- *decreasing in the repression of the political regime;*
- *uncertain in σ^2_θ, σ^2_ε and σ^2_η: it can be increasing, decreasing or non monotonic, depending on the values of the other parameters.*

Using simulations, we are able to derive the following result.

Result 2.24 *The probability of successful protest is*

- *decreasing in opacity unless the political regime is responsive and the society is radicalized or heterogenous where is increasing for small level of opacity and then decreasing;*
- *increasing in diversity unless the political regime is unresponsive but tolerant and the society is radicalized;*
- *increasing in radicalization unless the political regime is responsive and tolerant and the society diverse.*

The following table sum up our results:

Socio-pol. Var	**Socio political situation**		
	any		
repression	↘		
	any		
responsiveness	↗		
	R&R	D&R	other
opacity	↗	↗↘	↘
	D&RT		other
radicalization	↘		↗
	R&UT		other
diversity	↘		↗
	any		
unexp activism	↗		
	any		
antigovernment sentiment	↗		

Table 15: probability of successful protest and sociopolitical variables

2.7.4 The Probability of Failed Protest

Result 2.25 *The probability of positive mobilization but failed protest has no clear monotone relationship with the responsiveness and the repression of the political regime, however our simulations show that the relations is*

- *increasing in responsiveness when the political regime is intolerant or tolerant and the society homogeneous, otherwise is not monotonic, first decreasing and increasing*
- *decreasing in repression unless the political regime is responsive and opaque when the relation is not monotonic, first increasing and then decreasing;*
- *decreasing in opacity unless the political regime is tolerant and the society radicalized when the relation is increasing;*
- *decreasing in diversity unless the political regime is unresponsive but tolerant and the society homogenous when the relation is increasing or when the political regime is responsive and tolerant and the society moderate when the relations is not monotonic, but first increasing and then decreasing;*
- *increasing in radicalization unless the political regime is responsive and tolerant, and the society is homogenous: in this case the relation is not monotonic, but first increasing and then decreasing.*

The following figure represents some interesting results of our simulations:

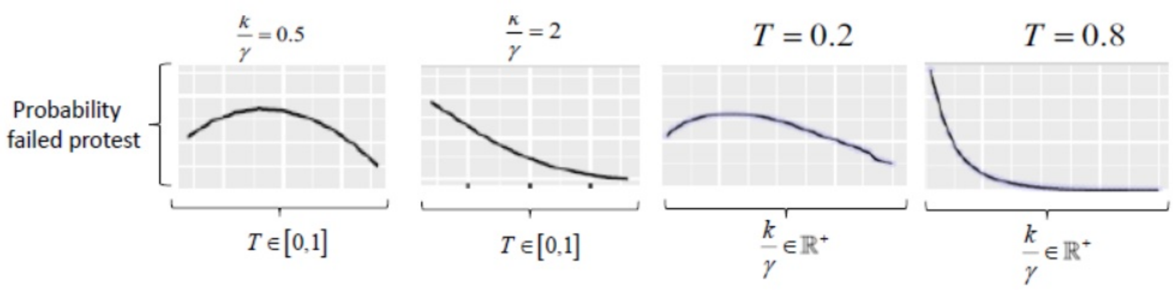

Figure 2.13: Probability of failed protest as function of T and of $\frac{k}{\gamma}$

The following table sum up these results:

Socio-pol. Var	Socio political situation			
	RO			any
repression	↗↘			↘
	D&T	R&TO		other
responsiveness	↘↗			↗
	D&T	R&T	R&R	other
opacity	↗			↘
	H&R			other
radicalization	↗↘			↗
	UO	M&RT		other
diversity	↗	↗↘		↘
	any			
antigovernment sentiment	↗↘			
	any			
antigovernment sentiment	↗↘			

Table 16: probability of failed protest and sociopolitical variables

To understand the reason for the non monotonicity in responsiveness, consider an increase in T. As shown, the mass of citizens protesting $\mathcal{N}\left(\theta,\widehat{\theta}\left(v-t^*\right)\right)$ is

$$\mathcal{N}\left(\theta,\widehat{\theta}_L\left(v-t^*;p,s\right)\right)=1-\Phi\left(\frac{\widehat{\theta}_L\left(v-t^*;p,s\right)-\theta}{\sigma_\varepsilon}\right)$$

which is decreasing in $\widehat{\theta}_L\left(v-t^*;p,s\right)$, which in turn is increasing in T. Thus the number of people revolting is decreasing as T increases, however also the threshold such that the protest is successful is increasing, thus it is possible that the probability of unsuccessful protest increases depending of which of the two factors is prevailing. The reason for the non monotonicity in $\frac{k}{\gamma}$ is similar.

2.8 Conclusion

The aim of this paper was to use the [6] model to investigate the causes and the consequences of citizens' protests. In particular, our aim was to analyze the complex interaction between political regimes and countries' social characteristics. Let we sum up our main findings.

2.8.1 Causes

The analysis of the causes of citizens' protests can be analyzed considering the effects of our sociopolitical variables on the number of protesting citizens and on the probability of protests.

The Number of Protesting Citizens

In general, an increase in the common antigovernment sentiment and of the activism of vanguards increases the number of protesting citizens, whatever the political regime. Similarly, an increase in the democratic dimensions of a polity - responsiveness, tolerance and transparence - increases the number of protesting citizens, apart from a paradoxical effect of transparency that reduces the number of protesting citizens in the most democratic polity.
Finally, an increase in country diversity increases the number of protesting citizens only in democratic regimes, while an increase in country radicalization produces the same effect unless the polity is responsive, but opaque and the country heterogenous.

The Probability of Protests

In general, an increase in the responsiveness and tolerance of the political regime increases the probability of protests, whatever the political regime, while an increase in transparency increases the probability of protests only if the polity is responsive, tolerant and the country heterogeneous or the polity is responsive and the country radicalized, otherwise the probability of protests is decreasing in transparence.
An increase in country diversity increases the probability of protests in responsive regimes in radicalized countries or in unresponsive, opaque but tolerant regimes, otherwise reduces such probability, while radicalization decreases this probability in responsive and tolerant regimes with heterogenous countries, otherwise the probability of protests is increasing.

2.8.2 Consequences

The analysis of the consequences of citizens' protests can be analyzed considering the effects of our sociopolitical variables on the probability of successful and of failed protests.

Successful Protests

A protest is successful if it is able to induce the government to change policy.
The probability of successful protests is usually increasing in our political and social variables, i.e. in responsiveness, tolerance, transparency, diversity and radicalization, apart from some interesting cases. Transparency, in the case of responsive regimes and heterogenous countries has a non monotonic effect, first decreasing and then increasing the probability of successful protests, and a decreasing relationship in the case of responsive regimes and radicalized countries. An increase in diversity decreases the probability of successful protests in an unresponsive but tolerant regime in a radicalized country, while an increase in radicalization reduces the probability of successful protests when the political regime is responsive and tolerant in a heterogenous country.

Failed Protests

A protest fails when the political regimes decides not to change policy notwithstanding the protests.
The probability of failed protests is usually increasing in responsiveness unless the political regimes is tolerant and the society heterogenous or tolerant but opaque and the society radicalized where this probability is first decreasing and then increasing. Similarly for tolerance, where its effects are non monotonic in a responsive but opaque regime. Transparency has a complex effect on this probability, which is usually increasing apart from the case of a tolerant political regime in a heterogenous or radicalized society or of a responsive regime in a radicalized society when its increase induces a reduction in this probability. An increase in diversity induces a decrement in this probability unless the political regime is unresponsive and opaque, or responsive and tolerant in a moderated society, when increments in diversity induce first an increment and then a decrement in this probability. Finally, an increment in radicalization generates a reduction in such probability, unless the regime is responsive and the society homogenous when first there is an increment and then a reduction.

2.9 Future Works

We believe that this paper has shown the efficacy of this model to analyze the causes and consequences of citizens' protests, extending Bueno de Mesquita analysis. On the other hand, according to our view, the main limitation of this model to analyze citizens' political behavior is twofold. First, it limits citizens' political behavior to a dual choice, whether to protest or not, while also the intensity of protests matters. Second, it limits citizens' political behavior to one dimension, while, beside protesting, citizens have other ways of dealing with public policies, for example voting or using violent tools. This notwithstanding, we believe this paper is a step towards a theory of how citizens come to political choices depending on different political and social settings, and how these choices affect the possible political outcomes.

2.10 Appendix

Proof of result 2.1

Since[15]

$$\theta|\theta_i \sim N(\lambda\theta_i, \lambda\sigma_\varepsilon^2) \text{ where } \lambda = \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}$$

- $$\frac{\partial E\left(\theta|\theta_i;p,s\right)}{\partial\theta_i} = \frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2} > 0$$

- If $\theta_i < 0$

$$\frac{\partial E\left(\theta|\theta_i;p,s\right)}{\partial\sigma_\varepsilon^2} = -\frac{\sigma_\theta^2}{(\sigma_\theta^2+\sigma_\varepsilon^2)^2}\,\theta_i > 0$$

$$\frac{\partial E\left(\theta|\theta_i;p,s\right)}{\partial\sigma_\theta^2} = \frac{\sigma_\varepsilon^2}{(\sigma_\theta^2+\sigma_\varepsilon^2)^2}\,\theta_i < 0$$

- If $\theta_i > 0$

$$\frac{\partial E\left(\theta|\theta_i;p,s\right)}{\partial\sigma_\varepsilon^2} = -\frac{\sigma_\theta^2}{(\sigma_\theta^2+\sigma_\varepsilon^2)^2}\,\theta_i < 0$$

$$\frac{\partial E\left(\theta|\theta_i;p,s\right)}{\partial\sigma_\theta^2} = \frac{\sigma_\varepsilon^2}{(\sigma_\theta^2+\sigma_\varepsilon^2)^2}\,\theta_i > 0$$

- $$\lim_{\sigma_\theta^2\to+\infty}\left(\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}\right)\theta_i = \lim_{\sigma_\theta^2\to+\infty}\left(\frac{1}{1+\frac{\sigma_\varepsilon^2}{\sigma_\theta^2}}\right)\theta_i = \left(\frac{1}{1+0}\right)\theta_i = \theta_i$$

- $$\lim_{\sigma_\varepsilon^2\to+\infty}\left(\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}\right)\theta_i = \left(\frac{1}{1+\infty}\right)\theta_i = 0.$$

Proof of result 2.2

Since[16]

$$\theta|\theta_i, v-t^* \sim N(\psi(v-t^*)+(1-\psi)\lambda\theta_i, \psi\sigma_\eta^2) \text{ where } \psi = \frac{\sigma_\theta^2\sigma_\varepsilon^2}{\sigma_\theta^2\sigma_\varepsilon^2+\sigma_\theta^2\sigma_\eta^2+\sigma_\eta^2\sigma_\varepsilon^2}$$

- $$\frac{\partial E\left(\theta|\theta_i, v-t^*;p,s\right)}{\partial(v-t^*)} = \psi > 0$$

$$\frac{\partial E\left(\theta|\theta_i, v-t^*;p,s\right)}{\partial\theta_i} = (1-\psi)\lambda > 0$$

- $$\frac{\partial E\left(\theta|\theta_i, v-t^*;p,s\right)}{\partial\sigma_\eta^2} = \frac{(\sigma_\varepsilon^2+\sigma_\theta^2)(\sigma_\varepsilon^2\sigma_\theta^2)(\lambda\theta_i-(v-t^*))}{(\sigma_\theta^2\sigma_\varepsilon^2+\sigma_\eta^2\sigma_\varepsilon^2+\sigma_\theta^2\sigma_\eta^2)^2} > 0 \Leftrightarrow v-t^* < \frac{\sigma_\theta^2\theta_i}{\sigma_\theta^2+\sigma_\varepsilon^2}$$

[15] See for example DeGroot 1970.

[16] See for example DeGroot 1970.

- $$\frac{\partial E\left(\theta|\theta_i, v-t^*; p, s\right)}{\partial \sigma_\theta^2} = \frac{\sigma_\varepsilon^2\sigma_\eta^2(\sigma_\varepsilon^2(v-t^*)+\sigma_\eta^2(\theta_i))}{(\sigma_\theta^2\sigma_\varepsilon^2+\sigma_\eta^2\sigma_\varepsilon^2+\sigma_\theta^2\sigma_\eta^2)^2} > 0 \Leftrightarrow v-t^* > -\frac{\theta_i\sigma_\eta^2}{\sigma_\varepsilon^2}$$

- $$\frac{\partial E\left(\theta|\theta_i, v-t^*; p, s\right)}{\partial \sigma_\varepsilon^2} = \frac{\sigma_\theta^2\sigma_\eta^2(\sigma_\theta^2(v-t^*-\theta_i)+\sigma_\eta^2(-\theta_i))}{(\sigma_\theta^2\sigma_\varepsilon^2+\sigma_\eta^2\sigma_\varepsilon^2+\sigma_\theta^2\sigma_\eta^2)^2} > 0 \Leftrightarrow v-t^* > \frac{\theta_i(\sigma_\theta^2+\sigma_\eta^2)}{\sigma_\theta^2}.$$

Proof of result 2.3

- If σ_η^2 increases $-\frac{\sigma_\eta^2}{\sigma_\varepsilon^2}$ decreases, while $\frac{\sigma_\eta^2+\sigma_\theta^2}{\sigma_\theta^2}$ increases shrinking the space of strongly incendiary/moderating signals;
- If σ_θ^2 increases $-\frac{\sigma_\eta^2}{\sigma_\varepsilon^2}$ does not vary, while $\frac{\sigma_\eta^2+\sigma_\theta^2}{\sigma_\theta^2}$ decreases, therefore the space of strongly incendiary/moderating signals is shrinking when country radicalization is decreasing;
- If σ_ε^2 increases $-\frac{\sigma_\eta^2}{\sigma_\varepsilon^2}$ increases, while $\frac{\sigma_\eta^2+\sigma_\theta^2}{\sigma_\theta^2}$ does not vary, therefore the space of strongly incendiary/moderating signals is shrinking when country diversity is decreasing.

Proof of result 2.4

If the protest is not successful, the optimal choice is always $a_i = 0$ because:

$$k > 0 \Rightarrow 0 > -k$$

If the protest is successful the citizens choose $a_i = 0$ if and only if:

$$(1-\gamma)\theta_i > \theta_i - k \Leftrightarrow \theta_i < \frac{k}{\gamma}$$

Therefore $a_i = 0$ is a dominant strategy fo those citizens characterized by $\theta_i < \frac{k}{\gamma}$. The strategy $a_i = 1$ can not be a dominant strategy because if the protest is not successful the optimal choice does not depend on θ_i and it is always $a_i = 0$.

Proof of result 2.5

- $$\frac{\partial \Phi\left(\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon}; p, s\right)}{\partial \frac{k}{\gamma}} = \phi\left(\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon}; p, s\right)\frac{1}{\sigma_\varepsilon} > 0$$

- $$\frac{\partial \Phi\left(\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon}; p, s\right)}{\partial \theta} = \phi\left(\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon}; p, s\right)\left(-\frac{1}{\sigma_\varepsilon}\right) < 0$$

- $$\frac{\partial \Phi\left(\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon};p,s\right)}{\partial \sigma_\varepsilon} = \phi\left(\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon};p,s\right)\left(-\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon^2}\right) < 0 \Leftrightarrow \frac{k}{\gamma}-\theta > 0$$

  In this case, as σ_ε increases, the mass of silent citizens decreases with lower limit $\frac{1}{2}$. If $\sigma_\varepsilon \to 0$ the mass of silent citizens becomes equal to 1.

- $$\frac{\partial \Phi\left(\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon};p,s\right)}{\partial \sigma_\varepsilon} = \phi\left(\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon};p,s\right)\left(-\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon^2}\right) > 0 \Leftrightarrow \frac{k}{\gamma}-\theta < 0$$

  In this case, as σ_ε increases, the mass of silent citizens increases with upper limit $\frac{1}{2}$. If $\sigma_\varepsilon \to 0$ the mass of silent citizens becomes equal to 0.

Proof of result 2.6

- Protests are impossible if the population is composed only by silent citizens therefore when:

  $$P\left(\theta_i < \frac{k}{\gamma}|\theta\right) = 1 \Leftrightarrow \Phi\left(\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon};p,s\right) = 1 \Leftrightarrow \frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon} = +\infty \Leftrightarrow \frac{k}{\gamma}-\theta > 0$$

- The necessary but not sufficient condition for the protest to be successful is that the mass of silent citizens is less than $1-T$ as there would be at least a potential T portion of participants

  $$P\left(\theta_i < \frac{k}{\gamma}|\theta\right) \le 1-T \Leftrightarrow \Phi\left(\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon};p,s\right) \le 1-T \Leftrightarrow \frac{k}{\gamma} \le \theta + \sigma_\varepsilon \Phi^{-1}(1-T)$$

  - Given $\frac{k}{\gamma} > 0$ if $T > \Phi\left(\frac{\theta}{\sigma_\varepsilon}\right)$ protests can not succeed because if:

    $$T > \Phi\left(\frac{\theta}{\sigma_\varepsilon}\right) \Rightarrow \theta + \sigma_\varepsilon \Phi^{-1}(1-T) < 0$$

    and therefore to observe a winning protest, $\frac{k}{\gamma}$ would need to be less than a negative value

  - The protests is not impossible if:

    $$\frac{k}{\gamma} - \theta \le \sigma_\varepsilon \Phi^{-1}(1-T)$$

    If $\frac{k}{\gamma} > \theta$ in order to observe successful protests $\sigma_\varepsilon \Phi^{-1}(1-T)$ must be positive hence:

    $$\sigma_\varepsilon \Phi^{-1}(1-T) > 0 \Leftrightarrow 1-T > \frac{1}{2}.$$

Proof of Conclusion 2.1

-
  - $$\lim_{1-T\to 1}\widehat{\theta}\left(v-t^*;p,s\right)-\sigma_\varepsilon\Phi^{-1}\left(1-T\right)=\widehat{\theta}\left(v-t^*;p,s\right)-\infty=-\infty$$
  - $$\lim_{1-T\to 0}\widehat{\theta}\left(v-t^*;p,s\right)-\sigma_\varepsilon\Phi^{-1}\left(1-T\right)=\widehat{\theta}\left(v-t^*;p,s\right)+\infty=+\infty$$
  - $$\theta^*\left(\widehat{\theta}\left(v-t^*;p,s\right);1-T=\frac{1}{2},\sigma_\varepsilon^2\right)=\widehat{\theta}\left(v-t^*;p,s\right)-0=\widehat{\theta}\left(v-t^*;p,s\right)$$
  - $$\theta^*\left(\widehat{\theta}\left(v-t^*;p,s\right);1-T=\Phi^{-1}\left(\frac{\widehat{\theta}\left(v-t^*;p,s\right)}{\sigma_\varepsilon}\right),\sigma_\varepsilon^2\right)= \\ =\widehat{\theta}\left(v-t^*;p,s\right)-\widehat{\theta}\left(v-t^*;p,s\right)=0$$

- $$\frac{\partial\theta^*\left(\widehat{\theta}\left(v-t^*;p,s\right);1-T,\sigma_\varepsilon^2\right)}{\partial\sigma_\varepsilon}=-\Phi^{-1}(1-T)>0\Leftrightarrow 1-T<\frac{1}{2}$$

-
  - $$\widehat{\theta}\left(v-t^*;p,s\right)\geq\frac{k}{\gamma}\Rightarrow\theta^*\left(\widehat{\theta}\left(v-t^*;p,s\right);1-T,\sigma_\varepsilon^2\right)\geq\frac{k}{\gamma}-\sigma_\varepsilon\Phi^{-1}(1-T)$$
  - $$\lim_{\widehat{\theta}(v-t^*)\to\infty}\theta^*\left(\widehat{\theta}\left(v-t^*;p,s\right);1-T,\sigma_\varepsilon^2\right)=\infty-\sigma_\varepsilon\Phi^{-1}(1-T)=+\infty$$
  - $$\widehat{\theta}\left(v-t^*;p,s\right)=\sigma_\varepsilon\Phi^{-1}(1-T)$$ $$\Rightarrow\theta^*\left(\widehat{\theta}\left(v-t^*;p,s\right);1-T,\sigma_\varepsilon^2\right)=\sigma_\varepsilon\Phi^{-1}(1-T)-\sigma_\varepsilon\Phi^{-1}(1-T)=0.$$

Proof of result 2.7

Consider i's subjective belief about the probability of policy change:

$$1-\Phi\left(\frac{\widehat{\theta}\left(v-t^*;p,s\right)-\sigma_\varepsilon\Phi^{-1}\left(1-T\right)-\psi\left(v-t^*\right)-\left(1-\psi\right)\lambda\theta_i}{\sqrt{\psi\sigma_\eta^2}};p,s\right)$$

- If $1-T$ grows the argument of $\Phi()$ decreases and therefore i's subjective belief about the probability of policy change increases.
- If $v-t^*$ grows the argument of $\Phi()$ decreases and therefore i's subjective belief about the probability of policy change increases.
- If θ_i grows the argument of $\Phi()$ decreases and therefore i's subjective belief about the probability of policy change increases.
- The uncertainty is related to the fact that that trends vary depending on the values assumed by $T, v-t^*, \theta_i, \sigma_\varepsilon, \sigma_\theta, \sigma_\eta$ jointly.

Proof of result 2.9

The equilibrium condition is:

$$1-\Phi\left(\frac{\widehat{\theta}\left(v-t^*;p,s\right)(1-(1-\psi)\lambda)-\sigma_\varepsilon\Phi^{-1}(1-T)-\psi\left(v-t^*\right)}{\sqrt{\psi\sigma_\eta^2}};p,s\right)=\frac{k}{\gamma\widehat{\theta}\left(v-t^*;p,s\right)}$$

If $\widehat{\theta}\left(v-t^*;p,s\right)<\frac{k}{\gamma}$ it would be impossible to solve the equation because the equilibrium condition would require a probability strictly greater than the unit. For this reason, the equilibrium condition by construction has as its solution in terms of equilibrium cutoff values greater than $\frac{k}{\gamma}$.

Proof of lemma 2.1

- $$\frac{\partial f\left(\widehat{\theta};p,s\right)}{\partial\widehat{\theta}}=\frac{\sigma_\eta}{\sqrt{\psi}}\frac{1}{\phi\left[\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right)\right]}\frac{k}{\gamma}\left(-\frac{1}{\widehat{\theta}^2}\right)+\frac{[1-(1-\psi)\lambda]}{\psi}=$$

  $$=\frac{\sigma_\eta}{\sqrt{\psi}}\frac{1}{\frac{1}{\sqrt{2\pi}}e^{-\frac{\left[\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right)\right]^2}{2}}}\frac{k}{\gamma}\left(-\frac{1}{\widehat{\theta}^2}\right)+\frac{[1-(1-\psi)\lambda]}{\psi}=$$

  $$=\frac{\sigma_\eta}{\sqrt{\psi}}\sqrt{2\pi}e^{\frac{\left[\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right)\right]^2}{2}}\frac{k}{\gamma}\left(-\frac{1}{\widehat{\theta}^2}\right)+\frac{[1-(1-\psi)\lambda]}{\psi}$$

  Since

  $$\frac{\sigma_\eta}{\sqrt{\psi}}\sqrt{2\pi}e^{\frac{\left[\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right)\right]^2}{2}}\frac{k}{\gamma}\left(-\frac{1}{\widehat{\theta}^2}\right)+\frac{[1-(1-\psi)\lambda]}{\psi}\geq 0\Leftrightarrow$$

  $$\Leftrightarrow e^{\frac{\left[\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right)\right]^2}{2}}\left(\frac{1}{\widehat{\theta}^2}\right)\leq\frac{[1-(1-\psi)\lambda]}{\psi}\frac{\gamma}{k}\frac{1}{\sqrt{2\pi}}\frac{\sqrt{\psi}}{\sigma_\eta}$$

  then, consider the function

  $$g(\widehat{\theta})=e^{\frac{\left[\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right)\right]^2}{2}}\left(\frac{1}{\widehat{\theta}^2}\right):$$

  - $g(\widehat{\theta})$ is a continuous function
  - $g(\widehat{\theta})$ has domain $\left(\frac{k}{\gamma},\infty\right)$ and codomain $(0,+\infty)$
  - $g(\widehat{\theta})$ is an injective function
  - $g(\widehat{\theta})$ is monotonically decreasing in $\widehat{\theta}$ along the entire domain with $\lim_{\widehat{\theta}\to\frac{k}{\gamma}}g(\widehat{\theta})=\infty$ and $\lim_{\widehat{\theta}\to\infty}g(\widehat{\theta})=0$.

  Moreover

  $$\frac{[1-(1-\psi)\lambda]}{\psi}\frac{\gamma}{k}\frac{1}{\sqrt{2\pi}}\frac{\sqrt{\psi}}{\sigma_\eta}\in(0,+\infty)$$

Therefore there exist an unique $\widehat{\theta}(\gamma,k)=\widehat{\theta}^*(\gamma,k)$ such that

$$g(\widehat{\theta})\leq\frac{[1-(1-\psi)\lambda]}{\psi}\frac{\gamma}{k}\frac{1}{\sqrt{2\pi}}\frac{\sqrt{\Psi}}{\sigma_\eta}\Leftrightarrow\widehat{\theta}\geq\widehat{\theta}^*(\gamma,k).$$

Therefore

$$min_{\widehat{\theta}}f\left(\widehat{\theta};p,s\right)=f\left(\widehat{\theta}^*\left(p,s\right),p,s\right).$$

Since the derivative is positive, once it passes the value where it is null $\widehat{\theta}^*\left(p,s\right)$ it will be a point of minimum and the function will have a U-shape.

- $$\frac{\partial f\left(\widehat{\theta};p,s\right)}{\partial T}=\frac{\partial}{\partial T}\left[\frac{\sigma_\varepsilon}{\psi}\Phi^{-1}(T)\right]=\frac{\sigma_\varepsilon}{\psi}\frac{1}{\phi[\Phi^{-1}(T)]}>0$$

  Since it is increasing in T it will be decreasing in $1-T$

- $$\frac{\partial f\left(\widehat{\theta};p,s\right)}{\partial\frac{k}{\gamma}}=\frac{\partial}{\partial\frac{k}{\gamma}}\left[\frac{\sigma_\eta}{\sqrt{\Psi}}\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right)\right]=\frac{\sigma_\eta}{\sqrt{\Psi}}\frac{1}{\phi[\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right)]}\frac{1}{\widehat{\theta}}>0$$

- The uncertainty is related to the fact that that trends vary depending on the values assumed by $T,\theta_i,\sigma_\varepsilon,\sigma_\theta,\sigma_\eta$ jointly

Proof of lemma 2.2

$\hat{\theta}^*$ is derived by:

$$\frac{df(\hat{\theta}|...)}{d\hat{\theta}}=0$$

$$\frac{\sigma_\eta}{\sqrt{\Psi}}\frac{1}{\phi\left(\Phi^{-1}\left(\frac{k}{\gamma\hat{\theta}}\right)\right)}\left(-\frac{k}{\gamma\hat{\theta}^2}\right)+\frac{[1-(1-\psi)\lambda]}{\psi}=0$$

- T is not involved within the equation $\Rightarrow\hat{\theta}^*$ does not depend on T

- Consider the level of violence necessary for a generic $\hat{\theta}$ to be equilibrium:

  $$f\left(\widehat{\theta}\left(p,s\right);p,s\right)=\frac{\sigma_\eta}{\sqrt{\Psi}}\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right)+\frac{[1-(1-\psi)\lambda]}{\psi}\widehat{\theta}+\frac{\sigma_\epsilon}{\psi}\Phi^{-1}(T)$$

  If $\frac{k}{\gamma}$ grows the asymptote of the curve in $\widehat{\theta}\left(p,s\right)$ grows. In addition, each $\widehat{\theta}\left(p,s\right)$ will be characterized by a greater value, so the curve not only shifts to the right due to the asymptote, but also grows upwards. Consequently given the U-shape of the curve and this transformation $\widehat{\theta}^*\left(p,s\right)$ increases

- Consider the condition useful to derive $\widehat{\theta}^*\left(p,s\right)$:

  $$\frac{\sigma_\eta}{\sqrt{\Psi}}\frac{1}{\phi\left(\Phi^{-1}\left(\frac{k}{\gamma\hat{\theta}}\right)\right)}\left(-\frac{k}{\gamma\hat{\theta}^2}\right)+\frac{[1-(1-\psi)\lambda]}{\psi}=0$$

Can be rewritten as

$$\frac{1}{\phi\left(\Phi^{-1}\left(\frac{k}{\gamma\hat{\theta}}\right)\right)}\frac{1}{\hat{\theta}^2} = \frac{[1-(1-\psi)\lambda]}{\sqrt{\psi}}\frac{\gamma}{k}\frac{1}{\sigma_\eta}$$

$$\frac{1}{\phi\left(\Phi^{-1}\left(\frac{k}{\gamma\hat{\theta}}\right)\right)}\frac{1}{\hat{\theta}^2} = \frac{\sigma_\theta^2\sigma_\epsilon + \sigma_\eta^2\sigma_\epsilon}{\sqrt{\sigma_\theta^2\sigma_\epsilon^2 + \sigma_\theta^2\sigma_\eta^2 + \sigma_\eta^2\sigma_\epsilon^2}}\frac{1}{\sigma_\eta\sigma_\theta}\frac{\gamma}{k}$$

$$\frac{1}{\phi\left(\Phi^{-1}\left(\frac{k}{\gamma\hat{\theta}}\right)\right)}\frac{1}{\hat{\theta}^2} = A$$

Only the right-hand member (called A) depends on $\sigma_\epsilon, \sigma_\eta, \sigma_\theta$

$$\frac{dA}{d\sigma_\epsilon} = \frac{\frac{2(\sigma_\theta^2+\sigma_\eta^2)(\sigma_\theta^2\sigma_\eta^2)}{2\left(\sqrt{\sigma_\theta^2\sigma_\epsilon^2+\sigma_\theta^2\sigma_\eta^2+\sigma_\eta^2\sigma_\epsilon^2}\right)}}{\left(\sqrt{\sigma_\theta^2\sigma_\epsilon^2 + \sigma_\theta^2\sigma_\eta^2 + \sigma_\eta^2\sigma_\epsilon^2}\right)^2} = \frac{2(\sigma_\theta^2+\sigma_\eta^2)(\sigma_\theta^2\sigma_\eta^2)}{2\left(\sqrt{\sigma_\theta^2\sigma_\epsilon^2 + \sigma_\theta^2\sigma_\eta^2 + \sigma_\eta^2\sigma_\epsilon^2}\right)^3} \geq 0$$

If σ_ϵ increases $\Rightarrow A$ increases and therefore since the left member is not affected by the variation of $\sigma_\epsilon \Rightarrow \hat{\theta}^*$ decreases

- $$\frac{dA}{d\sigma_\theta} = \frac{1}{\sigma_\theta\sigma_\eta}\frac{-2\sigma_\eta^4\sigma_\epsilon^3 - 4\sigma_\eta^4\sigma_\theta^2\sigma_\epsilon - 2\sigma_\theta^2\sigma_\epsilon^3\sigma_\eta^2}{2\left(\sqrt{\sigma_\theta^2\sigma_\epsilon^2 + \sigma_\theta^2\sigma_\eta^2 + \sigma_\eta^2\sigma_\epsilon^2}\right)^3} < 0$$

  If σ_θ increases $\Rightarrow A$ decreases and therefore since the left member is not affected by the variation of $\sigma_\theta \Rightarrow \hat{\theta}^*$ increases

- Similarly to the previous case if σ_η increases $\Rightarrow A$ decreases and therefore $\hat{\theta}^*$ increases.

Proof of result 2.11

The function $f\left(\widehat{\theta}\left(p,s\right);p,s\right)$ is U-shaped in $\widehat{\theta}\left(p,s\right)$ therefore there is an equilibrium cutoff $\widehat{\theta}^*\left(p,s\right)$ that requires the minimum level of violence $f\left(\widehat{\theta}^*\left(p,s\right);p,s\right)$.

- If $v-t^*$ is below this level no finite cutoff can be the equilibrium, because all the finite cutoffs in order to be the equilibrium require a level of violence greater than $f\left(\widehat{\theta}^*\left(p,s\right);p,s\right)$.
- If $v-t^*$ is exactly equal to this level the equilibrium cutoff will be exactly that finite value which requires the minimum level of violence therefore $\widehat{\theta}^*\left(p,s\right)$.
- If $v-t^*$ exceeds this value then given its U-shape there will exist two finite equilibrium values.

Proof of result 2.12

- By setting the level of $v-t^*$ above the minimum level required to observe a finite equilibrium cutoff, we will observe two equilibrium cutoffs. Since the curve is U-shaped as the level of $v-t^*$ decreases the two solutions will come closer together until they coincide into one when the violence reaches the compatibility minimum. Therefore the lower cutoff will increase towards $\widehat{\theta}^*(p,s)$ while the upper cutoff will decrease towards $\widehat{\theta}^*(p,s)$ as $v-t^*$ decreases. Consequently, the lower cutoff grows as $v-t^*$ decreases.

- Consider the level of violence necessary for a generic $\hat{\theta}$ to be equilibrium:

$$f\left(\widehat{\theta}(p,s);p,s\right)=\frac{\sigma_\eta}{\sqrt{\psi}}\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right)+\frac{[1-(1-\psi)\lambda]}{\psi}\widehat{\theta}+\frac{\sigma_\epsilon}{\psi}\Phi^{-1}(T)$$

  If T increases the curve is shifted upwards then fixed a level of violence $v-t^*$ given the U-shape of the curve we will observe two new solutions closer together, in other words the lower equilibrium cutoff will be higher, the upper one lower. Therefore the lower equilibrium cutoff is increasing in T and decreasing in $1-T$.

- Consider the level of violence necessary for a generic $\hat{\theta}$ to be equilibrium:

$$f\left(\widehat{\theta}(p,s);p,s\right)=\frac{\sigma_\eta}{\sqrt{\psi}}\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right)+\frac{[1-(1-\psi)\lambda]}{\psi}\widehat{\theta}+\frac{\sigma_\epsilon}{\psi}\Phi^{-1}(T)$$

  If $\frac{k}{\gamma}$ grows the asymptote of the curve in $\widehat{\theta}(p,s)$ grows. In addition, each $\widehat{\theta}(p,s)$ will be characterized by a greater value, so the curve not only shifts to the right due to the asymptote, but also grows upwards. Consequently given the U-shape of the curve fixed a level of violence $v-t^*$ we will have both solutions increased. While when $\frac{k}{\gamma}$ decreases the situation is the opposite therefore the lower cutoff is increasing in $\frac{k}{\gamma}$.

- The uncertainty is related to the fact that that trends vary depending on the values assumed by $T,\sigma_\varepsilon,\sigma_\theta,\sigma_\eta$ jointly.

Proof of result 2.14

- If θ increases the argument of $\Phi()$ decreases therefore the percentage of protesting citizens increases

- If $v-t^*$ increases $\widehat{\theta}_L\left(v-t^*;p,s\right)$ decreases (Result 13), the argument of $\Phi()$ decreases therefore the percentage of protesting citizens increases

- If T increases $\widehat{\theta}_L\left(v-t^*;p,s\right)$ increases (Result 13), the argument of $\Phi()$ increases therefore the percentage of protesting citizens decreases

- If $\frac{k}{\gamma}$ increases $\widehat{\theta}_L\left(v-t^*;p,s\right)$ increases (Result 13), the argument of $\Phi()$ increases therefore the percentage of protesting citizens decreases

- $\widehat{\theta}_L\left(v-t^*;p,s\right)$ is increasing in opacity unless the political regime is responsive, tolerant and the society is radicalized or the political regime is responsive and the society diverse (Result 14) therefore in these scenarios the percentage of protesting citizens decreases because the argument of $\Phi()$ increases

- $\widehat{\theta}_L\left(v-t^*;p,s\right)$ is increasing in diversity unless the political regime is responsive (Result 14) therefore in these scenarios the percentage of protesting citizens decreases rapidly since $\widehat{\theta}_L\left(v-t^*;p,s\right)<\theta$ because the argument of $\Phi()$ is negative and increasing and after this threshold the the argument of $\Phi()$ is positive and decreasing with upper limit $\frac{1}{2}$

- $\widehat{\theta}_L\left(v-t^*;p,s\right)$ is decreasing in radicalization unless the political regime is responsive but opaque and the society diverse (Result 14) therefore in these scenarios the percentage of protesting citizens increases because the argument of $\Phi()$ decreases.

Proof of result 2.15

- $$\frac{\partial\left(\Phi\left(\frac{\widehat{\theta}_L(v-t^*;p,s)-\theta}{\sigma_\varepsilon}\right)-\Phi\left(\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon}\right)\right)}{\partial\theta}=\frac{1}{\sigma_\varepsilon\sqrt{2\pi}}\left[-e^{-\left(\frac{\hat{\theta}-\theta}{\sigma_\varepsilon}\right)^2}+e^{-\left(\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon}\right)^2}\right]\geq 0\Leftrightarrow$$
$$\Leftrightarrow-\left(\frac{\hat{\theta}-\theta}{\sigma_\varepsilon}\right)^2\leq-\left(\frac{\frac{k}{\gamma}-\theta}{\sigma_\varepsilon}\right)^2\Leftrightarrow\theta\leq\frac{1}{2}\left[\hat{\theta}+\frac{k}{\gamma}\right]$$

- If $v-t^*$ increases $\widehat{\theta}_L\left(v-t^*;p,s\right)$ decreases (Result 13) and therefore the first member decreases while the second does not vary hence the percentage decreases.

- If T increases $\widehat{\theta}_L\left(v-t^*;p,s\right)$ increases (Result 13) and therefore the first member increases while the second does not vary hence the percentage increases. Clearly it is the opposite for $1-T$.

- If $\frac{k}{\gamma}$ increases both member increases therefore initially the percentage increases, but after a certain threshold it decreases

- $\widehat{\theta}_L\left(v-t^*;p,s\right)$ is increasing in opacity unless the political regime is responsive and the society radicalized (Result 14) therefore in these scenarios the first member increases while the second does not vary hence the percentage increases

- $\widehat{\theta}_L\left(v-t^*;p,s\right)$ is increasing in diversity unless the political regime is responsive (Result 14) therefore in these scenarios the percentage initially increases and then it decreases because as the diversity increases both members tends to $\frac{1}{2}$

- $\widehat{\theta}_L\left(v-t^*;p,s\right)$ is decreasing in radicalization unless the political regime is responsive but opaque and the society diverse (Result 14) therefore in these scenarios the first member decreases while the second does not vary hence the percentage decreases.

Proof of lemma 2.3

$$\mathring{\theta} := f\left(\widehat{\theta}^*\left(p,s\right);p,s\right)-\eta^* =: \mathring{\theta}\left(\eta^*;\left(\widehat{\theta}^*\left(p,s\right)\right);p,s\right)$$

- $\widehat{\theta}^*\left(p,s\right)\in\left(\frac{k}{\gamma},\infty\right)$ is a fixed value derived by the realization of the exogenous parameters $T,k,\gamma,\sigma_\varepsilon,\sigma_\theta,\sigma_\eta$ hence also $f\left(\widehat{\theta}^*\left(p,s\right);p,s\right)\in\mathbb{R}$ is a fixed value. Consequently the function is a straight line because it generates θ translating $-\eta^*$ by a fixed amount $f\left(\widehat{\theta}^*\left(p,s\right);p,s\right)$. Since η^*,θ are drawn by a normal distribution with support $\mathbb{R}$ the domain and the codomain of the function must be the set $\mathbb{R}$. The slope is the coefficient assigned to η^* hence -1, while the intercept is the value obtained when $\eta^*=0$ hence $f\left(\widehat{\theta}^*\left(p,s\right);p,s\right)$

- All the points of the curve are characterized by the same level of violence

$$\theta+\eta^*=f\left(\widehat{\theta}^*\left(p,s\right);p,s\right)$$

  That is the level of violence associated to the cutoff $\widehat{\theta}^*\left(p,s\right)$, therefore they are all characterized by the same cutoff $\widehat{\theta}^*\left(p,s\right)$

- The function $f\left(\widehat{\theta}\left(p,s\right);p,s\right)$ is increasing in $\frac{k}{\gamma}$ (Lemma 1) therefore its minimum value would be higher as $\frac{k}{\gamma}$ increases, consequently every point will be characterized by a higher level of violence. Moreover also $\widehat{\theta}^*\left(p,s\right)$ is increasing in $\frac{k}{\gamma}$ (Lemma 2) consequently every point will be characterized by a higher cutoff.

- The function $f\left(\widehat{\theta}\left(p,s\right);p,s\right)$ is decreasing in $1-T$ (Lemma 1) therefore its minimum value would be lower as $1-T$ increases, consequently every point will be characterized by a lower level of violence. Moreover $\widehat{\theta}^*\left(p,s\right)$ is independent from $1-T$ (Lemma 2) consequently every point will be characterized by the same cutoff.

- Follows from Result 2.10 and Lemma 2.1.

Proof of lemma 2.4

$$\eta^*\left(\theta;p,s\right)=\frac{\left[\left(1-\psi\right)\left(1-\lambda\right)\right]\theta+\sqrt{\psi\sigma_\eta^2}\Phi^{-1}\left(\frac{k}{\gamma\left[\theta+\sigma_\varepsilon\Phi^{-1}(1-T)\right]}\right)}{\psi}-\frac{\sigma_\varepsilon(1-\psi)\lambda\Phi^{-1}(1-T)}{\psi}$$

- The argument of $\Phi^{-1}()$ must be contained in the set $(0,1)$ therefore $\theta\geq\frac{k}{\gamma}-\sigma_\varepsilon\Phi^{-1}(1-T)$, while there are no constraints on the codomain of the function

- $$\frac{\partial \eta^*()}{\partial \theta} = \frac{(1-\psi)(1-\lambda)}{\psi} + \frac{\sigma_\eta}{\sqrt{\psi}}\sqrt{2\pi} e^{\frac{\left(\Phi^{-1}\left(\frac{k}{\gamma(\theta-\Phi^{-1}(T)\sigma_\varepsilon)}\right)\right)^2}{2}} \left(-\frac{k}{\gamma(\theta-\Phi^{-1}(T)\sigma_\epsilon)^2}\right) \geq 0 \Leftrightarrow$$

  $$\frac{\sigma_\eta}{\sqrt{\psi}}\sqrt{2\pi} e^{\frac{\left(\Phi^{-1}\left(\frac{k}{\gamma(\theta-\Phi^{-1}(T)\sigma_\varepsilon)}\right)\right)^2}{2}} \left(\frac{k}{\gamma(\theta-\Phi^{-1}(T)\sigma_\epsilon)^2}\right) \leq \frac{(1-\psi)(1-\lambda)}{\psi}$$

  It is possible to observe that

  $$\frac{(1-\psi)(1-\lambda)}{\psi}$$

  is positive and fixed

  $$\lim_{\theta-\Phi^{-1}(T)\sigma_\epsilon \to \frac{k}{\gamma}} \frac{\sigma_\eta}{\sqrt{\psi}}\sqrt{2\pi} e^{\frac{\left(\Phi^{-1}\left(\frac{k}{\gamma(\theta-\Phi^{-1}(T)\sigma_\varepsilon)}\right)\right)^2}{2}} \left(\frac{k}{\gamma(\theta-\Phi^{-1}(T)\sigma_\epsilon)^2}\right) = +\infty$$

  $$\lim_{\theta-\Phi^{-1}(T)\sigma_\epsilon \to +\infty} \frac{\sigma_\eta}{\sqrt{\psi}}\sqrt{2\pi} e^{\frac{\left(\Phi^{-1}\left(\frac{k}{\gamma(\theta-\Phi^{-1}(T)\sigma_\varepsilon)}\right)\right)^2}{2}} \left(\frac{k}{\gamma(\theta-\Phi^{-1}(T)\sigma_\epsilon)^2}\right) = 0^+$$

  Therefore there must be a $\overline{\theta}$ such that when $\theta \geq \overline{\theta}$ the inequality is satisfied hence $\overline{\theta}$ is the minimum

- Follows from point 1

  $$\frac{\partial^2 \eta^*()}{\partial \theta^2} = \frac{\sigma_\eta}{\sqrt{\psi}}\sqrt{2\pi} e^{\frac{\left(\Phi^{-1}\left(\frac{k}{\gamma(\theta-\Phi^{-1}(T)\sigma_\varepsilon)}\right)\right)^2}{2}} \left(\frac{k}{\gamma(\theta-\Phi^{-1}(T)\sigma_\epsilon)^3}\right) \times$$

  $$\times \left(2 + e^{\frac{\left(\Phi^{-1}\left(\frac{k}{\gamma(\theta-\Phi^{-1}(T)\sigma_\varepsilon)}\right)\right)^2}{2}} \left(\frac{k\Phi^{-1}\left(\frac{k}{\gamma(\theta-\Phi^{-1}(T)\sigma_\varepsilon)}\right)}{\gamma(\theta-\Phi^{-1}(T)\sigma_\epsilon)^3}\right)\right)$$

  is always positive hence the function is strictly convex if we consider the lower cutoff

- $$\eta^*() = -\frac{\sigma_\varepsilon(1-\psi)\lambda\Phi^{-1}(1-T)}{\psi} + \frac{\left[(1-\psi)(1-\lambda)\right]\theta + \sqrt{\psi\sigma_\eta^2}\Phi^{-1}\left(\frac{k}{\gamma[\theta+\sigma_\varepsilon\Phi^{-1}(1-T)]}\right)}{\psi}.$$

- $$\frac{\partial \eta^*(\theta; p, s)}{\partial \frac{k}{\gamma}} = \frac{\sigma_\eta}{\sqrt{\psi}} \frac{1}{\phi\left(\Phi^{-1}\left(\frac{k}{\gamma[\theta+\sigma_\varepsilon\Phi^{-1}(1-T)]}\right)\right)} \frac{1}{\theta+\sigma_\varepsilon\Phi^{-1}(1-T)} > 0$$

  When $\frac{k}{\gamma}$ increase the curve increases therefore all the points will be characterized by a higher level of violence and higher cutoff

- $$\frac{\partial \eta^*(\theta; p, s)}{\partial T} = \frac{\lambda(1-\psi)\sigma_\varepsilon}{\psi} \frac{1}{\phi(\Phi^{-1}(T))} > 0$$

  When T increases the curve is positively shifted on the η^* axes hence every point will be characterized by the same cutoff and a higher level of violence

- see appendix B.

Proof of result 2.16

- Points characterised by a level of violence below the minimum compatible with equilibrium by construction do not allow mobilization by citizens to be observed because there would be no participation rule. Consequently, the points on curve $\eta^*\left(\theta;p,s\right)$ that guarantee exactly T participation will always be above the points on curve $\eta^0\left(\theta;p,s\right)$ for any θ
- The two curves are equivalent at the point where the minimum violence level guarantees participation equal to T. The curve $\eta^0\left(\theta;p,s\right)$ is characterized by equilibrium cutoff $\widehat{\theta}^*$, the participation is exactly equal to T when is satisfied the condition $\theta = \hat{\theta} - \sigma_\varepsilon \Phi^{-1}(1-T)$, therefore in the point $\theta = \widehat{\theta}^* - \sigma_\varepsilon \Phi^{-1}(1-T)$. the curve $\eta^0\left(\theta;p,s\right)$ shows participation equal to T.

Proof of result 2.17

- Since the curve $\mathring{\theta}\left(\eta^*|\widehat{\theta}^*\left(p,s\right),p,s\right)$ has codomain $\mathbb{R}$ and $\widehat{\theta}^*\left(p,s\right)$ is a unique fixed value in the interval $\left(\frac{k}{\gamma},\infty\right)$ there must be a point in which the curve is equivalent to $\widehat{\theta}^*\left(p,s\right) - \sigma_\varepsilon \Phi^{-1}(1-T)$
- All the points of the curve $\mathring{\theta}\left(\eta^*|\widehat{\theta}^*\left(p,s\right),p,s\right)$ are characterized by the lowest level of violence compatible with equilibrium and equilibrium cutoff $\widehat{\theta}^*\left(p,s\right)$. The participation is exactly equal to T when is satisfied the condition $\theta = \hat{\theta} - \sigma_\varepsilon \Phi^{-1}(1-T)$, therefore in the point (η_0^*,θ_0) the participation is exactly equal to T.
- If $\eta^* > \eta_0^*$ the curve returns a level of θ strictly less than θ_0 and therefore a level of participation strictly less than T.

$$\eta^* > \eta_0^* \Rightarrow \mathring{\theta}\left(\eta^*|\widehat{\theta}^*\left(p,s\right),p,s\right) < \theta_0 = \widehat{\theta}^*\left(p,s\right) + \Phi^{-1}(T)\sigma_\varepsilon$$

- If $\eta^* < \eta_0^*$ the curve returns a level of θ strictly greater than θ_0 and therefore a level of participation strictly greater than T.

$$\eta^* < \eta_0^* \Rightarrow \mathring{\theta}\left(\eta^*|\widehat{\theta}^*\left(p,s\right),p,s\right) > \theta_0 = \widehat{\theta}^*\left(p,s\right) + \Phi^{-1}(T)\sigma_\varepsilon.$$

- Finally, these inequalities imply that (η_0^*,θ_0) is the only point on the curve $\mathring{\theta}\left(\eta^*|\widehat{\theta}^*\left(p,s\right),p,s\right)$ such that the citizens' participation to protest is exactly equal to T, thus point 5 follows.

Proof of result 2.18

Consider the points

- $O = \left(\eta_0^*,\mathring{\theta}\left(\eta_0^*|\widehat{\theta}^*\left(p,s\right),p,s\right)\right) = (\eta_0^*,\theta_0)$

- $A = \left(\eta_1^*, \mathring{\theta}\left(\eta_1^* | \widehat{\theta}^*\left(p,s\right), p, s\right)\right)$
- $B = \left(\eta_2^*, \mathring{\theta}\left(\eta_2^* | \widehat{\theta}^*\left(p,s\right), p, s\right)\right)$

where

- $\eta_0^* < \eta_1^* < \eta_2^*$
- $\mathring{\theta}\left(\eta_2^* | \widehat{\theta}^*\left(p,s\right), p, s\right) < \mathring{\theta}\left(\eta_1^* | \widehat{\theta}^*\left(p,s\right), p, s\right) < \theta_0$.

All the points belong to the curve $\mathring{\theta}\left(\eta^* | \widehat{\theta}^*\left(p,s\right), p, s\right)$, and for this reason they are characterized by the same level of activism ($f\left(\widehat{\theta}^*; p, s\right)$) and cutoff ($\widehat{\theta}^*\left(p,s\right)$). Consider the point O, where by construction the citizens' participation to protest is T. On the other hand, in A the cutoff is the same of O ($\widehat{\theta}^*\left(p,s\right)$), but

$$\mathring{\theta}\left(\eta_1^* | \widehat{\theta}^*\left(p,s\right), p, s\right) < \theta_0 = \widehat{\theta}^*\left(p,s\right) + \Phi^{-1}(T)\sigma_\varepsilon,$$

therefore the citizens' participation to the protest in A is clearly less than T. In order to observe participation equal to T for $\theta = \mathring{\theta}\left(\eta_1^* | \widehat{\theta}^*\left(p,s\right), p, s\right)$ it is necessary to have an higher level of activism that induces a lower equilibrium cutoff $\widehat{\theta} < \widehat{\theta}^*\left(p,s\right)$ such that

$$\mathring{\theta}\left(\eta_1^* \widehat{\theta}^*\left(p,s\right), p, s\right) = \widehat{\theta} + \Phi^{-1}(T)\sigma_\varepsilon.$$

Then, there exist a unique

$$\eta_A^* > \eta_1^*$$

that, for a fixed $\theta = \mathring{\theta}\left(\eta_1^* | \widehat{\theta}^*\left(p,s\right), p, s\right)$, there is a higher level of activism such that

$$\eta_A^* + \mathring{\theta}\left(\eta_1^* | \widehat{\theta}^*\left(p,s\right), p, s\right) >$$

$$> \eta_1^* + \mathring{\theta}\left(\eta_1^* | \widehat{\theta}^*\left(p,s\right), p, s\right) = f\left(\widehat{\theta}^*; p, s\right)$$

so that in the point $A_1 = (\eta_A^*, \mathring{\theta}\left(\eta_1^* | \widehat{\theta}^*\left(p,s\right), p, s\right))$ the participation to protest is T. Obviously the point A_1 lies on the east of A, because they show the same level of θ and $\eta_A^* > \eta_1^*$. The line $\mathring{\theta}\left(\eta^* | \widehat{\theta}^*\left(p,s\right), p, s\right)$ translated by $\eta_A^* - \eta_1^*$ represents all the combinations of θ and η^* for which we observe this new higher level of activism $\eta_A^* + \mathring{\theta}\widehat{\theta}^*\left(p,s\right), p, s$ and the equilibrium cutoff along this line will be

$$\mathring{\theta}\left(\eta_1^* | \widehat{\theta}^*\left(p,s\right), p, s\right) - \Phi^{-1}(T)\sigma_\varepsilon.$$

Similarly, in B the cutoff is the same of O ($\widehat{\theta}^*\left(p,s\right)$), but

$$\mathring{\theta}\left(\eta_2^* | \widehat{\theta}^*\left(p,s\right), p, s\right) < \theta_0 = \widehat{\theta}^*\left(p,s\right) + \Phi^{-1}(T)\sigma_\varepsilon,$$

therefore the participation in B is less than T. Similarly to the case of point A, there exists a unique

$$\eta_B^* > \eta_2^*$$

such that in the point $B_2 = \left(\eta_B^*, \mathring{\theta}\left(\eta_2^* | \widehat{\theta}^*\left(p,s\right), p, s\right)\right)$ the participation is equal to T. Moreover, note that

$$\eta_B^* - \eta_2^* > \eta_A^* - \eta_1^*.$$

If we translate the point B by $\eta_A^* - \eta_1^*$ (r_1) we obtain the point B_1 on the line r_1, because A and B are characterized by the same level of activism and A_1 and B_1 are their translation for a common value. The point B_1 is characterized by the same level of activism of A_1 and $\theta = \mathring{\theta}\left(\eta_2^* | \widehat{\theta}^*\left(p,s\right), p, s\right)$, but on the line r_1 the participation T is gained for $\mathring{\theta}\left(\eta_1^* | \widehat{\theta}^*\left(p,s\right), p, s\right) > \mathring{\theta}\left(\eta_2^* | \widehat{\theta}^*\left(p,s\right), p, s\right)$, therefore the participation T for $\theta = \mathring{\theta}\left(\eta_2^* | \widehat{\theta}^*\left(p,s\right), p, s\right)$ is observed in the point B_2 in the east of B and B_1 where the level of activism is higher than the one observed on the line r_1 and the cutoff is lower:

$$\eta_B^* + \mathring{\theta}\left(\eta_2^* | \widehat{\theta}^*\left(p,s\right), p, s\right) >$$

$$> \eta_A^* + \mathring{\theta}\left(\eta_1^* | \widehat{\theta}^*\left(p,s\right), p, s\right) >$$

$$> \eta_1^* + \mathring{\theta}\left(\eta_1^* \widehat{\theta}^*\left(p,s\right), p, s\right).$$

Then, the line $\mathring{\theta}\left(\eta^* | \widehat{\theta}^*\left(p,s\right), p, s\right)$ translated by $\eta_B^* - \eta_2^*$ (r_2) represents all the combinations of θ and η^* for which we observe this new higher level of activism $(\eta_B^* + \mathring{\theta}\left(\eta_2^* | \widehat{\theta}^*\left(p,s\right), p, s\right))$, and the equilibrium cutoff on this line will be

$$\mathring{\theta}\left(\eta_2^* | \widehat{\theta}^*\left(p,s\right), p, s\right) - \Phi^{-1}(T)\sigma_\varepsilon.$$

In conclusion if we consider all the θ for which $\eta^*\left(\theta; p, s\right)$ is defined and we iterate the above reasoning considering new starting points of $\mathring{\theta}\left(\eta^* | \widehat{\theta}^*\left(p,s\right), p, s\right)$ characterized by descending values of θ we get the lemma.

Proof of result 2.19

The probability of the outcome "mobilization" is given by:

$$1 - \frac{1}{2}(\alpha^* + \beta^*) =$$

$$= 1 - \frac{1}{2}\Phi\left(\frac{\widehat{\theta}^*\left(k, \gamma, \sigma_\theta^2, \sigma_\varepsilon^2, \sigma_\eta^2\right) + \Phi^{-1}(T)\sigma_\varepsilon}{\sigma_\theta}\right) +$$

$$-\frac{1}{2}\Phi\left(\frac{f\left(\widehat{\theta}^*\left(k, \gamma, \sigma_\theta^2, \sigma_\varepsilon^2, \sigma_\eta^2\right) | T, k, \gamma, \sigma_\theta^2, \sigma_\varepsilon^2, \sigma_\eta^2\right) - \widehat{\theta}^*\left(k, \gamma, \sigma_\theta^2, \sigma_\varepsilon^2, \sigma_\eta^2\right) - \Phi^{-1}(T)\sigma_\varepsilon}{\sigma_\eta}\right).$$

- The term α^* is clearly increasing in T. The term β^* is increasing in T since the numerator of the argument of $\Phi()$ can be rewritten as

$$\frac{\sigma_\eta}{\sqrt{\psi}}\Phi^{-1}\left(\frac{k}{\gamma\hat{\theta}^*}\right) + \frac{[1-(1-\psi)\lambda]}{\psi}\hat{\theta}^* + \frac{\sigma_\epsilon}{\psi}\Phi^{-1}(T) - \hat{\theta}^* - \Phi^{-1}(T)\sigma_\epsilon =$$

$$= \frac{\sigma_\eta}{\sqrt{\psi}}\Phi^{-1}\left(\frac{k}{\gamma\hat{\theta}^*}\right) + \hat{\theta}^*\left(\frac{[1-(1-\psi)\lambda]}{\psi} - 1\right)\hat{\theta}^* + \Phi^{-1}(T)\left(\sigma_\epsilon\left(\frac{1}{\psi} - 1\right)\right)$$

  Consequently the probabilty of "mobilization" is decreasing in T and therefore increasing in $1-T$

- Since $\widehat{\theta}^*$ is increasing in $\frac{k}{\gamma}$ the term α^* is increasing in $\frac{k}{\gamma}$. Regard the term β^* if we look at the numerator of the argument of $\Phi()$

$$= \frac{\sigma_\eta}{\sqrt{\psi}}\Phi^{-1}\left(\frac{k}{\gamma\hat{\theta}^*}\right) + \hat{\theta}^*\left(\frac{[1-(1-\psi)\lambda]}{\psi} - 1\right)\hat{\theta}^* + \Phi^{-1}(T)\left(\sigma_\epsilon\left(\frac{1}{\psi} - 1\right)\right)$$

when $\frac{k}{\gamma}$ increases $\widehat{\theta}^*$ increases therefore the second addend increases, while when $\frac{k}{\gamma}$ also the first addend increases similarly as we shown in previous results therefore β^* is increasing in $\frac{k}{\gamma}$ and the overall probability must be decreasing in $\frac{k}{\gamma}$

- with regard to the trends for these quantities, it is necessary to investigate them as they vary with the values assumed by the other exogenous variables.

Proof of result 2.21

The probability of the outcome "no mobilization" is given by:

$$\frac{1}{2}(\alpha^* + \beta^*) =$$

$$= \frac{1}{2}\Phi\left(\frac{\widehat{\theta}^*\left(k,\gamma,\sigma^2_\theta,\sigma^2_\varepsilon,\sigma^2_\eta\right) + \Phi^{-1}(T)\sigma_\varepsilon}{\sigma_\theta}\right) +$$

$$+\frac{1}{2}\Phi\left(\frac{f\left(\widehat{\theta}^*\left(k,\gamma,\sigma^2_\theta,\sigma^2_\varepsilon,\sigma^2_\eta\right)|T,k,\gamma,\sigma^2_\theta,\sigma^2_\varepsilon,\sigma^2_\eta\right) - \widehat{\theta}^*\left(k,\gamma,\sigma^2_\theta,\sigma^2_\varepsilon,\sigma^2_\eta\right) - \Phi^{-1}(T)\sigma_\varepsilon}{\sigma_\eta}\right).$$

- The term α^* is clearly increasing in T. The term β^* is increasing in T since the numerator of the argument of $\Phi()$ can be rewritten as

$$\frac{\sigma_\eta}{\sqrt{\psi}}\Phi^{-1}\left(\frac{k}{\gamma\hat{\theta}^*}\right) + \frac{[1-(1-\psi)\lambda]}{\psi}\hat{\theta}^* + \frac{\sigma_\epsilon}{\psi}\Phi^{-1}(T) - \hat{\theta}^* - \Phi^{-1}(T)\sigma_\epsilon =$$

$$= \frac{\sigma_\eta}{\sqrt{\psi}}\Phi^{-1}\left(\frac{k}{\gamma\hat{\theta}^*}\right) + \hat{\theta}^*\left(\frac{[1-(1-\psi)\lambda]}{\psi} - 1\right)\hat{\theta}^* + \Phi^{-1}(T)\left(\sigma_\epsilon\left(\frac{1}{\psi} - 1\right)\right)$$

Consequently the probabilty of "no mobilization" is increasing in T and therefore decreasing in $1-T$.

- Since $\widehat{\theta}^*$ is increasing in $\frac{k}{\gamma}$ the term α^* is increasing in $\frac{k}{\gamma}$. Regard the term β^* if we look at the numerator of the argument of $\Phi()$

$$= \frac{\sigma_\eta}{\sqrt{\psi}}\Phi^{-1}\left(\frac{k}{\gamma\hat{\theta}^*}\right) + \hat{\theta}^*\left(\frac{[1-(1-\psi)\lambda]}{\psi} - 1\right)\hat{\theta}^* + \Phi^{-1}(T)\left(\sigma_\epsilon\left(\frac{1}{\psi} - 1\right)\right)$$

when $\frac{k}{\gamma}$ increases $\widehat{\theta}^*$ increases therefore the second addend increases, while when $\frac{k}{\gamma}$ also the first addend increases similarly as we shown in previous results therefore β^* is increasing in $\frac{k}{\gamma}$ and the overall probability is increasing in $\frac{k}{\gamma}$.

- with regard to the trends for these quantities, it is necessary to investigate them as they vary with the values assumed by the other exogenous variables.

Proof of result 2.23

The probability of the outcome "successful protest" is given by:

$$(1-\alpha^*)(1-\beta^*)+\frac{1}{2}(1-\alpha^*)\beta^*+\delta(1-\rho)\frac{1}{2}\alpha^*(1-\beta^*)=$$

$$=1-\beta^*-\alpha^*+\alpha^*\beta^*+\frac{1}{2}\beta^*-\frac{1}{2}\alpha^*\beta^*+\frac{1}{2}\alpha^*\delta-\frac{1}{2}\alpha^*\delta\beta^*-\frac{1}{2}\alpha^*\delta\rho+\frac{1}{2}\alpha^*\delta\rho\beta^*=$$

$$=1-\frac{1}{2}\beta^*+\alpha^*(-1+\delta\frac{1}{2}-\delta\rho\frac{1}{2})+\frac{1}{2}\alpha^*\beta^*(1-\delta+\delta\rho)$$

where[17]

$$\alpha^*=\Phi\left(\frac{\widehat{\theta}^*\left(k,\gamma,\sigma_\theta^2,\sigma_\varepsilon^2,\sigma_\eta^2\right)+\Phi^{-1}(T)\sigma_\varepsilon}{\sigma_\theta}\right)$$

$$\beta^*=\Phi\left(\frac{f\left(\widehat{\theta}^*\left(k,\gamma,\sigma_\theta^2,\sigma_\varepsilon^2,\sigma_\eta^2\right)|T,k,\gamma,\sigma_\theta^2,\sigma_\varepsilon^2,\sigma_\eta^2\right)-\widehat{\theta}^*\left(k,\gamma,\sigma_\theta^2,\sigma_\varepsilon^2,\sigma_\eta^2\right)-\Phi^{-1}(T)\sigma_\varepsilon}{\sigma_\eta}\right)$$

Moreover

$$\frac{\partial(1-\frac{1}{2}\beta^*+\alpha^*(-1+\delta\frac{1}{2}-\delta\rho\frac{1}{2})+\frac{1}{2}\alpha^*\beta^*(1-\delta+\delta\rho))}{\partial\beta^*}=-\frac{1}{2}+\frac{1}{2}\alpha^*(1-\delta+\delta\rho)>0\Leftrightarrow$$

$$\alpha^*(1-\delta+\delta\rho)>1$$

Since the condition is not verifiable as both factors are strictly less than 1, the probability is decreasing in β^*.

$$\frac{\partial(1-\frac{1}{2}\beta^*+\alpha^*(-1+\delta\frac{1}{2}-\delta\rho\frac{1}{2})+\frac{1}{2}\alpha^*\beta^*(1-\delta+\delta\rho))}{\partial\alpha^*}=\left(-1+\delta\frac{1}{2}-\delta\rho\frac{1}{2}\right)+\frac{1}{2}\beta^*(1-\delta+\delta\rho)$$

$$(-1+\delta\frac{1}{2}-\delta\rho\frac{1}{2})+\frac{1}{2}\beta^*(1-\delta+\delta\rho)>0\Leftrightarrow\beta^*>\frac{2-\delta+\delta\rho}{1-\delta+\delta\rho}$$

Since the condition is not verifiable because the right member i greater than 1, the probability is decreasing in α^*.

- The probability is decreasing in both α^*,β^*, we have shown that both terms are increasing in T, therefore the overall probability is increasing in $1-T$. However when $\sigma_\theta^2,\sigma_\eta^2$ are high and σ_ε^2 is low there is initially a little non-monotonic effect due to the trend of ρ with respect to T.
- The probability is decreasing in both α^*,β^*, we have shown that both terms are increasing in $\frac{k}{\gamma}$ therefore the overall probability is decreasing in $\frac{k}{\gamma}$.
- with regard to the trends for these quantities, it necessary to investigate them as they vary with the values assumed by the other exogenous variables.

2.10.1 Simulations

The simulations mentioned in the chapter are written in R and the relevant code is available is available on Github following the path: *goergefil/PhD-Thesis-Simulations*

[17] ρ is derived by a precise probability ratio

SECTION

3 Regime Change Without Revolutionary Entrepreneurs

3.1 Introduction

In [6] Bueno de Mesquita proposed an important model to study why revolutionary vanguards might use violence to mobilize a mass of citizens against a regime. His analysis is focused on how vanguard's violence, affecting the population sentiment on a regime, may help its overthrown. In the paper Bueno de Mesquita proposes a particular global game with one-sided limit dominance that, differently from the usual global games with two-sided-limit dominance, have multiple equilibria, arguing for selecting one of these equilibria. The selected equilibrium has three possible probabilistic outcomes relative to citizens' protests: one where there is no mobilization, one with insufficient mobilization, and one with successful mobilization. Actually, in the Bueno de Mesquita model, the revolutionary entrepreneurs are modelled as sender of a public message related to the country's general antigovernment sentiment. Hence, if we want to understand the role of the vanguard in this context, i.e. the role of the public signal, it is interesting to analyze the model without the public signal. This is the aim of this work: to discuss the effectiveness of the Bueno de Mesquita model to understand the role of revolutionary vanguards.

The paper is organized as follows: initially we present and discuss the model and the interpretation of the exogenous variables, then in the following sections we reviews the properties of the citizens' beliefs, i.e. of a country public opinion, while section 4 derives citizens' behavior and we derive the possible equilibrium outcomes and discuss the relationships between the sociopolitical variables and the outcome probabilities.

This chapter will be presented during the 6th International Conference on the political economy of democracy and dictatorship (University of Münster, Germany, February 2023)

3.2 The Equilibria

3.2.1 The Equilibrium Concept

Denote by p, s the vector of political and social variables of the model:

$$(p,s) \in \left\{ p := \left(1-T, \frac{k}{\gamma}\right) \in (0,1] \times \mathbb{R}_+ \right\} \times \left\{ s := \left(\sigma_\theta^2, \sigma_\varepsilon^2\right) \in \mathbb{R}_+^2 \right\}.$$

The equilibrium concept of the game is pure strategy perfect Bayesian equilibrium (PBE) as in [6], where the set of equilibria is restricted in two ways:

- The players use cutoff strategies such that:

$$s_i(\theta_i) = \begin{cases} 1 & if \quad \theta_i \geq \widehat{\theta}(p,s) \\ 0 & if \quad \theta_i < \widehat{\theta}(p,s) \end{cases}$$

- For some values of the game variables the revolution stage is characterized by multiple finite equilibria in cutoff strategies; in this scenario the citizens play the same selection in terms of equilibrium.

Remark 3.1 *The difference with respect to [6] is that the equilibrium cutoff is no longer function of the public signal. Moreover as shown in the previous section* $\widehat{\theta} \in \left[\frac{k}{\gamma}, +\infty\right)$ *because any citizen with type* $\theta_i \in \left(-\infty, \frac{k}{\gamma}\right)$ *has a dominant strategy not to participate whatever* $i's$ *private signal.*

3.2.2 Citizens' Equilibrium Behavior

We derive the citizens' equilibrium behavior following four steps, as in [6].

Step 1: The Cutoff Rule

Let conjecture there exists a cutoff:

$$\widehat{\theta} \in \left[\frac{k}{\gamma}, +\infty\right]$$

such that in equilibrium:

$$s_i(\theta_i; p, s) = \begin{cases} 1 & if \quad \theta_i \geq \widehat{\theta}(p,s) \\ 0 & if \quad \theta_i < \widehat{\theta}(p,s) \end{cases} \iff$$

$$\iff s_i(\theta_i; p, s) = \begin{cases} 1 & if \quad \varepsilon_i \geq \widehat{\theta}(p,s) - \theta \\ 0 & if \quad \varepsilon_i < \widehat{\theta}(p,s) - \theta. \end{cases}$$

Step 2: i**'s Beliefs about the Probability of Regime Change**

From player i's perspective, if all other players use the cutoff rule $\widehat{\theta}(p,s)$, then j (with $j \neq i$) protests if and only if $\varepsilon_j \geq \widehat{\theta}(p,s) - \theta$. Thus, the mass of protesting citizens is [18]:

$$P(\varepsilon_j \geq \widehat{\theta}(p,s) - \theta) = 1 - \Phi\left(\frac{\widehat{\theta}(p,s) - \theta}{\sigma_\varepsilon}\right),$$

so that in equilibrium the protest succeeds if:

$$1 - \Phi\left(\frac{\widehat{\theta}(p,s) - \theta}{\sigma_\varepsilon}\right) \geq T \iff \Phi\left(\frac{\widehat{\theta}(p,s) - \theta}{\sigma_\varepsilon}\right) \leq 1 - T.$$

Since participation is increasing in θ, for any given cutoff rule $\widehat{\theta}(p,s)$ there exists a minimal level of θ, such that the protest is successful, which is the solution of the following equation in θ^*:

$$\Phi\left(\frac{\widehat{\theta}(p,s) - \theta^*}{\sigma_\varepsilon}\right) = 1 - T \iff \theta^*\left(\widehat{\theta}(p,s); T, \sigma_\varepsilon^2\right) = \widehat{\theta}(p,s) - \sigma_\varepsilon \Phi^{-1}(1-T) \in \mathbb{R}.$$

Conclusion 3.1 *The minimal level of the country antigovernment sentiment, such that the protest is successful when the common cutoff rule is* $\widehat{\theta}(p,s)$ *and there are only private signals is:*

$$\theta^*\left(\widehat{\theta}(p,s); T, \sigma_\varepsilon^2\right) = \widehat{\theta}(p,s) - \sigma_\varepsilon \Phi^{-1}(1-T),$$

which is:

- *decreasing in the responsiveness of the political regime, and such that:*
  - $\lim_{1-T\to 1} \theta^*\left(\widehat{\theta}(p,s); T, \sigma_\varepsilon^2\right) = -\infty$
  - $\lim_{1-T\to 0} \theta^*\left(\widehat{\theta}(p,s); T, \sigma_\varepsilon^2\right) = \infty$
  - $\theta^*\left(\widehat{\theta}(p,s); 1-T = \frac{1}{2}, \sigma_\varepsilon^2\right) = \widehat{\theta}(p,s) \geq \frac{k}{\gamma}$
  - $\theta^*\left(\widehat{\theta}(p,s); 1-T = \Phi\left(\frac{\widehat{\theta}(p,s)}{\sigma_\varepsilon}\right), \sigma_\varepsilon^2\right) = 0$
- *linearly increasing or decreasing in the diversity of the country depending whether the political regime is unresponsive or responsive*
- *linearly increasing in the common cutoff* $\widehat{\theta}(p,s)$, *such that:*
  - *the minimum is* $\frac{k}{\gamma} - \sigma_\varepsilon \Phi^{-1}(1-T)$
  - $\lim_{\widehat{\theta}(p,s)\to\infty} \theta^*\left(\widehat{\theta}(p,s); T, \sigma_\varepsilon^2\right) = \infty$
  - $\theta^*\left(\widehat{\theta}(p,s); T, \sigma_\varepsilon^2\right) = 0$ *if and only if* $\widehat{\theta}(p,s) = \sigma_\varepsilon \Phi^{-1}(1-T)$

[18] Denote by $\Phi(\cdot)$ the cumulative distribution function of the standard normal

Remark 3.2 *The previous results show that the removal of the public signal does not change the properties of the equilibrium cutoff, apart from the facts that:*

- $\widehat{\theta}(p,s)$ *does not depend on the public signal, which is random, hence in this setting the equilibrium cutoff is deterministic once the private signal is realized*
- $\widehat{\theta}(p,s)$ *and* $\theta^*\left(\widehat{\theta}(p,s);T,\sigma_\varepsilon^2\right)$ *may have different values*

At this point it is possible to evaluate i's subjective belief about the probability of regime change, given the private signal and the belief that all other players j (with $j \neq i$) participate if and only if $\theta_j \geq \widehat{\theta}(p,s)$:

$$\mathbb{P}\left(N \geq T|\theta_i, \widehat{\theta}(p,s);p,s\right) = \mathbb{P}\left(\theta \geq \theta^*\left(\widehat{\theta}(p,s)\right)|\theta_i;p,s\right) =$$
$$= 1 - \Phi\left(\frac{\widehat{\theta}(p,s) - \sigma_\varepsilon \Phi^{-1}(1-T) - \lambda\theta_i}{\sqrt{\lambda\sigma_\varepsilon^2}};p,s\right)$$

Result 3.1 *i's subjective belief about the probability of regime change, given the private signal and the belief that all other players j participate if and only if* $\theta_j \geq \widehat{\theta}(p,s)$*, is*

- *increasing in the responsiveness of the political regime*
- *increasing in the private signal*
- *increasing in diversity unless the political regime is unresponsive, monotonically if the political regime is sufficiently responsive*
- *increasing in radicalization unless the political regime is overly responsive*

The result is summed up in the following table:

Socio-pol. Var	**Socio political situation**	
	any	
private signal	↗	
	any	
responsiveness	↗	
	overly R	other
radicalization	↘	↗
	R	other
diversity	↗	↘

Table 4: *i*'s belief about the probability of policy change

Remark 3.3 *The previous results show that the removal of the public signal does not change the properties of i's subjective belief about the probability of regime change even if this probability can be different since* $\widehat{\theta}(p,s)$ *and* $\theta^*\left(\widehat{\theta}(p,s)\right)$ *can be different from the case with public signal.*

Step 3: Participating Citizens

A player i who believes that everyone else is using the cutoff rule $\widehat{\theta}(p,s)$ will participate to protests if and only if:

$$E\left[U_i(1,R);p,s\right] \geq E\left[U_i(0,R);p,s\right] \Longleftrightarrow \mathbb{P}\left(N \geq T|\theta_i,\widehat{\theta}(p,s)\right)\gamma\theta_i \geq k \Longleftrightarrow$$

$$\Longleftrightarrow s_i\left(\theta_i;p,s\right) = 1 \Longleftrightarrow \left[1-\Phi\left(\frac{\widehat{\theta}(p,s)-\sigma_\varepsilon\Phi^{-1}(1-T)-\lambda\theta_i}{\sqrt{\lambda\sigma_\varepsilon^2}};p,s\right)\right]\gamma\theta_i \geq k.$$

Step 4: Citizens' Equilibrium Behavior

Let define $i's$ expected incremental benefit (*IB*) from protesting when i is expecting the same cutoff behavior from the other citizens as:

$$E\left[U_i\left(1,R;p,s\right)\right] =: IB\left(\theta_i,\widehat{\theta}(p,s);p,s\right) = \left[1-\Phi\left(\frac{\widehat{\theta}(p,s)-\sigma_\varepsilon\Phi^{-1}(1-T)-\lambda\theta_i}{\sqrt{\lambda\sigma_\varepsilon^2}};p,s\right)\right]\gamma\theta_i$$

The equilibrium requires that a citizen whose type is right at the equilibrium cutoff (i.e. $\theta_i = \widehat{\theta}$) is exactly indifferent between participating and not. In other words:

$$IB\left(\widehat{\theta}(p,s),\widehat{\theta}(p,s);p,s\right) = k.$$

Let define:

$$\widehat{IB}\left(\widehat{\theta};p,s\right) := IB\left(\widehat{\theta},\widehat{\theta};p,s\right)$$

Then a generic $\widehat{\theta}$ is the equilibrium cutoff if and only if:

$$\widehat{IB}\left(\widehat{\theta};p,s\right) = k \Longleftrightarrow$$

$$\Longleftrightarrow \widehat{\theta} = -\frac{1}{1-\lambda}\left[\sqrt{\lambda\sigma_\varepsilon^2}\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right) - \sigma_\varepsilon\Phi^{-1}(1-T)\right]$$

Definition 3.1 *Let define:*

$$g\left(\widehat{\theta};p,s\right) := -\frac{1}{1-\lambda}\left[\sqrt{\lambda\sigma_\varepsilon^2}\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right) - \sigma_\varepsilon\Phi^{-1}(1-T)\right]$$

Result 3.2 $g\left(\widehat{\theta};p,s\right)$ *is:*

- *increasing in the responsiveness of the political regime*
- *decreasing in the repression of the political regime*

Result 3.3 $g\left(\widehat{\theta};p,s\right)$ *is:*

- *increasing in diversity when* $\widehat{\theta} \leq \widehat{\theta}^*$ *where* $\widehat{\theta}^*$ *is increasing in responsiveness*
- *increasing in radicalization when* $\widehat{\theta} \geq \widehat{\theta}^*$ *where* $\widehat{\theta}^*$ *is decreasing in responsiveness*

The results are summed up in the following table:

Socio-pol. Var	**Socio political situation**	
	any	
repression	↘	
	any	
responsiveness	↗	
	$\widehat{\theta} \geq \widehat{\theta}^*$	other
radicalization	↗	↘
	$\widehat{\theta} \leq \widehat{\theta}^*$	other
diversity	↘	↗

Table 5: $g\left(\widehat{\theta};\mathbf{p},s\right)$ **and the sociopolitical variables**

In conclusion, we can say that there may exist either zero or two solutions to the equation $\widehat{\theta} = g\left(\widehat{\theta};p,s\right)$ depending on the values of p,s. The figure 6 shows the situation.

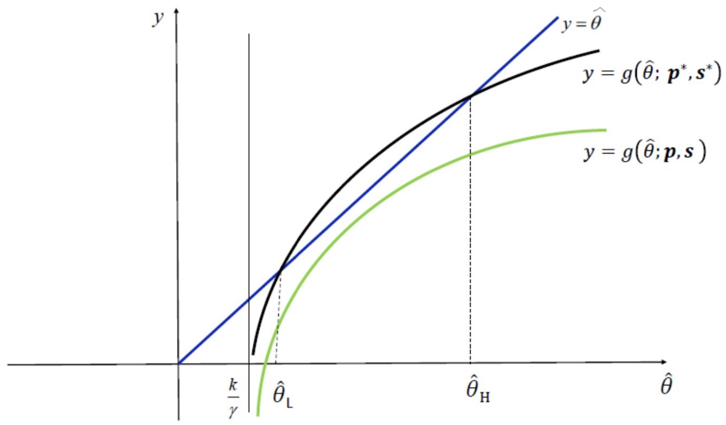

Figure 3.1: Existence of finite equilibrium cutoff

Result 3.4 *Any possible equilibrium cutoff satisfies the following restriction*

$$\widehat{\theta}\left(p,s\right) \in \left[\frac{k}{\gamma},\infty\right]$$

According to the previous result, any equilibrium cutoff satisfies the condition of individual rationality, i.e. $\widehat{\theta} \geq \frac{k}{\gamma}$. Moreover, any $\widehat{\theta} \in \left[\frac{k}{\gamma},\infty\right]$ can be an equilibrium cutoff for certain combinations of the exogenous variables.

Definition 3.2 *Let define :*

$$PS$$

the set of

$$\left(p, s\right) \in [0,1] \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}$$

such that there are two solutions to the equation :

$$\widehat{\theta} = g\left(\widehat{\theta}; p, s\right):$$

$$PS = \left\{\left(p, s\right) \in P \times S : \exists \widehat{\theta} \in \left(\frac{k}{\gamma}, +\infty\right) \mid \widehat{\theta} \leq g\left(\widehat{\theta}; p, s\right)\right\}.$$

Remark 3.4 *Obviously, we can partition the set*

$$P \times S = [0,1] \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}$$

in two sets PS *and its complement*

$$PS^C = \left\{\left(p, s\right) \in P \times S : \forall \widehat{\theta} \in \left(\frac{k}{\gamma}, +\infty\right) \ \widehat{\theta} > g\left(\widehat{\theta}; p, s\right)\right\}$$

so that

$$P \times S = PS \cup PS^C = [0,1] \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}.$$

Therefore, we can state the following result:

Result 3.5 *A strategy profile, where all citizens use the same strategy*

$$s : \mathbb{R} \to \{0,1\}$$

which is not identically 0*, i.e. never participate to the protest, is consistent with a cutoff equilibrium if and only if:*

$$s\left(\theta_i; p, s\right) = \begin{cases} 1 & if \quad \theta_i \geq \widehat{\theta}\left(p, s\right) \\ 0 & \quad otherwise \end{cases}$$

and

- $\left(p, s\right) \in PS$;
- $\widehat{\theta}\left(p, s\right)$ *is continuous in* $\left(p, s\right)$.

It is now possible to characterize the citizens' equilibrium behavior in the protest stage.

Proposition 3.1 *When*

- $\left(p, s\right) \in PS$*, then there are three strategies for the citizens that are consistent with a cutoff equilibrium of the full game:*

  –

$$s^{\infty}\left(\theta_i; p, s\right) = 0 \quad for\ all\ \theta_i;$$

–

$$s^M\left(\theta_i;p,s\right)=\begin{cases}1 & if \quad \theta_i \geq \widehat{\theta}_H\left(p,s\right)\\ 0 & if \quad otherwise\end{cases}$$

–

$$s^L\left(\theta_i;p,s\right)=\begin{cases}1 & if \quad \theta_i \geq \widehat{\theta}_L\left(p,s\right)\\ 0 & if \quad otherwise\end{cases}$$

where

$$\widehat{\theta}_L\left(p,s\right) \leq \widehat{\theta}_H\left(p,s\right);$$

- $\left(p,s\right) \in PS^C$, *then there is one cutoff equilibrium strategy for the citizens:*

$$s^{\infty}\left(\theta_i;p,s\right)=0 \quad \textit{for all } \theta_i.$$

Following the approach by [6], we make the following assumption:

Assumption 3.1 *When $\left(p,s\right) \in PS$, the space of possible equilibrium cutoff $\widehat{\theta}$ is restricted to $\widehat{\theta}_L$ so that citizens do not play the equilibrium strategy $s^M\left(\theta_i;p,s\right)$ or $s^{\infty}\left(\theta_i;p,s\right)$.*

As a result of this assumption, when $\left(p,s\right) \in PS$, we will observe a single finite equilibrium cutoff $\widehat{\theta}_L\left(p,s\right)$ with the following properties, that derives immediately from the previous characterization:

Result 3.6 *The finite equilibrium cutoff $\widehat{\theta}_L\left(p,s\right)$ is:*

- *decreasing in the responsiveness of the political regime*
- *increasing in the repression of the political regime*

Result 3.7 *The finite equilibrium cutoff $\widehat{\theta}_L\left(p,s\right)$ is:*

- *increasing in diversity unless the political regime is responsive*
- *decreasing in radicalization unless the political regime is responsive*

The following table sum up this results

Socio-pol. Var	**Socio political situation**	
	any	
repression	↗	
	any	
responsiveness	↘	
	R	other
radicalization	↗	↘
	R	other
diversity	↘	↗

Table 6: cutoff and the sociopolitical variables

We can conclude the analysis of the cutoff equilibria of the game with the following conclusion.

Conclusion 3.2 *Under our assumptions, depending on the exogenous variables, there are two different citizens's equilibrium behavior:*

- *when* $(p,s) \in PS$, *then there is a cutoff equilibrium strategy profile:*

$$s^L(\theta_i; p, s) = \begin{cases} 1 & if \quad \theta_i \geq \widehat{\theta}_L(p,s) \\ 0 & if \quad otherwise \end{cases}$$

  such that there are two possible equilibrium outcomes: successful protests or failed protests;

- *when* $(p,s) \in PS^C$, *then there is one cutoff equilibrium strategy profile:*

$$s^{\infty}(\theta_i; p, s) = 0 \quad for\ all\ \theta_i$$

  such that the unique equilibrium outcome is no protest.

3.3 Equilibrium Outcomes without Revolutionary Vanguards

3.3.1 The Set of Possible Socio-Political Regimes

According to the previous analysis, for any possible socio-political regime:

$$(p,s) \in PS \cup PS^C$$

there are three possible outcomes: no protest when $(p,s) \in PS^C$, and possibly failed protests and successful protests when $(p,s) \in PS$. Thus is important to get an idea of the characteristics of these two sets in order to understand when we get the above possible outcomes, and with which probability. Then, let we try to derive some qualitative property of the set PS.
By definition:

$$(p,s) \notin PS \Leftrightarrow \left[1 - \Phi\left(\frac{\widehat{\theta} - \sigma_\varepsilon \Phi^{-1}(1-T) - \lambda\widehat{\theta}}{\sqrt{\lambda \sigma_\varepsilon^2}}\right)\right] \gamma\widehat{\theta} < k$$

To have some intuition on PS, let consider the projection of this set on its political variables $\left(1-T, \frac{k}{\gamma}\right)$, i.e., fix a vector of sociopolitical variables $\overline{s} = \left(\sigma_\theta^2, \sigma_\varepsilon^2\right)$ and consider the set:

$$P(\overline{s}) = \left\{(p,\overline{s}) \in P \times \{\overline{s}\} : \exists \widehat{\theta} \in \left(\frac{k}{\gamma}, +\infty\right) \mid \widehat{\theta} \geq g\left(\widehat{\theta}; p, \overline{s}\right)\right\},$$

Similarly, let consider the projection of this set on its socio-political variables $\left(\sigma_\theta^2, \sigma_\varepsilon^2\right)$, i.e., fix a vector of sociopolitical variables $\overline{p} = \left(1-T, \frac{k}{\gamma}\right)$ and consider the set:

$$S(\overline{p}) = \left\{(s,\overline{p}) \in S \times \{\overline{p}\} : \exists \widehat{\theta} \in \left(\frac{k}{\gamma}, +\infty\right) \mid \widehat{\theta} \geq g\left(\widehat{\theta}; s, \overline{p}\right)\right\},$$

Result 3.8 *The area of socio-political regimes without protests PS^C is:*

- *increasing in repression*
- *decreasing in responsiveness*
- *decreasing in radicalization unless the political regime is responsive*
- *increasing in diversity unless the political regime is responsive*

Result 3.9 *If $\frac{k}{\gamma} \sim U(0, \hat{\theta})$ and $1 - T \sim U(0,1)$ the area of socio-political regimes without protests PS^C is greater than the area of socio-political regimes with protests PS*

The figure highlights what is contained in the last result:

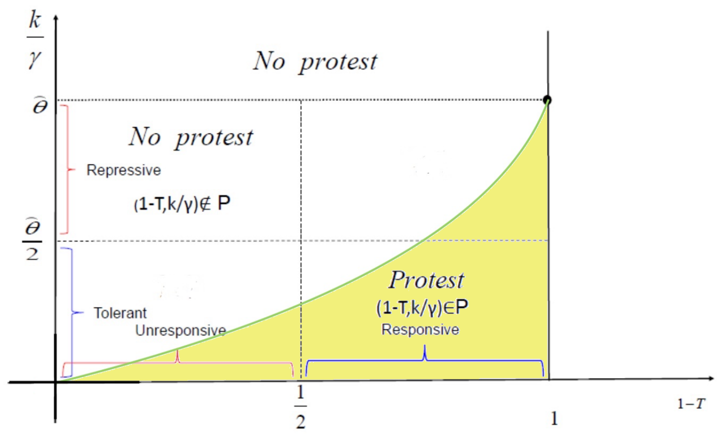

Figure 3.2: Areas of Socio-political regimes

3.3.2 Protesting Citizens

Consider the set where the socio-political variables are such that citizens in equilibrium have an incentive to protest, i.e. such that:

$$(p, s) \in PS.$$

The previous assumptions imply the following immediate result.

Result 3.10 *The percentage of **protesting citizens** is*

$$1 - \Phi\left(\frac{\widehat{\theta}_L(p,s) - \theta}{\sigma_\varepsilon}\right),$$

which is:

- *increasing in the antigovernment sentiment*
- *increasing in the responsiveness of the political regime*

- *decreasing in the repression of the political regime*
- *decreasing in diversity unless the political regime is responsive when $\widehat{\theta}_L(p,s) < \theta$, after this threshold is increasing with upper limit $\frac{1}{2}$*
- *increasing in radicalization unless the political regime is responsive*

The following table sum up these results:

Socio-pol. Var	**Socio political situation**	
	any	
repression	↘	
	any	
responsiveness	↗	
	R	other
radicalization	↘	↗
	R	other
diversity	↗	↘
	any	
antigovernment sentiment	↗	

Table 7: protesting citizens and sociopolitical variables

3.3.3 Private Signal and Successful Protests

Since the mass of protesting citizens is :

$$1 - \Phi\left(\frac{\widehat{\theta}_L(p,s) - \theta}{\sigma_\varepsilon}\right)$$

protests are successful if and only if:

$$1 - \Phi\left(\frac{\widehat{\theta}_L(p,s) - \theta}{\sigma_\varepsilon}\right) \geq T \Longleftrightarrow \widehat{\theta}_L(p,s) - \theta \leq \sigma_\varepsilon \Phi^{-1}(1-T) \Longleftrightarrow$$
$$\Longleftrightarrow \theta \geq \widehat{\theta}_L(p,s) - \sigma_\varepsilon \Phi^{-1}(1-T),$$

which implies the following result:

Result 3.11 *The protests will succeed with probability*

$$1 - \Phi\left(\frac{\widehat{\theta}_L(p,s) - \sigma_\varepsilon \Phi^{-1}(1-T)}{\sigma_\theta}\right).$$

The situation is represented in the following figure:

From previous results and considering the properties of $\widehat{\theta}_L(p,s)$, is possible to derive the following result:

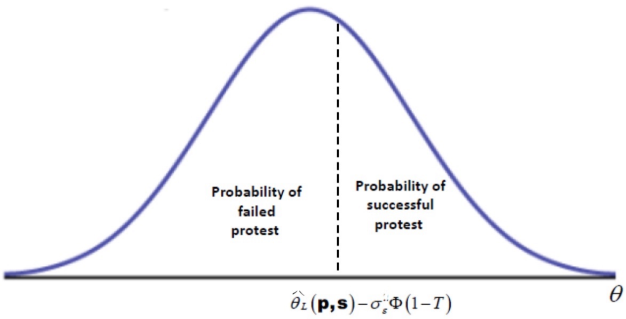

Figure 3.3: The probabilities of the two possible equilibrium outcomes

Result 3.12 *The probability of successful protest is:*

- *increasing in the responsiveness of the political regime*
- *decreasing in the repression of the political regime*
- *decreasing in diversity unless the political regime is responsive*
- *increasing in radicalization unless the political regime is responsive*

3.3.4 Political regimes and Equilibrium Outcomes

Consider the point of view of an external observer on the possible outcomes: if the external observer knows the specific social and political characteristics of the regimes, then either there is no protests with probability 1 or there are protests again with probability 1, which fail with probability $\Phi\left(\frac{\widehat{\theta}_L(p,s)-\sigma_\varepsilon\Phi^{-1}(1-T)}{\sigma_\theta^2}\right)$ and succeed with probability $1-\Phi\left(\frac{\widehat{\theta}_L(p,s)-\sigma_\varepsilon\Phi^{-1}(1-T)}{\sigma_\theta^2}\right)$.

On the other hand, it is possible that the external observer is uncertain on some specific characteristics of the regime. In this case, the socio-political variables can be considered random variables, so that there is a positive but not unitary probability α that $(p,s)\in PS$, and $1-\alpha$ that $(p,s)\in PS^C$. This is the approach we follow in the next sections, limiting our analysis to the political variables responsiveness and repression, $(1-T)$ and $\frac{k}{\gamma}$.

Uncertain Regime's Responsiveness

To perform this analysis, we need to introduce some new concepts and notation. Let we fix a partial vector of sociopolitical variables

$$\left(\overline{\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2}\right),$$

so to actually consider the projection of the set $P\times S$ on its first dimension $(1-T)\in[0,1]$ given a fixed vector $\left(\overline{\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2}\right)$. This means to consider the set:

$$\{1-T\}_{\left(\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right)} := \left\{\left(1-T,\overline{\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2}\right)\in[0,1]\times\left\{\overline{\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2}\right\}\right\}$$

This set, in turns can be partitioned in two sets such that there is or not protests:

$$\{1-T\}_{\left(\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right)} := \left\{(1-T)\in\{1-T\}_{\left(\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right)} : \exists\widehat{\theta}\in\left(\frac{k}{\gamma},+\infty\right)\ s.t.\ \widehat{\theta}\geq g\left(\widehat{\theta};1-T,\overline{\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2}\right)\right\}$$

and

$$\{1-T\}^C_{\left(\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right)} := \left\{(1-T)\in\{1-T\}_{\left(\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right)} : \forall\widehat{\theta}\in\left(\frac{k}{\gamma},+\infty\right)\ \widehat{\theta}< g\left(\widehat{\theta};1-T,\overline{\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2}\right)\right\},$$

Let we now introduce the simplest possible assumption on the randomness of the regime's responsiveness.

Hypothesis 3.1 *Suppose that an external observer consider a regime's responsiveness as uniformly distributed in* $[0,1]$:

$$1-T\sim U(0,1).$$

This assumption does not affect the distribution of $\theta|\theta_i$, thus we can work on the previous equilibrium condition defining equilibrium cutoff:

$$\left[1-\Phi\left(\frac{\widehat{\theta}-\sigma_\varepsilon\Phi^{-1}(1-T)-\lambda\widehat{\theta}}{\sqrt{\lambda\sigma_\varepsilon^2}}\right)\right]\gamma\widehat{\theta}=k.$$

Form this equation, is possible to derive an explicit function for $1-T$:

$$\left[1-\Phi\left(\frac{(1-\lambda)\widehat{\theta}-\sigma_\varepsilon\Phi^{-1}(1-T)}{\sqrt{\lambda\sigma_\varepsilon^2}}\right)\right]\gamma\widehat{\theta}=k\Rightarrow$$

$$\Longleftrightarrow 1-T=\Phi\left(\frac{\sigma_\varepsilon}{\sigma_\theta^2+\sigma_\varepsilon^2}\widehat{\theta}-\sqrt{\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}}\Phi^{-1}\left(1-\frac{k}{\gamma\widehat{\theta}}\right)\right).$$

Let define the following function:

$$h\left(\widehat{\theta}|\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right):=\Phi\left(\frac{\sigma_\varepsilon}{\sigma_\theta^2+\sigma_\varepsilon^2}\widehat{\theta}-\sqrt{\frac{\sigma_\theta^2}{\sigma_\theta^2+\sigma_\varepsilon^2}}\Phi^{-1}\left(1-\frac{k}{\gamma\widehat{\theta}}\right)\right)$$

which as a function of $\widehat{\theta}$ has a minimum:

$$(1-T)^*_{\left(\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right)}=\min_{\widehat{\theta}} h\left(\widehat{\theta}|\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right)$$

so that

$$\widehat{\theta}^*_{\left(\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right)} = \arg\min h\left(\widehat{\theta}|\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right)$$

Then

$$\{1-T\}^C_{\left(\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right)} = \left\{(1-T)\in[0,1] : (1-T) < (1-T)^*_{\left(\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right)}\right\}$$

Then, we are then able to derive the probabilities of the equilibrium outcomes.

Proposition 3.2 *In a cutoff equilibrium, the outcomes have the following probabilities:*

$$\mathbb{P}\left\{No\ revolt|\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right\} = (1-T)^*_{\left(\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right)}$$

$$\mathbb{P}\left\{Failed\ revolt|\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right\} = \left[\frac{\Phi\left(\frac{\widehat{\theta}_L\left(1-T,\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right)-\sigma_\varepsilon\Phi^{-1}(1-T)}{\sigma_\theta}\right)}{\sigma_\theta\Phi\left(\frac{\widehat{\theta}_L\left((1-T)^*,\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right)-\sigma_\varepsilon\Phi^{-1}((1-T)^*)}{\sigma_\theta}\right)}\right](T)^*_{\left(\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right)}$$

$$\mathbb{P}\left\{Successful\ revolt|\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right\} = \left[1-\frac{\Phi\left(\frac{\widehat{\theta}_L\left(1-T,\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right)-\sigma_\varepsilon\Phi^{-1}(1-T)}{\sigma_\theta}\right)}{\sigma_\theta\Phi\left(\frac{\widehat{\theta}_L\left((1-T)^*,\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right)-\sigma_\varepsilon\Phi^{-1}((1-T)^*)}{\sigma_\theta}\right)}\right](T)^*_{\left(\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right)}$$

The situation is represented in the following figure:

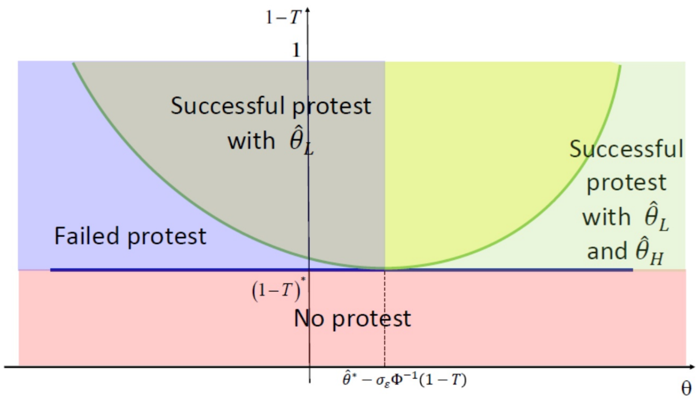

Figure 3.4: The space of the possible equilibrium outcomes

From the study of derivatives or from simple simulations it is possible to observe:

Proposition 3.3 *The threshold* $\{1-T\}^*_{\left(\frac{k}{\gamma},\sigma^2_\theta,\sigma^2_\varepsilon\right)}$ *is:*

- *Decreasing in radicalization*
- *Decreasing in diversity if the society is not radicalized*
- *Increasing in repression*

From this result and previous comparative statics results on the quantities involved in the game and by simulations, it can be stated that:

Corollary 3.1 *The probability of*

- *no mobilization is:*
  - *Increasing in participation cost*
  - *Decreasing in inclusiveness*
  - *Decreasing in redicalization*
  - *Decreasing in diversity if the society is not radicalized*
- *failed revolt is:*
  - *Decreasing in participation cost*
  - *Generally increasing in responsiveness except in one section where it is decreasing*
  - *Increasing in inclusiveness*
  - *Increasing in radicalization if the government is responsive or unresponsive with diversity*
  - *Decreasing in diversity if government is responsive while decreasing if it is unresponsive except in one section where it is increasing. Moreover if the government is responsive and the society not radicalized is decreasing except in one section where it is increasing*
- *successful revolt is:*
  - *Decreasing in participation cost*
  - *Tends to be increasing in responsiveness*
  - *Increasing in inclusiveness*
  - *Increasing in radicalization if the government is unresponsive or responsive with diversity*
  - *Decreasing in diversity if government is unresponsive while decreasing if it is responsive except in one section where it is increasing. Moreover if the government is responsive and the society not radicalized is decreasing*

Uncertain Regime's Political Repression

To perform this analysis, we need to introduce some new concepts and notation. Let we fix a partial vector of sociopolitical variables

$$\left(\overline{1-T,\sigma_\theta^2,\sigma_\varepsilon^2}\right),$$

so to actually consider the projection of the set $P \times S$ on its second dimension $\frac{k}{\gamma} \in [0,1]$ given a fixed vector $\left(\overline{1-T,\sigma_\theta^2,\sigma_\varepsilon^2}\right)$. This means to consider the set:

$$\left\{\frac{k}{\gamma}\right\}_{\left(\overline{1-T,\sigma_\theta^2,\sigma_\varepsilon^2}\right)} := \left\{\left(\overline{1-T},\frac{k}{\gamma},\overline{\sigma_\theta^2,\sigma_\varepsilon^2}\right) \in \left\{\overline{1-T}\right\} \times \mathbb{R}_+ \times \left\{\overline{\sigma_\theta^2,\sigma_\varepsilon^2}\right\}\right\}.$$

This set, in turns can be partitioned in two sets such that there is or not protests:

$$\{R\}_{\left(\overline{1-T,\sigma_\theta^2,\sigma_\varepsilon^2}\right)} := \left\{\frac{k}{\gamma} \in \left\{\frac{k}{\gamma}\right\}_{\left(\overline{1-T,\sigma_\theta^2,\sigma_\varepsilon^2}\right)} : \exists\widehat{\theta} \in \left(\frac{k}{\gamma},+\infty\right)\ s.t.\ \widehat{\theta} \geq g\left(\widehat{\theta};\overline{1-T},\frac{k}{\gamma},\overline{\sigma_\theta^2,\sigma_\varepsilon^2}\right)\right\}$$

and

$$\{R\}^C_{\left(\overline{1-T,\sigma_\theta^2,\sigma_\varepsilon^2}\right)} := \left\{\frac{k}{\gamma} \in \left\{\frac{k}{\gamma}\right\}_{\left(\overline{1-T,\sigma_\theta^2,\sigma_\varepsilon^2}\right)} : \forall\widehat{\theta} \in \left(\frac{k}{\gamma},+\infty\right)\ \widehat{\theta} < g\left(\widehat{\theta};\overline{1-T},\frac{k}{\gamma},\overline{\sigma_\theta^2,\sigma_\varepsilon^2}\right)\right\}.$$

Let we now introduce the simplest possible assumption on the randomness of the regime's responsiveness, once we assume that the set of possible repression values is bounded.

Hypothesis 3.2 *Suppose that there is a huge upper bound on the possible values of* $\frac{k}{\gamma}$, *so that*

$$\frac{k}{\gamma} \in [0,M].$$

Hypothesis 3.3 *Suppose that an external observer consider a regime's political repression is uniformly distributed on* $[0,M]$:

$$\frac{k}{\gamma} \sim U(0,M).$$

This assumption does not affect the distribution of $\theta|\theta_i$, thus we can work on the previous equilibrium condition defining equilibrium cutoff

$$\left[1-\Phi\left(\frac{\widehat{\theta}-\sigma_\varepsilon\Phi^{-1}(1-T)-\lambda\widehat{\theta}}{\sqrt{\lambda\sigma_\varepsilon^2}}\right)\right]\gamma\widehat{\theta} = k.$$

Form this equation, is possible to derive an explicit function for $\frac{k}{\gamma}$:

$$\frac{k}{\gamma} = \left[1-\Phi\left(\frac{(1-\lambda)\widehat{\theta}-\sigma_\varepsilon\Phi^{-1}(1-T)}{\sqrt{\lambda\sigma_\varepsilon^2}}\right)\right]\widehat{\theta} =$$

$$= \left[1-\Phi\left(\sqrt{\frac{\sigma_\varepsilon^2}{\sigma_\theta^2}\frac{1}{\sigma_\theta^2+\sigma_\varepsilon^2}}\widehat{\theta}-\sqrt{1+\frac{\sigma_\varepsilon^2}{\sigma_\theta^2}}\Phi^{-1}(1-T)\right)\right]\widehat{\theta}$$

Let define the following function:

$$r\left(\widehat{\theta}|1-T,\sigma_\theta^2,\sigma_\varepsilon^2\right) := \left[1-\Phi\left(\sqrt{\frac{\sigma_\varepsilon^2}{\sigma_\theta^2}\frac{1}{\sigma_\theta^2+\sigma_\varepsilon^2}}\widehat{\theta}-\sqrt{1+\frac{\sigma_\varepsilon^2}{\sigma_\theta^2}}\Phi^{-1}(1-T)\right)\right]\widehat{\theta}$$

which as a function of $\widehat{\theta}$ has a maximum:

$$\frac{k^*}{\gamma}_{\left(1-T,\sigma_\theta^2,\sigma_\varepsilon^2\right)} = \max_{\widehat{\theta}} r\left(\widehat{\theta}|1-T,\sigma_\theta^2,\sigma_\varepsilon^2\right)$$

so that

$$\widehat{\theta}^*_{\left(1-T,\sigma_\theta^2,\sigma_\varepsilon^2\right)} = \arg\max r\left(\widehat{\theta}|1-T,\sigma_\theta^2,\sigma_\varepsilon^2\right).$$

Then

$$R^C_{\left(1-T,\sigma_\theta^2,\sigma_\varepsilon^2\right)} = \left\{\frac{k}{\gamma}\in\mathbb{R}_+ : \frac{k}{\gamma} > \frac{k^*}{\gamma}_{\left(1-T,\sigma_\theta^2,\sigma_\varepsilon^2\right)}\right\}.$$

Then, we are able to derive the probabilities of the equilibrium outcomes.

Proposition 3.4 *In a cutoff equilibrium, the outcomes have the following probabilities:*

$$\mathbb{P}\left\{No\ revolt|1-T,\sigma_\theta^2,\sigma_\varepsilon^2\right\} = \left(M-\frac{k^*}{\gamma}_{\left(1-T,\sigma_\theta^2,\sigma_\varepsilon^2\right)}\right)$$

$$\mathbb{P}\left\{Failed\ revolt|1-T,\sigma_\theta^2,\sigma_\varepsilon^2\right\} = \left[\frac{\Phi\left(\frac{\widehat{\theta}_L\left(1-T,\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right)-\sigma_\varepsilon\Phi^{-1}(1-T)}{\sigma_\theta}\right)}{\sigma_\theta\Phi\left(\frac{\widehat{\theta}_L\left((1-T)^*,\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right)-\sigma_\varepsilon\Phi^{-1}((1-T)^*)}{\sigma_\theta}\right)}\right]\frac{k^*}{\gamma}_{\left(1-T,\sigma_\theta^2,\sigma_\varepsilon^2\right)}$$

$$\mathbb{P}\left\{Successful\ revolt|1-T,\sigma_\theta^2,\sigma_\varepsilon^2\right\} = \left[1-\frac{\Phi\left(\frac{\widehat{\theta}_L\left(1-T,\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right)-\sigma_\varepsilon\Phi^{-1}(1-T)}{\sigma_\theta}\right)}{\sigma_\theta\Phi\left(\frac{\widehat{\theta}_L\left((1-T)^*,\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right)-\sigma_\varepsilon\Phi^{-1}((1-T)^*)}{\sigma_\theta}\right)}\right]\frac{k^*}{\gamma}_{\left(1-T,\sigma_\theta^2,\sigma_\varepsilon^2\right)}$$

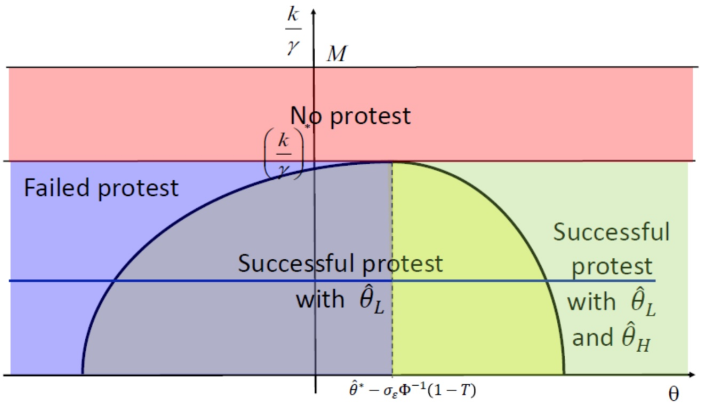

Figure 3.5: The space of the possible equilibrium outcomes

The situation is represented in the following figure:

The comparative statics of the probabilities of the different outcomes are similar to those in the previous case, except that in this scenario the chosen random variable is the repression instead of responsiveness.

Alternative distributions for repression and responsiveness

Consider the variable T, we specified the standard uniform ($U(0,1)$ as the starting distribution, which is the simplest solution that provides equiprobability for all values of the support. However, it is possible to use alternative distributions that instead favor certain values over others. For instance:

$$T \sim \text{Triangular}\left(\alpha = 0, \beta = 1, 0 < \delta < 1\right)$$

This solution retains the original support of the variable T, however the support values are no longer equiprobable, while the value with the maximum probability becomes $T = \delta$. In this case, the probability of no mobilization would become:

$$\mathbb{P}\left\{No\ revolt \,\middle|\, \frac{k}{\gamma}, \sigma_\theta^2, \sigma_\varepsilon^2\right\} = \frac{\left((1-T)^*_{\left(\frac{k}{\gamma}, \sigma_\theta^2, \sigma_\varepsilon^2\right)}\right)^2}{\delta} \quad \text{if } \delta \geq (1-T)^*_{\left(\frac{k}{\gamma}, \sigma_\theta^2, \sigma_\varepsilon^2\right)}$$

$$\mathbb{P}\left\{No\ revolt \,\middle|\, \frac{k}{\gamma}, \sigma_\theta^2, \sigma_\varepsilon^2\right\} = 1 - \frac{\left((T)^*_{\left(\frac{k}{\gamma}, \sigma_\theta^2, \sigma_\varepsilon^2\right)}\right)^2}{1-\delta} \quad \text{if } \delta < (1-T)^*_{\left(\frac{k}{\gamma}, \sigma_\theta^2, \sigma_\varepsilon^2\right)}$$

An additional alternative could be:

$$T \sim U(0, 1 - \Phi(v - t^*))$$

This solution, on the other hand, as the value of unexpected component of the public signal increases, provides that the support of the variable shrinks toward the lower limit 0 while maintaining equiprobability between the support values. In this case, the probability of no mobilization would become:

$$\mathbb{P}\left\{No\ revolt \,\middle|\, \frac{k}{\gamma}, \sigma_\theta^2, \sigma_\varepsilon^2\right\} = 1 - \frac{\left((T)^*_{\left(\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right)}\right)^2}{1-\Phi(v-t^*)} \ \text{ if } 1-\Phi(v-t^*) > (1-T)^*_{\left(\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right)}$$

$$\mathbb{P}\left\{No\ revolt \,\middle|\, \frac{k}{\gamma}, \sigma_\theta^2, \sigma_\varepsilon^2\right\} = 1 \ \text{ if } 1-\Phi(v-t^*) \leq (1-T)^*_{\left(\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right)}$$

Also in the case of repression, it is possible to consider an alternative distribution to the uniform, such as a law that considers positive values without a fixed upper bound such as the exponential random variable:

$$\frac{k}{\gamma} \sim \text{Exponential}\ (\lambda)$$

This solution unlike the uniform does not require specifying an upper limit and disfavors high realizations.
In this case, the probability of no mobilization would become:

$$\mathbb{P}\left\{No\ revolt \,\middle|\, 1-T, \sigma_\theta^2, \sigma_\varepsilon^2\right\} = 1 - e^{-\lambda \frac{k}{\gamma}^*\left(1-T,\sigma_\theta^2,\sigma_\varepsilon^2\right)}$$

Again, the interesting aspect would be to consider the possibility that the actors taking part in the game may have an influence on the λ parameter by favoring or disfavoring different outcomes of the game as in the previous case for responsiveness.

3.4 Conclusion

The chapter we have presented represents an attempt to understand the role of the unexpected component of the public signal within the original game [6] by studying its behaviour once the public signal observable by citizens has been removed.

The analysis has shown that the main effect of the activity of the revolutionary entrepreneurs is to randomize the equilibrium cutoff which in its absence, whether it exists, becomes a deterministic function of the exogenous parameters of the game.
Moreover the introduction of a revolutionary vanguard allows the support of the mobilization and non-mobilization probability to be transformed from a binary set $\{0,1\}$ to a continuous set with real values defined on the interval $[0,1]$.

The introduction of a distribution linked to one of the exogenous parameters of the original game, such as responsiveness or repression, independent from the other parameters of the game allows even in the absence of the activity of the revolutionary vanguard to recover these two advantages. This suggests that a different role for the vanguard within the game can be evaluated by, for example, integrating it into the payoff matrix, identifying a functional relationship with participation costs, or specifying a dependency between the random distribution of one of the exogenous parameters and its observable level by citizens.

3.5 Appendix

Proof of Result 3.1

Consider i's subjective belief about the probability of regime change:

$$1-\Phi\left(\frac{\widehat{\theta}}{\sqrt{\lambda\sigma_\varepsilon^2}}-\frac{\sigma_\varepsilon\Phi^{-1}(1-T)}{\sqrt{\lambda\sigma_\varepsilon^2}}-\frac{\lambda\theta_i}{\sqrt{\lambda\sigma_\varepsilon^2}};p,s\right)$$

- If $1-T$ increases also $\Phi^{-1}(1-T)$ increases consequently the argument of the distribution function decreases, increasing the overall probability
- If θ_i increases the argument of the distribution function decreases, increasing the overall probability since $1-\lambda>0$
- Consider $\theta_i \geq \widehat{\theta}$ where by construction the cutoff is positive. If σ_ε^2 increases the first term is positive but decreasing towards $\frac{\widehat{\theta}}{\sigma_\theta}$ since the denominator grows tending to σ_θ, the last term is decreasing in absolute value towards 0 since λ is decreasing in σ_ε, if the political regime is responsive the second term is negative and increasing in absolute value since λ is decreasing in σ_ε therefore it generally induces global probability increase, if the political regime is sufficiently responsive the increase is monotonic
- Consider $\theta_i \geq \widehat{\theta}$ where by construction the cutoff is positive. If σ_θ^2 increases the first term is positive but decreasing since the denominator grows tending to σ_ε, the last term is increasing in absolute value since λ is increasing in σ_θ, if the political regime is unresponsive the second term is also positive and decreasing due to the growth of the denominator and thus the overall probability is monotonically increasing since the argument of the function is decreasing. If the political regime is responsive the second term is negative and increasing so the overall effect is doubtful, decreasing prevails if the political regime is not overly responsive

Proof of Result 3.2

$$g\left(\widehat{\theta};p,s\right):=-\frac{1}{1-\lambda}\left[\sqrt{\lambda\sigma_\varepsilon^2}\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right)-\sigma_\varepsilon\Phi^{-1}\left(1-T\right)\right]$$

- If $1-T$ increases also $\Phi^{-1}(1-T)$ increases consequently since the associated multiplier is positive the function grows
- If $\frac{k}{\gamma}$ increases fixed a certain $\widehat{\theta}$ $\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right)$ increases, however since the associated multiplier is always negative globally the function decreases

Proof of Result 3.3

Consider:

$$g\left(\widehat{\theta};p,s\right):=-\left(\frac{\sigma_\theta\sqrt{\sigma_\theta^2+\sigma_\varepsilon^2}}{\sigma_\varepsilon}\right)\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right)+\left(\frac{\sigma_\theta^2+\sigma_\varepsilon^2}{\sigma_\varepsilon}\right)\Phi^{-1}\left(1-T\right)$$

- If $1-T=\frac{1}{2}$ and σ_ε^2 increases the function increases only for $\widehat{\theta}\leq\frac{2k}{\gamma}$ because the weight in absolute value is decreasing in σ_ε^2 and $\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right)$ is positive only for $\widehat{\theta}\geq\frac{2k}{\gamma}$. When $1-T>\frac{1}{2}$ the behavior is similar, however, a positive addend with increasing weight in σ_ε^2 is added therefore the function is increasing for $\widehat{\theta}\leq\widehat{\theta}^*>\frac{2k}{\gamma}$, whereas when $1-T<\frac{1}{2}$ the function is increasing for $\widehat{\theta}\geq\widehat{\theta}^*<\frac{2k}{\gamma}$ because a negative addend with increasing weight in σ_ε^2 is added

- If $1-T=\frac{1}{2}$ and σ_θ^2 increases the function increases only for $\widehat{\theta}\geq\frac{2k}{\gamma}$ because the weight in absolute value is increasing in σ_θ^2 and $\Phi^{-1}\left(\frac{k}{\gamma\widehat{\theta}}\right)$ is positive only for $\widehat{\theta}\geq\frac{2k}{\gamma}$. When $1-T>\frac{1}{2}$ the behavior is similar, however, a positive addend with increasing weight in σ_θ^2 is added therefore the function is increasing for $\widehat{\theta}\geq\widehat{\theta}^*<\frac{2k}{\gamma}$, whereas when $1-T<\frac{1}{2}$ the function is increasing for $\widehat{\theta}\geq\widehat{\theta}^*>\frac{2k}{\gamma}$ because a negative addend with increasing weight in σ_θ^2 is added

Proof of Result 3.4

The equilibrium cutoff is identified by the equation:

$$\left[1-\Phi\left(\frac{\widehat{\theta}\left(p,s\right)-\sigma_\varepsilon\Phi^{-1}\left(1-T\right)-\lambda\theta_i}{\sqrt{\lambda\sigma_\varepsilon^2}};p,s\right)\right]\gamma\widehat{\theta}=k$$

Since the first factor of the left-hand member is a probability with upper bound 1, it is impossible to solve the equation for $\widehat{\theta}<\frac{k}{\gamma}$.

Proof of Result 3.6

The equilibrium cutoff is identified by the equation:

$$\left[1-\Phi\left(\frac{\widehat{\theta}\left(p,s\right)-\sigma_\varepsilon\Phi^{-1}\left(1-T\right)-\lambda\widehat{\theta}}{\sqrt{\lambda\sigma_\varepsilon^2}};p,s\right)\right]\widehat{\theta}=\frac{k}{\gamma}$$

- If $1-T$ increases also $\Phi^{-1}(1-T)$ increases consequently the argument of the distribution function decreases, increasing the overall probability. If the probability of each $\widehat{\theta}$ increases then the left member of the equation increases by reducing the lower equilibrium cutoff and increasing the higher one.

- If $\frac{k}{\gamma}$ increases the horizontal line intersecting the function grows leading the lower cutoff to grow to a maximum beyond which it no longer exists.

Proof of Result 3.7

The equilibrium cutoff is identified by the equation:

$$\left[1-\Phi\left(\frac{\widehat{\theta}\left(p,s\right)-\sigma_{\varepsilon}\Phi^{-1}\left(1-T\right)-\lambda\widehat{\theta}}{\sqrt{\lambda\sigma_{\varepsilon}^{2}}};p,s\right)\right]\widehat{\theta}=\frac{k}{\gamma}$$

- If σ_{ε}^{2} increases the object $\frac{\widehat{\theta}(1-\lambda)}{\sqrt{\lambda}\sigma_{\varepsilon}}$ is always positive and increasing, if the political regime is unresponsive $-\frac{\Phi^{-1}(1-T)\sigma_{\varepsilon}}{\sqrt{\lambda}\sigma_{\varepsilon}}$ is positive and increasing therefore the probability is monotonically decreasing favoring an increase of the lower cutoff

- If σ_{θ}^{2} increases the object $\frac{\widehat{\theta}(1-\lambda}{\sqrt{\lambda})\sigma_{\varepsilon}}$ decreases towards 0^{+}, if the political regime is unresponsive $-\sigma_{\varepsilon}\Phi^{-1}(1-T)$ is a fixed positive value therefore the overall probability is monotonically increasing favoring a decrease of the lower cutoff

Proof of Result 3.8

The probability related to the area of socio-political regimes without protests moves in the same direction as the equilibrium cutoff whose trend is demonstrated in result 3.6 and 3.7

Proof of Result 3.9

Given the structure of the game it is assumed that $1-T\in(0,1)$ and $\widehat{\theta}\geq\frac{k}{\gamma}$, consequently the area of all possible outcomes given the two variables is given by the rectangle of unit base and height equal to $\widehat{\theta}$. The area of non-protest decreases in responsiveness therefore the northwest section is related to non-protest and the southeast section to mobilization. If the two areas are equivalent to each other at $1-T=\frac{1}{2}$ we observe mobilization where $\frac{k}{\gamma}\leq\frac{\widehat{\theta}}{2}$. However when $1-T=\frac{1}{2}$ the probability contained in the function $IB()$ is strictly less than $\frac{1}{2}$ because $\widehat{\theta}(1-\lambda)>0$ consequently the equilibrium cutoff is lower than $\frac{\widehat{\theta}}{2}$, therefore in conclusion if repression and responsiveness are uniformly distributed the area of mobilization is lower than the one without protests

Proof of Result 3.10

Consider the percentage of protesting citizens:

$$1-\Phi\left(\frac{\widehat{\theta}_{L}\left(p,s\right)-\theta}{\sigma_{\varepsilon}}\right),$$

- If θ increases the argument of the distribution function decreases by increasing protesting citizens

- If $1-T$ increases the equilibrium cutoff decreases, therefore the argument of the distribution function decreases by increasing the protesting citizens

- If $\frac{k}{\gamma}$ increases the equilibrium cutoff increases, therefore the argument of the distribution function increases by decreasing the protesting citizens
- If the political regime is unresponsive and σ_ε^2 increases the equilibrium cutoff increases, therefore if $\widehat{\theta} < \theta$ the argument of the distribution function increases because its denominator increases, after this threshold the numerator becomes positive and therefore the the argument of the distribution function decreases towards $\frac{1}{2}$ because the argument decreases towards 0^+.
- If the political regime is unresponsive and σ_θ^2 increases the equilibrium cutoff decreases, therefore the argument of the distribution function decreases by increasing protesting citizens

Proof of Result 3.12

Consider the probability of successful protest:

$$1 - \Phi\left(\frac{\widehat{\theta}_L\left(p, s\right) - \sigma_\varepsilon \Phi^{-1}\left(1 - T\right)}{\sigma_\theta}\right).$$

- If $1 - T$ increases the argument of the distribution function decreases inducing a growth in the overall probability both because a decrease in the equilibrium cutoff is induced and because the object $\Phi^{-1}(1 - T)$ increases
- If $\frac{k}{\gamma}$ increases the argument of the distribution function increases inducing a decrease in the overall probability because an increase in the equilibrium cutoff is induced
- If σ_ε^2 increases and the political regime is unresponsive the equilibrium cutoff is always positive and increasing, $-\sigma_\varepsilon \Phi^{-1}(1-T)$ is positive and increasing, while the denominator is fixed, therefore the probability is decreasing
- If σ_θ^2 increases and the political regime is unresponsive the equilibrium cutoff is always positive and decreasing therefore the numerator is always positive and decreasing because the denominator increases, therefore the probability is increasing

Proof of Proposition 3.2

Consider the following equalities

$$\mathbb{P}\left\{No\ revolt|\frac{k}{\gamma}, \sigma_\theta^2, \sigma_\varepsilon^2\right\} = \mathbb{P}\left\{1 - T < (1 - T)^*_{\left(\frac{k}{\gamma}, \sigma_\theta^2, \sigma_\varepsilon^2\right)}\right\} = (1 - T)^*_{\left(\frac{k}{\gamma}, \sigma_\theta^2, \sigma_\varepsilon^2\right)}$$

because $1 - T$ is uniformly distributed in $[0, 1]$. Then

$$\mathbb{P}\left\{Failed\ revolt|\frac{k}{\gamma}, \sigma_\theta^2, \sigma_\varepsilon^2\right\} =$$

$$= \mathbb{P}\left\{\left\{1-T \geq (1-T)^*_{\left(\frac{k}{\gamma},\sigma^2_\theta,\sigma^2_\varepsilon\right)}\right\} \cap \left\{\theta < \widehat{\theta}_L\left((1-T)^*, \frac{k}{\gamma}, \sigma^2_\theta, \sigma^2_\varepsilon\right) - \sigma_\varepsilon \Phi^{-1}\left((1-T)^*\right)\right\}\right\} =$$

$$= \mathbb{P}\left\{1-T \geq (1-T)^*_{\left(\frac{k}{\gamma},\sigma^2_\theta,\sigma^2_\varepsilon\right)}\right\} \times$$

$$\times \mathbb{P}\left\{\left\{\theta < \widehat{\theta}_L\left((1-T)^*, \frac{k}{\gamma}, \sigma^2_\theta, \sigma^2_\varepsilon\right) - \sigma_\varepsilon \Phi^{-1}\left((1-T)^*\right)\right\} | \left\{1-T \geq (1-T)^*_{\left(\frac{k}{\gamma},\sigma^2_\theta,\sigma^2_\varepsilon\right)}\right\}\right\} =$$

$$= \left[\frac{\Phi\left(\frac{\widehat{\theta}_L\left(1-T,\frac{k}{\gamma},\sigma^2_\theta,\sigma^2_\varepsilon\right)-\sigma_\varepsilon\Phi^{-1}(1-T)}{\sigma_\theta}\right)}{\sigma_\theta \Phi\left(\frac{\widehat{\theta}_L\left((1-T)^*,\frac{k}{\gamma},\sigma^2_\theta,\sigma^2_\varepsilon\right)-\sigma_\varepsilon\Phi^{-1}((1-T)^*)}{\sigma_\theta}\right)}\right] (T)^*_{\left(\frac{k}{\gamma},\sigma^2_\theta,\sigma^2_\varepsilon\right)}$$

using a truncated normal since $\theta \sim N\left(0, \sigma^2_\theta\right)$. Finally

$$\mathbb{P}\left\{Successful\ revolt | \frac{k}{\gamma}, \sigma^2_\theta, \sigma^2_\varepsilon\right\} =$$

$$= \mathbb{P}\left\{\left\{1-T \geq (1-T)^*_{\left(\frac{k}{\gamma},\sigma^2_\theta,\sigma^2_\varepsilon\right)}\right\} \cap \left\{\theta \geq \widehat{\theta}_L\left((1-T)^*, \frac{k}{\gamma}, \sigma^2_\theta, \sigma^2_\varepsilon\right) - \sigma_\varepsilon \Phi^{-1}\left((1-T)^*\right)\right\}\right\} =$$

$$= \mathbb{P}\left\{1-T \geq (1-T)^*_{\left(\frac{k}{\gamma},\sigma^2_\theta,\sigma^2_\varepsilon\right)}\right\} \times$$

$$\times \mathbb{P}\left\{\left\{\theta \geq \widehat{\theta}_L\left((1-T)^*, \frac{k}{\gamma}, \sigma^2_\theta, \sigma^2_\varepsilon\right) - \sigma_\varepsilon \Phi^{-1}\left((1-T)^*\right)\right\} | \left\{1-T \geq (1-T)^*_{\left(\frac{k}{\gamma},\sigma^2_\theta,\sigma^2_\varepsilon\right)}\right\}\right\} =$$

$$= \left[1 - \frac{\Phi\left(\frac{\widehat{\theta}_L\left(1-T,\frac{k}{\gamma},\sigma^2_\theta,\sigma^2_\varepsilon\right)-\sigma_\varepsilon\Phi^{-1}(1-T)}{\sigma_\theta}\right)}{\sigma_\theta \Phi\left(\frac{\widehat{\theta}_L\left((1-T)^*,\frac{k}{\gamma},\sigma^2_\theta,\sigma^2_\varepsilon\right)-\sigma_\varepsilon\Phi^{-1}((1-T)^*)}{\sigma_\theta}\right)}\right] (T)^*_{\left(\frac{k}{\gamma},\sigma^2_\theta,\sigma^2_\varepsilon\right)}$$

again using the truncated normal.

Proof of Proposition 3.3

Consider:

$$\widehat{\theta}^*_{\left(\frac{k}{\gamma},\sigma^2_\theta,\sigma^2_\varepsilon\right)} = \arg\min h\left(\widehat{\theta} | \frac{k}{\gamma}, \sigma^2_\theta, \sigma^2_\varepsilon\right)$$

- If σ^2_θ increases the first term is positive and decreasing because its weight is decreasing while the weight of second term (associated with negative sign) is increasing, therefore the argument of the function is uniformly decreasing $\forall \widehat{\theta}$ and the minimum is lower
- If σ^2_ε increases the first term is positive and decreasing because its weight is decreasing, the weight of the second term (associated with negative sign) is decreasing, but the effect of the first one is greater (this is also visible through simulations) therefore the argument of the function is uniformly decreasing $\forall \widehat{\theta}$ and the minimum is lower. If σ^2_θ is sufficiently high the effect is the opposite

- If $\frac{k}{\gamma}$ increases the first term is invariant, while the second term (associated with negative sign) is decreasing therefore the argument of the function is uniformly increasing $\forall\widehat{\theta}$ and the minimum is higher

Proof of Proposition 3.4

Consider the following equalities

$$\mathbb{P}\left\{No\ revolt|1-T,\sigma_\theta^2,\sigma_\varepsilon^2\right\} = \mathbb{P}\left\{\frac{k}{\gamma} > \frac{k^*}{\gamma}_{\left(1-T,\sigma_\theta^2,\sigma_\varepsilon^2\right)}\right\} = \left(M - \frac{k^*}{\gamma}_{\left(1-T,\sigma_\theta^2,\sigma_\varepsilon^2\right)}\right)$$

because $\frac{k}{\gamma}$ is uniformly distributed in $[0, M]$. Then

$$\mathbb{P}\left\{Failed\ revolt|1-T,\sigma_\theta^2,\sigma_\varepsilon^2\right\} =$$

$$= \mathbb{P}\left\{\left\{\frac{k}{\gamma} \leq \frac{k^*}{\gamma}_{\left(1-T,\sigma_\theta^2,\sigma_\varepsilon^2\right)}\right\} \cap \left\{\theta < \widehat{\theta}_L\left(1-T,\frac{k^*}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right) - \sigma_\varepsilon\Phi^{-1}(1-T)\right\}\right\} =$$

$$= \mathbb{P}\left\{\frac{k}{\gamma} \leq \frac{k^*}{\gamma}_{\left(1-T,\sigma_\theta^2,\sigma_\varepsilon^2\right)}\right\} \times$$

$$\times \mathbb{P}\left\{\left\{\theta < \widehat{\theta}_L\left(1-T,\frac{k^*}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right) - \sigma_\varepsilon\Phi^{-1}(1-T)\right\} | \left\{\frac{k}{\gamma} \leq \frac{k^*}{\gamma}_{\left(1-T,\sigma_\theta^2,\sigma_\varepsilon^2\right)}\right\}\right\} =$$

$$= \left[\frac{\Phi\left(\frac{\widehat{\theta}_L\left(1-T,\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right)-\sigma_\varepsilon\Phi^{-1}(1-T)}{\sigma_\theta}\right)}{\sigma_\theta\Phi\left(\frac{\widehat{\theta}_L\left(1-T,\frac{k^*}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right)-\sigma_\varepsilon\Phi^{-1}(1-T)}{\sigma_\theta}\right)}\right] \frac{k^*}{\gamma}_{\left(1-T,\sigma_\theta^2,\sigma_\varepsilon^2\right)}$$

using a truncated normal since $\theta \sim N\left(0,\sigma_\theta^2\right)$. Finally

$$\mathbb{P}\left\{Successful\ revolt|\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right\} =$$

$$= \mathbb{P}\left\{\left\{\frac{k}{\gamma} \leq \frac{k^*}{\gamma}_{\left(1-T,\sigma_\theta^2,\sigma_\varepsilon^2\right)}\right\} \cap \left\{\theta \geq \widehat{\theta}_L\left(1-T,\frac{k^*}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right) - \sigma_\varepsilon\Phi^{-1}(1-T)\right\}\right\} =$$

$$= \mathbb{P}\left\{\frac{k}{\gamma} \leq \frac{k^*}{\gamma}_{\left(1-T,\sigma_\theta^2,\sigma_\varepsilon^2\right)}\right\} \times$$

$$\times \mathbb{P}\left\{\left\{\theta \geq \widehat{\theta}_L\left(1-T,\frac{k^*}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right) - \sigma_\varepsilon\Phi^{-1}(1-T)\right\} | \left\{\frac{k}{\gamma} \leq \frac{k^*}{\gamma}_{\left(1-T,\sigma_\theta^2,\sigma_\varepsilon^2\right)}\right\}\right\} =$$

$$= \left[1 - \frac{\Phi\left(\frac{\widehat{\theta}_L\left(1-T,\frac{k}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right)-\sigma_\varepsilon\Phi^{-1}(1-T)}{\sigma_\theta}\right)}{\sigma_\theta\Phi\left(\frac{\widehat{\theta}_L\left(1-T,\frac{k^*}{\gamma},\sigma_\theta^2,\sigma_\varepsilon^2\right)-\sigma_\varepsilon\Phi^{-1}(1-T)}{\sigma_\theta}\right)}\right] \frac{k^*}{\gamma}_{\left(1-T,\sigma_\theta^2,\sigma_\varepsilon^2\right)}$$

again using the truncated normal.

3.5.1 Simulations

The simulations mentioned in the chapter are written in R and the relevant code is available is available on Github following the path: *goergefil/PhD-Thesis-Simulations*

4

SECTION

Regime Change and Citizens' Individual Preferences

4.1 Notes

This chapter was presented during the 6th International Conference on the political economy of democracy and dictatorship (University of Münster, Germany, February 2023) and at the European Public Choice Society 2023 (Hannover, Germany, March 2023).

4.2 Introduction

A global game of "policy change"[19] is a coordination game with incomplete information in which each player chooses whether to attack a policy and the policy is changed if and only if enough players attack. Many phenomena of both economic and political interest, such as currency attacks, bank runs, debt crises, revolutions, protests and so on, can be modeled as policy change games (see, for examples, [33]; [19]; [11]; [**?**]; [20]; [10]; [23]; [24]; [27]; [6]). In applications, policy change games are typically studied under technical assumptions on the informational environment that assure that the game satisfies the two-sided limit dominance property of "global games". That is, players' beliefs must assign positive probability to a state of the world in which some players have a dominant strategy to attack and must assign positive probability to a state of the world in which some players have a dominant strategy not to attack ([16],[9]). These assumptions are attractive because, as well known, despite being coordination games, global games have a unique equilibrium. In the absence of two-sided limit dominance, equilibrium uniqueness need not obtain in coordination games with incomplete information ([16]).

In the first part of the chapter, we build on the global game of policy change by [6], which aims to explain how a revolutionary vanguard might use violence to mobilize a mass public. The model used to analyze this problem is a specific "policy change" game with one side limit dominance, which means that players assign positive probability to a state of the world in which some players have a dominant strategy not to attack, but there is no state of the world where some players have to attack as dominant strategy. A consequence of this structure is that this game has multiple equilibria, two equilibria in which there may be protests and another in which there is certain not to be. Between the first two equilibria, Bueno De Mesquita assumes that the one associated to the highest cutoff point is never played because of its not intuitive properties. We don't think this is argument is fully satisfactory, hence within this structure we analyze the role of the payoff matrix proposing different alternative formulations and exploring their implications on game balance and outcomes. Thus, our results provide useful insights on the working of policy change games and on the role of expected payoff to generate equilibria multiplicity in this class of games.

The role of the payoff matrix as a factor for improvement towards a more interpretable equilibrium is studied in the first part of the chapter, starting with the critical and unconvincing aspects of the original game [6]. Similarly, in the previous chapter, the game was studied by proposing a different structural modification, namely the omission of the public signal. In the second part of the chapter, the structure of the original game is maintained, while the modification is applied to the specification of the cost function. In detail, we study the cost function proposed in the original article, identify the

[19]We prefer the name "policy change game" with respect to the more usual "regime change game" because we believe that this last name restricts unnecessarily the field of possible application of this model, as argued in [18]

desirable and reasonable characteristics of a generic cost function in order for it to be interpretable, and propose two potential alternatives capable of improving the derivation of the equilibrium and the general interpretability of the game.

4.3 Original Payoff Matrix

Consider the payoff matrix designed in [6]:

	$N < T$	$N \geq T$
$a_i = 0$	0	$(1-\gamma)\theta_i$
$a_i = 1$	$-k$	$\theta_i - k$
Payoff matrix 1		

As outlined in [7] this game retains the most attractive features of global games of regime change (smoothness property) while maintaining the multiplicity of equilibria typical of mass uprisings models as coordination problems, therefore it is consistent with the literatures that follow the arguments of [35] and [37]. Smoothness means that small changes in players' beliefs or payoffs may cause small changes in the equilibrium thereby inducing small changes in the level of mobilization or the probability of the regime falling. However the game is characterized by two criticalities:

- finite equilibria when they exist are multiple, this poses a problem as participation is linked to citizens presenting a private signal above the equilibrium cutoff, and in the absence of a precise rule selecting one of the cutoffs, it is difficult to understand whether citizens between its equilibria will participate. In addition, the higher equilibrium grows in $v - t^*$, whereas it would be logical to assume that an increase in perceived violence in society would favour the mobilization of citizens
- players with type $\theta_i \in \left(0, \frac{k}{\gamma}\right)$ have $a_i = 0$ as dominant strategy, but unlike the other players their potential payoffs are always non negative. This creates a curious mechanism whereby this portion of citizens is not only guaranteed not to suffer losses whatever the outcome of the protest, but even if they do not participate, they hope for the success of the protest as they would benefit in terms of payoff. The ideal scenario would be that no citizen would be faced with a choice characterized by both payoffs not negative. In general, it would be preferable for those who have a dominant strategy to profit more from the outcome logically associated to it.

The reformulation of the game, driven by the definition of alternative payoff matrix, has the dual purpose of maintaining the advantageous properties of the original game while obviating its criticalities.

4.4 Alternative Matrix I

4.4.1 Specification

If θ_i determines how much a citizen values a policy change, then for citizens supporting the policy, a failure in the protests might have a positive impact on their payoffs. Moreover, by symmetry, if $\gamma \in (0,1]$ is the portion of the value of policy change that i appropriates by attacking, then it should also be the portion of the value of maintaining the status quo by its supporters when not protesting. A possibility to model these ideas is to consider the following payoff matrix:

	$N < T$	$N \geq T$
$a_i = 0$	$-\gamma\theta_i$	$(1-\gamma)\theta_i$
$a_i = 1$	$-k$	$\theta_i - k$
Payoff matrix 2		

The policy change game with payoff matrix 2 is characterized by two sided limit dominance:

- citizens with $\theta_i \in \left(-\infty, \frac{k}{\gamma}\right)$ have $a_i = 0$ as dominant strategy
- citizens with $\theta_i \in \left(\frac{k}{\gamma}, +\infty\right)$ have $a_i = 1$ as dominant strategy

To verify this property it is sufficient to observe that:

$$\theta_i - k \geq (1-\gamma)\theta_i \Longleftrightarrow \theta_i \geq \frac{k}{\gamma}$$

$$-k \geq -\gamma\theta_i \Longleftrightarrow \theta_i \geq \frac{k}{\gamma}$$

In this case, the cutoff equilibrium is finite, unique and trivial, since there is no player who does not have a dominant strategy and the upper limit of the space of θ_i for which $a_i = 0$ is dominant coincides with the lower limit of the space of θ_i for which $a_i = 1$ is.

Proposition 4.1 *The policy change game with payoff matrix 2 has a unique equilibrium strategy:*

$$s^*(\theta_i) = \begin{cases} 0 & \theta_i < \frac{k}{\gamma} \\ 1 & \theta_i \geq \frac{k}{\gamma} \end{cases}$$

The advantages of this approach are:

- the smoothness property is preserved since the equilibrium cutoff is affected by the variation of the parameters k and γ, moreover given that the protest is successful where $\theta \geq \frac{k}{\gamma} - \sigma_\varepsilon \Phi^{-1}(1-T)$ variations in the other parameters $(T, \sigma_\theta, \sigma_\varepsilon)$ also influence the outcome of the game. This property is crucial as it later allows us to determine how the parameters of the game influence the equilibrium and the final outcome in a framework of comparative statics

- the game is characterized by a unique finite fixed equilibrium cutoff $\widehat{\theta} = \frac{k}{\gamma}$. As pointed out above, having a single finite equilibrium avoids the creation of doubts as to the population that actually mobilises

- when a player makes a choice, he never has both non-negative potential payoffs since $-k < 0$ and $-\gamma\theta_i$ always has opposite sign to $(1-\gamma)\theta_i$ given any level of θ_i. Instead, this aspect avoids the creation of segments of citizens who cannot register losses or who have hopes that run counter to their behaviour

To verify the validity of the third point it is sufficient to observe that:

$$-\gamma\theta_i > 0 \Longleftrightarrow \theta_i < 0 \; (0 < \gamma < 1)$$

$$(1-\gamma)\theta_i > 0 \Longleftrightarrow \theta_i > 0 \; (0 < \gamma < 1)$$

The drawbacks of this approach are:

- the multiplicity of equilibria is lost since the finite equilibrium is unique and there are no infinite equilibria and therefore there is no provision for the absence of mobilization or total mobilization

- the outcome of the game no longer depends on the level of unexpected component of the public signal as opposed to the other game parameters, because the cutoff equilibrium is fixed and the level of θ above which successful protest is observed is not affected by variations of this component

4.4.2 Properties of Equilibrium and Outcomes

The game always presents an unique finite equilibrium represented by the cutoff:

$$\widehat{\theta} = \frac{k}{\gamma}$$

Result 4.1 *The equilibrium cutoff of the policy game with payoff matrix 2 is:*

- *Increasing in the protesting cost k*

- *Decreasing in inclusiveness γ*

- *Independent from the other game variables*

Since there is always only an unique finite equilibrium, the game is characterized by two outcomes:

- Failed protest

- Successful protest

The protest is successful if the mass of citizens characterised by $\theta_i \geq \frac{k}{\gamma}$ is greater than T, therefore:

$$P(\text{Successful protest}) = P\left(\theta \geq \frac{k}{\gamma} - \sigma_\varepsilon \Phi^{-1}(1-T)\right) = 1 - \Phi\left(\frac{\frac{k}{\gamma} - \sigma_\varepsilon \Phi^{-1}(1-T)}{\sigma_\theta}\right)$$

Result 4.2 *The probability of successful protest in the policy game with payoff matrix 2 is:*

- *Decreasing in repression*
- *Increasing in responsiveness*
- *Increasing in diversity unless the regime is unresponsive*
- *Increasing in radicalization if* $\frac{k}{\gamma} > \sigma_\varepsilon \Phi^{-1}(1-T)$ *with upper limit* $\frac{1}{2}$ *otherwise decreasing with lower limit* $\frac{1}{2}$

The protest fails if the mass of citizens characterised by $\theta_i \geq \frac{k}{\gamma}$ is less than T, therefore:

$$P(\text{Failed protest}) = P\left(\theta < \frac{k}{\gamma} - \sigma_\varepsilon \Phi^{-1}(1-T)\right) = \Phi\left(\frac{\frac{k}{\gamma} - \sigma_\varepsilon \Phi^{-1}(1-T)}{\sigma_\theta}\right)$$

Result 4.3 *The probability of failed protest in the policy game with payoff matrix 2 is:*

- *Increasing in repression*
- *Decreasing in responsiveness*
- *Decreasing in diversity unless the regime is unresponsive*
- *Decreasing in radicalization if* $\frac{k}{\gamma} > \sigma_\varepsilon \Phi^{-1}(1-T)$ *with lower limit* $\frac{1}{2}$ *otherwise increasing with upper limit* $\frac{1}{2}$

Remark 4.1 *The equilibrium cutoff is influenced solely by the level of repression, unlike the probabilities of outcomes, which are influenced by the parameters of the game with the exception of the unexpected component of the public signal. If the realization of the parameter* θ *were not independent, but influenced by the realization of* $v - t^*$*, then the outcomes would also be influenced by the unexpected component of public signal*

4.5 Alternative Matrix I: review

4.5.1 Specification

However there is a modification of the proposed payoff matrix capable of ensuring the presence of a unique infinite equilibrium of zero-participation, maintaining an active role for the level of unexpected component of the public signal (in encouraging the mobilization) and the cited positive properties. It would be sufficient to consider the matrix:

	$N<T$	$N\geq T$
$a_i=0$	$-\gamma[(v-t^*)-(e^{-\theta_i}-1)]$	$(1-\gamma)\,[(v-t^*)-(e^{-\theta_i}-1)]$
$a_i=1$	$-k$	$[(v-t^*)-(e^{-\theta_i}-1)]-k$
Payoff matrix 3		

In this scenario, if we denote as α the probability of successful protest, the equilibrium condition becomes:

$$[(v-t^*)-(e^{-\widehat{\theta}}-1)]\alpha-k=-\gamma[(v-t^*)-(e^{-\widehat{\theta}}-1)](1-\alpha)+(1-\gamma)[(v-t^*)-(e^{-\widehat{\theta}}-1)]\alpha$$

$$\left[(v-t^*)-(e^{-\widehat{\theta}}-1)\right]=\frac{k}{\gamma}$$

Given that $\frac{k}{\gamma}$ is a positive fixed value and $-e^{-\widehat{\theta}}+1$ is an increasing function in $\widehat{\theta}$ (with upper limit 1), then if $v-t^*$ strictly exceeds a certain threshold there is a unique finite equilibrium decreasing in $v-t^*$, while on the contrary mobilization is absent since the left member is always less than $\frac{k}{\gamma}$.
Finite equilibrium where it exists will be characterized by the analytic form:

$$\widehat{\theta}=-log\left((v-t^*)-\frac{k}{\gamma}+1\right)$$

Proposition 4.2 *The policy change game with payoff matrix 3 has two strategies consistent with a cutoff equilibrium:*

if $v-t^*>\frac{k}{\gamma}-1$*:*

$$s^*(\theta_i)=\begin{cases}0 & \theta_i<-log\left((v-t^*)-\frac{k}{\gamma}+1\right)\\ 1 & \theta_i\geq -log\left((v-t^*)-\frac{k}{\gamma}+1\right)\end{cases}$$

where the equilibrium cutoff is decreasing in the level of unexpected component of the public signal.

If $v-t^*\leq\frac{k}{\gamma}-1$*:*

$$s^*(\theta_i)=0\ \forall\theta_i$$

The application of development proves useful because:

- the smoothness property is preserved since the equilibrium is affected by the variation of the parameters k,γ and $v-t^*$, moreover given that the protest is successful when $\theta\geq\frac{k}{\gamma}-\sigma_\varepsilon\Phi^{-1}(1-T)$ variations in the other parameters also influence the outcome of the game
- the multiplicity of equilibria is recovered but it is due to the presence of a unique zero-participation infinite equilibrium and a unique finite equilibrium decreasing in the level of unexpected component of the public signal. This is crucial be-

cause when the cutoff equilibrium is finite there is no doubt about the participants, while the mobilization is not always guaranteed and there is the possibility of a zero-participation scenario

- when a player makes a choice, he never has both non-negative potential payoffs since $-k < 0$ and $-\gamma[(v - t^*) - (e^{-\theta_i} - 1)]$ always has opposite sign to $(1-\gamma)[(v - t^*) - (e^{-\theta_i} - 1)]$

To verify the validity of the third point it is sufficient to observe that:

$$-\gamma[(v - t^*) - (e^{-\theta_i} - 1)] > 0 \Longleftrightarrow v - t^* < (e^{-\theta_i} - 1) \ (0 < \gamma < 1)$$

$$(1-\gamma)[(v - t^*) - (e^{-\theta_i} - 1)] > 0 \Longleftrightarrow v - t^* > (e^{-\theta_i} - 1) \ (0 < \gamma < 1)$$

Thus the gain over the previous case is the recovery of the multiplicity of equilibria while maintaining the uniqueness of the finite one and the recovery of the positive role of the vanguard on mobilization.

4.5.2 Interpretability

What is the economic interpretation of this new payoff matrix?

When a citizen takes part in the protest but it fails, he/she will suffer a loss equal to the fixed cost of mobilization ($-k$).
Where the protest is successful, the participating citizen has to bear the cost of mobilization, but obtains a prize equal to $v - t^* - (e^{\theta_i} - 1)$. The prize is composed of the level of unexpected violence and a function of personal anti-regime sentiment. The prize will increase as the violence expressed by the vanguard in society increases (as it makes regime change more likely), the functional form of the second member is crucial because it allows us to have a positive and increasing second part of the premium (with upper limit 1) for extremists (pro-protest), while negative and decreasing for moderates. The trade-off between unexpected violence and cost can make the prize more or less advantageous.
What a non-participating citizen gets from a winning protest is a smaller payoff in absolute value than in the scenario in which he/she had participated (hence a portion), without incurring the costs of participation.
Finally, the payoff of the non-participating citizen in the event of an unsuccessful protest has an inverse trend to the unexpected violence of the vanguard (as it makes regime change more likely), however the functional form of the second member within it allows us to identify a positive and increasing factor (with upper limit 1) for moderates and a negative and decreasing factor for extremists (pro-protest).
Finally, the payoff obtained by a non-participating citizen in a failed protest will be decreasing in the level of unexpected violence (as it encourages mobilisation) and decreasing in the level of personal anti-regime sentiment (the more extremist I am, the more incentive I will have to participate).

4.5.3 Properties of Equilibrium and Outcomes

The finite equilibrium of the game is represented by the cutoff:

$$\widehat{\theta} = -log\left((v - t^*) - \frac{k}{\gamma} + 1\right)$$

Result 4.4 *The equilibrium cutoff of the policy game with payoff matrix 3 is:*

- *Increasing in the protesting cost k*
- *Decreasing in inclusiveness γ*
- *Decreasing in the unexpected component of the public signal*

Since the game is characterized by a finite equilibrium or an infinite equilibrium with zero participation, the game is characterized by three outcomes:

- Failed protest
- Successful protest
- No protest

Protest does not occur when $v - t^* \leq \frac{k}{\gamma} - 1$, therefore:

$$P(\text{No protest}) = P\left(v - t^* \leq \frac{k}{\gamma} - 1\right) = \Phi\left(\frac{\frac{k}{\gamma} - 1 - \theta}{\sigma_\eta}\right)$$

Result 4.5 *The probability of no protest in the policy game with payoff matrix 3 is:*

- *Increasing in repression*
- *Decreasing in the antigovernment sentiment*
- *Increasing in opacity if $\theta > \frac{k}{\gamma} - 1$ with upper limit $\frac{1}{2}$ otherwise decreasing with lower limit $\frac{1}{2}$*

The protest is successful when there is mobilization and the mass of participants is greater than T:

$$P(\text{Successful protest}) = P\left(\left[\theta \geq \widehat{\theta} - \sigma_\varepsilon \Phi^{-1}(1-T)\right] \cap \left[v - t^* > \frac{k}{\gamma} - 1\right]\right) =$$

$$= P\left(\left[\theta \geq \widehat{\theta} - \sigma_\varepsilon \Phi^{-1}(1-T)\right] \mid \left[v - t^* > \frac{k}{\gamma} - 1\right]\right) P\left(v - t^* > \frac{k}{\gamma} - 1\right)$$

Result 4.6 *The probability of successful protest in the policy game with payoff matrix 3 is:*

- *Decreasing in repression*
- *Decreasing in the equilibrium cutoff*
- *Tends to be increasing in responsiveness*
- *Increasing in the level of unexpected component of public signal*
- *decreasing in opacity unless the political regime is responsive and the society is radicalized or heterogenous where is increasing for small level of opacity and then decreasing;*
- *increasing in diversity unless the political regime is unresponsive but tolerant and the society is radicalized;*
- *increasing in radicalization unless the political regime is responsive and tolerant and the society diverse.*

The protest fails when there is mobilization and the mass of participants is less than T:

$$P(\text{Failed protest}) = P\left(\left[\theta < \widehat{\theta} - \sigma_\varepsilon \Phi^{-1}(1-T)\right] \cap \left[v - t^* > \frac{k}{\gamma} - 1\right]\right) =$$
$$= P\left(\left[\theta < \widehat{\theta} - \sigma_\varepsilon \Phi^{-1}(1-T)\right] \mid \left[v - t^* > \frac{k}{\gamma} - 1\right]\right) P\left(v - t^* > \frac{k}{\gamma} - 1\right)$$

Result 4.7 *The probability of failed protest in the policy game with payoff matrix 3 is:*

- *Increasing in the equilibrium cutoff*
- *First increasing and then decreasing in the level of unexpected component of public signal*
- *increasing in responsiveness when the political regime is intolerant or tolerant and the society homogeneous, otherwise is not monotonic, first decreasing and increasing*
- *decreasing in repression unless the political regime is responsive and opaque when the relation is not monotonic, first increasing and then decreasing;*
- *decreasing in opacity unless the political regime is tolerant and the society radicalized when the relation is increasing;*
- *decreasing in diversity unless the political regime is unresponsive but tolerant and the society homogeneous when the relation is increasing or when the political regime is responsive and tolerant and the society moderate when the relations is not monotonic, but first increasing and then decreasing;*
- *increasing in radicalization unless the political regime is responsive and tolerant, and the society is homogeneous: in this case the relation is not monotonic, but first increasing and then decreasing.*

4.6 Alternative Matrix II

4.6.1 Specification

A second alternative to handle the critical issues contained in the original game is represented by the matrix:

	$N < T$	$N \geq T$
$a_i = 0$	$-(1-\gamma)\theta_i$	$(1-\gamma)\theta_i$
$a_i = 1$	$-k$	$\theta_i - k$
Payoff matrix 4		

The policy change game with payoff matrix 4 is characterized by two sided limit dominance:

- citizens with $\theta_i \in \left(-\infty, \min\left\{\frac{k}{\gamma}, \frac{k}{1-\gamma}\right\}\right)$ have $a_i = 0$ as dominant strategy
- citizens with $\theta_i \in \left(\max\left\{\frac{k}{\gamma}, \frac{k}{1-\gamma}\right\}, +\infty\right)$ have $a_i = 1$ as dominant strategy

To verify this property it is sufficient to observe that:

$$\theta_i - k \geq (1-\gamma)\theta_i \Longleftrightarrow \theta_i \geq \frac{k}{\gamma}$$

$$-k \geq -(1-\gamma)\theta_i \Longleftrightarrow \theta_i \geq \frac{k}{1-\gamma}$$

Consider α as the probability of winning protest:

$$\alpha = \left[1 - \Phi\left(\frac{\widehat{\theta} - \sigma_\varepsilon \Phi^{-1}(1-T) - \psi(v - t^*) - \lambda(1-\psi)\widehat{\theta}}{\sqrt{\psi\sigma_\eta^2}}\right)\right]$$

Then i's rational behavior in the supposed equilibrium is represented by the equation:

$$\alpha\widehat{\theta} - k = (1-\gamma)\alpha\widehat{\theta} - (1-\gamma)(1-\alpha)\widehat{\theta}$$

$$\alpha\widehat{\theta} - k = \alpha\widehat{\theta} - \widehat{\theta} + \gamma\widehat{\theta} - 2\gamma\alpha\widehat{\theta}$$

$$\widehat{\theta}(1 - \gamma - \alpha + 2\gamma\alpha) = k$$

If $\gamma = \frac{1}{2}$ the equilibrium cutoff is finite and has a fixed value of $2k$ regardless of the values of the game parameters because the equation would become:

$$\widehat{\theta}\left(1 - \frac{1}{2}\right) = k \Rightarrow \widehat{\theta} = 2k$$

Proposition 4.3 *If $\gamma = \frac{1}{2}$ the policy change game with payoff matrix 4 has a unique equilibrium strategy:*

$$s^*(\theta_i) = \begin{cases} 0 & \theta_i < 2k \\ 1 & \theta_i \geq 2k \end{cases}$$

If $\gamma \in \left(0, \frac{1}{2}\right)$:

- the object $1-\gamma$ correspond to a positive fixed value strictly greater than $\frac{1}{2}$
- $2\gamma\alpha - \alpha$ is negative for any value of $\widehat{\theta}$ since $2\gamma < 1$
- $\frac{k}{1-\gamma} < \frac{k}{\gamma}$
- the marginal profit derived from participating in a successful protest γ is small

Given the definition of α its value decreases as $\widehat{\theta}$ increases (because $\lambda(1-\psi) < 1$) consequently the object $(1-\gamma+2\gamma\alpha-\alpha)$ increases in $\widehat{\theta}$ because:

- $(1-\gamma)$ is a positive fixed value
- $(2\gamma\alpha-\alpha)$ is negative with absolute value decreasing in $\widehat{\theta}$

At this point if we consider the previous equilibrium equation:

$$\widehat{\theta}(1-\gamma-\alpha+2\gamma\alpha) = k$$

the left-hand member:

- is the product between two increasing factors in $\widehat{\theta}$ ($\widehat{\theta}$ and $(1-\gamma-\alpha+2\gamma\alpha)$) therefore it is increasing in $\widehat{\theta}$
- $\lim_{\widehat{\theta}\to+\infty} \widehat{\theta}(1-\gamma-\alpha+2\gamma\alpha) = +\infty$
- $\lim_{\widehat{\theta}\to+\infty} \widehat{\theta}(1-\gamma-\alpha+2\gamma\alpha) = -\infty$
- $\widehat{\theta}(1-\gamma-\alpha+2\gamma\alpha) > 0 \Longleftrightarrow \widehat{\theta} > 0$
- $\widehat{\theta}(1-\gamma-\alpha+2\gamma\alpha) < 0 \Longleftrightarrow \widehat{\theta} < 0$

while k is a positive constant, therefore there is always exist a unique finite equilibrium such that:

$$\widehat{\theta} \in \left(\frac{k}{1-\gamma}, \frac{k}{\gamma}\right)$$

The increase of $v - t^*$ on the other hand generates a growth of the probability α and consequently of the negative part of the object $(1-\gamma-\alpha+2\gamma\alpha)$ for this reason within this scenario the value of the equilibrium cutoff increases in the unexpected component of the public signal.

Proposition 4.4 *If the marginal profit derived from participating in a winning protest is small* $\left(\gamma \in \left(0, \frac{1}{2}\right)\right)$ *the policy change game with payoff matrix 4 has a unique equilibrium strategy:*

$$s^*(\theta_i) = \begin{cases} 0 & \theta_i < \widehat{\theta}_0 \\ 1 & \theta_i \geq \widehat{\theta}_0 \end{cases}$$

where the equilibrium cutoff $\widehat{\theta}_0 \in \left(\frac{k}{1-\gamma}, \frac{k}{\gamma}\right)$ *is increasing in the unexpected component of the public signal*

If $\gamma \in \left(\frac{1}{2}, 1\right)$:

- the object $1-\gamma$ is a positive fixed value less than $\frac{1}{2}$
- $2\gamma\alpha - \alpha$ is positive for any value of $\widehat{\theta}$ since $2\gamma > 1$
- $\frac{k}{1-\gamma} > \frac{k}{\gamma}$
- the marginal profit derived from participating in a successful protest γ is large

Given the definition of α its value decreases as $\widehat{\theta}$ increases consequently the object $(1-\gamma+2\gamma\alpha-\alpha)$ decreases in $\widehat{\theta}$ because:

- $(1-\gamma)$ is a positive fixed value
- $(2\gamma\alpha-\alpha)$ is positive and decreasing in $\widehat{\theta}$

At this point if we consider the equilibrium equation:

$$\widehat{\theta}(1-\gamma-\alpha+2\gamma\alpha) = k$$

the left-hand member:

- is the product between a factor increasing in $\widehat{\theta}$ $(\widehat{\theta})$ and another one decreasing in $\widehat{\theta}$ $(1-\gamma-\alpha+2\gamma\alpha)$
- $\lim_{\widehat{\theta}\to+\infty} \widehat{\theta}(1-\gamma-\alpha+2\gamma\alpha) = +\infty$
- $\lim_{\widehat{\theta}\to+\infty} \widehat{\theta}(1-\gamma-\alpha+2\gamma\alpha) = -\infty$
- $\widehat{\theta}(1-\gamma-\alpha+2\gamma\alpha) > 0 \Longleftrightarrow \widehat{\theta} > 0$
- $\widehat{\theta}(1-\gamma-\alpha+2\gamma\alpha) < 0 \Longleftrightarrow \widehat{\theta} < 0$

while k is a fixed positive constant.
Observing the limits of the function, when $\widehat{\theta} \to +\infty$ the function definitely increases toward $+\infty$ while when $\widehat{\theta} \to -\infty$ its definitely decreases toward $-\infty$. However due to the opposite monotonicity of its factors the transition between these two limits does not occur in a monotonically increasing way. In other words, the function grows monotonically to a local maximum, decreases to a local minimum, and then grows monotonically. This result is supported by the fact that when $\gamma \in \left(\frac{1}{2}, 1\right)$ the derivative of the left-hand member in $\widehat{\theta}$ takes on at least in one interval a negative value:

$$\frac{\partial\, \widehat{\theta}(1-\gamma-\alpha+2\gamma\alpha)}{\partial\widehat{\theta}} = (1-\gamma) + (2\gamma-1)\left(\alpha + \frac{\partial\alpha}{\partial\widehat{\theta}}\widehat{\theta}\right)$$

$$\frac{\partial\alpha}{\partial\widehat{\theta}} = \phi\left(-\frac{\widehat{\theta} - \sigma_\varepsilon\Phi^{-1}(1-T) - \psi(v-t^*) - \lambda(1-\psi)\widehat{\theta}}{\sqrt{\psi\sigma_\eta^2}}\right)\left(\frac{-\widehat{\theta}(1-\lambda(1-\psi))}{\sqrt{\psi\sigma_\eta^2}}\right)$$

This happens because as $\widehat{\theta}$ increases:

- $2\gamma - 1 > 0$
- the probability α decreases from unit to zero as $\widehat{\theta}$ increases
- the object $\frac{\partial\alpha}{\partial\widehat{\theta}}\widehat{\theta}$ always takes on negative values (except when it is null for $\widehat{\theta} = 0$), assuming a W-shape and thus tending towards 0^- for $\widehat{\theta} \to \pm\infty$ with two equal minimum values due to fact that there is an even symmetry between negative and positive values of $\widehat{\theta}$

Consequently, the negativity of the derivative is related to the sign of the factor $\left(\alpha + \frac{\partial\alpha}{\partial\widehat{\theta}}\widehat{\theta}\right)$ which takes on negative values where the probability α is smallest i.e., in the space of positive $\widehat{\theta}$ because as mentioned α decreases monotonically, whereas $\frac{\partial\alpha}{\partial\widehat{\theta}}\widehat{\theta}$ is a negative function with symmetrical even behavior with respect to $\widehat{\theta} = 0$.
The nonmonotonic trend of the left-hand member of the equilibrium equation highlights the fact that there will always exist at least one finite equilibrium, however, for given values of participation costs there could be three finite equilibria. The presence of two sided limit dominance, on the other hand, rules out the presence of infinite equilibria.
Regarding $v - t^*$ its increase generates a growth of the probability α and consequently of the function, this induces a decrease in the lower and higher equilibrium while an increase of the intermediate one, however as violence increases there is a threshold for which only the lower equilibrium is observed.

Proposition 4.5 *If the marginal profit derived from participating in a winning protest is large* $\left(\gamma \in \left(\frac{1}{2}, 1\right)\right)$ *the policy change game with payoff matrix 4 has potentially three strategies consistent with a cutoff equilibrium:*

$$s^*(\theta_i) = \begin{cases} 0 & \theta_i < \widehat{\theta}_L \\ 1 & \theta_i \geq \widehat{\theta}_L \end{cases}$$

$$s^*(\theta_i) = \begin{cases} 0 & \theta_i < \widehat{\theta}_M \\ 1 & \theta_i \geq \widehat{\theta}_M \end{cases}$$

$$s^*(\theta_i) = \begin{cases} 0 & \theta_i < \widehat{\theta}_H \\ 1 & \theta_i \geq \widehat{\theta}_H \end{cases}$$

where $\widehat{\theta}_L < \widehat{\theta}_M < \widehat{\theta}_H$*; the increase of the unexpected component of the public signal induces the reduction only of the first and the latter, but above a certain critical threshold it automatically selects the lower equilibrium*

Summarizing what emerged as γ varies infinite equilibria are not observed, only for $\gamma \in \left(\frac{1}{2}, 1\right)$ it is possible to observe multiple finite equilibria with the selection of the lower one by the growth of the unexpected component of the public signal, and the latter plays a positive role toward participation only when $\gamma \in \left(\frac{1}{2}, 1\right)$.

The advantages of this approach are:

- the smoothness property is preserved since the otucome of the game and the equilibria are affected by the variation of the game parameters
- multiplicity of equilibria is preserved when $\gamma \in \left(\frac{1}{2}, 1\right)$
- when a player makes a choice, he never has both non-negative payoffs since $-k < 0$ and $(1-\gamma)\theta_i$ always has opposite sign to $-(1-\gamma)\theta_i$ given any level of θ_i

To verify the validity of the third point is sufficient to observe that:

$$-(1-\gamma)\theta_i > 0 \Longleftrightarrow \theta_i < 0$$

$$(1-\gamma)\theta_i > 0 \Longleftrightarrow \theta_i > 0$$

The drawbacks are:

- the multiplicity of equilibria is related to finite equilibria
- not all finite equilibria are decreasing in the level of unexpected component of the public signal

However, what alleviates the weaknesses of this approach is the fact that in the presence of multiple finite equilibria the growth in the level of unexpected component of the public signal selects the lower one (decreasing in it),therefore there is a kind of selection mechanism in cases of multiplicity; while the presence of a unique finite equilibrium increasing in the level of unexpected violence occurs when the marginal profit derived from participating in a successful protest is small, that is, in a scenario at the interpretive level unfavorable to the protest. In this framework there is a mechanism to select an equilibrium between many and the their multiplicity is not random, but driven by the parameter γ that also plays a role in the behavior of the equilibria.

4.6.2 Properties of Equilibria and Outcomes

If $\gamma \in \left(\frac{1}{2}, 1\right)$ the finite equilibria of the game are:

$$\widehat{\theta}_L < \widehat{\theta}_M < \widehat{\theta}_H$$

If $\gamma \in \left(0, \frac{1}{2}\right)$ the finite equilibrium $\widehat{\theta}_L$ is unique.

Result 4.8 *If $\gamma = \frac{1}{2}$ the equilibrium cutoff of the policy game with payoff matrix 4 is:*

- *Increasing in protesting cost k*
- *Independent from the other game parameters*

If $\gamma \in \left(0, \frac{1}{2}\right)$ the equilibrium cutoff of the policy game with payoff matrix 4 is:

- *increasing in the unexpected component of the public signal $v - t^*$;*

- *increasing in the responsiveness of the political regime;*
- *decreasing in the repression of the political regime;;*
- *decreasing in opacity unless the political regime is responsive and tolerant;*
- *decreasing in diversity unless the political regime is responsive;*
- *increasing in radicalization unless the political regime is responsive but opaque and the society diverse.*

If $\gamma \in \left(\frac{1}{2}, 1\right)$ *the equilibrium cutoff of the policy game with payoff matrix 4 is (we consider the lower cutoff since it is the only one that is selected by the increase in the unexpected component of public signal):*

- *decreasing in the unexpected component of the public signal* $v - t^*$*;*
- *decreasing in the responsiveness of the political regime;*
- *increasing in the repression of the political regime;;*
- *increasing in opacity unless the political regime is responsive and tolerant;*
- *increasing in diversity unless the political regime is responsive;*
- *decreasing in radicalization unless the political regime is responsive but opaque and the society diverse.*

As the parameter γ varies, there is always a finite equilibrium cutoff, so there are two outcomes of the game:

- Failed protest
- Successful protest

The protest is successful if the mass of participants is greater than T, therefore:

$$P(\text{Successful protest}) = P(\theta \geq \widehat{\theta} - \sigma_\varepsilon \Phi^{-1}(1-T)) = 1 - \Phi\left(\frac{\widehat{\theta} - \sigma_\varepsilon \Phi^{-1}(1-T)}{\sigma_\theta}\right)$$

Result 4.9 *The probability of successful protest in the policy game with payoff matrix 4 is:*

- *If* $\gamma = \frac{1}{2}$*:*
  - *Decreasing in the protesting cost k*
  - *Increasing in responsiveness*
  - *Increasing in diversity unless the regime is unresponsive*

  - *Increasing in radicalization if* $2k > \sigma_\varepsilon \Phi^{-1}(1-T)$ *with lower limit* $\frac{1}{2}$*, otherwise decreasing with upper limit* $\frac{1}{2}$

- *If* $\gamma \in \left(\frac{1}{2}, 1\right)$*:*
  - *Decreasing in the protesting cost k*
  - *Increasing in responsiveness*
  - *Increasing in the unexpected component of the public signal* $v - t^*$
  - *Decreasing in opacity unless the political regime is responsive and tolerant*
  - *Increasing in diversity unless the regime is unresponsive*
  - *If the cutoff decreases in radicalization the probability is decreasing with lower limit* $\frac{1}{2}$ *or it is first increasing (if the cutoff is high) and then decreasing with lower limit* $\frac{1}{2}$*. If the cutoff increases in radicalization the probability is increasing with upper limit* $\frac{1}{2}$ *or it is first decreasing (if the cutoff is low) and then decreasing with upper limit* $\frac{1}{2}$*.*

- *If* $\gamma \in \left(0, \frac{1}{2}\right)$*:*
  - *Increasing in the protesting cost k*
  - *Doubt in responsiveness as it causes the cutoff to increase*
  - *Decreasing in the unexpected component of the public signal* $v - t^*$
  - *Increasing in opacity unless the political regime is responsive and tolerant*
  - *Doubt in diversity as its effect on cutoff and responsiveness is opposite*
  - *If the cutoff decreases in radicalization the probability is decreasing with lower limit* $\frac{1}{2}$ *or it is first increasing (if the cutoff is high) and then decreasing with lower limit* $\frac{1}{2}$*. If the cutoff increases in radicalization the probability is increasing with upper limit* $\frac{1}{2}$ *or it is first decreasing (if the cutoff is low) and then decreasing with upper limit* $\frac{1}{2}$*.*

With two complementary outcomes, the probability of a failing protest will be the complementary of the probability of a winning protest and consequently the comparative statics will be opposite.

4.7 Alternative Matrix III

4.7.1 Specification

It is possible to construct an analogous case to matrix 3 by exploiting the cumulative distribution function instead of the exponential function:

	$N < T$	$N \geq T$
$a_i = 0$	$\left[\frac{1}{2} - \Phi(\theta_i)\right] - (v - t^*)$	$-\left[\frac{1}{2} - \Phi(\theta_i)\right] + (v - t^*)$
$a_i = 1$	$-k$	$2(v - t^*) - 2\left[\frac{1}{2} - \Phi(\theta_i)\right] - k$
Payoff matrix 5		

In this scenario, if we denote as α the probability of successful protest, the equilibrium condition becomes:

$$2(v-t^*)\alpha - 2[1-\Phi(\widehat{\theta})]\alpha - k = -[1-\Phi(\widehat{\theta})]\alpha + (v-t^*)\alpha + [1-\Phi(\widehat{\theta})](1-\alpha) - (v-t^*)(1-\alpha)$$
$$-[1-\Phi(\widehat{\theta})] + (v-t^*) = k$$

The left-hand member is composed by two components:

- $-[1-\Phi(\widehat{\theta})]$ is a negative function increasing in $\widehat{\theta}$ with $\lim_{\widehat{\theta}\to-\infty} -[1-\Phi(\widehat{\theta})] = -1^+$, $\lim_{\widehat{\theta}\to+\infty} -[1-\Phi(\widehat{\theta})] = 0^-$
- $v - t^* \in \mathbb{R}$

Therefore it takes positive values if and only if $v - t^* > 0$. Given that k is a positive fixed value a finite equilibrium will be observed if and only if $k < v - t^* < k + 1$.
Finite equilibrium where it exists will be characterized by the analytic form:

$$\widehat{\theta} = \Phi^{-1}(k - (v - t^*) + 1)$$

This scenario retains all the advantageous properties of the aforementioned case:

- the smoothness property is preserved since the otucome of the game and the equilibrum are affected by the variation of the game parameters
- multiplicity of equilibria with a unique finite equilibrium decreasing in the unexpected component of the public signal and two infinite equilibria one of zero-participation and the other of full participation
- when a player makes a choice he never has the both non-negative payoffs since $-k < 0$ and $[1-\Phi(\theta_i)] - (v-t^*)$ always has opposite sign to $-[1-\Phi(\theta_i)] + (v-t^*)$ given any level of $\theta_i, v - t^*$

To verify the validity of the third point is sufficient to observe that:

$$[1-\Phi(\theta_i)] - (v-t^*) > 0 \Longleftrightarrow v - t^* < [1-\Phi(\theta_i)]$$
$$-[1-\Phi(\theta_i)] + (v-t^*) > 0 \Longleftrightarrow v - t^* > [1-\Phi(\theta_i)]$$

The interesting aspect with respect to the case involving the use of the exponential function is that while finite equilibrium where it exists is single (and decreasing in the unexpected component of the public signal) infinite equilibrium is dual therefore mass shifts toward one or the other strategy are possible, a behavior that is related to the level of unexpected component of the public signal.

Proposition 4.6 *The policy change game with payoff matrix 5 has three strategies consistent with a cutoff equilibrium: strategy:*

if $k < v - t^* < k + 1$*:*

$$s^*(\theta_i) = \begin{cases} 0 & \theta_i < \Phi^{-1}(k - (v - t^*) + 1) \\ 1 & \theta_i \geq \Phi^{-1}(k - (v - t^*) + 1) \end{cases}$$

where $\Phi^{-1}(k - (v - t^*) + 1)$ *is decreasing in the level of unexpected component of the public signal.*

If $v - t^* \leq k$*:*

$$s^*(\theta_i) = 0 \; \forall \theta_i$$

If $v - t^* \geq k + 1$*:*

$$s^*(\theta_i) = 1 \; \forall \theta_i$$

4.7.2 Interpretability

The interpretation of the payoff matrix 5 is analogous to the payoff matrix 3, the difference is that in this scenario we employ the function $\Phi()$ instead of the exponential function to generate the second member of the citizens' payoff. This modification allows the game to be characterized also by the outcome of total mobilization differently to the original game because this component has an upper and a lower limit.

4.7.3 Properties of Equilibrium and Outcomes

The finite equilibrium of the game is represented by the cutoff:

$$\widehat{\theta} = \Phi^{-1}(k - (v - t^*) + 1)$$

Result 4.10 *The equilibrium cutoff of the policy game with payoff matrix 5 is:*

- *Increasing in the protesting cost k*
- *Decreasing in the unexpected component of the public signal*

Since the finite equilibrium is unique, but its existence is not always guaranteed (due to the presence of infinite equilibria in both directions), the game is characterized by four outcomes:

- Failed protest
- Successful protest
- No protest
- Total mobilization

Protest does not occur when $v - t^* \leq k$, therefore:

$$P(\text{No protest}) = P\left(v - t^* \leq k\right) = \Phi\left(\frac{k-\theta}{\sigma_\eta}\right)$$

Result 4.11 *The probability of no protest in the policy game with payoff matrix 5 is:*

- *Increasing in protesting cost k*
- *Decreasing in the antigovernment sentiment*
- *Increasing in opacity if $\theta > k$ with upper limit $\frac{1}{2}$ otherwise decreasing with lower limit $\frac{1}{2}$*

Total mobilization occurs when $v - t^* \geq k + 1$, therefore:

$$P(\text{Total mobilization}) = P\left(v - t^* \geq k+1\right) = 1 - \Phi\left(\frac{k+1-\theta}{\sigma_\eta}\right)$$

Result 4.12 *The probability of total mobilization in the policy game with payoff matrix 5 is:*

- *Decreasing in protesting cost k*
- *Increasing in the antigovernment sentiment*
- *Decreasing in opacity if $\theta > k+1$ with upper limit $\frac{1}{2}$ otherwise increasing with lower limit $\frac{1}{2}$*

The protest is successful when there is mobilization and the mass of participants is greater than T:

$$P(\text{Successful protest}) = P\left(\left[\theta \geq \widehat{\theta} - \sigma_\varepsilon \Phi^{-1}(1-T)\right] \cap \left[v - t^* > k\right]\right) =$$
$$= P\left(\left[\theta \geq \widehat{\theta} - \sigma_\varepsilon \Phi^{-1}(1-T)\right] \mid \left[v - t^* > k\right]\right) P\left(v - t^* > k\right)$$

Result 4.13 *The probability of successful protest in the policy game with payoff matrix 5 is:*

- *Decreasing in protesting cost k*
- *Decreasing in the equilibrium cutoff*
- *Tends to be ncreasing in responsiveness*
- *Increasing in the level of unexpected component of public signal*
- *decreasing in opacity unless the political regime is responsive and the society is radicalized or heterogenous where is increasing for small level of opacity and then decreasing;*

- *increasing in diversity unless the political regime is unresponsive but tolerant and the society is radicalized;*
- *increasing in radicalization unless the political regime is responsive and tolerant and the society diverse.*

The protest fails when there is mobilization and the mass of participants is less than T:

$$P(\text{Failed protest}) = P\left(\left[\theta < \widehat{\theta} - \sigma_\varepsilon \Phi^{-1}(1-T)\right] \cap \left[v - t^* > k\right]\right) =$$
$$= P\left(\left[\theta < \widehat{\theta} - \sigma_\varepsilon \Phi^{-1}(1-T)\right] \mid \left[v - t^* > k\right]\right) P\left(v - t^* > k\right)$$

Result 4.14 *The probability of failed protest in the policy game with payoff matrix 5 is:*

- *Increasing in the equilibrium cutoff*
- *First increasing and then decreasing in the level of unexpected component of public signal*
- *increasing in responsiveness when the political regime is intolerant or tolerant and the society homogeneous, otherwise is not monotonic, first decreasing and increasing*
- *decreasing in repression unless the political regime is responsive and opaque when the relation is not monotonic, first increasing and then decreasing;*
- *decreasing in opacity unless the political regime is tolerant and the society radicalized when the relation is increasing;*
- *decreasing in diversity unless the political regime is unresponsive but tolerant and the society homogenous when the relation is increasing or when the political regime is responsive and tolerant and the society moderate when the relations is not monotonic, but first increasing and then decreasing;*
- *increasing in radicalization unless the political regime is responsive and tolerant, and the society is homogenous: in this case the relation is not monotonic, but first increasing and then decreasing.*

4.8 Alternative Matrix IV

4.8.1 Specification

In the original game the participants in the protest incur a cost k, if the protest is successful all citizens get $(1-\gamma)\theta_i$ regardless of their strategy, however if they are participants they get an additional payoff $\gamma\theta_i$.
The formulation in this section introduces a modification: participating always induces a cost, but regardless of the outcome of the protest also a gain (derived from belonging

to a movement) positive, increasing in the level of private signal, but always less than the participation cost.
Thus the cost function is strictly positive and constant, while the utility of being part of the protest (regardless of the outcome) is also positive, but increasing in the level of private signal.

	$N < T$	$N \geq T$
$a_i = 0$	0	$(1-\gamma)\theta_i$
$a_i = 1$	$-k + k\Phi(\theta_i)$	$\theta_i - k + k\Phi(\theta_i)$
Payoff matrix 6		

In terms of interpretation the function $\Phi(\theta_i)$ indicates the mass to the left of the value of θ_i considered a standard normal distribution. It is a probability, so it will always be an object less than 1 increasing in the level of personal anti-regime sentiment. Having an always positive estimate with upper bound 1 multiplied by the fixed share of the participation costs will guarantee that the upper bound will be represented by k. In other words the underlying utility of participation with this functional form will be stably contained below the mobilization cost. This property is useful in order to retain the scenario of one-sided limit dominance for the strategy $a_i = 0$.

The policy change game with payoff matrix 6 is characterized by one-sided limit dominance because:

$$-k + k\Phi(\theta_i) < 0 \ \forall \theta_i$$

$$\theta_i - k + k\Phi(\theta_i) < (1-\gamma)\theta_i \Longleftrightarrow \theta_i < \theta_i^*$$

In this scenario, if we denote as α the probability of successful protest, the equilibrium condition becomes:

$$\alpha\gamma\theta_i + k\Phi(\theta_i) = k$$

By simple simulations it is possible to show that if $v - t^*$ is less than a certain threshold $(v - t_M^*)$ the the left-hand member of the equation is always less than k leading to the presence only of an infinite equilibrium of zero participation, otherwise it is greater than k for $\widehat{\theta} \geq \widehat{\theta}_L$ where this cutoff is decreasing in the level of the unexpected component of the public signal.

This scenario retains the useful properties of the original game:

- One-sided limit dominance only for the strategy $a_i = 0$
- Smoothness property because the equilibrium and therefore the outcome of the game are affected by the variation of the game parameters

Moreover it preserves the multiplicity of equilibria with an unique finite equilibrium decreasing in the unexpected component of public signal and one infinite equilibrium of zero participation.

Proposition 4.7 *The policy change game with payoff matrix 6 has two strategies consistent with a cutoff equilibrium: strategy:*

if $v - t^* \geq v - t^*_M$*:*

$$s^*(\theta_i) = \begin{cases} 0 & \theta_i < \widehat{\theta}_L \\ 1 & \theta_i \geq \widehat{\theta}_L \end{cases}$$

where $\widehat{\theta}_L$ *is decreasing in the level of unexpected component of the public signal.*

If $v - t^* < v - t^*_M$*:*

$$s^*(\theta_i) = 0 \ \ \forall \theta_i$$

4.8.2 Properties of Equilibrium and Outcomes

Result 4.15 *The finite equilibrium cutoff of the policy game with payoff matrix 6 is:*

- *decreasing in the unexpected component of the public signal* $v - t^*$
- *decreasing in the responsiveness of the political regime*
- *increasing in the repression of the political regime*
- *increasing in opacity unless the political regime is responsive and tolerant*
- *increasing in diversity unless the political regime is responsive*
- *decreasing in radicalization unless the political regime is responsive but opaque and the society diverse.*

Since the finite equilibrium is unique, but its existence is not always guaranteed (due to the presence of an infinite equilibrium of zero participation), the game is characterized by three outcomes:

- Failed protest
- Successful protest
- No protest

Protest does not occur when $v - t^* \leq v - t^*_M$, therefore:

$$P(\text{No protest}) = P\left(v - t^* \leq v - t^*_M\right) = \Phi\left(\frac{v - t^*_M - \theta}{\sigma_\eta}\right)$$

Result 4.16 *The probability of no protest in the policy game with payoff matrix 6 is:*

- *Increasing in protesting cost* k
- *Increasing in* T

- *increasing in diversity unless the political regime is responsive*
- *decreasing in radicalization unless the political regime is responsive but opaque and the society diverse*
- *Decreasing in the antigovernment sentiment*
- *Increasing in opacity if $\theta > v - t^*_M$ with upper limit $\frac{1}{2}$ otherwise decreasing with lower limit $\frac{1}{2}$*

The protest is successful when there is mobilization and the mass of participants is greater than T:

$$P(\text{Successful protest}) = P\left(\left[\theta \geq \widehat{\theta} - \sigma_\varepsilon \Phi^{-1}(1-T)\right] \cap \left[v - t^* > v - t^*_M\right]\right) =$$

$$= P\left(\left[\theta \geq \widehat{\theta} - \sigma_\varepsilon \Phi^{-1}(1-T)\right] \mid \left[v - t^* > v - t^*_M\right]\right) P\left(v - t^* > v - t^*_M\right)$$

Result 4.17 *The probability of successful protest in the policy game with payoff matrix 6 is:*

- *Decreasing in protesting cost k*
- *Decreasing in the equilibrium cutoff*
- *Tends to be ncreasing in responsiveness*
- *Increasing in the level of unexpected component of public signal*
- *decreasing in opacity unless the political regime is responsive and the society is radicalized or heterogenous where is increasing for small level of opacity and then decreasing;*
- *increasing in diversity unless the political regime is unresponsive but tolerant and the society is radicalized;*
- *increasing in radicalization unless the political regime is responsive and tolerant and the society diverse.*

The protest fails when there is mobilization but the mass of participants is less than T:

$$P(\text{Failed protest}) = P\left(\left[\theta < \widehat{\theta} - \sigma_\varepsilon \Phi^{-1}(1-T)\right] \cap \left[v - t^* > v - t^*_M\right]\right) =$$

$$= P\left(\left[\theta < \widehat{\theta} - \sigma_\varepsilon \Phi^{-1}(1-T)\right] \mid \left[v - t^* > v - t^*_M\right]\right) P\left(v - t^* > v - t^*_M\right)$$

Result 4.18 *The probability of failed protest in the policy game with payoff matrix 6 is:*

- *Increasing in the equilibrium cutoff*

- *First increasing and then decreasing in the level of unexpected component of public signal*

- *increasing in responsiveness when the political regime is intolerant or tolerant and the society homogeneous, otherwise is not monotonic, first decreasing and increasing*

- *decreasing in repression unless the political regime is responsive and opaque when the relation is not monotonic, first increasing and then decreasing;*

- *decreasing in opacity unless the political regime is tolerant and the society radicalized when the relation is increasing;*

- *decreasing in diversity unless the political regime is unresponsive but tolerant and the society homogenous when the relation is increasing or when the political regime is responsive and tolerant and the society moderate when the relations is not monotonic, but first increasing and then decreasing;*

- *increasing in radicalization unless the political regime is responsive and tolerant, and the society is homogenous: in this case the relation is not monotonic, but first increasing and then decreasing.*

4.9 The General Case

Consider a general payoff matrix:

	$\mathcal{N} < T \Leftrightarrow R = 0$	$\mathcal{N} \geq T \Leftrightarrow R = 1$
$a_i = 0$	$\pi_1(\theta_i)$	$\pi_2(\theta_i)$
$a_i = 1$	$\pi_3(\theta_i) - k$	$\pi_4(\theta_i) - k$
General payoff matrix		

Let we make the following assumption on the payoff functions:

Hypothesis 4.1 *The payoff functions $\pi_j(\theta_i)$, $j = 1,2,3,4$, are continuous in θ_i.*

Moreover, we assume that these functions satisfy the following qualitative properties:

- $\pi_1(\theta_i)$ is associated to $a_i = 0$ and $R = 0$, then a supporter of the policy, i.e. such that $\theta_i < 0$, should have a positive payoff, the opposite if i is an opponent. Moreover, we can assume that the stronger the support of the policy, the greater is the payoff, hence $\pi_1(\theta_i)$ should satisfy the following conditions:

$$\pi_1(\theta_i) = \begin{cases} > 0 & \text{if } \theta_i < 0 \\ = 0 & \text{if } \theta_i = 0 \\ < 0 & \text{if } \theta_i > 0 \end{cases} \quad \text{and} \quad \frac{\partial \pi_1(\theta_i)}{\partial \theta_i} < 0.$$

- $\pi_2(\theta_i)$ is associated to $a_i = 0$ and $R = 1$, then a supporter of the policy, i.e. such that $\theta_i < 0$, should have a negative payoff, but also an opponent should be not fully satisfied since he/she didn't help to change the policy. Moreover, we can assume that the stronger the opposition to the policy, the greater is the payoff, hence $\pi_2(\theta_i)$ should satisfy the following conditions:

$$\pi_2(\theta_i) = \begin{cases} <0 & \text{if } \theta_i < 0 \\ =0 & \text{if } \theta_i = 0 \\ >0 & \text{if } \theta_i > 0 \end{cases} \quad \text{and} \quad \frac{\partial \pi_2(\theta_i)}{\partial \theta_i} > 0.$$

- $\pi_3(\theta_i)$ is associated to $a_i = 1$ and $R = 0$, then a supporter of the policy, i.e. such that $\theta_i < 0$, should have a slightly positive payoff, also an opponent should be disappointed since his/her support didn't help to change the policy. Moreover, we can assume that the stronger the support for the policy, the greater is the payoff, hence $\pi_3(\theta_i)$ should satisfy the following conditions:

$$\pi_3(\theta_i) = \begin{cases} >0 & \text{if } \theta_i < 0 \\ =0 & \text{if } \theta_i = 0 \\ <0 & \text{if } \theta_i > 0 \end{cases} \quad \text{and} \quad \frac{\partial \pi_3(\theta_i)}{\partial \theta_i} < 0.$$

- $\pi_4(\theta_i)$ is associated to $a_i = 1$ and $R = 1$, then a supporter of the policy, i.e. such that $\theta_i < 0$, should have an extremely negative payoff, while an opponent should be fully satisfied since he/she did help to change the policy. Moreover, we can assume that the stronger the opposition to the policy, the smaller is the payoff, hence $\pi_4(\theta_i)$ should satisfy the following conditions:

$$\pi_4(\theta_i) = \begin{cases} <0 & \text{if } \theta_i < 0 \\ =0 & \text{if } \theta_i = 0 \\ >0 & \text{if } \theta_i > 0 \end{cases} \quad \text{and} \quad \frac{\partial \pi_4(\theta_i)}{\partial \theta_i} > 0.$$

- Referring to the previous considerations, we can order the citizens payoffs as follows

- when $\theta_i < 0$, then

$$\pi_1(\theta_i) > \pi_3(\theta_i) > \pi_2(\theta_i) > \pi_4(\theta_i)$$

- when $\theta_i > 0$, then

$$\pi_1(\theta_i) < \pi_3(\theta_i) < \pi_2(\theta_i) < \pi_4(\theta_i).$$

4.10 Conclusions I

The purpose of this chapter was to investigate the role of the payoff matrix within the game designed in [6] and in general in regime change games with uncertainty related to citizens' payoffs.

Initially, we shed light on the criticality of the starting matrix from the point of view of

both interpretability and the identification of multiple finite equilibria without a selection mechanism.
The first solution was to create an alternative matrix characterised by two-sided limit dominance without swing citizens. This solves the original interpretability problems, but eliminates the 'positive' multiplicity, i.e. excludes the possibility that exogenous characteristics are such that mobilization is impossible.
Keeping swing citizens on the other side can lead to scenarios with an unique finite equilibrium or where the finite equilibria are multiple (analogous to games where uncertainty is linked to the threshold T as in [27],[2],[3]), where the unexpected component of public signal influences the equilibrium cutoffs differently. The interesting aspect is that the scenarios are driven by the value of inclusiveness and that where multiple finite equilibria are observed the growth of the unexpected component of public signal induces the selection of the lower one, thus there is a static selection mechanism. The role of inclusiveness is associated with the thick tails property described in [16], [9], [8], [26]
The inclusion within the payoffs of the unexpected component of public signal level, on the other hand, potentially allows for the identification of an unique finite equilibrium with desirable characteristics at the level of interpretability (e.g. decreasing in the unexpected component of public signal level), while retaining the possibility of observing an equilibrium of total mobilization or absent mobilization. In other words, it is possible to maintain the original structure of one-sided limit dominance by reducing the multiplicity of finite equilibria to an unique one with similar behaviour to the lower one in the original game [6].
This desirable scenario can also be achieved by specifying a participation benefit regardless of the outcome of the protest, which is positive, increasing in the private signal, but always lower than the costs of participation.
These specifications have the considerable advantage of moving from a scenario without finite equilibria to one where the conditions of two-sided limit dominance and thick tails are met leading to an unique finite equilibrium ([16], [9], [8], [26]).
In conclusion, the payoff matrix is crucial, and the number, type and interpretability of the equilibria depend closely on its specification.

4.11 The Original Cost Function

In the game contained in [6], the cost function is:

- a constant $k > 0$
- k value is exogenous and independent of any other variable in the game, therefore it is constant for all citizens whatever their private signal

This formulation is not convincing because it is more plausible that the individual cost of protesting is decreasing in the number of protesters because the amount of government's repression is shared among a greater number of citizens.

Moreover since the equilibrium condition in [6] is:

$$\left[1-\Phi\left(\frac{\widehat{\theta}\left(v-t^{*};p,s\right)(1-(1-\psi)\lambda)-\sigma_{\varepsilon}\Phi^{-1}(1-T)-\psi\left(v-t^{*}\right)}{\sqrt{\psi\sigma_{\eta}^{2}}}\right)\right]\gamma\widehat{\theta}\left(v-t^{*};p,s\right)=k$$

Given the shape of the left-hand member, using the original cost function generates an infinite equilibrium, or two finite equilibria, whereas the desirable scenario would be to generate eventual infinite equilibria, but only one eventual finite equilibrium.
The objective becomes to construct an alternative cost function (with constraints on interpretability) that is able to generate multiplicity of equilibria with an unique finite equilibrium while maintaining the original payoff matrix.

4.12 Interpretability constraints

The cost function must satisfy two constraints:

- Participating in the protest requires an effort on the part of the citizen to come out of his or her silent position, so it must be a strictly positive measure for every level of private signal
- The higher the proportion of participants, the lower the portion of cost borne by each of them, so the cost function must be decreasing in the number of participants

4.13 Alternative Cost Function I

4.13.1 Specification

Consider the equilibrium condition:

$$\left[1-\Phi\left(\frac{\widehat{\theta}\left(v-t^{*};p,s\right)(1-(1-\psi)\lambda)-\sigma_{\varepsilon}\Phi^{-1}(1-T)-\psi\left(v-t^{*}\right)}{\sqrt{\psi\sigma_{\eta}^{2}}}\right)\right]\gamma\widehat{\theta}\left(v-t^{*};p,s\right)=k$$

There is a minimum level of unexpected component of public signal $(v - t^*_M)$ such that the equation has only one solution $\widehat{\theta}_0(v - t^*) > \frac{k}{\gamma}$. [20]

Consider the following cost function with discontinuous rates for the citizens:

$$k(\widehat{\theta}) = \begin{cases} k & \text{if } \widehat{\theta} <\leq \widehat{\theta}_0(v - t^*) \\ \widehat{\theta} & \text{if } \widehat{\theta} > \widehat{\theta}_0(v - t^*) \end{cases}$$

Proposition 4.8 *The alternative cost function:*

- *is strictly positive*
- *is increasing in* $\widehat{\theta}$
- *If* $v - t^* < v - t^*_M$ *dominates the function* $IB()$ $\forall \widehat{\theta}$
- *If* $v - t^* \geq v - t^*_M$ *there is only one intersection between the cost function* $IB()$ *in the space* $\widehat{\theta} \leq \widehat{\theta}_0$

Result 4.19 *This formulation guarantees:*

- *A strictly positive and increasing cost function in the private signal*
- *The presence of a zero infinite equilibrium where the level of unexpected component of public signal is not sufficiently high and the presence of an unique finite equilibrium where the level of unexpected component of public signal is sufficiently high*

Proof.
The function $IB()$ is by construction always less than $\widehat{\theta}$ (because $0 < \gamma < 1$) therefore there will never be a solution greater than $\widehat{\theta}_0$. If the level of $v - t^*$ is too small the function $IB()$ will be strictly dominated by k even in the space $(-\infty, \widehat{\theta}_0)$ and thus only an infinite zero-participation equilibrium cutoff will exist. If the level of $v - t^*$ reaches the minimum level $v - t^*_M$ the $IB()$ function will be equal to k at $\widehat{\theta}_0$ which will represent the finite equilibrium cutoff. The moment the level of $v - t^*$ exceeds this minimum value, the $IB()$ function will dominate k in the interval $(\widehat{\theta}_L, \widehat{\theta}_0)$ and thus the finite equilibrium cutoff will be $\widehat{\theta}_L < \widehat{\theta}_0$. ■

In other words, not only do we have a more interpretable cost function, but the equilibria of the game with respect to the original case are more reasonable, as the finite equilibrium if it exists is unique and decreasing in the level of the unexpected component of public signal.

[20] this property is proven in Chapter 2

4.13.2 Interpretability

Consider the cost function of the original game [6]. all citizens are aware that those characterized by $\theta_i < \frac{k}{\gamma}$ have $a_i = 0$ as their dominant strategy.
Citizens are aware that the growth of the $v - t^*$ level induces the growth of the probability of successful protest by encouraging mobilization. In detail, when $v - t^*$ reaches a minimum level equal to $v - t^*_M$ one goes from a scenario with no mobilization to one with mobilization and consequently the equilibrium cutoff makes a discontinuous jump from an infinite positive value ($\widehat{\theta} = +\infty$) to a finite positive one ($\widehat{\theta} = \widehat{\theta}_0 > 0$).
Citizens therefore know that where the level of $v - t^*$ is equal to the minimum necessary to ensure positive mobilisation the equilibrium cutoff is $\widehat{\theta}_0$ and are aware that further growth of the level of $v - t^*$ raises mobilization and with it the probability of successful protest. Consequently, they strategically regard $\widehat{\theta}_0$ as the upper limit of the finite equilibrium cutoff. All citizens characterised by $\theta_i \geq \widehat{\theta}_0$ where mobilization exists will always participate, while the share of undecideds will be relative to those characterised by $\frac{k}{\gamma} \leq \theta_i < \widehat{\theta}_0$.
The alternative cost function we have specified follows perfectly the reasoning described above, avoiding the creation of an additional finite equilibrium with illogical behaviour, i.e. increasing in the level of $v - t^*$ observed by citizens (by contracting the mass of participants). In addition, its discontinuity is also perfectly reasonable given the parallel discontinuity of the equilibrium cutoff as a function of $v - t^*$ in the transition from a scenario with no mobilization to one with positive mobilization.

4.14 Alternative Cost Function II

Consider the following cost function:

$$k(\widehat{\theta}) = \begin{cases} k & \text{if } v - t^* < v - t^*_M \\ h(\widehat{\theta}) & \text{if } v - t^* \geq v - t^*_M \end{cases}$$

Proposition 4.9 *The function $h(\widehat{\theta})$ is characterized by the properties:*

- *$h(\widehat{\theta})$ is strictly positive*
- *$h(\widehat{\theta})$ is increasing in the private signal*
- *$h(\widehat{\theta})$ is tangent to the function $IB()$ at the point where it equals k. There would be two solutions, but given the form of the function $IB()$ and imposing that the cost function is increasing in the private signal, the lower tangency point is identified*

Result 4.20 *This formulation guarantees:*

- *A strictly positive and increasing cost function in the private signal*

- *The presence of a zero infinite equilibrium where the level of unexpected component of public signal is not sufficiently high and the presence of an unique finite equilibrium where the level of unexpected component of public signal is sufficiently high*

Proof.
By imposing this formulation where the level of $v - t^*$ is too small, the function $IB()$ will not exceed the value k and therefore the cost function, for this reason the only equilibrium will be the infinite zero participation. Where $v - t^*$ takes a value equal to or greater than $v - t_M^*$ the cost curve being tangent to $IB()$ and increasing in the private signal will identify a unique finite equilibrium cutoff. As $v - t^*$ the curve $IB()$ increases, the (lower) point for which it is equal to k decreases, therefore being it the point of tangency, the equilibrium cutoff is decreasing in $v - t^*$. ■

Compared to the previous case, a strong assumption is made, namely the property of tangency between the cost function and the $IB()$ curve, however the continuity of the cost function, which was previously discontinuous at one point, is recovered.

4.15 Properties of Equilibrium and Outcomes

The behaviour of the equilibrium cutoff and the probabilities of game's outcomes with the presented modifications of the cost function is summarized by the following results:

Result 4.21 *The finite equilibrium cutoff in the policy game with alternative specification of cost function (I or II) is:*

- *decreasing in the unexpected component of the public signal $v - t^*$*
- *decreasing in the responsiveness of the political regime*
- *increasing in the repression of the political regime*
- *increasing in opacity unless the political regime is responsive and tolerant*
- *increasing in diversity unless the political regime is responsive*
- *decreasing in radicalization unless the political regime is responsive but opaque and the society diverse.*

Result 4.22 *The probability of no protest in the policy game with alternative specification of cost function (I or II) is::*

- *decreasing in the responsiveness of the political regime*
- *increasing in the repression of the political regime*
- *uncertain in country radicalization, diversity and public information opacity*
- *has no clear trend in opacity, even if responsiveness seems to induce a decreasing trend*

- *is decreasing in diversity unless the country is radicalized and the political regime is responsive but intolerant and opaque or unresponsive but tolerant and opaque*
- *is decreasing in radicalization unless the political regime is responsive and tolerant and the society heterogenous*

Result 4.23 *The probability of successful protest in the policy game with alternative specification of cost function (I or II) is::*

- *Decreasing in protesting cost k*
- *Decreasing in the equilibrium cutoff*
- *Tends to be ncreasing in responsiveness*
- *Increasing in the level of unexpected component of public signal*
- *decreasing in opacity unless the political regime is responsive and the society is radicalized or heterogenous where is increasing for small level of opacity and then decreasing;*
- *increasing in diversity unless the political regime is unresponsive but tolerant and the society is radicalized;*
- *increasing in radicalization unless the political regime is responsive and tolerant and the society diverse.*

Result 4.24 *The probability of failed protest in the policy game with alternative specification of cost function (I or II) is::*

- *Increasing in the equilibrium cutoff*
- *First increasing and then decreasing in the level of unexpected component of public signal*
- *increasing in responsiveness when the political regime is intolerant or tolerant and the society homogeneous, otherwise is not monotonic, first decreasing and increasing*
- *decreasing in repression unless the political regime is responsive and opaque when the relation is not monotonic, first increasing and then decreasing;*
- *decreasing in opacity unless the political regime is tolerant and the society radicalized when the relation is increasing;*
- *decreasing in diversity unless the political regime is unresponsive but tolerant and the society homogenous when the relation is increasing or when the political regime is responsive and tolerant and the society moderate when the relations is not monotonic, but first increasing and then decreasing;*

- *increasing in radicalization unless the political regime is responsive and tolerant, and the society is homogenous: in this case the relation is not monotonic, but first increasing and then decreasing.*

The behaviour of the finite equilibrium (in both scenarios) is the same as the lower finite equilibrium of the original game, so it will respect the comparative statics presented in Chapter 2 and in particular will be decreasing in the level of the unexpected component of public signal with lower limit $\frac{k}{\gamma}$.
Since the outcomes of the game are linked to the equilibrium cutoff, the comparative states of their probabilities will also be the same as in Chapter 2.

4.16 Conclusions II

Given the criticality of the original game, the specification of an alternative payoff matrix makes it possible to modify the analytical form of the incremental benefit function with the aim of identifying from the intersection with the cost curve an unique interpretable finite equilibrium, without excluding the possibility of an infinite equilibrium.
The presented chapter handles the problem from the opposite perspective, maintaining the original payoff matrix and proposing alternative specifications of the cost function. Initially, the criticalities of the original cost function are identified, after which alternatives are presented that respect the function's interpretability constraints so that the problems of the original specification are solved.
The interesting aspect is that the proposed alternatives identify the possibility of observing an interpretable finite equilibrium or an infinite equilibrium of zero participation. However, the limitation of this approach is that the solution may require a discontinuous function at a point, or very stringent assumptions concerning its form.
In conclusion, this exploration, net of its analytical limitations, shows two advantages, since on the one hand it allows a reasonable equilibrium to be obtained without modifying the original payoff matrix, while on the other hand it proposes a more interpretable specification of the cost function.

4.17 Appendix

Proof of result 4.1

$\widehat{\theta} = \frac{k}{\gamma}$ is solely a function of k, γ and is therefore independent of the other parameters. Since both parameters are positive as k increases and decreases as γ increases.

Proof of result 4.2

- If $\frac{k}{\gamma}$ increases, the argument of the function $\Phi()$ increases by decreasing the overall probability
- If $1 - T$ increases, the argument of the function $\Phi()$ decreases by increasing the overall probability
- If the regime is responsive $\Phi^{-1}(1-T) > 0$ therefore as σ_ε increases the argument of the function $\Phi()$ decreases by increasing the overall probability, otherwise decreases
- If the numerator of the function $\Phi()$ is positive as radicalization increases, it decreases towards 0 ($\Phi(0) = \frac{1}{2}$) increasing the overall probability, otherwise the argument increases towards 0 decreasing the overall probability

Proof of result 4.3

The outcome of failed protest is complementary to that of successful protest so its comparative statics will be opposite

Proof of result 4.4

$\theta = -log\left((v - t^*) - \frac{k}{\gamma}\right)$ is solely a function of $k, \gamma, v - t^*$ and is therefore independent of the other parameters. As repression increases, the argument of the logarithm decreases and due to the minus sign the cutoff increases overall, conversely it decreases as $v - t^*$ increases.

Proof of result 4.5

- If $\frac{k}{\gamma}$ increases, the argument of the function $\Phi()$ increases by increasing the overall probability
- If θ increases, the argument of the function $\Phi()$ decreases by decreasing the overall probability
- If the numerator of the function $\Phi()$ is positive as opacity increases, it decreases towards 0 ($\Phi(0) = \frac{1}{2}$) increasing the overall probability, otherwise the argument increases towards 0 decreasing the overall probability

Proof of result 4.6, 4.7

For both results, the demonstration can be carried out analytically as we know the specification of the second factor, while that of the first can be derived by means of the a posterior distribution of $\theta | v - t^*$ and subsequently employing a truncated normal distribution. However, the probabilistic problem is the same in terms of structure as in [6] in that mobilization occurs where $v - t^*$ exceeds a critical threshold and the successful protest is observed when $\theta \geq \widehat{\theta} - \sigma_\varepsilon \Phi^{-1}(1 - T)$ consequently the results of the comparative statics are the same as those derived analytically and by means or simulations in Chapter 2 (results 2.23,2.24,2.25). This also ties in with the fact that the behaviour of the finite equilibrium cutoff is the same as the lower finite equilibrium cutoff selected in [6].

Proof of result 4.8

If $\gamma = \frac{1}{2}$ the cutoff is equal to $2k$ so it depends solely on the costs of the protest. If γ is below this threshold there is an unique equilibrium that has the same behaviour as the upper equilibrium in the game contained in [6] (results 2.12,2.13). If γ is above this threshold there is a lower equilibrium that has the same behaviour as the lower one in the game contained in [6] (opposite of results 2.12,2.13).

Proof of result 4.9

Variation of opacity or $v - t^*$ only have an influence on the equilibrium cutoff so the probability will move contrary to it (result 4.8), in the case of $\gamma = \frac{1}{2}$ therefore they will have no influence as the cutoff is fixed. Variations of the other variables influence both the cutoff (result 4.8) and the other quantities within the function $\Phi()$, where the influence on all components favours the decrease or growth of the function's argument it is possible to identify the overall probability trend, otherwise the effect is doubtful.

Proof of result 4.10

$\widehat{\theta} = \Phi^{-1}(k - (v - t^*) + 1)$ is solely a function of $k, \gamma, v - t^*$ and is therefore independent of the other parameters. As k increases, the function's argument grows, while conversely as $v - t^*$ increases, it decreases.

Proof of result 4.11

- If k increases, the argument of the function $\Phi()$ increases by increasing the overall probability
- If θ increases, the argument of the function $\Phi()$ decreases by decreasing the overall probability

- If the numerator of the function $\Phi()$ is positive as opacity increases, it decreases towards 0 ($\Phi(0) = \frac{1}{2}$) decreasing the overall probability, otherwise the argument increases towards 0 increasing the overall probability

Proof of result 4.12

- If $k+1$ increases, the argument of the function $\Phi()$ increases by decreasing the overall probability

- If θ increases, the argument of the function $\Phi()$ decreases by increasing the overall probability

- If the numerator of the function $\Phi()$ is positive as opacity increases, it decreases towards 0 ($\Phi(0) = \frac{1}{2}$) increasing the overall probability, otherwise the argument increases towards 0 decreasing the overall probability

Proof of result 4.13, 4.14

For both results, the demonstration can be carried out analytically as we know the specification of the second factor, while that of the first can be derived by means of the a posterior distribution of $\theta | v - t^*$ and subsequently employing a truncated normal distribution. However, the probabilistic problem is the same in terms of structure as in [6] in that mobilization occurs where $v - t^*$ exceeds a critical threshold and the successful protest is observed when $\theta \geq \widehat{\theta} - \sigma_\varepsilon \Phi^{-1}(1 - T)$ consequently the results of the comparative statics are the same as those derived analytically and by means or simulations in Chapter 2 (results 2.23,2.24,2.25). This also ties in with the fact that the behaviour of the finite equilibrium cutoff is the same as the lower finite equilibrium cutoff selected in [6]

Proof of result 4.15

The finite equilibrium cutoff has the same behaviour of the lower one in the game contained in [6] (results 2.12, 2.13)

Proof of result 4.16

- If k increases the level $v - t_M^*$ increases (because the equilibrium cutoff is increasing in repression, result 4.15) therefore the probability increases

- If T increases the level $v - t_M^*$ increases (because the equilibrium cutoff is decreasing in responsiveness, result 4.15) therefore the probability increases

- If diversity increases and the regime in unresponsive the overall probability increases (because the equilibrium cutoff in this scenario is increasing and therefore $v - t_M^*$, result 4.15)

- If radicalization increases and the regime is unresponsive, but opaque and the society diverse the overall probability decreases (because the equilibrium cutoff in this scenario is decreasing and therefore $v - t_M^*$, result 4.15)
- If θ increases the argument of the function decreases decreasing the overall probability
- If the numerator of the function $\Phi()$ is positive as opacity increases, the argument of the function decreases towards 0 ($\Phi(0) = \frac{1}{2}$) decreasing the overall probability, otherwise the argument increases towards 0 increasing the overall probability

Proof of result 4.17, 4.18

The same reasoning applies as for the results 4.13, 4.14.

4.17.1 Simulations

The simulations mentioned in the chapter are written in R and the relevant code is available is available on Github following the path: *goergefil/PhD-Thesis-Simulations*

5

SECTION

Study on the Cumulative Distribution Function

5.1 Introduction

The cumulative distribution function is an interesting function to investigate as it is a key factor in numerous results related to probability theory, statistics and mathematics. In detail, given the centrality of the standard normal distribution, the study of its cumulative distribution function is particularly relevant.
Given the close connection between these fields and game theory, and more generally various branches of economics, a thorough study can lead to the identification of considerable advantages for these studies as well.
The original purpose of this chapter, in its current preliminary form, is to identify appropriate properties of this function that can generate advantages in the study of the original game [6] and models derived from it with the relevant modifications presented. Among the results, we consider both theoretical properties useful for the study mainly of comparative statics and those derived from simulation studies that are more suited to the specification of approximations of the starting function.
Although the objective is specific, as the results are general in nature, the study does not exclude the possibility that they can also be applied to different contexts in order to facilitate greater interpretability.

5.2 The function $\Phi(x)$

Consider a random variable X distributed as a Standard Normal:

$$X \sim N(0,1)$$

The cumulative distribution function of X denominated $\Phi(x)$, given $x \in \mathbb{R}$ returns the associated cumulative probability:

$$\Phi(x) = P(X \leq x)$$

The function $\Phi(x)$ is continuous and it is defined as:

$$\Phi(x) : \mathbb{R} \to [0, 1]$$

The function's analytical specification is:

$$\Phi(x) = \frac{1}{2}\left(1 + erf\left(\frac{x}{\sqrt{2}}\right)\right)$$

The function is:

1. strictly monotonically increasing on its domain

$$\frac{\partial \Phi(x)}{\partial x} > 0 \; \forall x \in \mathbb{R}$$

   with limits represented by:

$$\lim_{x \to -\infty} \Phi(x) = 0^+ \qquad \lim_{x \to +\infty} \Phi(x) = 1^-$$

2. convex for $x \in (-\infty, 0)$ and concave for $x \in (0, +\infty)$

$$\frac{\partial^2 \Phi(x)}{\partial x^2} > 0 \iff x < 0$$

3. strictly positive $\forall x \in \mathbb{R}$

4. $\Phi(x) = 1 - \Phi(-x)$

5.3 Additional function's properties

Given the known properties of the function $\Phi(x)$, we present two additional properties

5.3.1 Property I

Given $\alpha \in \mathbb{R}^+$ and the difference $[\Phi(x) - \Phi(x - \alpha)]$ it is possible to state that:

$$\underset{x}{\operatorname{argmax}}[\Phi(x) - \Phi(x - \alpha)] = \frac{\alpha}{2}$$

$$\Phi(x) - \Phi(x - \alpha) \leq 2\Phi\left(\frac{\alpha}{2}\right) - 1$$

Proof.

$$\frac{\partial[\Phi(x)-\Phi(x-\alpha)]}{\partial x}=\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}-\frac{1}{\sqrt{2\pi}}e^{-\frac{(x-\alpha)^2}{2}}=\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}\left(1-e^{-\frac{\alpha^2}{2}+\alpha x}\right)$$

$$\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}\left(1-e^{-\frac{\alpha^2}{2}+\alpha x}\right)\geq 0 \Longleftrightarrow -\frac{\alpha^2}{2}+\alpha x\geq 0 \Longleftrightarrow x\geq\frac{\alpha}{2}$$

The usefulness of this property is evident for example in deriving the comparative statics of the mass of swing citizens (result 2.15).

■

5.3.2 Property II

Given $\alpha \in \mathbb{R}^+$ and the difference $[\Phi(\alpha x)-\Phi(x)]$ it is possible to state that:

If $\alpha > 1$:

$$\underset{x}{\operatorname{argmax}}[\Phi(\alpha x)-\Phi(x)]=\sqrt{\frac{-2ln(\alpha)}{(1-\alpha^2)}}$$

$$\underset{x}{\operatorname{argmin}}[\Phi(\alpha x)-\Phi(x)]=-\sqrt{\frac{-2ln(\alpha)}{(1-\alpha^2)}}$$

If $\alpha < 1$:

$$\underset{x}{\operatorname{argmin}}[\Phi(\alpha x)-\Phi(x)]=\sqrt{\frac{-2ln(\alpha)}{(1-\alpha^2)}}$$

$$\underset{x}{\operatorname{argmax}}[\Phi(\alpha x)-\Phi(x)]=-\sqrt{\frac{-2ln(\alpha)}{(1-\alpha^2)}}$$

Proof.

$$\frac{\partial[\Phi(\alpha x)-\Phi(x)]}{\partial x}=\frac{\alpha}{\sqrt{2\pi}}e^{-\frac{(\alpha x)^2}{2}}-\frac{1}{\sqrt{2\pi}}e^{-\frac{(x)^2}{2}}$$

$$\frac{\alpha}{\sqrt{2\pi}}e^{-\frac{(\alpha x)^2}{2}}-\frac{1}{\sqrt{2\pi}}e^{-\frac{(x)^2}{2}}=0 \Longleftrightarrow \frac{1}{2}(1-\alpha^2)x^2=-ln(\alpha)$$

$$\Longleftrightarrow x=\pm\sqrt{\frac{-2ln(\alpha)}{(1-\alpha^2)}}$$

If $\alpha > 1$ the derivative is positive for internal values, otherwise if $\alpha < 1$ for external values.

■

5.4 Approximation of the function

In the previous chapters, or in papers dealing with similar models and problems ([6], [7], [3]) it is often necessary to handle linear equations where the cumulative distribution function is involved. For instance in the equilibrium condition of the game presented in [6]:

$$\left[1-\Phi\left(\frac{\widehat{\theta}\left(v-t^{*};p,s\right)(1-(1-\psi)\lambda)-\sigma_{\varepsilon}\Phi^{-1}(1-T)-\psi\left(v-t^{*}\right)}{\sqrt{\psi\sigma_{\eta}^{2}}}\right)\right]\gamma\widehat{\theta}\left(v-t^{*};p,s\right)=k$$

In these scenarios, deriving an explicit analytical solution is often not possible other than graphically or by means of simulations, so it is useful to have a less complex approximation capable of overcoming this criticality that is both accurate and therefore provides reliable and precise solutions.
Most of the proposals in the literature () have accuracy of approximation as their main objective, which is why they are generally in such an elaborate form that they are ineffective in solving the problem we have highlighted. In this section, therefore, the aim is to identify an approximation that is not necessarily the best performing, but which manifests consistent accuracy while maintaining a relatively simple analytical form.

Given the symmetry of the function, we evaluate an approximation of the function $\Phi(x)$ in the set $x \in (0, +\infty)$. Consider the generic function:

$$f(\alpha,\beta,\gamma,x)=1-\frac{1}{2}\left(\frac{\alpha}{\gamma x^{\beta}+\alpha}\right)$$

The function has the following properties:

- $f(\alpha,\beta,\gamma,0)=\frac{1}{2}$
- $\lim_{x\to+\infty} f(\alpha,\beta,\gamma,x)=1$
- $f(\alpha,\beta,\gamma,x)$ is monotonically increasing

The specification of the final approximation requires the estimation of the parameters α,β,γ, where:

- $\alpha,\gamma > 0$
- $\beta \in (1,2)$ in order to avoid overly complex approximations

Parameter estimation is performed in two steps:

- By specifying the two possible values of β and a set of possible values of the parameter α and γ

- For each possible candidate function, the MAPE and MSE values are calculated, a set of candidates with the smallest values in the two indices is then identified, and from these, the function with the smallest deviations from the objective function in terms of maximum and minimum is selected

The simulation identifies the quadratic function:

$$f(x) = 1 - \frac{1}{2}\left(\frac{0.60}{1.60x^2 + 0.60}\right)$$

Where:

- $MSE\,(\Phi(x), f(x)) = 0.0003$
- $MAPE\,(\Phi(x), f(x)) = 0.017$
- $Max\,(|\Phi(x) - f(x)|) = 0.03$

From the figure it is possible to observe the goodness of the approximation represented by the red curve:

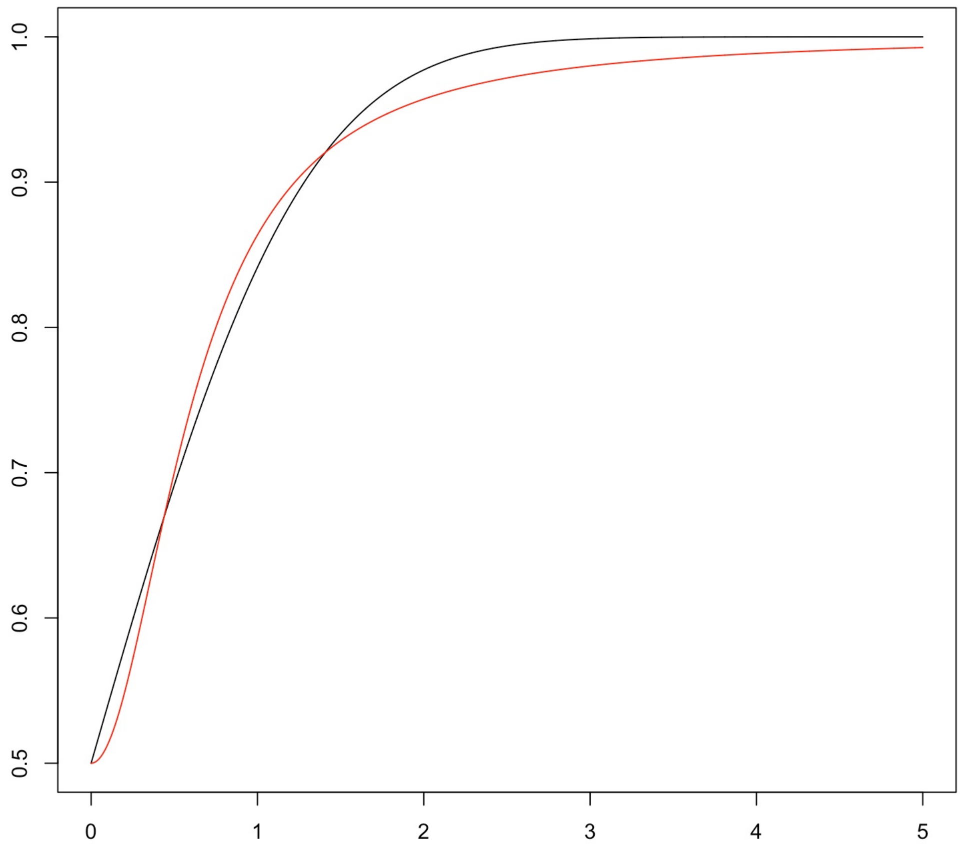

Figure 5.1: Approximation of $\Phi(x)$ with quadratic function $f(x)$

It is possible to apply the approximation obtained in the context of the equilibrium condition to verify its usefulness in order to identify the solutions:

$$\left[1-\Phi\left(\frac{\widehat{\theta}\left(v-t^{*};p,s\right)(1-(1-\psi)\lambda)-\sigma_{\varepsilon}\Phi^{-1}(1-T)-\psi\left(v-t^{*}\right)}{\sqrt{\psi\sigma_{\eta}^{2}}}\right)\right]\gamma\widehat{\theta}\left(v-t^{*};p,s\right)=k$$

$$\left[1-\Phi\left(\frac{\widehat{\theta}\left(v-t^{*};p,s\right)(1-(1-\psi)\lambda)+\rho}{\delta}\right)\right]\gamma\widehat{\theta}\left(v-t^{*};p,s\right)=k$$

$$\frac{1}{2}\left(\frac{0.6}{1.6\left(\frac{\widehat{\theta}\left(v-t^{*};p,s\right)(1-(1-\psi)\lambda)+\rho}{\delta}\right)^{2}+0.6}\right)\gamma\widehat{\theta}\left(v-t^{*};p,s\right)\approx k$$

$$0.3\gamma\widehat{\theta}\left(v-t^{*};p,s\right)-k\left(1.6\left(\frac{\widehat{\theta}\left(v-t^{*};p,s\right)(1-(1-\psi)\lambda)+\rho}{\delta}\right)^{2}+0.6\right)\approx 0$$

$$0.3\gamma\widehat{\theta}\left(v-t^{*};p,s\right)-k\left(1.6\left(\frac{\widehat{\theta}\left(v-t^{*};p,s\right)(1-(1-\psi)\lambda)}{\delta}\right)^{2}\right)+k\left(3.2\left(\frac{\widehat{\theta}\left(v-t^{*};p,s\right)(1-(1-\psi)\lambda)\rho}{\delta^{2}}\right)\right)$$
$$-k\left(1.6\left(\frac{\rho}{\delta}\right)^{2}\right)-0.6k\approx 0$$

$$\widehat{\theta}\left(v-t^{*};p,s\right)^{2}W_{1}+\widehat{\theta}\left(v-t^{*};p,s\right)W_{2}+W_{3}\approx 0$$

The two roots of the equation are represented by[21]:

$$\frac{[\left(\frac{3.2k(1-(1-\psi)\lambda)\rho}{\delta^{2}}\right)+(0.3\gamma)]\pm\sqrt{[\left(\frac{3.2k(1-(1-\psi)\lambda)\rho}{\delta^{2}}\right)+(0.3\gamma)]^{2}-4\left(1.6k\left(\frac{(1-(1-\psi)\lambda)}{\delta}\right)^{2}\right)\left(k\left(1.6\left(\frac{\rho}{\delta}\right)^{2}\right)+0.6k\right)}}{2\left(1.6k\left(\frac{(1-(1-\psi)\lambda)}{\delta}\right)^{2}\right)}$$

The lower solution is the one selected within the model proposed in [6]. Simulations comparing the $IB()$ target function and that obtained by applying the approximation show that the reliability of the solutions increases as the repression increases, and that the lower solution (i.e. the one of interest) is generally accurate as opposed to the upper one, since the distance between the target function and the approximation expands as $\widehat{\theta}$ increases due to the multiplicative effect of this factor. The following figure summarises this reasoning (the red curve represents the approximation):

[21] $\rho=-\sigma_{\varepsilon}\Phi^{-1}(1-T)-\psi(v-t^{*})$

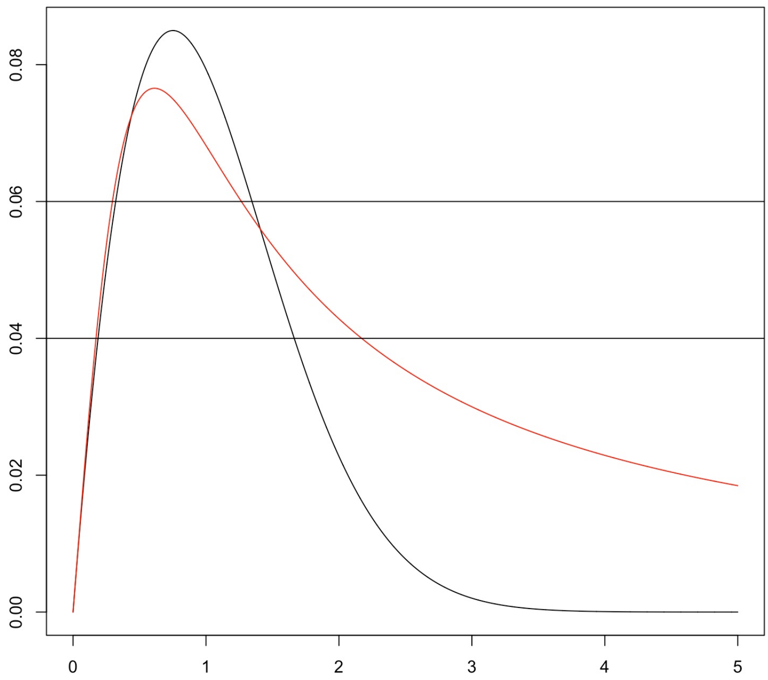

Figure 5.2: Approximation of equilibrium condition with quadratic function $f(x)$

Increasing the polynomial order of the approximation from quadratic to cubic induces an improvement in both the approximation of the $\Phi(x)$ function and the $IB()$ function.

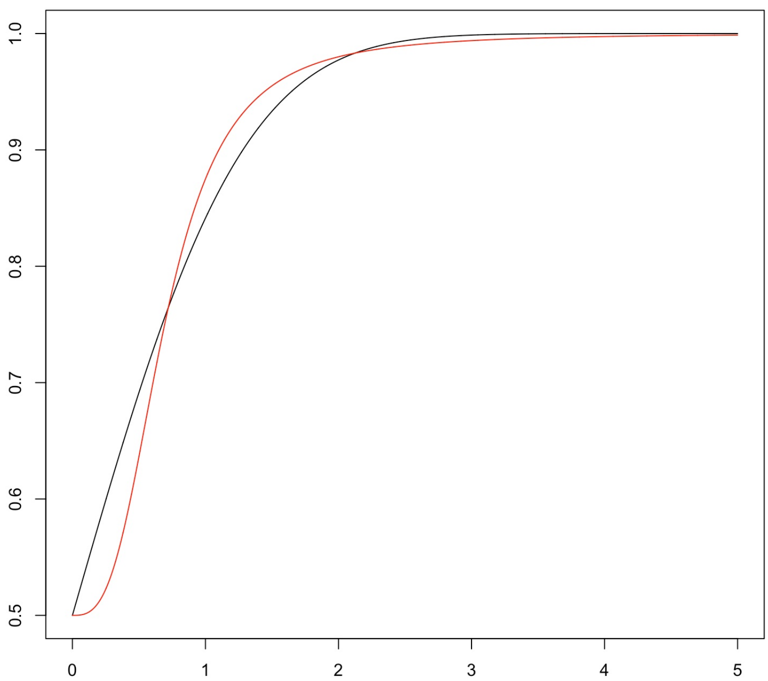

Figure 5.3: Approximation of $\Phi(x)$ with cubic function

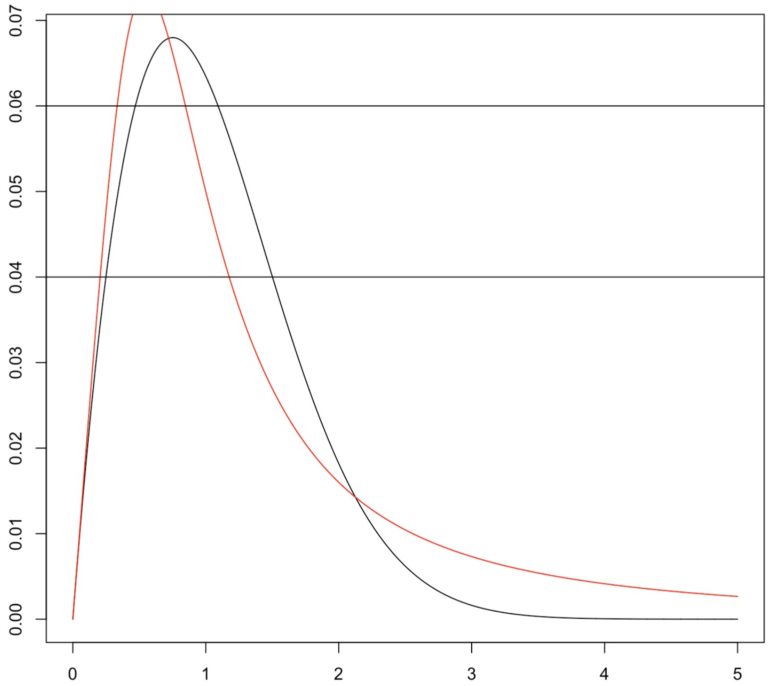

Figure 5.4: Approximation of equilibrium condition with cubic function

However, the growth in accuracy is not as significant, and the approximation of the solution of interest (i.e. the lower one) is already accurate with the less complex function, therefore in this context the complexity/accuracy trade-off rewards the former approximation.

The analysis contained in [29] suggests that where the value of x is small an accurate approximation of the error function is represented by the function:

$$erf(x) \approx \frac{2x}{\sqrt{\pi}}$$

Therefore the approximation of the function $\Phi(x)$ becomes:

$$\Phi(x) \approx \frac{1}{2}\left(1 + \sqrt{\frac{2}{\pi}}x\right)$$

Which theoretically should be optimal where the value of x is small as the following graph obtained by simulations suggests:

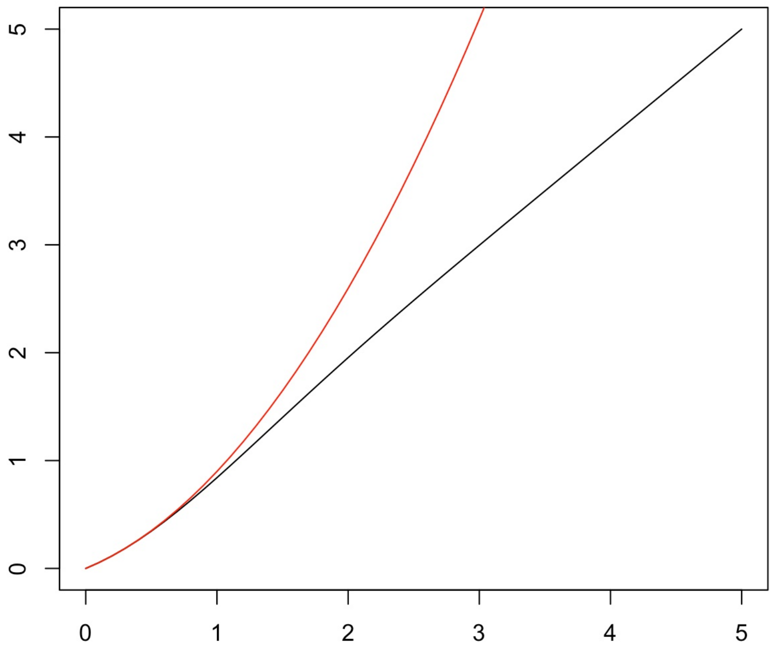

Figure 5.5: Approximation of $\Phi(x)$ with $erf(x) \approx \frac{2x}{\sqrt{\pi}}$

Given this approximation, it is interesting to note that, despite its low efficiency when x is large, the approximation of the function $IB()$ is such that it always generates an accurate estimate of the lower solution of the original problem contained in [6] for different levels of repression, while the higher one is always underestimated as the following figure suggests:

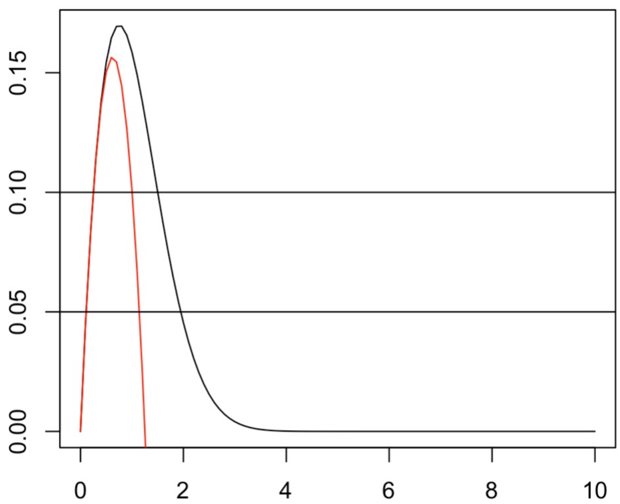

Figure 5.6: Approximation of equilibrium condition with $erf(x) \approx \frac{2x}{\sqrt{\pi}}$

Consequently, in the context of the original game and in the case of some of its modifications that do not lead to the mutation of the form of the $IB()$ function, this approximation, although not particularly efficient with respect to the cumulative distribution function, proves to be effective in approximating the lower solution while maintaining

a reduced complexity.
This suggests that, depending on the characteristics of the problem under consideration, rather than identifying an approximation along the entire domain of the cumulative distribution function, it is preferable to estimate a reduced-complexity approximation in a subset of the domain of interest (in our case x small).
Given this approximation, the lower solution of the equilibrium condition would become:

$$\left[1-\Phi\left(\frac{\widehat{\theta}\left(v-t^{*};p,s\right)(1-(1-\psi)\lambda)-\sigma_{\varepsilon}\Phi^{-1}(1-T)-\psi(v-t^{*})}{\sqrt{\psi\sigma_{\eta}^{2}}}\right)\right]\gamma\widehat{\theta}\left(v-t^{*};p,s\right)=k$$

$$\left[\frac{1}{2}-\frac{1}{\sqrt{2\pi}}\left(\frac{\widehat{\theta}\left(v-t^{*};p,s\right)(1-(1-\psi)\lambda)-\sigma_{\varepsilon}\Phi^{-1}(1-T)-\psi(v-t^{*})}{\sqrt{\psi\sigma_{\eta}^{2}}}\right)\right]\gamma\widehat{\theta}\left(v-t^{*};p,s\right)=k$$

$$\sqrt{\pi}\gamma\widehat{\theta}-\sqrt{2}\gamma\widehat{\theta}\left(\frac{\widehat{\theta}(1-(1-\psi)\lambda)-\sigma_{\varepsilon}\Phi^{-1}(1-T)-\psi(v-t^{*})}{\sqrt{\psi\sigma_{\eta}^{2}}}\right)-2\sqrt{\pi}k=0$$

$$+\widehat{\theta}^{2}\left(\frac{\sqrt{2}\gamma(1-(1-\psi)\lambda)}{\sqrt{\psi\sigma_{\eta}^{2}}}\right)-\widehat{\theta}\left(\sqrt{\pi}\gamma-\sqrt{2}\gamma\left(\frac{-\sigma_{\varepsilon}\Phi^{-1}(1-T)-\psi(v-t^{*})}{\sqrt{\psi\sigma_{\eta}^{2}}}\right)\right)+2\sqrt{\pi}k=0$$

$$\frac{\left(\sqrt{\pi}\gamma-\sqrt{2}\gamma\left(\frac{-\sigma_{\varepsilon}\Phi^{-1}(1-T)-\psi(v-t^{*})}{\sqrt{\psi\sigma_{\eta}^{2}}}\right)\right)-\sqrt{\left(\sqrt{\pi}\gamma-\sqrt{2}\gamma\left(\frac{-\sigma_{\varepsilon}\Phi^{-1}(1-T)-\psi(v-t^{*})}{\sqrt{\psi\sigma_{\eta}^{2}}}\right)\right)^{2}-4(2\sqrt{\pi}k)\left(\frac{\sqrt{2}\gamma(1-(1-\psi)\lambda)}{\sqrt{\psi\sigma_{\eta}^{2}}}\right)}}{2\left(\frac{\sqrt{2}\gamma(1-(1-\psi)\lambda)}{\sqrt{\psi\sigma_{\eta}^{2}}}\right)}$$

Alternatively it is possible to explore additional simple approximations of the cumulative distribution function for x small, able to generate accurate lower solutions related to the problem contained in the equilibrium condition of the orignal game [6].

5.5 Conclusion

The study of the properties and approximations of the cumulative distribution function of the Standard Normal distribution may prove very useful given its frequent application within problems related to game theory and other branches of economics.
Within this chapter it was possible to identify two properties related to the linear combinations of this function that proved useful already within the previous chapters for investigating comparative statics in game theory models and that constitute potentially useful tools also for work with different objectives and contexts.
In addition, accurate but not overly complex approximations of the function capable of

solving equations in which it is involved and which would otherwise not be analytically solvable have been identified using a protocol that includes some simulations. This in the context of the model considered is useful for deriving the analytical form of the finite equilibrium as a function of the variables in the game in order to study its properties and comparative statics, but may prove equally useful for any problem considering equations of the same character.

5.6 Simulations

The procedure for obtaining the approximations and graphs was written in R and the relevant code is available is available on Github following the path: *goergefil/PhD-Thesis-Simulations*

SECTION

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