Microstates of Accelerating and Supersymmetric AdS_4 Black Holes from the Spindle Index

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(Received 16 April 2024; accepted 17 June 2024; published 18 July 2024)

We provide a first principles derivation of the microscopic entropy of a very general class of supersymmetric, rotating, and accelerating black holes in AdS_4. This is achieved by analyzing the large-N limit of the spindle index and completes the construction of the first example of a holographic duality involving supersymmetric field theories defined on orbifolds with conical singularities.

DOI: 10.1103/PhysRevLett.133.031603

Introduction.—The explanation of the microscopic origin of the entropy of supersymmetric black holes in anti–de Sitter (AdS) is one of the most spectacular successes of the holographic duality. This was first accomplished in [1] for a class of AdS_4 black holes through the study of the large-N limit of the topologically twisted index [2]. The landscape of supersymmetric black holes was significantly broadened in [3], which constructed a supersymmetric, rotating, and accelerating black hole with *spindle* horizon, displaying a number of remarkable features. Most strikingly, in this solution supersymmetry is preserved via a novel mechanism, referred to as antitwist. It is was later noted that supersymmetry on the spindle may be preserved by means of a more standard topological twist [4,5]. Utilizing the insight of [6], it was shown in [7] that the on-shell action of a supersymmetric and complex deformation of the black hole of [3] takes the form of an *entropy function*, whose extremization yields the Bekenstein-Hawking entropy. A generalization of such entropy function was conjectured in [8], where it was proposed that it can be expressed in terms of gravitational blocks [9], as in all previous examples of black holes. The block decomposition of the gravitational entropy function was proved in [10] using the formalism of [11] and then in [12] employing equivariant localization in supergravity.

Motivated by these developments, Refs. [13,14] computed the localized partition function of $\mathcal{N}=2$ Chern-Simons-matter theories defined on $\Sigma \times S^1$, where $\Sigma = \mathbb{WCP}^1_{[n_+,n_-]}$ is the spindle, with either twist or anti-twist for the R-symmetry connection A:

$$\int_{\Sigma} \frac{\mathrm{d}A}{2\pi} = \frac{1}{2}\left(\frac{1}{n_-} + \frac{\sigma}{n_+}\right) \equiv \frac{\chi_\sigma}{2}, \qquad (1)$$

with $\sigma = \pm 1$. The result can be expressed by a single formula, dubbed *spindle index* [13], which can be defined [7] as a flavored Witten index,

$$Z_{\Sigma \times S^1} = \mathrm{Tr}_{\mathcal{H}[\Sigma]}\left[\mathrm{e}^{-\mathrm{i}\sum_{\alpha=1}^{\mathfrak{d}} \varphi_\alpha Q_\alpha + \mathrm{i}\epsilon J}\right], \qquad (2)$$

where Q_α are the generators of global symmetries of rank $\mathfrak{d}$, J generates angular momentum on Σ, $\mathcal{H}[\Sigma]$ is the Hilbert space of Bogomol'nyi-Prasad-Sommerfield states on the spindle, and the complex chemical potentials are related by the constraint

$$\sum_{\alpha=1}^{\mathfrak{d}} \varphi_\alpha + \frac{\chi_{-\sigma}}{2}\epsilon = 2\pi n, \qquad n \in \mathbb{Z}. \qquad (3)$$

In this Letter we will demonstrate that the large-N limit of the spindle index reproduces the entropy functions associated to the supersymmetric and accelerating AdS_4 black holes. Explicitly, the entropy of a black hole with electric charges Q_α and angular momentum J is obtained by extremizing with respect to the variables φ_α and ϵ the entropy function

$$\mathcal{S} \equiv \log Z_{\Sigma \times S^1} + \mathrm{i}\sum_{\alpha=1}^{\mathfrak{d}} \varphi_\alpha Q_\alpha - \mathrm{i}\epsilon J, \qquad (4)$$

under the constraint (3), setting $n=1$ and requiring that J, Q_α, and $\mathcal{S}$ are real. In the case $n_+ = n_- = 1$ our result encompasses the large-N limit of both the topologically twisted index and the generalized superconformal index. More details and generalizations will be discussed in [15].

Spindle index matrix model.—We consider $\mathcal{N}=2$ Chern-Simons-matter quiver gauge theories with gauge group $\mathcal{G}=\prod_{a=1}^{|\mathcal{G}|}\mathrm{U}(N)_a$ and chiral multiplets transforming in either bifundamental or adjoint representations of the gauge group factors. The index has been derived using supersymmetric localization in [13,14] and it is written as the matrix model,

$$Z_{\Sigma\times S^1}(\varphi,\mathfrak{n},\epsilon)=\sum_{\mathfrak{m}\in\Gamma_{\mathfrak{h}}}\oint_{\mathcal{C}}\frac{\mathrm{d}u}{|W_{\mathcal{G}}|}\hat{Z}(u,\mathfrak{m}|\varphi,\mathfrak{n},\epsilon),\quad (5)$$

where $\mathfrak{h}$, $\Gamma_{\mathfrak{h}}$, and $W_{\mathcal{G}}$ denote the Cartan algebra, the coroot lattice, and the Weyl group of the gauge group $\mathcal{G}$, respectively, while $\mathcal{C}$ is a suitable integration contour for u. Here we have collectively expressed by $u\in\mathfrak{h}$ and $\mathfrak{m}\in\Gamma_{\mathfrak{h}}$ the gauge holonomies on S^1 and fluxes through Σ, respectively. Similarly, φ and $\mathfrak{n}$ are flavor or topological charges and fluxes, with (5) implicitly depending on the spindle data n_+,n_- and the twist parameter σ.

We focus on theories whose gauge group and matter content can be represented by a quiver diagram with $|\mathcal{G}|$ nodes, where an arrow from node a to node b corresponds to a bifundamental field in the representation $\mathbf{N}_a\otimes\bar{\mathbf{N}}_b$, and $a=b$ indicates the adjoint representation. For each $\mathrm{U}(N)_a$ factor there are N holonomies and fluxes $(u_i^a,\mathfrak{m}_i^a)_{i=0}^{N-1}$; for each arrow we assign flavor charges and fluxes $(\varphi_I,\mathfrak{n}_I)$, where the index I runs over *all* the $|\mathcal{R}|$ chiral multiplets of the theory. If the corresponding arrow stretches from a node a to a node b, we write $I\in(a,b)$. Moreover, for each node we assign charges or fluxes $(\varphi_m^a,\mathfrak{n}_m^a)$ for the topological symmetries. As in [13,14] we consider a choice of R symmetry that assigns even charges to the chiral multiplets: $r_I\in2\mathbb{Z}$. For a chiral multiplet the corresponding chemical potential φ_I is related to the flavor holonomy u_I^F via

$$\varphi_I=2\pi u_I^F+\left(\pi n-\frac{\epsilon}{4}\chi_{-\sigma}\right)r_I,\quad (6)$$

where, for each monomial term W in the superpotential,

$$\sum_{I\in W}u_I^F=\sum_{I\in W}\mathfrak{n}_I=0,\qquad\sum_{I\in W}r_I=2,\quad (7)$$

so that

$$\sum_{I\in W}\varphi_I+\frac{\chi_{-\sigma}}{2}\epsilon=2\pi n.\quad (8)$$

Note that the index I runs over the fields belonging to a superpotential term, while in (3) the index α labels the generators of the global symmetries of the theory. For the Aharony-Bergman-Jafferis-Maldacena (ABJM) model, that is the main focus in this Letter, these two sets coincide. More general quivers will be discussed in [15].

The integrand of (5) is the product of a classical part and the one-loop determinants of chiral and vector multiplets. In order to write it explicitly we need to introduce some further notation [13]. First, we define the symbols $\sigma_+=\sigma$ and $\sigma_-=-1$. Then, we set

$$\mathfrak{b}_{ij}^I=1-\frac{\mathfrak{m}_i^a-\mathfrak{m}_j^b}{n_+n_-}-\frac{\mathfrak{n}_I}{n_+n_-}-\frac{r_I}{2}\chi_\sigma-\mathcal{A}_{I;ij}^--\sigma\mathcal{A}_{I;ij}^+,$$
$$\mathfrak{c}_{ij}^I=\mathcal{A}_{I;ij}^--\sigma\mathcal{A}_{I;ij}^+,\quad (9)$$

for each arrow $I\in(a,b)$, with

$$\mathfrak{l}_{a;i}^\pm=n_\pm\left\{\frac{\sigma_\pm a_\pm\mathfrak{m}_i^a}{n_\pm}\right\},$$
$$\mathcal{A}_{I;ij}^\pm=\left\{\frac{\mathfrak{l}_{a;i}^\pm-\mathfrak{l}_{b;j}^\pm+\sigma_\pm a_\pm\mathfrak{n}_I-r_I/2}{n_\pm}\right\},\quad (10)$$

and $a_\pm\in\mathbb{Z}$ such that $n_+a_--n_-a_+=1$. Moreover, $\{x\}\equiv x-\lfloor x\rfloor$. Note that $\mathfrak{b}_{ij}^I\in\mathbb{Z}$, while $\mathfrak{l}_{a;i}^\pm,n_\pm\mathcal{A}_{I;ij}^\pm\in\mathbb{Z}_{n_\pm}$. Denoting by

$$y_{ij}^I=\mathrm{e}^{-\mathrm{i}\varphi_I-2\pi\mathrm{i}(u_i^a-u_j^b)}\cdot q^{(1/2)\mathfrak{c}_{ij}^I},\qquad q=\mathrm{e}^{\mathrm{i}\epsilon},\quad (11)$$

the gauge holonomies, the one-loop determinant contribution of the chiral multiplets (CMs) can be written as [13,14]

$$Z_{1\mathrm{L}}^{\mathrm{CM}}=\prod_{I=1}^{|\mathcal{R}|}\prod_{i,j=0}^{N-1}\zeta_q^\sigma(y_{ij}^I,\mathfrak{b}_{ij}^I),\quad (12)$$

in terms of the function

$$\zeta_q^\sigma(y,\mathfrak{b})\equiv(-y)^{(1-\sigma-2\mathfrak{b})/4}q^{[(1-\sigma)(\mathfrak{b}-1)]/8}\frac{(q^{(1+\mathfrak{b})/2}y^{-1};q)_\infty}{(q^{(1-\mathfrak{b})/2\sigma}y^{-\sigma};q)_\infty},\quad (13)$$

where $(z;q)_\infty$ is the q-Pochhammer symbol, $y,q\in\mathbb{C}$, $\mathfrak{b}\in\mathbb{Z}$, and $\sigma=\pm1$. This is the one-loop determinant of a single chiral multiplet in an Abelian theory, satisfying

$$\zeta_q^\sigma(y,\mathfrak{b})=\zeta_q^\sigma(y^{-\sigma},1-\sigma-\mathfrak{b})^{-\sigma}.\quad (14)$$

The one-loop determinant of all vector multiplets reads

$$Z_{1\mathrm{L}}^{\mathrm{VM}}=\prod_{a=1}^{|\mathcal{G}|}\prod_{i,j=0}^{N-1}\zeta_q^\sigma(y_{ij}^a,\mathfrak{b}_{ij}^a),\quad (15)$$

where y_{ij}^a, $\mathfrak{b}_{ij}^a$, $\mathfrak{c}_{ij}^a$, and $\mathcal{A}_{a;ij}^\pm$ are defined as in (9)–(11), with all the instances of I and b replaced by a, and with the following identifications: $r_a\equiv2$, $\mathfrak{n}_a\equiv0$, $\varphi_a\equiv2\pi n-(\epsilon/2)\chi_{-\sigma}$.

The classical part receives contributions from the Chern-Simons terms, which can be written as [15]

$$Z_{\mathrm{eff}}^{\mathrm{CS}}=\prod_{a=1}^{|\mathcal{G}|}\prod_{i=0}^{N-1}(-y_i^a)^{\mathrm{k}_a(\mathfrak{b}_i^a-1)},\quad (16)$$

where we defined

$$y_i^a = e^{-2\pi i u_i^a} \cdot q^{\mathfrak{l}^-_{a;i}/2n_- - \sigma(\mathfrak{l}^+_{a;i}/2n_+)},$$
$$\mathfrak{b}_i^a = 1 - \frac{\mathfrak{m}_i^a}{n_+ n_-} - \frac{\mathfrak{l}^-_{a;i}}{n_-} - \sigma \frac{\mathfrak{l}^+_{a;i}}{n_+}. \tag{17}$$

In this Letter we restrict to the case where $\sum_a k_a = 0$, corresponding to $\mathcal{N} = 2$ Chern-Simons-matter quiver gauge theories with an M theory dual $AdS_4 \times M_7$. The topological symmetries also contribute to the classical part, but the explicit expression will not be needed in this Letter.

Holomorphic block factorization.—The spindle index factorizes into the product of dual holomorphic blocks [16]. It is convenient to use a choice of factorization that breaks the Weyl symmetry of the gauge group, generalizing the one introduced in [17] for the superconformal index. Starting from (12), we split the product over i, j into a product over $i < j$ and one over $i > j$, ignoring the diagonal terms that are subleading at large N, then we apply (14) to the $i > j$ terms and we find

$$Z^{\rm CM}_{\rm 1L} = \prod_{I=1}^{|\mathcal{R}|} \Psi_I \cdot \mathcal{B}_I^+ \cdot \mathcal{B}_I^-, \tag{18}$$

where for $I \in (a, b)$ we defined

$$\Psi_I = \prod_{i<j} (y_{ij}^I)^{(1-\sigma-2\mathfrak{b}_{ij}^I)/4} (y_{ji}^I)^{-(1-\sigma-2\mathfrak{b}_{ji}^I)/4} \cdot q^{[(1-\sigma)(\mathfrak{b}_{ij}^I - \mathfrak{b}_{ji}^I)]/8},$$
$$\mathcal{B}_I^\pm = \prod_{\substack{\mathcal{B}^-: i<j \\ \mathcal{B}^+: i>j}} \frac{\left(\left(\frac{z^\pm_{a;i}}{z^\pm_{b;j}}\right)^{\pm\sigma_\pm} e^{-i\sigma_\pm \Delta_I^\pm} q^{1-\mathcal{A}^\pm_{I;ij}}; q\right)_\infty}{\left(\left(\frac{z^\pm_{a;j}}{z^\pm_{b;i}}\right)^{\mp\sigma_\pm} e^{i\sigma_\pm \Delta_I^\pm} q^{\mathcal{A}^\pm_{I;ji}}; q\right)_\infty}. \tag{19}$$

Note that here we have swapped the role of i, j in the blocks for convenience. The Ψ_I will turn out to be subleading after the cancellation of the long-range forces. The blocks $\mathcal{B}_I^\pm$ depend on the combinations

$$\Delta_I^\pm = \varphi_I \pm \frac{\epsilon}{2}\left(\frac{\mathfrak{n}_I}{n_+ n_-} + \frac{\chi_\sigma}{2} r_I\right),$$
$$z^\pm_{a;i} = e^{\mp 2\pi i u_i^a} q^{-\mathfrak{m}_i^a/2n_+ n_-}. \tag{20}$$

Note that the variables $\Delta_I^\pm$ satisfy the constraints

$$\sum_{I \in W} \Delta_I^\pm = 2\pi n + \frac{\sigma_\pm \epsilon}{n_\pm}. \tag{21}$$

We derive the vector-multiplet counterparts of (18) and (19) by replacing the indices I and b with a and applying standard identifications.

Strategy for the large-N limit.—We will implement the large-N limit of the spindle index by relying on its factorization into holomorphic blocks, generalizing the approach of [17–19]. For the partition functions on $S^2 \times S^1$ the sum over all the possible values of the gauge fluxes $\mathfrak{m} \in \Gamma_\mathfrak{h} \equiv \mathbb{Z}^{|\mathcal{G}|N}$ is usually approximated at large N by promoting the fluxes $\mathfrak{m}$ to continuous variables. However, for $\sum \times S^1$ this approximation is hindered by the presence of fractional parts in (10). To take care of this, we split each gauge flux as $\mathfrak{m}_i^a \equiv n_+ n_- (\mathfrak{m}')_i^a + \mathfrak{r}_i^a$, with $(\mathfrak{m}')_i^a \in \mathbb{Z}$ and $\mathfrak{r}_i^a \in \mathbb{Z}_{n_+ n_-}$. We then observe that there is a one-to-one correspondence between the possible values of $\mathfrak{l}^\pm_{a;i}$ and $\mathfrak{r}_i^a$:

$$\frac{\mathfrak{l}^\pm_{a;i}}{n_\pm} = \left\{\frac{\sigma_\pm a_\pm \mathfrak{r}_i^a}{n_\pm}\right\}, \qquad \frac{\mathfrak{r}_i^a}{n_+ n_-} = \left\{-\frac{\mathfrak{l}^-_{a;i}}{n_-} - \sigma \frac{\mathfrak{l}^+_{a;i}}{n_+}\right\}. \tag{22}$$

We can therefore split the sum over $\mathfrak{m}_i^a$ as

$$\sum_{\mathfrak{m}_i^a \in \mathbb{Z}} = \sum_{\mathfrak{l}^-_{a;i}=0}^{n_- - 1} \sum_{\mathfrak{l}^+_{a;i}=0}^{n_+ - 1} \sum_{(\mathfrak{m}')_i^a \in \mathbb{Z}}, \tag{23}$$

and in the large-N limit we may promote the $(\mathfrak{m}')_i^a$ to be continuous variables while keeping the $\mathfrak{l}^\pm_{a;i}$ discrete. Thus, we approximate the integration measure of (5) by

$$\sum_{\mathfrak{m} \in \Gamma_\mathfrak{h}} \oint_\mathcal{C} \frac{du}{|W_\mathcal{G}|} \longrightarrow \sum_{\mathfrak{l}^\pm \in (\mathbb{Z}_{n_\pm})^{|\mathcal{G}|N}} \int_{\mathcal{C}^+} dz^+ \int_{\mathcal{C}^-} dz^-, \tag{24}$$

at large N, where the variables $z^\pm$ were defined in (20), $\mathcal{C}^\pm$ are appropriate middle-dimensional contours in $\mathbb{C}^{|\mathcal{G}|N}$, and the order of the Weyl group can be ignored since $\log|W_\mathcal{G}| = \mathcal{O}(N \log N)$.

Since the $\mathcal{B}_I^\pm$ blocks depend separately on $z^\pm$, we will be able to perform the saddle point approximation in z^- and z^+ independently of one another. However, the right-hand side of (24) also features a sum over the vectors of integers $\mathfrak{l}^\pm$, which can take a total of $(n_+ n_-)^{|\mathcal{G}|N}$ possible values, exponentially growing with N. At large N only one value of the $\mathfrak{l}^\pm$ is expected to dominate at any given region of the parameter space: in particular, there will be one such value associated to the saddle point that reproduces the accelerating AdS_4 black holes. Two observations are in order to find the correct ansatz for $\mathfrak{l}^\pm$. First, we need to restrict our attention to the set of possible choices of $\mathfrak{l}^\pm$ that, up to an appropriate permutation of the index i, are periodic under shifts $i \to i + T$ for some $T \ll N$. This assumption is necessary in order to be able to take (partially) the continuum limit: splitting the index i as $i = Ti' + \tilde{\imath}$, with $\tilde{\imath} \in \{0, \ldots, T-1\}$, makes the fluxes $\mathfrak{l}^\pm_{a;i}$ depend only on the index $\tilde{\imath}$, $\mathfrak{l}^\pm_{a;i} \equiv \mathfrak{l}^\pm_{a;\tilde{\imath}}$. Hence, at large N the index i' can be replaced with a continuous variable t. Second, all the known methods for computing 3D partition functions at large N [1,20,21] require that terms with $i \sim j$ dominate over the terms with $|i - j| \gg 1$. The latter are called "long-range forces" and with the appropriate assumptions they cancel out at leading order, at least for a class of quiver theories that we shall discuss momentarily. The cancellation of long-range forces constrains the possible choices of $\mathfrak{l}^{\pm}$, although in general the constraint is complicated and it involves the value of $z^{\pm}$ as well. Remarkably, the special value

$$\mathfrak{l}^{\pm}_{a;i} = i \text{ mod } n_{\pm} \tag{25}$$

makes the long-range forces vanish for any $z^{\pm}$. We also anticipate that (25), along with a simple ansatz for $z^{\pm}$, reproduces the entropy of accelerating AdS_4 black holes. Curiously, (25) exhibits a strong similarity to the ansatz reproducing the entropy of AdS_5 black holes with arbitrary momenta, as discussed in [22,23].

Long-range forces cancellation.—In (19) the prefactors Ψ_I encode long-range forces among the variables $z^{\pm}$ that could spoil the large-N limit. As in previous work on $3d$ theories, we cancel the long-range forces by restricting to "nonchiral" quivers, where for any bifundamental connecting the nodes a and b there is a bifundamental connecting b and a and

$$\sum_{I\in(a)} \mathfrak{n}_I = \sum_{I\in(a)} u_I^F = \sum_{I\in(a)} (r_I - 1) + 2 = 0 \tag{26}$$

at each node a, where the sum is taken over all the arrows in the quiver with an end point at the node a, with adjoint chirals counting twice. In a four-dimensional quiver this condition would be equivalent to the absence of ABJ anomalies for any symmetry. The conditions (26) also imply that Tr $Q = 0$ for any global or R-symmetry with generator Q, where the trace is taken over all the fermions in the theory.

Using the periodicity relation $\mathfrak{l}^{\pm}_{a;i} = \mathfrak{l}^{\pm}_{a;i+T}$ that we have assumed, for the nonchiral quivers satisfying (26) the product of all the prefactor terms (19) at large N can be simplified down to

$$\prod_{I=1}^{|\mathcal{R}|} \Psi_I \cdot \prod_{a=1}^{|\mathcal{G}|} \Psi_a \longrightarrow \prod_{I=1}^{|\mathcal{R}|} \tilde{\Psi}_I \cdot \prod_{a=1}^{|\mathcal{G}|} \tilde{\Psi}_a, \tag{27}$$

where for $I \in (a, b)$,

$$\tilde{\Psi}_I = \prod_{s=\pm} \prod_{i,j=0}^{N-1} \left(\frac{z^s_{a;i}}{z^s_{b;j}} \right)^{(\sigma_s/4)(1-1/n_s-2\mathcal{A}^s_{I;ij})\mathrm{sgn}(i-j)}, \tag{28}$$

and a similar definition holds for $\tilde{\Psi}_a$. Requiring the right-hand side of (27) to vanish yields a mixed constraint on $\mathfrak{l}^{\pm}$ and $z^{\pm}$. Crucially, the ansatz (25) is the only one that satisfies the property

$$\frac{1}{n_{\pm}} \sum_{j=j_0}^{j_0+n_{\pm}-1} \mathcal{A}^{\pm}_{I;ij} = \frac{1}{2}\left(1 - \frac{1}{n_{\pm}}\right) \tag{29}$$

(and a similar relation with i, j inverted) ensuring that the long-range forces coming from (27) vanish for any $z^{\pm}$. Thanks to the Weyl-symmetry breaking factorization that we have used, the blocks $\mathcal{B}^{\pm}_I$ will not produce any long-range term at leading order, as we will now show.

Holomorphic blocks at large N.—In order to compute the large-N limit of the blocks $\mathcal{B}^{\pm}_I$, we will first consider the usual ansatz for the saddle point distribution of $z^{\pm}$ [1,20,21],

$$\log z^{\pm}_{a;i} = -\sigma_{\pm} N^{\alpha} t_i \mp \mathrm{i} y^{\pm}_a(t_i), \tag{30}$$

where $t_i, y_a(t_i)$ are real and are assumed to be ordered so that $t_i \le t_j$ for $i < j$. The power of N must be set to $\alpha = \frac{1}{2}$, otherwise the one-loop contributions and the Chern-Simons terms would grow with a different power law at large N and it would not be possible to find nontrivial critical points. When we take the continuum limit we split the index $i \equiv Ti' + \tilde{\imath}$: assuming that the eigenvalues t_i conform to a single continuous distribution at large N allows us to make the replacements $t_i \equiv t_{i'} \equiv t$ and define the eigenvalue density $\rho^{\pm}(t)$ such that

$$\frac{1}{N}\sum_{i=0}^{N-1} \bullet \longrightarrow \frac{1}{T}\sum_{\tilde{\imath}=0}^{T-1} \int \mathrm{d}t \rho^{\pm}(t) \bullet, \qquad \int \mathrm{d}t \rho^{\pm}(t) = 1. \tag{31}$$

Expanding the q-Pochhammer symbols in terms of polylogarithms at all orders in ϵ and taking the large-N limit of each term as in [1,20,21] yields

$$\log \mathcal{B}^{\pm}_I = N^{3/2} \sum_{k=0}^{2} \epsilon^{k-1} \frac{B_k}{k!} \frac{1}{T^2} \sum_{\tilde{\imath},\tilde{\jmath}=0}^{T-1} \int \mathrm{d}t \rho^{\pm}(t)^2 \times g_{3-k}[-\sigma_{\pm}\delta y^{\pm}_{ab}(t) - \sigma_{\pm}\Delta^{\pm}_I - \epsilon \mathcal{A}^{\pm}_{I;\tilde{\imath}\tilde{\jmath}}] + o(N^{3/2}), \tag{32}$$

with $B_k = B_k(1) = \{1, \frac{1}{2}, \frac{1}{6}, \ldots\}$ and

$$g_k(x) = \frac{(2\pi)^k}{k!} B_k\left(\frac{x}{2\pi} + \nu\right), \tag{33}$$

where $B_k(w)$ are the Bernoulli polynomials and the integer ν in (33) must be chosen so that $\mathrm{Im}(1/\epsilon) < \mathrm{Im}\{(1/\epsilon) \times [(x/2\pi) + \nu]\} < 0$. We are using the notation $\delta y^{\pm}_{ab}(t) \equiv y^{\pm}_a(t) - y^{\pm}_b(t)$.

Large-N limit of the spindle index.—We assume that the index is dominated by the configuration (25), which leads to a consistent large-N limit. The large-N limit of the classical Chern-Simons terms simplifies to

$$\log Z^{\mathrm{CS}}_{\mathrm{eff}} = N^{3/2} \sum_a \sum_{s=\pm} \frac{\sigma_s}{\epsilon} k_a \int \mathrm{d}t \rho^s(t) y^s_a(t). \tag{34}$$

Consistently with fact that for saddles with gravity duals flux quantization implies $N/(n_+ n_-) \in \mathbb{N}$ [3], we can take

$T = n_+ n_-$. Moreover, in order to compare with the black hole solutions, we need to take $n = 1$ [7]. Finally, we also need to choose a determination: we assume that $\mathrm{Im}(1/\epsilon) < \mathrm{Im}[(1/2\pi\epsilon)(y^{\pm}_{ab}(t) + \varphi_I)] < 0$. After some algebra, the explicit expression for $\mathfrak{l}^{\pm}_{a;i}$ and the conditions (26) yield

$$\log Z_{\Sigma\times S^1} = -\sum_{s=\pm} \sigma_s \frac{F(\rho^s, \delta y^s_{ab}, \Delta^s_I)}{\epsilon}, \tag{35}$$

with

$$\frac{F(\rho^{\pm}, \delta y^{\pm}_{ab}, \Delta^{\pm}_I)}{N^{3/2}} = -\sum_a k_a \int \mathrm{d}t t\rho^{\pm}(t) y^{\pm}_a(t) + \sum_{I\in(a,b)} \int \mathrm{d}t \rho^{\pm}(t)^2 G^{\pm}_3[\delta y^{\pm}_{ab}(t) + \Delta^{\pm}_I], \tag{36}$$

where $G^{\pm}_3(x) = \frac{1}{6}x(x - \sum_{I\in W}\Delta^{\pm}_I/2)(x - \sum_{I\in W}\Delta^{\pm}_I)$. The functions $G^{\pm}_3(x)$ are obtained by $g_3(x)$ in the range $\mathrm{Re}x \in [0, 2\pi]$ by replacing all occurrences of π with $\sum_{I\in W}\Delta^{\pm}_I/2$. The two terms in (35) depend on different variables and they can be extremized independently.

For example, for the ABJM theory dual to $\mathrm{AdS}_4 \times S^7$, with $|\mathcal{G}| = 2$, Chern-Simons level $k_1 = 1$ and $k_2 = -1$, and four bifundamental fields transforming as $\mathbf{N}_1 \otimes \bar{\mathbf{N}}_2$ for $I = 1, 2$ and as $\mathbf{N}_2 \otimes \bar{\mathbf{N}}_1$ for $I = 3, 4$, we find

$$(36) = \int \mathrm{d}t t\rho^{\pm}\delta y^{\pm}_{21} - \frac{1}{2}\int \mathrm{d}t(\rho^{\pm})^2 \Big(\sum_I \Delta^{\pm}_I (\delta y^{\pm}_{21})^2 - 2(\Delta^{\pm}_1\Delta^{\pm}_2 - \Delta^{\pm}_3\Delta^{\pm}_4)\delta y^{\pm}_{21} - \sum_{I<J<K} \Delta^{\pm}_I\Delta^{\pm}_J\Delta^{\pm}_K\Big). \tag{37}$$

This functional coincides with the large-N limit of the effective twisted superpotential for the ABJM theory derived in [1], expressed in terms of $\pm$ quantities, and its extremization is straightforward [24]. The explicit expressions for $\rho^{\pm}$ and $\delta y^{\pm}_{21}$ can be found, for example, in [[1], (2.70)–(2.75)]. The critical value is

$$F(\rho^{\pm}, \delta y^{\pm}_{21}, \Delta^{\pm}_I)|_{\mathrm{crit}} = \frac{2}{3}N^{3/2}\sqrt{2\Delta^{\pm}_1\Delta^{\pm}_2\Delta^{\pm}_3\Delta^{\pm}_4}. \tag{38}$$

Using (35) we recover the gravitational block form [8,25] of the entropy function obtained in [7] and more generally conjectured in [5]. The density of eigenvalues $\rho^{\pm}$ also agrees with the gravitational analysis performed in [26].

We can extend the result to more general quivers: indeed, the term of order zero of F in the ϵ expansion coincides with the large-N limit of the effective twisted superpotential of the $\mathcal{N} = 2$ theory [27],

$$\mathrm{i}\frac{\mathcal{W}(\rho, \delta y_{ab}, \Delta_I)}{N^{3/2}} = -\sum_a k_a \int \mathrm{d}t t\rho(t) y_a(t) + \sum_{I\in(a,b)} \int \mathrm{d}t \rho(t)^2 g_3[\delta y_{ab}(t) + \Delta_I], \tag{39}$$

where $\Delta_I = \Delta^{\pm}_I|_{\epsilon=0} = 2\pi u^F_I + \pi r_I$ and we are ignoring topological symmetries for simplicity. This agrees with well-known asymptotic behavior of the holomorphic blocks [16]: $\log(\mathrm{block}) = \mathrm{i}(\mathcal{W}/\epsilon) + O(\epsilon)$. We then observe that (36) is a *homogeneous* form of the large-N limit of the effective twisted superpotential $\mathcal{W}$ obtained by replacing Δ_I with $\Delta^{\pm}_I$ and all occurrences of π with $\sum_{I\in W}\Delta^{\pm}_I/2$. The extremization of (36) is then equivalent to the extremization of $\mathcal{W}$ with the constraint $\sum_{I\in W}\Delta_I = 2\pi$. One concludes that the entropy function has always a block form

$$\log Z_{\Sigma\times S^1} = \frac{F_{\mathrm{crit}}(\Delta^-_I)}{\epsilon} - \sigma\frac{F_{\mathrm{crit}}(\Delta^+_I)}{\epsilon}. \tag{40}$$

We also see that the block function $F_{\mathrm{crit}}(\Delta_I)$, up to factors, is the homogeneous form of the large-N on-shell value of the effective twisted superpotential $\mathcal{W}$. This has been computed for many examples in [28]. At large N, $\mathcal{W}$ coincides with the S^3 partition function of the $\mathcal{N} = 2$ theory [27] and, for theories with an $\mathrm{AdS}_4 \times M_7$ dual, the latter is in turn related [20,21] to the Sasakian volume [29,30] of M_7. Using this chain of equalities, one provides a field theory derivation of the gravitational block decomposition obtained in [10,12,31] for configurations with a "mesonic" (or "flavor") twist [32]. More details about topological symmetries and issues with baryonic symmetries will be discussed in [15].

Discussion.—In this Letter, we solved the fundamental problem of elucidating the microscopic origin of the Bekenstein-Hawking entropy of the most general class of rotating Bogomol'nyi-Prasad-Sommerfield black holes currently known in four dimensions. Specifically, our findings demonstrate that the microstates contributing to the entropy of accelerating black holes in four-dimensional anti–de Sitter space-time are precisely mirrored by the physical degrees of freedom characterizing three-dimensional gauge theories quantized on a spindle. To successfully solve this problem we developed a novel approach tailored to deal with the degrees of freedom of gauge theories on orbifolds. This technique holds vast potential impact as it applies to supersymmetric systems in any number of dimensions, including, e.g., three-dimensional orbifold partition functions [14] and four-dimensional orbifold indices [33]. Our results complete the construction of the first duality between a gravitational theory and a quantum field theory defined on an orbifold, paving the way for a reenergized research program in holography.

The work of E. C. and D. M. is supported in part by a grant Trapezio (2023) of the Fondazione Compagnia di San Paolo. A. Z. is partially supported by the MUR-PRIN Grant No. 2022NY2MXY. E. C., D. M., A. P., and A. Z. acknowledge partial support by the INFN. S. M. H. is supported in part by the STFC Consolidated Grants No. ST/T000791/1 and No. ST/X000575/1.

[1] F. Benini, K. Hristov, and A. Zaffaroni, J. High Energy Phys. 05 (2016) 054.

[2] F. Benini and A. Zaffaroni, J. High Energy Phys. 07 (2015) 127.

[3] P. Ferrero, J. P. Gauntlett, J. M. Perez Ipiña, D. Martelli, and J. Sparks, Phys. Rev. D **104**, 046007 (2021).

[4] P. Ferrero, J. P. Gauntlett, and J. Sparks, J. High Energy Phys. 01 (2022) 102.

[5] P. Ferrero, M. Inglese, D. Martelli, and J. Sparks, Phys. Rev. D **105**, 126001 (2022).

[6] A. Cabo-Bizet, D. Cassani, D. Martelli, and S. Murthy, J. High Energy Phys. 10 (2019) 062.

[7] D. Cassani, J. P. Gauntlett, D. Martelli, and J. Sparks, Phys. Rev. D **104**, 086005 (2021).

[8] F. Faedo and D. Martelli, J. High Energy Phys. 02 (2022) 101.

[9] S. M. Hosseini, K. Hristov, and A. Zaffaroni, J. High Energy Phys. 12 (2019) 168.

[10] A. Boido, J. P. Gauntlett, D. Martelli, and J. Sparks, Commun. Math. Phys. **403**, 917 (2023).

[11] C. Couzens, J. P. Gauntlett, D. Martelli, and J. Sparks, J. High Energy Phys. 01 (2019) 212.

[12] P. Benetti Genolini, J. P. Gauntlett, Y. Jiao, A. Lüscher, and J. Sparks, J. High Energy Phys. 05 (2024) 152.

[13] M. Inglese, D. Martelli, and A. Pittelli, J. Phys. A **57**, 085401 (2024).

[14] M. Inglese, D. Martelli, and A. Pittelli, arXiv:2312.17086.

[15] E. Colombo, S. M. Hosseini, D. Martelli, A. Pittelli, and A. Zaffaroni (2024).

[16] C. Beem, T. Dimofte, and S. Pasquetti, J. High Energy Phys. 12 (2014) 177.

[17] S. Choi, C. Hwang, and S. Kim, arXiv:1908.02470.

[18] S. Choi and C. Hwang, J. High Energy Phys. 03 (2020) 068.

[19] S. M. Hosseini and A. Zaffaroni, J. High Energy Phys. 12 (2022) 025.

[20] C. P. Herzog, I. R. Klebanov, S. S. Pufu, and T. Tesileanu, Phys. Rev. D **83**, 046001 (2011).

[21] D. Martelli and J. Sparks, Phys. Rev. D **84**, 046008 (2011).

[22] F. Benini, E. Colombo, S. Soltani, A. Zaffaroni, and Z. Zhang, Classical Quantum Gravity **37**, 215021 (2020).

[23] E. Colombo, J. High Energy Phys. 12 (2022) 013.

[24] The functional $\mathcal{W}$ written in [1] has a critical point under the condition $\sum_I \Delta_I = 2\pi$. We are using a homogeneous form of $\mathcal{W}$ where this condition has been used to eliminate all occurrences of π. The variables Δ_I in [1] correspond to our $\Delta_I^{\pm}|_{\epsilon=0}$.

[25] To compare with the formulas in [8] we set $n = 1$ and identify the variables as follows: $\pi\Delta_i^{\pm}|_{\text{there}} = \Delta_I^{\mp}|_{\text{here}}$, $-2\pi\epsilon|_{\text{there}} = \epsilon|_{\text{here}}$, $\pi\varphi_i|_{\text{there}} = \varphi_I|_{\text{here}}$, $r_i|_{\text{there}} = r_I|_{\text{here}}$, $\mathfrak{n}_i|_{\text{there}} = \mathfrak{n}_I|_{\text{here}}/(n_+n_-) + r_I\chi_\sigma/2$.

[26] A. Boido, A. Lüscher, and J. Sparks, J. High Energy Phys. 05 (2024) 226.

[27] S. M. Hosseini and A. Zaffaroni, J. High Energy Phys. 08 (2016) 064.

[28] S. M. Hosseini and N. Mekareeya, J. High Energy Phys. 08 (2016) 089.

[29] D. Martelli, J. Sparks, and S.-T. Yau, Commun. Math. Phys. **268**, 39 (2006).

[30] D. Martelli, J. Sparks, and S.-T. Yau, Commun. Math. Phys. **280**, 611 (2008).

[31] A. Boido, J. P. Gauntlett, D. Martelli, and J. Sparks, Phys. Rev. Lett. **130**, 091603 (2023).

[32] S. M. Hosseini and A. Zaffaroni, J. High Energy Phys. 07 (2019) 174.

[33] A. Pittelli, arXiv:2403.12318.