From a mathemagic trick to number bases: using cryptography and the software WIMS to stimulate algebraic thinking in primary school

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Introduction

Kieran (2004) suggests that early algebraic thinking is linked to problem solving, modeling, working with generalizable patterns, justifying and proving, making predictions and conjectures, analyzing relationships, and identifying structure. Many studies reveal that algebraic thinking can be developed at an early stage, as long as it is stimulated by appropriate interventions by the teachers (e.g. Blanton & Kaput, 2005). Indeed, the application of student-active methodologies plays a fundamental role in the teaching process, especially in mathematics, as described by Schoenfeld (1992). In this view, our first aim of research is to provide support to improve teachers' practice toward engaging pupils in algebraic activities. We aim to achieve this goal by designing and sharing ready to use materials, which are realizing a *problem-solving approach* and involve modern technologies. The resulting effect on students' learning is our second aim of research: to examine if this type of instruction truly affects the pupils' capacity for justifying and proving.

We present an activity based on the modern cryptography technique of Hamming codes for error correction that is built around a magic trick and is supported by the software WIMS (WWW Interactive Multipurpose Server, e.g., see Cazzola, et al., 2020). The various tasks involved push the students into a learning path that can be described in short as follows:

- Participation in the magic trick
- Question: "How does it work?" (*problem solving, predictions and conjectures*)
- First analysis of the trick (*analyzing relationships and identifying structure)*
- Sharing of observations (*justifying and proving*)
- Conclusion (*generalizations*)
- Bridge to the study of number bases.

The last step is an added value of our activity: students approach a typical algebraic topic associating it to a fun and intriguing game, strengthening their motivation to learn about it. A detailed description of the experience can be found in (Cazzola & Grazian, 2021).

Description of the activity

The game we present is called "7 questions and 1 lie". A volunteer is asked to think about a number from 0 to 15 and to answer 7 yes/no questions about it, with the chance to lie only once. Looking at the answers, the mathemagician will be able to guess not only the mysterious number, but also if there was a lie and the question it corresponds to. Once the pupils' attention is caught, the real work starts, leading the students to find an answer to the obvious question "how does it work?".

The solution involves the use of a particular scheme (Figure 1) containing 7 numbered circles. Each circle corresponds to the question with the associated number and the instruction is to blacken it if the answer is “yes” and to leave it blank otherwise.

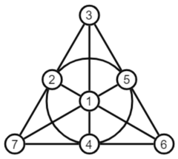

Figure 1: Scheme to solve the “7 question and 1 lie” magic trick

The analysis of the patterns will reveal the solution. Also, if we impose that a blackened circle corresponds to 1 and a blank one corresponds to 0, once the lies are detected and the coloring is fixed, the expression of the mysterious number in base 2 is given by the sequence of 0’s and 1’s given by the first 4 circles. This unusual approach to the study of number bases captures students’ attention and motivates the pupils in discovering more about this algebraic topic. Our activity can be presented using pen and paper only but becomes even more interesting if supported by its digital version, which we elaborated using the software WIMS. The dedicated WIMS page contains the magic trick, a step-by-step solution and a series of games aimed at practicing the description of numbers using different number bases.

Our experience

We presented the activity to four classes of 8-9 years old Italian students, for a total of 78 pupils, in Spring 2021. We conducted classroom observations and collected written students’ feedbacks. During the activity we recognized that pupils were actively engaged and carried out various examples of algebraic reasoning (especially in the form of making and motivating conjectures). This encourages us to further explore this approach and to deepen our investigation on its effects on pupils’ argumentation abilities. Furthermore, we could observe that pupils and teachers reacted enthusiastically to the tasks, strengthening our belief that this kind of proposal can also contribute to the diffusion of problem-solving and student-centered methodologies in primary school education.

References

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