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Monopolistic competition, as you like it

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Abstract

We explore monopolistic competition with asymmetric preferences over a variety of goods provided by heterogeneous firms, and compute equilibria (approximating Cournot and Bertrand equilibria when market shares are negligible) through average Morishima elasticities of substitution. Further results concerning pricing and entry emerge under homotheticity and when demands depend on a common aggregator, as with Generalized Additively Separable preferences. Under additivity we can determine which goods are going to be provided under free entry, as well as the selection effects associated with changes in market size, consumers' income, aggregate productivity, and preference parameters.

KEYWORDS

asymmetric preferences, generalized separability, heterogeneous firms, monopolistic competition, variable markups

JEL CLASSIFICATION

D11, D43, L11

1 | INTRODUCTION

Which products and at which prices will be provided by markets where heterogeneous firms sell differentiated goods? This is a core question of modern economic theories that depart from the perfectly competitive paradigm and adopt the monopolistic competition set up pioneered by Chamberlin (1933). Most of these theories rely on symmetric, Constant Elasticity of Substitution (CES) preferences based on Dixit and Stiglitz (1977: Section I), which delivers constant markups, either across countries and among firms in trade models (Krugman, 1980; Melitz, 2003) or over time in macroeconomic applications with flexible prices (see Barro & Sala-i-Martin, 2004 and Woodford, 2003). Few applications use more general but still symmetric preferences (Bertoletti & Etro, 2016; Dixit & Stiglitz, 1977: Section II), even when considering variable productivity across firms (as in Arkolakis et al., 2019; Melitz & Ottaviano, 2008; Parenti et al., 2017) and over time (as in Bilbiie et al., 2012 or Kimball, 1995). In an attempt to capture the features of monopolistic competition in the spirit of Chamberlin,[1] we study a large industry with heterogeneous firms supplying

Abbreviations: AIDS, Almost Ideal Demand System; CES, Constant Elasticity of Substitution; DA, directly additive; GAS, Generalized Additively Separable; IA, indirectly additive; MEC, Morishima Elasticity of Complementarity; MES, Morishima Elasticity of Substitution; QMOR, Quadratic Mean of Order R.

genuinely differentiated commodities, and explore methodologies to study monopolistic competition in such a setting.[2] This generates markups variable across markets and goods of different quality, possibly depending on aggregate variables, and allows for some progress concerning the way markets select not only how many, but also which goods are going to be provided, which is relevant for applied analysis.[3]

Consider demand systems derived from asymmetric preferences over a variety of different commodities that can be represented by well-behaved utility functions and that are produced by firms under idiosyncratic marginal and fixed costs: in this environment, we explore which strategies are adopted by these firms. The starting point is the analysis of Cournot and Bertrand equilibria in which firms choose either their quantities or their prices taking as given the strategies of competitors and the demand systems. We generalize the familiar pricing conditions by expressing the equilibrium markups of the firms in terms of their market shares and of the substitutability of their own products with those sold by competitors. Substitutability is measured by the average of the Morishima Elasticities of Substitution, as rediscovered and formalized by Blackorby and Russell (1981).[4]

Competition among a large number of firms with negligible market shares (Dixit & Stiglitz, 1977, 1993; Spence, 1976) corresponds to the concept of monopolistic competition. Here, we define it as the market structure in which firms "perceive" the demand elasticity as given by the average Morishima elasticity (which approximately coincides with the actual one when market shares are indeed small enough). Introducing free entry we can also ask which products are provided by the market, and what kind of selection is associated with changes in market size (for instance due to opening up markets), expenditure (due to a demand shock), aggregate productivity (due to a supply shock or technological growth) as well as preference parameters. The answers are simple under asymmetric CES preferences, because the set of goods provided by the market is not affected by changes of aggregate productivity, and an increase of expenditure or market size delivers new goods but without affecting the entry sequence (as implicit in endogenous growth models *à la* Romer, 1990). This is not the case in general (when also multiplicity of equilibria cannot be excluded), however we show that the irrelevance of common productivity shocks is preserved under homothetic preferences, the neutrality of expenditure under directly additive (DA) preferences, and the neutrality of the market size under indirectly additive (IA) preferences.

In practice, demand systems are often assumed to depend on simple aggregators of firm choices. Therefore, we study in further depth monopolistic competition for the Generalized Additively Separable (GAS) preferences introduced by Gorman (1970) and Pollak (1972), which deliver demand systems depending on a common aggregator. Intuition suggests that to take the aggregator as given should be approximately correct (i.e., profit maximizing) when the market shares are negligible: we show that this is indeed the case, in the sense that when these shares are negligible the impact of a single firm on the aggregator is negligible too, and the perceived demand elasticities are approximately equal to the average Morishima measures (which in turn are close to the actual ones). In addition, the equilibrium strategies do not depend on whether prices or quantities are chosen by firms, implying that imperfectly competitive choices do actually "converge" to those of monopolistic competition. This approach provides a simple way to solve for asymmetric equilibria, and can be extended to other demand functions that depend on multiple aggregators.

Free-entry equilibria can be naturally defined as those where firms make both entry and pricing decisions anticipating the value of the aggregator and taking it as given. Under additivity of preferences, we can actually show that the free entry equilibrium is essentially unique in spite of asymmetries between goods. For the class of DA preferences, we show that an increase in the market size favors the entry of firms producing goods with a less elastic demand, which enjoy the largest unit profitability. At the same time, these firms are the less favored by an increase in (aggregate) productivity, while changes in expenditure are neutral on the entry sequence. For the class of IA preferences, equilibrium pricing is independent across firms and the price of each firm only depends on its marginal cost, product substitutability, and consumers' expenditure. Moreover, we show that an increase of either expenditure or productivity affects proportionally more the firms that face the most elastic demands, which make the best of them in terms of their survival ability, while an increase of market size has a proportional impact on all firms and is neutral on pricing as well as on the entry sequence.

This work is related to different literatures. We generalize the analysis of imperfect competition with differentiated products, usually studied under quasilinear preferences (Vives, 1999) by reframing it in terms of the Morishima elasticities. After the seminal contribution of Spence (1976), only few papers have analyzed monopolistic competition with asymmetric preferences. The work of Dixit and Stiglitz (1977: Section III) only dealt with a specific example with intersectoral perfect substitutability. The earliest treatment we are aware of is in a work of Pascoa (1997), mainly focused on an example with Stone–Geary preferences and a continuum of goods. More recently, D'Aspremont and Dos Santos Ferreira (2016, 2017) have provided a related analysis of asymmetric preferences with an outside good adopting an alternative equilibrium concept (but their monopolistic competition limit is consistent with ours). The trade literature with heterogeneous firms, started by Melitz (2003) and Melitz and Ottaviano (2008), has usually considered monopolistic competition with symmetric preferences; only a few works have added asymmetries to model quality differentials among goods (for instance Baldwin & Harrigan, 2011; Crozet et al., 2012; Feenstra & Romalis, 2014), but retaining a CES structure. Heterogeneity in demand and costs is instead at the basis of the empirical literature of industrial organization on the impact of market size on entry (Bresnahan & Reiss, 1987; Campbell & Hopenhayn, 2005). We build a bridge between these distant literatures considering asymmetric preferences that generate different markups among goods affecting the determinants of market selection.[5] Our companion paper (Bertoletti & Etro, 2021) analyzes in further detail monopolistic competition with GAS preferences under heterogeneous firms, but retaining the symmetry of preferences.

The work is organized as follows. Section 2 presents alternative equilibria of imperfect competition for the same demand microfoundation. Sections 3 and 4 study monopolistic competition respectively when preferences are homothetic and when the demand system depends on simple aggregators. Section 5 concludes. All proofs are in the Appendix.

2 | THE MODEL

We consider L identical consumers with preferences over a finite number n of commodities represented by well-behaved direct and indirect utility functions:

$$U = U(\mathbf{x}) \quad \text{and} \quad V = V(\mathbf{s}), \tag{1}$$

where $\mathbf{x}$ is the n-dimensional vector of quantities and $\mathbf{s} = \mathbf{p}/E$ is the corresponding vector of prices normalized by exogenous expenditure E. We assume that the utility maximizing choices are unique, interior ($\mathbf{x}, \mathbf{p} > \mathbf{0}$) and characterized by the first-order conditions for utility maximization. Therefore, the inverse and direct demand systems are delivered by Hotelling-Wold's and Roy's identities:

$$s_i(\mathbf{x}) = \frac{U_i(\mathbf{x})}{\tilde{\mu}(\mathbf{x})}, \qquad x_i(\mathbf{s}) = \frac{V_i(\mathbf{s})}{\mu(\mathbf{s})}, \tag{2}$$

where

$$\tilde{\mu}(\mathbf{x}) = \sum_{j=1}^{n} U_j(\mathbf{x}) x_j, \quad \mu(\mathbf{s}) = \sum_{j=1}^{n} V_j(\mathbf{s}) s_j \tag{3}$$

and U_i and V_i denote marginal utilities, $i = 1, .., n$. Here $\tilde{\mu}$ is the marginal utility of income *times* the expenditure level, and it holds that $|\mu(\mathbf{s})| = \tilde{\mu}(\mathbf{x}(\mathbf{s}))$, as can be verified by adding up the market shares $b_j = s_j x_j$. As a simple example we will occasionally refer to the asymmetric CES preferences, that can be represented by:

$$U = \sum_{j=1}^{n} \tilde{q}_j x_j^{1-\epsilon} \quad \text{and} \quad V = \sum_{j=1}^{n} q_j s_j^{1-\varepsilon}, \tag{4}$$

where $q_j = \tilde{q}_j^{\varepsilon} > 0$ can be interpreted as an idiosyncratic quality index for good j, and $\epsilon = 1/\varepsilon \in [0, 1)$ is the parameter that governs substitutability among goods.

Firm i produces good i at a constant marginal cost $c_i = \tilde{c}_i / A > 0$, where the common parameter $A > 0$ represents aggregate productivity: the variable profits of firm i are then given by:

$$\pi_i = (p_i - c_i) x_i L, \tag{5}$$

and we will later introduce fixed costs of entry. We begin by studying market equilibria in which firms correctly perceive the demand system and choose their profit-maximizing strategies. In the partial equilibrium tradition of industrial organization we have to consider two cases, with each firm choosing either its production level (Cournot competition) or its price (Bertrand competition). Throughout this work, we assume that the first-order condition for profit maximization characterizes firm behavior. Of course, to behave well market equilibria may also require that the demand system satisfies other regularity conditions (for a related discussion see Vives, 1999, Ch. 6). We assume that these equilibria are well defined (but see our results on existence and uniqueness under additivity in Section 4), and use them to study a generalized form of monopolistic competition and to discuss free entry conditions.

2.1 | Cournot competition

Let us consider firms that choose their quantities on the basis of the inverse demand functions $s_i(\mathbf{x})$ in Equation (2). As usual, each firm i chooses x_i to equate its marginal revenue to its marginal cost c_i taking as given the strategies of the other firms which must be the profit-maximizing ones in equilibrium (i.e., $\mathbf{x}$ is a Nash equilibrium). The relevant (per-consumer) marginal revenue of firm i is $MR_i = \partial(p_i x_i)/\partial x_i$, where $p_i(\mathbf{x}) = s_i(\mathbf{x})E$. It can be written as:

$$MR_i(\mathbf{x}) = \frac{[U_i(\mathbf{x}) + U_{ii}(\mathbf{x})x_i]\tilde{\mu}(\mathbf{x}) - U_i(\mathbf{x})x_i\left[U_i(\mathbf{x}) + \sum_{j=1}^{n} U_{ji}(\mathbf{x})x_j\right]}{\tilde{\mu}(\mathbf{x})^2} E$$

$$= p_i(\mathbf{x})\left[1 - s_i(\mathbf{x})x_i - \sum_{j=1}^{n} \epsilon_{ij}(\mathbf{x})s_j(\mathbf{x})x_j\right],$$

where we have defined the (gross) Morishima Elasticity of Complementarity, henceforth MEC, between varieties i and j as follows[6]:

$$\epsilon_{ij}(\mathbf{x}) = -\frac{\partial \ln\{s_i(\mathbf{x})/s_j(\mathbf{x})\}}{\partial \ln x_i} = \frac{U_{ji}(\mathbf{x})x_i}{U_j(\mathbf{x})} - \frac{U_{ii}(\mathbf{x})x_i}{U_i(\mathbf{x})}. \tag{6}$$

Notice that this inverse measure of substitutability depends on preferences and not on the specific utility function which is chosen to represent them. Since substitutability can differ among goods, let us compute the weighted average of the MECs for good i with respect to all the other goods j, with weights based on the expenditure shares $b_j(\mathbf{x}) = s_j(\mathbf{x})x_j$, namely:

$$\bar{\epsilon}_i(\mathbf{x}) = \sum_{j \neq i}^{n} \epsilon_{ij}(\mathbf{x}) \frac{b_j(\mathbf{x})}{1 - b_i(\mathbf{x})}. \tag{7}$$

It is then immediate to verify that the marginal revenue above can be rewritten as $MR_i = p_i(1 - b_i)(1 - \bar{\epsilon}_i)$, and that the Cournot equilibrium quantities satisfy the system:

$$p_i(\mathbf{x}) = \frac{c_i}{1 - \epsilon_i^C(\mathbf{x})} \quad \text{for } i = 1, 2, ..., n, \tag{8}$$

where the left hand side comes from the inverse demand given in Equation (2) and the right hand side depends on:

$$\epsilon_i^C(\mathbf{x}) = b_i(\mathbf{x}) + [1 - b_i(\mathbf{x})]\bar{\epsilon}_i(\mathbf{x}). \tag{9}$$

Here, ϵ_i^C is an increasing function of the market share of firm i and of its average Morishima elasticity $\bar{\epsilon}_i$ (which we assume to be smaller than unity). Intuitively, a firm's markup is higher when it supplies a good that is on average less substitutable with the other goods (high $\bar{\epsilon}_i$), and its market share is larger (high b_i). In the CES example (Equation 4), ϵ corresponds to the common and constant MEC, and the market shares depend on the idiosyncratic quality and cost parameters q_j and c_j, but closed form equilibrium solutions can be obtained only in simple cases.

2.2 | Bertrand competition

Consider now firms that choose their prices on the basis of the direct demand $x_i(\mathbf{s})$ in Equation (2), while correctly anticipating the prices of the competitors (i.e., $\mathbf{p}$ is a Nash equilibrium). The elasticity of the Marshallian direct demand of firm i can be computed as:

$$\left|\frac{\partial \ln x_i(\mathbf{s})}{\partial \ln p_i}\right| = -\frac{s_i}{x_i(\mathbf{s})}\frac{V_{ii}(\mathbf{s})\mu(\mathbf{s}) - V_i(\mathbf{s})\left[V_i(\mathbf{s}) + \sum_{j=1}^n V_{ji}(\mathbf{s})s_j\right]}{\mu(\mathbf{s})^2}.$$

Let us consider the (gross) Morishima Elasticity of Substitution, or MES, between goods i and j[7]:

$$\varepsilon_{ij}(\mathbf{s}) = -\frac{\partial \ln\{x_i(\mathbf{s})/x_j(\mathbf{s})\}}{\partial \ln s_i} = \frac{s_i V_{ji}(\mathbf{s})}{V_j(\mathbf{s})} - \frac{s_i V_{ii}(\mathbf{s})}{V_i(\mathbf{s})}, \tag{10}$$

which again depends on preferences and not on their specific utility representation, and compute the weighted average:

$$\bar{\varepsilon}_i(\mathbf{s}) \equiv \sum_{j\neq i}^n \varepsilon_{ij}(\mathbf{s})\frac{b_j(\mathbf{s})}{1-b_i(\mathbf{s})}, \tag{11}$$

which is assumed larger than unity, and where, with a little abuse of notation, $b_j(\mathbf{s}) = s_j x_j(\mathbf{s})$ is now the expenditure share of firm i as a function of normalized prices. We can now rewrite demand elasticity $|\partial \ln x_i/\partial \ln p_i|$ as:

$$\varepsilon_i^B(\mathbf{s}) = b_i(\mathbf{s}) + [1-b_i(\mathbf{s})]\bar{\varepsilon}_i(\mathbf{s}), \tag{12}$$

to define the Bertrand equilibrium through the following system:

$$p_i = \frac{\varepsilon_i^B(\mathbf{s})c_i}{\varepsilon_i^B(\mathbf{s})-1} \quad \text{for } i = 1,2,..,n. \tag{13}$$

Firms set higher markups if their goods are on average less substitutable than those of competitors (low $\bar{\varepsilon}_i$) and their market shares larger (high b_i). In the CES example (Equation 4), the parameter ε is the constant and common MES and is the reciprocal of the common MEC.

2.3 | Generalized monopolistic competition

The remainder of this work is dedicated to analyze large markets of monopolistic competition under asymmetric preferences. There are alternative ways to make sense of this concept but, in the spirit of Dixit and Stiglitz's (1993) reply to Yang and Heidra (1993), we interpret monopolistic competition as the result of having firms that correctly perceive market shares as negligible. In fact, what Dixit and Stiglitz (1977) did in their symmetric setting amounts to neglect any term of order $1/n$ in the demand elasticities, where n was a number of firms assumed sufficiently large to make the omitted terms small. Similarly, in our setting, when there are many goods we expect consumers to spread their expenditure if preferences are well behaved and not too asymmetric, so that the market shares should be small for all goods.[8] On this basis, our previous results suggest to approximate the relevant demand elasticities with the corresponding averages of the Morishima measures.

Accordingly, we define as monopolistically competitive an environment where market shares are negligible, that is $b_i \approx 0$ for any $i = 1,..,n$, and where firms, correctly anticipating the value of actual demands, "perceive" the relevant elasticities as given by the average Morishima elasticities. This approach actually leads to two approximations according to whether we refer either to quantity or to price competition. In the first case, we can approximate Equation (8) by using the following system of pricing rules:

$$p_i(\mathbf{x}) = \frac{c_i}{1-\bar{\varepsilon}_i(\mathbf{x})} \quad \text{for } i=1,2,..,n. \tag{14}$$

In the second case, we can approximate Equation (13) with the pricing rules:

$$p_i = \frac{\bar{\varepsilon}_i(\mathbf{p}/E)c_i}{\bar{\varepsilon}_i(\mathbf{p}/E)-1} \quad \text{for } i=1,2,..,n. \tag{15}$$

These simplified systems need to be solved to derive the prices and quantities which arise in a monopolistic competition equilibrium (that ought to imply negligible market shares). Once we depart from symmetry this may still be a formidable task, but in next sections we will consider a methodology that allows one to obtain explicit solutions for some classes of asymmetric preferences.

We can learn something more about this approach to monopolistic competition by considering the relevant cross demand elasticities. They can be computed as ($i \neq j$):

$$\begin{aligned}\frac{\partial \ln p_j(\mathbf{x})}{\partial \ln x_i} &= \frac{U_{ji}(\mathbf{x})x_i}{U_j(\mathbf{x})} - \sum_{h=1}^{n} \frac{U_{hi}(\mathbf{x})x_i}{U_h(\mathbf{x})} b_h(\mathbf{x}) \\ &= \epsilon_{ij}(\mathbf{x}) - \bar{\epsilon}_i(\mathbf{x}) + b_i(\mathbf{x})\bar{\epsilon}_i(\mathbf{x}),\end{aligned} \tag{16}$$

$$\begin{aligned}\frac{\partial \ln x_j(\mathbf{s})}{\partial \ln s_i} &= \varepsilon_{ij}(\mathbf{s}) - \left|\frac{\partial \ln x_i(\mathbf{s})}{\partial \ln p_i}\right| \\ &= \varepsilon_{ij}(\mathbf{s}) - \bar{\varepsilon}_i(\mathbf{s}) - b_i(\mathbf{s})(1-\bar{\varepsilon}_i(\mathbf{s})).\end{aligned} \tag{17}$$

When shares are indeed negligible the cross effects should be perceived as negligible too whenever the differences $\epsilon_{ij} - \bar{\epsilon}_i$ and $\varepsilon_{ij} - \bar{\varepsilon}_i$ are small and the perceived own elasticities are not very large. Apparently, this is the case that Dixit and Stiglitz (1993) had in mind, and we expect it to apply to the typical monopolistic competition equilibrium with positive markups. Notice that the former condition is satisfied in any equilibrium of a symmetric environment. However, both conditions might be violated in our asymmetric setting: in similar cases the perceived cross demand elasticities can be large, and associated to a large own demand elasticity and therefore to small equilibrium markups. In other words, it can happen that goods are perceived as highly substitutable and that monopolistic competition pricing approximates marginal cost pricing as in a perfectly competitive setting.[9] We will exemplify this possibility in the next section for the case of translog preferences, and in Appendix F for the case of restricted *Almost Ideal Demand System* (AIDS) preferences (Deaton & Muellbauer, 1980).

Notice that in the CES case (Equation 4), the conditions (Equations 14 and 15) exactly characterize the *same* monopolistic competition solution:

$$\widehat{p}_i = \frac{c_i}{1-\epsilon} = \frac{\varepsilon c_i}{\varepsilon - 1}, \tag{18}$$

and that in such a case, the cross effects (Equations 16 and 17) actually vanish when market shares become negligible.

2.4 | Entry

Which set of goods will be provided in a monopolistic competition equilibrium, and how is the latter affected by market fundamentals? In this section, we introduce these questions by considering free entry equilibria when the production of each good requires a positive fixed cost. It may be useful to remind the reader that without fixed costs, a perfectly competitive market would provide all the suitable goods by pricing them at marginal cost: the question of which goods are actually introduced becomes relevant under fixed costs, imperfect competition, and asymmetries between goods (with symmetry it simplifies to the question of which *number* of goods should be provided, already explored in the literature).

Let us assume that preferences are defined over a large but finite set Ω of N different commodities, and that each good $i \in \Omega$ can be produced by a single firm only after paying a fixed entry cost $F_i > 0$. In the spirit of Chamberlin (1933), one can think of firms entering the market as long as they can price above the average cost.[10] Namely, in a monopolistic competition equilibrium with free entry there are $n \leq N$ active firms which all get nonnegative profits: the other $N - n$ firms would not obtain a positive profit by entering the market. The prices of the goods produced by the former firms are set at their monopolistic competition levels, say $\widehat{\mathbf{p}}$, and the prices of the goods of the latter firms should be set above their choke levels (if any), or equivalently at ∞. The variable profits of an active firm $i = 1, ..., n$ can be written as $\pi_i = \frac{p_i - c_i}{p_i} b_i EL$. By using equilibrium pricing condition (Equation 15) and defining $\phi_i \equiv -\partial \ln V / \partial \ln s_i$ as the price elasticity of utility of commodity i, with average $\phi \equiv \frac{1}{n}\sum_{j=1}^{n} \phi_j$, we can express equilibrium variable profits as:

$$\widehat{\pi}_i = \frac{\phi_i(\widehat{\mathbf{s}})EL}{\phi(\widehat{\mathbf{s}})\bar{\varepsilon}_i(\widehat{\mathbf{s}})n} \tag{19}$$

(a corresponding formula can be obtained from the dual representation of preferences through the average MEC). Since EL/n is a common term across firms, this implies that active firms with a lower average MES $\bar{\varepsilon}_i$ and a higher ratio ϕ_i/ϕ have higher variable profits because they can set higher markups and conquer larger market shares (these elasticities determine the intensive and extensive profit margins). In a free entry equilibrium only firms covering fixed costs with their variable profits can be active.[11]

At the present level of generality, we cannot exclude a multiplicity of market equilibria. However, in Section 4 we will make some progress (and prove existence and uniqueness) under further assumptions (in particular, by assuming that preferences are additive and firms take aggregators as given). Here, let us reconsider the CES example (Equation 4) as a benchmark, and let $\widehat{\Gamma} \subseteq \Omega$ be a set of goods provided in a free-entry equilibrium at prices (Equation 18).[12] We can directly compute profits (Equation 19) for a given market size as:

$$\widehat{\pi}_i = \frac{q_i \tilde{c}_i^{1-\varepsilon} EL}{\varepsilon \sum_{j \in \widehat{\Gamma}} q_j \tilde{c}_j^{1-\varepsilon}}, \tag{20}$$

which is independent from aggregate productivity A (an increase of productivity reduces prices and unit costs while increasing proportionally demand so that profits, and thus $\widehat{\Gamma}$, remain unchanged), and linear with respect to the total market size EL (for a given set of firms). Thus, the condition of a nonnegative profit for good i,

$$\frac{q_i \tilde{c}_i^{1-\varepsilon}}{F_i} \geq \frac{\varepsilon \sum_{j \in \widehat{\Gamma}} q_j \tilde{c}_j^{1-\varepsilon}}{EL}, \tag{21}$$

uniquely defines a ranking among firms based on the value of the left-hand side of Equation (21): it is natural to think of it as establishing the sequence of market introduction. Thus, as we will prove formally in Section 4, the asymmetric CES preferences generate a free entry equilibrium such that the identity of the goods introduced is independent from aggregate productivity A and it is determined by the total market size EL, while the sequence of introduction is unaffected from either expenditure E or market size L. We will see that some of the special properties of the CES example extend to more general classes of preferences. In particular, the irrelevance of productivity shocks will be preserved under homothetic preferences (Section 3), the neutrality of expenditure under DA preferences (Section 4.1), and the neutrality of the market size under IA preferences (Section 4.2).

3 | MONOPOLISTIC COMPETITION WITH HOMOTHETICITY

Monopolistic competition with symmetric homothetic preferences has been studied by Benassy (1996) and others.[13] Here, we are concerned with the more general case of asymmetric homothetic preferences, because they are crucial for representative agent models and provide an interesting application of our proposed equilibria. Let us normalize the indirect utility function to be:

$$V = \frac{E}{P(\mathbf{p})}, \tag{22}$$

where $P(\mathbf{p})$ is homogeneous of degree 1 and represents a fully-fledged price index. For instance $P = \left[\sum_{j=1}^{n} q_j p_j^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$ in the CES case (Equation 4). The Roy's identity delivers direct demands $x_i = P_i(\mathbf{s})/P(\mathbf{s})$ and market shares $b_i = s_i P_i(\mathbf{s})/P(\mathbf{s})$, which are homogeneous respectively of degree -1 and 0. This allows us to compute the MES:

$$\varepsilon_{ij}(\mathbf{s}) = \frac{s_i P_{ji}(\mathbf{s})}{P_j(\mathbf{s})} - \frac{s_i P_{ii}(\mathbf{s})}{P_i(\mathbf{s})},$$

which is homogeneous of degree 0, being the difference of two functions that are both homogeneous of that degree. Therefore also the average MES $\bar{\varepsilon}_i(\mathbf{s})$ is homogeneous of degree zero, which implies immediately that pricing is independent from the expenditure level (for a given set of firms).[14] Similar results can be derived starting from the direct utility (which can be written as a consumption index) and using the inverse demand system and the average MEC to study quantity competition.

3.1 | Examples

We now consider equilibrium pricing for two specifications of homothetic preferences.

3.1.1 | Translog preferences

As a first example, let us consider the homothetic translog preferences (Christensen et al., 1975) represented by the following price index:

$$P(\mathbf{s}) = \exp\left[\ln \alpha_0 + \sum_i \alpha_i \ln s_i + \frac{1}{2}\sum_i \sum_j \beta_{ij} \ln s_i \ln s_j\right], \tag{23}$$

where we assume without loss of generality $\beta_{ij} = \beta_{ji}$, and we need $\sum_i \alpha_i = 1$ and $\sum_j \beta_{ij} = 0$ to satisfy the linear homogeneity of P (a symmetric version of these preferences is used by Feenstra, 2003). The direct demand for good i is:

$$x_i(\mathbf{s}) = \frac{\alpha_i + \sum_j \beta_{ij} \ln s_j}{s_i},$$

which delivers the market share $b_i = \alpha_i + \sum_j \beta_{ij} \ln s_j$. Maximization of profits provides the Bertrand equilibrium conditions:

$$p_i = c_i\left(1 + \frac{b_i}{\beta_i}\right), \tag{24}$$

where the positiveness of $\beta_i \equiv -\beta_{ii}$ is necessary to ensure $\varepsilon_i^B = 1 + \beta_i/b_i > 1$.

We can obtain the same result, as well as the monopolistic competition equilibrium, by deriving the Morishima elasticity between goods i and j as:

$$\varepsilon_{ij} = 1 + \frac{\beta_i}{b_i} + \frac{\beta_{ji}}{b_j},$$

so that the average MES is:

$$\bar{\varepsilon}_i = \sum_{j \neq i}^{n} \varepsilon_{ij} \frac{b_j}{1-b_i} = 1 + \frac{\beta_i}{(1-b_i)b_i}.$$

This allows one to get Equation (24) from Equation (13), and to obtain the monopolistic competition prices:

$$p_i = c_i \left[1 + \frac{(1-b_i)b_i}{\beta_i} \right] \tag{25}$$

from Equation (15). Notice that these prices of monopolistic competition are below the Bertrand prices (Equation 24) for given market shares, and that, when market shares are negligible ($b_i \approx 0$), the average MES is large, goods are highly substitutable and prices must be close to the marginal costs (i.e., $\widehat{p}_i \approx c_i$), approaching the case of perfect competition.

3.1.2 | Generalized linear preferences

Let us now consider an example of homothetic preferences due to Diewert (1971).[15] Suppose that preferences can be represented by the following direct utility/consumption index:

$$U = \sqrt{\mathbf{x}}' \mathbf{A} \sqrt{\mathbf{x}} = \sum_i \sum_j \sqrt{x_i} a_{ij} \sqrt{x_j} \tag{26}$$

where, without loss of generality, we can take the matrix $\mathbf{A}$ to be symmetric. To satisfy the standard regularity conditions we assume that $a_{ij} \geq 0$ for any i,j. Here we obtain $U_i = \sum_j a_{ij} \sqrt{x_j}/\sqrt{x_i}$ and $\tilde{\mu} = U$, with market shares $b_i = (\sqrt{x_i} \sum_j a_{ij} \sqrt{x_j})/U$. Since the MECs can be computed as:

$$\epsilon_{ij} = \frac{1}{2} \left[1 + \frac{a_{ij}\sqrt{x_i}}{\sum_h a_{jh}\sqrt{x_h}} - \frac{a_{ii}\sqrt{x_i}}{\sum_h a_{ih}\sqrt{x_h}} \right],$$

we obtain the average MEC:

$$\bar{\epsilon}_i = \frac{1}{2} \left\{ 1 - \frac{a_{ii}\sqrt{x_i}}{\sum_h a_{ih}\sqrt{x_h}} + \frac{b_i - a_{ii}x_i/U(\mathbf{x})}{1-b_i} \right\},$$

which allows us to determine the equilibrium conditions.[16] Here $\bar{\epsilon}_i$ is strictly positive for every good, implying positive markups, unless $a_{ij} = 0$ for any $i \neq j$ (in which case commodities would be perfect substitutes).

A simple case emerges when $a_{ii} = 0$ for any i, which implies $\bar{\epsilon}_i = 1/[2(1-b_i)]$. This allows us to express Cournot prices as:

$$p_i = \frac{2c_i}{1-2b_i}, \tag{27}$$

and monopolistic competition prices as:

$$p_i = \frac{2(1-b_i)c_i}{1-2b_i}. \tag{28}$$

With these preferences markups do not vanish when market shares are negligible, but rather approach to twice the marginal cost: indeed we get $\widehat{p}_i \approx 2c_i$ when $b_i \approx 0$.

3.2 | Entry

As discussed in Section 1.4, in general changes in market size, individual expenditure and productivity affect the set of active firms (and the free-entry equilibrium need not to be unique). However, under homotheticity the equilibrium variable profit (Equation 19) can be computed as:

$$\widehat{\pi}_i = \frac{\widehat{p}_i P_i(\widehat{\mathbf{p}}) EL}{\bar{\varepsilon}(\widehat{\mathbf{p}}) P(\widehat{\mathbf{p}}),} \tag{29}$$

where $\widehat{p}_i = \frac{\bar{\varepsilon}(\widehat{\mathbf{p}}) c_i}{\bar{\varepsilon}(\widehat{\mathbf{p}}) - 1}$ for $i \in \widehat{\Gamma}$ (with infinite prices for $i \notin \widehat{\Gamma}$), and one can verify that is independent from the productivity component A, and linear with respect to EL for a given set $\widehat{\Gamma}$. Thus changes in aggregate productivity do not affect $\widehat{\Gamma}$, while increases in market size and individual expenditures exert the same expansionary effect on it. We summarize these facts as follows (see the proof in Appendix A):

> **Proposition 1** *When preferences are homothetic, the identity of the goods provided in a free entry equilibrium does not depend on aggregate productivity, and is symmetrically affected by expenditure and market size.*

In practice, general purpose technological progress or aggregate shocks reducing marginal costs do not expand the set of goods provided by the market (unless they also affect the fixed costs) and do not change markups. Instead, an increase of the aggregate market size is likely to generate the provision of new goods (and, possibly, the replacement of some), independently from whether its source is higher spending or more consumers, as in endogenous growth models *à la* Romer (1990), and this entry process might affect markups as well as profitability.[17] As we will see in the next section, the impact of supply shocks, spending and population on entry is radically different under non-homothetic preferences.

4 | MONOPOLISTIC COMPETITION WITH AN AGGREGATOR

Although well-behaved demands can depend on prices in a general way, the demand systems adopted in usual theoretical and empirical applications are simpler and depend on price aggregators or quantity indices (as in the CES case). For these cases we can study an alternative approach to monopolistic competition and verify its consistency with our previous proposal. In this section we mainly explore preferences that generate direct demand functions depending on the own price and a common aggregator of all prices or, equivalently, inverse demand functions that depend on the own quantity and a common aggregator of all quantities, but we will later discuss how the same approach can be used with more aggregators (see Section 4.3).

Pollak (1972) has defined GAS preferences as those exhibiting demand functions that can be written as:

$$s_i = s_i(x_i, \xi(\mathbf{x})) \quad \text{and} \quad x_i = x_i(s_i, \rho(\mathbf{s})), \tag{30}$$

where $\partial s_i/\partial x_i$, $\partial x_i/\partial p_i < 0$ and $\xi(\mathbf{x})$ and $\rho(\mathbf{s})$ are common functions ("aggregators") of respectively quantities and prices. Notice that we can write $s_i = x_i^{-1}(x_i; \xi(\mathbf{x}))$ with $\xi(\mathbf{x}) = \rho(\mathbf{s}(\mathbf{x}))$, so that $s_i(\cdot)$ is the partial inverse of $x_i(\cdot)$ with respect to its first argument.

Gorman (1970, 1987) has shown that GAS preferences encompass an extension of additive preferences that we call "Gorman-Pollak preferences."[18] They can be represented by the utility functions:

$$U = \sum_{j=1}^{n} u_j(x_j\xi) - \phi(\xi) \text{ and } V = \sum_{j=1}^{n} v_j(s_j\rho) - \theta(\rho), \tag{31}$$

where ξ and ρ are implicitly defined by $\phi'(\xi) \equiv \sum_{j=1}^{n} u_j'(x_j\xi)x_j$ and $\theta'(\rho) \equiv \sum_{j=1}^{n} v_j'(s_j\rho)s_j$, under suitable restrictions on the good-specific sub-utilities u_j and v_j and the common functions ϕ and θ. The demand system satisfies $s_i = u_i'(x_i\xi)/\phi'(\xi)$ and $x_i = v_i'(s_i\rho)/\theta'(\rho)$.[19]

GAS preferences provide an ideal setting to study monopolistic competition, since we can naturally define it as the environment in which each firm correctly anticipates the value of the aggregators ρ and ξ, but takes ("perceives") them as given while choosing its strategy to maximize profits:

$$\pi_i = (s_i E - c_i)x_i(s_i, \rho)L = (s_i(x_i, \xi)E - c_i)x_i L. \tag{32}$$

It is important to stress that in this case the price and quantity equilibria of monopolistic competition do coincide. Since the "perceived" inverse demand of a commodity is just the inverse of the "perceived" direct demand, the corresponding elasticities:

$$\epsilon_i = \frac{\partial \ln s_i(x_i, \xi)}{\partial \ln x_i} \quad \text{and} \quad \varepsilon_i = \frac{\partial \ln x_i(s_i, \rho)}{\partial \ln s_i} \tag{33}$$

are simply related by the exact condition $\varepsilon_i = 1/\epsilon_i$, as in a monopoly. For instance, with preferences (Equation 31) these elasticities are given by $\epsilon_i = -\frac{x_i u_i''(x_i\xi)}{u_i'(x_i\xi)}$ and $\varepsilon_i = -\frac{s_i v_i''(s_i\rho)}{v_i'(s_i\rho)}$ (e.g., see Appendix B for the case of the so-called self-dual addilog preferences). This approach is entirely consistent with that adopted by Dixit and Stiglitz (1977) who suggested to neglect the impact of an individual firm on marginal utility of income (the relevant aggregator in their setting), provided that this is sufficiently small to make this behavior approximately "correct" (i.e., profit maximizing). In fact, we can prove that, provided that the market shares are negligible, the impact of a single firm on the aggregator is negligible too, and to take the aggregator as given is approximately profit maximizing for firms, since the perceived demand elasticity is approximately equal to the average Morishima measure. Formally, we have (see Appendix B for a proof):

Proposition 2 *When preferences are of the GAS type and market shares become negligible, the impact of a single firm on the aggregator vanishes and the perceived demand elasticity approximates the average Morishima elasticity.*

Accordingly, a monopolistic competition equilibrium where firms take aggregators as given approximates the imperfect competition equilibria of Section 1, which in this sense do "converge," when market shares become negligible. The conditions for profit maximization of Equation (32) taking as given either ρ or ξ define a system of pricing or production rules as:

$$p_i = \underline{p}_i(c_i, \rho) \quad \text{and} \quad x_i = \underline{x}_i(c_i, \xi). \tag{34}$$

These rules, together with the budget constraint $\sum_j p_j x_j = E$ and the assumption that firms correctly anticipate the actual demands, can be used to derive the equilibrium value of the aggregators as a function of the marginal cost vector $\mathbf{c}$ and of expenditure E, and therefore the equilibrium prices $\widehat{p}_i(\mathbf{c}, E)$ and quantities $\widehat{x}_i(\mathbf{c}, E)$.

Moreover, under GAS preferences entry decisions can be studied by assuming that in a free entry equilibrium *each firm decides to enter taking as given the relevant aggregator*, which is approximatively correct when market share are negligible (due to Proposition 2). To make further progress in the analysis of free entry, we focus on the simpler case of additive preferences,[20] for which we are able to characterize which goods are going to be provided, and the selection effects associated to changes in market size (i.e., opening up to free trade), expenditure (i.e., a demand shock), aggregate productivity (i.e., technological growth) and preference parameters.

4.1 | DA preferences

DA preferences can be represented by a direct utility that is additive as in:

$$U = \sum_{j=1}^{n} u_j(x_j), \tag{35}$$

where the sub-utility functions u_j are potentially all different but always increasing and concave. The inverse demand system is given by

$$s_i(x_i, \xi(\mathbf{x})) = \frac{u_i'(x_i)}{\xi(\mathbf{x})},$$

where $\xi = \tilde{\mu} = \sum_j x_j u_j'$ and $x_i(s_i, \rho) = u_i'^{-1}(s_i \xi)$. These preferences clearly belong to the GAS type, and were originally used by Dixit and Stiglitz (1977: Section II) in the symmetric version with $u_j(x) = u(x)$ for all j.[21] We can express the variable profits of firm i as:

$$\pi_i = \left[\frac{u_i'(x_i)E}{\xi} - c_i\right] x_i L. \tag{36}$$

The profit-maximizing condition with respect to x_i, taking ξ as given, can be rearranged in the pricing conditions:

$$p_i(x_i) = \frac{c_i}{1 - \epsilon_i(x_i)}, \quad i = 1, 2, ..., n, \tag{37}$$

where $p_i(x_i) = u_i'(x_i)E/\xi$ and we define the elasticity of the marginal subutility $\epsilon_i(x) \equiv -x u_i''(x)/u_i'(x)$, which corresponds to the elasticity of the inverse demand $s_i(x, \xi)$ for given ξ. In this case, ϵ_i is also the MEC ϵ_{ij} between good i and any other good $j \neq i$, therefore it coincides also with the average MEC $\bar{\epsilon}_i$ discussed in Section 2. In general, the markups can either increase or decrease in the consumption, depending on whether $\epsilon_i(x)$ is increasing or decreasing.

Given a set of n active firms, a monopolistic competition equilibrium is a vector $(\mathbf{x}, \xi)$ that satisfies the $n+1$ equations $u_i'(x_i)E = \xi c_i/[1 - \epsilon_i(x_i)]$ for each $i = 1, .., n$ and $\xi = \sum_j x_j u_j'(x_j)$. Asymmetries of preferences and costs complicate its derivation because the quantity of each good depends on the quantities of all the other goods through the inverse demand system. However, under assumptions that guarantee that the profit-maximization problem is well defined for all firms and any value of $c_i \xi / E$ (essentially, assuming that all marginal revenues are positive and decreasing), in Appendix C we show that it exists a unique equilibrium. Formally, we have:

Proposition 3 *Assume that preferences are DA and that* $r_i'(x) > 0 > r_i''(x)$, *where* $r_i(x) \equiv x u_i'(x)$, *with* $\lim_{x \to 0} r_i'(x) = \infty$ *and* $\lim_{x \to \infty} r_i'(x) = 0$ *for* $i = 1, 2, ..., n$. *Then for that set of firms it exists a unique equilibrium of monopolistic competition pricing.*

We can easily study the comparative statics of this equilibrium. In particular, an increase in the expenditure level E increases all quantities, and raises the markup of firm i if and only if $\epsilon_i'(x) > 0$: this allows one to obtain different forms of "pricing to market" for different goods depending on their MEC functions. A rise of the marginal cost c_i decreases the quantity x_i, inducing an "incomplete pass-through" on the price of firm i if and only if its MEC is increasing. Also the indirect effect on the markups of the other firms (taking place through the change of the aggregator) depends on whether their MECs are increasing or decreasing. Finally, when a new good is introduced in the market through entry of an additional firm (for given E and L), the production of all other commodities decreases and therefore the markup of a firm decreases if and only if its MEC is increasing.

4.1.1 | Examples

Power sub-utility

A simple case of DA preferences is based on the sub-utility power function:

$$u_i(x_i) = \tilde{q}_i x_i^{1-\epsilon_i}, \tag{38}$$

where both the MEC $\epsilon_i \in [0, 1)$ and the shift parameters $\tilde{q}_i > 0$ can differ among goods. These preferences are a special instance of the "direct addilog" preferences discussed by Houthakker (1960). They are neither CES nor homothetic unless the exponents are all identical.[22] Under monopolistic competition, since the MECs are constant, markups are also constant and different across firms, and the equilibrium prices are:

$$\widehat{p}_i = \frac{c_i}{1-\epsilon_i}, \tag{39}$$

which shows a full pass-through of changes in the marginal cost and independence from the pricing behavior of competitors and the expenditure level.

Stone–Geary sub-utility

Consider a simple version of the well-known Stone–Geary preferences (see Geary, 1950-51; Stone, 1954) where:

$$u_i(x_i) = \log(x_i + \bar{x}_i), \tag{40}$$

with every $\bar{x}_i$ positive but small enough to insure a positive demand.[23] Solving for the elasticity of the perceived inverse demand, we get $\epsilon_i(x) = x/(x+\bar{x}_i)$ and then the pricing condition:

$$p_i(x_i) = c_i\left(1+\frac{x_i}{\bar{x}_i}\right).$$

The right-hand side is decreasing in $\bar{x}_i$ because a higher value of it increases demand elasticity. However, the equilibrium price of each firm cannot be derived independently from the behavior of competitors: the interdependence between firms created by demand conditions requires the following computation. By the Hotelling-Wold's identity we have:

$$s_i(x_i, \xi) = \frac{1}{(x_i+\bar{x}_i)\xi},$$

where $\xi = \sum_j x_j/(x_j+\bar{x}_j)$. Combining this with the pricing condition, we can compute the quantity $x_i = \sqrt{\bar{x}_i E/(c_i \xi)} - \bar{x}_i$ and the price rule $s_i = \sqrt{c_i/(\bar{x}_i E \xi)}$ for firm i. Defining $\Psi = \sum_{j=1}^n \sqrt{\bar{x}_j c_j}$ and using the adding up constraint we obtain the condition $n/\xi - \Psi/\sqrt{E\xi} = 1$, which can be solved for the equilibrium value of the aggregator:

$$\widehat{\xi} = \frac{\left[\sqrt{\Psi^2+4nE}-\Psi\right]^2}{4E}.$$

Replacing $\widehat{\xi}$ in the price rule, we finally get the closed-form solution for the monopolistic competition price of any firm i:

$$\widehat{p}_i = \frac{2E\sqrt{\frac{c_i}{\bar{x}_i}}}{\sqrt{\Psi^2+4nE}-\Psi}. \tag{41}$$

In this example, the price of each firm i increases less than proportionally in its marginal cost c_i and decreases in the preference parameter $\bar{x}_i$ (which reduces the relevant MEC). Moreover, an increase in expenditure raises the markup of each good less than proportionally. Note that each price is increasing in Ψ, therefore an increase of the marginal cost c_j of a competitor or an increase of the preference parameter $\bar{x}_j$ (which reduces the associated marginal utility) induce, albeit indirectly, a small increase in the markup of firm i.

4.1.2 | Entry

Let us now move to the question of which set of goods is actually introduced in the market under monopolistic competition, and of its comparative statistics. Firm i can survive in a monopolistic competition equilibrium only if $\pi_i \geq F_i$ or, using Equation (36), if:

$$\frac{E}{\widehat{\xi}} \geq \frac{c_i \widehat{x}_i + F_i/L}{\widehat{x}_i u_i'(\widehat{x}_i)}.$$

Extending the approach of Spence (1976) we can (inversely) rank firms according to their "survival coefficient":

$$S_i \equiv \underset{x_i}{Min}\left\{\frac{c_i x_i + F_i/L}{x_i u_i'(x_i)}\right\}, \tag{42}$$

and show (see Appendix C) that the free-entry equilibrium is essentially unique (i.e., assuming that firms differ according to their coefficients S_i).[24] The survival coefficient is computed at the survival quantity level that minimizes the ratio between average cost and average revenue, and therefore it is only a function of exogenous parameters concerning technology (c_i and F_i) and preferences (depending on the subutility) as well as of market size (L), and captures the ability to survive in the market, allowing us to identify exactly the set of active firms in equilibrium. Only firms with the smallest coefficients can survive in a market equilibrium. We can therefore think of the survival ranking as determining the actual sequence of market introduction, which allows us to study how entry is affected by a change of market size, expenditure, aggregate productivity, and also different preferences parameters. The main result concerning the free-entry equilibrium of monopolistic competition is the following:

Proposition 4 *When preferences are DA, the identity of the goods provided in the free entry equilibrium of monopolistic competition is uniquely determined, and an increase of the market size or a fall of productivity favor firms with the largest values of the MEC (computed at survival quantity), while a change of expenditure is neutral on the survival ranking.*

With the expression "to favor" we refer to improvements of the survival ranking (which apply to the marginal firm selected by the market to be active at the equilibrium), while the referred values of the MECs are computed at the quantities which define the survival coefficients (therefore again depending only on exogenous parameters).

The rationale for these results is the following. An increase of the market size (for given individual quantities) increases profitability more for firms that have high markups due to a less elastic demand, which makes them relatively more likely to enter. Analogously, and increase of aggregate productivity increases proportionally more the profits of the firms that start with lower markups because they face a more elastic demand, while a fall of productivity favors firms facing a less elastic demand. Finally, the neutrality of expenditure relies on the fact that this has a proportional impact on the revenue of all firms: therefore an expansion of demand attracts new firms in the market, but without altering their survival ability.[25]

Our examples provide an illustration of these results and of the impact of exogenous preference parameters. The power sub-utility delivers the survival coefficients:

$$S_i = \left(\frac{F_i}{L}\right)^{\epsilon_i} \frac{(1-\epsilon_i)^{\epsilon_i - 2}}{\tilde{q}_i c_i^{\epsilon_i - 1} \epsilon_i^{\epsilon_i}},$$

and we remind the reader that only the firms with the smallest values of these coefficients can be active in the market. A larger market size promotes entry (by decreasing all survival coefficients), but favors the introduction of goods with a less elastic demand, that is those with a high ϵ_i (since the impact of market size is stronger). In the Stone–Geary case, the survival coefficient can be computed as:

$$S_i = \left(\sqrt{\frac{F_i}{L}} + \sqrt{\bar{x}_i c_i}\right)^2$$

defined at $\tilde{x}_i = \sqrt{\bar{x}_i F_i / c_i L}$, with $\epsilon_i(\tilde{x}_i) = \left[1 + \sqrt{\bar{x}_i c_i L / F_i}\right]^{-1}$. Here, it is the combination of preference and cost parameters that affects the survival index and the relevant elasticity. In particular, a larger market size favors entry of goods with the lowest value of $\bar{x}_i c_i / F_i$, which are the goods produced with high fixed costs and low marginal costs and facing a less elastic demand (due to a low $\bar{x}_i$).

4.2 | IA preferences

Preferences with an indirect utility that is additive (IA preferences) can be represented by:

$$V = \sum_{j=1}^{n} v_j(s_j), \tag{43}$$

with sub-utilities v_j decreasing and convex (Houthakker, 1960). Elsewhere we have used the symmetric version of these preferences for the analysis of monopolistic competition.[26] The direct demand function is given by:

$$x_i(s_i, \rho(\mathbf{s})) = \frac{|v_i'(s_i)|}{\rho(\mathbf{s})},$$

with $\rho = |\mu| = -\sum_{j=1}^{n} s_j v_j'$ and $s_i(x_i, \xi) = v_i'^{-1}(-x_i\rho)$, which confirms that these preferences belong to the GAS type. Here, we can express the variable profits of firm i as:

$$\pi_i = \frac{(p_i - c_i)|v_i'(p_i/E)|L}{\rho}. \tag{44}$$

For a given value of price aggregator ρ the elasticity of perceived demand $x_i(s_i, \rho)$ is given by $\varepsilon_i(s) = -sv_i''(s)/v_i'(s)$, which is also the MES ε_{ij} between goods i and j $(i \neq j)$ and thus coincides with the average MES $\bar{\varepsilon}_i$ of Section 2. The profit-maximizing price for each firm is then given by the solution to the price condition:

$$p_i = \frac{\varepsilon_i(p_i/E)c_i}{\varepsilon_i(p_i/E) - 1}, \quad i = 1, 2, \ldots, n \tag{45}$$

which requires $\varepsilon_i > 1$. Under weak conditions, a solution to Equation (45) exists, and it is unique under the assumption that $2\varepsilon_i > -s_i v_i'''/v_i''$ for any i, which ensures that the second-order condition for profit maximization is satisfied.

Remarkably, each condition (Equation 45) is now sufficient to determine the monopolistic competition price of each firm in function of its own marginal cost and consumers' expenditure. This means that for the entire class of IA preferences each firm i can choose its price $\widehat{p}_i(c_i, E)$ independently from the behavior of competitors, as well as from their cost conditions or from parameters concerning their goods (e.g., from their "qualities"). An increase of expenditure increases the price of a good, and changes in its marginal cost are undershifted on the price if and only if the MES is increasing, which means that the demand is perceived as less elastic when expenditure is higher. These prices, for a set of n firms, together with the corresponding value of the aggregator, provide an equilibrium of monopolistic competition. Formally, this is a vector $(\mathbf{s}, \rho)$ that satisfies the $n+1$ equations $s_i E = \varepsilon_i(s_i)c_i/[\varepsilon_i(s_i) - 1]$ for each $i = 1, .., n$ and $\rho = -\sum_j s_j v_j'(v_j)$. In Appendix D, we prove the following:

Proposition 5 *Assume that preferences are IA, that* $2\varepsilon_i(s) > -sv_i'''(s)/v_i''(s)$ *and that a solution to the profit maximization problem exists for each member of a given set of firms* $i = 1, 2, \ldots, n$ *and for any value of* c_i *and* E. *Then for that set of firms it exists a unique equilibrium of monopolistic competition pricing.*

All the equilibrium quantities (and the other firm-level variables, such as sales and profits) as well as welfare measures can then be recovered from the direct demand functions. For this reason, this class of preferences can be naturally employed in applied industrial organization (and trade) models, whereas the effects of differential (trade) costs, qualities and demand elasticities can be empirically assessed. A natural outcome of this environment is that goods of higher quality or lower substitutability generate higher revenues in a given market and therefore are more likely to be sold in more distant countries. Similar *Alchian-Allen effects* ("shipping the good apples out") have been explored in recent works by Baldwin and Harrigan (2011), Crozet et al. (2012), Feenstra and Romalis (2014) and others, but always retaining a CES structure that generates identical markups across goods. The IA class allows us to move easily beyond the case of common markups, and to endogenize quality differentiation across firms and across destinations within firms, whose empirical relevance has been pointed out in Manova and Zhang (2012).

4.2.1 | Examples

Power sub-utility
Consider a power sub-utility as:

$$v_i(s_i) = q_i s_i^{1-\varepsilon_i}, \tag{46}$$

where heterogeneity derives from the shift parameter $q_i > 0$ and the constant MES parameter $\varepsilon_i > 1$, implying that preferences are neither CES nor homothetic (unless all the exponents are identical).[27] The pricing of firm i under monopolistic competition is immediately derived as:

$$\hat{p}_i = \frac{\varepsilon_i c_i}{\varepsilon_i - 1}, \tag{47}$$

which implies again full pass-through of changes of the marginal cost. It is straightforward to derive the equilibrium quantity (for a given set of active firms):

$$\hat{x}_i = \frac{q_i \left[\frac{(\varepsilon_i - 1)E}{c_i \varepsilon_i}\right]^{\varepsilon_i}}{\sum_{j=1}^n q_j \left[\frac{(\varepsilon_j - 1)E}{\varepsilon_j c_j}\right]^{\varepsilon_j - 1}},$$

and consequently sales and profits. Clearly, q_i is a shift parameter capturing the quality of good i, that leaves unchanged the price but increases profit by increasing sales. The relative productions, sales and profits of firms depend on the relative quality of their goods, on their cost efficiency and demand elasticity, and on the level of expenditure in simple ways that can be exploited in empirical work. We can also solve for the associated equilibrium welfare as:

$$\hat{V} = \sum_{j=1}^n \frac{q_j (\varepsilon_j c_j)^{1-\varepsilon_j} E^{\varepsilon_j - 1}}{(\varepsilon_j - 1)^{2-\varepsilon_j}},$$

which allows one to analyze the welfare impact of any parameter change.

Translated power sub-utility
Consider the following sub-utility:

$$v_i(s) = \frac{(a_i - s)^{1+\gamma_i}}{1 + \gamma_i}, \tag{48}$$

with willingness-to-pay parameter $a_i > 0$, such that $v_i(s) = 0$ if $s > a_i$, and with $\gamma_i > 0$. It delivers simple perceived demand functions, including the case of a linear demand (for $\gamma_i = 1$) and the limit cases of a perfectly rigid demand ($\gamma_i \to 0$) and a perfectly elastic demand ($\gamma_i \to \infty$). These preferences have been recently applied by Bertoletti et al. (2018) and Macedoni and Weinberger (2021) to study the welfare impact of trade liberalization and quality regulation in multicountry models with heterogeneous firms. Since the MES for good i is $\varepsilon_i(s) = \gamma_i s/(a_i - s)$, the price of firm i can be computed as:

$$\hat{p}_i = \frac{a_i E + \gamma_i c_i}{1 + \gamma_i}, \tag{49}$$

which shows incomplete pass-through of marginal cost changes (parametrized by the firm-specific parameter γ_i) and markups increasing in the intensity of preference for each good (as captured by willingness-to-pay a_i) and in the expenditure level.

4.2.2 | Entry

Contrary to what happens under alternative preferences, under IA the entry of a new firm does not change the prices of the pre-existing goods, but just reduces their production (through an increase of ρ). Firm i can survive in a monopolistic competition equilibrium only if $\pi_i \geq F_i$, that is, from Equation (44) only if:

$$\frac{L}{\rho} \geq \frac{F_i}{(p_i - c_i)\left[-v_i'(p_i/E)\right]}.$$

This allows us to characterize an essentially unique free entry equilibrium in terms of a survival ranking, which is simply given by an index of relative (total) profitability:

$$\tilde{S}_i = \underset{p_i}{Min}\left\{\frac{F_i}{v_i'(p_i/E)(c_i - p_i)}\right\}. \tag{50}$$

that is again a function of exogenous technological and preference parameters as well as income (uniqueness requires that firms differ according to their coefficients $\tilde{S}_i$). In this case, the survival coefficient $\tilde{S}_i$ is defined at the equilibrium price of monopolistic competition $\widehat{p}_i$ which is the relevant one to survive in the market. Accordingly, the introduction of commodities follows this ranking, in the sense that goods with a lower value of $\tilde{S}_i$ are always more profitable and are introduced before others. In Appendix D, we prove:

Proposition 6 *When preferences are IA, the identity of the goods provided in the free entry equilibrium of monopolistic competition is uniquely determined, and an increase of the expenditure level or a rise of productivity favors firms with the largest values of the MES (computed at equilibrium prices), while an increase of market size is neutral on the survival ranking.*

The intuition is the following. An increase of expenditure exerts its impact (for given prices) through an expansion of the demand size, which is stronger for firms facing a more elastic demand, which therefore manage to expand more their gross profits and are more likely to cover the entry costs. Analogously, an increase of aggregate productivity has an impact on unit profitability (for given prices and given demand) that is larger for firms starting with lower markups and therefore facing a more elastic demand, which again induces a larger expansion of the gross profits. Finally, an increase of market size does not affect prices or individual demand and has a proportional impact on all firms without altering the sequence of entry.[28] These results should be contrasted with those obtained under direct additivity, stressing the crucial role of preferences in driving entry patterns.

Again we can use our examples to illustrate the fact that a positive shock to the expenditure level favors firms facing relatively more elastic demands. The power sub-utility delivers the survival coefficients:

$$\tilde{S}_i = F_i\left(\frac{\varepsilon_i}{E}\right)^{\varepsilon_i}\frac{c_i^{\varepsilon_i - 1}}{q_i}(\varepsilon_i - 1)^{-\varepsilon_i},$$

and we remind the reader that a lower coefficient makes it easier to survive. In this case, a higher expenditure favors firms with the higher parameters ε_i, that is facing a more elastic demand. The translated power sub-utility provides:

$$\tilde{S}_i = F_i\left(\frac{E}{\gamma_i}\right)^{\gamma_i}\left(\frac{1+\gamma_i}{a_i E - c_i}\right)^{1+\gamma_i},$$

and higher expenditure favors goods with the largest values of the equilibrium MES, which can be computed here as $\varepsilon_i(\widehat{s}_i) = \frac{a_i E + \gamma_i c_i}{a_i E - c_i}$ and depends on all the exogenous preference parameters and income. Since this value is increasing in c_i and γ_i, and decreasing with respect to a_i, an income expansion favors entry of goods produced with a high marginal cost, characterized by a relatively low willingness to pay of consumers, and facing a highly elastic demand.

4.3 | Multiple aggregators

We conclude by noting that in principle the approach to monopolistic competition that we have explored when preferences are separable can be extended to other cases in which each demand function depends on multiple aggregators, generalizing Equation (30) as in:

$$s_i = s_i(x_i, \boldsymbol{\xi}(\mathbf{x})) \quad \text{and} \quad x_i = x_i(s_i, \boldsymbol{\rho}(\mathbf{s})),$$

where $\boldsymbol{\xi}$ and $\boldsymbol{\rho}$ are now vectors. In fact, the associated procedure to determine the equilibrium can be applied to any system of well defined "perceived" demands as soon as the alleged behavioral rules (based on the perceived demand elasticities) are consistent with the demand system, so that firms can be seen as correctly anticipating the actual demands. However, to argue that taking all aggregators as given is approximately profit-maximizing for firms, one has to verify that when the market shares become negligible the perceived demand elasticity does converge to the relevant Morishima measure (this basically requires that the impact of each firm on the aggregators vanishes).

A type of preferences generating demand systems with two aggregators is given by the implicitly additive preferences of Hanoch (1975), for which we can easily extend the convergence result of Proposition 2.[29] Other examples include preferences for which the marginal utility of each good is separable, as in the generalization of the preferences of Melitz and Ottaviano (2008) that we present in Appendix E, and a restricted version of the preferences of Deaton and Muellbauer (1980) generating the AIDS, that we present in Appendix F: in both cases we show how one can apply the methodology of this section in the presence of multiple aggregators.

5 | CONCLUSION

We have analyzed imperfect competition when consumers have asymmetric preferences over many differentiated commodities and firms are heterogeneous in costs. Defining monopolistic competition as the market structure which arises when market shares are negligible, we have obtained a well-defined and workable characterization of monopolistic competition pricing. Moreover, we have presented a simple and consistent approach to the functioning of a market with monopolistic competition when demand functions depend on common aggregators. Notice, however, that in this work we have assumed throughout an exogenously given individual expenditure level. In our setting one can safely take the wage as the *numéraire*, but in fully-fledged general equilibrium applications it would also be necessary to account for the redistribution of profits to consumers, so that total income would then be given by labor income plus the per-capita aggregate profit. This can be done by adding the assumption, in the tradition of monopolistic competition, that firms also ignore the impact of their choices on consumers' income, that is that they neglect the so-called "Ford effect" discussed in the oligopoly literature (see Azar & Vives, 2021 and Neary, 2003).

Our main message is that changes of aggregate productivity due to technological shocks or growth, changes of expenditure due to demand shocks and changes of the market size affect not only the distribution of markups of the goods provided in free-entry equilibria, but even the same identity of the goods that are provided. For the case of additive preferences, we have been able to determine the exact direction of these selection effects, which could inform future empirical research. Our approach can be usefully applied in industrial organization to study the introduction and pricing of multiple products and services with interdependent demands. The research on the impact of market size on entry (Bresnahan & Reiss, 1987; Campbell & Hopenhayn, 2005) usually accounts for heterogeneity on the demand and cost size but without providing a theoretical microfoundation (for instance of the differential impact of consumers' income and market size) that should drive empirical research. Instead, most of the recent research on trade with heterogeneous firms is actually based on symmetric preferences (since Melitz, 2003), which is hardly realistic, especially to analyze empirical issues. And the macroeconomic applications of monopolistic competition have mostly focused on symmetric homothetic aggregators (as in Woodford, 2003). Building on more general microfoundations of imperfect competition would allow one to examine markup variability among goods, across markets and over time, and its influence on welfare in industrial organization analysis as well as in other fields.

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ENDNOTES

[1] Chamberlin (1933) defined monopolistic competition with reference to factors affecting the shape of the demand curve, and certainly did not intend to limit his analysis to the case of symmetric goods. He saw no discontinuity between its own market theory and the theory of monopoly as familiarly conceived, claiming inter alia that "monopolistic competition embraces the whole theory of monopoly. But it also looks beyond, and considers the interrelations, wherever they exist, between monopolists who are in some appreciable degree of competition with each other." (Chamberlin, 1937, pp. 571–572).

[2] See also the seminal work of Spence (1976), who explicitly deals with the problem of product selection, focusing on quasi-linear preferences.

[3] Important empirical works on partial equilibrium entry in function of the market size include Bresnahan and Reiss (1987) and Campbell and Hopenhayn (2005). See also Cosman and Schiff (2019) on a market closer to monopolistic competition such as restaurants in New York City.

[4] The Morishima Elasticity of Substitution was originally proposed by Morishima (1967). We also employ the related concept of the Morishima Elasticity of Complementarity.

[5] See also Mrázová and Neary (2019) on selection effects with heterogeneous firms and Hottman et al. (2016) for an empirical approach based on a nested-CES utility system.

[6] See Blackorby and Russell (1981) on the corresponding *net* measure which applies to compensated demands. The larger is ϵ_{ij}, the smaller is the possibility of good j to substitute for good i. Notice that $\epsilon_{ii} = 0$ and that in general $\epsilon_{ij} \neq \epsilon_{ji}$ for $i \neq j$.

[7] See Blackorby and Russell (1981) and Blackorby et al. (2007) for a comparison between alternative measures of substitutability between goods. The higher is ε_{ij} the greater is the possibility of good j to substitute for good i. Notice that $\varepsilon_{ii} = 0$ and that in general $\varepsilon_{ij} \neq \varepsilon_{ji}$ for $i \neq j$.

[8] Sufficient conditions on preferences to deliver this result are studied in Vives (1987).

[9] Notice that, in general, the value of these cross demand elasticities need not be negligible in a strategic setting. In fact, if they were null there would be no reason for strategic interaction and we could think of those producers as "isolated monopolists."

[10] One can also consider an entry process *à la* Melitz (2003) that exhausts *expected* profits: given an ex ante probability distribution over parameters indexing the goods, firms would enter the market until they expect profits to cover the entry cost. This would leave unchanged the competition stage whenever costs and market size attract a number of firms large enough to justify the assumption of negligible market shares.

[11] A social planner maximizing utility under a resource constraint $EL = \sum_{j=1}^{n}(c_j x_j L + F_j)$ would set a common markup on all goods: this implies that a market equilibrium tends to provide too much of the goods with a low average MES. Without loss of generality, the optimal prices can be set at the marginal costs when the fixed costs are directly paid out of individual expenditure. Accordingly, the social planner chooses the set Γ^* of goods solving:

$$\max_{\Gamma \subseteq \Omega} V\left(\underset{i \in \Gamma}{s_i} = \frac{c_i}{E - \sum_{i \in \Gamma} F_i / L}, \underset{i \notin \Gamma}{s_i} = \infty \right).$$

As long as unproduced goods become less costly (or either E or L or A increases) they can enter the set of optimally provided goods. However, there is no general reason why the market should be expected to either provide the optimal set of goods or to introduce them in the optimal order (see Spence, 1976, on some special cases).

[12] In Section 3, we will show that the set $\widehat{\Gamma}$ is essentially unique under additive (encompassing CES) preferences.

[13] See Feenstra (2003) on translog preferences, and Feenstra (2018) for their generalization to the case of the so-called "quadratic mean of order r" (QMOR) preferences with heterogeneous firms.

[14] When preferences are homothetic and symmetric, this also implies that Morishima elasticities and markups in a symmetric equilibrium can be at most a function of the number of goods. While this result has been used elsewhere (for instance in Bilbiie et al., 2012), we are not aware of a formal proof (we are thankful to Mordecai Kurz for pointing this out).

[15] They belong to the more general, so-called QMOR class: see, for example, Feenstra (2018).

16 Notice that in the special, fully symmetric case with $a_{ij} = a > 0$ and $x_i = x$ for $i, j = 1, .., n$, one gets $\epsilon_{ij} = 1/2$.

17 For an application to endogenous growth models *à la* Romer (1990) departing from CES production function and allowing for general technologies see Etro (2020).

18 GAS preferences also include the class of implicit CES preferences (Blackorby & Russell, 1981; Hanoch, 1975). See Bertoletti and Etro (2021) for a discussion.

19 These preferences are *homothetic* when $\phi(\xi) = \ln \xi$ and $\theta(\rho) = -\ln \rho$, a case which covers the GAS demand systems investigated in Matsuyama and Ushchev (2017). They are *directly additive* when $\theta(\rho) = -\rho$ (so that $\xi = 1$). Finally, they are *indirectly additive* when $\phi(\xi) = \xi$ (so that $\rho = 1$). These functional forms have to satisfy further regularity conditions explored by Fally (2020).

20 However, an anonymous referee has suggested an interesting extention of our free entry analysis to the general GAS preferences under a few regularity conditions.

21 For a further analysis of symmetric DA preferences see Zhelobodko et al. (2012), as well as Arkolakis et al. (2019) and Bertoletti and Epifani (2014) for applications to trade, and Cavallari and Etro (2020) and Latzer et al. (2020) for applications to macroeconomics.

22 They have been often used in applications with perfect competition. Dhrymes and Kurz (1964) is an early example of these functional forms as production technologies. More recently, Fieler (2011) has used them as utility functions in a trade model.

23 Simonovska (2015) has recently used a symmetric version of these preferences to study monopolistic competition among heterogeneous firms.

24 If two firms share the same survival coefficient they may be active in two equilibria which are trivially different.

25 This provides a rationale for the results on the selection effects of globalization derived by Bertoletti and Epifani (2014) and Zhelobodko et al. (2012) in a setting with symmetric goods and firms with heterogeneous marginal costs. They show that a market size increase has an impact on efficiency which depends on whether the MEC is increasing or decreasing. In fact, in that setting the firms facing the largest MECs are either the most or the least efficient firms according to the MEC being either decreasing or increasing in consumption.

26 Also see Bertoletti et al. (2018) and Macedoni and Weinberger (2021) for applications to trade and Anderson et al. (2018) and Boucekkine et al. (2017) for applications to macroeconomics.

27 This generalization of the CES case is a special instance of the "indirect addilog" preferences of Houthakker (1960) and differs from the one based on DA power sub-utilities (presented in Section 4.1.1).

28 This provides a rationale for results derived by Bertoletti and Etro (2017) in a setting with symmetric goods and firms with heterogeneous marginal costs. They show that an increase of expenditure has an impact on efficiency which depends on whether the MES is increasing or decreasing. In fact, the firms facing the largest MESs are in that setting either the least or the most efficient firms according to the MES being increasing or decreasing.

29 The implicitly additive preferences include also the implicit CES preferences generating one aggregator (whose application to monopolistic competition and optimality properties have been explored in Bertoletti and Etro [2021]), and others employed in applications by Feenstra and Romalis (2014), Kimball (1995), and Matsuyama (2019). See also Fally (2020) for futher analysis of preferences generating two aggregators.

30 This formally assumes that not all the demand own elasticities *and* the quantity aggregator are too small.

31 Since $(h \neq i \neq j)$

$$\epsilon_{ij} - \epsilon_{ih} = \left[\frac{\partial \ln s_j(x_j, \xi(\mathbf{x}))}{\partial \ln \xi} - \frac{\partial \ln s_h(x_i, \xi(\mathbf{x}))}{\partial \ln \xi} \right] \frac{\partial \ln \xi(\mathbf{x})}{\partial \ln x_i},$$

from Equation (16) cross demand effects are approximately zero when market shares are negligible, unless the own demand elasticities are indeed large.

32 Sufficient but not necessary conditions are $a_i E > c_i$ and $\lim_{s \to a_i} \theta_i(s_i) > a_i E / (a_i E - c_i)$, where $a_i > 0$ is a (finite or infinite) choke off (normalized) price for which $v_i(s) = 0$ for $s \geq a_i$, and $\lim_{s \to a_i} v_i(s) = \lim_{s \to a_i} v_i'(s) = 0$, $i = 1, 2, ..., n$.

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APPENDIX

A | Monopolistic competition with homothetic preferences

Proof of Proposition 1: Due to the homogeneity of degree zero of both the average MES $\bar{\varepsilon}(\mathbf{p})$ and the market share $b_i = p_i P_i(\mathbf{p}) EL/P(\mathbf{p})$, changes in aggregate productivity A do not alter the monopolistic competition profits of a given set of firms, given by Equation (29). Since A affects neither the equilibrium profits of the active firms, nor the profit that each other firm may get by entering the market, it does not affect the free-entry equilibrium set $\widehat{\Gamma}$, which on the contrary possibly depends on EL. □

B | Monopolistic competition with GAS preferences

Proof of Proposition 2: Taking as given the relevant aggregator of GAS preferences, firms compute the perceived (inverse) demand elasticity according to $\epsilon_i = -\partial \ln s_i(x_i, \xi)/\partial \ln x_i$ in Equation (33). We now show that, when market shares are negligible, to take the aggregator ξ as given approximately coincides with using the average Morishima measures as the relevant demand elasticities, and is thus approximately profit maximizing. Let us start by computing the MEC between commodities i and j $(i \neq j)$:

$$\begin{aligned}\epsilon_{ij} &= -\frac{\partial \ln\{s_i(\mathbf{x})/s_j(\mathbf{x})\}}{\partial \ln x_i} \\ &= \left[\frac{\partial \ln s_j(x_j, \xi(\mathbf{x}))}{\partial \ln \xi} - \frac{\partial \ln s_i(x_i, \xi(\mathbf{x}))}{\partial \ln \xi}\right]\frac{\partial \ln \xi(\mathbf{x})}{\partial \ln x_i} - \frac{\partial \ln s_i(x_i, \xi(\mathbf{x}))}{\partial \ln x_i}.\end{aligned}$$

This implies that the average MEC is:

$$\bar{\epsilon}_i = \left[\sum_{j\neq i}\frac{\partial \ln s_j(x_j,\xi(\mathbf{x}))}{\partial \ln \xi}\frac{b_j(\mathbf{x})}{1-b_i(\mathbf{x})} - \frac{\partial \ln s_i(x_i,\xi(\mathbf{x}))}{\partial \ln \xi}\right]\frac{\partial \ln \xi(\mathbf{x})}{\partial \ln x_i} - \frac{\partial \ln s_i(x_i,\xi(\mathbf{x}))}{\partial \ln x_i}.$$

By differentiating the identity $\sum_j s_i(x_j,\xi)x_j = 1$, we can also compute:

$$\frac{\partial \ln \xi(\mathbf{x})}{\partial \ln x_i} = -\frac{\frac{\partial \ln s_i(x_i,\xi(\mathbf{x}))}{\partial \ln x_i}+1}{\sum_{j=1}^n \frac{\partial \ln s_j(x_j,\xi(\mathbf{x}))}{\partial \ln x_j}}\frac{b_i(\mathbf{x})}{\xi(\mathbf{x})^2}.$$

This shows that when $b_i \approx 0$ then $\partial \ln \xi/\partial \ln x_i \approx 0$: accordingly we have $\bar{\epsilon}_i \approx \epsilon_i \approx \epsilon_{ij}$ when $b_i \approx 0$.[30] Notice that $\bar{\epsilon}_i = \epsilon_i = \epsilon_{ij}$ even when shares are *not* negligible if both preferences and the consumption bundle (and then the price vector) are symmetric.[31] Analogously, one can derive the MES and show that with GAS preferences small market shares imply $\bar{\varepsilon}_i \approx \varepsilon_i \approx \varepsilon_{ij}$. Thus to take the aggregator ρ as given while choosing the own price is approximately correct when market shares are indeed negligible. □

Self-dual addilog preferences As an example, the family of "self-dual addilog" preferences introduced by Houthakker (1965) and investigated by Pollak (1972) belongs to the Gorman-Pollak class Equation (31). For this family of preferences the direct demand system is given by:

$$x_i(\mathbf{s}) = q_i\frac{s_i^{-\varepsilon_i}}{\rho(\mathbf{s})^{\varepsilon_i+\frac{\delta-1}{\delta}}},$$

where $q_i > 0$ is a shift parameter reflecting the quality of good i, $\varepsilon_i > 1$ governs the perceived elasticity of demand and $\rho(\mathbf{s})$ is implicitly defined by the condition $\sum_{i=1}^n q_i s_i^{1-\varepsilon_i}\rho^{\frac{1-\delta}{\delta}-\varepsilon_i} = 1$. We assume $\delta \in (0,1)$, and $\varepsilon_i \neq \varepsilon_j$ for some i and j (otherwise preferences are CES). Moreover, the inverse demand system is:

$$s_i(\mathbf{x}) = \tilde{q}_i\frac{x_i^{-\epsilon_i}}{\xi(\mathbf{x})^{\epsilon_i+\frac{\tilde{\delta}-1}{\tilde{\delta}}}},$$

where $\xi(\mathbf{x})$ is implicitly defined by the condition $\sum_{i=1}^n \tilde{q}_i x_i^{1-\epsilon_i}\xi^{\frac{1-\tilde{\delta}}{\tilde{\delta}}-\epsilon_i} = 1$, with $\epsilon_i = \frac{1}{\varepsilon_i} > 0$, $q_i = \tilde{q}_i^{\varepsilon_i}$ and $\delta = 1-\tilde{\delta}$. Pollak (1972) showed that the underlying preferences can be represented for $\delta \neq 1/2$ by:

$$U = \sum_{j=1}^n \frac{\tilde{q}_j(x_j\xi)^{1-\epsilon_j}}{1-\epsilon_j} - \frac{\tilde{\delta}\xi^{\frac{2\tilde{\delta}-1}{\tilde{\delta}}}}{2\tilde{\delta}-1} \quad \text{and} \quad V = \sum_{j=1}^n \frac{q_j(s_j\rho)^{1-\varepsilon_j}}{\varepsilon_j-1} + \frac{\delta\rho^{\frac{2\delta-1}{\delta}}}{2\delta-1}.$$

In the special case with $\delta = 1/2$ preferences are homothetic and ϕ and θ take a logarithmic form with respect to the corresponding aggregators.

Given the inverse and direct demand systems, when firms maximize profits taking as given the aggregators, we immediately obtain the following prices under monopolistic competition:

$$\hat{p}_i = \frac{c_i}{1-\epsilon_i} = \frac{\varepsilon_i c_i}{\varepsilon_i - 1},$$

where the idiosyncratic markups are constant as in our additive, power sub-utility examples (see Sections 4.1.1 and 4.2.1). In fact, we can also derive the equilibrium quantities as:

$$\hat{x}_i = \frac{q_i(\varepsilon_i-1)^{\varepsilon_i}E^{\varepsilon_i}}{c_i^{\varepsilon_i}\varepsilon_i^{\varepsilon_i}\rho(\hat{\mathbf{s}})^{\frac{\delta-1}{\delta}+\varepsilon_i}}.$$

These results make this family the natural extension of the power additive preferences. The availability of a homothetic version, with the associated well-defined price and consumption indexes, and the flexibility of the general specification provide interesting advantages for applications.

C | Monopolistic competition with DA preferences

Proof of Proposition 3: It follows immediately from the first-order condition for profit maximization rewritten as:

$$r_i'(x_i)\frac{E}{\xi} = c_i$$

that for any $c_i\xi/E$ it exists a unique profit-maximizing quantity $x_i > 0$ for any firm $i = 1, 2, ..., n$, and that this quantity decreases with respect to ξ. Moreover, it also follows from Equation (37) that:

$$p_i x_i = \frac{u_i'(x_i)x_i}{\xi}E = \frac{c_i r_i(x_i)}{r_i'(x_i)}.$$

The right-hand side of latter expression implies that the profit-maximizing revenue of a monopolistically competitive firm is increasing with respect to its equilibrium quantity, rising from zero to infinite. Since total revenue must be equal to the expenditure level E, and in such a case it holds that $\xi = \sum_j x_j u_j'(x_j)$, it follows that it exists a single value of ξ which characterizes a unique equilibrium for a given set of firms (a level of expenditure E and a vector of marginal cost $\mathbf{c}$). □

Proof of Proposition 4: Consider free entry under DA of preferences. By using Equation (36), we can write the condition of a nonnegative profit as:

$$\frac{E}{\xi} \geq \frac{c_i x_i + F_i/L}{x_i u_i'(x_i)}.$$

Following Spence (1976), let us rank firms increasingly according to their *survival coefficient* ($S_N \geq S_{N-1} \geq ... \geq S_1$):

$$S_i = \underset{x_i}{Min}\left\{\frac{c_i x_i + F_i/L}{x_i u_i'(x_i)}\right\}.$$

Notice that the ranking is independent from the values of expenditure and ξ. Assume that all firms differ according to their survival coefficients. The entry equilibrium can be described as follows: for a given E/ξ, any active firm maximizes profit by setting its Lerner index equal to the MEC ϵ_i, independently from F_i/L. This determines the whole set of quantities for the active firms, and then the aggregator ξ. After any entry, the value of aggregator ξ must increase to reduce the expenditure in the incumbent commodities, making room for the entrant and survival more difficult for all firms. In a free-entry equilibrium, all active firms get nonnegative profits, and their quantities are consistent with the value of the aggregator ξ. All the other firms do not expect a positive profit if entering the market, taking as given the equilibrium value of the aggregator ξ. It is then the case that $S_i \leq E/\xi$ for all firms $i \in \widehat{\Gamma} = \{1, .., n\}$, and $S_i \geq E/\xi$ for $i \notin \widehat{\Gamma}$. Notice that firm k cannot belong to the equilibrium set $\widehat{\Gamma}$ if firm $j < k$ does not. Differentiating S_i and using the envelope theorem, we have:

$$\frac{\partial \ln S_i}{\partial \ln L} = -\epsilon_i(\tilde{x}_i), \quad \frac{\partial \ln S_i}{\partial \ln E} = 0 \quad \text{and} \quad \frac{\partial \ln S_i}{\partial \ln A} = \epsilon_i(\tilde{x}_i) - 1,$$

where ϵ_i is evaluated at the quantity:

$$\tilde{x}_i = \frac{F_i}{c_i L}\frac{1 - \epsilon_i(\tilde{x}_i)}{\epsilon_i(\tilde{x}_i)}$$

which defines S_i. Accordingly, an increase of market size (which has a positive impact on all firms) or in a common component of the marginal cost (a reduction of productivity) alters the survival ranking favoring firms producing varieties with the largest MECs (thus facing steeper perceived demand functions), while expenditure is neutral. □

D | Monopolistic competition with IA preferences

Proof of Proposition 5: Assume that a solution to the first-order condition for profit-maximization (Equation 45) exists for any value of c_i and E,[32] and that $2\varepsilon_i(s_i) > -s_i v_i'''(s_i)/v_i''(s_i)$, for $i = 1, 2, ..., n$. Than the second-order condition condition is satisfied and there is a unique solution to Equation (45). It follows immediately that this solution for each commodity together with $\rho = -\sum_{j=1}^{n} s_j v_j'(s_i)$ characterizes a unique monopolistic competition equilibrium for a given set of firms (a level of expenditure E and a vector of marginal cost $\mathbf{c}$). □

Proof of Proposition 6: The free entry equilibrium under IA preferences can be characterized starting from the nonnegative profit condition, derived from Equation (44) as:

$$\frac{L}{\rho} \geq \frac{F_i}{(c_i - p_i)v_i'(p_i/E)},$$

Let us define the *survival coefficient*:

$$\tilde{S}_i = \underset{p_i}{Min}\left\{\frac{F_i}{(c_i - p_i)v_i'(s_i)}\right\},$$

which is proportional to the ratio of fixed cost and variable profit, and it has to be evaluated at the profit-maximizing value of p. Assume that all firms differ according to their survival coefficients. Firms can then be ranked in terms of their survival coefficient: after any entry, the value of aggregator ρ must increase, to reduce demand and expenditure on the incumbent firms, making survival more difficult for all firms. A free entry equilibrium is determined by the unique value of ρ that makes all active firms to get nonnegative profits (and an aggregate revenue equal to EL), while all other firms expect a nonpositive profit if entering the market (taking ρ as given). The market equilibrium then satisfies $\tilde{S}_i \leq L/\rho$ for all firms $i \in \widehat{\Gamma} = \{1, .., n\}$, and $\tilde{S}_i \geq L/\rho$ for $i \notin \widehat{\Gamma}$. Since

$$\frac{\partial \ln \tilde{S}_i}{\partial \ln L} = 0, \quad \frac{\partial \ln \tilde{S}_i}{\partial \ln E} = -\varepsilon_i(\widehat{s}_i) \quad \text{and} \quad \frac{\partial \ln \tilde{S}_i}{\partial \ln A} = 1 - \varepsilon_i(\widehat{s}_i)$$

an increase of expenditure or aggregate productivity unambiguously favors firms with larger MESs (evaluated at the profit-maximizing normalized price $\widehat{s}_i$), while an increase in market population is neutral on the ranking. □

E | Monopolistic competition with generalized Melitz-Ottaviano preferences

As an example of preferences for which the demand system depends on two aggregators, let us consider the direct utility:

$$U(\mathbf{x}) = \sum_{j=1}^{n} u_j(x_j) - \frac{\eta}{2}\xi(\mathbf{x})^2,$$

where $\eta \geq 0$ and $\xi = \sum_{j=1}^{n} x_j$. One can recognize here a generalization of the quadratic specification used by Melitz and Ottaviano (2008). This class of preferences satisfies the separability of the marginal utility in the quantity of a good and an aggregator $\xi(\mathbf{x})$. The perceived demand elasticity is:

$$\epsilon_i(x, \xi) = \frac{-u_i''(x)x}{u_i'(x) - \eta\xi},$$

which allows one to compute the pricing conditions. It is useful to sketch how in principle one can apply the solution procedure of Section 4 in the presence of two aggregators. Consider the example of a quadratic subutility $u_i(x) = \alpha_i x - \frac{\gamma_i}{2}x^2$ where the parameters α_i and γ_i change across goods. The pricing condition becomes:

$$p_i(x_i) = c_i\left(\frac{\alpha_i - \gamma_i x_i - \eta\xi}{\alpha_i - 2\gamma_i x_i - \eta\xi}\right),$$

and by the Hotelling-Wold's identity we have:

$$s_i(x_i, \xi, \psi) = \frac{\alpha_i - \gamma_i x_i - \eta\xi}{\psi},$$

where $\psi = \sum_j x_j\left(\alpha_j - \gamma_j x_j - \eta\xi\right)$. Thus, we can compute the quantity rules $x_i = (\alpha_i - \eta\xi - \psi c_i/E)/2\gamma_i$ and the pricing rules $s_i = (\alpha_i - \eta\xi + c_i\psi/E)/2\psi$. Using the budget constraint one can solve for the marginal utility of income $\psi(\xi)$ in function of the other aggregator and for the quantity $x_i(\xi, \psi(\xi))$. Using the definition $\xi = \sum_{j=1}^n x_j(\xi, \psi(\xi))$ we can solve for $\widehat{\xi}(\mathbf{c}, E)$ and then $\psi(\widehat{\xi}(\mathbf{c}, E))$, and derive the prices:

$$\widehat{p}_i = \frac{c_i}{2} + \frac{\left(\alpha_i - \eta\widehat{\xi}(\mathbf{c}, E)\right)E}{2\psi(\widehat{\xi}(\mathbf{c}, E))}$$

which can be expressed implicitly after replacing the aggregator. Finally, notice that with $\eta = 0$ these preferences become DA, generating prices:

$$\widehat{p}_i = \frac{c_i}{2} + \frac{\alpha_i \Upsilon}{4\left(\sqrt{E^2 + \frac{\Upsilon\Phi}{4}} - E\right)}$$

where $\Upsilon = \sum_{j=1}^n \frac{c_j^2}{\gamma_j}$ and $\Phi = \sum_{j=1}^n \frac{\alpha_j^2}{\gamma_j}$. In such a case, the price of each firm i increases less than proportionally in its marginal cost c_i as well as in the intensity of preferences for the good α_i and in the expenditure level E.

F | Monopolistic Competition with restricted *Almost Ideal Demand System* preferences

We present another example of preferences for which the demand system depends on two aggregators, based on the so-called *Almost Ideal Demand System* introduced by Deaton and Muellbauer (1980). Consider preferences represented by the following indirect utility:

$$V(\mathbf{s}) = -\frac{\rho(\mathbf{s})}{\zeta(\mathbf{s})},$$

where the aggregators are defined by $\rho \equiv \alpha_0 + \sum_j \alpha_j \ln s_j + \sum_j \sum_i \frac{\gamma_{ij}}{2} \ln s_j \ln s_i$ and $\zeta \equiv \beta_o \prod_j s_j^{\beta_j}$, and assume $\sum_j \gamma_{ij} = \sum_j \beta_j = 0$ and $\sum_{j=1}^n \alpha_j = 1$ to satisfy the regularity conditions, and $\gamma_{ij} = \gamma_{ji}$ without loss of generality. For any commodity i the marginal disutility is:

$$V_i(\mathbf{s}) = -\frac{\alpha_i + \sum_j \gamma_{ij} \ln s_j - \beta_i \rho(\mathbf{s})}{s_i \zeta(\mathbf{s})},$$

and the direct demand functions can be derived by Roy's identity as:

$$x_i(\mathbf{s}) = \frac{\alpha_i + \sum_j \gamma_{ij} \ln s_j - \beta_i \rho(\mathbf{s})}{s_i}.$$

This demand system does not depend in general just on one or two common aggregators, thus we cannot use it to study monopolistic competition as an environment where firms set prices taking aggregators as given as in Section 4. However, consider the following additional restrictions that introduce some symmetry between goods: $\gamma_{ij} = \gamma\gamma_i\gamma_j$ for $i \neq j$, $\sum_{j=1}^n \gamma_j = 1$, $\gamma_{ii} = \gamma\gamma_i(\gamma_i - 1)$ with $\gamma > 0$. In this case, the marginal disutility becomes separable in three aggregators, namely $\rho(\mathbf{s})$, $\zeta(\mathbf{s})$ and $\omega(\mathbf{s}) \equiv \sum_j \gamma_j \ln s_j$, and the direct demand functions read as:

$$x_i(s_i, \rho(\mathbf{s}), \omega(\mathbf{s})) = \frac{\alpha_i + \gamma\gamma_i[\omega(\mathbf{s}) - \ln s_i] - \beta_i \rho(\mathbf{s})}{s_i},$$

where only two aggregators remain. Notice that the aggregator ρ also disappears when $\beta_i = 0$ for any i, delivering the homothetic translog demand considered in Matsuyama and Ushchev (2017) and whose utility representation was derived in Bertoletti and Etro (2021) through the GAS functional form Equation (31). Otherwise, perceived demands and demand elasticities depend on two aggregators. Taking both of them as given, this elasticity can be computed as:

$$\varepsilon_i = 1 + \frac{\gamma\gamma_i}{b_i},$$

and it grows unboundedly when the market share becomes negligible, implying $\widehat{p}_i \approx c_i$ when $b_i \approx 0$. Of course, this outcome is consistent with what one could obtain using the average Morishima elasticities to study strategic interactions as in Section 2.