Modeling the Techno-Economic Interactions of Infrastructure and Service Providers in 5G Networks with a Multi-Leader-Follower Game

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***Abstract*—The decoupling of infrastructure from services, which has been so far a mainstream paradigm in the computational and storage domain, is now becoming a paradigm also for mobile networks. Indeed, 5G must provide a variety of services with very diverse requirements, such as throughput, latency, or reliability, and decoupling infrastructure from service provisioning allows to deal with such heterogeneity. In this context, a new business model, involving two different stakeholders, Infrastructure Providers and Service Providers, has emerged. Besides addressing the technical issues, it is also important to study the economic feasibility and behavior of such new paradigm and the techno-economic interactions among the different stakeholders that play different roles in the mobile network market. In this paper, we propose a multi-leader multi-follower variant of the Stackelberg game to model the considered environment. The proposed model is then fed with realistic data and used to analyze the system behavior and the impact of the technological features of the stakeholders on their competitiveness.**

***Index Terms*—multi-stage games, Stackelberg game, 5G, multi-tenancy, network slicing**

I. Introduction

The decoupling of infrastructure from services, a mainstream paradigm in the computational and storage domain, is now being materialized also for mobile networks with the advent of 5G. Up to date, a typical pre-5G Mobile Network Operator (MNO) owns and manages by itself the network resources (infrastructure and spectrum) and provisions services for its end users. However, over time, there has been a progressive deviation from this typical MNO business model which can be witnessed through, e.g., the Mobile Virtual Network Operator (MVNO) business model and infrastructure and/or spectrum sharing agreements among MNOs [1]–[3]. The emergence of such new business models, even prior to 5G, has been mainly driven by the need to cut down on infrastructure cost (so as to improve the return on investment) and to increase resource utilization (e.g., when scarce such as spectrum).

As for 5G networks, in addition to delivering higher throughput mobile broadband services, they are also expected to provide support for the Internet of Things and for vertical industries: the International Telecommunication Union Radio-communication Sector has identified three usage scenarios for the International Mobile Telecommunications (IMT) for 2020 and beyond [4], namely enhanced Mobile BroadBand (eMBB), Ultra-Reliable and Low Latency Communications (URLLCs) and massive Machine Type Communications (mMTCs). In these lines, unlike the previous generations, 5G networks will have to provision heterogeneous services with very distinct requirements in terms of throughput, latency, reliability, connection density, type of end user devices, etc. As a means to deal with such heterogeneity and open up the mobile network to verticals, the decoupling of network infrastructure and resources from service provisioning is considered a design principle by several entities, initiatives and research projects involved/contributing in the 5G architecture definition and standardization [5]–[7] with Network Function Virtualization and Software Defined Networking being two key technical enablers. In this context, a key 5G concept is that of network slicing [6], which allows to create logically separated networks (slices) over the set of shared physical network resources where each such slice will be tailored to the service requirements of a specific tenant (i.e., a business entity which provides eMBB/URLLC/mMTC services to end users).

Apart from the architectural aspects of 5G, there is a need to address its economic viability [5] which, by far, has been studied from the point of view of a single MNO [8]–[10]. However, one of the implications of the 5G architecture is the emergence of new stakeholders that play different roles in the mobile network market such as, e.g., infrastructure providers and mobile service providers, in addition to the so-called tenants (see, e.g., [6], [10]). The techno-economic interactions among these new stakeholders (such as resource demand and pricing, provider selection, etc.) give rise to new competitive scenarios for the mobile network market requiring suitable models to be studied, which is the object of this work.

In this paper, we devise a mathematical model to capture the technological and economic features of the considered scenarios and the techno-economic interactions among stakeholders. We feed the model with realistic technological and economic parameters describing different network configurations (either 4G or 5G) and end user services (5G usage scenarios). Then, the developed model and data are used to deeply analyze the interactions among stakeholders of the same type and those playing different roles and how their features, both technological and economic, influence their behavior and the resulting mobile market setting.

In this work, we consider two types of stakeholders: Infrastructure Providers (InPs) and Service Providers (SPs), while

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end users are represented implicitly. An InP is an entity which owns spectrum licenses, deploys and manages the infrastructure of the mobile network and rents/sells its resources to SPs, but does not provision services for end users. In turn, an SP[1] is an entity which does not own network resources but provisions services to end users through leased/acquired resources. Here the resource sold by InPs (acquired by SPs) is the cell capacity at base station (BS) level and the problem we address is the pricing of the cell capacity from the InPs' perspective and the selection of an InP from which to acquire cell capacity from the SPs' perspective.

Specifically, we consider a dense urban area where there are multiple InPs that own mobile networks and multiple SPs, each provisioning a single type of service to a given number of end users in the area. The SPs provision services for their own end users in the cell area by acquiring cell capacity from only one of the InPs (i.e., from the BS of one of the InPs) while each InP can host multiple SPs. All InPs and SPs are considered profit-maximizers, i.e., each InP offers a cell capacity unit price that maximizes its profit from the amount of cell capacity sold to SPs that select the InP, while each SP selects an InP from which to acquire cell capacity so as to maximize its profit (revenue from own users given the acquired cell capacity minus cost of the latter). As the cell capacity of each InP is fixed and finite, SPs compete among them in selecting an InP from which to acquire cell capacity, whereas InPs compete among them over the cell capacity unit prices to be selected by SPs. In this setting, we formulate the problem of cell capacity pricing from the InP perspective and InP selection from the SP perspective as a multi-leader multi-follower extension of the basic (one-leader one-follower) Stackelberg game [12]; we will refer to the proposed model as the multi-leader-follower game (MLFG) as in [13].

We have applied the proposed MLFG to several realistic scenarios in which services provisioned by SPs are inspired from usage scenarios for IMT for 2020 and beyond [4] and characterized by their respective performance requirements (such as user target rates, connection densities, etc.) as in [14], while we vary the InPs' network technology (whether 4G or 5G) and their spectrum bandwidth availability. To devise meaningful pricing strategies for the InPs across the different scenarios, we propose an InP cost model that accounts for the InP's network technology type and available spectrum bandwidth based on [10], whereas the SP revenue function is based on a noted function in literature [15] that allows to represent how the end user responds to the fee offered by its SP based on the utility achieved from resources assigned by the latter [15]. The proposed MLFG has been instrumental to derive insights concerning these scenarios. Indeed, for all the considered instances, it is possible to compute either an equilibrium or an approximation of the equilibrium. Results show that the technological features of the InPs have a significant impact on their competitiveness.

[1]An SP is equivalent to a tenant in the 5G literature terminology. For instance, in [11], a tenant is either an MVNO, a vertical industry or an Over The Top provider (OTT). In this paper, we have opted for the term SP since the focus of our work is not on the 5G architecture.

The layout of this paper is the following. In Section II, we identify and review works in the mobile networks literature which are related to ours in terms of methodology and/or in application. The proposed framework and the mathematical models behind it are presented in Section III. Then, in Section IV we explain how the framework has been applied in the context of migrating from 4G to 5G through the characterization of InPs and services provisioned by the SPs and how we set up several scenarios/problem instances for our computational tests. Numerical results concerning these problem instances are presented and analyzed in Section V, whereas conclusions are drawn in Section VI.

II. Related work

Stackelberg games are widely used in the literature to model the interaction among multiple self-interested entities in the field of resource management problems in 5G networks [16]; specific application arena include Heterogenous Networks (HetNets) [17], [18], edge caching [19], edge computing [20], device-to-device communications [21], cognitive networks [22], Cloud Radio Access Networks (C-RANs) [23].

Whilst the aforementioned work is similar to ours only in terms of the adopted methodology, the work in [24]–[32] share with ours the same context and application arena targeting the techno-economic interactions arising among multiple stakeholders of mobile radio networks. Among [24]–[32], [25]–[29] also resort to variants of the Stackelberg game.

[24] resorts to congestion games to address the problem of partitioning the RAN resources of a Telco Operator (TO) (analogous to an InP in our framework) among multiple MVNOs (analogous to SPs in our framework), each with a fixed number of users. In details, the TO's RAN consists of a set of heterogeneous Remote Radio Heads (RRHs) which the TO leases to MVNOs at a fixed RRH-specific price. Then, each MVNO decides how to distribute its own set of users over these RRHs so as to minimize its total cost. Our work differs in the following aspects: (i) in [24] MVNOs compete over a set of RRHs whereas our framework applies to the single BS and (ii) the congestion game proposed by [24] models competition only among MVNOs while the TO is not a player of the game; differently, our MLFG allows to model all involved InPs and SPs as players of the game and in particular thus capturing competition also among multiple InPs (while a single TO is considered in [24]).

In [25], an InP owns a virtualized RAN which hosts multiple MVNOs (analogous to SPs in our framework) each with a fixed number of users. The InP faces the problem of pricing and allocating its available BS resources among the users of all MVNOs so as to maximize its own profit, while satisfying Service Level Agreements signed with the MVNOs which are given in terms of a minimum number of subcarriers per MVNO and a maximum total rate over all MVNO users. The problem is formulated as a one-leader multi-follower Stackelberg game (OLMFSG) with the InP acting as the leader and MVNOs acting as followers. A single InP is considered, whereas we model competition among multiple InPs.

In [26], multiple service providers with distinct wireless access technologies (either a Wireless Metropolitan Area Network (WMAN), a cellular network or a Wireless Local Area Network (WLAN)) and fixed amount of available bandwidth compete among them over prices per unit of bandwidth to be selected by users in a common coverage area. The user sensitivity to changes in price and the user churn among service providers are incorporated in the service providers' payoff functions. The authors propose multiple formulations for the problem, among which a one-leader two-follower Stackelberg game, assuming one of the service providers announces its price before the others. The most significative differences with our approach are: (i) in our framework the selection of an InP by SPs is modeled explicitly as a game (subgame of the proposed MLFG), whereas in [26] the selection of a service provider by users is modeled implicitly (through the service provider payoff function) and (ii) in the MLFG of our framework, InPs announce their prices simultaneously, whereas in the Stackelberg game proposed in [26] one of the service providers moves first.

Rose et al. ([28]) address the problem of service selection from the end user perspective and service pricing from a Network Service Provider (NSP) perspective. They consider multiple NSPs, each providing multiple types of services, and multiple users with different Quality of Service (QoS) evaluation. The NSPs price their offered services so as to maximize their profit, while each user selects a unique service from a single NSP so as to maximize its payoff given by the difference between its evaluation of the QoS of the selected service and its price. The problem is formulated as a MLFG with NSPs acting as leaders (by announcing the prices of their offered services) and users as followers (each selecting a service and an NSP in response to the service prices offered by NSPs). A similar modeling approach is used in [27] which though focuses on the emerging machine type communications (MTC) and introduces in the framework MTC service providers. Differently than our approach, [28] and [27] assume a continuum of end users (which makes each subgame of stage 2 of the MLFG therein a non-atomic game) while the set of SPs in our work is assumed discrete and finite.

Along the same lines, [29] proposes a similar MLFG which however also accounts for a Small Cell Provider (SCP) (analogous to an InP in our framework), which leases small cell BSs to the NSPs. The interaction among the SCP and the NSPs is modeled through an additional OLMFSG in which the SCP acts as the leader by announcing the spectrum price per small cell BS and NSPs are followers deciding the amount of spectrum to purchase to maximize their individual payoffs. The work in [29] is substantially different from ours: (i) a single SCP is considered in [29], while we have multiple InPs; (ii) since the SCP available spectrum is not bounded in [29], given the spectrum price offered by the SCP, each NSP can derive its optimal amount of spectrum independently, i.e., there is no real competition among the NSPs at stage 2 of the OLMFSG; (iii) while in [29] end users select a service from one of the NSPs, in our framework the user – SP association is given[2].

In [32], the available Physical Resource Blocks (PRBs) of a BS in a C-RAN have to be split among an eMBB, a mMTC slice and an URLLC slice, each requesting a minimum amount. The authors model this problem as bankruptcy game and apply the Shapley value to determine the number of PRBs assigned to each slice. The problem bears similarities with a subproblem of our framework, namely the InP capacity assignment problem (see Section III-C) in which each InP has a fixed amount of capacity per BS cell and SPs that choose to be served by a given InP (each providing either eMBB or mMTC services to a specific market segment of users) request a minimum and maximum of capacity per cell from the latter. While [32] opts for a cooperative game approach for the resource assignment problem, in our framework we propose a two-step lexicographic optimization problem as the assignment is handled in a centralized fashion by the InP, which aims to maximize the total amount of assigned (sold) cell capacity.

Even though our proposed framework is *per se* generic and bears conceptual and formulation similarities with [28] and [29], one of the core contributions of this work is the use of the proposed framework as a means to investigate realistic scenarios in terms of network technologies and related costs, mobile services and related performance requirements, and user tariffing in the context of migrating from 4G to 5G. To this extent, inspired from the usage scenarios for IMT for 2020 and beyond [4], we build up a methodology to evaluate the techno-economic impact of different dimensioning and architectural choices for 5G network. Along these lines, [8]–[10] also target a financially sustainable design and development of 5G networks to meet user requirements and envisioned demand for connectivity. However, [8]–[10] focus on the dimensioning of a single 5G network, while we address competition among multiple InPs with individual 4G/5G mobile networks.

III. Framework

To present our framework, we start by describing the problem it addresses in Section III-A. Next, we dwell on the interactions between an SP and its end users in Section III-B and between an InP and its hosted SPs in III-C. Specifically, in Section III-B we explain the utility function representing the QoS requirements of the service provisioned by each SP and define the SP revenue function based on a noted function in literature which relates the end user fee to its perceived utility, whereas in III-C we propose an optimization problem to model how an InP splits its available capacity among its hosted SPs given their requirements. Then, in Section III-D we formulate the addressed problem as a MLFG.

A. Problem statement

We consider a mobile ecosystem such that the network infrastructure and its resources are decoupled from service

[2]In our work, the interaction between an SP and its set of users is modeled through a noted function in literature [15] which represents the user response to the fee offered by the SP based on the utility perceived by the user from the amount of resources allocated by the SP. Hence, the optimal user fee is affected by the equilibrium of the MLFG or vice versa the user response to the fee offered by the SP affects its strategy in the MLFG.

provisioning for end users, which gives rise to two types of actors: InPs and SPs. An InP is the entity that deploys and maintains the cellular network whose resources it then sells/rents to one or multiple SPs. In turn, an SP provisions services for end users through resources acquired/rented from one of the InPs. From a technical point of view, an InP can support multi-tenancy, i.e., it can host multiple SPs over its network infrastructure and resources by relying on the network slicing paradigm [6]. We assume that InPs do not have end users of their own, whereas SPs do not own any network infrastructure.

We consider a geographical area with multiple InPs with individual RANs, and multiple SPs that provision mobile services to end users through RAN resources acquired from InPs. The RAN of each InP consists of a set of BSs and their respective back-hauling links to connect the former with the core network. The specific architecture of the BS is abstracted away to keep the modeling framework as general as possible[3].

The InPs' BSs are assumed to be co-located and their respective cells to overlap, hence we focus on the area of a single BS cell provisioned by all InPs simultaneously through their individual BSs. In turn, this means that an SP can select any of the InPs to serve its user demand within the cell area. The BS cell of a given InP is characterized by an average capacity which depends on the InP's network technology and configuration and its available spectrum resources. The network resources requested by an SP from an InP for a given cell are expressed in terms of average cell capacity.

Each InP offers its available cell capacity at a certain unit price lower bounded by its unit cost. Based on the InPs' available cell capacities and their offered unit prices, each SP selects an InP from which to acquire cell capacity so as to maximize its profit (difference between revenues from own users and cost incurred from the selected InP, both depending on the amount of acquired cell capacity). The objective of each InP is to maximize the profit from the total amount of cell capacity sold to SPs selecting it. It follows that SPs compete among them for the InPs' cell capacities (as these are finite), while InPs compete among them in cell capacity unit prices to be selected by SPs. Given that InPs and SPs are all self-interested payoff-maximizers, actions taken by any of the actors affect all the others (e.g., by lowering its offered unit price, an InP may be able to attract more SPs or sell more cell capacity to SPs that select it) and we assume that InPs announce their cell capacity unit prices simultaneously and SPs simultaneously select their serving InPs based on these announced prices, then we resort to hierarchical games to model the problem. Specifically, we formulate this problem as a multi-leader-follower game which is an extension of the basic (one-leader one-follower) Stackelberg game. In the proposed model, InPs act as leaders and SPs act as followers. The strategy of each leader is the price per unit of cell capacity which maximizes its profit from the total amount of sold capacity, whereas the strategy of each follower is the choice of an InP which maximizes its profit.

[3]The proposed model remains valid under different realization of the 5G BSs (e.g., a single Active Antenna Unit, a Radio Unit coupled with a Distributed Unit, etc.).

B. SP service characterization and revenue function

Let $\mathcal{V}$ denote the set of SPs. Each SP v is assumed to provision a single type of service and all end users of v, i.e., users subscribing to the service provisioned by v, are assumed identical. The QoS requirements of the service provisioned by v are given in terms of a minimum and a target user rate (both equal for all users of v). Then, the level of satisfaction of a user of v depends on the rate perceived by the user w.r.t. these minimum and target rates: we represent it by the utility function described in Section III-B1. In turn, we adopt the acceptance probability function proposed in [15] to model the user response to a fee offered by its SP depending on its achieved utility, as described in Section III-B2. Based on these two functions, in Section III-B3 we define the optimal SP revenue in terms of the amount of capacity acquired from its selected InP.

1) User utility function: Let x_v denote the amount of cell capacity acquired by SP v from its selected InP. Notice that the cell capacity of an InP is intended as its total cell rate (i.e., the product between its spectral efficiency and bandwidth) hence x_v can be a portion of/all the cell rate of the InP selected by v. Let N_v denote the number of users of v and η_v the activity factor of each user of v. We assume that SP v splits x_v uniformly among its identical N_v users. Let $\widetilde{N}_v$ denote the number of simultaneously active users of v which we determine[4] as $\widetilde{N}_v = \max\{1, \eta_v N_v\}$, then each user of v perceives a rate equal to $x_v/\widetilde{N}_v$. The level of satisfaction of a user of v from $x_v/\widetilde{N}_v$ is represented by a variant of the normalized sigmoid utility function [15], defined as

$$u_v(x_v) = \begin{cases} 0, & \text{if } 0 \le x_v \le \widetilde{N}_v \underline{\mathcal{X}}_v, \\ \dfrac{\left(\frac{x_v/\widetilde{N}_v - \underline{\mathcal{X}}_v}{\mathcal{X}_v - \underline{\mathcal{X}}_v}\right)^{\xi_v}}{1+\left(\frac{x_v/\widetilde{N}_v - \underline{\mathcal{X}}_v}{\mathcal{X}_v - \underline{\mathcal{X}}_v}\right)^{\xi_v}}, & \text{if } x_v > \widetilde{N}_v \underline{\mathcal{X}}_v, \end{cases} \tag{1}$$

where $\underline{\mathcal{X}}_v$ denotes the minimum user rate characterizing the service provisioned by v, $\mathcal{X}_v$ denotes the user rate which provides a utility value equal to 0.5, i.e., $u_v(\widetilde{N}_v \mathcal{X}_v) = 0.5$, while $\overline{\mathcal{X}}_v$ represents the target user rate of the service provisioned by v, that is the rate value that would make a user of v fully satisfied in practice[5], i.e., $u_v(\widetilde{N}_v \overline{\mathcal{X}}_v) = U$, where $0 < U < 1$ and $U \approx 1$. It follows that

$$\mathcal{X}_v = \underline{\mathcal{X}_v} + \left(\overline{\mathcal{X}}_v - \underline{\mathcal{X}}_v\right)\left(\frac{1-U}{U}\right)^{1/\xi_v},$$

where ξ_v denotes the utility elasticity to x_v (the higher the value of ξ_v, the more step-like the shape of the utility function).

2) Acceptance probability function: Let p_v denote the fee offered by SP v to each of its users and let $a_v(u_v(x_v), p_v)$ de-

[4]The max operator in $\widetilde{N}_v = \max\{1, \eta_v N_v\}$ makes sure that when $\eta_v N_v < 1$, the rate perceived by a user of v, i.e., $x_v/\widetilde{N}_v$, does not exceed the total available capacity/rate x_v of SP v.

[5]The utility function $u_v(x_v)$ is such that $\lim_{x_v \to \infty} u_v(x_v) = 1$.

note the user acceptance probability function proposed in [15] and defined as

$$a_v(u_v(x_v), p_v) = 1 - e^{-A_v u_v(x_v)^{\mu_v} p_v^{-\varepsilon_v}}. \tag{2}$$

$a_v(u_v(x_v), p_v)$ relates $u_v(x_v)$, i.e., the level of utility achieved by a user of v when SP v acquires x_v units of capacity (see Equation (1)), and p_v, where μ_v and ε_v denote the user sensitivity to changes in utility and to changes in the offered fee, respectively, whereas A_v is a normalizing constant. Assume users of SP v, characterized by μ_v and ε_v, achieve the maximum level of utility $\overline{u}_v$ and are offered the fee $\overline{p}_v$. Let $\overline{q}_v$ denote the probability with which these users reject[6] $\overline{p}_v$, i.e.,

$$\overline{q}_v = 1 - a_v(\overline{u}_v, \overline{p}_v) = e^{-A_v \overline{u}_v^{\mu_v} \overline{p}_v^{-\varepsilon_v}},$$

hence the normalizing constant $A_v = -\overline{u}_v^{-\mu_v} \overline{p}_v^{\varepsilon_v} \log(\overline{q}_v)$ and, as a result, $a_v(u_v(x_v), p_v)$ can be rewritten as

$$a_v(u_v(x_v), p_v) = 1 - \overline{q}_v^{(u_v/\overline{u}_v)^{\mu_v} (p_v/\overline{p}_v)^{-\varepsilon_v}}. \tag{3}$$

3) SP revenue function: Being $a_v(u_v(x_v), p_v)$ the probability that a user of SP v accepts the offered fee p_v when it achieves the level of utility $u_v(x_v)$, then $a_v(u_v(x_v), p_v)p_v$ represents the fee accepted by the user or, in other words, the expected revenue of v from the single user when v acquires x_v units of capacity. Then, as the number of users of SP v is equal to N_v, the total revenue of SP v from x_v units of capacity, when users are offered the fee p_v, can be determined as

$$r_v(x_v, p_v) = N_v a_v(u_v(x_v), p_v) p_v. \tag{4}$$

Let $p_v^*(u_v(x_v))$ denote the value of p_v which maximizes $r_v(x_v, p_v)$ for a given x_v and let $r_v^*(x_v)$ be the total optimal revenue of SP v for x_v, i.e. $r_v^*(x_v) = N_v a_v(u_v(x_v), p_v^*(u_v(x_v)))p_v^*(u_v(x_v))$.

If $x_v \le \widetilde{N}_v \underline{\mathcal{X}}_v$, then $r_v(x_v, p_v) = 0$ for any $p_v > 0$ as $u_v(x_v) = 0$ (see Equation 1) and $a_v(0, p_v) = 0$ for any $p_v > 0$[7] if $0 < A_v < \infty$, $0 < \mu_v < \infty$ and $0 < \varepsilon_v < \infty$ (see Equation 2 and Appendix A for the assumptions on A_v, μ_v and ε_v). This means that $p_v^*(u_v(x_v))$ is indeterminate for $x_v \le \widetilde{N}_v \underline{\mathcal{X}}_v$, but $r_v^*(x_v) = 0$.

Instead, for $x_v > \widetilde{N}_v \underline{\mathcal{X}}_v$, which implies $u_v(x_v) > 0$ (see Equation 1), we show in Appendix A that, when $0 < A_v < \infty$, $0 < u_v(x_v) < \infty$, $0 < \mu_v < \infty$ and $1 < \varepsilon_v < \infty$, we have

$$p_v^*(u_v(x_v)) = \overline{p}_v \left[\frac{\log(\overline{q}_v)}{W_{-1}\left(-\frac{1}{\varepsilon_v} e^{-\frac{1}{\varepsilon_v}}\right) + \frac{1}{\varepsilon_v}} \right]^{\frac{1}{\varepsilon_v}} \left[\frac{u_v(x_v)}{\overline{u}_v} \right]^{\frac{\mu_v}{\varepsilon_v}}, \tag{5}$$

where W_{-1} denotes the lower branch of the Lambert W function for the real numbers domain. It follows that for $x_v > \widetilde{N}_v \underline{\mathcal{X}}_v$, $0 < A_v < \infty$, $0 < \mu_v < \infty$ and $0 < \varepsilon_v < \infty$, we have

$$a_v(u_v(x_v), p_v^*(u_v(x_v))) = 1 - e^{W_{-1}\left(-\frac{1}{\varepsilon_v} e^{-\frac{1}{\varepsilon_v}}\right) + \frac{1}{\varepsilon_v}}, \tag{6}$$

that is, the acceptance probability at $p_v^*(u_v(x_v))$ is a function of only ε_v and independent of $u_v(x_v)$. Let $a_v^* = a_v(u_v(x_v), p_v^*(u_v(x_v))) = 1 - e^{W_{-1}\left(-\frac{1}{\varepsilon_v} e^{-\frac{1}{\varepsilon_v}}\right) + \frac{1}{\varepsilon_v}}$ and $\overline{a}_v = 1 - \overline{q}_v$. Hence, for $0 < A_v < \infty$, $0 < \mu_v < \infty$ and $1 < \varepsilon_v < \infty$, we have

$$r_v^*(x_v) = \begin{cases} 0, & \text{if } 0 \le x_v \le \widetilde{N}_v \underline{\mathcal{X}}_v, \\ N_v a_v^* \overline{p}_v \left(\frac{\log(1-\overline{a}_v)}{\log(1-a_v^*)} \right)^{\frac{1}{\varepsilon_v}} \left(\frac{u_v(x_v)}{\overline{u}_v} \right)^{\frac{\mu_v}{\varepsilon_v}}, & \text{if } x_v > \widetilde{N}_v \underline{\mathcal{X}}_v. \end{cases} \tag{7}$$

C. InP capacity assignment problem

In the proposed MLFG, the strategy of each SP consists solely in the choice of InP from which to acquire capacity. However, the amount of cell capacity acquired by an SP affects both its revenue (as explained in Section III-B) and its total cost (product of InP unit price with the amount of cell capacity acquired by the SP), hence its payoff (difference between the two). In these lines, given that SPs are rational, none of them can accept an amount of cell capacity which provides a negative payoff. We therefore assume that, given the cell capacity unit price offered by an InP, each SP selecting the InP communicates a minimum and a maximum amount of cell capacity that the SP finds suitable, i.e., the minimum amount of cell capacity that guarantees a non-negative payoff and the payoff-maximizing amount. Based on such cell capacity ranges, the InP determines which of the SPs that select it to serve and how to split its available cell capacity among them so as to maximize its own profit (payoff) while satisfying their cell capacity ranges. We refer to this procedure as the capacity assignment problem and formulate it as an optimization problem detailed in the following paragraphs.

First, we explain how an SP determines its suitable cell capacity range for a given cell capacity unit price. Consider an SP v and a cell capacity unit price $P > 0$ and let:

$$\overline{X}_v(P) = \begin{cases} 0, \quad \text{if } r_v^*(x_v) - P x_v \le 0, \ \forall\, x_v \ge \widetilde{N}_v \underline{\mathcal{X}}_v, \\ \underset{x_v \ge \widetilde{N}_v \underline{\mathcal{X}}_v}{\operatorname{argmax}} \ (r_v^*(x_v) - P x_v), \\ \qquad \text{if } \exists\, x_v > \widetilde{N}_v \underline{\mathcal{X}}_v \mid r_v^*(x_v) - P x_v > 0, \end{cases} \tag{8}$$

$$\underline{X}_v(P) = \begin{cases} 0 \qquad \text{if } \overline{X}_v(P) = 0, \\ x_v \in [\widetilde{N}_v \underline{\mathcal{X}}_v, \overline{X}_v(P)] \mid r_v^*(x_v) - P x_v = 0, \\ \qquad \text{if } \overline{X}_v(P) > 0, \end{cases} \tag{9}$$

where $r_v^*(x_v) - P x_v$ is the profit of SP v when it purchases x_v units of cell capacity at a unit price P. Notice that we consider $\underline{\mathcal{X}}_v > 0$ for each $v \in \mathcal{V}$. For $\underline{\mathcal{X}}_v > 0$, $r_v^*(x_v) = 0$

[6]The reference rejection probability $\overline{q}_v$ can be determined by polling a large set of users of SP v with known ε_v and μ_v on whether they accept the fee $\overline{p}_v$ when they achieve the maximum level of utility $\overline{u}_v$. Then, $\overline{q}_v$ is set equal to the fraction of users which reject $\overline{p}_v$ [15], [33].

[7]Notice that even for $0 < A_v < \infty$, $0 < \mu_v < \infty$ and $0 < \varepsilon_v < \infty$, $a_v(0, 0)$ is indeterminate (see Equation 2). However, in practice, $p_v = 0$ means that SP v obtains zero revenue hence we are interested in $p_v > 0$.

holds for any $x_v \in (0, \widetilde{N}_v \underline{\mathcal{X}}_v]$ (see Equations (1) and (7)), which implies $r_v^*(x) - Px < 0$ for any $x \in (0, \widetilde{N}_v \underline{\mathcal{X}}_v]$ as $P > 0$ (being the cell capacity unit price). Therefore, we look for the payoff-maximizing cell capacity of v, denoted as $\overline{X}_v(P)$, for $x \geq \widetilde{N}_v \underline{\mathcal{X}}_v$ (see Equation 8). If $r_v^*(x_v) - Px_v \leq 0$ for any $x_v \geq \widetilde{N}_v \underline{\mathcal{X}}_v$, then we impose $\overline{X}_v(P) = 0$ otherwise, if it exists $x_v > \widetilde{N}_v \underline{\mathcal{X}}_v$ such that $r_v^*(x_v) - Px_v > 0$, then $\overline{X}_v(P) > \widetilde{N}_v \underline{\mathcal{X}}_v > 0$.

In turn, $\underline{X}_v(P)$ (see Equation (9)) denotes the minimum amount of cell capacity that provides v with a non-negative payoff. If $r_v^*(x_v) - Px_v \leq 0$ for any $x_v \geq \widetilde{N}_v \underline{\mathcal{X}}_v$, for which we imposed $\overline{X}_v(P) = 0$, then we set $\underline{X}_v(P) = 0$ as well. Otherwise, if $\overline{X}_v(P) > \widetilde{N}_v \underline{\mathcal{X}}_v > 0$, then $\underline{X}_v(P)$ is set equal to the unique[8] root of equation $r_v^*(x_v) - Px_v = 0$ in the interval $[\widetilde{N}_v \underline{\mathcal{X}}_v, \overline{X}_v(P)]$. Hence, one has $0 < \widetilde{N}_v \underline{\mathcal{X}}_v < \underline{X}_v(P) < \overline{X}_v(P)$. In summary, for the considered SP payoff function, for $\underline{\mathcal{X}}_v > 0$ and for any unit price $P > 0$, either $\underline{X}_v(P) = \overline{X}_v(P) = 0$ or $\overline{X}_v(P) > \underline{X}_v(P) > 0$.

Let $\mathcal{K}$ denote the set of InPs and C_k the cell capacity of an InP k. C_k is assumed to be a fixed positive quantity. Now consider an InP k which offers a cell capacity unit price $P_k > 0$. Suppose that k is selected by the set of SPs $\mathcal{V}_k \subseteq \mathcal{V}$. Recall that $\underline{X}_v(P_k)$ and $\overline{X}_v(P_k)$ denote the the minimum and maximum amount of capacity requested by SP v at the cell capacity unit price P_k, respectively. Let $\widehat{\mathcal{V}}_k = \{v \in \mathcal{V}_k \mid \overline{X}_v(P_k) > \underline{X}_v(P_k) > 0\}$. The InP assigns a null capacity to any SP $v \in \mathcal{V}_k \backslash \widehat{\mathcal{V}}_k$ as $\underline{X}_v(P_k) = \underline{X}_v(P_k) = 0$. In turn, for $\widehat{\mathcal{V}}_k \neq \emptyset$, the capacity assignment problem is formalized as follows: as C_k is fixed and finite, given the cell capacity ranges of all SPs in $\mathcal{V}_k$, InP k has to decide:

(1) which SPs in $\mathcal{V}_k$ to serve, represented by the binary variables z_{vk}, for any $v \in \mathcal{V}_k$,
(2) how much capacity to allocate to each SP $v \in \mathcal{V}_k$, represented by non-negative variables x_{vk},

so that its profit, $P_k \left(\sum_{v \in \mathcal{V}_k} x_{vk} \right)$, is maximized while the cell capacity ranges of served SPs are satisfied (i.e., if $z_{vk} = 1$, $\underline{X}_v(P_k) \leq x_{vk} \leq \overline{X}_v(P_k)$, otherwise $x_{vk} = 0$) and its available capacity is not exceeded, i.e., $\sum_{v \in \mathcal{V}_k} x_{vk} \leq C_k$. As P_k is fixed in the context of the capacity assignment problem, then the objective function of InP k reduces to $\sum_{v \in \mathcal{V}_k} x_{vk}$.

We opted for a two-step lexicographic approach to formulate the capacity assignment problem. In the first step, InP k solves problem (10)–(14) to determine the maximum amount of cell capacity it can sell, i.e., $C'_k = \sum_{v \in \mathcal{V}_k} x'_{vk}$ where x'_{vk} denotes the value of variable x_{vk} in the optimal solution of (10)–(14).

$$\max \sum_{v \in \mathcal{V}_k} x_{vk} \tag{10}$$

$$x_{vk} \geq \underline{X}_v(P_k) z_{vk}, \qquad \forall v \in \mathcal{V}_k, \tag{11}$$

$$x_{vk} \leq \overline{X}_v(P_k) z_{vk}, \qquad \forall v \in \mathcal{V}_k, \tag{12}$$

$$\sum_{v \in \mathcal{V}_k} x_{vk} \leq C_k, \tag{13}$$

$$x_{vk} \geq 0, \; z_{vk} \in \{0,1\}, \qquad \forall v \in \mathcal{V}_k. \tag{14}$$

[8] This is always the case for the considered SP payoff function for each $v \in \mathcal{V}$ and for each considered instance. A few examples of the payoff function are provided in Appendix B.

However, there may be multiple equivalent optimal solutions to problem (10)–(14) such that $\sum_{v \in \mathcal{V}_k} x'_{vk} = C'_k$; these solutions are equivalent from the InP perspective but not necessarily from the SPs' perspective (which may obtain a different amount of capacity in each of these solutions and hence a possibly different payoff value). When the first step of the capacity assignment problem, i.e., problem (10)–(14), does not have a unique solution, then the InP solves the second step of the capacity assignment problem, represented by problem (15)–(20):

$$\min \; \zeta_k - \sum_{v \in \mathcal{V}_k} z_{vk} \tag{15}$$

$$x_{vk} \geq \underline{X}_v(P_k) z_{vk}, \qquad \forall v \in \mathcal{V}_k, \tag{16}$$

$$x_{vk} \leq \overline{X}_v(P_k) z_{vk}, \qquad \forall v \in \mathcal{V}_k, \tag{17}$$

$$\sum_{v \in \mathcal{V}_k} x_{vk} = C'_k, \tag{18}$$

$$\zeta_k \geq z_{vk} - x_{vk}/\overline{X}_v(P_k), \qquad \forall v \in \mathcal{V}_k \mid \overline{X}_v(P_k) > 0, \tag{19}$$

$$x_{vk} \geq 0, \; z_{vk} \in \{0,1\}, \qquad \forall v \in \mathcal{V}_k, \qquad \zeta_k \geq 0. \tag{20}$$

The aim of the second step is to select among the multiple optimal solutions of the first step, one which satisfies a fairness criterion from the SPs' perspective while using up C'_k entirely (see Equation (18)) as C'_k is the optimal amount of the total assigned cell capacity for InP k determined in the first step. The fairness criterion consists of minimizing the highest among all SPs in $\mathcal{V}_k$ of the relative difference between the maximum amount of capacity requested by an SP (i.e., its payoff-maximizing capacity) and the amount assigned to the SP by the InP. In other words, the InP's capacity assignment accounts for the most "unsatisfied" SP among all. The highest relative difference is represented by the variable $0 \leq \zeta_k \leq 1$ and modeled through constraints (19). Consider an SP v with $\underline{X}_v(P_k) = \overline{X}_v(P_k) = 0$ (which means that it is unprofitable for v to purchase capacity from InP k at a unit price P_k): the corresponding optimal value of x_{vk} is equal to zero due to constraints (16) and (17), and since v is "fully-satisfied", we exclude it from the calculation of ζ_k (see constraints (19)). In turn, for an SP v with $\overline{X}_v(P_k) > \underline{X}_v(P_k) > 0$, if $x_{vk} = 0$ (which implies $z_{vk} = 0$ due to constraints (16)), the right hand side of constraints (19) equals 0, i.e., an SP which is willing to purchase capacity from InP k at a unit price P_k but it is not assigned any capacity is also considered as fully-satisfied to avoid ζ_k being stuck to its upper bound value equal to 1 regardless of the assignment of the other SPs. Therefore, only SPs $v \in \mathcal{V}_k$ such that $\overline{X}_v(P_k) > \underline{X}_v(P_k) > 0$ and $x_{vk} > 0$ (and hence $z_{vk} = 1$ due to constraints (17)) influence the value of ζ_k. The second term in the objective function, i.e., $\sum_{v \in \mathcal{V}_k} z_{vk}$, is introduced to deal with equivalent optimal solutions, although uniqueness cannot be guaranteed. Since $\zeta_k \leq 1$, an increase by one of the number of served SPs outweighs the increase of ζ_k from splitting the capacity over a larger set of SPs. Therefore, by minimizing $\zeta_k - \sum_{v \in \mathcal{V}_k} z_{vk}$, we select optimal solutions which are characterized by the largest possible number of served

SPs for the fixed capacity C'_k, while the capacity assignment follows the min-max fairness criterion. Notice that for an SP v with $\underline{X}_v(P_k) = \overline{X}_v(P_k) = 0$, although in the optimal solution $x_{vk} = 0$, z_{vk} is set to one by the objective function. However, this does not affect the optimal solution as such v does not use up any capacity given that its respective x_{vk} is equal to zero in optimal solution.

D. Multi-Leader-Follower Game

As mentioned, all InPs' BSs are co-located hence InPs compete among them to be selected by SPs on a per BS cell basis, which means that the proposed framework applies to each BS cell independently. We assume that each InP k announces its cell capacity unit price P_k to the SPs independently from all other InPs but simultaneously to them. In turn, once the InP unit price profile, $\boldsymbol{P} = \{P_k\}_{k\in\mathcal{K}}$, is known by the SPs, we further assume that also each SP v acts independently but simultaneously to all other SPs in deciding from which InP to acquire capacity in order to serve its users' demand in the area of the considered cell. All involved actors are assumed rational and self-interested, i.e., each of them aims to maximize its individual payoff. Moreover, actions of any actor can affect all other actors, e.g., the InP choice of an SP can affect not only the InPs' payoffs but also the SPs' payoffs given that the cell capacity of each InP is finite and has to be split among SPs selecting the InP. With this setting in mind, we propose a Multi-Leader-Follower game to model the interaction among InPs and SPs. In the proposed model, InPs act as *leaders* (i.e., as the subset of players that move first) by announcing their unit prices to the SPs, whereas SPs act as *followers* as they choose an InP from which to acquire capacity only after the InPs' unit prices have been announced. Formally, this game is a two stage game with observable actions [34]. The game is also of imperfect information since within a stage players move simultaneously, i.e., at stage 1 InPs announce their unit prices simultaneously and at stage 2, for a given InP unit price profile, SPs make their InP choices simultaneously.

As previously argued, since the cell capacity of each InP is fixed and finite and each InP splits its available capacity among SPs that select it, the InP choice of an SP can affect the choices of all other SPs. Hence, for a given InP unit price profile ($\boldsymbol{P} = \{P_k\}_{k\in\mathcal{K}}$) the independent but simultaneous choice of an InP by each SP can be represented by a simultaneous noncooperative game in pure strategies described by the tuple $G^{\mathcal{V}}(\boldsymbol{P}) = \{\mathcal{V}, \{\mathcal{Y}_v\}_{v\in\mathcal{V}}, \{g_v\}_{v\in\mathcal{V}}\}$, where the set of players coincides with the set $\mathcal{V}$ of SPs, $\mathcal{Y}_v$ denotes the strategy set of player v representing its choice of an InP, whereas g_v denotes the payoff of v which is defined for each SP strategy profile and depends on the InP unit price profile (i.e., $g_v = g_v(\boldsymbol{P}, \boldsymbol{y})$). Further, each InP k can anticipate[9] the outcome of $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$ for any $\boldsymbol{P}$, i.e., k can anticipate the subset of SPs that will select k at the Nash Equilibrium(a) of $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$ and consequently determine its payoff for $\boldsymbol{P}$. Therefore, InPs compete among them in cell capacity unit prices to be selected by the SPs: this can be represented by another simultaneous noncooperative game described by the tuple $\mathcal{G}^{\mathcal{K}} = \{\mathcal{K}, \{\mathcal{P}_k\}_{k\in\mathcal{K}}, \{G_k\}_{k\in\mathcal{K}}\}$, where the set of players coincides with the set $\mathcal{K}$ of InPs, $\mathcal{P}_k$ denotes the strategy set of player k representing a unit price range, whereas G_k denotes the payoff of k which is defined for each InP strategy profile (i.e., $G_k : \mathcal{P} \to \mathbb{R}$ where $\mathcal{P} = \prod_{k\in\mathcal{K}} \mathcal{P}_k$). We now detail $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$ and $\mathcal{G}^{\mathcal{K}}$ which hereon we will refer to as the SPs' game and InPs' game, respectively.

1) SPs' game: As detailed in Section III-B, each SP provides a single type of mobile service to a fixed number of end users. As none of SPs owns any network infrastructure/resources, then each SP, to provision the mobile service for its users in the area of a cell, acquires an aggregate amount of cell capacity from one of the InPs which it splits among all of its users in the cell area; the users are then charged by the SP based on the utility achieved from the amount of allocated capacity which results in a total amount of revenue per cell for the SP (see Section III-B3). Therefore, the goal of the SP is to select an InP from which to acquire cell capacity in order to maximize its profit (payoff) given by the difference between the cell revenues incurred from the amount of cell capacity assigned by the selected InP and the cost of the latter.

For each InP unit price profile $\boldsymbol{P} \in \mathcal{P}$, when selecting the InP from which to acquire capacity, SPs contend among them for the InPs' fixed and finite capacities; this gives rise to the SPs' game described by $G^{\mathcal{V}}(\boldsymbol{P})$. Formally, the strategy of an SP v is modeled by a set of binary variables $\boldsymbol{y}_v = (y_{vk})_{k\in\mathcal{K}}$ such that $y_{vk} \in \{0, 1\}$ for any $k \in \mathcal{K}$ and $\sum_{k\in\mathcal{K}} y_{vk} = 1$. Let $\boldsymbol{y} = \{\boldsymbol{y}_v\}_{v\in\mathcal{V}}$ denote a strategy profile of $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$. Then, let $x_{vk}(P_k, \boldsymbol{y})$ denote the amount of cell capacity obtained by SP v from InP k at unit price P_k given the SP strategy profile $\boldsymbol{y}$: if v does not select k in $\boldsymbol{y}$ (i.e., $y_{vk} = 0$) then clearly $x_{vk}(P_k, \boldsymbol{y}) = 0$, otherwise if v selects k in $\boldsymbol{y}$ (i.e., $y_{vk} = 1$) then $x_{vk}(P_k, \boldsymbol{y})$ is equal to the value of variable x_{vk} in the optimal solution of problem (15)–(20) when the capacity assignment problem is solved by InP k for the set $\mathcal{V}_k = \{v \in \mathcal{V} : y_{vk} = 1\}$, given the cell capacity ranges $[\underline{X}_v(P_k), \overline{X}_v(P_k)]$ for its offered unit price P_k (see Section III-C)).

The payoff of v from $\boldsymbol{y}$ is defined as

$$g_v(\boldsymbol{P}, \boldsymbol{y}) = \sum_{k\in\mathcal{K}} \left(r_v^*(x_{vk}(P_k, \boldsymbol{y})) - P_k x_{vk}(P_k, \boldsymbol{y})\right), \quad (21)$$

[9]The equilibrium(a) of the MLFG are determined by means of the *sub-game perfect equilibrium* solution concept which is an extension of the *backward induction* solution concept for the original one-leader, one-follower Stackelberg game. The idea behind *backward induction* is that the leader assumes that the follower is rational and it anticipates the follower's best response to each action of its own. Therefore, the leader's strategy consists of selecting the action that maximizes its own payoff given the best response of the follower. In case of the MLFG we propose here, leaders anticipate the outcome of the followers' game, i.e., its Nash equilibrium(a), for any action profile of their own, which is, in turn, the main idea behind the *sub-game perfect equilibrium* solution concept. Details concerning the calculation of the equilibrium(a) of the MLFG for the considered problem instances are presented in Section IV-E.

where $r_v^*(x_{vk}(P_k, \boldsymbol{y}))$ is the total optimal revenue of SP v (see Equation (7) and Section III-B3) for the amount of cell capacity $x_{vk}(P_k, \boldsymbol{y})$, whereas $P_k x_{vk}(P_k, \boldsymbol{y})$ is the cost incurred by SP v from purchasing the amount of cell capacity $x_{vk}(P_k, \boldsymbol{y})$ at unit price P_k, therefore $g_v(\boldsymbol{P}, \boldsymbol{y})$ is given in terms of the total profit[10] of v.

By definition, a strategy profile $\breve{\boldsymbol{y}} = [\breve{\boldsymbol{y}}_v, \breve{\boldsymbol{y}}_{-v}]$, where $\boldsymbol{y}_{-v}$ denotes the strategies of all SPs but v, is a Nash Equilibrium (NE) of the SPs' game $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$ if for each SP $v \in \mathcal{V}$, $\breve{\boldsymbol{y}}_v = \text{argmax}_{\boldsymbol{y}_v \in \mathcal{Y}_v} g_v(\boldsymbol{P}, [\boldsymbol{y_v}, \breve{\boldsymbol{y}}_{-v}])$, i.e., if no SP has an incentive to unilaterally deviate from $\breve{\boldsymbol{y}}$. Since there is one SPs' game $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$ for each InP unit price profile $\boldsymbol{P} \in \mathcal{P}$, hereon we will use the notation $\breve{\boldsymbol{y}}(\boldsymbol{P})$ to denote the NE strategy profile(s) of $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$.

2) InPs' game: Each InP unit price profile $\boldsymbol{P} \in \mathcal{P}$ may result in a distinct NE of the SPs' game, i.e., in a different partition of the set of SPs over the set of InPs and consequently in different profits for the InPs. To put it differently, InPs compete among them in cell capacity unit prices to be selected by the SPs, which we modeled as the game $\mathcal{G}^{\mathcal{K}} = \{\mathcal{K}, \{\mathcal{P}_k\}_{k\in\mathcal{K}}, \{G_k\}_{k\in\mathcal{K}}\}$, namely the InPs' game. The strategy set of each player k consists of a unit price range, i.e., $\mathcal{P}_k = [\underline{P}_k, \overline{P}]$ where $\underline{P}_k$ denotes the cell capacity unit cost for InP k and $\overline{P}$ denotes the minimum unit price for which no SP is willing to buy capacity. A strategy of InP k is then a cell capacity unit price $P_k \in \mathcal{P}_k$. We impose $P_k \geq \underline{P}_k$ as we assumed InPs to be rational, i.e., they will not accept gains lower than their costs and similarly, as all SPs are also assumed to be rational, they will not purchase cell capacity at a unit price resulting in a negative payoff; in other words, any InP k offering $P_k \geq \overline{P}$, would not sell any cell capacity. The payoff of player k from the strategy (unit price) profile $\boldsymbol{P} = \{P_k\}_{k\in\mathcal{K}}$ is defined as

$$G_k(\boldsymbol{P}) = P_k \left(\sum_{v\in\mathcal{V}} x_{vk}(P_k, \breve{\boldsymbol{y}}(\boldsymbol{P})) \right), \qquad (22)$$

that is, as the product between the cell capacity unit price of InP k and the total amount of capacity sold to SPs that select k at the NE $\breve{\boldsymbol{y}}(\boldsymbol{P})$ of the SPs' game $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$). Recall that, under the assumption that all SPs are rational, each InP can anticipate $\breve{\boldsymbol{y}}(\boldsymbol{P})$ of $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$. If for some $\boldsymbol{P} \in \mathcal{P}$, the NE of $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$ is not unique, we assume InPs are pessimistic and each of them independently considers the worst payoff achieved over all the NE of $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$. In turn, if $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$ has no NE in pure strategies, we would look for its NE in mixed strategies[11].

[10] As for each SP $v \in \mathcal{V}$, $\sum_{k\in\mathcal{K}} y_{vk} = 1$, then $r_v^*(x_{vk}(P_k, \boldsymbol{y})) - P_k x_{vk}(P_k, \boldsymbol{y}) \neq 0$ for at most one InP $k \in \mathcal{K}$.

[11] If there were no NE in pure strategies for $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$ for some $\boldsymbol{P} \in \mathcal{P}$, then we would look for its NE in mixed strategies: formally, we would relax variables $\boldsymbol{y}_v$, $\forall v \in \mathcal{V}$ representing the InP choice of SP v (see Section III-D1), i.e., an SP's mixed strategy for the game $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$ would be represented by variables $\boldsymbol{\gamma}_v = \{\gamma_{vk}\}_{k\in\mathcal{K}} \mid 0 \leq \gamma_{vk} \leq 1,\ \forall k \in \mathcal{K}, \sum_{k\in\mathcal{K}} \gamma_{vk} = 1, \forall v \in \mathcal{V}$ and the expected payoff of v from $\boldsymbol{\gamma}$ given $\boldsymbol{P}$, $\tilde{g}_v(\boldsymbol{P}, \boldsymbol{\gamma}) = \sum_{\boldsymbol{y}\in\mathcal{P}} P(\boldsymbol{y}|\boldsymbol{\gamma}) g_v(\boldsymbol{P}, \boldsymbol{y})$ where $P(\boldsymbol{y}|\boldsymbol{\gamma})$ is the probability of occurrence of the outcome represented by the pure strategy profile $\boldsymbol{y}$ (i.e., a partitioning of the set of SPs over the set of InPs) given $\boldsymbol{\gamma}$ (the InPs' expected payoff from the equilibrium mixed strategy profile $\breve{\boldsymbol{\gamma}}$ would be calculated in a similar fashion). However, for all considered instances (see Section V-B), there is at least one NE in pure strategies for $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$, $\forall \boldsymbol{P} \in \mathcal{P}$.

A strategy profile $\breve{\boldsymbol{P}} = [\breve{P}_k, \breve{\boldsymbol{P}}_{-k}]$, where $\breve{\boldsymbol{P}}_{-k}$ denotes the unit prices offered by all InPs but k, is an NE of the InPs' game $\mathcal{G}^{\mathcal{K}}$ if $\breve{P}_k = \text{argmax}_{P_k\in\mathcal{P}_k} G_k([P_k, \breve{\boldsymbol{P}}_{-k}])$ for any $k \in \mathcal{K}$, i.e., if no InP has an incentive to unilaterally deviate from $\breve{\boldsymbol{P}}$.

IV. Scenarios and computational tests

In this section, we describe the scenarios that we have addressed by means of the proposed framework. First, we explain how we set up different types of InPs based on their network technology, available spectrum bandwidth, etc., and propose a cost model largely based on [10] to derive a sensible cell capacity unit cost for each InP type (Section IV-A). In Sections IV-B and IV-C we dwell on the set of service types that we have set up based on usage scenarios for IMT for 2020 and beyond [4] and on the set of SPs providing such services; in particular, we report how these services have been characterized based on Key Performance Indicators (KPI) requirements from [14] and how SPs set up their user fees based on their service characteristics and user types. The set of scenarios (problem instances) addressed in our computational tests is defined in Section IV-D, whereas implementation details concerning these computational tests are reported in Section IV-E.

A. InPs

The considered set $\mathcal{K}$ of InPs consists of InPs which coexist in a dense urban area where each of them has either (i) deployed a legacy (pre-5G) heterogeneous network of macro cells (MCs) and small cells (SCs) prior to the beginning of the studied period and does not upgrade to 5G in the meantime or (ii) just deployed a 5G heterogeneous network of MCs and SCs. We refer to (i) and (ii) as InP types and denote them by $\mathcal{L}$ and $\mathcal{N}$, respectively. The type of each InP k is then represented by a binary parameter λ_k, which equals 1 if k is of type $\mathcal{L}$ and 0 if k is of type $\mathcal{N}$.

We assume that in the considered dense urban area both MCs and SCs of different InPs are colocated. In these lines, at the beginning of the studied period, a site is present in a MC candidate site (and can be used by any of the InPs) if at least one of the InPs has previously deployed a MC BS in it; instead, a site is present in a SC candidate site and can be used by a given InP if the InP itself has previously deployed a SC BS in it. Let $\pi_{MC,k}(\pi_{SC,k})$ denote the probability that InP k has not deployed a MC(SC) BS in a MC(SC) candidate site[12] prior the beginning of the studied period. Then, as MC sites are shared, $\pi_{MC} = \prod_{k\in\mathcal{K}} \pi_{MC,k}$ is the probability that none of the InPs has deployed a MC BS in a MC candidate site prior the beginning of the studied period, i.e., at the beginning of the studied period, a MC site has to be built with probability π_{MC}. In turn, since SC sites are not shared, at the beginning of the studied period, InP k has to build a SC site in a SC candidate site with probability $\pi_{SC,k}$. For an InP k of type $\mathcal{L}$, we consider $\pi_{MC,k} = \pi_{SC,k} = 0$, i.e., we assume k has deployed legacy MCs(SCs) BSs in all available MC(SC) candidate sites.

[12] $\pi_{MC,k}(\pi_{SC,k})$ is intended as an average probability for the considered area, hence it is the same for all MC(SC) candidate sites.

Instead for an InP k of type $\mathcal{N}$, $\pi_{MC,k} = \pi_{SC,k} = 1$ imply that InP k has not previously deployed a legacy network, whereas $0 \leq \pi_{MC,k}, \pi_{SC,k} < 1$ means that InP k has previously deployed a legacy network and can reuse its sites. However, we assume that, at the beginning of the studied period, type $\mathcal{N}$ InPs will deploy 5G MC and SC BSs in all available MC and SC candidate sites and that such InPs will compete with the other InPs solely through their new (5G) network while simply reusing sites of thier previously deployed legacy networks (if any).

For MC sites we have considered 3-sector antennas as in [10], [35], [36], whereas for SC sites, omnidirectional (i.e., 1-sector) ones.

Let B_k denote the total available bandwidth of InP k and $\widehat{B}_k \in [0, B_k]$ the amount of bandwidth associated with a spectrum license whose cost has already been amortized at the beginning of the studied period, while the remaining amount of bandwidth $B_k - \widehat{B}_k$ corresponds to a spectrum license acquired at the beginning of the studied period. In particular, $\widehat{B}_k = B_k$ if InP k is of type $\mathcal{L}$ and $\widehat{B}_k = 0$ if k is of type $\mathcal{N}$ and has no legacy network. We assume that for each InP k, B_k is dynamically shared between the MC and the SC layers and, within the MC/SC layer, the bandwidth is also dynamically shared (and not a priori partitioned) between the Downlink (DL) and Uplink (UL) of each MC sector/SC. Further, as we consider a very dense deployment of SCs, MCs can be assumed idle, hence B_k is then dynamically shared between the DL and the UL of each SC.

We have made the simplifying assumption that the DL and UL spectral efficiencies are equal. For InP k, let $\nu_{MC,k}(\nu_{SC,k})$ denote the MC(SC) average spectral efficiency[13] of both DL and UL. We assume InPs compete among them to be selected by SPs on a per SC basis[14], hence the capacity C_k characterizing InP k is set equal to its total (DL+UL) average capacity of a SC, i.e., $C_k = \nu_{SC,k} B_k$.

The cost per unit of capacity characterizing InP k, i.e., $\underline{P}_k$, is then set equal to the monthly[15] cost per unit of capacity provided in the area of a SC, i.e.,

$$\underline{P}_k = \frac{1}{12\,L\,C_k}\Big[(1-\lambda_k)\left(c^{cpx}_{SC,k} + (A_{SC}/A_{MC})\,c^{cpx}_{MC,k}\right) + \\ + c^{opx}_{SC,k}L + (A_{SC}/A_{MC})\,c^{opx}_{MC,k}L + c^{spec}_{SC,k}\Big], \tag{23}$$

where L denotes the duration of the studied period in years, $c^{cpx}_{MC,k}(c^{cpx}_{SC,k})$ and $c^{opx}_{MC,k}$ $(c^{opx}_{SC,k})$ denote the total CAPEX and total annual OPEX incurred by InP k per MC sector (SC site)[16], $A_{MC}(A_{SC})$ denotes the area of a MC sector (SC), respectively, whereas $c^{spec}_{SC,k}$ denotes the spectrum license cost normalized to the area of the SC and to the duration of the studied period. The per sector MC cost terms ($c^{cpx}_{MC,k}$ and $c^{opx}_{MC,k}$) are multiplied by A_{SC}/A_{MC}, i.e., the inverse of the number of SCs per MC sector, to uniformly split the cost of the MC sector among all SCs that overlay the MC sector. The $(1-\lambda_k)$ term sets the CAPEX terms to zero for an InP k of type $\mathcal{L}$ (for which $\lambda_k = 1$); however, k will incur the OPEX of its legacy network. Instead, an InP of type $\mathcal{N}$ (for which $\lambda_k = 0$), incurs both CAPEX and OPEX terms as it deploys its 5G network at the beginning of the study period.

In details, $c^{cpx}_{MC,k}$, $c^{cpx}_{SC,k}$, $c^{opx}_{MC,k}$, $c^{opx}_{SC,k}$ and $c^{spec}_{SC,k}$ are determined as follows:

$$c^{cpx}_{MC,k} = \frac{1}{3}\Big[(1/|\mathcal{K}|)\,\pi_{MC}c^{c,s}_{MC} + c^{c,a}_{MC,k} + c^{c,f}_{MC,k} + \\ + \lceil B_k/B_0\rceil c^{c,rf}_{MC,k} + \lceil m_k(B_k/B_0)\rceil c^{c,bp}_{MC,0} + c^{c,bh}_{MC,k}\Big], \tag{24}$$

$$c^{opx}_{MC,k} = \frac{1}{3}\Big[(1/|\mathcal{K}|)c^{o,s}_{MC} + c^{o,r\&u}_{MC} + c^{o,v}_{MC} + \\ + \Xi^{l\&m}_{MC}\left(\lceil B_k/B_0\rceil c^{c,rf}_{MC,k} + \lceil m_k(B_k/B_0)\rceil c^{c,bp}_{MC,0}\right) + c^{o,bh}_{MC,k}\Big], \tag{25}$$

$$c^{cpx}_{SC,k} = \pi_{SC,k}c^{c,s}_{SC} + c^{c,a}_{SC,k} + c^{c,f}_{SC,k} + c^{c,bh}_{SC,k}, \tag{26}$$

$$c^{opx}_{SC,k} = c^{o,s}_{SC} + c^{o,r\&u}_{SC} + c^{o,v}_{SC} + \Xi^{l\&m}_{SC}c^{c,a}_{SC,k} + c^{o,bh}_{SC,k}, \tag{27}$$

$$c^{spec}_{SC,k} = c^{spec}_0(B_k - \widehat{B}_k)A_{MC}L, \tag{28}$$

where the cost terms that make up $c^{cpx}_{MC,k}$, $c^{cpx}_{SC,k}$, $c^{opx}_{MC,k}$ and $c^{opx}_{SC,k}$ and values given to these cost terms are based on the cost model and respective values in [10]. Notice that in Equations (24) and (25), the 1/3 multiplier has been introduced to derive the cost per MC sector since each MC cost term therein refers to the total cost of all three sectors of a 3-sector MC site. As for Equations (26) and (27), the values obtained from [10] for the cost terms $c^{c,s}_{SC}$, $c^{c,f}_{SC,k}$, $c^{o,s}_{SC}$, $c^{o,r\&u}_{SC}$, $c^{o,v}_{SC}$ (which will be defined in the consecutive paragraphs) are costs per site for 2-sector small cell sites but also for 1-sector picocell sites whereas the values for $c^{c,a}_{SC,k}$ are costs per sector for 2-sector small cell sites, hence we deemed all these values to be reasonable also for the 1-sector SC sites considered here without introducing any multipliers.

We now define the individual cost terms involved in Equations (24)–(28) and explain how their values have been set in order to characterize the InPs considered here. First, $c^{c,s}_{MC}(c^{c,s}_{SC})$ denotes the MC(SC) site civil works and acquisition cost. $c^{c,s}_{MC}$ is weighted by π_{MC} and divided by the number of InPs since we assume that each MC site will be shared by all InPs[17]. In turn, for each InP k, $c^{c,s}_{SC}$ is weighted by the probability that k has to build a SC by itself at the beginning of the studied period ($\pi_{SC,k}$) given that SC sites are not shared. In these lines, also the MC site rental cost $c^{o,s}_{MC}$ is uniformly split among all InPs (see Equation (25)), whereas the SC site rental cost $c^{o,s}_{SC}$ is not (see Equation (27)).

$c^{c,a}_{MC,k}(c^{c,a}_{SC,k})$ is the MC(SC) antenna cost for InP k. $c^{c,f}_{MC,k}(c^{c,f}_{SC,k})$ are the feeder (cable connecting active antenna to equipment cabinet), install and test and commission cost per MC(SC) site for InP k. $c^{c,rf}_{MC,k}$ is a baseline Radio Frequency (RF) front end cost per MC site for a baseline bandwidth $B_0 = 20$ MHz for InP k which has to be scaled by $\lceil B_k/B_0\rceil$

[13]We refer to the average spectral efficiency definition in [37] or equivalently to the cell spectral efficiency definition in [38].

[14]When the average spectral efficiency is considered and the SPs' users can be assumed uniformly distributed over the considered geographical area, the same solution should apply to all SCs in the area.

[15]$\underline{P}_k$ is defined as a monthly cost to match the timescale of the SPs' user fee (see Section IV-C). Consequently, the InP price strategy $P_k \geq \underline{P}_k$ (see Section III-D2, is a monthly price per unit of average SC capacity.) In these lines, the payoff of each InP $k \in \mathcal{K}$, i.e., G_k, (as defined by Equation (22)) and the payoff of each SP $v \in \mathcal{V}$, i.e., g_v, (as defined by Equation (21)) also correspond to a one-month period.

[16]Notice that for SCs, we refer to the site costs since we consider 1-sector SC sites.

[17]Notice thatif there is at least one InP k with $\lambda_k = 1$, for which $\pi_{MC,k} = 0$, and hence $\pi_{MC} = \min_{k\in\mathcal{K}} \pi_{MC,k} = 0$, a site is already present in each MC candidate site hence $c^{c,s}_{MC}$ is not incurred.

(i.e., the ratio between the total bandwidth of InP k and the baseline bandwidth), whereas $c^{c,bp}_{MC,0}$ is a baseline baseband processing cost for 3 sectors of a $B_0 = 20$ MHz 2x2 MIMO channel (see Section 11.5.1.3 in [10]) that needs to be scaled by $\lceil m_k(B_k/B_0)\rceil$, where m_k is a factor[18] that allows to estimate the relative amount of base band processing for InP k for $B_0 = 20$ MHz units of bandwidth given its antenna MIMO order w.r.t. to the baseline (i.e., the $B_0 = 20$ MHz 2x2 MIMO channel). Notice that we have introduced the ceiling operator in the scaling factors of $c^{c,rf}_{MC,k}$ and $c^{c,bp}_{MC,0}$ in order to be conservative as in [10] there is no explicit expression of the cost scaling operation for none of the two.

In the following, we explain how starting from the cost model in [10], we set the values of $c^{c,a}_{MC,k}$, $c^{c,f}_{MC,k}$, $c^{c,rf}_{MC,k}$, m_k, $c^{c,a}_{SC,k}$ and $c^{c,f}_{SC,k}$ for each InP k depending on its type. For an InP k of type $\mathcal{L}$ ($\lambda_k = 1$), we have considered the following values for the MC and SC average spectral efficiency for both DL and UL: $\nu_{k,MC} = 2.2$ bps/Hz and $\nu_{SC,k} = 2.6$ bps/Hz, which are the required DL average spectral efficiency values for IMT-Advanced systems for base urban coverage and microcellular environments, respectively [38]. Instead, for an InP k of type $\mathcal{N}$ ($\lambda_k = 0$), we have set $\nu_{MC,k} = 6.6$ bps/Hz and $\nu_{SC,k} = 7.8$ bps/Hz as ITU-R expects the average spectral efficiency for IMT for 2020 and beyond to be three times higher than for IMT-Advanced [4],[37]. Since the 5G radio interface has not been defined yet, we cannot anticipate the spectral efficiency improvements it will bring about, hence we have assumed that the required spectral efficiency for IMT for 2020 and beyond will be achieved through high order MIMO antennas, although there are several factors that affect the achieved spectral efficiency [39]. We have considered the antenna configurations (MIMO order + frequency band) presented in [10] and, when possible, for each InP k we have selected antenna configurations that would best match its MC(SC) average spectral efficiency $\nu_{MC,k}$ ($\nu_{SC,k}$), otherwise we have associated[19] InPs of type $\mathcal{L}/\mathcal{N}$ with the least/most complex (and hence expensive) antenna configurations while some of the cost terms for InPs of type $\mathcal{N}$ have also been over-estimated so as to account for the factor-of-three difference between the average spectral efficiency of type $\mathcal{N}$ and $\mathcal{L}$ InPs. For instance, to choose the MC antenna configurations among those listed in [10], we were mainly driven by their respective average spectral efficiency values: the different 2x2 MIMO operation modes provide average spectral efficiency values in the range 2.23 – 2.88 bps/Hz whereas 64x2 MIMO ones provide average spectral efficiency values in the range 5.53 – 7.14 bps/Hz which makes the former suitable for an InP k of type $\mathcal{L}$ ($\nu_{MC,k} = 2.2$ bps/Hz), whereas the latter suitable for an InP k of type $\mathcal{N}$ ($\nu_{MC,k} = 6.6$ bps/Hz). Instead, for SCs, as the average spectral efficiencies of the two configurations listed in [10] are not provided, we associate InPs of type $\mathcal{L}/\mathcal{N}$ with the lowest/highest MIMO order configuration. Let M denote the MIMO order of an antenna. Specifically, for each InP k of type $\mathcal{L}$, we have assumed that its MCs operate only at sub-1GHz and low frequency bands with $M = 2$ antennas, whereas its SCs operate at low and medium unpaired bands with $M = 2$ antennas. Instead, for each InP of type $\mathcal{N}$, we have assumed that its MCs operate both at sub-1GHz and low frequencies with $M = 4$ antennas and at medium frequencies with $M = 64$ antennas, whereas its SCs operate at low and medium frequency bands with $M = 4$ antennas. Values of $c^{c,a}_{MC,k}$, $c^{c,f}_{MC,k}$, $c^{c,rf}_{MC,k}$, m_k, $c^{c,a}_{SC,k}$ and $c^{c,f}_{SC,k}$ depending on the antenna configuration(s) associated with the type of InP k are then set as reported in Table II based on [10]. Some details concerning these values follow. According to [10], for sub-1GHz and low frequencies multiband MC antennas are available, hence an InP k of type $\mathcal{L}$ ($\lambda_k = 1$), deploys only one $M = 2$ antenna per MC site. Instead, as MCs of an InP k of type $\mathcal{N}$ operate at two frequency band groups (i.e., sub-1GHz and low frequency bands and medium frequency bands) that require individual radio equipment, we set $c^{c,a}_{MC,k}$ equal to the sum of the antenna cost of the two frequency band groups and $c^{c,f}_{MC,k}$ equal to the sum of feeder, install and test and commission cost of the two frequency band groups. In turn, $\lceil B_k/B_0\rceil c^{c,rf}_{MC,k}$ and $\lceil m_k(B_k/B_0)\rceil c^{c,bp}_{MC,0}$ for k of type $\mathcal{N}$ have been overestimated by setting $c^{c,rf}_{MC,k}$ and m_k equal to the respective values for $M = 64$ antennas at medium frequency bands (which are both higher than the respective values for $M = 4$ antennas at sub-1GHz and low frequency bands).

$c^{o,r\&u}_{MC}$($c^{o,r\&u}_{SC}$) denotes the annual rates and utilities for a MC(SC) site, whereas $c^{o,v}_{MC}$($c^{o,v}_{SC}$) the annual vendor service fee. The annual licensing and maintenance cost per MC(SC) site are calculated as fraction $\Xi^{l\&m}_{MC}$($\Xi^{l\&m}_{SC}$) of the active equipment cost which in case of MC sites corresponds to the sum of the total RF front end cost ($\lceil (B_k/B_0) c^{c,rf}_{MC,k}\rceil$) and the total base band processing cost ($\lceil m_k(B_k/B_0) c^{c,bp}_{MC,0}\rceil$) whereas in case of SC sites it corresponds to the antenna cost ($c^{c,a}_{SC,k}$) as the RF front end and the baseband processing unit are part of the integrated active equipment [10].

In particular, the MC(SC) CAPEX and OPEX backhauling cost of InP k, $c^{c,bh}_{MC,k}$($c^{c,bh}_{SC,k}$), $c^{o,bh}_{MC,k}$($c^{o,bh}_{SC,k}$) depend on the type of backhauling selected by k. We have considered the set of backhauling options presented in [10] which we denote as $\mathcal{T}$. For each option $t \in \mathcal{T}$, the capacity (C^{bh}_t), CAPEX ($c^{c,bh}_t$) and annual OPEX ($c^{o,bh}_t$) per backhauling link are reported in Table III. We assume that each InP has deployed individual backhauling links for the SCs and MCs, i.e., there is no aggregation of the traffic of the SCs at the underlying MC site. Then, each InP k determines its best (minimum cost) option for the SCs, denoted by $t^*_{SC,k}$, as

$$t^*_{SC,k} = \arg\min_{t\in\mathcal{T}} \left\{ \left\lceil \frac{\nu_{SC,k}\, B_k}{C^{bh}_t} \right\rceil (c^{c,bh}_t + L c^{o,bh}_t) \right\}, \qquad (29)$$

hence $c^{c,bh}_{SC,k} = \left\lceil \left(\nu_{SC,k} B_k\right) / C^{bh}_{t^*_{SC,k}} \right\rceil c^{c,bh}_{t^*_{SC,k}}$ and $c^{o,bh}_{SC,k} = \left\lceil \left(\nu_{SC,k} B_k\right) / C^{bh}_{t^*_{SC,k}} \right\rceil c^{o,bh}_{t^*_{SC,k}}$. Similarly, the best backhauling

[18] Let $\mathcal{M}_k$ denote the product of the number of MIMO streams with the number of the spatial beams (see Section 11.4.2. in [10]) that correspond to MC antenna MIMO order for InP k and let $\mathcal{M}_0$ denote the value of this product for the baseline 2x2 MIMO channel where $\mathcal{M}_0 = 2$ (see Table 11-9 in [10]). We then set $m_k = \mathcal{M}_k/\mathcal{M}_0$ (see Section 11.5.1.3 in [10]).

[19] It is worth pointing out that the aforementioned association of InPs to antenna configurations based on their types has not been used for a network deployment simulation but it only serves to obtain an estimate of the cost incurred by an InP for providing a certain average cell capacity based on its available bandwidth and average spectral efficiency.

option $t^*_{MC,k}$ for the MC sites (i.e., for all three sectors per site) for InP k is determined as

$$t^*_{MC,k} = \arg\min_{t\in\mathcal{T}} \left\{ \left\lceil \frac{3\nu_{MC,k}\, B_k}{C^{bh}_t} \right\rceil (c^{c,bh}_t + Lc^{o,bh}_t) \right\}, \quad (30)$$

therefore, $c^{c,bh}_{MC,k} = \left\lceil (3\nu_{MC,k} B_k) / C^{bh}_{t^*_{MC,k}} \right\rceil c^{c,bh}_{t^*_{MC,k}}$ and $c^{o,bh}_{MC,k} = \left\lceil (3\nu_{MC,k} B_k) / C^{bh}_{t^*_{SC,k}} \right\rceil c^{o,bh}_{t^*_{MC,k}}$.

Finally, in Equation (28), c^{spec}_0 denotes the reference annual spectrum license cost per unit of bandwidth and unit of geographical area which, multiplied by the amount of bandwidth associated with the spectrum license acquired at the beginning of the studied period ($B_k - \widehat{B}_k$), the area of the SC (A_{SC}) and the studied period (L), provides the spectrum license cost $c^{spec}_{SC,k}$ per SC for the studied period. c^{spec}_0 was derived from the outcome of the 5G spectrum auction in the UK [40] by first calculating the average cost per MHz of the total auctioned spectrum and then dividing the latter with the area of the UK and the license duration (20 years) [41].

Values given to the cost terms and related parameters throughout Equations (24)–(28) are summarized in Table II. Notice that we have not considered cost inflation over time and that all values obtained from [10] and [40], originally in GBP currency, have been converted to EUR using a conversion rate 1.11 EUR/GBP.

B. Service types

In this work, we address the provision of two types of 5G services motivated by two usage scenarios identified by ITU-R for IMT for 2020 [4], namely eMBB and mMTC. We have characterized these services using KPIs of the 5GPPP project FANTASTIC-5G ([14]) for use cases defined therein. Specifically, for eMBB we consider the KPIs of use case 7 (dense urban society below 6 GHz) in [14], whereas for mMTC, KPIs of use case 3 (sensor networks) in [14]. Let $d_{eMBB}(d_{mMTC})$ denote the density of devices that request eMBB(mMTC) services. We have set $d_{\text{eMBB}} = 25000$ devices/km^2 and $d_{\text{mMTC}} = 600000$ devices/km^2 according to the device density values considered in [14] for the respective use cases. The average number of devices in the area of one SC that request services of a given type can be determined as the product of the device density with the area of the SC, i.e., $d_{eMBB}A_{SC}$ for eMMB and $d_{mMTC}A_{SC}$ for mMTC. In turn, the area of a MC sector (A_{MC}) and the area of a SC (A_{SC}), have been derived from their respective inter site distances, D_{MC} and D_{SC}. As mentioned, each MC site has three sectors. We also assume that the cells of a MC site (one per sector) are hexagonal and that MC sites are located at the corner of these cells, therefore the MC inter site distance (D_{MC}) is equal to three times the side of a hexagonal cell [10], [35], [36]. Instead, SC antennas are assumed to be omnidirectional hence the SC inter site distance (D_{SC}) is equal to twice the cell radius. Therefore $A_{MC} = (1/(2\sqrt{3}))D^2_{MC}$ and $A_{SC} = (1/4)\pi D^2_{SC}$. Values $D_{MC} = 0.5$ km and $D_{SC} = 0.05$ km have been used as suggested in [14] for a urban area for both use cases 3 and 7 therein.

C. SPs

We consider three market segments for eMBB services, each served by a unique SP, while a fourth SP provisions mMTC services. eMBB services are characterized only by their DL demand; the UL demand, being generally much lower, is assumed to be equal to zero. Instead, mMTC services are characterized only by their UL demand, as they are mainly UL biased [14], while their DL demand is set equal to zero. Values given to parameters characterizing the service and the users of each SP are reported in Table IV.

Concerning the user utility function (see Section III-B1), for SPs providing eMBB services, we set $\underline{\mathcal{X}}_v$ equal to the required value for the user experienced data rate[20] in the DL for use case 7 in [14] (same for all market segments), whereas $\overline{\mathcal{X}}_v$ varies across the eMBB market segments as reported in Table IV assuming users of different market segments have different target rates. For the fourth SP (which provides mMTC services), we set $\underline{\mathcal{X}}_4$ and $\overline{\mathcal{X}}_4$ equal to the minimum and maximum required value for the user experienced data rate in the UL for use case 3 in [14], respectively. Further, the elasticity parameter ξ_v was set to 2 for all eMBB SPs (minimum value considered in [15]) and to 20 for the mMTC SP (maximum value considered in [15]) to account for the fact that the eMBB traffic is more elastic than the mMTC one.

As for the acceptance probability function (see Section III-B2), we set the user sensitivity to changes in utility equal to the value considered in [15] for all SPs, i.e., $\mu_v = 2$, but we vary the user sensitivity to changes in the offered fee (ε_v) across SPs as reported in Table IV. We assume that mMTC users have a high sensitivity to changes in the offered fee ($\varepsilon_4 = 4$, which is the value considered in [15]), while the eMBB market segments served by SPs 1, 2 and 3 are assumed to have low, medium and high sensitivity to changes in the offered fee, respectively, represented by values $\varepsilon_1 = 2$, $\varepsilon_2 = 3$ and $\varepsilon_3 = 4$. Given that the considered utility function u_v is such that $0 \le u_v(x_v) \le 1$ for any x_v (see Equation (1)), then, by definition, the maximum utility level is equal to 1 for all SPs, i.e., $\overline{u}_v = 1$. It is reasonable to assume that each SP v will tailor its reference offered fee $\overline{p}_v$ to the service requirements of its own users (represented by the utility function here), hence we set $\overline{p}_v = 0.4\mathcal{X}_v\,(1 + 1/\xi_v)$. Recall that $\mathcal{X}_v$ is the rate value that provides a user of SP v with a utility value equal to 0.5 and that $\mathcal{X}_v = \underline{\mathcal{X}_v} + \left(\overline{\mathcal{X}}_v - \underline{\mathcal{X}}_v\right)\left((1-U)/U\right)^{1/\xi_v}$, where $U = 0.999$ has been considered (see Section III-B1). We set the values of $\overline{q}_v$ as reported in Table IV. We make the following assumptions on the behavior of the rejection probability $\overline{q}_v$ as a function of μ_v, ε_v, $\overline{u}_v$ and $\overline{p}_v$:

(i) for any two SPs $v, w \in \mathcal{V}$, such that $\mu_v = \mu_w$, $\varepsilon_v = \varepsilon_w$, $\overline{u}_v = \overline{u}_w$, we assume $\overline{q}_v = \overline{q}_w$ even if $\overline{p}_v \neq \overline{p}_w$, i.e., when users of v and w are equally sensitive to changes in utility and in the offered fee and they perceive a maximum level of utility, we expect them to reject the considered reference offered fee $\overline{p}_v$ and $\overline{p}_w$, respectively, with the same probability, since $\overline{p}_v$ and $\overline{p}_w$ reflect their respective service requirements;

[20]In [14], the user experienced data rate is defined as the 5 percentile user rate hence we use its required value as a minimum for the average user rate.

Notation	Definition	Unit
λ_k	binary parameter: 1 if InP k of type $\mathcal{L}$, 0 if of type $\mathcal{N}$	—
$\mathcal{L}$	label to represent an InP that has a legacy network & does not upgrade to 5G	—
$\mathcal{N}$	label to represent an InP that deploys a 5G network at the beginning of the studied period	—
B_k	total available bandwidth of InP k	MHz
$\widehat{B}_k$	amount of bandwidth of InP k associated with an amortized spectrum license	MHz
$\pi_{SC,k}$	probability that InP k has not deployed a legacy SC in a SC candidate site	—
$\pi_{MC,k}$	probability that InP k has not deployed a legacy MC in a MC candidate site	—
π_{MC}=$\min_{k\in\mathcal{K}}\pi_{MC,k}$	probability that a site has to be built in a MC candidate site	—
A_{MC}	area of a MC sector	km^2
A_{SC}	area of a SC	km^2
$c^{cpx}_{MC,k}$	per sector total CAPEX of a 3-sector MC site for InP k	EUR
$c^{cpx}_{SC,k}$	total CAPEX of a SC site for InP k	EUR
$c^{opx}_{MC,k}$	per sector total annual OPEX of a 3-sector MC site for InP k	EUR/year
$c^{opx}_{SC,k}$	total annual OPEX of a SC site for InP k	EUR/year
$c^{spec}_{SC,k}$	spectrum license cost normalized to A_{SC} and L for InP k	EUR
$\underline{P}_k$	monthly overall cost incurred by InP k to provide one unit (1 Mbps) of capacity	EUR/Mbps/month

TABLE I: InP related parameters

Notation	Definition	Value	Unit
B_0	baseline bandwidth	20	MHz
L	duration of the studied period	10	years
$c^{c,s}_{MC}$	site civil works & acquisition cost for a MC site	51282	EUR
$c^{c,a}_{MC,k}$	total antenna cost per MC site for InP k		
	$\lambda_k=1$: M=2 antenna at sub-1GHz & low bands	1776	EUR
	$\lambda_k=0$: M=4 antenna at sub-1GHz & low bands + M=64 antenna at medium band	10656	EUR
$c^{c,f}_{MC,k}$	total feeder, install, test and commission costs for all antennas of a MC site for InP k		
	$\lambda_k=1$: M=2 antenna at sub-1GHz & low bands	4884	EUR
	$\lambda_k=0$: M=4 antenna at sub-1GHz & low + M=64 antenna at medium band	9768	EUR
$c^{c,rf}_{MC,k}$	RF front end cost per 20MHz bandwidth per MC site for InP k		
	$\lambda_k=1$: M=2 antenna at sub-1GHz & low bands	12487.5	EUR
	$\lambda_k=0$: M=64 antenna at medium band	39960	EUR
$c^{c,bp}_{MC,0}$	baseline baseband processing cost for a 3-sector MC with 2×2 MIMO for each 20 MHz	4162.5	EUR
m_k	scaling factor for baseband processing cost (see Table 11-9 and Section 11.5.1.3 in [10])		
	$\lambda_k=1$: M=2 antenna configuration	1	–
	$\lambda_k=0$: M=64 antenna configuration	6	–
$c^{o,s}_{MC}$	annual MC site rental cost	22200	EUR
$c^{o,r\&u}_{MC}$	annual rates and utilities cost for a MC site	11100	EUR
$c^{o,v}_{MC}$	annual vendor services cost for a MC site	3552	EUR
$\Xi^{l\&m}_{MC}$	fraction of active equipment cost (total RF front end + BBU processing cost) to calculate annual licensing & maintenance cost for a MC site	0.1	–
$c^{c,s}_{SC}$	site civil works and acquisition cost for a SC site	5328	EUR
$c^{c,a}_{SC,k}$	antenna cost per SC site for InP k		
	$\lambda_k=1$: M=2 antenna at low & medium unpaired bands	277.5	EUR
	$\lambda_k=0$: M=4 antenna at low & medium unpaired bands	555	EUR
$c^{c,f}_{SC,k}$	feeder, install, test and commission costs per SC site for InP k		
	$\lambda_k=1$: M=2 antenna at low & medium unpaired bands	777	EUR
	$\lambda_k=0$: M=4 antenna at low & medium unpaired bands	777	EUR
$c^{o,s}_{SC}$	annual SC site rental cost	1110	EUR
$c^{o,r\&u}_{SC}$	annual rates and utilities cost for a SC site	599.4	EUR
$c^{o,v}_{SC}$	annual vendor services cost for a SC site	0	EUR
$\Xi^{l\&m}_{SC}$	fraction of active equipment cost (SC antenna cost) to calculate annual licensing & maintenance cost for a SC site	0.25	–
c^{spec}_0	spectrum license cost per MHz, unit of area and year [40], [41]	1.6331	EUR/MHz/km^2/year

TABLE II: Cost model parameters

Backhauling type (t)	Capacity per link (C^{bh}_t)	CAPEX ($c^{c,bh}_t$)	annual OPEX ($c^{o,bh}_t$)
dark fiber (1 Gbps)	1 Gbps	35409 EUR	1248.75 EUR
dark fiber (10 Gbps)	10 Gbps	36630 EUR	1248.75 EUR
dark fiber (100 Gbps)	100 Gbps	39405 EUR	1248.75 EUR
Ethernet Access Direct (EAD) Managed	1 Gbps	2331 EUR	3496.5 EUR

TABLE III: Capacity and cost of different backhauling options [10]

(ii) for any given μ_v, $\overline{u}_v$ and $\overline{p}_v$, we expect $\overline{q}_v$ to be non-decreasing in ε_v, $\lim_{\varepsilon_v\to 0}\overline{q}_v(\mu_v,\varepsilon_v,\overline{u}_v,\overline{p}_v)=0$ and $\lim_{\varepsilon_v\to\infty}\overline{q}_v(\mu_v,\varepsilon_v,\overline{u}_v,\overline{p}_v)=1$ for each $v\in\mathcal{V}$.

In [42]–[46], the normalizing constant A (see Equation (2) and Section III-B2) is set equal to 0.1 for $\varepsilon=4$, $\mu=2$, $\overline{p}=1$ and $\overline{u}=1$, therefore the corresponding reference rejection probability $\overline{q}=e^{-A}\approx 0.9$. We then set $\overline{q}_v=0.9$ for any SP v with $\varepsilon_v=4$, $\mu_v=2$, $\overline{u}_v=1$ and the considered $\overline{p}_v$, in line with assumption (i), whereas for SPs with ε_v equal to 3 and 2 we set $\overline{q}_v$ equal to 0.6 and 0.3, respectively, as per assumption (ii).

Concerning the number of users or, alternatively, devices[21] subscribing to each SP, first let σ_v denote the market share of SP v for the service offered by v. We assume that the eMBB market segment served by SP 1 makes up 20% of the eMBB market (i.e., $\sigma_1 = 0.2$), whereas the eMBB market segments served by SPs 2 and 3 make up 30% and 50%, respectively (i.e., $\sigma_2 = 0.3$ and $\sigma_3 = 0.5$). SP 4 is assumed to serve the entire mMTC market (i.e., $\sigma_4 = 1$). Then, the number of devices in the area of a SC that have subscribed to each SP are: $N_v = \sigma_v d_{eMBB} A_{SC}$, for any $v \in \{1, 2, 3\}$ (eMBB SPs), and $N_4 = \sigma_4 d_{mMTC} A_{SC}$ (mMTC SP), where d_{eMBB} and d_{mMTC} denote the eMBB and mMTC device density, respectively, whereas A_{SC} the area of a SC (see Section IV-B). Further, for eMBB SPs we consider a device activity factor equal to 0.1, i.e., $\eta_v = 0.1$ for any $v \in \{1, 2, 3\}$ as in [5], [14], whereas for the mMTC SP we assume $\eta_4 = 0.01$ (as sensors tend to become active less often).

D. Instances

In our numerical tests, for InPs of type $\mathcal{N}$, i.e., for InPs which deploy a 5G network, we have considered two particular cases, labeled as $\mathcal{N}^{(1)}$ and $\mathcal{N}^{(2)}$. Specifically, $\mathcal{N}^{(1)}$ refers to an InP k for which:

(1) $B_k \geq \widehat{B}_k = 20$ MHz, i.e., k has amortized the spectrum license cost of 20 MHz of bandwidth from its total available (B_k) and it may have acquired a new spectrum license (if $B_k - \widehat{B}_k > 0$);

(2) $\pi_{MC,k} = 0.3$ and $\pi_{SC,k} = 0.5$, i.e., k has not deployed a legacy MC BS in a MC candidate site with probability equal to 0.3 and analogously for SCs for which such probability is assumed equal to 0.5.

In turn, $\mathcal{N}^{(2)}$ refers to an InP k for which:

(1) $B_k > \widehat{B}_k = 0$, i.e., k does not own any spectrum license whose cost has been amortized but has acquired a new spectrum license of B_k units of bandwidth;

(2) $\pi_{MC,k} = \pi_{SC,k} = 1$, i.e., no legacy MC/SC BSs of k are present in any of the MC/SC candidate sites or, in other words, k has not previously deployed a legacy network.

Instead, as mentioned in Section IV-A, for an InP k of type $\mathcal{L}$ which does not upgrade to 5G we assume:

(1) $B_k = \widehat{B}_k > 0$, i.e., k has amortized the spectrum license cost of all its available bandwidth, meaning that k does not acquire any new spectrum licenses;

(2) $\pi_{MC,k} = \pi_{SC,k} = 0$, i.e., k has deployed legacy MC/SC BSs in all available MC/SC candidate sites hence it does not deploy additional MCs and SCs during the studied period.

We then set up several instances with two InPs ($|\mathcal{K}| = 2$) and four SPs ($|\mathcal{V}| = 4$). Across these instances, we vary the type and total available bandwidth of the two InPs, but consider the same set of four SPs (as described in Section IV-C). The instances are described and labeled in Table V where, e.g., for the instance labeled as A10, the first InP is of type $\mathcal{N}^{(1)}$ and its total available bandwidth B_1 is equal to 100 MHz, whereas the second InP is of type $\mathcal{L}$ and $B_2 = 100$ MHz.

[21] We use the terms device and user interchangeably.

E. Computational tests

The proposed framework was implemented in Matlab, whose solvers have been used in the implementation to calculate $\overline{X}_{vk}(P_k)$ and $\underline{X}_{vk}(P_k)$ according to Equations (8) and (9), respectively, and to determine an optimal solution of the capacity assignment problem formulated as a two-step optimization problem (see Section III-C).

The value of $\overline{P}$ (i.e., the minimum monthly price per unit of average SC capacity which is unprofitable for all SPs) has been determined as follows: for each SP v, let $\overline{P}_v$ denote the minimum value of P for which $\overline{X}_v(P) = 0$ (see Equation (8)) and let $\overline{P}^{\circ}_v$ denote an upper bound for $\overline{P}_v$ (which we calculate through a heuristic that provides $\overline{P}^{\circ}_v \leq \overline{P}_v + 0.001$); we then set $\overline{P} = \max_{v \in \mathcal{V}} \overline{P}^{\circ}_v$.

To solve the MLFG numerically, we have discretized the continuous InP price strategy sets $\mathcal{P}_k = [\underline{P}_k, \overline{P}]$, (see Section III-D2), i.e., hereon, $\mathcal{P}_k = \{\underline{P}_k, \ldots, \overline{P}\}$, for any $k \in \mathcal{K}$. Consequently, the resulting set of InP price profiles $\mathcal{P} = \prod_{k \in \mathcal{K}} \mathcal{P}_k$ is also discrete and finite. We determine the Subgame Perfect Equilibrium(a) (SPE) [34] of the two-stage MLFG as follows:

(1) for each InP price profile $\boldsymbol{P} \in \mathcal{P}$, we look for the NE in pure strategies of the corresponding SPs' game, i.e., for $\breve{\boldsymbol{y}}(\boldsymbol{P})$ of $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$, (see Section III-D1) from which we can calculate the payoff $G_k(\boldsymbol{P})$ of each InP for the price profile $\boldsymbol{P}$ according to Equation (22) – if there are multiple NE in pure strategies for $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$, then $G_k(\boldsymbol{P})$ is set equal to the minimum payoff attained by k among all these NE;

(2) we look for the NE in pure strategies[22] of the InPs' game, i.e., for $\breve{\boldsymbol{P}}$ of $\mathcal{G}^{\mathcal{K}}$ (see Section III-D2).

The NE of $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$ and of $\mathcal{G}^{\mathcal{K}}$ were determined through exhaustive search. In the definition of the NE in pure strategies for $\mathcal{G}^{\mathcal{K}}$ and for $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$ we have introduced an absolute margin $\Delta = 10^{-6}$ EUR (recall that the payoffs G_k and g_v are all given in EUR). For instance, the InP price profile $\breve{\boldsymbol{P}}$ is an NE of the $\mathcal{G}^{\mathcal{K}}$ iff

$$G_k([\breve{P}_k, \breve{\boldsymbol{P}}_{-k}]) \geq G_k([P_k, \breve{\boldsymbol{P}}_{-k})] - \Delta, \quad \forall\ P_k \in \mathcal{P}_k,\ \forall\ k \in \mathcal{K}, \tag{31}$$

where $\breve{\boldsymbol{P}}_{-k}$ denotes the prices of all other InPs but k. Δ was introduced to account for the inaccuracy caused by inherent tolerances of the Matlab solvers.

Concerning the discretization of the originally continuous InP unit price strategy sets $\mathcal{P}_k = [\underline{P}_k, \overline{P}]$, we initially created a unit price strategy set consisting of 30 logarithmically-spaced values in the range $[\underline{P}_k, \overline{P}]$. The MLFG resulting from these discrete InP unit price strategy sets has at least one SPE for all instances but B4 and B5. Instead, for both B4 and B5, although there is at least one NE in pure strategies for each SPs' game,

[22] As mentioned, the price strategy set $\mathcal{P}_k$ of each InP $k \in \mathcal{K}$ is discrete which means that the InPs' game $\mathcal{G}^{\mathcal{K}}$ is formally a non-cooperative game in strategic form, hence we look for its NE in pure strategies.

v	service type	utility function			acceptance probability			# users & activity factor		
1	eMBB	$\underline{\mathcal{X}}_1$=50 Mbps	$\overline{\mathcal{X}}_1$=5000 Mbps	ξ_1=2	μ_1=2	ε_1=2	$\overline{q}_1$=0.3	σ_1=0.2	N_1=9.82	η_1=0.1
2	eMBB	$\underline{\mathcal{X}}_2$=50 Mbps	$\overline{\mathcal{X}}_2$=2500 Mbps	ξ_2=2	μ_2=2	ε_2=3	$\overline{q}_2$=0.6	σ_2=0.3	N_2=14.73	η_2=0.1
3	eMBB	$\underline{\mathcal{X}}_3$=50 Mbps	$\overline{\mathcal{X}}_3$=500 Mbps	ξ_3=2	μ_3=2	ε_3=4	$\overline{q}_3$=0.9	σ_3=0.5	N_3=24.54	η_3=0.1
4	mMTC	$\underline{\mathcal{X}}_4$=0.00016 Mbps	$\overline{\mathcal{X}}_4$=1 Mpbs	ξ_4=20	μ_4=2	ε_4=4	$\overline{q}_4$=0.9	σ_4=1	N_4=1178.10	η_4=0.01

TABLE IV: Parameters characterizing the service and the users of each SP.

	(B_1, B_2)										
	(20,20)	(20,60)	(60,20)	(60,60)	(80,80)	(20,100)	(40,100)	(60,100)	(80,100)	(100,100)	(120,100)
$(\mathcal{N}^{(1)}, \mathcal{L})$	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11
$(\mathcal{N}^{(2)}, \mathcal{N}^{(1)})$	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	B11

TABLE V: Instances and respective labels

there is no NE in pure strategies for the InPs' game and thus no SPE for the MLFG. Then, for both B4 and B5, for each InP k, we created an alternative unit price strategy set consisting of 60 values in the range $[\underline{P}_k, \overline{P}]$ with the majority of these values in price ranges where we expected the NE of $\mathcal{G}^{\mathcal{K}}$ to be based on the best response mappings of $\mathcal{G}^{\mathcal{K}}$ resulting from the initial discrete InP unit price strategy sets (see Section V-B). As there was no NE for $\mathcal{G}^{\mathcal{K}}$ neither for B4 nor for B5 even for the MLFG resulting from the alternative discrete InP unit price strategy sets, we settled on suggesting as a solution for $\mathcal{G}^{\mathcal{K}}$ an InP unit price profile $\boldsymbol{P}^{\diamond}$ with a small (0.53% for B4 and 3.89% for B5) maximum relative payoff difference from the InPs' best response (see Equation (40) and Section V-B for details).

V. Numerical results analysis

In this section, we report and analyze numerical results concerning the equilibrium(a) of the considered problem for the instances defined in Section IV-D. To start with, in Section V-A we explain the notation used in reporting these results. Then, in Section V-B we discuss the existence and multiplicity of equilibria across these instances. Instead, in Section V-C we analyze the impact of the InPs' network technology and available spectrum bandwidth on the equilibrium strategies of the players, i.e., on the capacity unit price offered by the each InP and the InP choice of each SP.

A. Notation summary

For the sake of brevity, hereon we will simplify the terminology as follows:

- the term equilibrium will refer to equilibrium of the overall game, i.e., to the *sub-game perfect equilibrium* of the MLFG which consists of the Nash Equilibrium InP capacity unit price profile of the InPs' game, i.e., $\breve{\boldsymbol{P}}$ of $\mathcal{G}^{\mathcal{K}}$ and of the Nash Equilibrium SPs' choice of InP of the SPs' game resulting from $\breve{\boldsymbol{P}}$, i.e., $\breve{\boldsymbol{y}}(\breve{\boldsymbol{P}})$ of $\mathcal{G}^{\mathcal{V}}(\breve{\boldsymbol{P}})$, where both are Nash Equilibria in pure strategies;
- the term capacity will refer to the average SC capacity of an InP;
- the term spectral efficiency will refer to the average SC spectral efficiency of an InP;
- the term unit cost will refer to the total monthly cost per unit of average SC capacity of an InP;
- the term unit price will refer to the monthly price per unit of average SC capacity offered by an InP at the equilibrium.

For all considered instances A1–A11 and B1–B11, the values of the main parameters characterizing the InPs and the SPs and the equilibrium outcomes are reported in Tables VII–XIV, where Tables VII, IX, XI and XIII concern the InPs, whereas Tables VIII, X, XII and XIV, the SPs. The definitions and unit of measurements of the notation used across Tables VII–XIV are provided in Table VI. When reporting numerical values in the text, the respective units of measurement have been omitted. Notice also that:

- in Tables VIII, X, XII and XIV, for each SP v (column two), column three reports the InP selected by v at the equilibrium, i.e., k for which $\breve{y}_{vk}(\breve{\boldsymbol{P}}) = 1$; in particular, the symbol – has been reported in all the columns starting from the third one for each SP for which it is not profitable to purchase capacity from any of the InPs at their equilibrium capacity unit prices and hence it cannot provide services to its users;
- in Tables VII–XIV values for $\underline{P}_k$, $\breve{P}_k$, $\breve{G}_k$, $a^*_v p^*_v(\breve{u}_v)$, $\breve{g}_v$ and $\breve{r}^*_v/\breve{x}_{vk}$ are reported rounded to two decimals, whereas values for C_k, $\breve{C}'_k$, $\underline{X}_v(\breve{P}_k)$, $\breve{x}_{vk}$, $\overline{X}_v(\breve{P}_k)$ and $\breve{u}_v$ are reported rounded to three decimals to highlight the differences;
- in Tables VIII, X, XII and XIV, when the reported values for $\breve{x}_{4k}$ across different instances are distinct but the respective reported values for $\breve{u}_4$ are equal among them and/or the respective reported values for $a^*_4 p^*_4(\breve{u}_4)$) are equal among them, this is due to the aforementioned rounding. Consider e.g., instances A7 and A8 in Table X: for A7, $\breve{x}_{4k} = 10.343$ Mbps, whereas for A8, $\breve{x}_{4k} = 10.355$ Mbps, while for both of them $\breve{u}_4 = 0.987$ and $a^*_4 p^*_4(\breve{u}_4) = 0.12$ EUR/month. In fact, distinct values of $\breve{x}_{4k}$ for the two instances imply distinct values of the respective $\breve{u}_4$, but the latter differ from one another not before the fourth decimal; similarly, the respective values of $a^*_4 p^*_4(\breve{u}_4)$ differ not before the third decimal.

B. Existence and multiplicity of equilibria

For the considered instances, it is always possible to find an equilibrium when the InPs are different, either for the technology or for the available spectrum bandwidth. Instead,

Notation	Definition	Unit
$\underline{P}_k$	unit cost of InP k (see Equation (23))	EUR/Mbps/month
$\breve{P}_k \geq \underline{P}_k$	unit price offered by InP k (at the equilibrium)	EUR/Mbps/month
$\overline{P}$	minimum unit price unprofitable for all SPs (see Section IV-E)	EUR/Mbps/month
C_k	available capacity of InP k (see Section IV-A)	Mbps
$\breve{C}'_k \leq C_k$	capacity sold by InP k at the equilibrium	Mbps
$\breve{G}_k$	payoff of InP k at the equilibrium (see Equation (22))	EUR/month
$\mathcal{W}_k$	subset of SPs that select InP k at the equilibrium and are assigned non-zero capacity	—
$\underline{X}_v(\breve{P}_k)$	minimum capacity requested by SP v from the selected InP k at $\breve{P}_k$ (see Equation (9))	Mbps
$\overline{X}_v(\breve{P}_k)$	maximum capacity requested by SP v from the selected InP k at $\breve{P}_k$ (see Equation (8))	Mbps
$\breve{x}_{vk}$	capacity assigned to SP v by the selected InP k at the equilibrium (see Section III-D1)	Mbps
$\breve{u}_v$	utility obtained by a single user of SP of v at the equilibrium (see Equation (1))	—
$a^*_v p^*_v(\breve{u}_v)$	monthly fee accepted by a user of SP v for $\breve{u}_v > 0$ (see Section III-B3)	EUR/month
$\breve{g}_v$	payoff of SP v at the equilibrium (see Equation (21))	EUR/month
$\breve{r}^*_v$	total revenue of SP v at the equilibrium (see Equation (7))	EUR/month
$\breve{r}^*_{vk}/\breve{x}_{vk}$	revenue per unit of purchased capacity for SP v at the equilibrium	EUR/Mbps/month

TABLE VI: Summary of notation used in Tables VII–XIV

if the InPs are very similar, it might be difficult to find an equilibrium, unless the spectrum bandwidth is very low or very high. Concerning the equilibria multiplicity, which results[23] from the equilibria multiplicity of the InPs' game at stage 1 and/or of the SPs' game stage 2, the multiple equilibria are always equivalent for all players (i.e., for all InPs and all SPs) since each player obtains the same payoff in all of them, hence they represent the same system behavior; at stage 1, the equilibria multiplicity occurs because there is an InP which is not selected by any SP for any offered unit price, whereas at stage 2 it occurs because some SPs are not provided with capacity in any of the equilibria, therefore it is not relevant which InP they select.

Specifically, no equilibrium was found for instances B4 and B5 (see Section IV-E); however, for both of them, it is possible to determine an approximate equilibrium as explained in Appendix C. In turn, a single equilibrium was found for instances A8–A11 and B7–B11 and multiple equivalent ones for the rest of the instances. As for the equilibria equivalence for instances with multiple equilibria, some illustrative examples follow.

Consider instance A7 (see Tables IX and X) for which the equilibria multiplicity derives from stage 2. In details, for A7, the InPs' game at stage 1 has a unique NE $\breve{\boldsymbol{P}} = (\breve{P}_1 = 1.87, \breve{P}_2 = 1.80)$, whereas the SPs' game at stage 2 for $\breve{\boldsymbol{P}}$ has two NE denoted by (i) and (ii) in Table X: in (i) SP 3 selects InP 1, whereas in (ii) it selects InP 2, while in both (i) and (ii) SP 1 selects InP 2 whereas SPs 2 and 4 select InP 1. In (i), SP 3 requests a minimum amount of capacity equal to 173.051 Mbps and a maximum of 175.857 Mbps from InP 1 given $\breve{P}_1 = 1.87$ whereas in (ii) SP 3 requests a minimum of 160.391 Mbps and a maximum of 176.817 Mbps from InP 2 given $\breve{P}_2 = 1.80$. However, SP 3 is allocated a null capacity in both (i) and (ii); in fact, in (i), InP 1 (which serves SPs 2 and 4) does not have enough spare capacity to serve SP 3 ($C_1 - \breve{C}'_1 = 97.257 < \underline{X}_3(\breve{P}_1) = 173.051$ Mbps), whereas in (ii) InP 2 has allocated all its available capacity to SP 1 ($\breve{x}_{12} = \breve{C}'_2 = C_2 = 260$ Mbps). Formally, the unique NE of of the game at stage 1 and the NE (i) and (ii) of the game at stage 2 imply two equilibria for instance A7. However, it can easily be seen that these two equilibria are equivalent for all SPs: each of the SPs 1, 2 and 4 is served by the same InP, at the same unit price and with the same amount of capacity in both equilibria hence each of them obtains the same payoff in both, while SP 3 is not served in neither equilibria resulting in a null payoff in both. The two equilibria are equivalent also from the InPs' perspective: each InP sells the same amount of capacity at the same unit price in both equilibria thus obtaining the same payoff in both.

[23]Let $n^{\mathcal{K}}$ denote the number of NE in pure strategies of $\mathcal{G}^{\mathcal{K}}$ where $n^{\mathcal{K}} \geq 1$, and let $\breve{\boldsymbol{P}}_i$ denote the unit price profile of the i-th NE of $\mathcal{G}^{\mathcal{K}}$ where $1 \leq i \leq n^{\mathcal{K}}$. Then let $n_i^{\mathcal{V}}$ denote the number of NE in pure strategies of $\mathcal{G}^{\mathcal{V}}(\breve{\boldsymbol{P}}_i)$ where $n_i^{\mathcal{V}} \geq 1$. The number of *sub-game perfect equilibria* of the MLFG is then equal to $\sum_{i=1}^{n^{\mathcal{K}}} n_i^{\mathcal{V}}$.

For instances A1, A2 and B1 as well, the equilibria multiplicity derives from stage 2. However, for these instances, unlike for A7, the multiplicity of NE for the stage 2 game is due to there being at least one SP for which it is not profitable to buy capacity from any InP , hence each such SP is indifferent to the InP choice. Recall that for such SPs, in Tables VIII, X and XII we report the symbol – in all columns starting from the third one. Consider, for instance, instance B1: it is SPs 1, 2 and 3 for which it is not profitable to purchase capacity from any of the InPs, while SP 4 selects and is fully served by InP 2 (see Table XII). Formally, there are 8 equilibria for B1 since the stage 1 game has a unique NE (see Table XI), whereas the stage 2 game has 8 NE resulting from SPs 1, 2 and 3 selecting either InP 1 or InP 2 but acquiring a null capacity from either, while in all these NE SP 4 is served with the same amount of capacity and at the same unit price by InP 2. Clearly, payoff-wise, these equilibria are equivalent for all SPs and all InPs.

For instances A5 and B6 the equilibria multiplicity derives instead from stage 1. Let us consider instance A5 (similarly then for B6). For A5, $\mathcal{G}^{\mathcal{K}}$ has multiple NE which are all unit price profiles $\breve{\boldsymbol{P}} = (\breve{P}_1, \breve{P}_2)$ such that $\breve{P}_1 = 1.77$, whereas $\breve{P}_2$ can take any value in the considered discrete unit price strategy set of InP 2, i.e., $\mathcal{P}_2 = \{\underline{P}_2 = 2.24, \ldots, \overline{P} = 14.86\}$ (see Table VII). Although each such $\breve{\boldsymbol{P}}$ induces a distinct stage 2 game $\mathcal{G}^{\mathcal{V}}(\breve{\boldsymbol{P}})$, all these stage 2 games have the same unique NE reported in Table XII in which all SPs select and are served by InP 1, hence InP 2 sells a null capacity and obtains a null payoff. Thus, formally, there are $|\mathcal{P}_2|$ equilibria for instance A5, but each player obtains the same payoff in all of them.

In turn, instances A3, A4, A6, B2 and B3 have multiple NE at both stages. For the NE multiplicity at stage 1 (stage

2), similar observations to those made for instances A5 and A6 (A1, A2 and B1) apply from which one can easily see the equivalence among the resulting equilibria. Nevertheless, it is worth clarifying that for A3, A4, A6, B2 and B3, each distinct NE unit price profile at stage 1 results in the same set of NE at stage 2, which are *per se* equivalent among them. For example, instance A3 has $2|\mathcal{P}_2|$ equilibria since each NE unit price profile $\breve{\boldsymbol{P}} = (\breve{P}_1, \breve{P}_2)$ with $\breve{P}_1 = 1.99$ and $\breve{P}_2 \in \mathcal{P}_2 = \{\underline{P}_2 = 8.90, \ldots, \overline{P} = 14.86\}$ at stage one (see Table VII) results in two NE at stage 2, due to SP 3 not finding it profitable to purchase capacity from any of InPs hence being indifferent to the InP choice (see Table VIII).

C. Technology and spectrum availability impact on competition among InPs

Recall that for each InP k, 1) its network technology type and 2) its available spectrum bandwidth (B_k) affect its average SC capacity (C_k) and its total cost per unit of average SC capacity ($\underline{P}_k$) as explained in Section IV-A. In the following paragraphs we will then analyze the impact of 1) and 2) on the competition among InPs to be selected by SPs.

Let us first consider instances A1–A11 for which InP 1 has a new (5G) network (type $\mathcal{N}^{(1)}$) whereas InP 2 has a legacy (4G) network (type $\mathcal{L}$), while their available spectrum bandwidths vary across the instances. As for instances A1–A5 (see Tables VII and VIII), InP 2 does not sell capacity to any SP, thus obtaining a null payoff, in all the instances but A2, even when SPs are not fully satisfied from InP 1 (e.g., in case of instance A5). InP 1, instead, always serves at least one SP (but in instance A2). Indeed, for A1, A3, A4 and A5, InP 1 is preferred to InP 2 by at least one SP because InP 1 can offer a lower unit price since it is more cost-efficient, i.e., it has a lower unit cost ($\underline{P}_1 < \underline{P}_2$) and because it has sufficient available capacity. InP 1 has a lower unit cost and a higher capacity than InP 2 due to InP 1 having a higher spectral efficiency (resulting in a higher cell capacity for equal amounts of spectrum bandwidth) and due to $B_1 \geq B_2$. However, notice that for equal amounts of bandwidth, InP 1 incurs a higher total cost per cell than InP 2 to attain a higher spectral efficiency and the total cell cost increases with the spectrum bandwidth. Instead for instance A2, $B_2 = 3B_1$ hence $C_1 = C_2$ while $\underline{P}_1 > \underline{P}_2$ (i.e., $\underline{P}_1 C_1 > \underline{P}_2 C_2$), meaning that the legacy (4G) InP 2, which owns the triple of spectrum holdings of the new (5G) InP 1, provides the same amount of capacity as the latter but more cost-efficiently.

As for the SPs, when the spectrum bandwidth is low, and the unit costs and hence the (equilibrium) unit prices are high, only SP 4 is served and provided with the maximum amount of requested capacity from the selected InP (instances A1 and A2). With the increasing spectrum bandwidth and decreasing unit costs and hence unit prices (instances A3 and A4), other SPs are served and provided with the maximum requested capacity. Finally, in instance A5 all the SPs are served. Although the maximum requested capacity is not provided to any of them by InP 1, they all obtain a higher payoff from selecting InP 1 due to its lower unit price.

Instances A6–A11 are such that for all of them $B_2 = 100$ MHz (see Table V) and hence $C_2 = 260$ Mbps (see Table IX) given that InP 2 is of type $\mathcal{L}$. Instead, B_1 increases with a step of 20 MHz from A6 to A11 starting from $B_1 = 20$ MHz for A1 (see Table V), therefore C_1 increases accordingly from 156 Mbps for A6 to 936 Mbps for A11 (see Table IX) given that InP 1 is of type $\mathcal{N}^{(1)}$. Among A6–A11, only for instance A6, the legacy (4G) InP is more cost-efficient than the new (5G) InP (i.e., $\underline{P}_2 < \underline{P}_1$) and has a higher capacity (i.e., $C_2 > C_1$). Indeed, InP 1 always provides capacity to at least one SP, but in instance A6. On the contrary, for A7–A11 one has $\underline{P}_1 < \underline{P}_2$ and $C_1 > C_2$. With the increasing spectrum bandwidth of InP 1 and its decreasing unit price, InP 1 serves an increasing number of SPs. When the spectrum bandwidths are comparable or InP 1 has a greater amount (A10 and A11), InP 1 serves all the SPs. In general, InP 1 is able to offer a unit price higher than its unit cost, while InP 2 is always selling at a unit price equal to its unit cost but for instance A6. InP 1 does not sell all its available capacity but in instance A9, when it first serves SP 1, which is served by InP 2 as long as the spectrum bandwidth of InP 1 is below 80 Mhz.

As for the SPs, SP 1 and SP 4 are always served. SP 2 and SP 3 are not served in instance A6 as they cannot afford the offered unit prices. Instead, in instance A7, SP 3 can actually afford the unit prices of both InPs but it is not served as neither InP has sufficient available capacity to satisfy its minimum requested capacity. When an SP is served, it is usually provided with the maximum requested capacity. Exceptions are SP 1 in instance A7 and A8, where SP 1 cannot be provided with the maximum requested capacity due to the limited capacity of InP 2, and instance A9, where the available capacity of InP 1 makes it impossible for it to serve completely the three SPs that select it.

Instances B1–B11 (see Tables XI and XIV) are analogous to the respective A1–A11 in terms of spectrum bandwidth availabilities of the two InPs, but for B1–B11 both InPs have deployed a new (5G) network and InP 1 is of type $\mathcal{N}^{(2)}$, i.e., a sheer new entrant, whereas InP 2 is of type $\mathcal{N}^{(1)}$, i.e., InP 2 reuses available sites and spectrum licenses from its legacy (4G) network when it upgrades to the new (5G) network. In particular, for B1–B11, when the spectrum bandwidths of the two InPs are equal (i.e., $B_1 = B_2$) then also their capacities are equal (i.e., $C_1 = C_2$), which is the case for instances B1, B4, B5 and B10. However, for these latter instances, the unit cost of InP 1 is slightly higher than the one of InP 2 (i.e., $\underline{P}_1 > \underline{P}_2$), reflecting the disadvantage of InP 1 for being a new entrant.

For instances B1–B5, when the capacity is low and the unit cost high, the least cost-efficient InP sells no capacity and hence obtains a null payoff: this is the case of InP 1 in B1 and B2, and of InP 2 in instance B3. Moreover, for B2 and B3, the least cost-efficient InP induces no competition (similarly to A3, A4 and A5), therefore the unit price of the other (most cost-efficient) InP is determined solely by the SPs' demand and its own available capacity. Concerning the SPs, some of them are not served across B1–B3 because neither $\breve{P}_1$ nor $\breve{P}_2$ are profitable for them.

As for instances B4 and B5, we recall that we did not find a NE for the InPs' game, hence we suggested as a solution the unit price profile $\boldsymbol{P}^{\diamond} = (P_1^{\diamond}, P_2^{\diamond})$ which is calculated according to Equation (40) and it can be considered an approximate NE (see Appendix C). In these lines, for both instances, values reported under $\breve{P}_1$ and $\breve{P}_2$ in Table XI, which are marked by the symbol $\diamond$, are in fact the values of $P_1^\diamond$ and $P_2^\diamond$, respectively. For B4, the SPs' game for $\boldsymbol{P}^\diamond$ has two distinct NE in pure strategies denoted as (**i**) and (**ii**) when reported in Tables XI and XII. This NE multiplicity is due to the fact that the two InPs are very similar ($P_1^\diamond = 1.23$, $P_2^\diamond = 1.22$ and $C_1 = C_2 = 468$). However, neither NE is preferred by all InPs or all SPs. In fact, in both (**i**) and (**ii**) SP 4 is served by InP 2 at the same unit price ($P_2^\diamond = 1.22$) and with the same amount of capacity ($\breve{x}_{42} = 10.558$) hence SP 4 is indifferent between the two NE. Instead, SPs 2 and 3 prefer (**ii**), in the sense that they attain a higher payoff from (**ii**), whereas SP 1 prefers (**i**) which means that SPs 1, 2 and 3 are all better off in the NE in which they are served by the cheapest InP[24]. In turn, InP 1 prefers (**i**), whereas InP 2 prefers (**ii**) since each InP is able to sell more capacity and hence attain a higher payoff when serving both SPs 2 and 3 instead of SP 1. For instance B5 instead, the SPs' game for $\boldsymbol{P}^\diamond$ has a unique NE. In particular, this $\boldsymbol{P}^\diamond$ is such that $P_1^\diamond = 1.09 > P_2^\diamond = 0.94$ despite $\underline{P}_1 = 0.94 > \underline{P}_2 = 0.90$ which shows that InP 1 leverages the fact that C_2 is not sufficiently large for all SPs to be served by InP 2. In fact, even though $P_1^\diamond > P_2^\diamond$, at the unique NE of SPs' game for $\boldsymbol{P}^\diamond$, SP 3 selects and is served by InP 1 from which it obtains $\breve{x}_{31} = 190.370$ at $P_1^\diamond = 1.09$. If SP 3 were to select InP 2 while SPs 1, 2 and 4 still selected and were served by InP 2, then InP 2 would split $C_2 = 624$ among all SPs and SP 3 would obtain an amount of capacity equal to 147.825 at $P_2^\diamond = 0.94$ which would lower its payoff value by 35.86% w.r.t. the value attained in the NE.

Concerning instances B6–B11, it results that InP 1 becomes more cost-efficient than InP 2 only for instance B11 for which $B_2 > B_1$. Nevertheless, InP 2 is unaffected by the presence of InP 1 only for instance B6 for which InP 1 has only 20 MHz of spectrum bandwidth resulting in a high unit cost ($\underline{P}_1 = 3.55$ as opposed to $\underline{P}_2 = 0.73$). Specifically, for B6, all SPs select and are served by InP 2 and $\breve{P}_2$ is determined solely by the SPs' demand and the available capacity of InP 2. Instead, in instances B7–B8, although all SPs still select and are served by InP 2, the unit price offered by InP 2 at the equilibrium is dictated by the unit cost of InP 1 ($\breve{P}_2$ is the highest discrete unit price value lower than $\underline{P}_1$). Indeed, as the spectrum bandwidth of InP 1 increases, its capacity increases whereas its unit cost decreases making InP 1 more competitive hence forcing InP 2 to lower its offered unit price which in turn increases the amount of capacity requested by the SPs. When the spectrum bandwidths are comparable or InP 1 has a greater amount (B10–B11), the SPs move from InP 2 to InP 1 and InP 2 is forced to sell at its unit cost. As for the SPs, they are always fully served, but in instance B9, where SPs 1, 2 and 3 select InP 2 which is not able to fully serve them whereas SP 4 opts for InP 1, despite its higher unit price, so as to obtain all the requested capacity.

On the overall, we notice that there is more head-to-head competition when InPs are of the same type. Indeed, more recent 5G InPs are preferred w.r.t. older ones (4G ones), but if the latter provide much more spectrum bandwidth, thus resulting more cost-efficient. In this case 5G InP is either less cost-efficient or does not have sufficient capacity for all SPs. Further, there should be sufficient bandwidth even for a 5G InP to be affordable for all 5G services given realistic user fees.

VI. Conclusion

In this work, we address a mobile ecosystem in which the network infrastructure and resources are decoupled from services provisioned for end users giving rise to two types of stakeholders: InPs and SPs. InPs deploy and manage the mobile network and sell their resources to SPs through which the latter provision services for the end users. We consider a case in which there are multiple InPs and multiple SPs and the resource sold/purchased by InPs/SPs is the amount of capacity per BS cell assuming the cell area is provisioned by each InP through its individual BS. We model the problem of cell capacity pricing from the InP perspective and of the choice of an InP from which to acquire capacity from the SP perspective as a multi-leader-follower game. The proposed model has been applied in the context of migration from 4G to 5G for several scenarios in which InPs are characterized by different network technologies and available spectrum bandwidths, whereas SPs provide different 5G mobile services. To set up realistic scenarios, the InP cost structure and the service characterization are based on recent 5G literature.

The analysis of the obtained equilibria shows that more recent InPs are preferred w.r.t. older ones. Older InPs can be competitive if they provide much more spectrum bandwidth, thus resulting more cost-efficient. When the InPs have the same technology, the new entrant ones are less preferred. Indeed, they incur a slightly higher unit cost thus being less competitive.

Appendix A
Optimal user fee derivation

In the following, we derive the optimal user fee from the SP perspective, i.e., the fee *offered* to a user by its SP so that the SP revenue is maximized. We show that this fee is a function of the level of utility perceived by the user which is *per se* a function of the amount of capacity allocated to the user by the SP. As explained in Section III-B, each SP $v \in \mathcal{V}$ splits its available cell capacity x_v uniformly among its users and the utility perceived by the single user from the allocated capacity, i.e., $u_v(x_v)$ is represented by Equation (1). Further, if a user of SP v perceiving the utility $u_v(x_v)$ is offered a fee p_v, it will accept it with a probability $a_v(p_v, u_v(x_v))$ given by Equation (2) (or equivalently (3)), therefore, $a_v(p_v, u_v(x_v))p_v$ represents the fee *accepted* by the user. It follows that, for a given amount of cell capacity x_v, which implies a level

[24] In fact, as reported in Table XII, SPs 1, 2 and 3 attain a higher payoff when served by InP 2 since in addition to InP 2 offering a lower unit price than InP 1 (i.e., $P_2^\diamond < P_1^\diamond$), the maximum amount of capacity requested by SPs 1, 2 and 3 at $P_2^\diamond$ is higher than the respective one at $P_1^\diamond$ (i.e., $\overline{X}_v(P_2^\diamond) > \overline{X}_v(P_1^\diamond)$, $\forall v \in \{1,2,3\}$) and C_2 is sufficiently large for InP 2 to provide each SP that selects it in each of these NE with its maximum requested capacity (i.e., $\breve{x}_{12} = \overline{X}_1(P_2^\diamond)$ in NE (**i**) and $\breve{x}_{22} = \overline{X}_2(P_2^\diamond)$ and $\breve{x}_{32} = \overline{X}_3(P_2^\diamond)$ in NE (**ii**)).

Instance	$\underline{P}_1$	$\underline{P}_2$	$\check{P}_1$	$\check{P}_2$	C_1	C_2	$\check{C}'_1$	$\check{C}'_2$	$\check{G}_1$	$\check{G}_2$	$\mathcal{W}_1$	$\mathcal{W}_2$
A1	3.41	8.90	8.50	8.90	156	52	9.592	0	81.55	0	{4}	∅
A2	3.41	2.98	3.41	3.33	156	156	0	10.055	0	33.52	∅	{4}
A3	1.18	8.90	1.99	{8.90,...,$\overline{P}$=14.86}	468	52	460.326	0	917.48	0	{1,2,4}	∅
A4	1.18	2.98	1.99	{2.98,...,$\overline{P}$=14.86}	468	156	460.326	0	917.48	0	{1,2,4}	∅
A5	0.90	2.24	1.77	{2.24,...,$\overline{P}$=14.86}	624	208	624.000	0	1105.80	0	{1,2,3,4}	∅

TABLE VII: Key equilibrium outcomes related to the InPs — instances A1–A5

Instance	v	k	$\underline{X}_v(\check{P}_k)$	$\check{x}_{vk}$	$\overline{X}_v(\check{P}_k)$	$\check{u}_v$	$a^*_v p^*_v(\check{u}_v)$	$\check{g}_v$	$\check{r}^*_v/\check{x}_{vk}$
A1	1	–	–	–	–	–	–	–	–
	2	–	–	–	–	–	–	–	–
	3	–	–	–	–	–	–	–	–
	4	1	7.799	9.592	9.592	0.942	0.12	60.03	14.76
A2	1	–	–	–	–	–	–	–	–
	2	–	–	–	–	–	–	–	–
	3	–	–	–	–	–	–	–	–
	4	2	6.949	10.055	10.055	0.977	0.12	110.61	14.33
A3	1	1	190.156	251.008	251.008	0.622	54.02	30.02	2.11
	2	1	163.424	199.007	199.007	0.547	28.08	16.86	2.08
	3	–	–	–	–	–	–	–	–
	4	1	6.556	10.311	10.311	0.986	0.12	124.24	14.04
A4	1	1	190.157	251.008	251.008	0.622	54.02	30.02	2.11
	2	1	163.424	199.007	199.007	0.547	28.08	16.86	2.08
	3	–	–	–	–	–	–	–	–
	4	1	6.556	10.311	10.311	0.986	0.12	124.24	14.04
A5	1	1	161.214	250.977	266.747	0.622	54.01	85.48	2.11
	2	1	140.492	196.525	208.873	0.537	27.74	60.22	2.08
	3	1	158.099	166.741	177.217	0.613	12.57	12.94	1.85
	4	1	6.471	9.757	10.370	0.958	0.12	125.47	14.63

TABLE VIII: Key equilibrium outcomes related to the SPs — instances A1–A5

Instance	$\underline{P}_1$	$\underline{P}_2$	$\check{P}_1$	$\check{P}_2$	C_1	C_2	$\check{C}'_1$	$\check{C}'_2$	$\check{G}_1$	$\check{G}_2$	$\mathcal{W}_1$	$\mathcal{W}_2$
A6	3.41	1.80	{3.41,...,$\overline{P}$=14.86}	2.08	156	260	0	255.376	0	531.72	∅	{1,4}
A7	1.74	1.80	1.87	1.80	312	260	214.743	260.000	401.40	468.00	{2,4}	{1}
A8	1.18	1.80	1.83	1.80	468	260	393.144	260.000	718.04	468.00	{2,3,4}	{1}
A9	0.90	1.80	1.77	1.80	624	260	624.000	10.362	1105.80	18.65	{1,2,3}	{4}
A10	0.73	1.80	1.68	1.80	780	260	675.940	0	1136.71	0	{1,2,3,4}	∅
A11	0.62	1.80	1.66	1.80	936	260	678.429	0	1129.28	0	{1,2,3,4}	∅

TABLE IX: Key equilibrium outcomes related to the InPs — instances A6–A11

of utility for the single user equal to $u_v(x_v)$, the optimal user fee for SP v is the value of the p_v which maximizes $a_v(p_v, u_v(x_v))p_v$.

For ease of notation, hereon, we drop the argument x_v of the utility function $u_v(x_v)$ as we derive the optimal offered fee for a fixed level of utility. We also drop the SP subscript v from all parameters and variables since the optimal fee derivation is analogous for all SPs. We assume $0 < \varepsilon < \infty$, $0 < \mu < \infty$, $0 < \overline{p} < \infty$, $0 < \overline{u} < \infty$, $0 < \overline{q} < 1$, $0 < p < \infty$, and $0 < u < \infty$. It can be easily argued that these are all sensible assumptions. First, recall that an SP polls a large set of its own users characterized by ε and μ (i.e., the sensitivities to changes in price and utility, respectively) on whether they accept the fee $\overline{p}$ when they perceive a maximum level of utility ($\overline{u}$) and then it sets $\overline{q}$ equal to the fraction of users that reject it. As explained in Section III-B2, the normalizing constant $A = -\overline{p}^{\varepsilon}\overline{u}^{-\mu}\log(\overline{q})$, therefore for the assumed values of $\overline{p}$, ε, $\overline{u}$, μ and $\overline{q}$, we have $0 < A < \infty$. Recall also that $a(p,u) = 1 - e^{-Ap^{-\varepsilon}u^{\mu}} = 1 - \overline{q}^{(\overline{p}/p)^{\varepsilon}(u/\overline{u})^{\mu}}$ given the definition of A (as detailed in Section III-B2). Concerning ε and μ, they are both assumed positive constants in [15] and positive bounded values are considered in literature ([15], [33], [42]–[46]). In fact, ε and μ should be estimated through realistic measurements [15], hence, in practice, it cannot be that $\varepsilon = \infty$ or $\mu = \infty$ as users cannot be infinitely sensitive to changes in the offered fee or the perceived utility. Consider the equivalent definition of $a(p,u)$, i.e., $a(p,u) = 1-\overline{q}^{(\overline{p}/p)^{\varepsilon}(u/\overline{u})^{\mu}}$ and suppose that $0 < \mu < \infty$, $0 < \overline{p} < \infty$, $0 < \overline{u} < \infty$, $0 < \overline{q} < 1$, $0 < p < \infty$, and $0 < u < \infty$ but $\varepsilon = \infty$. It follows that

$$a(p,u) = \begin{cases} 1 & \text{if } p < \overline{p}, \\ 0 & \text{if } p > \overline{p}, \\ \text{indeterminate} & \text{otherwise,} \end{cases}$$

hence $a(p,u)p$ is maximized by a fee equal to $\overline{p} - \Delta$, where Δ is an infinitely small positive constant. Now, suppose that $0 < \varepsilon < \infty$, $0 < \overline{p} < \infty$, $0 < \overline{u} < \infty$, $0 < \overline{q} < 1$, $0 < p < \infty$,

Instance		v	k	$\underline{X}_v(\check{P}_k)$	$\check{x}_{vk}$	$\overline{X}_v(\check{P}_k)$	$\check{u}_v$	$a^*_v p^*_v(\check{u}_v)$	$\check{g}_v$	$\check{r}^*_v/\check{x}_{vk}$
A6		1	2	213.666	245.087	245.087	0.608	52.79	7.95	2.11
		2	–	–	–	–	–	–	–	–
		3	–	–	–	–	–	–	–	–
		4	2	6.589	10.289	10.289	0.985	0.12	123.33	14.07
A7	**(i)**	1	2	164.024	260.000	264.666	0.643	55.78	79.64	2.11
		2	1	148.471	204.400	204.400	0.568	28.79	41.85	2.07
		3	1	173.051	0	175.857	0	–	0	–
		4	1	6.509	10.343	10.343	0.987	0.12	125.52	14.01
	(ii)	1	2	164.024	260.000	264.666	0.643	55.78	79.64	2.11
		2	1	148.471	204.400	204.400	0.568	28.79	41.85	2.07
		3	2	160.391	0	176.817	0	–	0	–
		4	1	6.509	10.343	10.343	0.987	0.12	125.52	14.01
A8		1	2	164.024	260.000	264.666	0.643	55.78	79.64	2.11
		2	1	144.708	206.343	206.343	0.575	29.03	50.64	2.07
		3	1	163.186	176.446	176.446	0.703	13.45	7.84	1.87
		4	1	6.493	10.355	10.355	0.987	0.12	125.97	13.99
A9		1	1	161.214	254.964	266.747	0.631	54.81	86.25	2.11
		2	1	140.492	199.647	208.873	0.549	28.17	60.98	2.08
		3	1	158.099	169.389	177.217	0.641	12.84	15.04	1.86
		4	2	6.482	10.362	10.362	0.987	0.12	126.24	13.98
A10		1	1	152.987	273.711	273.711	0.671	58.26	111.63	2.09
		2	1	134.438	213.265	213.265	0.599	29.85	81.00	2.06
		3	1	152.816	178.567	178.567	0.719	13.60	33.53	1.87
		4	1	6.433	10.396	10.396	0.988	0.12	127.47	13.94
A11		1	1	151.554	275.071	275.071	0.674	58.49	116.33	2.09
		2	1	133.396	214.124	214.124	0.602	29.95	84.66	2.06
		3	1	152.028	178.832	178.832	0.721	13.62	36.59	1.87
		4	1	6.426	10.402	10.402	0.988	0.12	127.65	13.94

TABLE X: Key equilibrium outcomes related to the SPs — instances A6–A11

Instance	$\underline{P}_1$	$\underline{P}_2$	$\check{P}_1$	$\check{P}_2$	C_1	C_2		$\check{C}'_1$	$\check{C}'_2$	$\check{G}_1$	$\check{G}_2$	$\mathcal{W}_1$	$\mathcal{W}_2$
B1	3.55	3.41	3.55	3.41	156	156		0	10.043	0	34.27	$\emptyset$	{4}
B2	3.55	1.18	{3.55,...,$\overline{P}$=14.86}	1.99	156	468		0	460.143	0	917.88	$\emptyset$	{1,2,4}
B3	1.23	3.41	2.06	{3.41,...,$\overline{P}$=14.86}	468	156		453.265	0	932.84	0	{1,2,4}	$\emptyset$
B4	1.23	1.18	$1.23^{\diamond}$	$1.22^{\diamond}$	468	468	(i)	426.404	326.953	525.89	399.29	{2,3}	{1,4}
							(ii)	315.067	438.082	388.58	535.01	{1}	{2,3,4}
B5	0.94	0.90	$1.09^{\diamond}$	$0.94^{\diamond}$	624	624		190.370	624.000	207.35	584.04	{3}	{1,2,4}

TABLE XI: Key equilibrium outcomes related to the InPs — instances B1–B5

and $0 < u < \infty$ but $\mu = \infty$. It follows that

$$a(p,u) = \begin{cases} 0 & \text{if } u < \overline{u}, \\ 1 & \text{if } u > \overline{u}, \\ \text{indeterminate} & \text{otherwise,} \end{cases}$$

therefore, if $u < \overline{u}$, $a(p,u)p = 0$ for any offered price $p \neq \infty$, i.e., any $p \neq \infty$ generates zero revenue for the SP, whereas for $u > \overline{u}$, $a(p,u)p$ is maximized by any $p \neq \infty$. Further, if $\overline{u} = 0$ (where by definition $\overline{u}$ is the maximum utility perceived by the user), then it would make no sense to look for the optimal fee, as no rational user would be willing to pay for a service which provides no utility. For the considered utility function (see Equation (1)), $\overline{u} \leq 1$ hence $\overline{u} < \infty$. Even if we were to consider a different utility function, it would still be reasonable to assume that $\overline{u} < \infty$ since the utility is a function of the allocated capacity which is *per se* physically limited. As for $\overline{p}$, the value $\overline{p} = \infty$ is impractical whereas $\overline{p} = 0$ would result in $\overline{q} = 0$ (as no rational user would reject a service providing a maximum level of utility $\overline{u}$ when offered for free) and therefore $a(p,u) = 1 - \overline{q}^{(\overline{p}/p)^{\varepsilon}(u/\overline{u})^{\mu}}$ would be indeterminate for any value of u and p, which means that the SP cannot make use of the acceptance probability function if it were to poll its users with $\overline{p} = 0$. Next, for $0 < \varepsilon < \infty$, $0 < \mu < \infty$, $0 < \overline{p} < \infty$, and $0 < \overline{u} < \infty$, $\overline{q} = 0$ would result in $a(p,u) = 1 - \overline{q}^{(\overline{p}/p)^{\varepsilon}(u/\overline{u})^{\mu}} = 1$, $\forall u \in (0,\infty)$, $\forall p \in (0,\infty)$ and vice versa, $\overline{q} = 1$ would result in $a(p,u) = 0$, $\forall u \in (0,\infty)$, $\forall p \in (0,\infty)$, which means that in both cases the SP cannot make use of the acceptance probability function. In practice, if an SP estimated $\overline{q} = 0$ ($\overline{q} = 1$), we would expect it to re-poll the users with a higher (lower) value of $\overline{p}$ until it attains[25] a value of $\overline{q}$ in (0,1). As for p, while $p = \infty$ is impractical, for the assumed parameter values, $p = 0$ would instead result in $a(p,u) = 1, \forall u \in (0,\infty)$ and, as a result, in $a(p,u)p = 0, \forall u \in (0,\infty)$ which is the minimal value of $a(p,u)p$ hence we look for $p \in (0,\infty)$. Finally, we have assumed $0 < u < \infty$, where $u < \infty$ can be justified in the same fashion as $\overline{u} < \infty$ since by definition $u \leq \overline{u}$, whereas $u = 0$ is not interest: for the assumed parameter values, when $u = 0$, $a(p,u) = 0$, $\forall p \in (0,\infty)$, and, as a result, $a(p,u)p =$

[25]In practice, it should be unlikely for the SP to attain $\overline{q} = 0$ for $\overline{p} \to \infty$. Instead, if the SP attained $\overline{q} = 1$ for $\overline{p} \to 0$, it means the service it proposes has no market.

Instance	v	k	$\underline{X}_v(\check{P}_k)$	$\check{x}_{vk}$	$\overline{X}_v(\check{P}_k)$	$\check{u}_v$	$a_v^* p_v^*(\check{u}_v)$	$\check{g}_v$	$\check{r}_v^*/\check{x}_{vk}$
B1	1	–	–	–	–	–	–	–	–
	2	–	–	–	–	–	–	–	–
	3	–	–	–	–	–	–	–	–
	4	2	6.967	10.043	10.043	0.976	0.12	109.82	14.35
B2	1	2	190.471	250.896	250.896	0.622	53.99	29.60	2.11
	2	2	163.699	198.937	198.937	0.546	28.07	16.53	2.08
	3	–	–	–	–	–	–	–	–
	4	2	6.557	10.311	10.311	0.986	0.12	124.23	14.04
B3	1	1	205.287	246.667	246.667	0.612	53.12	13.86	2.11
	2	1	178.664	196.304	196.304	0.536	27.71	4.03	2.08
	3	–	–	–	–	–	–	–	–
	4	1	6.580	10.295	10.295	0.985	0.12	123.57	14.06
B4 (i)	1	2	122.136	316.395	316.395	0.743	64.51	246.93	2.00
	2	1	113.199	239.532	239.532	0.679	32.43	182.18	1.99
	3	1	139.726	186.872	186.872	0.771	14.09	115.34	1.85
	4	2	6.207	10.558	10.558	0.991	0.12	132.29	13.75
(ii)	1	1	122.811	315.067	315.067	0.741	64.34	243.12	2.00
	2	2	112.740	240.378	240.378	0.681	32.50	185.07	1.99
	3	2	139.485	187.145	187.145	0.773	14.10	117.59	1.85
	4	2	6.207	10.558	10.558	0.991	0.12	132.29	13.75
B5	1	2	107.215	351.059	352.994	0.787	68.32	342.14	1.91
	2	2	102.825	262.305	263.751	0.732	34.11	256.80	1.91
	3	1	137.031	190.370	190.370	0.789	14.25	142.52	1.84
	4	2	6.026	10.636	10.695	0.992	0.12	135.32	13.66

TABLE XII: Key equilibrium outcomes related to the SPs — instances B1–B5

Instance	$\underline{P}_1$	$\underline{P}_2$	$\check{P}_1$	$\check{P}_2$	C_1	C_2	$\check{C}_1'$	$\check{C}_2'$	$\check{G}_1$	$\check{G}_2$	$\mathcal{W}_1$	$\mathcal{W}_2$
B6	3.55	0.73	{3.55,...,$\overline{P}$=14.86}	1.87	156	780	0	650.537	0	1214.45	$\emptyset$	{1,2,3,4}
B7	1.81	0.73	1.81	1.68	312	780	0	675.761	0	1137.25	$\emptyset$	{1,2,3,4}
B8	1.23	0.73	1.23	1.11	468	780	0	778.145	0	864.84	$\emptyset$	{1,2,3,4}
B9	0.94	0.73	0.94	0.90	624	780	10.694	780.000	10.02	704.49	{4}	{1,2,3}
B10	0.76	0.73	0.76	0.73	780	780	201.031	684.881	153.28	502.69	{3}	{1,2,4}
B11	0.65	0.73	0.72	0.73	936	780	892.532	0	642.29	0	{1,2,3,4}	$\emptyset$

TABLE XIII: Key equilibrium outcomes related to the InPs — instances B6–B11

Instance	v	k	$\underline{X}_v(\check{P}_k)$	$\check{x}_{vk}$	$\overline{X}_v(\check{P}_k)$	$\check{u}_v$	$a_v^* p_v^*(\check{u}_v)$	$\check{g}_v$	$\check{r}_v^*/\check{x}_{vk}$
B6	1	2	171.470	259.799	259.799	0.642	55.74	62.26	2.11
	2	2	148.251	204.506	204.506	0.568	28.80	42.33	2.07
	3	2	171.615	175.889	175.889	0.698	13.41	0.72	1.87
	4	2	6.509	10.344	10.344	0.987	0.12	125.55	14.00
B7	1	2	153.092	273.613	273.613	0.671	58.24	111.30	2.09
	2	2	134.515	213.203	213.203	0.599	29.85	80.74	2.06
	3	2	152.875	178.548	178.548	0.719	13.60	33.31	1.87
	4	2	6.433	10.396	10.396	0.988	0.12	127.46	13.94
B8	1	2	116.180	329.198	329.198	0.761	66.03	282.38	1.97
	2	2	108.730	248.547	248.547	0.701	33.15	211.92	1.96
	3	2	137.423	189.794	189.794	0.787	14.23	138.29	1.84
	4	2	6.143	10.606	10.606	0.992	0.12	133.45	13.69
B9	1	2	105.594	340.210	358.017	0.774	67.23	352.74	1.94
	2	2	101.776	253.685	266.963	0.713	33.53	264.58	1.95
	3	2	134.005	186.105	195.846	0.767	14.05	176.76	1.85
	4	1	6.027	10.694	10.694	0.993	0.12	135.31	13.59
B10	1	2	97.388	387.952	387.952	0.823	71.46	416.81	1.81
	2	2	96.566	286.109	286.109	0.776	35.46	312.24	1.83
	3	1	131.953	201.031	201.031	0.834	14.65	206.33	1.79
	4	2	5.866	10.820	10.820	0.995	0.12	137.49	13.44
B11	1	1	96.699	390.870	390.870	0.826	71.68	422.40	1.80
	2	1	96.137	287.976	287.976	0.779	35.56	316.36	1.82
	3	1	131.360	202.855	202.855	0.840	14.71	214.98	1.78
	4	1	5.853	10.830	10.830	0.995	0.12	137.65	13.43

TABLE XIV: Key equilibrium outcomes related to the SPs — instances B6–B11

0, $\forall p \in (0, \infty)$, i.e., there is no optimal fee as the SP incurs no revenue when the user achieves no utility.

Now, for a given u, we look for p which maximizes $a(p,u)p$, which we denote as $p^*(u)$. Recall that $a(p,u) = 1 - e^{-Ap^{-\varepsilon}u^{\mu}}$ (see Equation (2)), therefore to determine $p^*(u)$ we solve

$$\frac{\partial\left((1 - e^{-Ap^{-\varepsilon}u^{\mu}})p\right)}{\partial p} = 0,$$

that is, the equation

$$1 - e^{-Ap^{-\varepsilon}u^{\mu}} - \varepsilon Ap^{-\varepsilon}u^{\mu}e^{-Ap^{-\varepsilon}u^{\mu}} = 0. \tag{32}$$

First, let $z = \varepsilon Ap^{-\varepsilon}u^{\mu} + 1$. Equation (32) becomes equivalent to

$$1 - ze^{(1-z)/\varepsilon} = 0. \tag{33}$$

Then let $y = -z/\varepsilon$ which allows to rewrite (33) as

$$ye^{y} = (-1/\varepsilon)e^{(-1/\varepsilon)}. \tag{34}$$

If we denote a solution of Equation (34) by y^*, then the optimal offered price for the givel level of utility u, i.e., $p^*(u)$, corresponding to y^* is

$$p^*(u) = \left[(Au^{\mu}) / (-y^* - 1/\varepsilon)\right]^{1/\varepsilon}. \tag{35}$$

It follows that the acceptance probability of $p^*(u)$, given u, is

$$\begin{aligned} a(p^*(u),u) &= 1 - e^{-A(p^*(u))^{-\varepsilon}u^{\mu}} \\ &= 1 - e^{-AA^{-1}(-y^*-1/\varepsilon)u^{-\mu}u^{\mu}} \\ &= 1 - e^{y^*+1/\varepsilon}. \end{aligned} \tag{36}$$

Notice that $a(p^*(u),u)$ is independent of u and it only depends on ε (as from (34) y^* only depends on ε) hence, hereon, we refer to $a(p^*(u),u)$ by a^*. Equation (34) has one easily identifiable solution, $y^* = -1/\varepsilon$, for which, however, $p^*(u) = \infty$ as $A \neq 0$, $u \neq 0$, $\mu \neq \infty$ and $0 < \varepsilon < \infty$ (see Equation (35)), whereas $a^* = 0$ (see Equation (36)) and, as a result $a^*p^*(u) = 0 \times \infty$ (i.e., the optimal accepted fee is indeterminate). However, depending on the value of ε, $y^* = -1/\varepsilon$ may not be the only solution of (34). To determine all solutions of Equation (34), we proceed as follows. Let $\alpha = (-1/\varepsilon)e^{-1/\varepsilon}$. Equation (34) becomes equivalent to

$$ye^{y} = \alpha, \tag{37}$$

whose solutions are given by the noted Lambert W function. As here $y = -z/\varepsilon = -Ap^{-\varepsilon}u^{\mu} - 1/\varepsilon \in \mathbb{R}$, we consider the real-valued variant of the Lambert W function which we denote as $W : \alpha \to y$, where $\alpha \in [-1/e, +\infty)$. The lower bound of α is due to the fact that the minimum value of the function $f(y) = ye^{y}$, attained at $y = -1$, is equal to $-1/e$. Since $\alpha = -(1/\varepsilon)e^{-1/\varepsilon}$ and $0 < \varepsilon < \infty$, here we also have that $\alpha < 0$. W is single valued for $\alpha = -1/e$, whereas for $\alpha \in (-1/e, 0)$, it is double-valued as illustrated by Figure 1. The upper branch of W (for which $W \geq -1$), is denoted as W_0, whereas the lower branch (for which $W \leq -1$) as W_{-1}, where both W_0 and W_{-1} are *per se* single-valued functions of α. It follows that for $\varepsilon = 1$, which implies a value of $\alpha = (-1/\varepsilon)e^{-1/\varepsilon} = -1/e$, Equation (37) admits a single

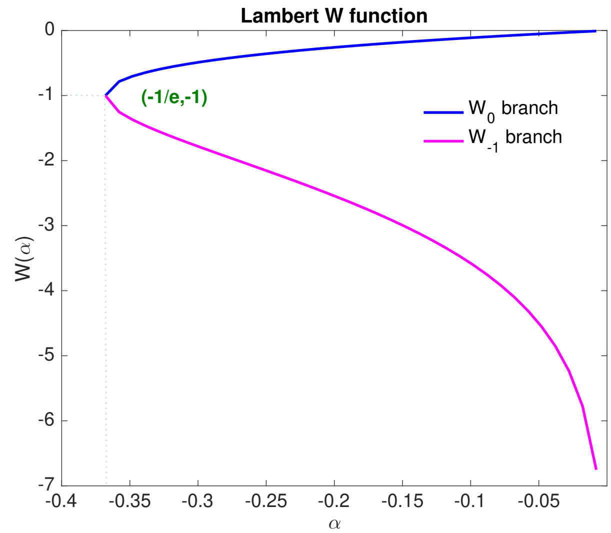


Fig. 1: Lambert W function for $\alpha \in [-1/e, 0)$.

solution $y^* = W(-1/e) = W_0(-1/e) = W_{-1}(-1/e) = -1$ (see Figure 1) which coincides with the solution $y^* = -1/\varepsilon = -1$ of the equivalent Equation (34) obtained by inspection. Instead, for $\varepsilon \in (0, \infty)$ with $\varepsilon \neq 1$, for which $\alpha \in (-1/e, 0)$, Equation (37) admits two solutions: $y_0^* = W_0(\alpha)$ and $y_{-1}^* = W_{-1}(\alpha)$. In summary, based on the value of ε which results in $\alpha = -(1/\varepsilon)e^{-(1/\varepsilon)}$, there are three cases concerning the solution(s) of Equation (37):

1) For $\varepsilon = 1$, which implies $\alpha = -(1/\varepsilon)e^{-(1/\varepsilon)} = -1/e$, Equation (37) admits a single solution $y^* = W(-1/e) = W_0(-1/e) = W_{-1}(-1/e) = -1$ (see Figure 1)). From (35), the optimal fee for the given level of utility u corresponding to $y^* = -1$, i.e.,

$$\begin{aligned} p^*(u) &= \left[(Au^{\mu}) / (-y^* - 1/\varepsilon)\right]^{1/\varepsilon} \\ &= (Au^{\mu}) / (1 - 1) \\ &= \infty, \end{aligned}$$

as $A > 0$, $u > 0$, $\mu < \infty$. Then from (36), the acceptance probability of $p^*(u)$ for the given level of utility u, i.e.,

$$a^* = 1 - e^{y^*+1/\varepsilon} = 1 - e^{-1+1} = 0$$

and, as a result, $a^*p^*(u) = 0 \times \infty$, i.e, the optimal accepted fee for the given level of utility u is indeterminate.

2) For $0 < \varepsilon < 1$, which implies $\alpha = -(1/\varepsilon)e^{-(1/\varepsilon)} \in (-1/e, 0)$ and hence $W(\alpha)$ being double-valued, Equation (37) admits two solutions: $y_0^* = W_0(\alpha) > -1$ and $y_{-1}^* = W_{-1}(\alpha) = -1/\varepsilon < -1$. Let a_0^* denote the acceptance probability of $p_0^*(u)$ for the given level of utility u and, analogously, a_{-1}^*, the acceptance probability of $p_{-1}^*(u)$ for u. From (35), we have

$$\begin{aligned} p_0^*(u) &= \left[(Au^{\mu}) / (-y_0^* - 1/\varepsilon)\right]^{1/\varepsilon} \\ &= \left[(Au^{\mu}) / (-W_0(\alpha) - 1/\varepsilon)\right]^{1/\varepsilon}. \end{aligned}$$

Due[26] to $-1 < W_0(\alpha) < 0$ and $0 < \varepsilon < 1$, which imply $-\infty < -W_0(\alpha) - 1/\varepsilon < 0$ and given that $0 < A < \infty$, $0 < u < \infty$, $\mu < \infty$ and $0 < \varepsilon < 1$, then

$$p_0^*(u) \in \begin{cases} (0,\infty) & \text{if } 1/\varepsilon \text{ is an even integer,} \\ (-\infty,0) & \text{if } 1/\varepsilon \text{ is an odd integer,} \\ \mathcal{C} & \text{otherwise,} \end{cases}$$

which means that if $1/\varepsilon$ is not an even integer, then $p_0^*(u)$ is an infeasible solution. From (36),

$$a_0^* = 1 - e^{y_0^* + 1/\varepsilon} = 1 - e^{W_0(\alpha)+1/\varepsilon} \in (-\infty, 0)$$

as $0 < W_0(\alpha) + 1/\varepsilon < \infty$ for which $1 < e^{W_0(\alpha)+1/\varepsilon} < \infty$. As $-\infty < a_0^* < 0$, we can conclude that even when $1/\varepsilon$ is an even integer, $p_0^*(u)$ is infeasible as it cannot be accepted with a negative probability.

As for $y_{-1}^* = W_{-1}(\alpha) = -1/\varepsilon$, we have that

$$\begin{aligned} p_{-1}^*(u) &= \left[(Au^\mu) / \left(-y_{-1}^* - 1/\varepsilon\right)\right]^{1/\varepsilon} \\ &= \left[(Au^\mu) / (1/\varepsilon - 1/\varepsilon)\right]^{1/\varepsilon} = \infty \end{aligned}$$

as $A > 0$, $u > 0$, $\mu < \infty$ and $0 < \varepsilon < 1$ whereas

$$a_{-1}^* = 1 - e^{y_{-1}^* + 1/\varepsilon} = 1 - e^{-1/\varepsilon + 1/\varepsilon} = 0,$$

and, as a result, $a_{-1}^* p_{-1}^* = 0 \times \infty$.

3) For $1 < \varepsilon < \infty$, $\alpha = -(1/\varepsilon)\, e^{-(1/\varepsilon)} \in (-1/e, 0)$, therefore $W(\alpha)$ is double-valued and consequently Equation (37) admits two solutions: $y_0^* = W_0(\alpha) = -1/\varepsilon > -1$ and $y_{-1}^* = W_{-1}(\alpha) < -1$. As for case 2, a_0^* and a_{-1}^* denote the acceptance probability of $p_0^*(u)$ and $p_0^*(u)$, respectively, for the given level of utility u. From (35) and (36), we get

$$\begin{aligned} p_0^*(u) &= \left[(Au^\mu) / (-y_0^* - 1/\varepsilon)\right]^{1/\varepsilon} \\ &= \left[(Au^\mu) / (1/\varepsilon - 1/\varepsilon)\right]^{1/\varepsilon} \\ &= \infty \end{aligned}$$

as $A > 0$, $u > 0$, $\mu < \infty$ and $1 < \varepsilon < \infty$, while

$$a_0^* = 1 - e^{y_0^* + 1/\varepsilon} = 1 - e^{-1/\varepsilon + 1/\varepsilon} = 0,$$

hence $a_0^* p_0^* = 0 \times \infty$.

Concerning the solution $y_{-1}^* = W_{-1}(\alpha)$,

$$\begin{aligned} p_{-1}^*(u) &= \left[(Au^\mu) / \left(-y_{-1}^* - 1/\varepsilon\right)\right]^{1/\varepsilon} \\ &= \left[(Au^\mu) / (-W_{-1}(\alpha) - 1/\varepsilon)\right]^{1/\varepsilon} \in (0,\infty), \end{aligned}$$

as $0 < A < \infty$, $0 < u < \infty$, $\mu < \infty$, $0 < -W_{-1}(\alpha) - 1/\varepsilon < \infty$ (due[27] to $-\infty < W_{-1}(\alpha) < -1$ and $1 < \varepsilon < \infty$) and $\varepsilon > 0$. In turn, as $-\infty < W_{-1}(\alpha) + 1/\varepsilon < 0$ and, as a result, $0 < e^{W_{-1}(\alpha)+1/\varepsilon} < 1$,

$$a_{-1}^* = 1 - e^{y_{-1}^* + 1/\varepsilon} = 1 - e^{W_{-1}(\alpha)+1/\varepsilon} \in (0, 1).$$

Then, due to $0 < p_{-1}^*(u) < \infty$ and $0 < a_{-1}^* < 1$, $0 < a_{-1}^* p_{-1}^*(u) < \infty$.

[26]Recall that here $\alpha < 0$ and since $W_0(\alpha)$ is strictly increasing in α, then $W_0(\alpha) < W_0(0) = 0$.

[27]Recall that here $\alpha < 0$ and since $W_{-1}(\alpha)$ is strictly decreasing in α then $W_{-1}(\alpha) > \lim_{\alpha \to 0^-} W_{-1}(\alpha) = -\infty$.

$$\begin{aligned} p^*(u) &= \left(\frac{Au^\mu}{-W_{-1}\left(-\frac{1}{\varepsilon}e^{-\frac{1}{\varepsilon}}\right) - \frac{1}{\varepsilon}} \right)^{\frac{1}{\varepsilon}} \\ &= \overline{p} \left(\frac{\log(\overline{q})}{W_{-1}\left(-\frac{1}{\varepsilon}e^{-\frac{1}{\varepsilon}}\right) + \frac{1}{\varepsilon}} \right)^{\frac{1}{\varepsilon}} \left(\frac{u}{\overline{u}}\right)^{\frac{\mu}{\varepsilon}} \in (0,\infty), \end{aligned}$$

which is accepted with a probability

$$a(p^*(u), u) = 1 - e^{W_{-1}\left(-\frac{1}{\varepsilon}e^{-\frac{1}{\varepsilon}}\right) + \frac{1}{\varepsilon}} \in (0, 1),$$

hence the optimal accepted fee $a(p^*(u), u)p^*(u) \in (0, \infty)$.

Appendix B
Examples of the SPs' payoff function

The payoff of an SP (defined in Equation (21)) is the difference between its revenue (see Equations (7) and (1)) and its cost for a given amount of acquired cell capacity at a given cell capacity unit price. We drop the SP subscript v from the aforementioned formulas and write in extensive form the SP payoff as function of the amount of acquired cell capacity x at a given cell capacity unit price P as follows:

$$g(x) = \begin{cases} -Px, \quad \text{if } 0 \le x \le \widetilde{N}\underline{\mathcal{X}}, \\ Na^*\overline{p}\left[\frac{\log(1-\overline{a})}{\log(1-a^*)}\right]^{1/\varepsilon} \left(\frac{1}{\overline{u}}\right)^{\mu/\varepsilon} \left[\frac{\left(\frac{x/\widetilde{N} - \underline{\mathcal{X}}}{\mathcal{X} - \underline{\mathcal{X}}}\right)^\xi}{1 + \left(\frac{x/\widetilde{N} - \underline{\mathcal{X}}}{\mathcal{X} - \underline{\mathcal{X}}}\right)^\xi}\right]^{\mu/\varepsilon} - Px, \\ \quad \text{if } x > \widetilde{N}\underline{\mathcal{X}}. \end{cases} \tag{38}$$

Let $\mu = \varepsilon = 2$, $Na^*\overline{p}\left(\frac{\log(1-\overline{a})}{\log(1-a^*)}\right)^{1/\varepsilon} = 1$, $\overline{u} = 1$, $\widetilde{N} = 1$, $\underline{\mathcal{X}} = 1$ and $\mathcal{X} = 10$. For these values of parameters, in Figures 2 and 3 we plot $g(x)$ in terms of x for all combinations of two different values for each of the remaining parameters: i.e., for values 2 and 20 for ξ (the utility elasticity) and for values 0.03 and 0.1 for P (the cell capacity unit price). Values 2 and 20 are the minimum and maximum values considered for ξ in this work (see Section IV-C) and also in literature [15]. As for P, given the considered values for all other parameters, values 0.03 and 0.1 are simply two values that allow to illustrate the two different cases concerning the calculation of the minimum and maximum amount of cell capacity requested by the SP for a given cell capacity unit price P, i.e., $\underline{X}(P)$ and $\overline{X}(P)$, where

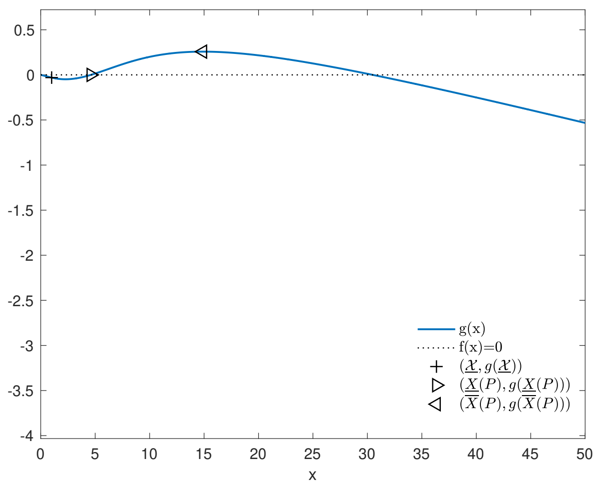
$$\overline{X}(P) = \begin{cases} 0, & \text{if } g(x) \le 0,\ \forall x \ge \widetilde{N}\underline{\mathcal{X}}, \\ \underset{x \ge \widetilde{N}\underline{\mathcal{X}}}{\operatorname{argmax}}\ g(x), & \text{if } \exists\, x > \widetilde{N}\underline{\mathcal{X}} \mid g(x) > 0, \end{cases}$$

$$\underline{X}(P) = \begin{cases} 0, & \text{if } \overline{X}(P) = 0, \\ x \in [\widetilde{N}\underline{\mathcal{X}}, \overline{X}(P)] \mid g(x) = 0, & \text{if } \overline{X}(P) > 0. \end{cases}$$

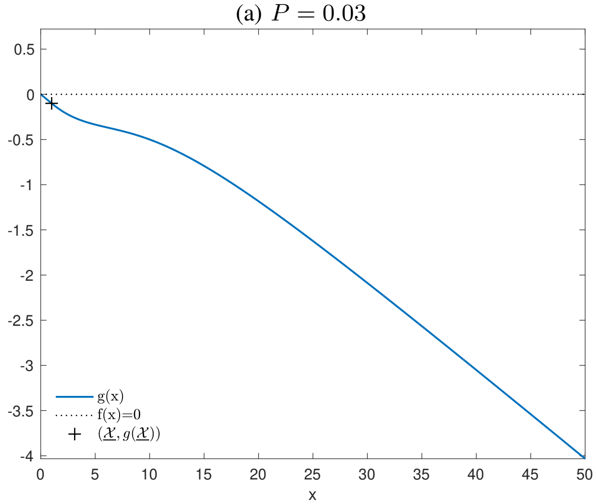
Notice that for $0 < x \le \widetilde{N}\underline{\mathcal{X}}$, one has $g(x) < 0$ as in this range $g(x) = -Px$ (see Equation (38), Figures 2 and 3). Hence, we look for $\underline{X}(P)$ and $\overline{X}(P)$ for $x > \widetilde{N}\underline{\mathcal{X}}$. From Figures 2b and 3b, we can see that for both values of ξ, when $P = 0.1$,

$g(x) < 0$, $\forall x \geq \widetilde{N}\underline{\mathcal{X}}$, as a result, we force $\underline{X}(P) = \overline{X}(P) = 0$. Instead, from Figures 2a and 3a we can see that, for both values of ξ when $P = 0.03$, one has:

(1) there exists $x > \widetilde{N}\underline{\mathcal{X}}$ such that $g(x) > 0$, therefore, $\overline{X}(P) > \widetilde{N}\underline{\mathcal{X}} > 0$ and $g(\overline{X}(P)) > 0$;
(2) $g(x)$ has a unique zero in $[\widetilde{N}\underline{\mathcal{X}}, \overline{X}(P)]$, hence there is a unique value for $\underline{X}(P)$;
(3) $\overline{X}(P) > \underline{X}(P) > \widetilde{N}\underline{\mathcal{X}} > 0$, as $g(\widetilde{N}\underline{\mathcal{X}}) < 0$ whereas $g(\overline{X}(P)) > 0$.

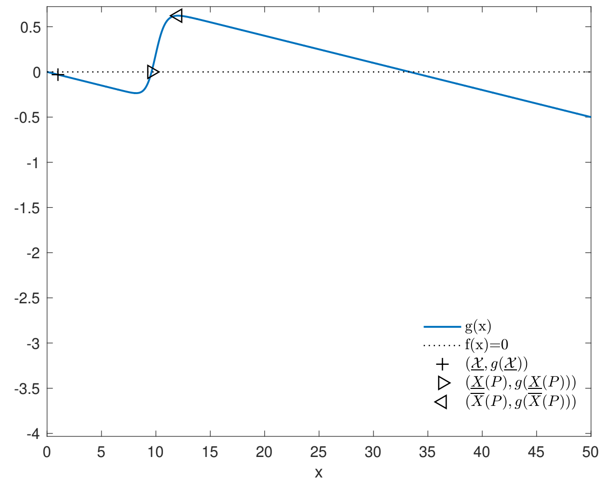


(a) $P = 0.03$

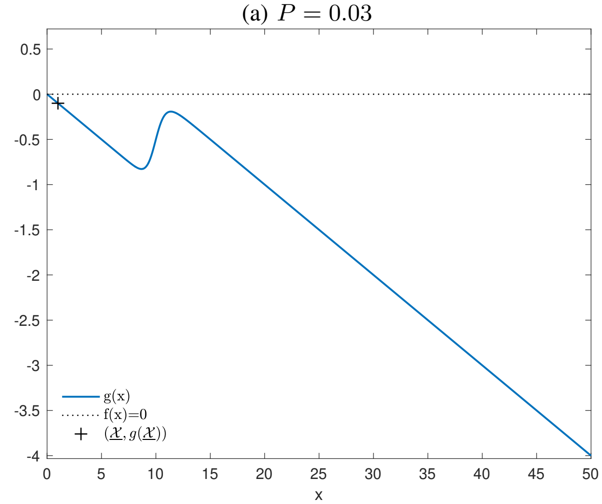


(b) $P = 0.1$

Fig. 2: SP payoff function examples for utility elasticity $\xi = 2$.



(a) $P = 0.03$



(b) $P = 0.1$

Fig. 3: SP payoff function examples for utility elasticity $\xi = 20$.

Appendix C
Approximated Equilibria

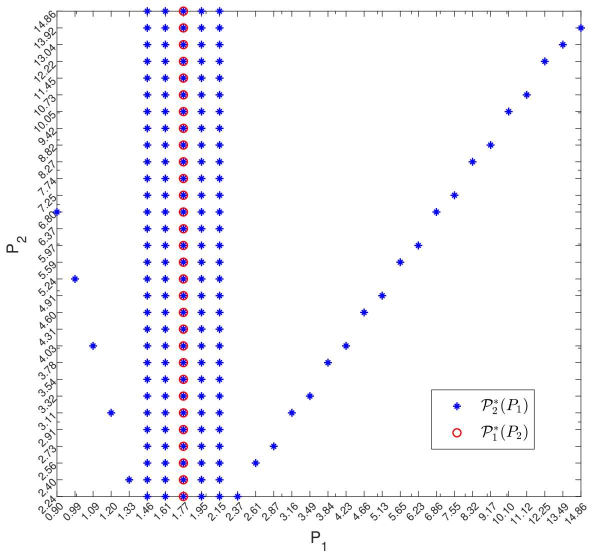
For the InPs' game $\mathcal{G}^{\mathcal{K}}$, since there are two players (InPs) for each considered instance, the existence and multiplicity of its NE in pure strategies can be also depicted graphically through the InPs' best response functions. The best response $\mathcal{P}_k^*$ of any InP k is the set of strategies

$$\begin{aligned} \mathcal{P}_k^*(\boldsymbol{P}_{-k}) = \{ & P_k \in \mathcal{P}_k \mid \\ & G_k([P_k, \boldsymbol{P}_{-k}]) \geq \max_{P_k' \in \mathcal{P}_k} G_k([P_k', \boldsymbol{P}_{-k}]) - \Delta\}, \\ & \forall\, \boldsymbol{P}_{-k} \in \textstyle\prod_{j \in \mathcal{K}\setminus\{k\}} \mathcal{P}_j, \end{aligned} \tag{39}$$

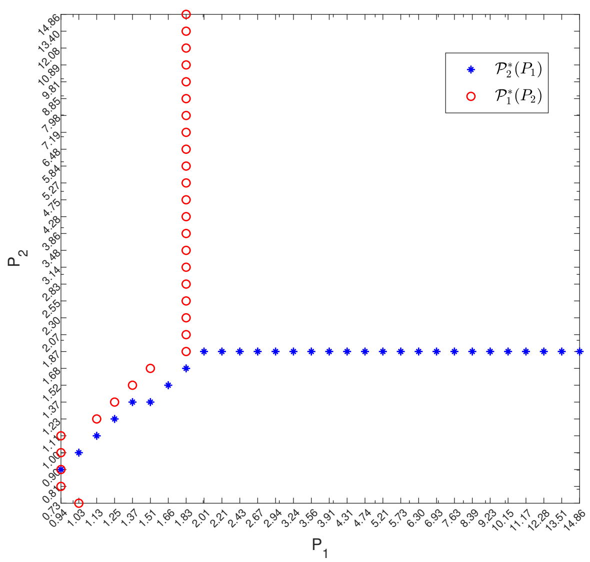
that is, $\mathcal{P}_k^*(\boldsymbol{P}_{-k})$ is the set of all InP k unit prices that maximize its payoff within a margin[28] Δ (see Section IV-E) given $\boldsymbol{P}_{-k}$, i.e., given the unit prices offered by all other InPs but k.

For two illustrative instances, A5 and B9, in Figures 4a and 4b, we have plotted in blue ($*$ markers) the best response function of InP 2, that is the payoff-maximizing unit price(s) for InP 2 for each possible unit price that InP 1 can offer (i.e., $\mathcal{P}_2^*(P_1)$ for any $P_1 \in \mathcal{P}_1 = \{\underline{P}_1, \ldots, \overline{P}\}$) and in red ($\circ$ markers) the best response function of InP 1, that is the payoff-maximizing unit price(s) for InP 1 for each possible unit price that InP 2 can offer (i.e., $\mathcal{P}_1^*(P_2)$ for any $P_2 \in \mathcal{P}_2 = \{\underline{P}_2, \ldots, \overline{P}\}$). The NE InP unit price profile(s) $\check{\boldsymbol{P}}$ of the InPs' game for A5 and B9 are then represented by all intersections between $\mathcal{P}_1^*$ and $\mathcal{P}_2^*$ in Figures 4a and 4b, respectively. For instance A5 (see Figure 4a), as also reported in Table VII, there are $|\mathcal{P}_2| = 30$ NE such that $\check{P}_1 = 1.77$

[28]The absolute payoff margin $\Delta = 10^{-6}$ EUR introduced in the NE definition (see Equation (31)) to deal with numerical issues brought about by solver tolerances (as explained in Section IV-E) has been applied to best response definition accordingly.



(a) Instance A5



(b) Instance B9

Fig. 4: InPs' best response functions for $\mathcal{G}^{\mathcal{K}}$ — example of multiple NE (a) and unique NE (b).

EUR/Mbps/month and $\breve{P}_2 \in \mathcal{P}_2 = \{\underline{P}_2 = 2.24, \ldots, \overline{P} = 14.86\}$ EUR/Mbps/month. In fact, as previously mentioned, all these NE are equivalent for both InPs in terms of achieved payoffs ($\breve{G}_1 = 1105.80$, $\breve{G}_2 = 0$) EUR/month: in each such NE, i.e., for each such $\breve{\boldsymbol{P}}$, InP 1 offers the unit price $\breve{P}_1 = 1.77$ EUR/Mbps/month (which is strictly lower than $\underline{P}_2 = 2.24$ EUR/Mbps/month, i.e., the lowest unit price that InP 2 can offer) and is selected by all four SPs in the unique NE of the respective $\mathcal{G}^{\mathcal{V}}(\breve{\boldsymbol{P}})$ (see Table VIII), while InP 2, not being selected by any SP and therefore not selling any capacity even when it offers $\breve{P}_2 = \underline{P}_2$, is indifferent between all unit prices it can offer, each proving it with zero payoff (i.e., for InP 2 any unit price $P_2 \in \mathcal{P}_2$ is a best response to $\breve{P}_1$). Instead, for instance B9 (see Figure 4b) there is a single intersection between $\mathcal{P}_1^*$ and $\mathcal{P}_2^*$, therefore a unique NE for $\mathcal{G}^{\mathcal{K}}$, $\breve{\boldsymbol{P}} = (\breve{P}_1 = 0.94, \breve{P}_2 = 0.90)$ EUR/Mbps/month, as reported in Table XIII as well; at the unique NE of the respective $\mathcal{G}^{\mathcal{V}}(\breve{\boldsymbol{P}})$, InP 1 is selected only by SP 4 to which it sells capacity at its minimum unit price $\breve{P}_1 = \underline{P}_1 = 0.94$ EUR/Mbps/month, i.e., at a unit price equal to its unit cost, while all the other SPs (1, 2 and 3) select the more cost-efficient InP (2) which at the equilibrium offers a unit price $\underline{P}_2 < \breve{P}_2 < \underline{P}_1$ (see Tables XIII and XIV).

As anticipated in Section IV-E, for instances B4 and B5, there is no NE in pure strategies for the InPs' game $\mathcal{G}^{\mathcal{K}}$ resulting from the initial discrete InP unit price strategy sets $\mathcal{P}_k$ (each made up of 30 logarithmically-spaced discrete values in $[\underline{P}_k, \overline{P}]$), although there is at least one NE in pure strategy for each SPs' game $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P})$ for any $\boldsymbol{P} \in \mathcal{P}$. For this setting, the absence of NE for $\mathcal{G}^{\mathcal{K}}$ for B4 and B5 can be witnessed in Figures 5a and 5b, respectively, where the InPs' best response functions, i.e., $\mathcal{P}_1^*$ and $\mathcal{P}_2^*$, do not intersect.

If we were to linearly interpolate $\mathcal{P}_1^*$ and $\mathcal{P}_2^*$ depicted in Figure 5a and determine the intersection of their interpolations, then, for instance B4, we would expect the NE InP unit prices to be within the following ranges: $\breve{P}_1 \in [\underline{P}_1 = 1.23, 1.34]$ EUR/Mbps/month and $\breve{P}_2 \in [\underline{P}_2 = 1.18, 1.29]$ EUR/Mbps/month. Analogously for B5 (see Figure 5b), we would expect $\breve{P}_1 \in [1.03, 1.13]$ EUR/Mbps/month and $\breve{P}_2 \in [\underline{P}_2 = 0.90, 0.99]$ EUR/Mbps/month. On this basis, for each InP $k \in \mathcal{K}$, we set up[29] an alternative unit price strategy set $\mathcal{P}_k$ made up of 60 discrete values in $[\underline{P}_k, \overline{P}]$ such that the vast majority of these values lie in the respective aforementioned range where we expect the NE unit price to be for InP k. However, even for the MFSG resulting from these alternative discrete InP unit price strategy sets, for both B4 and B5, there still is no NE in pure strategies for $\mathcal{G}^{\mathcal{K}}$ although, there is at least one NE in pure strategies for $\mathcal{G}^{\mathcal{K}}(\boldsymbol{P})$ for any $\boldsymbol{P} \in \mathcal{P}$. In absence of an NE for $\mathcal{G}^{\mathcal{K}}$, we consider as a solution for $\mathcal{G}^{\mathcal{K}}$ the InP unit price profile(s) denoted by $\boldsymbol{P}^{\diamond}$ and determined as

$$\boldsymbol{P}^{\diamond} = \underset{\boldsymbol{P}=[P_k, \boldsymbol{P}_{-k}] \in \mathcal{P}}{\operatorname{argmin}} \left[\max_{k \in \mathcal{K}} \ \delta_k([P_k, \boldsymbol{P}_{-k}]) \right], \tag{40}$$

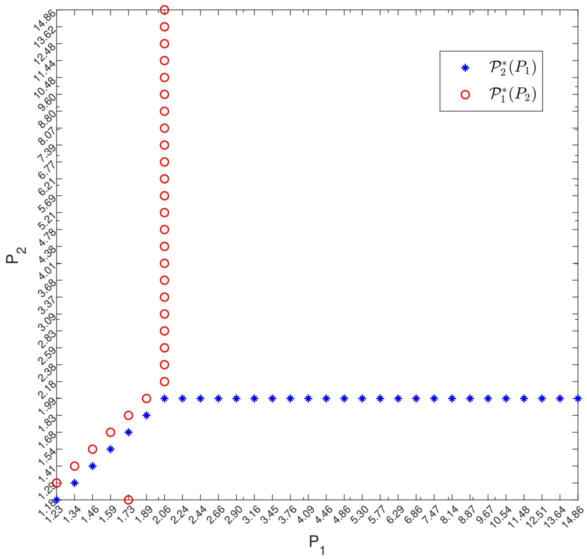
where

$$\delta_k([P_k, \boldsymbol{P}_{-k}]) = \frac{\max\limits_{P'_k \in \mathcal{P}_k} G_k([P'_k, \boldsymbol{P}_{-k}]) - G_k([P_k, \boldsymbol{P}_{-k}])}{\max\limits_{P'_k \in \mathcal{P}_k} G_k([P'_k, \boldsymbol{P}_{-k}])}$$

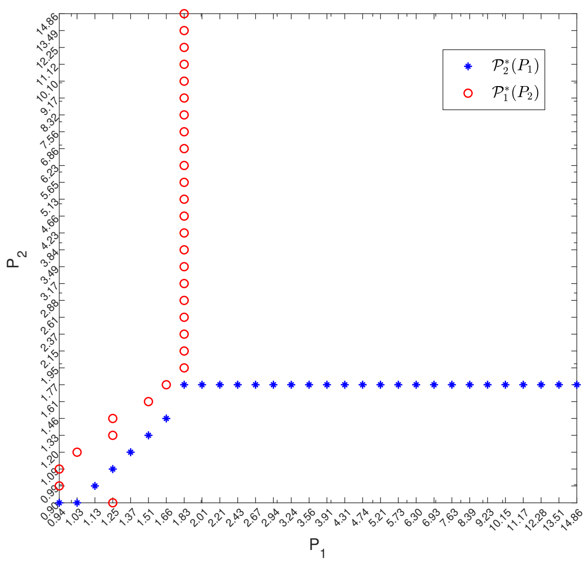
is the relative difference between the payoff of InP k from $\boldsymbol{P} = [P_k, \boldsymbol{P}_{-k}]$ and the maximum payoff that k can obtain by unilaterally deviating from $\boldsymbol{P}$. In Equation (40), we set $\boldsymbol{P}^{\diamond}$ equal to the InP unit price profile(s) which provide the minimum value for $\max_{k \in \mathcal{K}} \delta_k([P_k, \boldsymbol{P}_{-k}])$[30].

[29]The alternative discrete InP unit price strategy sets were set up as follows. For instance B4, $\mathcal{P}_1$ consists of: 50 linearly-spaced values in $[\underline{P}_1 = 1.23, 1.34]$, 5 linearly-spaced values in $[1.35, 2.06]$ and 5 linearly-spaced values in $[2.07, \overline{P} = 14.86]$ whereas $\mathcal{P}_2$ consists of: 50 linearly-spaced in values in $[\underline{P}_2 = 1.18, 1.29]$, 5 linearly-spaced values in $[1.3, 2.18]$ and 5 linearly-spaced values in $[2.19, \overline{P} = 14.86]$. For instance B5, $\mathcal{P}_1$ consists of: $\underline{P}_1 = 0.94$, $(\underline{P}_1 + 1.03)/2$, 50 linearly-spaced values in $[1.03, 1.13]$, 3 linearly-spaced values in $[1.14, 1.83]$ and 5 linearly-spaced values in $[1.84, \overline{P} = 14.86]$ whereas $\mathcal{P}_2$ consists of: 50 linearly-spaced values in $[\underline{P}_2 = 0.90, 0.99]$, 5 linearly-spaced values in $[1, 1.95]$ and 5 linearly-spaced values in $[1.96, \overline{P} = 14.86]$.

[30]If $\mathcal{G}^{\mathcal{K}}$ had a NE $\breve{\boldsymbol{P}}$, then $\boldsymbol{P}^{\diamond} = \breve{\boldsymbol{P}}$ and $\min\limits_{\boldsymbol{P} \in \mathcal{P}} \max\limits_{k \in \mathcal{K}} \delta_k(\boldsymbol{P}) = 0$.



(a) Instance B4



(b) Instance B5

Fig. 5: InP best response functions for $\mathcal{G}^{\mathcal{K}}$ — initial, logarithmically-spaced sets $\mathcal{P}_k$ for any $k \in \mathcal{K}$.

For both B4 and B5, we have calculated $\boldsymbol{P}^{\diamond}$ for the MFSG resulting from the alternative $\mathcal{P}_k$ described above (i.e., for the $\mathcal{P}_k$, $\forall k \in \mathcal{K}$ made up of 60 discrete values in $[\underline{P}_k, \overline{P}]$ with the vast majority of these values where we expect the NE to be by looking at Figures 5a and 5b, respectively). It results that for both B4 and B5 there is a unique $\boldsymbol{P}^{\diamond}$. For B4, $\boldsymbol{P}^{\diamond} = (P_1^{\diamond} = 1.23, P_2^{\diamond} = 1.22)$ EUR/Mbps/month with $\max_{k \in \mathcal{K}} \delta_k(\boldsymbol{P}^{\diamond}) = 0.53$, whereas for B5, $\boldsymbol{P}^{\diamond} = (P_1^{\diamond} = 1.09, P_2^{\diamond} = 0.94)$ EUR/Mbps/month with $\max_{k \in \mathcal{K}} \delta_k(\boldsymbol{P}^{\diamond}) = 3.89$, hence we deemed these $\boldsymbol{P}^{\diamond}$ as reasonable solutions for $\mathcal{G}^{\mathcal{K}}$. Notice also that, although these $\boldsymbol{P}^{\diamond}$ are not NE of $\mathcal{G}^{\mathcal{K}}$ for B4 and B5, it turns out that for B4, $P_1^{\diamond} = 1.23 \in [\underline{P}_1 = 1.23, 1.34]$ EUR/Mbps/month and $P_2^{\diamond} = 1.22 \in [\underline{P}_2 = 1.18, 1.29]$ EUR/Mbps/month, and for B5, $P_1^{\diamond} = 1.09 \in [1.03, 1.13]$ EUR/Mbps/month and $P_2^{\diamond} = 0.94 \in [\underline{P}_2 = 0.90, 0.99]$ EUR/Mbps/month, which are the InP unit price ranges where we would expect the NE of $\mathcal{G}^{\mathcal{K}}$ to be by looking at the best response functions of $\mathcal{G}^{\mathcal{K}}$ for the initial MFSG illustrated in Figures 5a and 5b, respectively.

For both B4 and B5, the respective values of $P_1^{\diamond}/P_2^{\diamond}$ are reported in Table XI under $\breve{P}_1/\breve{P}_2$, whereas the outcomes of the respective SPs' game $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P}^{\diamond})$ in Table XII. In particular, for B4, $\mathcal{G}^{\mathcal{V}}(\boldsymbol{P}^{\diamond})$ turns out to have two distinct NE in pure strategies denoted by **(i)** and **(ii)** in Tables XI and XII and analyzed in Section V-C.

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