Ramsey pricing: a simple example of a subordinate commodity*

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Abstract

We present preferences exhibiting a so-called subordinate good, namely a commodity such that the willingness to pay for it increases when the consumption of all goods increases proportionally, and thus receives a negative price-cost margin according to Ramsey pricing. We also show that its Bertrand equilibrium price is above its Cournotian price.

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1 Introduction

It is well known that in the case of a multiproduct firm Ramsey pricing (of which monopolistic pricing is an example) may involve (some) negative price-cost margin, and that this requires some complementarity among goods: see e.g. Tirole (1988: section 1.1.2) and Belleflamme and Peitz (2015: section 2.2.2). The optimal pricing literature has longly provided an explanation of this possibility based on the, rather involved, so-called "superelasticities" of demand: see e.g. Brown and Sibley (1986: chapter 3).

However, Armstrong and Vickers (2018) have recently showed that the condition of having a commodity with a negative Ramsey margin boils down to consumer surplus being (locally) decreasing with respect to the quantity of that good. In addition, Bertoletti (2018) has argued that this is equivalent to that commodity having (locally) a negative (inverse) "outside substitutability", the latter being measured by (minus) the scale elasticity of its inverse demand,

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meaning that the willingness to pay for it increases when the consumption of all goods increases proportionally. In the case of two goods (in addition to the outside commodity), Bertoletti (2018) has also showed that a subordinate commodity has a relatively poor substitutability, a relatively small budget share and it is a luxury (in terms of preferences over inside commodities).

The intuitive idea is that the consumption of similar commodities is subordinated to the consumption of other goods: think for example of mountain climbing equipment which is of no (or little) use without suitable mountain clothes, and possibly such that the willingness to pay for the former increases with their joint consumption. For this reason, Bertoletti (2018) has suggested to classify similar goods as "subordinates". However, we are not aware of any example of preferences delivering such a commodity.[1] The aim of this note is to provide such an example, exploiting a simple linear demand system with two goods. In addition, we show that the monopoly price of the other commodity is larger than the corresponding Bertrand price, a result due to strategic substitutability of prices. Also related to this feature of our setting is the fact that the Bertrand price of the subordinate commodity is larger than the Cournotian one. Finally, it turns out that Ramsey quantities are proportional to efficient ones: in fact, preferences belong to the class studied by Armstrong and Vickers (2018).

2 A simple model

Consider the quasi-linear preferences represented by the direct utility function:

$$U(\mathbf{x}, x_0) = u(\mathbf{x}) + x_0 = ax_1 + x_1 x_2 - x_1^2 - \frac{x_2^2}{2} + x_0, \tag{1}$$

where $a > 0$ and x_0 is the quantity of the *numéraire* (with price $p_0 = 1$). Assuming a positive consumption of the latter commodity (and more generally restricting attention to the case of interior solutions, meaning $\mathbf{x}$, $\mathbf{p} > \mathbf{0}$), direct differentiation of (1) delivers the *inverse* demand system:

$$p_1(\mathbf{x}) = \frac{\partial u(\mathbf{x})}{\partial x_1} = a - 2x_1 + x_2, \tag{2}$$

$$p_2(\mathbf{x}) = \frac{\partial u(\mathbf{x})}{\partial x_2} = x_1 - x_2. \tag{3}$$

Notice that $a + x_2$ is the maximum willingness to pay for commodity 1, while x_1 is the maximum willingness to pay for commodity 2. Accordingly, the consumed amount of commodity 2 is always smaller than that of commodity 1 (otherwise the marginal utility of the former commodity would be negative).

$u(\mathbf{x})$ can be written as $h(\mathbf{x}) + g(q(\mathbf{x}))$, where $h(\mathbf{x}) = ax_1$ and $q(\mathbf{x}) = \sqrt{\mathbf{x}'\mathbf{M}\mathbf{x}}$ with $\mathbf{M} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$ are linear homogeneous, and $g(t) = -t^2/2$ is concave,

[1] On the contrary, it is well known that access pricing and two-sided markets can exhibit negative price-cost margins: see e.g. Belleflamme and Peitz (2015: chapter 22).

showing that (1) represents preferences that belong to the class, studied by Armstrong and Vickers (2018), whose associated consumer surplus, as a function of quantities, is homothetic.[2] In fact, consumer surplus is given by:

$$\begin{aligned} s(\mathbf{x}) &= u(\mathbf{x}) - \mathbf{p}(\mathbf{x})'\mathbf{x} \\ &= x_1^2 - x_2 x_1 + \frac{x_2^2}{2}, \end{aligned}$$

and thus is homogeneous of degree 2.

Now notice that $\frac{\partial s}{\partial x_2} = x_2 - x_1 < 0$ (whenever $p_2 > 0$). Equivalently, the measure of *outside substitutability* proposed by Bertoletti (2018), namely, the "scale elasticity" $\mu_i(\mathbf{x}) = -\frac{\partial \ln p_i(\lambda \mathbf{x})}{\partial \ln \lambda}|_{\lambda=1}$, is actually negative for commodity 2: $\mu_2 = (x_2 - x_1)/(x_1 - x_2) = -1$. Accordingly, consumer surplus s actually *decreases* with respect to the consumption of commodity 2, and the willingness to pay for it, $p_2(\mathbf{x})$, *increases* when the consumption of all goods increases proportionally.

Finally, the system (2)-(3) can be easily inverted to provide the *direct* demand system

$$x_1(\mathbf{p}) = a - p_1 - p_2, \tag{4}$$

$$x_2(\mathbf{p}) = a - p_1 - 2p_2. \tag{5}$$

Note that commodities are complements (i.e., $\frac{\partial x_i}{\partial p_j} < 0$ for $x_i > 0$, i, $j = 1$, 2, $i \neq j$) and $x_1 > x_2$ (indeed, they are somehow close to the case of perfect complements).[3] It is easily verified by Roy's identity that these demands follows from the following indirect utility function, dual to (1):

$$V(\mathbf{p}, E) = S(\mathbf{p}) + E = \frac{(a - p_1 - p_2)^2 + p_2^2}{2} + E, \tag{6}$$

where E is consumer expenditure:[4] notice that $S(\mathbf{p})$ is decreasing and (strictly, whenever strictly decreasing) convex for $a \geq p_1 + 2p_2$, and then a legitimate consumer surplus measure.

2.1 Ramsey pricing

Suppose that commodities 1 and 2 are produced with constant unit costs $c_1 \geq 0$ and $c_2 > 0$. The corresponding profit functions, given by $\pi_i(\mathbf{p}) = (p_i - c_i)\, x_i(\mathbf{p})$, $i = 1$, 2, are concave, and so it is overall profit $\Pi = \pi_1 + \pi_2$. In what follows we assume that a is sufficiently large to make feasible all the market allocations considered (a *sufficient* condition is $a > c_1 + 4c_2$).

[2] Armstrong and Vickers (2018: p. 1458) mention that this kind of preferences may deliver subordinate commodities, but do not provide an example.

[3] With perfect complements Ramsey prices would not be uniquely defined: see e.g. Tirole (1988: p. 71, Exercise 1.5).

[4] The functional form of $S(\mathbf{p})$ is close to the "translated-power" considered in Bertoletti and Etro (2021).

Ramsey prices (see e.g. Bertoletti, 2018) maximize $W(\mathbf{p}) = \Pi(\mathbf{p}) + \alpha S(\mathbf{p})$ for $1 \geq \alpha \geq 0$ (the case of monopoly pricing arises for $\alpha = 0$, while $\alpha = 1$ delivers marginal cost pricing). Notice that $W(\mathbf{p})$ is concave. The FOCs can be written as:

$$\begin{aligned}
(p_1 - c_1)\frac{\partial x_1(\mathbf{p})}{\partial p_1} + (p_2 - c_2)\frac{\partial x_2(\mathbf{p})}{\partial p_1} &= -(1-\alpha)\, x_1(\mathbf{p}), \\
(p_1 - c_1)\frac{\partial x_1(\mathbf{p})}{\partial p_2} + (p_2 - c_2)\frac{\partial x_2(\mathbf{p})}{\partial p_2} &= -(1-\alpha)\, x_2(\mathbf{p}),
\end{aligned}$$

and by manipulating them we get the following Ramsey prices:

$$p_1^R(\alpha) = \frac{c_1 + (1-\alpha)\, a}{2-\alpha} \geq c_1,\; p_2^R(\alpha) = \frac{c_2}{2-\alpha} \leq c_2,$$

which show that commodity 2 is indeed “subordinate” (see Bertoletti, 2018 for a discussion). Notice that $\frac{dp_2^R}{d\alpha} > 0$ and $\frac{dp_1^R}{d\alpha} < 0$, with $p_i^R(1) = c_i$, $p_1^R(0) = p_1^m = \frac{c_1+a}{2} > c_1$ and $p_2^R(0) = p_2^m = \frac{c_2}{2} < c_2$, where p_i^m denotes the price a two-product monopolist would adopt for commodity i, and that p_2^R does not depend on the willingness-to-pay parameter a.

It is also easily computed that:

$$\begin{aligned}
x_1^R(\alpha) &= \frac{a - c_1 - c_2}{2-\alpha},\; x_2^R(\alpha) = \frac{a - c_1 - 2c_2}{2-\alpha}, \\
\Pi^R(\alpha) &= \frac{1-\alpha}{(2-\alpha)^2}\left[(a - c_1 - c_2)^2 + c_2^2\right], \\
S^R(\alpha) &= \frac{1}{2(2-\alpha)^2}\left[(a - c_1)(a - c_1 - 2c_2) + 2c_2^2\right].
\end{aligned}$$

Notice that Ramsey quantities $\mathbf{x}^R(\alpha)$ are proportional to efficient quantities $\mathbf{x}^R(1)$, i.e., x_1^R/x_2^R does not depend on α: see Armstrong and Vickers (2018) for a discussion.

2.2 Duopoly Competition

Consider a duopoly counterpart of the previous setting in which each commodity is produced only by an independent firm.

2.2.1 Bertrand

When firms compete by setting simultaneously their own price, the best responses are given by:

$$\begin{aligned}
p_1 &= \frac{a - p_2 + c_1}{2}, \\
p_2 &= \frac{a - p_1 + 2c_2}{4}.
\end{aligned}$$

Note that prices are strategic substitutes. In the unique Bertrand-Nash equilibrium:

$$p_1^B = \frac{3a + 2\left(2c_1 - c_2\right)}{7} > c_1,\ p_2^B = \frac{a + 4c_2 - c_1}{7} > c_2,$$
$$x_1^B = \frac{3a - 3c_1 - 2c_2}{7} > 0,\ x_2^B = 2\frac{a - c_1 - 3c_2}{7} > 0,$$
$$\pi_1^B = \frac{\left(3a - 2c_2 - 3c_1\right)^2}{49},\ \pi_2^B = 2\frac{\left(a - 3c_2 - c_1\right)^2}{49}.$$

2.2.2 Cournot

When firms compete by setting simultaneously their own quantity, the best responses are given by:

$$x_1 = \frac{a + x_2 - c_1}{4},$$
$$x_2 = \frac{x_1 - c_2}{2}.$$

Note that quantities are strategic complements. In the unique Cournot-Nash equilibrium:

$$x_1^C = \frac{2a - 2c_1 - c_2}{7} > 0,\ x_2^C = \frac{a - 4c_2 - c_1}{7} > 0,$$
$$p_1^C = \frac{4a + 3c_1 - 2c_2}{7} > c_1,\ p_2^C = \frac{a - c_1 + 3c_2}{7} > c_2,$$
$$\pi_1^C = 2\frac{\left(2a - c_2 - 2c_1\right)^2}{49},\ \pi_2^C = \frac{\left(a - 4c_2 - c_1\right)^2}{49}.$$

2.2.3 Discussion

Notice that $p_1^m > p_1^B$. This perhaps surprising (given complementarity between goods) result arises out of price strategic substitutability, since $p_2^m < p_2^B$. Moreover, $p_1^C > p_1^m$ and $p_2^C < p_2^B$. The latter result is in contrast with the widespread opinion that price competition leads to lower prices (see e.g. Belleflamme and Peitz, 2015: p. 66, Lesson 3.11), and it is again due to price strategic substitutability. The intuition at the basis of the common Bertrand-versus-Cournot price ranking comes from the fact that demand elasticity *tends* to be larger in Bertrand setting (independently from the type of substitutability among goods).[5] However, with variable demand elasticities, and an asymmetric demand system, the standard ranking between Bertrand and Cournot equilibrium prices also hinges upon price strategic complementarity (i.e., supermodularity of the price game), a condition which does not hold in our setting: see

[5] Demand (own) elasticity in a Bertrand setting is locally larger than in its Cournotian counterpart if $sign\left\{\frac{\partial x_i}{\partial p_j}\right\} = -sign\left\{\frac{\partial p_i}{\partial q_j}\right\}$ for all i and j, $i \neq j$, a condition which always holds with 2 goods: see e.g. Vives (1999: chapter 6).

Vives (1999: paragraph 6.3). Since quantities are strategic complements, both Bertrand quantities are anyway larger than the Cournotian ones (see Vives, 1999: Remark 3, p. 156), which explains why the Bertrand price of commodity 1 must be lower than its Cournotian counterpart. One can also verify that profits are higher in the Bertrand equilibrium.

We leave to future research to investigate the general relationship (if any) between a subordinate commodity and its Bertrand-versus-Cournot price ranking. However, it is worth mentioning that the linear demand system, which implies that Bertrand prices are lower than the Cournotian ones under the parameter restrictions considered by Vives (1999: chapter 6), is under the same conditions also inconsistent with negative price-cost margins *à la* Ramsey.[6] Our example, on the contrary, shows that a linear demand system with a symmetric, negative definite Jacobian (i.e., which can be derived by a fully-fledged consumer surplus measure)[7] under complementarity among goods can well exhibit both a subordinate commodity and Bertrand prices lower than their Cournotian counterparts even with just two commodities.

3 Conclusions

We have presented a simple example of preferences exhibiting a *subordinate* commodity, namely a good such that the willingness to pay for it increases when the consumption of all commodities increases proportionally, and thus it is priced below its marginal cost according to Ramsey pricing. This subordinate good is a complement to another commodity that a two-product monopolist would price more than in the corresponding Bertrand equilibrium (with competitors producing a single product). Moreover, its Cournot equilibrium price is below its Bertrand equilibrium value, contrary to a widespread opinion. Finally, Ramsey quantities enjoy the property of being proportional to efficient ones: see Armstrong and Vickers (2018).

[6] Considers the general demand system (two goods)

$$\begin{aligned} x_1(\mathbf{p}) &= a_1 - b_1 p_1 + d p_2, \\ x_2(\mathbf{p}) &= a_2 + d p_1 - b_2 p_2, \end{aligned}$$

and assume constant marginal costs c_i, $i = 1, 2$. Ramsey prices are then given by

$$\begin{bmatrix} p_1^R \\ p_2^R \end{bmatrix} = \frac{1}{\left(b_2 b_1 - d^2\right)(2-\alpha)} \begin{bmatrix} \left(b_2 b_1 - d^2\right) c_1 + (1-\alpha)\left(b_2 a_1 + d a_2\right) \\ \left(b_1 b_2 - d^2\right) c_2 + (1-\alpha)\left(d a_1 + b_1 a_2\right) \end{bmatrix}.$$

Under the restrictions used by Vives (1999: chapter 6): $b_i = \beta_j/\Delta$, $a_i = \left(\alpha_i \beta_j - \alpha_j \gamma\right)/\Delta$, $d = \gamma/\Delta$, $\Delta = \beta_i \beta_j - \gamma^2 > 0$, β_i, $\alpha_i > 0$ and $\alpha_i > c_i$, Cournot prices are higher than their Bertrand counterparts and no Ramsey price involves a negative price-cost margin.

[7] Also notice that the Jacobian of the demand system (4)-(5) does not have a dominant diagonal: see Okuguchi (1987).

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