symmetry

MDPI

Review

CP Violation in the Quark Sector: Mixing Matrix Unitarity

Maurizio Martinelli [1,2]

[1] Dipartimento di Fisica "Giuseppe Occhialini", Università degli Studi di Milano-Bicocca, 20126 Milano, Italy; maurizio.martinelli@unimib.it
[2] INFN Sezione di Milano-Bicocca, 20126 Milano, Italy

Abstract: Since its discovery in the 1960s, the violation of *CP* symmetry has intrigued scientists and stimulated the advancement of knowledge in particle physics. Numerous experiments were designed and built to study it in increasingly deeper detail. Nowadays, the phenomenon is well framed within the Standard Model of Particle Physics. Nevertheless, new results are being produced by modern experiments at colliders that challenge the current understanding of the model. In this article, the current status of *CP* violation studies and the role of *CP* violation in the search for effects beyond the Standard Model are described together with the prospects for ongoing and future experiments.

Keywords: particle physics; flavor physics; *CP* violation

Citation: Martinelli, M. *CP* Violation in the Quark Sector: Mixing Matrix Unitarity. *Symmetry* **2024**, *16*, 950. https://doi.org/10.3390/sym16080950

Academic Editors: Giulia Brunetti, Marta Torti and Ignatios Antoniadis

Received: 19 April 2024
Revised: 7 July 2024
Accepted: 16 July 2024
Published: 24 July 2024

1. Introduction

The Standard Model of Particle Physics (SM) describes the interaction of quarks and leptons with utmost precision. Developed over the course of the 20th century, the SM has been tested in a wide range of experiments showing remarkable predictive power. While extremely successful in describing the interaction of elementary particles, the SM is not able to explain some of the most fundamental questions in physics. For instance, limiting ourselves to the energy scale for which the SM is best suited, it does not explain the size of the asymmetry between matter and antimatter in the Universe, why there are exactly three generations of fermions, and the origin of the masses of neutrinos.

These compelling questions indicate that the SM is not the ultimate theory of particle physics, rather an excellent approximation of a more fundamental theory. The SM is, therefore, considered as an effective theory, valid up to a certain energy scale, Λ, where new physics is expected to appear. Probing higher energy scales is, therefore, a key aspect in the search for new physics. There are two main approaches to this problem: direct and indirect searches. The former consists in looking for new particles directly by producing them in a particle collider, while the latter consists in looking for deviations from the SM predictions in precision measurements. At the Large Hadron Collider (LHC) both these approaches are pursued thanks to the unprecedented energy and luminosity of the proton–proton collisions.

In this review, we will focus on the indirect searches for new physics in the flavor sector, in particular, those concerning the study of the matter–antimatter (*CP*) asymmetry in hadron decays.

2. *CP* Violation in the Standard Model

The charge–parity (*CP*) symmetry is a fundamental symmetry of the SM of particle physics, being the combination of two fundamental symmetry operations, charge-conjugation (*C*) and parity (*P*). Before the discovery of *CP* violation in the neutral kaon system [1], it was believed that the *CP* symmetry was an exact symmetry of nature. The necessity to describe this phenomenon in the SM led to the introduction of an irreducible complex phase in the Cabibbo–Kobayashi–Maskawa (CKM) quark mixing matrix [2], a complex matrix representing the probability of each *up-type* quark (u, c, t) to interact with a *down-type* quark (d, s, b) by means of a W boson emission or absorption.

This phase appears in the CKM matrix predominantly in the elements V_{ub} and V_{td} that in the Wolfenstein parametrization [3] are expressed as

$$\begin{aligned} V_{ub} &\approx A\lambda^3(\rho - i\eta), \\ V_{td} &\approx A\lambda^3(1 - \rho - i\eta), \end{aligned} \tag{1}$$

which leads to an intuitive way to show *CP* violation by drawing *unitarity triangles* in the complex plane. The unitarity ($VV^\dagger = I$) of the CKM matrix implies column and row orthogonality

$$\sum_i V_{ij}V^*_{ik} = \delta_{jk} \quad \text{and} \quad \sum_j V_{ij}V^*_{kj} = \delta_{ik},$$

and each of these conditions can be represented as a triangle in the complex plane. The most common triangle is built from the unitarity relation

$$V_{ud}V^*_{ub} + V_{cd}V^*_{cb} + V_{td}V^*_{tb} = 0 \tag{2}$$

that can be rewritten as

$$1 - \frac{|V_{ud}|}{|V_{cd}||V^*_{cb}|}V^*_{ub} - \frac{|V^*_{tb}|}{|V_{cd}||V^*_{cb}|}V_{td} = 0 \tag{3}$$

when isolating the complex terms. The sum of three complex numbers can be drawn as a triangle in the complex plane with the three sides of the triangle corresponding to the three terms in the sum, as shown in Figure 1. The angles of the triangle are related to the *CP*-violating complex phase. In this case, they are defined as

$$\alpha = \arg\left(-\frac{V_{td}V^*_{tb}}{V_{ud}V^*_{ub}}\right) \qquad \beta = \arg\left(-\frac{V_{cd}V^*_{cb}}{V_{td}V^*_{tb}}\right) \qquad \gamma = \arg\left(-\frac{V_{ud}V^*_{ub}}{V_{cd}V^*_{cb}}\right). \tag{4}$$

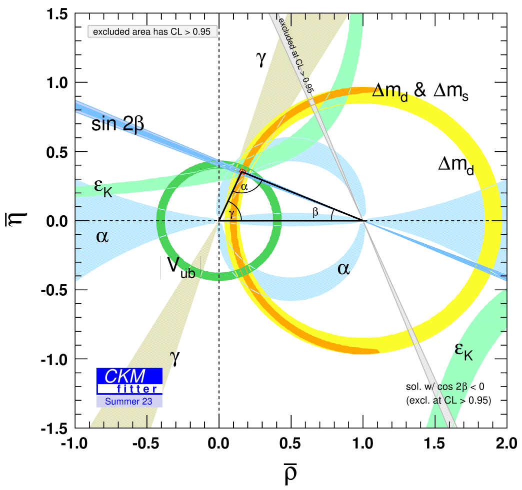

Figure 1. Plot of the CKM unitarity triangle in the complex plane from the CKMFitter group [4] made in the summer of 2023. The labels superimposed on the plots and the corresponding shaded areas show the various measurements of *CP* violation and the constraints they pose on the triangle.

A fundamental ingredient for observing *CP* violation is the presence of at least two amplitudes that can interfere. These amplitudes can be of a different nature; for example, in the neutral meson system, the two amplitudes are the direct decay and the decay after oscillation. With charged particles, these amplitudes are represented by the different resonant decay paths that lead to the same final state. Different amplitudes show not only a difference in the *weak* (*CP*-violating) phase, but also in the *strong* (*CP*-conserving) phase. The non-zero difference between these two phases is what allows for the interference between the two amplitudes and the observation of *CP* violation [5,6].

CP violation first manifested itself in the decays of neutral kaons, but has become widely studied in the decays of B mesons. In Figure 1, there are many labels superimposed on the plot, each corresponding to a different measurement of *CP* violation. ϵ_K is the sole constraint imposed by measurements in the neutral kaons system, while all the other measurements are related to B mesons decays.

The aforementioned unitarity triangle is the most commonly drawn since it is the one that shows the largest *CP* violation effects and is also the most constrained one. Alternative triangles can be drawn when focusing on the transitions involving B_s^0 and D decays but they are not as clear as the B one, since they are characterized by smaller *CP* violation effects that lead to triangles flattened on the real axis. Nevertheless, observations of *CP* violation in the B_s^0 and D systems have been made. In the upcoming section, we will discuss the measurements of *CP* violation in the B and D systems.

3. Experimental Status of *CP* Violation

Measurements of *CP* violation were first made in the 1960s by studying neutral kaon decays [1], but it was not until the discovery of the b quark that an extensive experimental program could start. Once B mesons were discovered and the technology was ready to produce them in large quantities, an extensive research program was undertaken by two competing experiments, BaBar [7] and Belle [8], that ran in the early 2000s at the asymmetric e^+e^- colliders PEP-II and KEKB, respectively.

3.1. *CP* Violation in B^0 Decays

The first observation of *CP* violation in the B meson system was made by two collaborations in 2001 [9,10] by measuring

$$\sin(2\beta) = 0.59 \pm 0.14 \pm 0.05 \quad \text{and} \quad \sin(2\beta) = 0.99 \pm 0.14 \pm 0.06, \tag{5}$$

respectively. They combined the analyses of $B^0 \to (c\bar{c})K_S^0$ ($c\bar{c} = J/\psi$, $\psi(2S)$, χ_{c1}) and $B^0 \to J/\psi K_L^0$ decays in this result.

CP violation is observed as an asymmetry in the decay time distribution of the B^0 mesons decaying to $J/\psi K_S^0$ and $J/\psi K_L^0$, as shown in Figure 2

$$\begin{aligned} A_{CP}(\Delta t) &\equiv \frac{N_{\bar{B}^0}(\Delta t) - N_{B^0}(\Delta t)}{N_{\bar{B}^0}(\Delta t) + N_{B^0}(\Delta t)} \\ &= \eta_f \sin(2\beta)\sin(\Delta m_d \Delta t), \end{aligned} \tag{6}$$

with η_f being the *CP* eigenvalue of the final state and $\Delta t = t_{CP} - t_{\text{tag}}$ the decay time difference between the reconstructed and tagging B mesons. It is interesting to note that *CP* violation at the B Factories is measured through the measurement of the difference in the decay time of the two B mesons in the event. Such a measurement is possible thanks to the entanglement of the B mesons, i.e., being produced from the same $e^+e^- \to b\bar{b}$ interaction, the large acceptance of the detectors covering almost the whole solid angle, and to the spatial separation between the two B mesons decay vertices, which is achieved by the asymmetric energy of the beams that boosts the B mesons in the laboratory frame.

At hadron colliders, such as the LHC, the B mesons are still produced in pairs with a large boost, but most of the time, one of them is not reconstructed due to the detector's acceptances. In this case, *CP* violation is measured by studying the decay time distribution of the B meson that is reconstructed, and the rest of the event is used to infer the flavor of the B meson (*tagging* [11]). Larger yields have been collected since the first observation of *CP* violation in B^0 decays, improving the precision of the measurements and requiring Equation (6) to be extended to account for second-order effects

$$\begin{aligned} A^{CP}(t) &\equiv \frac{\Gamma(\bar{B}^0(t) \to f) - \Gamma(B^0(t) \to f)}{\Gamma(\bar{B}^0(t) \to f) + \Gamma(B^0(t) \to f)} \\ &= \frac{S\sin(\Delta m_d t) - C\cos(\Delta m_d t)}{\cosh\left(\frac{1}{2}\Delta\Gamma_d t\right) + \mathcal{A}_{\Delta\Gamma}\sinh\left(\frac{1}{2}\Delta\Gamma_d t\right)}. \end{aligned} \tag{7}$$

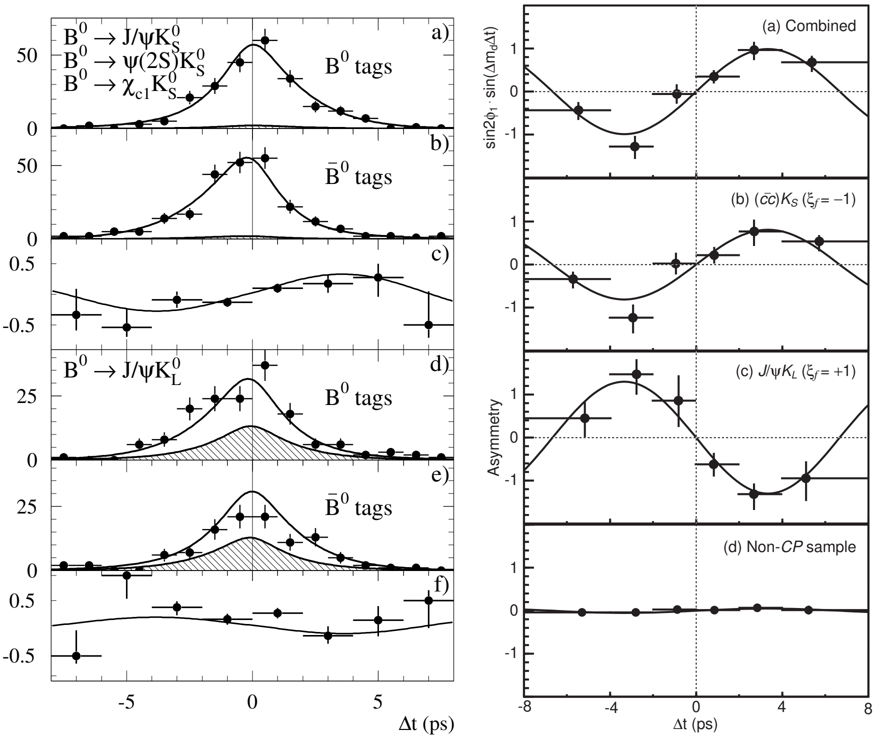

Figure 2. First observation of *CP* violation in B^0 decays as obtained by the BaBar (**left**) [9] and Belle (**right**) [10] collaborations. In the left plots, the cumulative decay time distributions of B mesons decaying to $J/\psi K^0_S$, $\psi(2S)K^0_S$, and $\chi_{c1}K^0_S$ are shown when identified as B^0 mesons (**a**) and $\bar{B}^0$ mesons (**b**). A shaded area represents the contribution of background events. The asymmetry in the decay time distributions of the signal candidates, which is a measure of *CP* violation, is shown in (**c**). Similar plots are made for $B \to J/\psi K^0_L$ decays in (**d–f**). In the right plots, (**a**) shows the asymmetry in the decay time distributions of B mesons decaying to $J/\psi K^0_S$, $\psi(2S)K^0_S$, $\chi_{c1}K^0_S$, $\eta_c K^0_S$, and $J/\psi K^0_L$, which is separated for B mesons decaying to $c\bar{c}K^0_S$ final states (**b**) and $J/\psi K^0_L$ final states (**c**), and (**d**) shows the asymmetry of the control samples. The black dots in the plots represent the data, while the solid lines represent the fit to the data. Shaded area represent the contribution of background events.

Here S, C, and $\mathcal{A}_{\Delta\Gamma}$ are the *CP* violation parameters, Δm_d and $\Delta\Gamma_d$ are the difference between the mass and the decay width of the two B^0 mass eigenstates, respectively. It is useful to note that the parameter S is related to the *CP* violation parameter $\sin(2\beta)$ by the relation $S = \sin(2\beta + \Delta\Phi_d + \Delta\Phi_d^{NP})$, where $\Delta\Phi_d$ is a contribution from loop (or *penguin* [12], see Appendix A) diagrams (suppressed in the SM), and $\Delta\Phi_d^{NP}$ is a contribution of the same type arising from phenomena beyond the SM. The B Factories have measured the *CP* violation parameters S and C in various $B^0 \to c\bar{c}K^0$ decays, obtaining

$$S = 0.687 \pm 0.028(stat) \pm 0.012(syst) \qquad C = 0.024 \pm 0.020(stat) \pm 0.016(syst), \tag{8}$$

$$S = 0.667 \pm 0.023(stat) \pm 0.012(syst) \qquad C = 0.006 \pm 0.016(stat) \pm 0.012(syst), \tag{9}$$

where the first set of values (Equation (8)) are measured by the BaBar experiment [13], and the second (Equation (9)) by Belle [14]. These results represents the legacy of the *B* Factories and were quite recently superseded by the LHCb experiment [15], whose latest measurement of this parameter is

$$\begin{aligned} S(\psi K^0_S) &= 0.717 \pm 0.013(stat) \pm 0.008(syst), \\ C(\psi K^0_S) &= 0.008 \pm 0.012(stat) \pm 0.003(syst), \end{aligned} \tag{10}$$

which is the most precise measurement of *CP* violation in B^0 decays to date [16].

Despite being the most precise, the measurement of *CP* violation in B^0 decays through the angle β is not the only one. The other two angles of the unitarity triangle, α and γ, can be measured through the study of other B^0 decays.

In particular, the angle α can be measured through the study of $b \to u$ transitions, like $B^0 \to \pi\pi$ and $B^0 \to \rho\rho$ decays. In these decays, the interference between the tree-level $b \to u$ and the box diagram of the B^0–$\bar{B}^0$ mixing result in the *CP* asymmetry parameters $S = \sin 2\alpha$ and $C = 0$. In reality, the penguin $b \to d$ diagrams also contribute and introduce theoretical uncertainties that need to be corrected (*penguin pollution*). A seminal paper by Gronau and London [17] provided the strategy to measure α in a model-independent way by combining measurements of *CP* asymmetries in the isospin-related $B^0 \to \pi^+\pi^-$, $B^0 \to \pi^0\pi^0$, and $B^+ \to \pi^+\pi^0$ decays. In this context, the experiments measure the *CP* asymmetry and the combination of the measurements allows to extract the angle α. The most precise measurement of *CP* violation in B^0 decays to $\pi^+\pi^-$, performed by the LHCb experiment [18] with a dataset of 4.7 fb^{-1} luminosity, is

$$\begin{aligned} C_{\pi\pi} &= -0.311 \pm 0.045, \\ S_{\pi\pi} &= -0.706 \pm 0.042, \end{aligned} \tag{11}$$

while BaBar and Belle studied the decays $B^0 \to \pi^0\pi^0$ and $B^+ \to \pi^+\pi^0$ with a sensitivity smaller than anticipated [19,20]. These results led the *B* Factories to pursue alternative avenues to measure the angle α by performing a time-dependent *CP* violation measurement in $B^0 \to \rho\rho$ and $B^0 \to \rho^0(\to \pi^+\pi^-)\pi^0$ decays [21–26]. It is interesting to note that nowadays the most stringent constraints to the determination of α come from the measurements of the $B^0 \to \rho\rho$ decays, as shown in Figure 3, which were not considered at the beginning of the *B* Factories program. It was at the time thought that the $B^0 \to \rho\rho$ were theoretically challenging due to the need of performing three isospin analyses for longitudinal and transverse polarizations of the ρ mesons. In reality, the longitudinal polarization dominates the decay [21,22,27]. Together with the relatively small branching ratio of the penguin-dominated $B^0 \to \rho^0\rho^0$ decay, the $B^0 \to \rho^+\rho^-$ decays provide a theoretically clean determination of the angle α.

The last angle of the unitarity triangle, γ, can be measured through the study of $B \to DX$ decays, in which the interference between the $b \to c$ and the $b \to u$ transitions gives access to $\gamma \equiv \arg\left[-V_{ud}V^*_{ub}/V_{cd}V^*_{cb}\right]$. Since it is measured through the study of tree-level decays, it has a very small irreducible theoretical uncertainty $\delta_\gamma/\gamma \leq 10^{-7}$ [28], which also makes it a promising ground for searches of physics beyond the SM effects. The experimental uncertainty on γ saw a significant reduction in recent years thanks to the efforts of the LHCb collaboration. Typically, measurements of γ are made by studying the $B^\pm \to \overset{(-)}{D} K^\pm$ decays. They proceed through the $b \to c\bar{u}s$ and $b \to u\bar{c}s$ transitions, whose ratio is $r_B e^{i(\delta_B \pm \gamma)}$, where r_B is the absolute ratio of the two amplitudes and δ_B is the strong phase difference between the two amplitudes. This ratio can be measured experimentally by studying the rate of B^+ mesons

$$\begin{aligned} \Gamma(B^\pm \to \overset{(-)}{D}[\to f]K^\pm) &= \left| r_D e^{-i\delta_D} + r_B e^{i(\delta_B \pm \gamma)} \right|^2 \\ &= r_D^2 + r_B^2 + 2\kappa r_D r_B \cos(\delta_D - \delta_B \pm \gamma), \end{aligned} \tag{12}$$

where no CP violation is assumed in the D meson decays; therefore, the only nuisance parameters are the absolute ratio (r_D) and the strong phase difference (δ_D) between the $D \to f$ and the $D \to \bar{f}$ decays. To account for the dilution of the interference in multibody B and D decays, the "coherence factor" κ is present in the formula, being 1 for two-body decays and $\kappa < 1$ for multibody decays. Since the measurement of γ reduces to the measurement of the decay rates of charged-conjugate decays, it is important to have a good control of the systematic uncertainties, especially the charged-particles detection asymmetries.

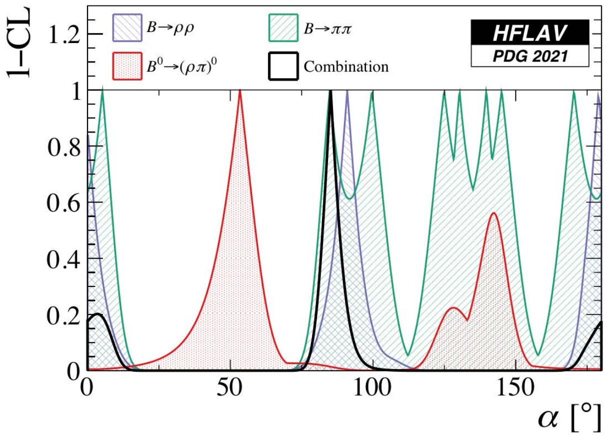

Figure 3. Constraints on the angle α from the measurements of CP violation in $B^0 \to \pi\pi$, $B^0 \to (\rho\pi)^0$, and $B^0 \to \rho\rho$ decays [16].

The simplest topology for the measurement of γ is the $B^\pm \to D^0 K^\pm$ decay, where the D^0 meson is reconstructed in two-body final states. Two methods are used to measure γ in these decays: the GLW method [29] and the ADS method [30]. In the former method, $D^0 \to \pi^+\pi^-, K^+K^-$ decays are used, offering $r_D^{hh} = 1$ and $\delta_D^{hh} = 0$, while in the latter, $D^0 \to K^+\pi^-, K^-\pi^+$ decays are used, which have $r_D^{K\pi} = 0.06$, and make best use of the similarity of $r_D^{K\pi}$ and $r_B^{D^0K}$($r_B^{D^0K} \approx c_f|V_{cs}V_{ub}^*/V_{us}V_{cb}^*| \approx 0.1$, where $c_f \approx 0.3$ is a color suppression factor), giving large interference effects and high sensitivity to the phase information.

Another class of measurements involves multibody D^0 decays, such as $D^0 \to K_S^0\pi^+\pi^-$, $D^0 \to K_S^0 K^+K^-$, and $D^0 \to K_S^0\pi^+\pi^-\pi^0$ decays. The technique used in this case is commonly referred to as GGSZ [31], and takes advantage of the resonant structure of the D^0 decay to acquire sensitivity to γ. As shown before, the strong phase difference between the D^0 and $\overline{D}^0$ decays to the same final state is an important ingredient to the sensitivity on γ. By studying multibody D^0 decays, one gets access to regions in which the strong phase difference is large, therefore enhancing the sensitivity. The price to pay is the need of a model to describe the resonant structure of the D^0 decays, which introduces a theoretical uncertainty in the measurement. Alternatively, external input is needed to constrain the strong phase difference in the regions of the Dalitz plot. This model-independent approach is becoming more and more popular, thanks to the synergy between the LHCb and the BESIII experiments. The large sample of quantum-correlated D^0–$\overline{D}^0$ pairs produced at the e^+e^- colliders allows to measure the strong phase difference in the D^0 decays, and detailed measurements across the Dalitz plot of various D^0 decays are performed [32].

Finally, the angle γ can also be measured through the study of $B^0 \to D^{(*)\mp}\pi^\pm$ decays. In the SM, these decays proceed through the $\bar{b} \to \bar{c}u\bar{d}$ and $b \to u\bar{c}d$ transitions, and the interference between the two amplitudes gives access to the angle γ. The same final states can also be reached after B^0–$\overline{B}^0$ mixing though, and the asymmetry between the decay rates gives access to $2\beta + \gamma$. The measurement of γ in these decays, therefore, relies on the knowledge of β from the $B^0 \to c\bar{c}K_S^0$ decays. In this case, a time-dependent measurement of the CP asymmetries allows to measure the angle γ [33–35]. Mesons initially produced as B^0 decay to the final states $f = D^{(*)-}\pi^+$ and $\bar{f} = D^{(*)+}\pi^-$ as

$$
\begin{aligned}
\Gamma(B^0(t) \to f) &= e^{-\Gamma_d t}\Big[1 + C_f \cos(\Delta m_d t) - S_f \sin(\Delta m_d t)\Big], \\
\Gamma(B^0(t) \to \bar{f}) &= e^{-\Gamma_d t}\Big[1 + C_{\bar{f}} \cos(\Delta m_d t) - S_{\bar{f}} \sin(\Delta m_d t)\Big].
\end{aligned} \tag{13}
$$

The time evolution of initially produced $\bar{B}^0$ mesons is the same except for the flipped sign of the C and S coefficients. These coefficients are related to the theoretical observables $r_{D\pi}$, δ, β, and γ by the relations

$$
\begin{aligned}
C_f &= \frac{1 - r_{D\pi}^2}{1 + r_{D\pi}^2} = -C_{\bar{f}}, \\
S_f &= -\frac{2 r_{D\pi} \sin[\delta - (2\beta + \gamma)]}{1 + r_{D\pi}^2}, \\
S_{\bar{f}} &= \frac{2 r_{D\pi} \sin[\delta + (2\beta + \gamma)]}{1 + r_{D\pi}^2},
\end{aligned} \tag{14}
$$

where $r_{D\pi}$ is the ratio of the $B^0 \to D^{(*)-}\pi^+$ and $B^0 \to D^{(*)+}\pi^-$ decay amplitudes and δ their strong phase difference. By constraining the values of $r_{D\pi}$ and β from external measurements, the angle γ can be extracted from the measurement of the S coefficients, since they only differ by a phase $2(2\beta + \gamma)$. Measurements of γ with this approach were made by the B Factories using $B^0 \to D^{(*)\mp}\pi^\pm$ and $B^0 \to D^\mp\rho^\pm$ decays [36–39]. The first measurement of this kind at a hadron collider was made by LHCb using $B^0 \to D^\mp\pi^\pm$ decays [40].

The LHCb and BaBar experiments have produced a compendium of all their measurements of γ and provided their own averages. Up to 2013, BaBar was the leader in the measurement of γ [41], with an average of $\gamma = (69^{+17}_{-16})^\circ$. This was outclassed by the LHCb experiment, whose latest average is $\gamma = (67 \pm 4)^\circ$ [42], clearly dominating the world average of $\gamma = (66.2^{+3.4}_{-3.6})^\circ$ [16]. A summary of the sensitivity to γ from the various measurements is shown in Figure 4. So far, the average is mostly constrained by the measurement of γ using $B^+ \to D^0K^+$ decays, with the GGSZ method.

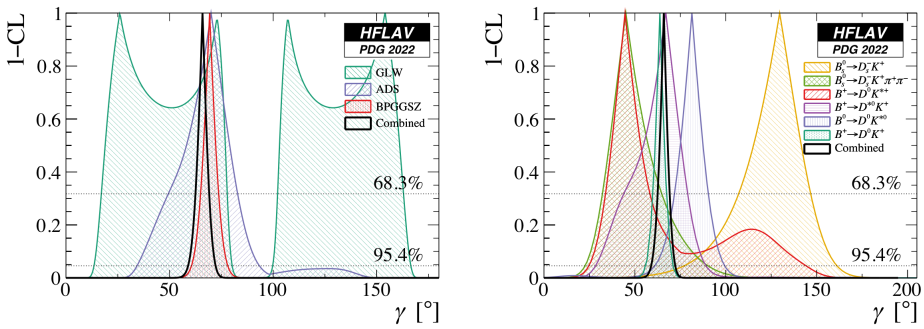

Figure 4. Constraints on the angle γ from the measurements of *CP* violation using various methods (**left**) and decay modes (**right**) [16].

To summarize the status of *CP* violation in B^0 decays, the combination of all the measurements of the angles α, β, and γ gives [16]

$$
\begin{aligned}
\alpha &= (85.2^{+4.8}_{-4.3})^\circ, \\
\beta &= (22.2 \pm 0.7)^\circ, \\
\gamma &= (66.2^{+3.4}_{-3.6})^\circ.
\end{aligned} \tag{15}
$$

When summing the angles of the unitarity triangle, the sum is [6]

$$\alpha + \beta + \gamma = (173 \pm 6)^\circ, \tag{16}$$

consistent with the SM expectations.

3.2. CP Violation in B_s^0 Decays

When discussing the measurement of γ, the possibility was omitted of measuring the angle by studying $B_s^0 \to D_s^\pm K^\mp$ [43] and $B_s^0 \to D_s^\pm K^\mp \pi^+ \pi^-$ [44] decays.

In these decays, the sensitivity to *CP* violation arises from the interference of the mixing and decay amplitudes [33–35,45], and the *CP*-violating parameters are a function of γ and $\beta_s \equiv \arg[-(V_{ts}V_{tb}^*)/(V_{cs}V_{cb}^*)]$, the weak phase of the B_s^0–$\bar{B}_s^0$ mixing. Equations (13) and (14) can be adapted to the B_s^0 system by replacing the B^0 with B_s^0, the $D^{(*)}$ with D_s^+ mesons, Δm with Δm_s, and the angle β with β_s.

The weak phase β_s is of particular interest since it is a sensitive probe of physics beyond the SM [46]. It is usually measured as $\phi_s = -2\beta_s$ and its value is predicted to be $\phi_s = -0.0368^{+0.0009}_{-0.0006}$ [47] using the known values of the CKM matrix elements. The most sensitive measurement of *CP* violation in B_s^0 decays is obtained through the study of $B_s^0 \to \psi hh$ decays ($h = \pi, K$). Since their final state particles may exhibit various polarizations depending on the hh resonances involved (e.g., $\phi(1020)$, $\rho(770)$), the decay amplitudes are studied on the transversity basis, where they are decomposed in terms of the helicity amplitudes, to disentangle the *CP*-odd and the *CP*-even contributions [48–50].

The specific topology of the $B_s^0 \to J/\psi\phi$ decay allows to measure ϕ_s with good precision at hadron colliders. The J/ψ and ϕ resonances have very small widths, which makes their identification easier even without a specific particle identification sub-system. Therefore, many experiments have measured the ϕ_s parameter using this decay mode [51–55]. The latest combination of these measurements is shown in Figure 5. As shown in the figure, the combination of the independent measurements from different experiments is consistent with the SM prediction.

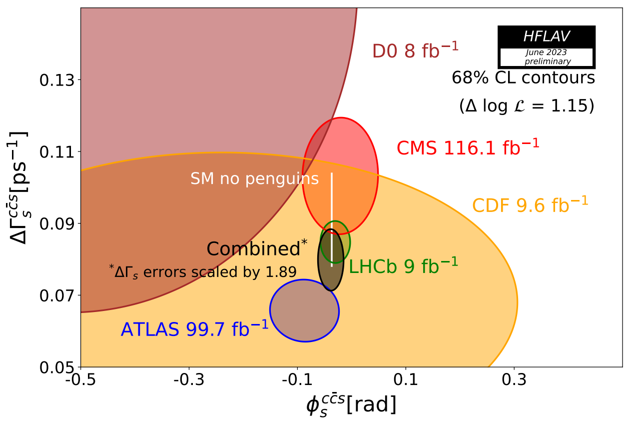

Figure 5. Constraints on the weak phase ϕ_s from the measurements of *CP* violation in B_s^0 decays from various experiments [16]. Please note that the latest CMS (preliminary) result [56] is not included in the combination.

3.3. CP Violation in Λ_b^0 Decays

Since the Λ_b^0 baryon is the lightest baryon containing a b quark, it is a good candidate to study *CP* violation in the baryon sector. Similar approaches as those developed for charged B mesons can be used to study *CP* violation in Λ_b^0 decays. In particular, the angle α can be measured by studying charmless Λ_b^0 decays, and the angle γ with final states involving charm mesons. Mixing does not happen in baryon decays; therefore, measurements of *CP* violation in the interference between decay and mixing (β) are not possible.

No measurements of *CP* violation in Λ_b^0 decays have been made so far, but the LHCb experiment is aiming at it with many searches on various decay modes. Evidence of *CP* violation in the decays of multibody $\Lambda_b^0 \to p\pi^-\pi^+\pi^-(K^+K^-)$ decays was reported by the LHCb experiment [57], but further data collected in Run2 of the LHC have not led to observation of the effect [58] yet. The technique used in this analysis searches for *CP* violation by measuring triple-product asymmetries [59–61]. In this case, asymmetries of kinematical distributions are measured separately on particle and anti-particle decays. These distributions are built to be odd under *CP* transformation, but may exhibit an asymmetry due to strong-phase differences. These strong-phase effects are canceled out by measuring the difference between the asymmetries of the two charged-conjugate states, which is a direct probe of *CP* violation.

Another class of measurements of *CP* violation in Λ_b^0 decays is the study of the $\Lambda_b^0 \to pK^-$ and $\Lambda_b^0 \to p\pi^-$ decays. These decays follow the same quark-level transitions of charmless B^0 and B_s^0 two-body decays, in which *CP* violation is established. Nevertheless, the LHCb experiment has not yet reported any measurement of *CP* violation in these decays with sensitivities as low as 2% [62]. A challenging aspect of this analysis is the effect of the Λ_b^0 production asymmetry in the LHC, which is known with comparable precision to the statistical precision of the *CP* asymmetry measurement [63].

3.4. CP Violation in Charm Decays

The study of *CP* violation in charm decays is a challenging task, since the *CP* violation effects in the charm sector are expected to be very small in the SM. Nevertheless the charm sector is unique in the SM, as it is the only up-type quark allowing a thorough study of flavor-changing neutral currents (FCNC) and searches for physics beyond the SM (top quarks undergo decay before they change to hadronize [64,65], and the lighter hadrons built with u and $\overline{u}$ are their own antiparticle).

Since the quarks involved in the box diagram for D^0 mixing are $m_u, m_s, m_b \ll m_W$, the process is highly suppressed in the SM. This results in a similar size of the difference of the mass and width eigenvalues [66].

Direct *CP* violation in charm decays arises from the interference of the tree-level and penguin $c \to u\bar{s}s$ diagrams, and is expected to be of the order of 10^{-3} or less [67–69]. Only recently, the LHCb experiment reported the first observation of *CP* violation in the decays of D^0 mesons [70]. This measurement showed a difference in the *CP* asymmetries of $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ decays of $(-15.4 \pm 2.9) \times 10^{-4}$. Since D^0 mesons can mix into $\overline{D}^0$ before decaying, and given the size of expected *CP* violation and mixing, the experimental asymmetry is a combination of the direct *CP* violation ($a_{CP}^{\rm dir}(f)$) and the mixing-induced *CP* violation

$$A_{CP}(f) \approx a_{CP}^{\rm dir}(f) - \frac{\langle t(f)\rangle}{\tau(D^0)} A_\Gamma(f), \tag{17}$$

where $\langle t(f)\rangle$ is the mean decay time of $D^0 \to f$ decays in the reconstructed sample, $\tau(D^0)$ is the lifetime of the D^0 meson, and $A_\Gamma(f)$ is the the asymmetry between the effective D^0 and $\overline{D}^0$ decay widths [71]. In the limit of U-spin symmetry $a_{CP}^{\rm dir}(K^+K^-) = -a_{CP}^{\rm dir}(\pi^+\pi^-)$, and by assuming A_Γ to be the same for the two decays, the difference between the *CP* asymmetries in the two decays is

$$\Delta A_{CP} \equiv A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-) \approx \Delta a_{CP}^{\rm dir} - \frac{\Delta\langle t\rangle}{\tau(D^0)} A_\Gamma, \tag{18}$$

where $\Delta\langle t\rangle$ is the difference in the mean decay times of the two decays. The experimental advantage of measuring the difference in the *CP* asymmetries is the cancellation of experimental asymmetries arising from the production and detection of the D^0 mesons. Furthermore, while A_Γ is of the same order of magnitude as ΔA_{CP}, the correction factor $\Delta\langle t\rangle/\tau(D^0) < 0.1$ since the experimental acceptance is typically similar between the two decay modes. Studies are ongoing to measure *CP* violation separately for $D^0 \to K^+K^-$

and $D^0 \to \pi^+\pi^-$ decays, which involves removal of the nuisance production and detection asymmetries by means of control samples. The latest result, based on 4.7 fb^{-1} data collected by the LHCb experiment [72], reported

$$a_{CP}^{\mathrm{dir}}(K^+K^-) = (7.7 \pm 5.7) \times 10^{-4},$$
$$a_{CP}^{\mathrm{dir}}(\pi^+\pi^-) = (23.2 \pm 6.1) \times 10^{-4}.$$

While the quest to measure direct *CP* violation from a single decay mode is ongoing, it is not the only challenge in the study of *CP* violation in charm decays. The mixing-induced *CP* violation in the charm sector is expected to be even smaller than the direct *CP* violation, and still escapes being measured. In the literature, *CP*-violating observables in charm decays are expressed in terms of the mixing parameters x and y, which are defined as

$$x = \frac{(m_1 - m_2)}{\Gamma},$$
$$y = \frac{\Gamma_1 - \Gamma_2}{\Gamma}, \tag{19}$$

where $m_{1,2}$ and $\Gamma_{1,2}$ are the mass and decay width of the two mass eigenstates $D_{1,2}$, respectively, and Γ is the average decay width. The two mass eigenstates can be written as a linear combination of the flavor eigenstates

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\overline{D}^0\rangle, \tag{20}$$

with the complex coefficients satisfying the condition $|p|^2 + |q|^2 = 1$. In this formalism, *CP* violation in mixing can manifest itself as a deviation of $|q/p|$ from unity, while interference of mixing and decay can give rise to a non-zero phase difference $\phi_f \equiv \arg(q\bar{A}_f/pA_f)$ between the D^0 (A_f) and $\overline{D}^0$ ($\bar{A}_f$) decay amplitudes. In the case of decays to the same final state, $\phi_f = \phi$. The time evolution of the decay rates of D^0 and $\overline{D}^0$ mesons can, therefore, be studied in terms of the mixing parameters x and y measured separately in D^0 and $\overline{D}^0$ decays. For convenience, these parameters are expressed in terms of the *CP*-averaged mixing parameters

$$x_{CP} = \frac{1}{2}\left[x\cos\phi\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right) + y\sin\phi\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right)\right], \tag{21}$$
$$y_{CP} = \frac{1}{2}\left[y\cos\phi\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right) - x\sin\phi\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right)\right], \tag{22}$$

and the *CP*-violating differences

$$\Delta x = \frac{1}{2}\left[x\cos\phi\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right) + y\sin\phi\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right)\right], \tag{23}$$
$$\Delta y = \frac{1}{2}\left[y\cos\phi\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right) - x\sin\phi\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right)\right]. \tag{24}$$

In the absence of *CP* violation ($\phi = 0$, $|q/p| = 1$), the mixing parameters $x_{CP} = x$ and $y_{CP} = y$ and the *CP*-violating differences Δx and Δy are zero. The latest average of the mixing and *CP* violation parameters in the charm sector is [16]

$$x = (4.07 \pm 0.44) \times 10^{-3},$$
$$y = (6.45^{+0.24}_{-0.23}) \times 10^{-3},$$
$$|q/p| = 0.994^{+0.016}_{-0.015},$$
$$\phi = -2.6^{+1.1}_{-1.2},$$

and the *CP* violation parameters are graphically shown in Figure 6. The data are so far compatible with the absence of *CP* violation in the charm sector up to 2.1 σ.

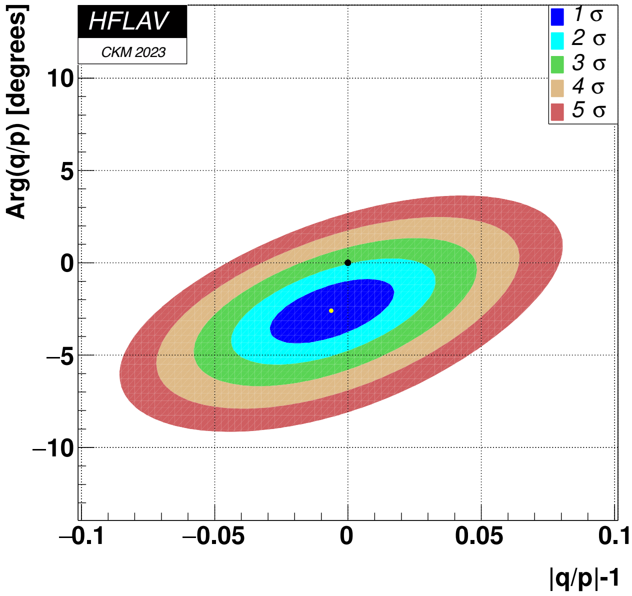

Figure 6. Constraints on the mixing and *CP* violation parameters in the charm sector [16].

The *golden* channel to measure mixing-induced *CP* violation in the charm sector is the $D^0 \to K^0_S \pi^+\pi^-$ decay. It gives access to all the aforementioned observables at once, and has a relatively large branching ratio. Seminal studies performed at the *B* Factories were made through a time-dependent amplitude analysis of the decay [73,74]. Such analyses require an excellent understanding of the decay amplitude of the D^0 meson and especially of the time-dependent reconstruction efficiency of the experiment. This has not been possible at LHCb so far, given the limited amount of simulated data available. Therefore, the LHCb experiment has pursued an alternative method [75] to study $D^0 \to K^0_S \pi^+\pi^-$ decays, which relies on the measurement of the strong phase differences from BESIII [32]. This model-independent approach avoids the need of a time-dependent amplitude analysis, at the cost of limited sensitivity to the parameters associated to the width difference (y_{CP} and Δy). By applying this technique, the LHCb experiment obtained the first observation of the mixing parameter x and the most precise determination of *CP* violation parameters in mixing at the time [76].

Complementarily to $D^0 \to K^0_S \pi^+\pi^-$ decays, there are ways of measuring the difference in the decay widths of D^0 and $\overline{D}^0$ mesons. By studying the evolution of the ratio of $D^0 \to K^+K^-(\pi^+\pi^-)$ over $D^0 \to K^-\pi^+$ decays over time, it is possible to measure the mixing-induced *CP* violation parameter y_{CP}

$$\frac{\hat{\Gamma}(D^0 \to f) + \hat{\Gamma}(\overline{D}^0 \to f)}{\hat{\Gamma}(D^0 \to K^-\pi^+) + \hat{\Gamma}(\overline{D}^0 \to K^+\pi^-)} - 1 = y^f_{CP} - y^{K\pi}_{CP} \tag{25}$$

The best measurement to date of this parameter is performed by the LHCb experiment [77] and gives

$$y^{\pi\pi}_{CP} - y^{K\pi}_{CP} = (6.57 \pm 0.53 \pm 0.16) \times 10^{-3},$$
$$y^{KK}_{CP} - y^{K\pi}_{CP} = (7.08 \pm 0.30 \pm 0.14) \times 10^{-3},$$

which is consistent with the world average of y reported above; therefore, no evidence of *CP* violation in mixing is found.

4. Experimental Status of Quark-Mixing Matrix Unitarity

An important aspect of the CKM matrix is that it is unitary, i.e., the sum of the squares of the elements in each row and column is equal to one. This indicates that there are only three families of quarks, and the total strength of the charged current couplings between each up-type (down-type) quark and all down-type (up-type) quarks is universally consistent. Within the SM, this is a consequence of the universality of non-abelian gauge couplings [5]. As such, it deserves experimental investigation that is achieved by measuring the strength of the couplings as the magnitude of the CKM matrix elements. The latest averages of the experimental measurements of the magnitudes of the CKM matrix elements are summarized in Table 1.

Table 1. The latest averages of the magnitudes of the CKM elements [6]. Each element is given as $|V_{ij}|$ with i, j being the row and column indices of the table.

	d	**s**	**b**
u	0.97373 ± 0.00031	0.2243 ± 0.0008	$(3.82 \pm 0.20) \times 10^{-3}$
c	0.221 ± 0.004	0.975 ± 0.006	$(40.8 \pm 1.4) \times 10^{-3}$
t	$(8.6 \pm 0.2) \times 10^{-3}$	$(41.5 \pm 0.9) \times 10^{-3}$	1.014 ± 0.029

A brief description of the most sensitive measurements of the CKM matrix elements is given below.

The most precise determination of $|V_{ud}|$ comes from the superallowed $0^+ \to 0^+$ nuclear β-decay transitions, which are mediated by the weak interaction. A complete review of the experimental and theoretical aspects of superallowed β-decays can be found in Ref. [78].

The value of $|V_{us}|$ is obtained from the study of semileptonic kaon decays of the type $K \to \pi\ell^+\nu$ ($\ell = e, \mu$), whose amplitude can be expressed as

$$\mathcal{M} = -i\frac{G_F}{\sqrt{2}}V_{us}L^\mu H_\mu \tag{26}$$

where G_F is the Fermi constant, L^μ is the leptonic current, and H_μ is the hadronic current:

$$L^\mu = \ell^+\gamma^\mu(1-\gamma^5)\nu_\ell, \tag{27}$$

$$H_\mu = \langle\pi(p')|\overline{u}\gamma_\mu(1-\gamma^5)s|K(p)\rangle, \tag{28}$$

where p and p' are the momenta of K and π, respectively. Their product leads to an effective Hamiltonian that can be expressed as

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}}V_{us}\left[\overline{u}\gamma_\mu s - \overline{u}\gamma_\mu\gamma_5 s\right]\ell^+\gamma^\mu(1-\gamma^5)\nu_\ell. \tag{29}$$

Since $K \to \pi\ell^+\nu$ is a pseudoscalar meson transition (K and π have $J^P = 0^-$), the axial-vector component of H_μ is zero due to constraints on the spin of the outgoing u quark. The vector component of H_μ can be expressed in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$ as

$$\langle\pi(p')|\overline{u}\gamma_\mu s|K(p)\rangle = f_+(q^2)\left(p_\mu + p'_\mu - \frac{m_K^2 - m_\pi^2}{q^2}q_\mu\right) + f_0(q^2)\frac{m_K^2 - m_\pi^2}{q^2}q^\mu, \tag{30}$$

where $q = p - p'$ is the momentum transfer. Analyses of $K \to \pi\ell^+\nu$ decays often assume a linear dependence of the form factors $f_{+,0}(q^2) = f_+(0)\left[1 + \lambda_{+,0}(q^2/m_\pi^2)\right]$ [6,79] and the decay rate can be expressed in terms of $|V_{us}|f_+(0)$. By averaging the results of $K_L^0 \to \pi e\nu$, $K_L^0 \to \pi\mu\nu$, $K^\pm \to \pi^0\ell^\pm\nu$, $K^\pm \to \pi^0\mu^\pm\nu$, and $K_S^0 \to \pi e\nu$, the Particle Data Group (PDG) obtained the value of $|V_{us}|f_+(0) = 0.21635 \pm 0.00038$ [6]. For the form factor, the PDG used the value $f_+(0) = 0.9698 \pm 0.0017$ [80], to obtain the value of $|V_{us}|$ reported in Table 1.

To complete the first row of the CKM matrix, the value of $|V_{ub}|$ is obtained from the study of the inclusive and exclusive semileptonic decays of the type $b \to u\ell^+\nu$ ($\ell = e, \mu$). The inclusive determination of $|V_{ub}|$ is based on the measurement of the total rate of $B \to X_u\ell^+\nu$ decays, where X_u is a hadronic system that contains a u quark. This measurement is challenging due to the presence of the large background from the $B \to X_c\ell^+\nu$ decays, where X_c are charm hadrons. The theoretical estimate of this effect is crucial to extract the value of $|V_{ub}|$ from the data. The B Factories and CLEO made significant advancements in the inclusive determination of $|V_{ub}|$ by using two different approaches. Initially they studied the inclusive electron momentum [81–83] to determine a partial decay rate near the kinematic endpoint. Once the number of $B\bar{B}$ pairs became large enough, they also developed a technique based on the full reconstruction of a (tagging) B meson and of the recoiling B decaying semileptonically [84–86].

Exclusive measurements of $|V_{ub}|$ are possible by studying various decay modes, $B \to \pi\ell^+\nu$ and $B \to \rho\ell^+\nu$ [87–91], $\overline{\Lambda}_b^0 \to \overline{p}\ell^+\nu$ [92], and $\bar{B}_s^0 \to K^-\ell^+\nu$ [93]. In the case of the B decays, the transition is described in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$ as in Equation (30). For the Λ_b^0 decays, four additional form factors are needed to account for the polarization of the baryons [94]. Since determining the absolute branching fraction of a decay at LHCb (and at hadron colliders in general) is quite challenging, measurements of $|V_{ub}/V_{cb}|$ are rather made by studying the ratio of branching fractions with respect to the $\bar{B}_s^0 \to D_s^-\ell^+\nu$ ($\overline{\Lambda}_b^0 \to \overline{\Lambda}_c^-\ell^+\nu$).

The averages of the inclusive and exclusive measurements of $|V_{ub}|$ are [6]:

$$
\begin{aligned}
|V_{ub}|_{\text{incl}} &= (4.13 \pm 0.12(\text{exp})^{+0.13}_{-0.14}(\text{theo}) \pm 0.18(\Delta\text{model})) \times 10^{-3}, \\
|V_{ub}|_{\text{excl}} &= (3.70 \pm 0.10 \pm 0.12) \times 10^{-3}.
\end{aligned}
$$

A tension between the two averages is observed. This is a matter of debate within the community; nevertheless, the two averages are combined after scaling the uncertainties to account for the tension [6], giving the result reported in Table 1.

Moving to the second row, $|V_{cd}|$ can be determined from $D \to \pi\ell^+\nu$ decays. Experimental measurements from BaBar [95], BESIII [96,97], CLEO [98], and Belle [99] have been combined in conjunction with input from lattice QCD calculations (needed to estimate the form factor $f_+^{D\pi}(0) = 0.612 \pm 0.035$ [80]) to extract the value of $|V_{cd}| = 0.2330 \pm 0.0029 \pm 0.0133$, where the first uncertainty is experimental and the second theoretical from the form factor determination. Alternative ways of determining $|V_{cd}|$ are from the study of $D^+ \to \mu^+\nu$ and $D^+ \to \tau^+\nu$ decays [100–102], and neutrino scattering data [103–105], yielding $|V_{cd}| = 0.2181 \pm 0.0049 \pm 0.0007$ [16] and 0.230 ± 0.011 [6], respectively.

The value of $|V_{cs}|$ is obtained directly from the branching fraction of $D_s^+ \to \mu^+\nu$ and $D_s^+ \to \tau^+\nu$ decays, using the lattice QCD calculation of the semileptonic D_s^+ decay constant [80], giving $|V_{cs}| = 0.984 \pm 0.012$ [6]. Another approach relies on lattice QCD calculations of the $D \to K\ell^+\nu$ form factors [80] and the experimental measurement of the branching fraction of $D \to K\ell^+\nu$ decays to obtain $|V_{cs}| = 0.972 \pm 0.007$ [6]. The average of these two values is reported in Table 1.

The magnitude of $|V_{cb}|$ is obtained from the study of inclusive and exclusive semileptonic decays of the type $B \to X_c\ell^+\nu$ ($\ell = e, \mu$). The form factors for the B decays are calculated with lattice QCD methods by various collaborations [106–111]. Exclusive determinations make use of the decays of B mesons to the ground states of D and D^* charm mesons. The most recent analyses of $B \to D^*\ell^+\nu$ decays have been performed by BaBar [112], Belle [113,114], and Belle II [115], and they all study the kinematic distribution of the decay products in a four-dimensional space to extract the value of $|V_{cb}|$. In the analysis of $B \to D\ell^+\nu$ decays, only the product of the four momenta of the initial- and final-state hadrons is studied to extract $|V_{cb}|$. BaBar [116] and Belle [117] obtained results compatible with the $B \to D^*\ell^+\nu$ decays.

Not only $B^{\pm}$ mesons can be used to determine $|V_{cb}|$, but also B_s^0, B_c^+, and Λ_b^0 hadrons. The LHCb collaboration studied $B_s^0 \to D_s^{(*)}\ell^+\nu$ [118] decays, measuring $|V_{cb}|$ with a precision comparable to the theoretical uncertainties, even though not competitive yet with the B measurements from the B Factories. In this perspective, $B_c^+ \to \tau^+\nu$ decays can also be used to determine the value of $|V_{cb}|$ with small theoretical uncertainties, but they are difficult to reconstruct at a hadron collider and they will be studied in a future e^+e^- facility [119].

The inclusive determination of $|V_{cb}|$ has been investigated by multiple experiments through the measurements of moments as a function of either the minimum lepton momentum [120–128], or the squared lepton invariant mass [129,130].

The averages of the exclusive and inclusive measurements of $|V_{cb}|$ are [6]:

$$|V_{cb}|_{\text{excl}} = (42.2 \pm 0.5) \times 10^{-3},$$
$$|V_{cb}|_{\text{incl}} = (39.8 \pm 0.6) \times 10^{-3}.$$

Marginal consistency between the two averages is observed, and the uncertainties are scaled to account for this before combining the two values [6], resulting in the reported values in Table 1.

The measurements of the matrix elements involving the t quark is challenging due to its large mass. Even when boosted to the energies of the LHC, the t quark does not hadronize or form bound states because its decay length is shorter than the typical scale of the hadronization process. Therefore, $|V_{td}|$ and $|V_{ts}|$ are not likely to be measured in tree-level decay processes, rather they are determined from B–$\bar{B}$ mixing processes, where the t quark is involved in the box diagram (see Appendix A). In particular, the mass differences between the two mass eigenstates of the B mesons, Δm_d and Δm_s, are related to the $|V_{td}|$ and $|V_{ts}|$ matrix elements, respectively, enabling their determination. Many experiments have measured Δm_d, whose average $0.5065 \pm 0.0019\,\text{ps}^{-1}$ [6] is dominated by the latest LHCb measurement using $B^0 \to D^{(*)-}\mu^+\nu_\mu X$ decays [131]. The average of the measurements of Δm_s is $17.765 \pm 0.006\,\text{ps}^{-1}$ [6], with the most precise measurement provided by the LHCb collaboration using $B_s^0 \to D_s^-\pi^+$ decays [132].

The value of $|V_{tb}|$ can be determined either by assuming the unitarity of the CKM matrix or without making this assumption. In the first case, the ratio of branching fractions $R = \mathcal{B}(t \to Wb)/\mathcal{B}(t \to Wq) = |V_{tb}|^2/(\sum_q |V_{tq}|^2) = |V_{tb}|^2$, where $q = b, s, d$. This measurement was made during Run II of the Tevatron by CDF [133] and D0 [134], and by CMS [135] at LHC obtaining $|V_{tb}| > 0.975$ at the 95% confidence level. In the second case, $|V_{tb}|$ can be measured from the single top quark production cross-section. Measurements of this cross-section have been made at Tevatron by CDF and D0 [136], and at LHC by ATLAS and CMS [137]. The value reported in Table 1 for $|V_{tb}|$ is the average of this second set of measurements.

Tests of the unitarity of the CKM matrix are made by verifying the equality to 1 of the sum of the squared of the matrix elements along each row and column:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985 \pm 0.0007,$$
$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1.001 \pm 0.012,$$
$$|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2 = 1.03 \pm 0.06,$$
$$|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 0.9971 \pm 0.0019,$$
$$|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 1.003 \pm 0.012,$$
$$|V_{tb}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1.03 \pm 0.06.$$

All of the unitarity relations are verified within the uncertainties, except the first row that shows a tension of 2.2 standard deviations. A smaller tension is also observed in the first column, but it is not statistically significant. In both cases the tension is driven by the determination of $|V_{ud}|$, and is generally called the Cabibbo angle anomaly.

5. The High Intensity Frontier

A tremendous improvement in the precision of the measurements of *CP* violation in the *B* and *D* meson systems has been achieved in the last decade. Figure 7 shows how much the constraints on the CKM unitarity triangle have improved since 2012. In particular, the uncertainty on the angle γ is reduced by more than a factor of 2 with the constraints from the analysis of LHCb data.

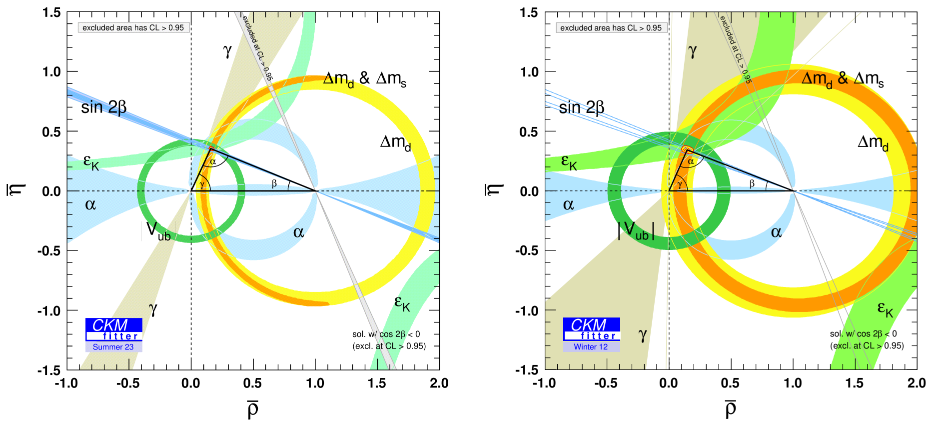

Figure 7. Plot of the CKM unitarity triangle in the complex plane from the CKMFitter group [4] as of 2023 (**left**) and 2012 (**right**). The labels superimposed on the plots and the corresponding shaded areas show the various measurements of *CP* violation and the constraints they pose on the triangle.

The quest for ultimate precision in flavor physics studies is not over yet, and the high intensity frontier is the next step in this direction. The LHCb experiment is already running its first upgrade [138] and the Belle II experiment is in the process of taking data at its design capabilities [139]. The two experiments are expected to provide a significant improvement in the precision of the measurements of *CP* violation and the CKM matrix elements in the *B* and *D* meson systems. In particular, the LHCb upgrade should further reduce the uncertainty on the angle γ down to less than 1°, measure ϕ_s with a precision of less than 20% of the Standard Model, and have sensitivity to evidence of *CP* violation in the interference between decay and mixing of D^0 decays [140].

Similarly, Belle II should improve the precision on the angles α and β by a factor of 2 [141]. Improved determinations of the CKM matrix elements will provide stringent tests of the CKM paradigm and will be sensitive to new physics effects at the loop level. Most importantly, the physics capabilities of the two experiments are complementary: LHCb excels in high efficiency for charged final states and benefits from larger cross-sections, while Belle II achieves high efficiency for neutral final states and offers a larger acceptance. Therefore, the combination of the two experiments will provide a complete picture.

Finally, the LHCb collaboration is proposing a second upgrade of the experiment, called LHCb Upgrade II [142], which will collect an integrated luminosity of 300 fb^{-1} to test the CKM paradigm with unprecedented precision by the end of 2041, when CERN will stop the LHC operations.

6. Conclusions

Flavor physics is a fundamental part of the Standard Model of particle physics, and the study of *CP* violation in the *B* and *D* meson systems is a key ingredient to test the CKM paradigm. In the last two decades *CP* violation has been established in the *B* meson system, and it has been observed recently also in the *D* meson system by the LHCb experiment.

Despite the tremendous improvement in the precision of the measurements of *CP* violation in the *B* and *D* meson systems, effects beyond the Standard Model have not been observed yet. Small inconsistencies in the measurements of *CP* violation in the *B* meson system are present, but they are not statistically significant. The quest for ultimate precision in *CP* violation studies is not over yet, and the high intensity frontier is the next step in this direction. The next decade will be crucial to test the CKM paradigm with unprecedented precision, and the LHCb and Belle II experiments are expected to play a key role in this quest, urging an update to this review. Possibly the last word will be given by the LHCb Upgrade II that will reach ultimate precision in flavor physics for our generation.

Funding: This research received no external funding.

Acknowledgments: The author would like to express his sincere gratitude to Marta Calvi and Martino Borsato for the time they took to read this review and provide comments and suggestions to improve the quality of this manuscript.

Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A. Tree, Box, and Loop Diagrams

Particle physics processes can be described in terms of Feynman diagrams. The simplest diagrams are the tree-level diagrams, which represent the leading order contributions to a process. They typically represent the exchange of a gauge boson between two particles, as shown in Figure A1 (left).

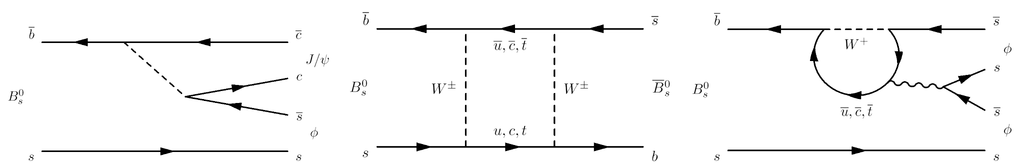

Figure A1. Tree-level (**left**), box (**center**), and loop (**right**) Feynman diagrams for B_s^0 decays. Courtesy of University of Zurich (accessed 7 July 2024).

Other than providing a visual representation of the decay, Feynman diagrams offer a way to calculate the amplitude of the process, since its probability is given by the product of the probabilities of each decay vertex. In general, the more the vertices, the less likely the process is to happen.

A box diagram is shown in the middle of Figure A1 and represents the process of oscillation of a B_s^0 meson into a $\overline{B}_s^0$ one. The exchange of two W bosons allows the B_s^0 meson to change its flavor. There are four vertices in the process and the comma between u, c, and t quarks indicates that the process can happen through many different ways, whose probabilities are summed up.

Loop diagrams, shown in Figure A1 (right), are very important in the search for effects beyond the Standard Model, since new particles could participate to the decay *virtually*, meaning that they will not need energy greater or equal to their mass to be produced. This quantum-mechanical effect gives access to energy scales which are not directly accessible by the collider.

References

1. Christenson, J.H.; Cronin, J.W.; Fitch, V.L.; Turlay, R. Evidence for the 2π Decay of the K_2^0 Meson. *Phys. Rev. Lett.* **1964**, *13*, 138–140. [CrossRef]
2. Kobayashi, M.; Maskawa, T. CP Violation in the Renormalizable Theory of Weak Interaction. *Prog. Theor. Phys.* **1973**, *49*, 652–657. [CrossRef]
3. Wolfenstein, L. Parametrization of the Kobayashi-Maskawa Matrix. *Phys. Rev. Lett.* **1983**, *51*, 1945. [CrossRef]
4. Charles, J.; Hocker, A.; Lacker, H.; Laplace, S.; Le Diberder, F.R.; Malcles, J.; Ocariz, J.; Pivk, M.; Roos, L. CP violation and the CKM matrix: Assessing the impact of the asymmetric *B* factories. *Eur. Phys. J. C* **2005**, *41*, 1–131. [CrossRef]
5. Bigi, I.I.; Sanda, A.I. *CP Violation*; Cambridge University Press: Cambridge, UK, 2009; Volume 9. [CrossRef]
6. Particle Data Group; Workman, R.L.; Burkert, V.D.; Crede, V.; Klempt, E.; Thoma, U.; Tiator, L.; Agashe, K.; Aielli, G.; Allanach, B.C.; et al. Review of particle physics. *Prog. Theor. Exp. Phys.* **2022**, *2022*, 083C01. [CrossRef]
7. Aubert, B.; Bazan, A.; Boucham, A.; Boutigny, D.; De Bonis, I.; Favier, J.; Gaillard, J.-M.; Jeremie, A.; Karyotakis, Y.; Le Flour, T.; et al. The BaBar detector. *Nucl. Instrum. Meth. A* **2002**, *479*, 1–116. [CrossRef]

8. Abashian, A.; Gotow, K.; Morgan, N.; Piilonen, L.; Schrenk, S.; Abe, K.; Adachi, I.; Alexander, J.; Aoki, K.; Behari, S.; et al. The Belle Detector. *Nucl. Instrum. Meth. A* **2002**, *479*, 117–232. [CrossRef]
9. Aubert, B.; Boutigny, D.; Gaillard, J.-M.; Hicheur, A.; Karyotakis, Y.; Lees, J.P.; Robbe, P.; Tisserand, V.; Palano, A.; Chen, G.P.; et al. Observation of CP violation in the B^0 meson system. *Phys. Rev. Lett.* **2001**, *87*, 091801. [CrossRef]
10. Abe, K.; Abe, R.; Adachi, I.; Ahn, B.S.; Aihara, H.; Akatsu, M.; Alimonti, G.; Asai, K.; Asai, M.; Asano, Y.; et al. Observation of large CP violation in the neutral B meson system. *Phys. Rev. Lett.* **2001**, *87*, 091802. [CrossRef]
11. Aaij, R.; The LHCb Collaboration; Beteta, C.A.; Adeva, B.; Adinolfi, M.; Adrover, C.; Affolder, A.; Ajaltouni, Z.; Albrecht, J.; Alessio, F.; et al. Opposite-side flavour tagging of B mesons at the LHCb experiment. *Eur. Phys. J. C* **2012**, *72*, 2022. [CrossRef]
12. Lingel, K.; Skwarnicki, T.; Smith, J.G. Penguin decays of B mesons. *Ann. Rev. Nucl. Part. Sci.* **1998**, *48*, 253–306. [CrossRef]
13. Aubert, B.; Karyotakis, Y.; Lees, J.P.; Poireau, V.; Prencipe, E.; Prudent, X.; Tisserand, V.; Tico, J.G.; Grauges, E.; Lopez, L.; et al. Measurement of Time-Dependent CP Asymmetry in B0 —> c anti-c K(*)0 Decays. *Phys. Rev. D* **2009**, *79*, 072009. [CrossRef]
14. Adachi, I.; Aihara, H.; Asner, D.M.; Aulchenko, V.; Aushev, T.; Aziz, T.; Bakich, A.M.; Bay, A.; Bhardwaj, V.; Bhuyan, B.; et al. Precise measurement of the CP violation parameter $\sin 2\phi_1$ in $B^0 \to (c\bar{c})K^0$ decays. *Phys. Rev. Lett.* **2012**, *108*, 171802. [CrossRef]
15. Aaij, R.; Abdelmotteleb, A.S.W.; Beteta, C.A.; Abudinén, F.; Ackernley, T.; Adeva, B.; Adinolfi, M.; Adlarson, P.; Afsharnia, H.; Agapopoulou, C.; et al. Measurement of *CP* violation in $B^0 \to \psi(\to \ell^+\ell^-)K^0_S(\to \pi^+\pi^-)$ decays. *Phys. Rev. Lett.* **2024**, *132*, 021801. [CrossRef] [PubMed]
16. Amhis, Y.; Banerjee, S.; Ben-Haim, E.; Bertholet, E.; Bernlochner, F.U.; Bona, M.; Bozek, A.; Bozzi, C.; Brodzicka, J.; Chobanova, V.; et al. Averages of b-hadron, c-hadron, and τ-lepton properties as of 2021. *Phys. Rev.* **2023**, *D107*, 052008.
17. Gronau, M.; London, D. Isospin analysis of CP asymmetries in B decays. *Phys. Rev. Lett.* **1990**, *65*, 3381–3384. [CrossRef] [PubMed]
18. Aaij, R.; The LHCb collaboration; Beteta, C.A.; Ackernley, T.; Adeva, B.; Adinolfi, M.; Afsharnia, H.; Aidala, C.A.; Aiola, S.; Ajaltouni, Z.; et al. Observation of *CP* violation in two-body $B^0_{(c)}$-meson decays to charged pions and kaons. *J. High Energy Phys.* **2021**, *3*, 75. [CrossRef]
19. Aubert, B.; Bona, M.; Boutigny, D.; Karyotakis, Y.; Lees, J.P.; Poireau, V.; Prudent, X.; Tisserand, V.; Zghiche, A.; Tico, J.G.; et al. Study of $B^0 \to \pi^0\pi^0$, $B^\pm \to \pi^\pm\pi^0$, and $B^\pm \to K^\pm\pi^0$ Decays, and Isospin Analysis of $B \to \pi\pi$ Decays. *Phys. Rev. D* **2007**, *76*, 091102. [CrossRef]
20. Duh, Y.T.; Wu, T.-Y.; Chang, P.; Mohanty, G.B.; Unno, Y.; Adachi, I.; Aihara, H.; Asner, D.M.; Aulchenko, V.; Aushev, T.; et al. Measurements of branching fractions and direct CP asymmetries for B$\to$Kπ, B$\to\pi\pi$ and B$\to$KK decays. *Phys. Rev. D* **2013**, *87*, 031103. [CrossRef]
21. Aubert, B.; Bona, M.; Boutigny, D.; Karyotakis, Y.; Lees, J.P.; Poireau, V.; Prudent, X.; Tisserand, V.; Zghiche, A.; Tico, J.G.; et al. A Study of $\mathbf{B}^0 \to \rho^+\rho^-$ Decays and Constraints on the CKM Angle alpha. *Phys. Rev. D* **2007**, *76*, 052007. [CrossRef]
22. Vanhoefer, P.; Dalseno, J.; Kiesling, C.; Abdesselam, A.; Adachi, I.; Aihara, H.; Al Said, S.; Arinstein, K.; Asner, D.M.; Atmacan, H.; et al. Study of $\mathbf{B}^0 \to \rho^+\rho^-$ decays and implications for the CKM angle ϕ_2. *Phys. Rev. D* **2016**, *93*, 032010; Erratum in *Phys. Rev. D* **2016**, *94*, 099903. [CrossRef]
23. Adachi, I.; Vanhoefer, P.; Dalseno, J.; Kiesling, C.; Aihara, H.; Asner, D.M.; Aulchenko, V.; Aushev, T.; Bakich, A.M.; Bala, A.; et al. Study of $B^0 \to \rho^0\rho^0$ decays, implications for the CKM angle ϕ_2 and search for other B0 decay modes with a four-pion final state. *Phys. Rev. D* **2014**, *89*, 072008; Erratum in *Phys. Rev. D* **2014**, *89*, 119903. [CrossRef]
24. Kusaka, A.; Ishino, H.; Abe, K.; Adachi, I.; Aihara, H.; Anipko, D.; Aulchenko, V.; Aushev, T.; Bakich, A.M.; Barberio, E.; et al. Measurement of CP Asymmetry in a Time-Dependent Dalitz Analysis of B0 —> (rho pi)0 and a Constraint on the CKM Angle phi(2). *Phys. Rev. Lett.* **2007**, *98*, 221602. [CrossRef] [PubMed]
25. Aubert, B.; Bona, M.; Boutigny, D.; Karyotakis, Y.; Lees, J.P.; Poireau, V.; Prudent, X.; Tisserand, V.; Zghiche, A.; Tico, J.G.; et al. Measurement of CP-violating asymmetries in $B^0 \to (\rho\pi)^0$ using a time-dependent Dalitz plot analysis. *Phys. Rev. D* **2007**, *76*, 012004. [CrossRef]
26. Aubert, B.; Bona, M.; Karyotakis, Y.; Lees, J.P.; Poireau, V.; Prencipe, E.; Prudent, X.; Tisserand, V.; Tico, J.G.; Grauges, E.; et al. Measurement of the Branching Fraction, Polarization, and CP Asymmetries in $B^0 \to \rho^0\rho^0$ Decay, and Implications for the CKM Angle α. *Phys. Rev. D* **2008**, *78*, 071104. [CrossRef]
27. Aaij, R.; Adeva, B.; Adinolfi, M.; Affolder, A.; Ajaltouni, Z.; Akar, S.; Albrecht, J.; Alessio, F.; Alexander, M.; Ali, S.; et al. Observation of the $B^0 \to \rho^0\rho^0$ decay from an amplitude analysis of $B^0 \to (\pi^+\pi^-)(\pi^+\pi^-)$ decays. *Phys. Lett.* **2015**, *B747*, 468. [CrossRef]
28. Brod, J.; Zupan, J. The ultimate theoretical error on γ from $B \to DK$ decays. *J. High Energy Phys.* **2014**, *1*, 51. [CrossRef]
29. Gronau, M.; Wyler, D. On determining a weak phase from CP asymmetries in charged B decays. *Phys. Lett. B* **1991**, *265*, 172–176. [CrossRef]
30. Atwood, D.; Dunietz, I.; Soni, A. Enhanced CP violation with B —> K D0 (anti-D0) modes and extraction of the CKM angle gamma. *Phys. Rev. Lett.* **1997**, *78*, 3257–3260. [CrossRef]
31. Giri, A.; Grossman, Y.; Soffer, A.; Zupan, J. Determining gamma using B+- —> DK+- with multibody D decays. *Phys. Rev. D* **2003**, *68*, 054018. [CrossRef]
32. Ablikim, M.; Achasov, M.N.; Adlarson, P.; Albrecht, M.; Aliberti, R.; Amoroso, A.; An, M.R.; An, Q.; Bai, X.H.; Bai, Y.; et al. Improved measurement of the strong-phase difference $\delta_D^{K\pi}$ in quantum-correlated $D\bar{D}$ decays. *Eur. Phys. J. C* **2022**, *82*, 1009. [CrossRef]

33. Dunietz, I.; Sachs, R.G. Asymmetry Between Inclusive Charmed and Anticharmed Modes in B0, Anti-b0 Decay as a Measure of CP Violation. *Phys. Rev. D* **1988**, *37*, 3186; Erratum in *Phys. Rev. D* **1989** *39*, 3515. [CrossRef] [PubMed]
34. Aleksan, R.; Dunietz, I.; Kayser, B. Determining the CP violating phase gamma. *Z. Phys. C* **1992**, *54*, 653–660. [CrossRef]
35. Fleischer, R. New strategies to obtain insights into CP violation through $B_s \to D_s^{\mp} K^{\pm}$, $D_s^{*\mp} K^{\pm}$, ... and $B_d \to D^{\pm}\pi^{\mp}$, $D^{*\pm}\pi^{\mp}$, ... decays. *Nucl. Phys. B* **2003**, *671*, 459–482. [CrossRef]
36. Aubert, B.; Barate, R.; Boutigny, D.; Couderc, F.; Karyotakis, Y.; Lees, J.P.; Poireau, V.; Tisserand, V.; Zghiche, A.; Grauges, E.; et al. Measurement of time-dependent CP-violating asymmetries and constraints on $\sin(2\beta+\gamma)$ with partial reconstruction of $B \to D^{*\mp}\pi^{\pm}$ decays. *Phys. Rev. D* **2005**, *71*, 112003. [CrossRef]
37. Aubert, B.; Barate, R.; Boutigny, D.; Couderc, F.; Karyotakis, Y.; Lees, J.P.; Poireau, V.; Tisserand, V.; Zghiche, A.; Grauges, E.; et al. Measurement of time-dependent CP asymmetries in $B^0 \to D^{(*)}$ +- $\pi^{\mp}$ and $B^0 \to D^{\pm}\rho^{\mp}$ decays. *Phys. Rev. D* **2006**, *73*, 111101. [CrossRef]
38. Ronga, F.J.; Trabelsi, K.; Kinoshita, K.; Arinstein, K.; Aulchenko, V.; Aushev, T.; Bakich, A.M.; Balagura, V.; Barberio, E.; Belous, K.; et al. Measurements of CP violation in B0 —> D*- pi+ and B0 —> D- pi+ decays. *Phys. Rev. D* **2006**, *73*, 092003. [CrossRef]
39. The LHCb collaboration; Aaij, R.; Adeva, B.; Adinolfi, M.; Ajaltouni, Z.; Akar, S.; Albicocco, P.; Albrecht, J.; Alessio, F.; Alexander, M.; et al. Measurements of time-dependent CP asymmetries in $B \to D^{*\mp}\pi^{\pm}$ decays using a partial reconstruction technique. *Phys. Rev. D* **2011**, *84*, 021101. [CrossRef]
40. The LHCb collaboration; Aaij, R.; Adeva, B.; Adinolfi, M.; Ajaltouni, Z.; Akar, S.; Albrecht, J.; Alessio, F.; Alexander, M.; Albero, A.A.; et al. Measurement of *CP* violation in $\mathrm{B}^0 \to D^{\pm}\pi^{\mp}$ decays. *J. High Energy Phys.* **2018**, *6*, 84. [CrossRef]
41. Lees, J.P.; Poireau, V.; Tisserand, V.; Grauges, E.; Palano, A.; Eigen, G.; Stugu, B.; Brown, D.N.; Kerth, L.T.; Kolomensky, Y.G.; et al. Observation of direct CP violation in the measurement of the Cabibbo-Kobayashi-Maskawa angle gamma with $B^{\pm} \to D^{(*)}K^{(*)\pm}$ decays. *Phys. Rev. D* **2013**, *87*, 052015. [CrossRef]
42. The LHCb Collaboration. Updated LHCb Combination of the CKM Angle γ. 2020. Available online: https://cds.cern.ch/record/2743058 (accessed on 18 April 2024)
43. The LHCb collaboration; Aaij, R.; Adeva, B.; Adinolfi, M.; Ajaltouni, Z.; Akar, S.; Albrecht, J.; Alessio, F.; Alexander, M.; Albero, A.A.; et al. Measurement of *CP* asymmetry in $\mathrm{B_c^0} \to D_s^{\mp}\mathrm{K}^{\pm}$ decays. *J. High Energy Phys.* **2018**, *3*, 59. [CrossRef]
44. Aaij, R.; The LHCb collaboration; Beteta, C.A.; Ackernley, T.; Adeva, B.; Adinolfi, M.; Afsharnia, H.; Aidala, C.A.; Aiola, S.; Ajaltouni, Z.; et al. Measurement of the CKM angle γ and $\mathrm{B_c^0} - \overline{\mathrm{B}}_\mathrm{c}^0$ mixing frequency with $\mathrm{B_c^0} \to \mathrm{D_c^{\mp}} h^{\pm}\pi^{\pm}\pi^{\mp}$ decays. *J. High Energy Phys.* **2021**, *3*, 137. [CrossRef]
45. De Bruyn, K.; Fleischer, R.; Knegjens, R.; Merk, M.; Schiller, M.; Tuning, N. Exploring $B_s \to D_s^{(*)\pm} K^{\mp}$ Decays in the Presence of a Sizable Width Difference $\Delta\Gamma_s$. *Nucl. Phys. B* **2013**, *868*, 351–367. [CrossRef]
46. Artuso, M.; Borissov, G.; Lenz, A. *CP* violation in the B_s^0 system. *Rev. Mod. Phys.* **2016**, *88*, 045002. [CrossRef]
47. Charles, J.; Deschamps, O.; Descotes-Genon, S.; Lacker, H.; Menzel, A.; Monteil, S.; Niess, V.; Ocariz, J.; Orloff, J.; Perez, A.; et al. Current status of the Standard Model CKM fit and constraints on $\Delta F = 2$ New Physics. *Phys. Rev. D* **2015**, *91*, 073007. [CrossRef]
48. Dighe, A.S.; Dunietz, I.; Lipkin, H.J.; Rosner, J.L. Angular distributions and lifetime differences in $B_s \to J/\psi\phi$ decays. *Phys. Lett. B* **1996**, *369*, 144–150. [CrossRef]
49. Dighe, A.S.; Dunietz, I.; Fleischer, R. Extracting CKM phases and $B_s - \bar{B}_s$ mixing parameters from angular distributions of nonleptonic *B* decays. *Eur. Phys. J. C* **1999**, *6*, 647–662. [CrossRef]
50. Dunietz, I.; Fleischer, R.; Nierste, U. In pursuit of new physics with B_s decays. *Phys. Rev. D* **2001**, *63*, 114015. [CrossRef]
51. Aaltonen, T.; González, B.; Amerio, S.; Amidei, D.; Anastassov, A.; Annovi, A.; Antos, J.; Apollinari, G.; Appel, J.A.; Arisawa, T.; et al. Measurement of the Bottom-Strange Meson Mixing Phase in the Full CDF Data Set. *Phys. Rev. Lett.* **2012**, *109*, 171802. [CrossRef]
52. Abazov, V.M.; Abbott, B.; Acharya, B.S.; Adams, M.; Adams, T.; Alexeev, G.D.; Alkhazov, G.; Alton, A.; Alverson, G.; Alves, G.A.; et al. Measurement of the CP-violating phase $\phi_s^{J/\psi\phi}$ using the flavor-tagged decay $B_s^0 \to J/\psi\phi$ in 8 fb^{-1} of $p\bar{p}$ collisions. *Phys. Rev. D* **2012**, *85*, 032006. [CrossRef]
53. Aad, G.; ATLAS Collaboration; Abbott, B.; Abbott, D.C.; Abdinov, O.; Abud, A.A.; Abeling, K.; Abhayasinghe, D.K.; Abidi, S.H.; AbouZeid, O.S.; et al. Measurement of the *CP*-violating phase ϕ_s in $B_s^0 \to J/\psi\phi$ decays in ATLAS at 13 TeV. *Eur. Phys. J. C* **2021**, *81*, 342. [CrossRef]
54. Sirunyan, A.M.; Tumasyan, A.; Adam, W.; Ambrogi, F.; Bergauer, T.; Dragicevic, M.; Erö, J.; Del Valle, A.E.; Frühwirth, R.; Jeitler, M.; et al. Measurement of the *CP*-violating phase ϕ_s in the $\mathrm{B_s^0} \to \mathrm{J}/\psi\,\phi(1020) \to \mu^+\mu^-\mathrm{K}^+\mathrm{K}^-$ channel in proton-proton collisions at $\sqrt{s} = 13$ TeV. *Phys. Lett. B* **2021**, *816*, 136188. [CrossRef]
55. Aaij, R.; Abdelmotteleb, A.S.W.; Beteta, C.A.; Abudinén, F.; Ackernley, T.; Adeva, B.; Adinolfi, M.; Adlarson, P.; Afsharnia, H.; Agapopoulou, C.; et al. Improved measurement of CP violation parameters in $\mathrm{B_c^0} \to \mathrm{J}/\psi\mathrm{K}^+\mathrm{K}^-$ decays in the vicinity of the $\phi(1020)$ resonance. *Phys. Rev. Lett.* **2024**, *132*, 051802. [CrossRef]
56. CMS Collaboration. *Measurement of Time-Dependent CP Violation in $B_s^0 \to J/\psi\,\phi(1020)$ Decays with the CMS Detector*; Technical Report; CERN: Geneva, Switzerland, 2024.
57. Aaij, R.; Camboni, A.; Coquereau, S.; Garrido Beltrán, L.; Gascón Fora, D.; Graciani Díaz, R.; Marin Benito, C.; LHCb Collaboration. Measurement of matter-antimatter differences in beauty baryon decays. *Nat. Phys.* **2017**, *13*, 391. [CrossRef]
58. Aaij, R.; Beteta, C.A.; Ackernley, T.; Adeva, B.; Adinolfi, M.; Afsharnia, H.; Aidala, C.A.; Aiola, S.; Ajaltouni, Z.; Akar, S.; et al. Search for *CP* violation and observation of *P* violation in $\Lambda_\mathrm{b}^0 \to p\pi^-\pi^+\pi^-$ decays. *Phys. Rev.* **2020**, *D102*, 051101. [CrossRef]

59. Durieux, G.; Grossman, Y. Probing CP violation systematically in differential distributions. *Phys. Rev. D* **2015**, *92*, 076013. [CrossRef]
60. Gronau, M.; Rosner, J.L. Triple product asymmetries in K, $D_{(s)}$ and $B_{(s)}$ decays. *Phys. Rev. D* **2011**, *84*, 096013. [CrossRef]
61. Datta, A.; Duraisamy, M.; London, D. Searching for New Physics with B-Decay Fake Triple Products. *Phys. Lett. B* **2011**, *701*, 357–362. [CrossRef]
62. Aaij, R.; Adeva, B.; Adinolfi, M.; Aidala, C.; Ajaltouni, Z.; Akar, S.; Albicocco, P.; Albrecht, J.; Alessio, F.; Alexander, M.; et al. Search for *CP* violation in $\Lambda_b^0 \to pK^-$ and $\Lambda_b^0 \to p\pi^-$ decays. *Phys. Lett.* **2018**, *B784*, 101. [CrossRef]
63. Aaij, R.; Adeva, B.; Adinolfi, M.; Ajaltouni, Z.; Akar, S.; Albrecht, J.; Alessio, F.; Alexander, M.; Ali, S.; Alkhazov, G.; et al. Measurement of B^0, B_c^0, B^+ and Λ_b^0 production asymmetries in 7 and 8 TeV proton-proton collisions. *Phys. Lett.* **2017**, *B774*, 139. [CrossRef]
64. Fujikawa, K. Heavy Fermions in the Standard Sequential Scheme. *Prog. Theor. Phys.* **1979**, *61*, 1186. [CrossRef]
65. Bigi, I.I.Y.; Dokshitzer, Y.L.; Khoze, V.A.; Kuhn, J.H.; Zerwas, P.M. Production and Decay Properties of Ultraheavy Quarks. *Phys. Lett. B* **1986**, *181*, 157–163. [CrossRef]
66. Petrov, A.A. Charm physics. *Eur. Phys. J. ST* **2024**, *233*, 439–456. [CrossRef]
67. Bigi, I.I.; Paul, A.; Recksiegel, S. Conclusions from CDF Results on CP Violation in $D^0 \to \pi^+\pi^-, K^+K^-$ and Future Tasks. *J. High Energy Phys.* **2011**, *6*, 89. [CrossRef]
68. Isidori, G.; Kamenik, J.F.; Ligeti, Z.; Perez, G. Implications of the LHCb Evidence for Charm CP Violation. *Phys. Lett. B* **2012**, *711*, 46–51. [CrossRef]
69. Brod, J.; Kagan, A.L.; Zupan, J. Size of direct CP violation in singly Cabibbo-suppressed D decays. *Phys. Rev. D* **2012**, *86*, 014023. [CrossRef]
70. Aaij, R.; Beteta, C.A.; Adeva, B.; Adinolfi, M.; Aidala, C.A.; Ajaltouni, Z.; Akar, S.; Albicocco, P.; Albrecht, J.; Alessio, F.; et al. Observation of *CP* violation in charm decays. *Phys. Rev. Lett.* **2019**, *122*, 211803. [CrossRef] [PubMed]
71. Kagan, A.L.; Silvestrini, L. Dispersive and absorptive *CP* violation in $D^0 - \overline{D^0}$ mixing. *Phys. Rev. D* **2021**, *103*, 053008. [CrossRef]
72. Aaij, R.; Abdelmotteleb, A.S.W.; Beteta, C.A.; Abudinén, F.; Ackernley, T.; Adeva, B.; Adinolfi, M.; Adlarson, P.; Afsharnia, H.; Agapopoulou, C.; et al. Measurement of the time-integrated *CP* asymmetry in $D^0 \to K^-K^+$ decays. *Phys. Rev. Lett.* **2023**, *131*, 091802. [CrossRef]
73. Peng, T.; Zhang, Z.P.; Abdesselam, A.; Adachi, I.; Aihara, H.; Arinstein, K.; Asner, D.M.; Aulchenko, V.; Aushev, T.; Ayad, R.; et al. Measurement of $D^0 - \bar{D}^0$ mixing and search for indirect CP violation using $D^0 \to K_S^0\pi^+\pi^-$ decays. *Phys. Rev. D* **2014**, *89*, 091103. [CrossRef]
74. del Amo Sanchez, P.; Lees, J.P.; Poireau, V.; Prencipe, E.; Tisserand, V.; Tico, J.G.; Grauges, E.; Martinelli, M.; Palano, A.; Pappagallo, M.; et al. Measurement of D0-antiD0 mixing parameters using D0 —> K(S)0 pi+ pi- and D0 —> K(S)0 K+ K- decays. *Phys. Rev. Lett.* **2010**, *105*, 081803. [CrossRef] [PubMed]
75. Di Canto, A.; Garra Ticó, J.; Gershon, T.; Jurik, N.; Martinelli, M.; Pilař, T.; Stahl, S.; Tonelli, D. Novel method for measuring charm-mixing parameters using multibody decays. *Phys. Rev. D* **2019**, *99*, 012007. [CrossRef]
76. Aaij, R.; Beteta, C.A.; Ackernley, T.; Adeva, B.; Adinolfi, M.; Afsharnia, H.; Aidala, C.A.; Aiola, S.; Ajaltouni, Z.; Akar, S.; et al. Observation of the mass difference between neutral charm-meson eigenstates. *Phys. Rev. Lett.* **2021**, *127*, 111801. [CrossRef]
77. Aaij, R.; Abdelmotteleb, A.S.W.; Beteta, C.A.; Abudinén, F.; Ackernley, T.; Adeva, B.; Adinolfi, M.; Afsharnia, H.; Agapopoulou, C.; Aidala, C.A.; et al. Measurement of the charm mixing parameter $y_{CP} - y_{CP}^{K\pi}$ using two-body D^0 meson decays. *Phys. Rev.* **2022**, *D105*, 092013. [CrossRef]
78. Hardy, J.C.; Towner, I.S. Superallowed $0^+ \to 0^+$ nuclear β decays: 2020 critical survey, with implications for V_{ud} and CKM unitarity. *Phys. Rev. C* **2020**, *102*, 045501. [CrossRef]
79. Gasser, J.; Leutwyler, H. Low-Energy Expansion of Meson Form-Factors. *Nucl. Phys. B* **1985**, *250*, 517–538. [CrossRef]
80. Aoki, Y.; Blum, T.; Colangelo, G.; Collins, S.; Della Morte, M.; Dimopoulos, P.; Dürr, S.; Feng, X.; Fukaya, H.; Golterman, M.; et al. FLAG Review 2021. *Eur. Phys. J. C* **2022**, *82*, 869. [CrossRef]
81. Bornheim, A.; Lipeles, E.; Pappas, S.P.; Shapiro, A.; Sun, W.M.; Weinstein, A.J.; Masek, G.; Paar, H.P.; Mahapatra, R.; Morrison, R.J.; et al. Improved measurement of |V(ub)| with inclusive semileptonic B decays. *Phys. Rev. Lett.* **2002**, *88*, 231803. [CrossRef] [PubMed]
82. Limosani, A.; Abe, K.; Adachi, I.; Aihara, H.; Asano, Y.; Aushev, T.; Bahinipati, S.; Bakich, A.; Barberio, E.; Bitenc, U.; et al. Measurement of inclusive charmless semileptonic B-meson decays at the endpoint of the electron momentum spectrum. *Phys. Lett. B* **2005**, *621*, 28–40. [CrossRef]
83. Lees, J.P.; Poireau, V.; Tisserand, V.; Grauges, E.; Palano, A.; Eigen, G.; Brown, D.N.; Kolomensky, Y.G.; Koch, H.; Schroeder, T.; et al. Measurement of the inclusive electron spectrum from B meson decays and determination of |Vub|. *Phys. Rev. D* **2017**, *95*, 072001. [CrossRef]
84. Lees, J.P.; Poireau, V.; Tisserand, V.; Tico, J.G.; Grauges, E.; Martinelli, M.; Milanes, D.A.; Palano, A.; Pappagallo, M.; Eigen, G.; et al. Study of $\bar{B} \to X_u \ell \bar{\nu}$ decays in $B\bar{B}$ events tagged by a fully reconstructed B-meson decay and determination of $\|V_{ub}\|$. *Phys. Rev. D* **2012**, *86*, 032004. [CrossRef]
85. Cao, L.; Sutcliffe, W.; Van Tonder, R.; Bernlochner, F.U.; Adachi, I.; Aihara, H.; Al Said, S.; Asner, D.M.; Atmacan, H.; Aushev, T.; et al. Measurements of Partial Branching Fractions of Inclusive $B \to X_u\, \ell^+\, \nu_\ell$ Decays with Hadronic Tagging. *Phys. Rev. D* **2021**, *104*, 012008. [CrossRef]

86. Hohmann, M.; Urquijo, P.; Adachi, I.; Aihara, H.; Asner, D.M.; Aushev, T.; Ayad, R.; Babu, V.; Banerjee, S.; Bauer, M.; et al. Measurement of the Ratio of Partial Branching Fractions of Inclusive $\overline{B}\to X_u\ell\overline{\nu}$ to $\overline{B}\to X_c\ell\overline{\nu}$ and the Ratio of their Spectra with Hadronic Tagging. *arXiv* **2023**, arXiv:2311.00458.
87. Adam, N.E.; Alexander, J.P.; Berkelman, K.; Cassel, D.G.; Duboscq, J.E.; Ehrlich, R.; Fields, L.; Gibbons, L.; Gray, R.; Gray, S.W.; et al. A Study of Exclusive Charmless Semileptonic B Decay and |V(ub)|. *Phys. Rev. Lett.* **2007**, *99*, 041802. [CrossRef]
88. Gray, R.; Edwards, K.W.; Briere, R.A.; Ferguson, T.; Tatishvili, G.; Vogel, H.; Watkins, M.E.; Rosner, J.L.; Adam, N.E.; Alexander, J.P.; et al. A Study of Exclusive Charmless Semileptonic B Decays and Extraction of $|V_{ub}|$ at CLEO. *Phys. Rev. D* **2007**, *76*, 012007; Erratum in *Phys. Rev. D* **2007**, *76*, 039901. [CrossRef]
89. del Amo Sanchez, P.; Lees, J.P.; Poireau, V.; Prencipe, E.; Tisserand, V.; Tico, J.G.; Grauges, E.; Martinelli, M.; Palano, A.; Pappagallo, M.; et al. Study of $B\to\pi\ell\nu$ and $B\to\rho\ell\nu$ Decays and Determination of $|V_{ub}|$. *Phys. Rev. D* **2011**, *83*, 032007. [CrossRef]
90. Lees, J.P.; Poireau, V.; Tisserand, V.; Tico, J.G.; Grauges, E.; Palano, A.; Eigen, G.; Stugu, B.; Brown, D.N.; Kerth, L.T.; et al. Branching fraction and form-factor shape measurements of exclusive charmless semileptonic B decays, and determination of $|V_{ub}|$. *Phys. Rev. D* **2012**, *86*, 092004. [CrossRef]
91. Ha, H.; Won, E.; Adachi, I.; Aihara, H.; Aziz, T.; Bakich, A.M.; Balagura, V.; Barberio, E.; Bay, A.; Belous, K.; et al. Measurement of the decay $B^0\to\pi^-\ell^+\nu$ and determination of $|V_{ub}|$. *Phys. Rev. D* **2011**, *83*, 071101. [CrossRef]
92. Aaij, R.; The LHCb Collaboration. Determination of the quark coupling strength $|V_{ub}|$ using baryonic decays. *Nature Phys.* **2015**, *11*, 743–747. [CrossRef]
93. Aaij, R.; Beteta, C.A.; Ackernley, T.; Adeva, B.; Adinolfi, M.; Afsharnia, H.; Aidala, C.A.; Aiola, S.; Ajaltouni, Z.; Akar, S.; et al. First observation of the decay $B_s^0\to K^-\mu^+\nu_\mu$ and Measurement of $|V_{ub}|/|V_{cb}|$. *Phys. Rev. Lett.* **2021**, *126*, 081804. [CrossRef]
94. Feldmann, T.; Yip, M.W.Y. Form factors for $\Lambda_b\to\Lambda$ transitions in the soft-collinear effective theory. *Phys. Rev. D* **2012**, *85*, 014035; Erratum in *Phys. Rev. D* **2012**, *86*, 079901. [CrossRef]
95. Lees, J.P.; Poireau, V.; Tisserand, V.; Grauges, E.; Palano, A.; Eigen, G.; Stugu, B.; Brown, D.N.; Kerth, L.T.; Kolomensky, Y.G.; et al. Measurement of the $D^0\to\pi^-e^+\nu_e$ differential decay branching fraction as a function of q^2 and study of form factor parameterizations. *Phys. Rev. D* **2015**, *91*, 052022. [CrossRef]
96. Ablikim, M.; Achasov, M.N.; Ai, X.C.; Albayrak, O.; Albrecht, M.; Ambrose, D.J.; Amoroso, A.; An, F.F.; An, Q.; Bai, J.Z.; et al. Study of Dynamics of $D^0\to K^-e^+\nu_e$ and $D^0\to\pi^-e^+\nu_e$ Decays. *Phys. Rev. D* **2015**, *92*, 072012. [CrossRef]
97. Ablikim, M.; Achasov, M.N.; Ahmed, S.; Ai, X.C.; Albayrak, O.; Albrecht, M.; Ambrose, D.J.; Amoroso, A.; An, F.F.; An, Q.; et al. Analysis of $D^+\to\bar{K}^0e^+\nu_e$ and $D^+\to\pi^0e^+\nu_e$ semileptonic decays. *Phys. Rev. D* **2017**, *96*, 012002. [CrossRef]
98. Besson, D.; Pedlar, T.K.; Xavier, J.; Cronin-Hennessy, D.; Gao, K.Y.; Hietala, J.; Kubota, Y.; Klein, T.; Poling, R.; Scott, A.W.; et al. Improved measurements of D meson semileptonic decays to pi and K mesons. *Phys. Rev. D* **2009**, *80*, 032005. [CrossRef]
99. Widhalm, L.; Abe, K.; Adachi, I.; Aihara, H.; Arinstein, K.; Asano, Y.; Aushev, T.; Bakich, A.M.; Balagura, V.; Barberio, E.; et al. Measurement of D0 —> pi l nu (Kl nu) Form Factors and Absolute Branching Fractions. *Phys. Rev. Lett.* **2006**, *97*, 061804. [CrossRef] [PubMed]
100. Ablikim, M.; Achasov, M.N.; Ai, X.C.; Albayrak, O.; Ambrose, D.J.; An, F.F.; An, Q.; Bai, J.Z.; Ferroli, R.B.; Ban, Y.; et al. Precision measurements of $B(D^+\to\mu^+\nu_\mu)$, the pseudoscalar decay constant f_{D^+}, and the quark mixing matrix element $|V_{\rm cd}|$. *Phys. Rev. D* **2014**, *89*, 051104. [CrossRef]
101. Eisenstein, B.I.; Karliner, I.; Mehrabyan, S.; Lowrey, N.; Selen, M.; White, E.J.; Wiss, J.; Mitchell, R.E.; Shepherd, M.R.; Besson, D.; et al. Precision Measurement of B(D+ —> mu+ nu) and the Pseudoscalar Decay Constant f(D+). *Phys. Rev. D* **2008**, *78*, 052003. [CrossRef]
102. Ablikim, M.; Achasov, M.N.; Adlarson, P.; Ahmed, S.; Albrecht, M.; Alekseev, M.; Amoroso, A.; An, F.F.; An, Q.; Bai, Y.; et al. Observation of the leptonic decay $D^+\to\tau^+\nu_\tau$. *Phys. Rev. Lett.* **2019**, *123*, 211802. [CrossRef]
103. Abramowicz, H.; Groot, J.G.H.; Knobloch, J.; May, J.; Palazzi, P.; Para, A.; Ranjard, F.; Rothberg, J.; Rüden, W.; Schlatter, W.D.; et al. Experimental Study of Opposite Sign Dimuons Produced in Neutrino and anti-neutrinos Interactions. *Z. Phys. C* **1982**, *15*, 19. [CrossRef]
104. CCFR Collaboration; Bazarko, A.O.; Arroyo, C.G.; Bachmann, K.T.; Bolton, T.; Foudas, C.; King, B.J.; Lefmann, W.C.; Leung, W.C.; Mishra, S.R.; et al. Determination of the strange quark content of the nucleon from a next-to-leading order QCD analysis of neutrino charm production. *Z. Phys. C* **1995**, *65*, 189–198. [CrossRef]
105. Vilain, P. Leading order QCD analysis of neutrino induced dimuon events. *Eur. Phys. J. C* **1999**, *11*, 19–34. [CrossRef]
106. Bailey, J.A.; Bazavov, A.; Bernard, C.; Bouchard, C.M.; DeTar, C.; Du, D.; El-Khadra, A.X.; Foley, J.; Freeland, E.D.; Gámiz, E.; et al. B→D$\ell\nu$ form factors at nonzero recoil and $|V_{cb}|$ from 2+1-flavor lattice QCD. *Phys. Rev. D* **2015**, *92*, 034506. [CrossRef]
107. Na, H.; Bouchard, C.M.; Lepage, G.P.; Monahan, C.; Shigemitsu, J. $B\to Dl\nu$ form factors at nonzero recoil and extraction of $|V_{cb}|$. *Phys. Rev. D* **2015**, *92*, 054510; Erratum in *Phys. Rev. D* **2016**, *93*, 119906. [CrossRef]
108. Bazavov, A.; DeTar, C.E.; Du, D.; El-Khadra, A.X.; Gámiz, E.; Gelzer, Z.; Gottlieb, S.; Heller, U.M.; Kronfeld, A.S.; Laiho, J.; et al. Semileptonic form factors for $B\to D^*\ell\nu$ at nonzero recoil from $2+1$-flavor lattice QCD: Fermilab Lattice and MILC Collaborations. *Eur. Phys. J. C* **2022**, *82*, 1141; Erratum in *Eur. Phys. J. C* **2023**, *83*, 21. [CrossRef]
109. Harrison, J.; Davies, C.T.H. B→D* and Bs→Ds* vector, axial-vector and tensor form factors for the full q2 range from lattice QCD. *Phys. Rev. D* **2024**, *109*, 094515. [CrossRef]
110. Aoki, Y.; Colquhoun, B.; Fukaya, H.; Hashimoto, S.; Kaneko, T.; Kellermann, R.; Koponen, J.; Kou, E. B→D*$\ell\nu\ell$ semileptonic form factors from lattice QCD with Möbius domain-wall quarks. *Phys. Rev. D* **2024**, *109*, 074503. [CrossRef]

111. Harrison, J.; Davies, C.T.H. Bs→Ds* form factors for the full q2 range from lattice QCD. *Phys. Rev. D* **2022**, *105*, 094506. [CrossRef]
112. Lees, J.P.; Poireau, V.; Tisserand, V.; Grauges, E.; Palano, A.; Eigen, G.; Brown, D.N.; Kolomensky, Y.G.; Fritsch, M.; Koch, H.; et al. Extraction of form Factors from a Four-Dimensional Angular Analysis of $\overline{B} \to D^{*}\ell^{-}\overline{\nu}_{\ell}$. *Phys. Rev. Lett.* **2019**, *123*, 091801. [CrossRef]
113. Prim, M.T.; Bernlochner, F.; Metzner, F.; Lieret, K.; Kuhr, T.; Adachi, I.; Aihara, H.; Al Said, S.; Asner, D.M.; Atmacan, H.; et al. Measurement of differential distributions of B→D*$\ell\nu^{-}\ell$ and implications on ∣Vcb∣. *Phys. Rev. D* **2023**, *108*, 012002. [CrossRef]
114. Ferlewicz, D.; Urquijo, P.; Waheed, E. Revisiting fits to $B^0 \to D^{*-}\ell^+\nu_\ell$ to measure $|V_{cb}|$ with novel methods and preliminary LQCD data at nonzero recoil. *Phys. Rev. D* **2021**, *103*, 073005. [CrossRef]
115. Adachi, I.; Aggarwal, L.; Ahmed, H.; Aihara, H.; Akopov, N.; Aloisio, A.; Ky, N.A.; Asner, D.M.; Atmacan, H.; Aushev, T.; et al. Determination of ∣Vcb∣ using B¯0→D*+$\ell-\nu^{-}\ell$ decays with Belle II. *Phys. Rev. D* **2023**, *108*, 092013. [CrossRef]
116. Aubert, B.; Karyotakis, Y.; Lees, J.P.; Poireau, V.; Prencipe, E.; Prudent, X.; Tisserand, V.; Tico, J.G.; Grauges, E.; Martinelli, M.; et al. Measurement of $|V_{cb}|$ and the Form-Factor Slope in $\bar{B} \to D\ell^{-} \bar{nu}_\ell$ Decays in Events Tagged by a Fully Reconstructed B Meson. *Phys. Rev. Lett.* **2010**, *104*, 011802. [CrossRef] [PubMed]
117. Glattauer, R.; Schwanda, C.; Abdesselam, A.; Adachi, I.; Adamczyk, K.; Aihara, H.; Al Said, S.; Asner, D.M.; Aushev, T.; Ayad, R.; et al. Measurement of the decay $B \to D\ell\nu_\ell$ in fully reconstructed events and determination of the Cabibbo-Kobayashi-Maskawa matrix element $|V_{cb}|$. *Phys. Rev. D* **2016**, *93*, 032006. [CrossRef]
118. Aaij, R.; Beteta, C.A.; Ackernley, T.; Adeva, B.; Adinolfi, M.; Afsharnia, H.; Aidala, C.A.; Aiola, S.; Ajaltouni, Z.; Akar, S.; et al. Measurement of $|V_{cb}|$ with $B^0_s \to D^{(*)-}_s \mu^+\nu_\mu$ decays. *Phys. Rev. D* **2020**, *101*, 072004. [CrossRef]
119. Amhis, Y.; Hartmann, M.; Helsens, C.; Hill, D.; Sumensari, O. Prospects for $B^+_c \to \tau^+\nu_\tau$ at FCC-ee. *J. High Energy Phys.* **2021**, *12*, 133. [CrossRef]
120. Csorna, S.E.; Bonvicini, G.; Cinabro, D.; Dubrovin, M.; Bornheim, A.; Lipeles, E.; Pappas, S.P.; Shapiro, A.; Weinstein, A.J.; Briere, R.A.; et al. Moments of the B meson inclusive semileptonic decay rate using neutrino reconstruction. *Phys. Rev. D* **2004**, *70*, 032002. [CrossRef]
121. Mahmood, A.H.; Csorna, S.E.; Bonvicini, G.; Cinabro, D.; Dubrovin, M.; Bornheim, A.; Lipeles, E.; Pappas, S.P.; Shapiro, A.; Weinstein, A.J.; et al. Measurement of the B-meson inclusive semileptonic branching fraction and electron energy moments. *Phys. Rev. D* **2004**, *70*, 032003. [CrossRef]
122. Aubert, B.; Barate, R.; Boutigny, D.; Couderc, F.; Hicheur, A.; Karyotakis, Y.; Lees, J.P.; Tisserand, V.; Zghiche, A.; Palano, A.; et al. Measurements of moments of the hadronic mass distribution in semileptonic B decays. *Phys. Rev. D* **2004**, *69*, 111103. [CrossRef]
123. Aubert, B.; Barate, R.; Boutigny, D.; Couderc, F.; Hicheur, A.; Karyotakis, Y.; Lees, J.P.; Tisserand, V.; Zghiche, A.; Palano, A.; et al. Measurement of the electron energy spectrum and its moments in inclusive $B \to Xe\nu$ decays. *Phys. Rev. D* **2004**, *69*, 111104. [CrossRef]
124. Schwanda, C.; Abe, K.; Adachi, I.; Aihara, H.; Anipko, D.; Aulchenko, V.; Barberio, E.; Bay, A.; Bedny, I.; Belous, K.; et al. Moments of the Hadronic Invariant Mass Spectrum in $B \to X_c\ell\nu$ Decays at BELLE. *Phys. Rev. D* **2007**, *75*, 032005. [CrossRef]
125. Urquijo, P.; Barberio, E.; Abe, K.; Adachi, I.; Aihara, H.; Anipko, D.; Aulchenko, V.; Aushev, T.; Barbero, M.; Belous, K.; et al. Moments of the electron energy spectrum and partial branching fraction of B —> X(c) e nu decays at Belle. *Phys. Rev. D* **2007**, *75*, 032001. [CrossRef]
126. Abdallah, J.; DELPHI Collaboration. Determination of heavy quark non-perturbative parameters from spectral moments in semileptonic B decays. *Eur. Phys. J. C* **2006**, *45*, 35–59. [CrossRef]
127. Acosta, D.; Adelman, J.; Affolder, T.; Akimoto, T.; Albrow, M.G.; Ambrose, D.; Amerio, S.; Amidei, D.; Anastassov, A.; Anikeev, K.; et al. Measurement of the moments of the hadronic invariant mass distribution in semileptonic B decays. *Phys. Rev. D* **2005**, *71*, 051103. [CrossRef]
128. Aubert, B.; Karyotakis, Y.; Lees, J.P.; Poireau, V.; Prencipe, E.; Prudent, X.; Tisserand, V.; Tico, J.G.; Grauges, E.; Martinelli, M.; et al. Measurement and interpretation of moments in inclusive semileptonic decays anti-B —> X(c) l- anti-nu. *Phys. Rev. D* **2010**, *81*, 032003. [CrossRef]
129. van Tonder, R.; Cao, L.; Sutcliffe, W.; Welsch, M.; Bernlochner, F.U.; Adachi, I.; Aihara, H.; Asner, D.M.; Aushev, T.; Ayad, R.; et al. Measurements of q^2 Moments of Inclusive $B \to X_c\ell^+\nu_\ell$ Decays with Hadronic Tagging. *Phys. Rev. D* **2021**, *104*, 112011. [CrossRef]
130. Abudinén, F.; Adamczyk, K.; Aggarwal, L.; Ahmed, H.; Aihara, H.; Akopov, N.; Aloisio, A.; Ky, N.A.; Asner, D.M.; Atmacan, H.; et al. Measurement of lepton mass squared moments in B→Xc$\ell\nu^{-}\ell$ decays with the Belle II experiment. *Phys. Rev. D* **2023**, *107*, 072002. [CrossRef]
131. Aaij, R.; Beteta, C.A.; Adeva, B.; Adinolfi, M.; Affolder, A.; Ajaltouni, Z.; Akar, S.; Albrecht, J.; Alessio, F.; Alexander, M.; et al. A precise measurement of the B^0 meson oscillation frequency. *Eur. Phys. J. C* **2016**, *76*, 412. [CrossRef]
132. LHCb collaboration; Aaij, R.; Beteta, C.A.; Ackernley, T.; Adeva, B.; Adinolfi, M.; Afsharnia, H.; Aidala, C.A.; Aiola, S.; Ajaltouni, Z.; et al. Precise determination of the B^0_s–$\overline{B}^0_s$ oscillation frequency. *Nat. Phys.* **2022**, *18*, 1–5. [CrossRef]
133. Aaltonen, T.A.; Amerio, S.; Amidei, D.; Anastassov, A.; Annovi, A.; Antos, J.; Apollinari, G.; Appel, J.A.; Arisawa, T.; Artikov, A.; et al. Measurement of $B(t \to Wb)/B(t \to Wq)$ in Top-Quark-Pair Decays Using Dilepton Events and the Full CDF Run II Data Set. *Phys. Rev. Lett.* **2014**, *112*, 221801. [CrossRef]

134. Abazov, V.M.; Abbott, B.; Acharya, B.S.; Adams, M.; Adams, T.; Alexeev, G.D.; Alkhazov, G.; Alton, A.; Alverson, G.; Alves, G.A.; et al. Precision measurement of the ratio $\mathrm{B}(t \to Wb)/\mathrm{B}(t \to Wq)$ and Extraction of V_{tb}. *Phys. Rev. Lett.* **2011**, *107*, 121802. [CrossRef] [PubMed]
135. Khachatryan, V.; Sirunyan, A.; Tumasyan, A.; Adam, W.; Bergauer, T.; Dragicevic, M.; Erö, J.; Fabjan, C.; Friedl, M.; Frühwirth, R.; et al. Measurement of the ratio $\mathcal{B}(t \to Wb)/\mathcal{B}(t \to Wq)$ in pp collisions at $\sqrt{s}$ = 8 TeV. *Phys. Lett. B* **2014**, *736*, 33–57. [CrossRef]
136. Aaltonen, T.A.; Abazov, V.M.; Abbott, B.; Acharya, B.S.; Adams, M.; Adams, T.; Agnew, J.P.; Alexeev, G.D.; Alkhazov, G.; Alton, A.; et al. Tevatron Combination of Single-Top-Quark Cross Sections and Determination of the Magnitude of the Cabibbo-Kobayashi-Maskawa Matrix Element $\mathbf{V_{tb}}$. *Phys. Rev. Lett.* **2015**, *115*, 152003. [CrossRef]
137. LHC Top Working Group Summary Plots, Single Top Quark Production. Available online: https://twiki.cern.ch/twiki/bin/view/LHCPhysics/LHCTopWGSummaryPlots (accessed on 18 April 2024).
138. Aaij, R.; Abdelmotteleb, A.S.W.; Beteta, C.A.; Abudinén, F.; Achard, C.; Ackernley, T.; Adeva, B.; Adinolfi, M.; Adlarson, P.; Afsharnia, H.; et al. The LHCb Upgrade I. *arXiv* **2023**, arXiv:2305.10515.
139. Abe, T.; Adachi, I.; Adamczyk, K.; Ahn, S.; Aihara, H.; Akai, K.; Aloi, M.; Andricek, L.; Aoki, K.; Arai, Y.; et al. Belle II Technical Design Report. *arXiv* **2010**, arXiv:1011.0352.
140. Aaij, R.; Beteta, C.A.; Adeva, B.; Adinolfi, M.; Adrover, C.; Affolder, A.; Ajaltouni, Z.; Albrecht, J.; Alessio, F.; Alexander, M.; et al. *Letter of Intent for the LHCb Upgrade*; Technical Report; CERN: Geneva, Switzerland, 2011.
141. Altmannshofer, W.; Kou, E.; Urquijo, P.; Beaujean, F.; Bell, G.; Beneke, M.; I Bigi, I.; Bishara, F.; Blanke, M.; Bobeth, C.; et al. The Belle II Physics Book. *Prog. Theor. Exp. Phys.* **2019**, *2019*, 123C01; Erratum in *Prog. Theor. Exp. Phys.* **2020**, *2020*, 029201. [CrossRef]
142. Physics case for an LHCb Upgrade II—Opportunities in flavour physics, and beyond, in the HL-LHC era. *arXiv* **2018**, arXiv:1808.08865.