

$\mathcal{N} = (1, 1)$ supersymmetric AdS_3 in 10 dimensions

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ABSTRACT: Warped AdS_3 solutions in 10 dimensional supergravity that preserve $\mathcal{N} = (1, 1)$ supersymmetry are considered. Sufficient geometric conditions for their existence, and to stop the AdS_3 factor experiencing an enhancement to AdS_4 , are presented. The internal space of such solutions decomposes as a foliation of M_6 over an interval where M_6 supports either an $\text{SU}(3)$ - or $\text{SU}(2)$ -structure. The former case is classified in terms of torsion classes and new solutions are found.

KEYWORDS: Extended Supersymmetry, Superstring Vacua, AdS-CFT Correspondence

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1 Introduction

In recent years the $\text{AdS}_3/\text{CFT}_2$ correspondence has received increased attention. For the canonical maximally supersymmetric cases of the duality,¹ the D1-D5 [2] and D1-D5+D1-D5 [3] near horizons, preserving either small or large $\mathcal{N} = (4, 4)$ algebras, a great deal of progress has been made from both the string and CFT perspectives. Finding the CFT dual to [3] had been a long standing open question which was finally answered in [4, 5] — the key contribution being an explicit computation of the (S-dual) Kaluza-Klein spectrum that had been lacking. It was realised how to quantise strings on the pure NS avatars of $\text{AdS}_3 \times \text{S}^3 \times \mathbb{T}^4$ and $\text{AdS}_3 \times \text{S}^3 \times \text{S}^3 \times \text{S}^1$ in [6, 7] and [4] respectively. These advancements have led to a deeper understanding of the $\text{AdS}_3/\text{CFT}_2$ correspondence, even leading to a proposed world-sheet derivation, for pure NS $\text{AdS}_3 \times \text{S}^3 \times \mathbb{T}^4$, in [8].

This progress motivates efforts to construct more general AdS_3 string vacua in $d = 10, 11$ that are dual to two dimensional CFTs. This is a wide and varied problem: as we will review shortly, a priori such vacua can come equipped with a wide array of possible superconformal algebras [9, 10], but most of the allowed possibilities have not been realized so far. In particular, except for the $\mathcal{N} = (4, 4)$ case, little systematic progress has been made studying geometries supporting both a left and right superconformal algebra. In this work we aim to provide some tools to redress this. Specifically we will derive a geometric framework for the construction of string vacua with $\mathcal{N} = (1, 1)$ supersymmetry. This case has been relatively unexplored so far (one known example being in [11]), but it may be a promising stepping stone towards more general $\mathcal{N} = (p, q)$ vacua.

As promised, we now give a short review of the general situation so far. Recall first that for some values of \mathcal{N} there are several possible superalgebras, both for the L and R copy. For $\mathcal{N} = 8$ there are four possibilities, with R-symmetry $\text{so}(8)$, $\text{spin}(7)$, $\text{u}(4)$, $\text{sp}(2) \oplus \text{sp}(1)$. For $\mathcal{N} = 7$ there are two possibilities: $\text{so}(7)$ (large) and \mathfrak{g}_2 (small). For $\mathcal{N} = 6$, $\text{so}(6)$ (large) and $\text{u}(3)$ (small); and finally for $\mathcal{N} = 4$, $\text{so}(4)$ (large) and $\text{so}(3)$ (small). In $\mathcal{N} = (4, 4)$ one can then have three possibilities, and the (large,small) case is sometimes also called “medium”.

At this point two maximal cases are completely classified. First the local form of all large $\mathcal{N} = (4, 4)$ solutions was found across [12–15]. Second, all $\mathcal{N} = (8, 0)$ solutions were recently found in [16] (see also [17] for the first such example and [18] for a 3d gauged supergravity perspective). The only true string vacua² among these lie within the U-duality orbit of the D1-D5+D1-D5 near horizon; however these works do find many interesting Janus-like solutions (putatively) dual to conformal defects within higher dimensional CFTs. The case of small $\mathcal{N} = (4, 4)$ is more sparsely populated: we are only aware of the U-duality orbit of the D1-D5 near horizon [2] and [19], with a geometrical characterization given in [20]. The status of the other cases with sixteen supercharges, $\mathcal{N} = (8 - n, n)$, is unknown. Of the less supersymmetric examples: there is an example of (small) $\mathcal{N} = (7, 0)$ in [17], while the status of $\mathcal{N} = (6, 0)$ and $\mathcal{N} = (5, 0)$ is unknown. Partial classifications results

¹Recall that the notation $\mathcal{N} = (p, q)$ indicates that the two copies of $\text{sl}(2, \mathbb{R})$ in the AdS_3 isometry algebra can be supersymmetrized independently. For AdS_3 solution in 10 or 11 dimensions, $p + q \leq 8$ [1]. The “small” and “large” labels will be reviewed soon.

²Ref. [15] find more, but these are $\mathcal{N} = (4, 0)$.

$\mathcal{N}_R \backslash \mathcal{N}_L$	1	2	3	4	5	6	7	8	
0	pcl	pcl	ex	pcl	?	?	?(1)	ex(s)	cl
1	ex	ex	?	?	?	?	?		
2		ex	?	ex	?	?			
3			ex	?	?				
4				cl(1) ?(m)	ex(s)				

Table 1. Summary of known AdS₃ classification: “cl” means that a complete classification exists; “pcl” means partial classification; “ex” means that some examples are known; “?” means that no examples are known. As recalled in the text, some cases have a large (l) and small (s) version; $\mathcal{N} = 4$ also has a medium (m) case. In $\mathcal{N} = 8$ the classification exists for all four versions, while for $\mathcal{N} = 6$ and 5 no example is known for either version.

and some particular solutions for large and small $\mathcal{N} = (4, 0)$ are given across [15, 21, 22] and [23–34] respectively. An example with $\mathcal{N} = (4, 2)$ is given in [35]. For $\mathcal{N} = (2, 2)$, an incomplete picture is provided by [20, 36, 37]; explicit examples are in [38, 39]. There are limited examples with $\mathcal{N} = (3, 0)$ and $\mathcal{N} = (3, 3)$ in [40–42]. There is one known example of $\mathcal{N} = (2, 1)$ solution in [39, section 2]. A systematic study of $\mathcal{N} = (2, 0)$ in IIB is provided by [43–45], and there exist several classifications of $\mathcal{N} = (1, 0)$ [17, 46–49]; for the latter two, many explicit examples exist. We summarize this discussion in table 1.

We will set up our study in terms of the pure spinor formalism (section 2, with a more detailed derivation in the appendices). As usual in this method, we reformulate supersymmetry in terms of equations in terms of pure forms and exterior algebra. We will find in general that the internal space M_7 is a fibration over an interval of a six-manifold M_6 . For a more detailed analysis we restrict to the easiest case, where the internal supersymmetry parameters are characterized by an $SU(3)$ -structure (J_2, Ω_3) (section 3). Here we identify several natural classes into which a solution must fall (sections 4, 5).

Two of these are particularly interesting: in IIA we find that M_6 has to be a half-flat manifold ($dJ_2^2 = d\text{Re}\Omega_3 = 0$), while in IIB we find an even less restrictive condition ($(d\text{Re}\Omega_3)_{2,2} = 0$). The former case is in particular not too surprising, as it already appears in a related context: G_2 -holonomy spaces can be obtained as foliations of half-flat manifolds [50, 51], a fact that was exploited for a proposal to extend mirror symmetry beyond the Calabi-Yau realm [52, 53]. There are further restrictions that relate these geometries to the other fields, but overall these are rather forgiving, and we are able to find several explicit examples, albeit mostly numerically (section 6). These include cases where M_6 is a sphere fibration over three- or two-dimensional manifold with constant curvature; they have fully localized and back-reacted O8- and O5-plane sources, as we summarize in section 6.6.

2 Geometric conditions for type II $\mathcal{N} = (1, 1)$ AdS₃ vacua

Here we shall define an AdS₃ vacua in type II supergravity to be any solution with bosonic fields that respect the decomposition

$$\begin{aligned} ds^2 &= e^{2A} ds^2(\text{AdS}_3) + ds^2(M_7), & H &= e^{3A} h_0 \text{vol}(\text{AdS}_3) + H_3, \\ F_{10} &= f_{\pm} + e^{3A} \text{vol}(\text{AdS}_3) \wedge \star_7 \lambda(f_{\pm}) \end{aligned} \quad (2.1)$$

where we take AdS₃ to have inverse radius m , the AdS₃ warp factor e^{2A} , dilaton Φ and magnetic components of the NS flux, and RR polyform (H_3, f_{\pm}) have support on M₇ only. Additionally $\lambda(X_n) = (-1)^{\lfloor \frac{n}{2} \rfloor} X_n$ and \pm labels forms of even/odd degree, so the upper signs should be taken in IIA the lower ones in IIB here and elsewhere. For a true AdS₃ string vacuum M₇ should also be some bounded manifold, but this is not something we shall impose at this point.

One can realise $\mathcal{N} = (1, 1)$ supersymmetry in an AdS₃ solution by assuming that the associated 10 dimensional Majorana-Weyl Killing spinors decompose as

$$\epsilon_1 = \zeta_+ \otimes \theta_+ \otimes \chi_+^1 + \zeta_- \otimes \theta_+ \otimes \chi_-^1, \quad \epsilon_2 = \zeta_+ \otimes \theta_{\mp} \otimes \chi_+^2 + \zeta_- \otimes \theta_{\mp} \otimes \chi_-^2 \quad (2.2)$$

where ζ_{\pm} are independent real Killing spinors on AdS₃ charged under $\text{SL}(2)_{\pm} \subset \text{SO}(2,2)$, which obey the Killing spinor equation

$$\nabla_{\mu} \zeta_{\pm} = \pm \frac{m}{2} \gamma_{\mu}^{(3)} \zeta_{\pm}, \quad (2.3)$$

$\chi_{\pm}^{1,2}$ are 4 independent Majorana spinors on M₇ whose norms can be taken to satisfy

$$|\chi_+^1|^2 + |\chi_+^2|^2 = |\chi_-^1|^2 + |\chi_-^2|^2 = 2e^A, \quad e^A (|\chi_+^1|^2 - |\chi_+^2|^2) = -e^A (|\chi_-^1|^2 - |\chi_-^2|^2) = c \quad (2.4)$$

without loss of generality, for c some arbitrary constant. Finally θ_{\pm} are two dimensional vectors that are always required when one decomposes an even dimensional space in terms of two odd ones, as such they contain the 10 dimensional chirality labeled here by the \pm subscript; the subscript on the internal $d = 7$ spinors is just a label. In appendix C we derive sufficient geometric conditions for an $\mathcal{N} = (1, 1)$ solution to exist in terms of bi-linears formed from $\chi_{\pm}^{1,2}$, namely

$$\Psi_+^{st} + i\Psi_-^{st} = \chi_s^1 \otimes \chi_t^{2\dagger}, \quad s, t = \pm, \quad \chi_s^1 \otimes \chi_t^{2\dagger} = \frac{1}{8} \sum_{n=1}^7 \frac{1}{n!} \chi_t^{2\dagger} \gamma_{a_n \dots a_1} \chi_s^1 e^{a_1 \dots a_n} \quad (2.5)$$

where γ_a are flat space 7 dimensional gamma matrices, e^a are a corresponding vielbein and the subscript on Ψ_{\pm}^{st} again refers to form degree. We shall now summarise these geometric conditions. First off, similar necessary and sufficient conditions for $\mathcal{N} = 1$ AdS₃ vacua are given in [17, 48], and it should be no surprise that when a geometry supports both an $\mathcal{N} = (1, 0)$ and an $\mathcal{N} = (0, 1)$ set of these one has necessary and sufficient conditions for $\mathcal{N} = (1, 1)$: these are

$$\begin{aligned} e^{3A} h_0 &= -mc, \\ d_{H_3}(e^{A-\Phi} \Psi_{\mp}^{++}) \pm \frac{c}{16} f_{\pm} &= 0, & d_{H_3}(e^{A-\Phi} \Psi_{\mp}^{--}) \mp \frac{c}{16} f_{\pm} &= 0, \end{aligned} \quad (2.6)$$

$$\begin{aligned}
 (\Psi_{\mp}^{++}, f_{\pm})_7 &= \mp \frac{1}{2} \left(m + \frac{1}{4} e^{-A} c h_0 \right) e^{-\Phi} \text{vol}(M_7), & (\Psi_{\mp}^{--}, f_{\pm})_7 &= \pm \frac{1}{2} \left(m + \frac{1}{4} e^{-A} c h_0 \right) e^{-\Phi} \text{vol}(M_7) \\
 d_{H_3}(e^{2A-\Phi} \Psi_{\pm}^{++}) &\mp 2m e^{A-\Phi} \Psi_{\mp}^{++} = \frac{1}{8} e^{3A} \star_7 \lambda(f_{\pm}), & d_{H_3}(e^{2A-\Phi} \Psi_{\pm}^{--}) &\pm 2m e^{A-\Phi} \Psi_{\mp}^{--} = \frac{1}{8} e^{3A} \star_7 \lambda(f_{\pm}),
 \end{aligned}$$

where $(X, Y)_7$ is the 7 dimensional Mukai pairing, namely $(X, Y)_7 = X \wedge \lambda(Y)|_7$. These conditions imply the Bianchi identity of the electric component of the RR flux, which when $dH_3 = 0$ is

$$d(e^{3A} \star_7 \lambda(f_{\pm})) + e^{3A} h_0 f_{\pm} = 0 \quad (2.7)$$

and when $c \neq 0$ imply the source free magnetic flux Bianchi identity

$$d_{H_3} f_{\pm} = 0. \quad (2.8)$$

In addition to (2.6) it is possible to derive several more conditions involving bi-linears that mix the $\mathcal{N} = (1, 0)$ and $\mathcal{N} = (0, 1)$ sub-sectors; these are implied by what we present so far, but are very useful for constraining the system. First off it is possible to show that an $\mathcal{N} = (1, 1)$ AdS₃ solution will experience an enhancement to AdS₄ at all regular points of the internal space unless

$$\chi_-^{1\dagger} \chi_+^1 + \chi_-^{2\dagger} \chi_+^2 = 0, \quad \chi_+^{1\dagger} \gamma_a \chi_-^1 \mp \chi_+^{2\dagger} \gamma_a \chi_-^2 = 0; \quad (2.9)$$

so, if one is interested in genuine AdS₃ solutions, these conditions should be satisfied.³ Under these restrictions one can show that several other conditions should be satisfied: these additionally involve

$$g = \chi_-^{1\dagger} \chi_+^1 - \chi_-^{2\dagger} \chi_+^2, \quad \tilde{\xi} = -i(\chi_+^{1\dagger} \gamma_a \chi_-^1 \pm \chi_+^{2\dagger} \gamma_a \chi_-^2) e^a, \quad (2.10)$$

neither of which can vanish globally, otherwise $\mathcal{N} = (1, 1)$ cannot be realised. The conditions are

$$\begin{aligned}
 d(e^A g) + m \tilde{\xi} &= 0, \\
 d_{H_3}(e^{2A-\Phi}(\Psi_{\pm}^{+-} + \Psi_{\pm}^{-+})) &= 0, \\
 d_{H_3}(e^{-\Phi}(\Psi_{\pm}^{+-} - \Psi_{\pm}^{-+})) &= \frac{1}{8} \tilde{\xi} \wedge f_{\pm}, \quad (2.11) \\
 d_{H_3}(e^{A-\Phi}(\Psi_{\mp}^{+-} + \Psi_{\mp}^{-+})) &\pm m e^{-\Phi}(\Psi_{\pm}^{+-} - \Psi_{\pm}^{-+}) = \mp \frac{1}{8} e^A g f_{\pm}, \\
 d_{H_3}(e^{3A-\Phi}(\Psi_{\mp}^{+-} - \Psi_{\mp}^{-+})) &\pm e^{3A-\Phi} h_0(\Psi_{\pm}^{+-} - \Psi_{\pm}^{-+}) \pm 3m e^{2A-\Phi}(\Psi_{\pm}^{+-} + \Psi_{\pm}^{-+}) = \pm \frac{1}{8} e^{3A} \tilde{\xi} \wedge \star_7 \lambda(f_{\pm})
 \end{aligned}$$

from which it follows that the Bianchi identity of the magnetic flux is in fact implied in general for $\mathcal{N} = (1, 1)$ AdS₃ vacua, when that of H is imposed. Much of (2.6), (2.11) can be shown to be redundant using some identities that the 7d bi-linears must obey, (C.26a)–(C.26h). Upon tuning the internal spinors such that (2.4) and (2.9) are satisfied, a sufficient

³The argument leading to this claim is local, but may hold in general. The metric is always locally AdS₄ when (2.9) is not imposed, but we have not ruled out the possibility of fluxes that contain terms that break the isometries of AdS₄ when their Bianchi identities are violated, either by partially localised or completely smeared sources.

system for $\mathcal{N} = (1, 1)$ supersymmetry is given by⁴

$$e^{3A}h_0 = -mc, \tag{2.12a}$$

$$d(e^A g) + m\tilde{\xi} = 0, \tag{2.12b}$$

$$d_{H_3}(e^{A-\Phi}(\Psi_{\mp}^{++} + \Psi_{\mp}^{--})) = 0, \tag{2.12c}$$

$$d_{H_3}(e^{2A-\Phi}(\Psi_{\pm}^{++} - \Psi_{\pm}^{--})) \mp 2me^{A-\Phi}(\Psi_{\mp}^{++} - \Psi_{\mp}^{--}) = 0, \tag{2.12d}$$

$$d_{H_3}(e^{A-\Phi}(\Psi_{\mp}^{++} - \Psi_{\mp}^{--})) \pm \frac{c}{8}f_{\pm} = 0, \tag{2.12e}$$

$$d_{H_3}(e^{2A-\Phi}(\Psi_{\pm}^{++} + \Psi_{\pm}^{--})) \mp 2me^{A-\Phi}(\Psi_{\mp}^{++} + \Psi_{\mp}^{--}) = \frac{1}{4}e^{3A}\star_7\lambda(f_{\pm}), \tag{2.12f}$$

$$d_{H_3}(e^{A-\Phi}(\Psi_{\mp}^{+-} + \Psi_{\mp}^{-+})) \pm me^{-\Phi}(\Psi_{\pm}^{+-} - \Psi_{\pm}^{-+}) = \mp \frac{1}{8}e^A g f_{\pm}, \tag{2.12g}$$

$$(\Psi_{\mp}^{++} - \Psi_{\mp}^{--}, f_{\pm})_7 = \pm \left(m + \frac{1}{4}e^{-A}ch_0 \right) e^{-\Phi} \text{vol}(M_7). \tag{2.12h}$$

To actually have a solution one must also solve the supergravity equations of motion, however well known integrability arguments tell us that these are implied for AdS₃ by supersymmetry and the Bianchi identities of f_{\pm} and H_3 . For $\mathcal{N} = (1, 1)$ specifically this amounts to just imposing

$$dH_3 = 0 \tag{2.13}$$

in regular regions of the internal space, with possible δ -function sources elsewhere. The remaining equations of motion then follow. Note also that a non trivial Romans mass in IIA requires $c = h_0 = 0$ due to (2.12d) while in IIB one can always assume one is in an SL(2,ℝ) duality frame where $h_0 = c = 0$ holds. Thus in the rest of the paper we will simply fix $c = 0$, with the understanding that the only IIB solutions outside this limit are S-dual to what we do find, and the only IIA solutions belong to classes better studied from an M-theory perspective.⁵

In the next section we shall proceed to parametrise the bi-linears appearing in (2.12a)–(2.12g) in the $c = 0$ limit.

3 Parametrising the bi-linears

As argued in the previous section, to end up with a solution that is not locally AdS₄ we should solve (2.9). We will assume from now on that $c = 0$, or in other words that the internal spinors have equal norm; this is often necessary in higher dimensions. Now (2.12a) is solved by $h_0 = 0$; this means that without loss of generality we can decompose our four Majorana spinors in a basis of two real unit norm vectors (V, \hat{V}) , three real functions

⁴It turns out that when $c = 0$ and the internal manifold supports an SU(3)-structure, the main focus of this work, (2.12f) and (2.12h) are actually implied. We suspect that this holds true in general but have not proved this.

⁵In other words, all such IIA solutions can be lifted to M-theory where they will have a U(1) flavour isometry. Such solutions may just be special cases of broader classes without this U(1), but one is blind to that in IIA.

(a, b, α) and a single unit norm spinor χ as

$$\begin{aligned}\chi_+^1 &= e^{\frac{A}{2}}\chi, & \chi_-^1 &= e^{\frac{A}{2}}(\cos\alpha + i\sin\alpha V)\chi, \\ \chi_+^2 &= e^{\frac{A}{2}}(a + ibV)\chi, & \chi_-^2 &= -e^{\frac{A}{2}}(\cos\alpha \mp i\sin\alpha V)(a + ib\hat{V})\chi,\end{aligned}\tag{3.1}$$

where $a^2 + b^2 = 1$, which immediately leads to

$$g = 2e^A \cos\alpha, \quad \tilde{\xi} = 2e^A \sin\alpha V\tag{3.2}$$

where neither $\sin\alpha$ nor $\cos\alpha$ can be set to zero globally. This means that (2.12b) becomes

$$d(e^{2A} \cos\alpha) + me^A \sin\alpha V = 0,\tag{3.3}$$

which implies that locally M_7 decomposes as a warped product of an interval spanned by V and some six-manifold M_6 . Locally one can solve (3.3) in terms of a function h of a local coordinate y as

$$e^{2A} \cos\alpha = h, \quad V = -\frac{h'}{me^A \sqrt{1 - e^{-4A}h^2}} dy\tag{3.4}$$

where h essentially parametrises diffeomorphism invariance in y , and $h' = \partial_y h$. The metric then decomposes as

$$ds^2(M_7) = \frac{(h')^2}{m^2 e^{2A} (1 - e^{-4A}h^2)} dy^2 + ds^2(M_6).\tag{3.5}$$

The specific form M_6 can take depends on precisely how the rest of the supersymmetry conditions are solved, in general its metric will depend on y .

In general we can take the spinor χ to be such that

$$\chi \otimes \chi^\dagger = \Psi_+^{(G_2)} + i\Psi_-^{(G_2)} = \frac{1}{8}(1 - i\Phi_3 - \star_7\Phi_3 + i\text{vol}_7), \quad \Phi_3 \wedge \star_7\Phi_3 = 7\text{vol}_7,\tag{3.6}$$

where Φ_3 is the 3 form associated to the G_2 -structure that a single $d = 7$ Majorana spinor supports. The actual G -structure supported by an $\mathcal{N} = (1, 1)$ solution will depend on how \hat{V} is aligned, if it is parallel to V then the solution will support an $SU(3)$ -structure on M_6 , otherwise (assuming $b \neq 0$) it will support an $SU(2)$ -structure there. In general we can decompose the G_2 -structure in terms of an $SU(3)$ -structure as

$$\Phi_3 = V \wedge J_2 - \text{Im}\Omega_3, \quad \star_7\Phi_3 = \frac{1}{2}J_2 \wedge J_2 - V \wedge \text{Re}\Omega_3\tag{3.7}$$

where (J_2, Ω_3) are the associated $(1, 1)$ and $(3, 0)$ forms that are orthogonal to V . When b is non trivial we can further decompose

$$\hat{V} = \tilde{\kappa}_\parallel V + \tilde{\kappa}_\perp U_1, \quad U_1 \lrcorner V = 0, \quad U_1 \lrcorner U_1 = 1, \quad \tilde{\kappa}_\parallel^2 + \tilde{\kappa}_\perp^2 = 1,\tag{3.8}$$

and then the $SU(3)$ -structure decomposes in terms of an $SU(2)$ -structure orthogonal to V as

$$J_2 = j_2 + U_1 \wedge U_2, \quad \Omega_3 = (U_1 + iU_2) \wedge \tilde{\omega}_2, \quad U_1 \lrcorner U_2 = 0, \quad U_2 \lrcorner U_2 = 1.\tag{3.9}$$

where $(j_2, \tilde{\omega}_2)$ define a 4d SU(2)-structure orthogonal to (U_1, U_2) which, when $b\kappa_{\parallel} \neq 0$, one can take to be two components of the 7 dimensional vielbein without loss of generality. One can then show that the bi-linears of the previous section decompose in terms of two bi-linears on M_6 $\Psi_{\pm}^{(6)}$ as

$$\begin{aligned}
 \Psi_+^{++} &= e^A \text{Re} \left(\Psi_+^{(6)} + V \wedge \Psi_-^{(6)} \right), & \Psi_-^{++} &= e^A \text{Im} \left(\Psi_-^{(6)} + V \wedge \Psi_+^{(6)} \right), \\
 \Psi_+^{-+} &= e^A \text{Re} \left(e^{i\alpha} (\Psi_+^{(6)} + V \wedge \Psi_-^{(6)}) \right), & \Psi_-^{-+} &= e^A \text{Im} \left(e^{i\alpha} (\Psi_-^{(6)} + V \wedge \Psi_+^{(6)}) \right), \\
 \Psi_+^{+-} &= -e^A \text{Re} \left(e^{\pm i\alpha} \Psi_+^{(6)} + e^{\mp i\alpha} V \wedge \Psi_-^{(6)} \right), & \Psi_-^{+-} &= -e^A \text{Im} \left(e^{\mp i\alpha} \Psi_-^{(6)} + e^{\pm i\alpha} V \wedge \Psi_+^{(6)} \right), \\
 \Psi_+^{--} &= -e^A \text{Re} \left(e^{2i\alpha_{\pm}} \Psi_+^{(6)} + e^{2i\alpha_{\mp}} V \wedge \Psi_-^{(6)} \right), & \Psi_-^{--} &= -e^A \text{Im} \left(e^{2i\alpha_{\mp}} \Psi_-^{(6)} + e^{2i\alpha_{\pm}} V \wedge \Psi_+^{(6)} \right),
 \end{aligned} \tag{3.10}$$

where $\alpha_+ = \alpha$ and $\alpha_- = 0$, which parameterise the difference between IIA and IIB bi-linears. The 6d bi-linears themselves may be expressed in general as

$$\Psi_+^{(6)} = \frac{e^{i\beta}}{8} (\kappa_{\parallel} e^{-ij_2} + \kappa_{\perp} \omega_2) \wedge e^{\frac{1}{2} U \wedge \bar{U}}, \quad \Psi_-^{(6)} = \frac{1}{8} U \wedge (\kappa_{\parallel} \omega_2 - \kappa_{\perp} e^{-ij_2}), \tag{3.11}$$

where we have simplified the parametrisation by introducing

$$(a + ib\tilde{\kappa}_{\parallel}) = e^{i\beta} \kappa_{\parallel}, \quad b\tilde{\kappa}_{\perp} = \kappa_{\perp}, \quad \tilde{\omega}_2 = e^{i\beta} \omega_2, \quad \kappa_{\parallel}^2 + \kappa_{\perp}^2 = 1, \quad U = U_1 + iU_2. \tag{3.12}$$

which are standard bi-linears for a 6d SU(2)-structure.

In this work we shall focus on the SU(3)-structure limit, $(\kappa_{\parallel}, \kappa_{\perp}) = (1, 0)$ where the bi-linears reduce to

$$\Psi_+^{(6)} = \frac{e^{i\beta}}{8} e^{-iJ_2}, \quad \Psi_-^{(6)} = \frac{1}{8} \Omega_3, \tag{3.13}$$

leaving the general case for future work. To this end it is helpful to review some features of SU(3)-structures in 7 dimensions. First we should make our conventions clear. We have taken (J_2, Ω_3) to be defined such that

$$\begin{aligned}
 J_2 \wedge \Omega_3 &= 0, & J_2 \wedge J_2 \wedge J_2 &= \frac{3i}{4} \Omega_3 \wedge \bar{\Omega}_3, \\
 \star_7 1 &= \text{vol}_7 = \frac{1}{3!} V \wedge J_2 \wedge J_2 \wedge J_2, & \star_7 \Omega_3 &= iV \wedge \Omega_3.
 \end{aligned} \tag{3.14}$$

This fixes the torsion classes of the SU(3)-structure to be [54]

$$\begin{aligned}
 dV &= RJ_2 + T_1 + \text{Re}(\iota_{\bar{V}_1} \Omega_3) + V \wedge W_0, \\
 dJ_2 &= \frac{3}{2} \text{Im}(\bar{W}_1 \Omega_3) + W_3 + W_4 \wedge J_2 + V \wedge \left(\frac{2}{3} \text{Re} E J_2 + T_2 + \text{Re}(\iota_{\bar{V}_2} \Omega_3) \right), \\
 d\Omega_3 &= W_1 J_2 \wedge J_2 + W_2 \wedge J_2 + \bar{W}_5 \wedge \Omega_3 + V \wedge \left(E \Omega_3 - 2V_2 \wedge J + S \right).
 \end{aligned} \tag{3.15}$$

The part without V in dJ_2 and $d\Omega_3$ reduces to the more familiar definition of torsion classes for SU(3)-structures in $d = 6$. (E, R) are complex and real functions respectively, $(V_{1,2}, W_5)$ are (1,0)-forms, (W_0, W_4) real one-forms, $(W_4, T_{1,2})$ are respectively complex

M_6	Vanishing torsion classes
Complex	$W_1 = W_2 = 0$
Symplectic	$W_1 = W_3 = W_4 = 0$
Half-flat	$\text{Re}W_1 = \text{Re}W_2 = W_4 = W_5 = 0$
Special Hermitian	$W_1 = W_2 = W_4 = W_5 = 0$
Nearly Kähler	$W_2 = W_3 = W_4 = W_5 = 0$
Almost Kähler	$W_1 = W_3 = W_4 = W_5 = 0$
Kähler	$W_1 = W_2 = W_3 = W_4 = 0$
Calabi-Yau	$W_1 = W_2 = W_3 = W_4 = W_5 = 0$

Table 2. Some notable SU(3)-structure six-manifolds in terms of vanishing torsion classes. Additionally a manifold may be conformally one of these when (W_4, W_5) are appropriately tuned.

and real primitive⁶ (1,1)-forms, S is a primitive (2,1) form and W_3 is the real part of such a form. These make up irreducible representations of SU(3), the details of which are immaterial for our purposes; what matters is that they must obey the identities

$$\begin{aligned}
 (S, W_3) \wedge J_2 &= (S, W_2, V_{1,2}, W_5) \wedge \Omega_3 = (W_4, T_{1,2}) \wedge J_2 \wedge J_2 = 0, \\
 \star_7 S &= -iV \wedge S, \quad \star_7(W_4, T_{1,2}) = -V \wedge J_2 \wedge (W_4, T_{1,2}), \\
 \star_7(V_{1,2}, W_5) &= \frac{i}{2}V \wedge J_2 \wedge J_2 \wedge (V_{1,2}, W_5),
 \end{aligned}
 \tag{3.16}$$

where W_4 behaves under Hodge duality as $\text{Re}S$ does. These facts will enable us to establish what manifolds can support $\mathcal{N} = (1, 1)$ supersymmetry in terms of an SU(3)-structure in the following sections. Solving the supersymmetry constraints will set some of W_i to zero, and will consequently also lead to constraints on the y -dependent M_6 in the decomposition (3.5). As we recall in table 2, many manifolds where some of the W_i vanish have been studied and have known names.⁷

Finally we should stress that for all our $\mathcal{N} = (1, 1)$ classes we necessarily have

$$R = V_1 = T_1 = 0, \tag{3.17}$$

with W_0 fixed universally due to (3.3), which holds in both IIA and IIB — we shall thus not comment further on these torsion classes.

We begin our analysis in type IIA in the next section.

4 SU(3)-structure in IIA

In this section we shall classify the possible solutions in IIA that support an SU(3)-structure. As we shall see there are two classes where the 7d internal space decomposes as a foliation over an interval whose leaves are either an almost Kähler or half-flat six-manifold.

⁶See [49] for a very good review of what this means in this context.

⁷Some of the names are rather common, some less so. We are mostly following notation in [55], which however was concerned with U(3)- (rather than SU(3)-) structures, where W_5 is not defined. Following [56], we have set $W_5 = 0$ except for the standard symplectic, complex and Kähler cases. The nearly and almost Kähler cases might perhaps have been called nearly and almost Calabi-Yau.

To begin with we find it helpful to redefine the bi-linears of the previous section slightly by mapping

$$\beta \rightarrow \beta - \alpha, \quad \Omega_3 \rightarrow e^{-i\frac{\pi}{2}} \Omega_3. \quad (4.1)$$

It is then possible to show that the necessary conditions (2.12b)–(2.12d) are implied in general by

$$d\left(e^{-A-\Phi} \frac{\cos \beta}{\cos \alpha}\right) = d(e^{A-\Phi} \sin \beta \cot \alpha) \wedge V = 0, \quad (4.2a)$$

$$d(e^{3A-\Phi} \cos \alpha \sin \beta J_2) - e^{3A-\Phi} \cos \alpha \cos \beta H_3 + 2me^{2A-\Phi} \sin \alpha \sin \beta V \wedge J_2 = 0, \quad (4.2b)$$

$$d(e^{2A-\Phi} (\text{Re} \Omega_3 + \cos \alpha \cos \beta V \wedge J_2) + e^{2A-\Phi} \cos \alpha \sin \beta H_3 \wedge V) = 0, \quad (4.2c)$$

$$\cos \alpha \sin \beta J_2 \wedge J_2 \wedge dJ_2 - (\cos \alpha \cos \beta J_2 \wedge J_2 - 3V \wedge \text{Im} \Omega_3) \wedge H_3 = 0, \quad (4.2d)$$

$$(\sin \alpha J_2 \wedge d\text{Re} \Omega_3 - \cos \beta d\alpha \wedge V \wedge J_2 \wedge J_2 = \sin \alpha H_3 \wedge \text{Re} \Omega_3 + \sin \beta d\alpha V \wedge J_2 \wedge J_2 = 0, \quad (4.2e)$$

$$d(e^{3A-\Phi} (\cos \alpha \cos \beta J_2 \wedge J_2 - 2V \wedge \text{Im} \Omega_3) + 2e^{3A-\Phi} \cos \alpha \sin \beta H_3 \wedge J_2 + 2me^{2A-\Phi} \sin \alpha \cos \beta V \wedge J_2 \wedge J_2 = 0. \quad (4.2f)$$

From (4.2b) we see that solutions with $\cos \beta = 0$ appear to be distinct from those with generic values of β : namely this constraint fixes H_3 uniquely unless $\cos \beta = 0$, in which case it is only more weakly constrained by the remaining conditions. This is a strong indication that $\cos \beta = 0$ is a physically distinct class to generic β . It thus makes sense to study these two scenarios separately; we shall continue our analysis for the $\cos \beta = 0$ case in the next section, and consider the more generic case in section 4.2.

4.1 Class I: restricted almost Kähler foliation $F_0 = 0$

This class is defined by fixing $\cos \beta = 0$, and in fact we can solve this without loss of generality as

$$\beta = \frac{\pi}{2}. \quad (4.3)$$

Clearly the conditions (4.2a)–(4.2f) truncate a fair bit under this restriction. First the conditions that remain non trivial in (4.2a) can be solved in terms of an arbitrary function $k(y)$ as

$$\frac{e^{-A-\Phi} h}{\sqrt{1 - e^{-4A} h^2}} = k, \quad (4.4)$$

where we use (3.4) to substitute for α here and elsewhere. To proceed it is helpful to decompose the NS flux as

$$H_3 = dy \wedge H_2 + \tilde{H}_3, \quad dy \lrcorner H_2 = dy \lrcorner \tilde{H}_3 = 0, \quad (4.5)$$

and to substitute for $(dJ_2, d\Omega_3)$ with their respective torsion classes (3.15) in (4.2b)–(4.2f). A careful analysis reveals that $A = A(y)$ and that most of the torsion classes must vanish, specifically one has

$$V_2 = \text{Im} E = T_1 = T_2 = W_1 = \text{Re} W_2 = W_3 = W_4 = W_5 = 0. \quad (4.6)$$

Comparing this to table 2 tells us that M_6 is a y -dependent almost Kähler manifold, further restricted by the requirement that $\text{Re}W_2 = 0$. These requirements are rather stringent, and in practice we will satisfy them in later sections only by taking conformal Calabi-Yau spaces.

The torsion classes that remain non trivial are

$$\text{Re}E = 3m \frac{\partial_y(e^A h)}{h h' \sqrt{1 - e^{-4A} h^2}}, \quad (4.7)$$

and $(S, \text{Im}W_2)$, which appear in the NS flux and get fixed as

$$H_3 = \frac{e^A h'}{m h \sqrt{1 - e^{-4A} h^2}} dy \wedge \text{Im}W_2 + \frac{e^{2A}}{h} \text{Re}S + \frac{m \partial_y(e^A h)}{e^{2A} h' \sqrt{1 - e^{-4A} h^2}} \text{Re}\Omega_3. \quad (4.8)$$

Finally the functions h, k are constrained to obey

$$\partial_y \log\left(\frac{k}{h}\right) = \frac{3}{1 - e^{-4A} h^2} \partial_y \log h, \quad (4.9)$$

at which point (4.2a)–(4.2f) are implied. To make further progress one needs to extract the magnetic components of the RR flux from (2.12g), which yields

$$\begin{aligned} f_0 = 0, \quad f_2 = \frac{m k e^{2A}}{h^2} \sqrt{1 - e^{-4A} h^2} J_2, \quad f_6 = \frac{m k \sqrt{1 - e^{-4A} h^2} \partial_y(e^{4A} h^3)}{3! e^{2A} h^4 h'} J_2 \wedge J_2 \wedge J_2, \quad (4.10) \\ f_4 = \frac{e^{3A} k}{h^2} (1 - e^{-4A} h^2) \text{Im}W_2 \wedge J_2 + \frac{e^{2A} k h'}{m h^2} \sqrt{1 - e^{-4A} h^2} dy \wedge \text{Im}S + \frac{k}{h} \partial_y(e^{-A} h) dy \wedge \text{Im}\Omega_3. \end{aligned}$$

These definitions of the flux actually imply (2.12f) and (2.12h), so all of the supersymmetry constraints are implied by (4.6), (4.7) and (4.9). The Bianchi identities of the RR fluxes and the remaining supergravity equations of motion now follow from the Bianchi identity of H_3 , but this is not implied for this class so one additionally needs to impose it.

4.2 Class II: half-flat foliation $F_0 \neq 0$

For this class we assume $\cos \beta \neq 0$, which means that we can divide by this quantity and further simplify (4.2a)–(4.2f). Similarly to the previous class one finds that

$$A = A(y), \quad \beta = \beta(y), \quad (4.11)$$

while this time the NS flux gets fixed such that it automatically solves its Bianchi identity as

$$H_3 = d(\tan \beta J_2). \quad (4.12)$$

The rest of (4.2a)–(4.2f) impose the following on the functions of the ansatz:

$$e^{A-\Phi} \cos \beta = c_1 h, \quad \beta' = \sin(2\beta) \frac{\partial_y \log(e^A h)}{1 - e^{-4A} h^2} \quad (4.13)$$

where c_1 is an integration constant and fix the torsion classes. The following torsion classes must vanish:

$$V_2 = \text{Im}E = \text{Re}W_1 = \text{Re}W_2 = W_4 = W_5 = 0, \quad (4.14)$$

making M_6 a y -dependent half-flat manifold. As we remarked in the introduction, the occurrence of such spaces is not surprising, because of the role they played in foliated G_2 -manifolds [50, 51]. As we will see in later sections, it is quite easy to provide explicit examples.

The non trivial torsion classes are expressed in terms of (T_2, S) , which remain arbitrary, and the functions of the ansatz as

$$\begin{aligned} \text{Im}W_1 &= -\frac{2m}{3} \frac{\cos\beta \partial_y(e^A h)}{e^{2A} h' \sqrt{1 - e^{-4A} h^2}}, & \text{Im}W_2 &= -\frac{h}{e^{2A} \cos\beta} T_2, \\ W_3 &= \frac{e^{2A} \cos\beta}{h} \text{Re}S, & \text{Re}E &= \frac{m(2 - \cos(2\beta)) \partial_y(e^A h)}{h h' \sqrt{1 - e^{-4A} h^2}} \end{aligned} \tag{4.15}$$

From (2.12g) we extract the magnetic components of the RR fluxes:

$$\begin{aligned} f_0 = F_0 &= c_1 m, & f_2 &= c_1 m \tan\beta J_2, & f_6 &= \frac{cm}{3!} \tan\beta \frac{\partial_y(e^{4A} h^3)}{e^{4A} h^2 h'} J_2 \wedge J_2 \wedge J_2, \\ f_4 &= \frac{ch\sqrt{1 - e^{-4A} h^2}}{e^A \cos^2\beta} J_2 \wedge T_2 + \frac{ch'}{m \cos\beta} dy \wedge \text{Im}S \\ &+ \frac{ch\partial_y(e^{-A} h)}{e^{2A} \cos\beta \sqrt{1 - e^{-4A} h^2}} dy \wedge \text{Im}\Omega_3 + \frac{cm}{3!} \frac{\partial_y(e^{4A} h)}{e^{4A} h'} J_2 \wedge J_2; \end{aligned} \tag{4.16}$$

these again imply (2.12f) and (2.12h). Thus as the Bianchi identities of the RR and NS fluxes are implied for this class one just needs to find half-flat manifolds consistent with (4.14) and (4.15) and solve (4.13) to have a solution. We note that $\beta = 0$, the trivial solution to (4.13), is a potentially interesting limit as the flux reduces to just (f_0, f_4) . This suggests a link to the AdS_6 compactification on hyperbolic spaces Σ_3 appearing in [57]; we will make this link more precise in section 6.5.1.

This concludes our analysis of the geometries and fluxes of $\mathcal{N} = (1, 1)$ solution preserving an $\text{SU}(3)$ -structure in IIA. In the next section we shall carry out a similar analysis in IIB, before giving some new solutions within these classes in section 6.

5 $\text{SU}(3)$ -structure in IIB

In this section we will classify $\text{SU}(3)$ -structure solutions in IIB. As with IIA, the internal 7-manifold will decompose as a foliation of M_6 over an interval. Again there are two cases: one with M_6 again an almost Kähler manifold, the other is a quite broad class with $\text{Re}W_1 = \text{Re}W_2 = 0$, but that is not merely conformally half-flat in general.

As with IIA it is helpful to redefine the $\text{SU}(3)$ -structure slightly, this time by just modifying the three-form as

$$\Omega_3 \rightarrow -ie^{-i\alpha} \Omega_3. \tag{5.1}$$

With some work it is then possible to show that the conditions (2.12b)–(2.12d) are implied

by

$$d(e^{2A-\Phi}\cos\beta) = d\left(e^{-\Phi}\frac{\sin\beta}{\sin(2\alpha)}\right) \wedge V = 0, \quad (5.2a)$$

$$d(e^{2A-\Phi}\sin\beta J_2) - e^{2A-\Phi}\cos\beta H_3 = 0, \quad (5.2b)$$

$$d(e^{3A-\Phi}(\cos\alpha\text{Re}\Omega_3 + \cos\beta J_2 \wedge V)) + e^{3A-\Phi}\sin\beta H_3 \wedge V + 2me^{2A-\Phi}\sin\alpha V \wedge \text{Re}\Omega_3 = 0, \quad (5.2c)$$

$$d(e^{2A-\Phi}(\cos\beta J_2 \wedge J_2 - 2\cos\alpha V \wedge \text{Im}\Omega_3)) + 2e^{2A-\Phi}\sin\beta J_2 \wedge H_3 = 0 \quad (5.2d)$$

$$\sin\beta d\alpha \wedge V \wedge J_2 \wedge J_2 + \sin\alpha H_3 \wedge \text{Re}\Omega_3 = 0, \quad (5.2e)$$

$$d(e^{2A-\Phi}\sin\beta) \wedge J_2 \wedge J_2 \wedge J_2 - 3e^{2A-\Phi}\cos\alpha V \wedge \text{Im}\Omega_3 \wedge H_3 = 0, \quad (5.2f)$$

in the $c = 0$ limit, and where we have made no assumption about β . Like in IIA, (5.2b) indicates that we have a branching of possible solutions depending on how β is tuned: this condition fixes H_3 such that its Bianchi identity is implied by the first of (5.2a) unless $\cos\beta = 0$. In this case H_3 is more weakly constrained by (5.2d)–(5.2f) and its Bianchi identity must be additionally imposed. We shall study these two cases in the next sections starting with $\cos\beta = 0$.

5.1 Class III: restricted almost Kähler foliation

In this section we consider the case where $\cos\beta = 0$, we will actually fix $\beta = \frac{\pi}{2}$ without loss of generality.

The analysis of this class runs pretty parallel to that of section 4.1. One can make progress solving (5.2a)–(5.2f) using the torsion classes of the SU(3)-structure (3.15) and the identities they must obey (3.16). One finds in general that

$$A = A(y), \quad \Phi = \Phi(y), \quad (5.3)$$

and that the following torsion classes must vanish:

$$V_2 = \text{Im}E = T_1 = T_2 = W_1 = \text{Re}W_2 = W_3 = W_4 = W_5 = 0, \quad (5.4)$$

which again makes M_6 a y -dependent almost Kähler manifold, of the restricted type we found in (4.6). The torsions $(\text{Re}E, S, \text{Im}W_2)$ remain non trivial, with the former fixed to

$$\text{Re}E = \frac{3m}{2} \frac{e^{A-\Phi}\sqrt{1-e^{-4A}h^2}\partial_y(e^\Phi h^2)}{h'(e^{4A}-2h^2)}, \quad (5.5)$$

while the latter two appear in the definition of the NS flux:

$$H_3 = e^{-2A}h\text{Re}S + \frac{hh'}{me^{3A}\sqrt{1-e^{-4A}h^2}}dy \wedge \text{Im}W_2 + \frac{me^{3A-\Phi}\sqrt{1-e^{-4A}h^2}}{2hh'(e^{4A}-2h^2)}\partial_y(h^2e^\Phi)\text{Re}\Omega_3. \quad (5.6)$$

In addition to this, (5.2a)–(5.2f) imply one further condition on the functions of the ansatz, namely

$$\partial_y \log\left(\frac{e^{2A-\Phi}}{\sqrt{1-e^{-4A}h^2}}\right) = 3\frac{e^{-4A}hh'}{1-e^{-4A}h^2}. \quad (5.7)$$

One can extract the magnetic components of the RR flux from (2.12g) which, given what has been derived thus far, may be expressed as

$$\begin{aligned}
 f_1 &= -2 \frac{e^{-2A-\Phi} h}{\sqrt{1-e^{-4A}h^2}} d \log(e^{2A}h), \\
 f_3 &= \frac{e^{2A-\Phi} \sqrt{1-e^{-4A}h^2}}{h} H_3 + \frac{m e^{A-\Phi}}{h} \text{Re} \Omega_3, \quad f_5 = \frac{e^{-2A-\Phi} h'}{\sqrt{1-e^{-4A}h^2}} dy \wedge J_2 \wedge J_2,
 \end{aligned}
 \tag{5.8}$$

where $f_7 = 0$ in general. Like the almost Kähler class in IIA, these fluxes imply both (2.12f) and (2.12h), so the supersymmetry equations are implied by just (5.4), (5.5) and (5.7). What remains to ensure that one actually has a solution is to impose the Bianchi identity of the NS three-form.

5.2 Class IV: a broad class

In this section we assume that $\cos \beta \neq 0$, as we shall see this leads to the least restrictive of our SU(3)-structure classes.

For this class we find it helpful to decompose the exterior derivative as

$$d = d_6 + dy \wedge \partial_y. \tag{5.9}$$

First off (5.2c) implies that

$$e^{2A-\Phi} \cos \beta = c_1, \quad d \left(\tan \beta (1 - e^{-4A}h^2) \right) + \frac{3}{2} h^2 \tan \beta d(e^{-4A}) = 0, \tag{5.10}$$

where c_1 is a constant, and from (5.2b) we get that the NS flux is

$$H_3 = d(\tan \beta J_2), \tag{5.11}$$

solving its Bianchi identity by definition. Substituting $(dJ_2, d\Omega_3)$ for their SU(3)-structure torsion classes in (5.2c)–(5.2f) it is possible to show that the only torsion classes that are necessarily trivial are

$$\text{Im} E = \text{Re} W_1 = \text{Re} W_2 = 0. \tag{5.12}$$

Remarkably, this is a broader class than any of those in table 2; the requirements on M_6 are rather lax. We will find some simple explicit examples in later sections, but we expect that a lot more may emerge in the future with more effort.

The torsion forms (T_2, S) remain completely arbitrary, while the rest are fixed in terms of these, α, β and the functions appearing in the metric ansatz as

$$\begin{aligned}
 \text{Re} E &= -\frac{m h^2 (2 - \cos(2\beta)) \partial_y \log(e^A h)}{e^{3A} h' \sqrt{1 - e^{-4A} h^2}}, \quad \text{Re} V_2 = -\frac{\cos \beta}{(1 - e^{-4A} h^2)} d_6(e^{-2A} h), \\
 \text{Im} W_1 &= -\frac{2m}{3} \frac{\cos \beta h}{e^A h' \sqrt{1 - e^{-4A} h^2}} \partial_y \log(e^A h), \quad \text{Im} W_2 = -\frac{e^{2A}}{h \cos \beta} T_2, \quad W_3 = e^{-2A} h \cos \beta \text{Re} S, \\
 W_4 &= \frac{e^{-2A} h \sin^2 \beta}{(1 - e^{-4A} h^2)} d_6(e^{-2A} h), \quad \text{Re} W_5 = -\frac{e^{-2A} h}{4(1 - e^{-4A} h^2)} (e^{4A} h^{-2} + \cos(2\beta) - 2) d_6(e^{-2A} h),
 \end{aligned}
 \tag{5.13}$$

where the imaginary components of (V_2, W_5) are implied since these are holomorphic one-forms. From (2.12g) we extract the magnetic fluxes

$$\begin{aligned}
 f_1 &= -\frac{2c_1}{3h \tan \beta} d(\sqrt{1 - e^{-4A}h^2} \tan^2 \beta), & f_5 &= -\frac{c_1 \tan \beta h'}{e^{4A}\sqrt{1 - e^{-4A}h^2}} dy \wedge J_2^2, \\
 f_3 &= \frac{c_1 \sqrt{1 - e^{-4A}h^2}}{h \sin \beta \cos \beta} H_3 - \frac{c_1 m}{e^A h \cos \beta} \text{Re} \Omega_3 - \frac{2c_1 h'}{e^{4A}\sqrt{1 - e^{-4A}h^2}} dy \wedge J_2, & (5.14) \\
 f_7 &= \frac{c_1}{3e^{4A}\sqrt{1 - e^{-4A}h^2}} dy \wedge J_2^3.
 \end{aligned}$$

Some care should be taken with the expression for f_3 when $\sin \beta = 0$, as while $H_3 = 0$ this is not true of $(\sin \beta)^{-1}H_3$ (which is however finite). Again (2.12f) and (2.12h) are implied, so solving the supersymmetry constraints amounts to finding a manifold consistent with (5.12), (5.13) and a solution to the β constraint in (5.10); as the NS flux is already closed by definition there are no further conditions to be solved. We note that $\beta = 0$ is again a potentially interesting limit, where this time the flux reduces to that of a D1-D5 system. We shall in fact recover the standard D1-D5 near horizon from this class in section 6.1, while we present a class generalising this and some new solutions in section 6.4.2.

This concludes our classification of type II solutions preserving $\mathcal{N} = (1, 1)$ supersymmetry in terms of an $SU(3)$ -structure. In the next section we shall construct some more refined classes within what we have presented, and find some old and new solutions.

6 Solutions

The aim of this section is to present some new solutions within the $SU(3)$ -structure classes derived in sections 4 and 5. We provide a summary of the physical solutions we find in section 6.6.

Before presenting some new examples let us first perform a sanity check and recover something well known to preserve at least $\mathcal{N} = (1, 1)$ supersymmetry in the next section.

6.1 Recovering the D1-D5 near horizon

An obvious solution that should lie within the $SU(3)$ -structure classification of the preceding sections is the D1-D5 near horizon geometry, preserving $\mathcal{N} = (4, 4)$. We know that this has a metric of the form

$$e^{2A} = L^2, \quad ds^2(M_7) = e^{2C} ds^2(S^3) + \lambda^2 ds^2(CY_2), \quad (6.1)$$

with (e^C, L, λ) , and the dilaton set to a constant. Its only non trivial flux is the RR three-form, which has both an electric and magnetic component, so f_3, f_7 should be the only non trivial magnetic fluxes. As such this solution should lie within the class of section 5.2, specialised to

$$\beta = 0. \quad (6.2)$$

As $dA = 0$ for this solution, it is convenient to dispense with the local coordinate y : going back to (3.4), we can write

$$h = L^2 \cos \alpha, \quad V = \frac{L}{m} d\alpha. \quad (6.3)$$

This makes it clear that we should take α to be a coordinate on M_7 . The $SU(3)$ -structure equations we need to solve are now

$$\begin{aligned} dJ_2 &= \cos \alpha \operatorname{Re} S + \frac{m}{\sin \alpha L} \operatorname{Re} \Omega_3 + d\alpha \wedge \left(\frac{2}{3} \cot \alpha J_2 + \frac{L}{m} T_2 \right), \\ d\operatorname{Im} \Omega_3 &= -\frac{2m}{3L \sin \alpha} J_2 \wedge J_2 - \frac{1}{\cos \alpha} J_2 \wedge T_2 + d\alpha \wedge \left(\frac{L}{m} \operatorname{Im} S + \cot \alpha \operatorname{Im} \Omega_3 \right), \\ d\operatorname{Re} \Omega_3 &= d\alpha \wedge \left(\frac{L}{m} \operatorname{Re} S + \cot \alpha \operatorname{Re} \Omega_3 \right). \end{aligned} \quad (6.4)$$

Given that V is aligned along $d\alpha$, it is natural to assume that is part of the three-sphere, so we decompose it as a foliation of S^2 over an interval:

$$ds^2(S^3) = d\alpha^2 + \sin^2 \alpha ds^2(S^2), \quad e^{2C} = \frac{L^2}{m^2}. \quad (6.5)$$

We parameterise the two-sphere in terms of embedding coordinates (y_1, y_2, y_3) such that $(y_a)^2 = 1$, so that we have the $SU(2)$ singlets⁸

$$ds^2(S^2) = (dy_a)^2, \quad \operatorname{vol}(S^2) = \frac{1}{2} \epsilon_{abc} y_a dy_b \wedge dy_c \quad (6.6)$$

and the $SU(2)$ triplets

$$y_a, \quad dy_a, \quad k_a = \epsilon_{abc} y_b dy_c, \quad dk_a = 2y_a \operatorname{vol}(S^2). \quad (6.7)$$

As there is a CY_2 factor in the metric, we have three closed two-forms, (j_1, j_2, j_3) at our disposal obeying

$$j_a \wedge j_b = \frac{1}{2} \delta_{ab} \operatorname{vol}(CY_2). \quad (6.8)$$

A natural ansatz for the $SU(3)$ -structure forms, giving rise to (6.1) and obeying (3.14) is then

$$J_2 = \frac{L^2}{m^2} \sin^2 \alpha \operatorname{vol}(S^2) + \lambda^2 y_a j_a, \quad \Omega_3 = \frac{L\lambda^2}{m} (dy_a - ik_a) \wedge j_a. \quad (6.9)$$

Plugging this into (6.4), we find that it is solved by

$$S = 0, \quad T_2 = \frac{4L}{3m} \cos \alpha \sin \alpha \operatorname{vol}(S^2) - \frac{2m\lambda^2}{3L} \cot \alpha y_a j_a. \quad (6.10)$$

It is quick to show that T_2 is a primitive $(1, 1)$ -form, $T_2 \wedge \Omega_3 = T_2 \wedge J_2 \wedge J_2 = 0$, by introducing a complex vielbein on M_6 ; so the supersymmetry conditions are solved. Substituting the ansatz into (5.14) yields the fluxes

$$f_3 = \frac{2c_1}{m^2} \sin^2 \alpha d\alpha \wedge \operatorname{vol}(S^2), \quad f_7 = -\frac{2c_1 \lambda^4}{m^2} \sin^2 \alpha d\alpha \wedge \operatorname{vol}(S^2) \wedge \operatorname{vol}(CY_2) \quad (6.11)$$

with all else zero, so we have indeed reproduced the D1-D5 near horizon. The fact that this actually preserves $\mathcal{N} = (4, 4)$ can be ascertained from the form of (6.9). If we rotate y_a

⁸The D1-D5 near horizon supports small $\mathcal{N} = (4, 4)$, as such it should support two sets of spinors charged under either a left or right $SU(2)$ subgroup of the $SO(4)$ global isometry of S^3 . The specific $SU(2)$ referenced here is the diagonal one formed of both these left and right $SU(2)$ s.

in these expressions by a constant element of $SO(3)$, (J_2, Ω_3) change, but this new $SU(3)$ -structure still solves (6.4) (for a rotated T_2) and still gives rise to (6.11). One can extract a further 3 independent $SU(3)$ -structures in this fashion, for a total of 4 that the D1-D5 near horizon supports, so $\mathcal{N} = (1, 1) \rightarrow (4, 4)$.

Having performed a non trivial check of the geometric conditions derived earlier, let us now construct some new solutions. We shall begin by assuming M_6 is conformally CY_3 .

6.2 M_6 conformally Calabi-Yau

The easiest way to find a solution within the new classes presented in sections 4–5 is to impose that M_6 is (conformally) a Calabi-Yau three-fold (CY_3). This leads to four new local solutions, one for each class. In general we take the following ansatz for the metric

$$ds^2 = e^{2A} ds^2(\text{AdS}_3) + V^2 + e^{2C} ds^2(\text{CY}_3), \quad V = -\frac{1}{m\sqrt{1 - e^{-4A}h(y)^2}} e^{-A} dy h'(y) \quad (6.12)$$

where we allow e^{2C} to depend on (y, CY_3) and decompose the $SU(3)$ -structure forms as

$$J_2 = e^{2C} \hat{J}_2, \quad \Omega_3 = e^{3C} \hat{\Omega}_3, \quad d\hat{J}_2 = d\hat{\Omega}_3 = 0. \quad (6.13)$$

We begin by studying the possibilities of class I in section 4.1, then turn our attention to class III in section 5.1. CY_3 classes are also possible for class II and class IV, but they are special cases of more general warp nearly Kähler manifolds that we consider in section 6.3.

6.2.1 CY_3 in IIA from class I

Applying our conformal CY_3 ansatz to class I in section 4.1, we can find that all the torsion classes must vanish except for $\text{Re}E$ which enters the defining ODE for $C = C(y)$. Supersymmetry holds when the following ODEs are solved:

$$\partial_y \log\left(\frac{k}{h}\right) = \frac{3}{1 - e^{-4A}h^2} \partial_y \log h, \quad (1 - e^{-4A}h^2) \partial_y C + \partial_y \log(e^A h) = 0. \quad (6.14)$$

For this class the Bianchi identity of the NS three-form is not implied, so we must impose it by hand. Away from the loci of possible NS sources this gives rise to the additional ODE

$$\frac{me^{-A+3C} \partial_y \log(e^A h)}{\sqrt{1 - e^{-4A}h^2}} = c \partial_y \log h, \quad (6.15)$$

where c is constant. A simple solution to this system is found by simply setting $\partial_y C = 0$, allowing us to integrate the ODEs with ease. By redefining

$$e^{2A} = \frac{h}{\cos \alpha} \quad (6.16)$$

where $\alpha = \alpha(y)$, we find the solution

$$h = L^2 \cos^{\frac{1}{3}} \alpha, \quad k = L\lambda \frac{\cos^{\frac{4}{3}} \alpha}{\sin \alpha}, \quad e^{2C} = 1, \quad c = 0, \quad (6.17)$$

where (L, λ) are integration constants. These imply that $H_3 = 0$ and at this point everything is a function of $\alpha(y)$, so we promote it to a coordinate. This is all that needs to be solved, the solutions takes the form

$$\begin{aligned}
 ds^2 &= \frac{L^2}{\cos^{\frac{2}{3}} \alpha} \left[ds^2(\text{AdS}_3) + \frac{1}{9m^2} d\alpha^2 \right] + ds^2(\text{CY}_3), \quad e^{-\Phi} = \lambda \cos^{\frac{2}{3}} \alpha, \\
 f_2 &= \frac{m\lambda}{L} \hat{J}_2, \quad f_4 = -\frac{2\lambda}{3} \cos^{\frac{2}{3}} \alpha d\alpha \wedge \text{Im} \hat{\Omega}_3, \quad f_6 = -\frac{m\lambda}{3!L} \hat{J}_2^3.
 \end{aligned}
 \tag{6.18}$$

The solution is bounded between $0 < \alpha \leq \frac{\pi}{2}$ and exhibits the following singular behavior at the extrema:

$$ds^2 \sim L^2 \left[\frac{1}{x^2} ds^2(\text{AdS}_3) + \frac{x^2}{m^2} dx^2 \right] + ds^2(\text{CY}_3),
 \tag{6.19}$$

where $|\alpha \pm \pi/2| = x^3$. This can be understood as the singularity associated to an intersection of several O-planes extended in AdS_3 that wrap and are smeared over various cycles in CY_3 .

To see this, recall first that in general O-planes have a metric of the form

$$ds^2 = H^{-1/2} ds_{\parallel}^2 + H^{1/2} ds_{\perp}^2,
 \tag{6.20}$$

where H is a function of the transverse coordinates, harmonic with respect to ds_{\perp}^2 . We now consider the cycle \mathcal{C} in the CY_6 whose Poincaré dual is \hat{J}_2 , and wrap a smeared O6 along it. This may consist of multiple components: for example when $\text{CY}_3 = T^6$, $\mathcal{C} = \sum_{i=1}^3 \mathcal{C}_i$, where $\mathcal{C}_1 = T_{1234}^2$, $\mathcal{C}_2 = T_{1256}^2$, $\mathcal{C}_3 = T_{3456}^2$, and in that case we are smearing three O6-planes wrapped along all three. On top of this set up one can place an O2 hole smeared over the whole of CY_3 . For $\text{CY}_3 = T^6$, the resulting metric takes the form

$$\begin{aligned}
 ds^2 &= \frac{1}{\sqrt{H_2 H_6^1 H_6^2 H_6^3}} ds^2(\text{AdS}_3) + \sqrt{H_2 H_6^1 H_6^2 H_6^3} dx^2 \\
 &+ \sqrt{\frac{H_6^1 H_2}{H_6^2 H_6^3}} ds^2(\mathcal{C}_1) + \sqrt{\frac{H_6^2 H_2}{H_6^1 H_6^3}} ds^2(\mathcal{C}_2) + \sqrt{\frac{H_6^3 H_2}{H_6^1 H_6^2}} ds^2(\mathcal{C}_3),
 \end{aligned}
 \tag{6.21}$$

the way the smearing has been performed means the radial dependence of each warp factor is the same:

$$H_2 = H_6^1 = H_6^2 = H_6^3 = 1 - \frac{x}{x_0}.
 \tag{6.22}$$

Expanding about $x = x_0$ to leading order we reproduce the behavior of (6.19).

The metric singularity in (6.19) is reproduced by any arrangement such that every M_6 direction is parallel to two smeared O-planes and transverse to two other O-planes; one should then look at the fluxes to decide what the correct O-plane system is. For supersymmetry one should also make sure that the number of directions that are transverse to one plane and parallel to another is always a multiple of four. Of course determining this precisely is not that important, since (partially) smeared O-planes such as these are not sensible in string theory.

6.2.2 CY₃ in IIB from class III

When we apply the conformal CY₃ ansatz to section 5.1 we find again that $C = C(y)$, and that the supersymmetry and NS flux Bianchi identity require

$$\begin{aligned} \partial_y \log \left(\frac{e^{2A-\Phi}}{\sqrt{1-e^{-4A}h^2}} \right) &= 3 \frac{e^{-4A}hh'}{1-e^{-4A}h^2}, & 2(e^{4A}-2h^2)\partial_y C + e^{-\Phi}\partial_y(e^\Phi h^2) &= 0, \\ \frac{me^{3A+3C-\Phi}\sqrt{1-e^{-4A}h^2}}{2hh'(e^{4A}-2h^2)}\partial_y(h^2e^\Phi) &= c, \end{aligned} \tag{6.23}$$

for c a constant. We again find a closed form solution when $\partial_y C = 0$; it is rather similar to that of the previous section. Again substituting for e^{2A} in favor of α through $e^{2A} = \frac{h}{\cos \alpha}$ we find that

$$h = L^2 \cos^{\frac{1}{3}} \alpha, \quad e^{-\Phi} = \lambda \cos^{\frac{2}{3}} \alpha, \quad e^{2C} = 1, \quad c = 0, \tag{6.24}$$

solves all the ODEs, setting $H_3 = 0$ in the process. The solution takes the form

$$\begin{aligned} ds^2 &= \frac{L^2}{\cos^{\frac{2}{3}} \alpha} \left[ds^2(\text{AdS}_3) + \frac{1}{9m^2} d\alpha^2 \right] + ds^2(\text{CY}_3), & e^{-\Phi} &= \lambda \cos^{\frac{2}{3}} \alpha, \\ f_1 &= \frac{2}{3} \lambda \cos^{\frac{2}{3}} \alpha d\alpha, & f_3 &= -\frac{m\lambda}{L} \text{Re}\hat{\Omega}_3, & f_5 &= \frac{\lambda}{3} \cos^{\frac{2}{3}} \alpha d\alpha \wedge \hat{J}_2^2. \end{aligned} \tag{6.25}$$

We again find a solution bounded between $0 \leq \alpha < \frac{\pi}{2}$ with the same singular behavior at the boundaries as in (6.19). Clearly the interpretation must be a little different from (6.21), as this time as O6- and O4-planes are not BPS in IIB. Inspecting the flux components suggests that the singularity should this time come from four intersecting O5-planes extended in AdS₃ and wrapping four distinct three-cycles in CY₃ (for example, when CY₃ = T⁶, the four independent components of Im $\hat{\Omega}_3$). When such objects are smeared over the rest of CY₃ their warp factors are the same and reproduce the singularity.

As in IIA, the same metric singularity can be created by other arrangements of smeared O-planes, and one should examine the fluxes to determine the appropriate one; in future examples however we will refrain from doing so, because of the limited physical interest of smeared O-planes.

6.3 M₆ conformally nearly Kähler

In this section we shall consider solutions that are foliations containing a nearly-Kähler 6 manifold NK₆:

$$ds^2 = e^{2A} ds^2(\text{AdS}_3) + V^2 + e^{2C} ds^2(\hat{M}_6), \quad V = -\frac{1}{m\sqrt{1-e^{-4A}h(y)^2}} e^{-A} dy h'(y) \tag{6.26}$$

where the SU(3)-structure forms are taken to be

$$\begin{aligned} J_2 &= e^{2C} \hat{J}_2, & \Omega_3 &= e^{3C} \hat{\Omega}_3, \\ d\hat{J}_2 &= 3n \text{Im}\hat{\Omega}_3, & d\hat{\Omega}_3 &= -4n \hat{J}_2 \wedge \hat{J}_2, \end{aligned} \tag{6.27}$$

where n is a constant. This ansatz is appropriate for classes II and IV in sections 4.2 and 5.2. For a NK₆ manifold we can actually fix $n = 1$ without loss of generality, however

when $n = 0$ we have conditions for CY₃ solutions not considered in the previous section, so we shall keep n arbitrary for now.

Only a few NK₆ manifolds are known so far: S^6 , $S^3 \times S^3$, $\mathbb{C}\mathbb{P}^3$, and the so-called flag manifold $\mathbb{F}(1, 2, 3)$. The latter two are homogeneous, while the first two admit both homogeneous and cohomogeneity-one nearly-Kähler structures [58].

6.3.1 IIA solutions

Applying the ansatz of the previous section to the class of section 4.2 we find that solutions in IIA are governed by the following ODEs:

$$\begin{aligned} \partial_y \left(\frac{e^{2A+6C} h^2}{\cos^2 \beta} \right) &= \frac{2ne^{A-C}}{m \cos \beta} \partial_y \left(\frac{e^{2A+6C} h^2 \sqrt{1 - e^{-4A} h^2}}{\cos^2 \beta} \right), \\ \partial_y \left(\frac{e^{2A+6C} h^2 \sqrt{1 - e^{-4A} h^2}}{\cos^2 \beta} \right) &= -3 \frac{e^{-2A+6C} h^3 h'}{\cos^2 \beta \sqrt{1 - e^{-4A} h^2}}, \quad \partial_y \left(\frac{e^{6C} \sin \beta}{\cos^3 \beta} \right) = 0, \end{aligned} \tag{6.28}$$

where $C = C(y)$. These imply supersymmetry and the Bianchi identities of the fluxes.

CY₃ limit. First off, it is a simple matter to establish that when $n = 0$, the CY₃ limit, the ODEs truncate to

$$\partial_y(e^A h) = 0, \quad \partial_y C = \partial_y \beta = 0, \tag{6.29}$$

which can be solved as

$$h = L^2 \cos^{\frac{1}{3}} \alpha, \quad e^{2C} = 1, \quad \beta = \text{constant} \tag{6.30}$$

where we have again made use of $e^{2A} \cos \alpha = h$. This gives rise to the following analytic CY₃ solution

$$\begin{aligned} ds^2 &= \frac{L^2}{\cos^{\frac{2}{3}} \alpha} \left[ds^2(\text{AdS}_3) + \frac{1}{9m^2} d\alpha^2 \right] + ds^2(\text{CY}_3), & e^{-\Phi} &= \frac{c_1 L \cos^{\frac{2}{3}} \alpha}{\cos \beta}, \\ f_0 &= c_1 m, & f_2 &= c_1 m \tan \beta \hat{J}_2, & f_6 &= -\frac{c_1 m}{3!} \tan \beta \hat{J}_2^3, \\ f_4 &= -\frac{c_1 m}{2} \hat{J}_2 \wedge \hat{J}_2 - \frac{2c_1 L \cos^{\frac{2}{3}} \alpha}{3 \cos \beta} d\alpha \wedge \text{Im} \hat{\Omega}_3, \end{aligned} \tag{6.31}$$

We see that the solution is bounded between singularities of the type discussed below (6.21); this time it can be explained as the intersection of a D8/O8 system and three O4 planes that each wrap a distinct two-cycle in CY₃ and are smeared over the rest of it.

NK₆ solutions. When $n \neq 0$, M₆ is a nearly-Kähler manifold. We take $n = 1$, without loss of generality. In this case we have not been able to find any analytic solutions to (6.28). One can make progress by studying it in a power series expansion. We have found it easier to study the system after fixing the y coordinate reparameterization freedom by demanding $h' = \sqrt{1 - h^2 e^{-4A}}$.

For the solution to be compact, y needs to belong to an interval $[y_-, y_+]$, and one or more of the other directions should shrink at its endpoints y_{\pm} . If we take $e^{C(y_+)} = 0$, M₆

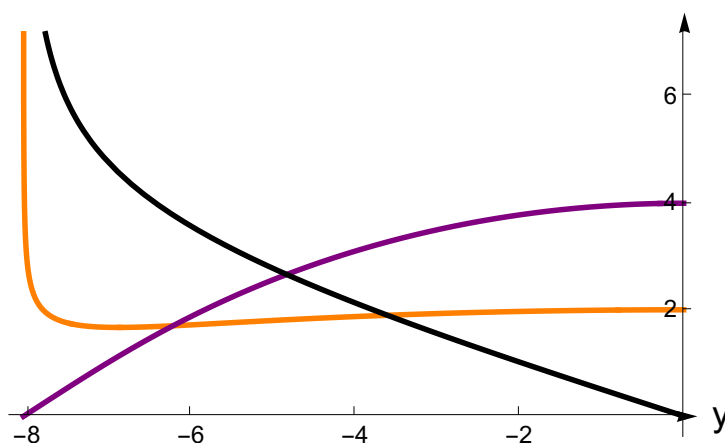


Figure 1. A numerical solution with M_6 nearly-Kähler, interpolating between an O8-plane on the left, and a regular (or conical G_2 singularity) on the right. The functions are e^A (orange), e^C (black), h (purple). The parameters in (6.32) in this case are $a_0 = 2$, $m = 1$.

shrinks; this is a regular point when $M_6 = S^6$, and is a conical G_2 singularity otherwise, which is believed to be allowed in string theory. By assuming a power series with integer coefficients for all the relevant functions, we find (for $y_+ = 0$)

$$e^A = a_0 - \frac{3}{49a_0^3}y^2 + O(y)^4, \quad e^C = -\frac{1}{ma_0}y - \frac{5}{98ma_0^5}y^3 + O(y)^5, \quad h = a_0^2 - \frac{3}{14a_0^2}y^2 + O(y)^4. \tag{6.32}$$

A standard procedure is to evaluate this at a small value of y , and evolve it numerically (towards negative y). This ends with a singularity, which upon further numerical inspection is revealed to be

$$ds^2 \sim (y - y_-)^{-1/2}ds^2(\text{AdS}_3) + (y - y_-)^{-1/2}ds^2(M_6) + (y - y_-)^{-1/2}dy^2 \quad [\text{O8}]. \tag{6.33}$$

Comparing with (6.20), we see that this can be straightforwardly identified as an O8-plane, since in that case H is linear.

We have also studied the system (6.28) numerically, by evolving from random initial conditions at values $y = y_0$ towards both smaller and larger y . The evolution continues on both sides until it stops at two types of singularities, which again can be further investigated by zooming around them. One type of solutions we obtained this way has an O8-plane (6.33) at the endpoint $y = y_-$, and

$$ds^2 \sim (y - y_+)^{-1/2}ds^2(\text{AdS}_3) + (y - y_+)^{1/2}ds^2(M_6) + (y - y_+)^{1/2}dy^2 \quad [\text{O2}] \tag{6.34}$$

at $y = y_+$. Recalling again (6.20), we see that this has the structure of an O2-plane: y is interpreted as the radial direction on the cone $C(M_6)$ over M_6 , and the harmonic function $H \sim 1 - (r_0/r)^5$, expanded around $r = r_0$, produces a linear $H \sim y$. This is an ordinary O2-plane when $M_6 = S^6$, and otherwise represents an O2 at the tip of a conical G_2 singularity.

A final, less meaningful type of numerical solutions has again an O8-plane at $y = y_-$, and

$$ds^2 \sim (y - y_0)^{-3/2} ds^2(\text{AdS}_3) + (y - y_0)^{1/2} ds^2(\text{M}_6) + (y - y_0)^{3/2} dy^2 \quad [3 \text{ smeared O4-planes}]. \quad (6.35)$$

This can be interpreted along similar lines to (6.21), as the result of three smeared O4-planes. Again all are parallel to AdS_3 and perpendicular to dy ; one extended along directions x^1, x^2 ; one along x^3, x^4 ; one along x^5, x^6 , in local coordinates on M_6 .

6.3.2 IIB solutions

Applying the NK_6 ansatz to the class of section 5.2 we find that solutions in IIB are in one to one correspondence with the following ODEs:

$$\begin{aligned} \partial_y \left(\frac{e^{6C}}{\cos^2 \beta} \right) &= \frac{2ne^{-A+C}}{m \cos \beta} \partial_y \left(\frac{e^{6C} \sqrt{1 - e^{-4A} h^2}}{\cos^2 \beta} \right), \\ \partial_y \left(\frac{e^{6C} \sqrt{1 - e^{-4A} h^2}}{\cos^2 \beta} \right) &= -3 \frac{e^{6C-4A} h h'}{\cos^2 \beta \sqrt{1 - e^{-4A} h^2}}, \quad \partial_y \left(\frac{e^{6C} \sin \beta}{\cos^3 \beta} \right) = 0, \end{aligned} \quad (6.36)$$

where again $C = C(y)$.

CY₃ limit. As in IIA, when we fix $n = 0$ we get CY_3 solutions, as before they are unwarped and an analogous calculations yields

$$\begin{aligned} ds^2 &= \frac{L^2}{\cos^{\frac{2}{3}} \alpha} \left[ds^2(\text{AdS}_3) + \frac{1}{9m^2} d\alpha^2 \right] + ds^2(\text{CY}_3), \quad e^{-\Phi} = \frac{c_1 \cos^{\frac{2}{3}} \alpha}{L^2 \cos \beta}, \\ f_1 &= -\frac{2c_1 \tan \beta}{3L^2} \cos^{\frac{2}{3}} \alpha d\alpha, \quad f_5 = \frac{c_1}{3L^2} \tan \beta \cos^{\frac{2}{3}} \alpha d\alpha \wedge \hat{J}_2^2, \quad f_7 = -\frac{c_1}{9L^2} \cos^{\frac{2}{3}} \alpha d\alpha \wedge \hat{J}_2^3, \\ f_3 &= \frac{c_1 m}{L^3 \cos \beta} \text{Re} \hat{\Omega}_3 + \frac{2c_1}{3L^2} \cos^{\frac{2}{3}} \alpha d\alpha \wedge \hat{J}_2. \end{aligned} \quad (6.37)$$

This time the singularity can be interpreted as that coming from four O5-planes that wrap four distinct 3-cycles in CY_3 and are smeared over the rest of it.

NK₆ solutions. In this case we have not found any analytic solutions. While it is possible to find a local regular (or conical G_2) solution similar to (6.32), its numerical evolution ends at points without a clear physical interpretation. The same issue presents itself when evolving from a random point internal to the y interval.

6.4 Foliations over $\text{Tw}(\text{M}_4)$

A natural generalisation of the D1-D5 near horizon $\text{SU}(3)$ -structure (6.9) is to gauge the $\text{SU}(2)$ of the two-sphere, such that our ansatz for the internal seven-manifold becomes

$$\begin{aligned} ds^2(\text{M}_7) &= V^2 + ds^2(\text{M}_6), \quad V = -\frac{1}{m \sqrt{1 - e^{-4A} h(y)^2}} e^{-A} dy h'(y) \\ ds^2(\text{M}_6) &= e^{2B} Dy_a Dy_a + e^{2C} ds^2(\text{M}_4), \quad Dy_a = dy_a + \epsilon_{abc} y_b A_c \end{aligned} \quad (6.38)$$

where we take (e^A, e^B, e^C) to be functions of y only. We take A_i to be the connection on the bundle of anti-self-dual forms on M_4 ; this makes M_6 the twistor bundle $\text{Tw}(M_4)$. A basis of two-forms j_i on M_4 then satisfies

$$dj_a = -\epsilon_{abc}A_b \wedge j_c. \tag{6.39}$$

We can introduce an $SU(3)$ -structure as

$$J_2 = \frac{1}{2}e^{2B}\epsilon_{abc}y_aDy_b \wedge Dy_c + e^{2C}y_a j_a, \quad \Omega_3 = e^{B+2C}(dy_a - i\epsilon_{abc}y_bDy_c) \wedge j_a. \tag{6.40}$$

We shall also assume that M_4 is Einstein and self-dual.⁹ This implies that the field strength $F_a = dA_a + \frac{1}{2}\epsilon_{abc}A_b \wedge A_c$ is proportional to j_a :¹⁰

$$F_a = bj_a, \quad db = 0, \tag{6.41}$$

so that we shall also recover conformal CY_2 solutions if we fix $b = 0$. More generically b is proportional to the constant curvature on M_4 , $b > 0$ is positive curvature, $b < 0$ negative. There are two cases where it makes sense to try this ansatz, namely the $\beta = \beta(y)$, $d_6\alpha = 0$ limit of cases II and IV.

6.4.1 IIA solutions

Plugging this into the torsion classes of section 4.2 we find

$$S = 0, \quad T_2 = \frac{2}{3}\cos\beta\frac{e^{2A+B}}{h}\left(e^{-2B} - 2be^{-2C}\right)\left(\frac{1}{2}e^{2B}\epsilon_{abc}y_aDy_b \wedge Dy_c - e^{2C}y_a j_a\right) \tag{6.42}$$

where T_2 is indeed a primitive (1,1)-form. We additionally get the following ODEs

$$\begin{aligned} \partial_y\left(\frac{e^{2B+2C}}{\cos^2\beta}\right) &= -\frac{2}{mh}\frac{e^{A+B+2C}h'}{\cos\beta\sqrt{1-e^{-4A}h^2}}, & \partial_y\left(\frac{e^{4C}}{\cos^2\beta}\right) &= 2b\partial_y\left(\frac{e^{2B+2C}}{\cos^2\beta}\right), \\ \partial_y(e^Ah) &= \frac{e^{2A}h'(e^{2C} + be^{2B})\sqrt{1-e^{-4A}h^2}}{me^{B+2C}\cos\beta}, & \partial_y\left(\frac{e^{2B+4C}\sin\beta}{\cos^3\beta}\right) &= 0, \end{aligned} \tag{6.43}$$

which imply supersymmetry and the EOM. When $\beta = 0$, the fluxes are those of a D4-D8 system with Romans mass $F_0 = c_1m$.

We did not find any analytic solutions to (6.43). As in 6.3, one can find local solutions with a power series expansion. In this case it is slightly easier to take the radial coordinate to be $y = h$ itself.

⁹There are only two smooth such manifolds, S^4 and $\mathbb{C}P^2$ [59], but many more with orbifold singularities [60].

¹⁰For more details on this set-up, see for example [61, section 9.2], or [62] in a different language for AdS₄ solutions. M_6 is half-flat more generally when M_4 is only self-dual and not Einstein, but in this case we have not been able to solve the remaining conditions on M_7 . It would be very interesting to find solutions with general M_4 ; we thank E. Witten for related discussions.

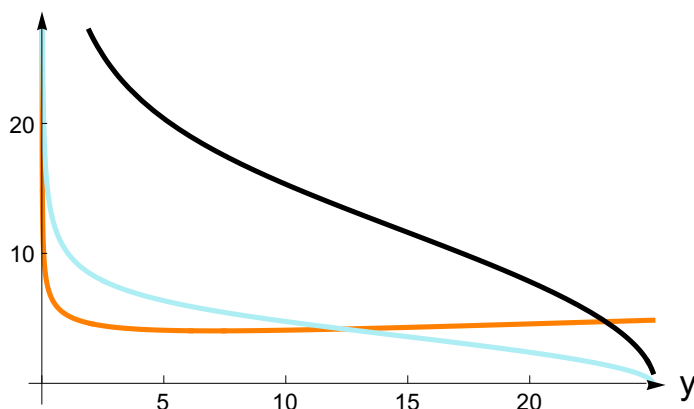


Figure 2. A numerical solution with $M_6 = \text{Tw}(M_4)$, interpolating between an O8-plane on the left, and a regular point on the right. The functions are e^A (orange), e^B (aqua), e^C (black). The parameters in (6.44) are $a_0 = 5$, $c_0 = 5$, $b = 1$, $m = 1$.

A regular endpoint y_+ of the y interval can now be obtained by making the S^2 shrink, which is achieved by imposing $e^B(y_+) = 0$. We find $y_+ = a_0^2$ and

$$\begin{aligned}
 e^A &= a_0 + O(a_0^2 - y)^2, & e^B &= \sqrt{2}m(a_0^2 - y)^{1/2} + \frac{7m^2c_0^2 - 8ba_0^2}{10\sqrt{2}m^3a_0^2c_0^2}(a_0^2 - y)^{3/2} + O(a_0^2 - y)^{5/2}, \\
 e^C &= c_0 + \frac{b}{c_0m^2}(a_0^2 - y) + O(a_0^2 - y)^2.
 \end{aligned}
 \tag{6.44}$$

A numerical evolution starting from this local solution ends with a singularity at $y = y_-$. For $b > 0$ this is again the O8-plane singularity (6.33), so this gives another class of solutions where an O8 is the only O-plane; we show an example in figure 2. For $M_4 = S^4$ and CP^2 , the $\text{Tw}(M_4)$ is CP^3 and a flag manifold respectively; both admit nearly-Kähler metrics. However, even these cases are distinct from the solutions in section 6.3, as we see by comparing the local solution (6.44) with (6.32). In particular, away from the O8 the present ones are fully regular for any M_4 , while those of section 6.3 are regular only when $M_6 = S^6$, and have a conical G_2 singularity otherwise.

For $b < 0$, a numerical evolution from (6.44) ends with the singularity at $y = y_-$:

$$ds^2 \sim (y - y_-)^{-1/2} \left(ds^2(\text{AdS}_3) + Dy^a Dy^a \right) + (y - y_-)^{1/2} (ds^2(M_4) + dy^2) \quad [(\text{smearred}) \text{O4}].
 \tag{6.45}$$

Comparing with (6.20), this would appear to have the structure of an O4, with $H \sim y$. At this point one might try to interpret y as the radial direction on the cone $C(M_4)$ over M_4 ; a harmonic function $h \sim 1 - (r_0/r)^3$, expanded around $r = r_0$, would produce a linear $h \sim y$. This cone $C(M_4)$ would be smooth for $M_4 = S^4$, but unfortunately this is excluded by the assumption $b < 0$. For other choices of four-manifolds, $C(M_4)$ is not a physically sensible singularity. Another possible interpretation is that y is not radial, but one of the transverse coordinates; (6.34) is then an O4 partially smeared along M_4 . This is unfortunately not

really sensible in string theory, although one may hope that it points to the existence of a solution where the O4 is fully localized.¹¹

Globally, we have been able to obtain numerical solutions where $y \in [y_-, y_+]$, that behave as O2-planes (6.34) on both sides, and others that behave as an O2 on one side and an O8 on the other.

6.4.2 IIB solutions

Plugging the twistor ansatz (6.38)–(6.41) into the torsion classes of section 5.2 we find

$$S = 0, \quad T_2 = \frac{2}{3} h e^{-2A+B} \cos \beta \left(e^{-2B} - 2b e^{-2C} \right) \left(\frac{1}{2} e^{2B} \epsilon_{abc} y_a D y_b \wedge D y_c - e^{2C} y_a j_a \right), \quad (6.46)$$

which differs from (6.42) only by a pre-factor. T_2 is again a primitive (1,1)-form, we also get the following ODEs

$$\begin{aligned} \partial_y \left(\frac{e^{2B+2C}}{\cos^2 \beta} \right) &= -\frac{2h}{m} \frac{e^{B+2C} h'}{e^{3A} \cos \beta \sqrt{1 - e^{-4A} h^2}}, & \partial_y \left(\frac{e^{4C}}{\cos^2 \beta} \right) &= 2b \partial_y \left(\frac{e^{2B+2C}}{\cos^2 \beta} \right), \\ \partial_y (e^A h) &= \frac{e^{2A} h' (b e^{2B} + e^{2C}) \sqrt{1 - e^{-4A} h^2}}{m e^{B+2C} \cos \beta}, & \partial_y \left(\frac{e^{2B+4C} \sin \beta}{\cos^3 \beta} \right) &= 0 \end{aligned} \quad (6.47)$$

which imply supersymmetry and all the EOM. Note that fixing $b = 0$ (so that the fibration becomes topologically trivial) gives a generalisation of the D1-D5 near horizon, additionally fixing $e^A = \text{constant}$. $\beta = 0$ reduces to it.

We applied to (6.47) the same procedure we saw in IIA. We did manage to find a regular local solution similar to (6.44), but numerical evolution from it now ends with a singularity of the type (6.19), with the CY_6 replaced by M_6 . Its possible interpretation in terms of smeared O-planes was discussed there and below (6.25).

Finally, numerical evolution from a random internal point results in solutions which have two solutions of the type (6.19) at both endpoints y_{\pm} .

6.5 S^3 fibered over Σ_3

In this section we shall consider solutions where M_6 is S^3 fibered over a 3-manifold of constant curvature. We shall take the metric to be

$$\begin{aligned} ds^2(M_7) &= V^2 + ds^2(M_6), & V &= -\frac{1}{m \sqrt{1 - e^{-4A} h(y)^2}} e^{-A} dy h'(y) \\ ds^2(M_6) &= e^{2B} (\mu^i)^2 + e^{2C} ds^2(\Sigma_3), & \mu_i &= \omega_i + A_i, \quad d\omega_i = \frac{1}{2} \epsilon_{ijk} \omega^j \wedge \omega^k, \end{aligned} \quad (6.48)$$

where (e^{2B}, e^{2C}) depend on y and the connection A_i is defined on Σ_3 , which we take to have constant curvature. We will impose that the $SU(2)$ field strength of the connection is

$$F_i = -\frac{R_0}{12} \epsilon_{ijk} e^j \wedge e^k \quad (6.49)$$

¹¹See [63, section 4.1.1] for a more extended discussion of the difference between genuine and smeared O-plane singularities.

where R_0 is the Ricci scalar on Σ_3 . We will take the SU(3)-structure forms to be

$$\begin{aligned} J_2 &= e^{B+C} \mu^i \wedge e^i, \\ \text{Re}\Omega_3 &= e^{3C} e^1 \wedge e^2 \wedge e^3 - \frac{1}{2} e^{2B+C} \epsilon_{ijk} \mu^i \wedge \mu^j \wedge e^k, \\ \text{Im}\Omega_3 &= e^{3B} \mu_1 \wedge \mu_2 \wedge \mu_3 - \frac{1}{2} e^{B+2C} \epsilon_{ijk} \mu^i \wedge e^j \wedge e^k. \end{aligned} \tag{6.50}$$

6.5.1 IIA solutions

Plugging the ansatz into for the SU(3)-structure of the previous section into the classification of section 4.2 we find that the non trivial torsion classes are fixed as

$$\begin{aligned} T_2 = 0, \quad S &= \frac{e^{-2A-2B-C} h (R_0 e^{2C} - 2e^{2B})}{8 \cos \beta} \left[-3e^{3C} e^1 \wedge e^2 \wedge e^3 - \frac{1}{2} e^{2B+C} \epsilon_{ijk} \mu^i \wedge \mu^j \wedge e^k \right. \\ &\quad \left. + i \left(3e^{3B} \mu_1 \wedge \mu_2 \wedge \mu_3 + \frac{1}{2} e^{B+2C} \epsilon_{ijk} \mu^i \wedge e^j \wedge e^k \right) \right] \end{aligned} \tag{6.51}$$

where S is indeed a primitive (2,1)-form. In addition to this we get the following ODEs for the functions of the ansatz:

$$\begin{aligned} \partial_y \left(\frac{e^{A+3C} h}{\cos \beta} \right) &= \frac{R_0}{2} \partial_y \left(\frac{e^{A+2B+C} h}{\cos \beta} \right), & \partial_y \left(\frac{e^{A+2B+C} h}{\cos \beta} \right) &= \frac{1}{m} \frac{e^{-2A+B+C} h^2 h'}{\cos^2 \beta \sqrt{1 - e^{-4A} h^2}}, \\ \partial_y \left(\frac{e^{2B+2C}}{\cos^2 \beta} \right) &= \frac{1}{6m} \frac{e^{A+B} (e^{2B} R_0 + 6e^{2C}) h'}{\cos \beta h \sqrt{1 - e^{-4A} h^2}}, & \partial_y \left(\frac{e^{3B+3C} \sin \beta}{\cos^3 \beta} \right) &= 0. \end{aligned} \tag{6.52}$$

These imply that

$$e^{3B+3C} \sin \beta = b_1 \cos^3 \beta, \quad e^{A+3C} h - \frac{R_0}{2} e^{A+2B+C} h = b_2 \cos \beta \tag{6.53}$$

where $b_{1,2}$ are arbitrary integration constants.

We find the following solution to these conditions:¹²

$$\begin{aligned} \beta &= 0, & R_0 &= -6, \\ e^{2B} &= \frac{4}{9m^2} e^{2A} (1 - e^{-4A} h^2), & e^{2C} &= \frac{4}{3m^2} e^{2A}, & h e^{4A} &= \frac{1}{2} (C + h^3), & dC &= 0. \end{aligned} \tag{6.54}$$

We can make more sense of the solution by fixing

$$C = 2^{\frac{3}{2}} L^6, \quad h = \sqrt{2} L^2 \cos^{\frac{2}{3}} y, \quad m c_1 = F_0. \tag{6.55}$$

The metric and dilaton then become

$$\begin{aligned} ds^2 &= \frac{L^2}{\cos^{\frac{1}{3}} y} \left[\sqrt{\Delta} \left(ds^2(\text{AdS}_3) + \frac{4}{3m^2} ds^2(\mathbb{H}_3) \right) + \frac{16}{9m^2 \sqrt{\Delta}} \left(\frac{1}{2} \Delta dy^2 + \frac{1}{4} \sin^2 y (\mu^i)^2 \right) \right], \\ e^{-\Phi} &= \frac{\sqrt{2} F_0 L \cos^{\frac{5}{6}} y}{m \Delta^{\frac{1}{4}}}, \quad \Delta = 1 + \cos^2 y. \end{aligned} \tag{6.56}$$

¹²Positivity of the metric fixes $R_0 < 0$, but it can then be set to -6 without loss of generality.

If we ignore the overall warping, the (y, μ_i) directions are topologically a 4-sphere with $0 < y < \pi$, however this gets restricted to the hemisphere $0 < y < \frac{\pi}{2}$ in the full space. One can easily see that $y = 0$ is regular zero while $y = \frac{\pi}{2}$ is a singular locus; this is in fact what one expects for D8 branes coincident to an O8 plane extended in all by the y direction. This suggests that we have derived a solution with a system of D4-D8 branes compactified on \mathbb{H}_3 (mod some discrete subgroup). In other words, this solution should be the uplift of the $\text{AdS}_3 \times \mathbb{H}_3$ vacua of Romans F(4) found in [57]; an uplift of this solution to IIB was also recently performed in [64]. The non trivial fluxes are the Romans mass F_0 and the 4-form which is purely magnetic, given by

$$F_4 = dC_3, \quad C_3 = \frac{8L^4 F_0}{27m^3} \cos^{\frac{4}{3}} y \left(\frac{1}{8} - \frac{1}{\Delta} \right) \mu^1 \wedge \mu^2 \wedge \mu^3 + \frac{3}{4} \epsilon_{ijk} e^i \wedge e^j \wedge \mu^k. \quad (6.57)$$

The analytic solution (6.56) is not the most general solution to the system (6.52). As we did in previous sections, we can look for more general solutions by imposing regularity at one of the endpoints, by demanding that the S^3 shrink there (just like $y = 0$ in (6.56)). This leads to a more general class, which we give in the gauge where $h' = \sqrt{1 - h^2 e^{-4A}}$:

$$\begin{aligned} e^A &= a_0 - \frac{3}{64a_0^3} y^2 + O(y)^4, & e^B &= \frac{1}{2a_0 m} y + \frac{9m^2 c_0^2 - 8a_0^2 R_0}{1152a_0^5 c_0^2 m^3} y^3 + O(y)^5 \\ e^C &= c_0 + \frac{8a_0^2 R_0 + 27c_0^2 m^2}{192a_0^4 c_0 m^2} y^2 + O(y)^4, & h &= a_0^2 - \frac{3}{8a_0^2} y^2 + O(y)^4. \end{aligned} \quad (6.58)$$

For $c_0 = \frac{2a_0}{\sqrt{3m}}$, this is an expansion of (6.54). A numerical evolution of this perturbative solution often results in an O8-plane singularity (6.33), just as in (6.56). So that solution has a two-parameter numerical generalization, also for $R_0 > 0$ ($\Sigma_3 = S^3$). We show one numerical example in figure 3.

For $R_0 < 0$, occasionally an evolution of (6.58) results instead in the singularity (6.35). As usual we also tried a numerical evolution from random values of the functions at an internal y . With this method, we found solutions that have (6.35) at y_+ and an O8 at y_- ; or solutions with (smeared) O2s on both sides.

6.5.2 IIB solutions

Applying the $S^3 \times \Sigma_3$ fibration ansatz to the IIB class of section 5.2 we find that the torsion classes are fixed in a similar (but not the same) fashion as for the IIA class, namely

$$\begin{aligned} T_2 = 0, \quad S &= \frac{e^{2A-2B-C} h^{-1} (R_0 e^{2C} - 2e^{2B})}{8 \cos \beta} \left[-3e^{3C} e^1 \wedge e^2 \wedge e^3 - \frac{1}{2} e^{2B+C} \epsilon_{ijk} \mu^i \wedge \mu^j \wedge e^k \right. \\ &\quad \left. + i \left(3e^{3B} \mu_1 \wedge \mu_2 \wedge \mu_3 + \frac{1}{2} e^{B+2C} \epsilon_{ijk} \mu^i \wedge e^j \wedge e^k \right) \right]. \end{aligned} \quad (6.59)$$

In addition to this we get the following ODEs for the functions of the ansatz:

$$\begin{aligned} \partial_y \left(\frac{e^{-A+3C} h^{-1}}{\cos \beta} \right) &= \frac{R_0}{2} \partial_y \left(\frac{e^{-A+2B+C} h^{-1}}{\cos \beta} \right), & \partial_y \left(\frac{e^{-A+2B+C} h^{-1}}{\cos \beta} \right) &= \frac{1}{m} \frac{e^{B+C} h'}{h^2 \cos^2 \beta \sqrt{1 - e^{-4A} h^2}}, \\ \partial_y \left(\frac{e^{2B+2C}}{\cos^2 \beta} \right) &= \frac{1}{6m} \frac{e^{-3A+B} (e^{2B} R_0 + 6e^{2C}) h h'}{\cos \beta \sqrt{1 - e^{-4A} h^2}}, & \partial_y \left(\frac{e^{3B+3C} \sin \beta}{\cos^3 \beta} \right) &= 0. \end{aligned} \quad (6.60)$$

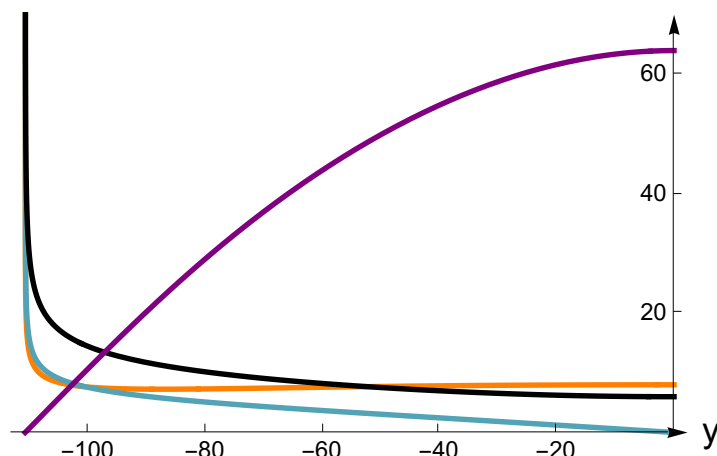


Figure 3. A numerical solution with M_6 an S^3 -fibration over a three-manifold Σ_3 , interpolating between an O8-plane on the left, and a regular point on the right. The functions are e^A (orange), e^B (aqua), e^C (black), h (purple). The parameters in (6.58) in this case are $a_0 = 8$, $c_0 = 4$, $R_0 = 6$, $m = 1$.

These imply that

$$e^{3B+3C} \sin \beta = b_1 \cos^3 \beta, \quad e^{3C} - \frac{R_0}{2} e^{2B+C} = b_2 e^A h \cos \beta \quad (6.61)$$

where $b_{1,2}$ are arbitrary integration constants.

We work in the gauge $h = y$. As in section 6.5.1, we have been able to find a local analytic solution where the S^3 shrinks smoothly at $y = a_0^2$:

$$\begin{aligned} e^A &= a_0 + \frac{1}{8a_0}(y - a_0^2) + O(a_0^2 - y)^2, & e^B &= -\frac{1}{m} \sqrt{\frac{2}{3}(a_0^2 - y)} + \frac{4R_0 a_0^2 - 8m^2 c_0^2}{108\sqrt{6}m^3} O(a_0^2 - y)^{3/2} \\ e^C &= c_0 + \frac{8a_0^2 R_0 - 27m^2 c_0^2}{72m^2 a_0^2 c_0} (a_0^2 - y) + O(a_0^2 - y)^2. \end{aligned} \quad (6.62)$$

This is almost identical to (6.58) at this level of approximation, but differs more markedly from it at higher orders. Evolving it numerically, the solution stops at a singularity with local metric

$$ds^2 \sim (y - y_0)^{-1/2} \left(ds^2(\text{AdS}_3) + ds^2(S^3) \right) + (y - y_0)^{1/2} (ds^2(\Sigma_3) + dy^2) \quad [(\text{smearred}) \text{ O5}]. \quad (6.63)$$

When Σ_3 is hyperbolic, we encounter the same problem we described below (6.45): the cone $C(\Sigma_3)$ is not physically sensible, so we cannot consider (6.63) as an O5 placed at its tip. Rather, we need to fall back on the alternative interpretation of an O5 smeared along Σ . But when $\Sigma = S^3$, (6.63) should represent a genuine O5-plane singularity, without any smearing. Both cases are possible numerically.

So in particular in this class we are able to find solutions whose only singularity consists of a fully localized O5-plane. We show an example in figure 4.

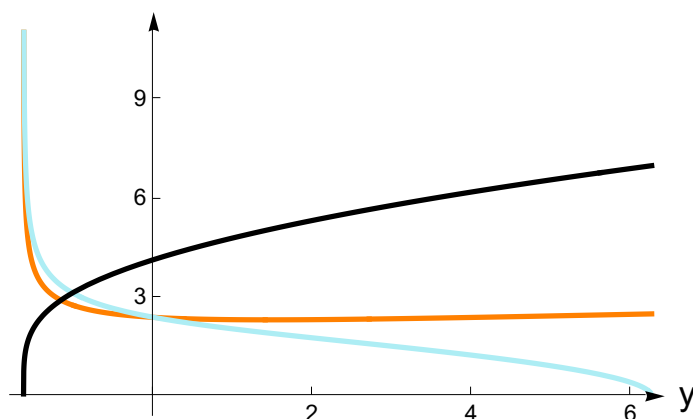


Figure 4. A numerical solution with M_6 an S^3 -fibration over a three-manifold Σ_3 , interpolating between an O5-plane on the left, and a regular point on the right. The functions are e^A (orange), e^B (aqua), e^C (black). The parameters in (6.62) in this case are $a_0 = 2.5$, $c_0 = 7$, $R_0 = 6$, $m = -1$.

A less interesting type of numerical solution we found has (6.63) at $y = y_-$, and the singularity with four smeared O p -planes (6.21) at $y = y_+$.

6.6 Summary of physical solutions

We found several types of concrete solutions in this section. We list here only those that have no smeared O-planes.

- In IIA, the solution in figure 1 has M_7 = a fibration of a nearly Kähler manifold M_6 over an interval $[y_-, y_+]$; it has an O8-plane at $y = y_-$, and a conical G_2 -singularity at $y = y_+$. When $M_6 = S^6$, the latter is a regular point.
- In IIA, the solution in figure 2 has now an $M_6 = S^2$ fibration over an Einstein self-dual M_4 , again all fibred over an interval. As in the previous case, there is an O8 at $y = y_-$, but $y = y_+$ is always fully regular.
- In IIA, for the analytic solution (6.56) the internal space is an S^3 fibration over a maximally symmetric Σ_3 , all fibred over an interval. Again there is an O8 at $y = y_-$, and a regular point at $y = y_+$. This is interpreted as a compactification on Σ_3 of the AdS₆ solution in [65]. However, we also found a two-parameter numerical generalization, shown in figure 3.
- In IIB, again with $M_6 =$ an S^3 -fibration over Σ_3 , fibred over an interval, the solution in figure 4 has an O5 at $y = y_-$, and is regular at $y = y_+$.

We should stress again that our search was far from exhaustive. We picked only some simple possibilities for M_6 ; it is very likely that many other solutions exist. Even for the ansätze we did try, it would have been possible to look for more general solutions, for example including D8-branes and/or O8-planes with a finite dilaton (as opposed to (6.33)),

where it diverges as $e^\phi \sim |y - y_-|^{-5/4}$). On the other hand, a more complete treatment would now be required to make sure that the supergravity approximation is under control, and that flux quantization can be imposed. For the types of solutions under consideration in this paper, both of these are expected to work easily, in part because of various rescaling symmetries of the supergravity equations of motion. We do not carry out this explicitly in this paper, but it would be needed to analyze the correspondence with CFT_2 's.

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A AdS₃ bi-linears

In this appendix we give details of the bi-linears following from the two Killing spinors on AdS₃ ζ_\pm charged under the $\text{SL}(2)_\pm$ subgroup of $\text{SO}(2,2)$. These solve the Killing spinor equation as

$$\nabla_\mu \zeta_\pm = \pm \frac{m}{2} \gamma_\mu \zeta_\pm \tag{A.1}$$

and we choose real gamma matrices, such that ζ_\pm can also be taken to be real we also take them to be unit norm without loss of generality. Using the Killing spinor equation, or via direct computation with respect to specific gamma matrices and specific solutions to the Killing spinor equation, one can establish that

$$\zeta_\pm \otimes \bar{\zeta}_\pm = \frac{1}{2} (v^\mp \mp f^{-1} u \wedge v^\mp), \quad \zeta_\pm \otimes \bar{\zeta}_\mp = \frac{1}{2} (\pm f - u + \frac{1}{2} f^{-1} v^+ \wedge v^- \mp f \text{vol}(\text{AdS}_3)) \tag{A.2}$$

where (v^\pm, u) are one-forms obeying the conditions

$$\begin{aligned} dv^\pm &= 2mf^{-1}v^\pm \wedge u, & df &= -mu, \\ u \lrcorner \text{vol}(\text{AdS}_3) &= \frac{1}{2} f^{-1} v^+ \wedge v^-, & v^\pm \lrcorner \text{vol}(\text{AdS}_3) &= \pm f^{-1} v^\pm \wedge u. \end{aligned} \tag{A.3}$$

(The \pm is just a label.) Further one can show that

$$\nabla_{(\mu} v_{\nu)}^\pm = 0, \quad \nabla_{(\mu} u_{\nu)} = -mf g(\text{AdS}_3)_{\mu\nu}, \tag{A.4}$$

so that $(v^\pm)^\mu \partial_\mu$ are both Killing vectors while $(u)^\mu \partial_\mu$ is only a conformal Killing vector. They obey the following interior product relations

$$v^\pm \lrcorner v^\pm = v^\pm \lrcorner u = 0, \quad v^\pm \lrcorner v^\mp = -2u \lrcorner u = -2f^2, \tag{A.5}$$

so v^\pm are null and orthogonal to u , such that any linear combination of v^\pm with positive coefficients is strictly time-like. This will eventually lead to a time-like $d = 10$ Killing vector in the next section.

A particular parameterisation of AdS₃ is

$$ds^2(\text{AdS}_3) = e^{2mr}(-dt^2 + dx^2) + dr^2. \quad (\text{A.6})$$

In terms of this, the Killing spinors that are Poincaré invariant in (t, x) are

$$\zeta_+ = e^{\frac{m}{2}r} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \zeta_- = e^{\frac{m}{2}r} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad (\text{A.7})$$

when we take $\gamma_\mu = (i\sigma_2, \sigma_1, \sigma_3)_\mu$. One then has

$$f = e^{mr}, \quad v^\pm = e^{2mr}(dt \pm dx), \quad u = -e^{mr} dr. \quad (\text{A.8})$$

Using a specific parameterisation such as this is probably the easiest way to derive the identities in this section.

B General $\mathcal{N} = (1, 0)$ conditions

In this appendix, for the first time, we present all the $\mathcal{N} = (1, 0)$ conditions for AdS₃ without assumption. We make use of an existing classification of generic supersymmetric solutions in type II supergravities presented in [66].

The $d = 10$ Majorana-Weyl Killing spinors for $\mathcal{N} = (1, 0)$ supersymmetric AdS₃ decompose as

$$\epsilon_1 = \zeta_+ \otimes \theta_+ \otimes \chi^1, \quad \epsilon_2 = \zeta_+ \otimes \theta_\mp \otimes \chi^2 \quad (\text{B.1})$$

where \pm in θ_\pm is labeling 10d chirality while on ζ_+ (the AdS₃ Killing spinor defined in appendix A) it labels the $\text{SL}(2)_+$ subgroup of $\text{SO}(2, 2) = \text{SL}(2)_+ \times \text{SL}(2)_-$. The bosonic fields decompose as in (2.1) so as to respect the $\text{SO}(2, 2)$ isometry of AdS₃. Following [66] one can define the following one-form bi-linears

$$K = \frac{1}{64}(\bar{\epsilon}_1 \Gamma_M \epsilon_1 + \bar{\epsilon}_2 \Gamma_M \epsilon_2) dX^M, \quad \tilde{K} = \frac{1}{64}(\bar{\epsilon}_1 \Gamma_M \epsilon_1 - \bar{\epsilon}_2 \Gamma_M \epsilon_2) dX^M \quad (\text{B.2})$$

where K^M are necessarily the components of a 10 dimensional Killing vector that is either time-like or null. Decomposing the 10 dimensional gamma matrices as

$$\Gamma_\mu = e^A \gamma_\mu \otimes \sigma_3 \otimes \mathbb{I}, \quad \Gamma_a = \mathbb{I} \otimes \sigma_1 \otimes \gamma_a, \quad i\gamma_{1234567} = 1 \quad (\text{B.3})$$

implies that the 10d intertwiner defining Majorana conjugation as $\epsilon^c = B^{(10)}\epsilon^*$ and chirality matrix $\hat{\Gamma}$ are respectively

$$B = \mathbb{I} \otimes \sigma_3 \otimes B, \quad \hat{\Gamma} = \mathbb{I} \otimes \sigma_2 \otimes \mathbb{I}, \quad (\text{B.4})$$

where $B\gamma_a B^{-1} = -\gamma_a^*$, $BB^* = \mathbb{I}$. As such we should take

$$\theta_+ = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}, \quad \theta_- = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}, \quad (\text{B.5})$$

ζ_+ to be real and $(\chi^{1,2})^c = B(\chi_{\pm}^{1,2})^* = (\chi^{1,2})$ so that $\epsilon_{1,2}$ are Majorana-Weyl as required. Using this we can now refine (B.2) as

$$K = \frac{e^A}{64}(|\chi_-^1|^2 + |\chi_-^2|^2)v^+, \quad \tilde{K} = \frac{e^A}{64}(|\chi_-^1|^2 - |\chi_-^2|^2)v^+, \quad (\text{B.6})$$

making K null. The first conditions for supersymmetry are that $K^M \partial_M$ should be Killing, and that NS the three-form H and the one-form \tilde{K}_M obey

$$d\tilde{K} = K \lrcorner H. \quad (\text{B.7})$$

These impose that

$$|\chi^1|^2 \pm |\chi^2|^2 = c_{\pm} e^{\pm A}, \quad c_+ e^{3A} h_0 = -2mc_- \quad (\text{B.8})$$

where c_+ , c_- are constants, the former strictly positive. Another set of necessary conditions is given by

$$d_H(e^{-\Phi} \Psi^{(10)}) = -(K \lrcorner + \tilde{K} \wedge) F_{10}, \quad \Psi^{(10)} = \epsilon_1 \otimes \bar{\epsilon}_2. \quad (\text{B.9})$$

It is not hard to show that the bi-linear is given by

$$4\Psi^{(10)} = \mp e^A v^- \wedge \Psi_{\mp} - f^{-1} e^{2A} u \wedge v^- \wedge \Psi_{\pm}, \quad \chi_1 \otimes \chi_2^{\dagger} = \Psi_+ + i\Psi_-, \quad (\text{B.10})$$

while the term involving the flux becomes

$$(K \lrcorner + \tilde{K} \wedge) F_{10} = \frac{1}{32} \left[\frac{c_-}{2} v^- \wedge f_{\pm} - \frac{c_+}{2} f^{-1} e^{3A} v^- \wedge u \wedge \star_7 \lambda(f_{\pm}) \right]. \quad (\text{B.11})$$

Given this, the 10d bi-linear constraints reduce to the 7d ones

$$d_{H_3}(e^{A-\Phi} \Psi_{\mp}) \pm \frac{c_-}{16} f_{\pm} = 0, \quad d_{H_3}(e^{2A-\Phi} \Psi_{\pm}) \mp 2me^{A-\Phi} \Psi_{\mp} = \frac{c_+}{16} e^{3A} \star_7 \lambda(f_{\pm}). \quad (\text{B.12})$$

The final necessary conditions are given by (3.1c) and (3.1d) of [66]. Dealing with these is a much more lengthy computation, so from here we shall only sketch the derivations giving some key intermediate results. Given the AdS₃ ansatz, (3.1c) for instance simplifies considerably, reducing to

$$\left(e_{1+} \Psi^{(10)} e_{2+}, \Gamma^{MN} \left[dA \wedge e_{2+} \Psi^{(10)} - \frac{1}{2} \left(e^{-A} m + \frac{1}{2} h_0 \right) P e_{2+} \Psi^{(10)} - 2e^{\Phi} f_{\pm} \right] \right) = 0 \quad (\text{B.13})$$

where

$$e_{a+} = -\frac{8e^A}{f^2 |\chi_a|^2} v^+, \quad P = \mathbb{I} \otimes \sigma_3 \otimes \mathbb{I}, \quad (\text{B.14})$$

and the bracket is $(\alpha, \beta) \equiv (\alpha \wedge \lambda(\beta))_{10}$. In principle (B.13) could give 3 independent conditions for (M, N) , i.e. those aligned along AdS₃, M₇ or mixed directions respectively. However, due to $v^+ \cdot v^+ = 0$, when $(M, N) = (a, b)$ this condition is trivial. With some effort one can show that $(M, N) = (\mu, a)$ is equivalent to

$$\left(\Psi_{\pm}, \gamma_a \left(e^{-\Phi} dA \wedge \Psi_{\mp} \pm \frac{|\chi_2|^2}{8} f_{\pm} \right) \right)_7 = 0, \quad (\text{B.15})$$

where the bracket is now $(X, Y)_7 = X \wedge \lambda(Y)|_7$. It is then possible to show that this is implied by (B.12) using properties of the pairing. Thus the only new conditions follow from the AdS₃ directions — one can show that this holds true for (3.1d) of [66] also. It turns that (3.1c) and (3.1d) of [66] yield conditions depending on $|\chi_2|^2$ and $|\chi_1|^2$ respectively, which given (B.8) are not actually independent; they are equivalent to a single condition

$$(\Psi_{\mp}, f_{\pm})_7 = \mp \frac{c_+ m + \frac{1}{2} e^{-A} c_- h_0}{4} e^{-\Phi} \text{vol}(M_7). \quad (\text{B.16})$$

In summary, in conventions where $i\gamma_{1234567} = 1$, the necessary conditions for $\mathcal{N} = (1, 0)$ supersymmetry in full generality are

$$d_{H_3}(e^{A-\Phi} \Psi_{\mp})_{\pm} \pm \frac{c_-}{16} f_{\pm} = 0, \quad d_{H_3}(e^{2A-\Phi} \Psi_{\pm}) \mp 2m e^{A-\Phi} \Psi_{\mp} = \frac{c_+}{16} e^{3A} \star_7 \lambda(f_{\pm}), \quad (\text{B.17})$$

$$(\Psi_{\mp}, f_{\pm})_7 = \mp \frac{c_+ m + \frac{1}{2} e^{-A} c_- h_0}{4} e^{-\Phi} \text{vol}(M_7), \quad |\chi^1|^2 \pm |\chi^2|^2 = c_{\pm} e^{\pm A}, \quad c_+ e^{3A} h_0 = -2m c_-,$$

where one can in fact fix c_+ to any value one chooses without loss of generality. Notice that for $(c_+ = 2, c_- = 0)$ we reproduce the conditions presented in [48] which shares our conventions. The conditions for $\mathcal{N} = (0, 1)$ are quite similar, they are given by sending $m \rightarrow -m$ everywhere it appears in the above expressions.

C Derivation of supersymmetry conditions for $\mathcal{N} = (1, 1)$ AdS₃

In this appendix we give a detailed derivation of the supersymmetry conditions for $\mathcal{N} = (1, 1)$ AdS₃ summarised in section 2. We again make use of [66] and use the same conventions as the preceding appendices.

The $d = 10$ Majorana-Weyl Killing spinors for $\mathcal{N} = (1, 1)$ supersymmetric AdS₃ decompose as

$$\epsilon_1 = \zeta_+ \otimes \theta_+ \otimes \chi_+^1 + \zeta_- \otimes \theta_+ \otimes \chi_-^1, \quad \epsilon_2 = \zeta_+ \otimes \theta_{\mp} \otimes \chi_+^2 + \zeta_- \otimes \theta_{\mp} \otimes \chi_-^2 \quad (\text{C.1})$$

where \pm in θ_{\pm} is labeling 10d chirality (so the upper/lower signs should be taken in IIA/IIB), while on ζ_{\pm} and $\chi_{\pm}^{1,2}$ it is just a label. Specifically ζ_{\pm} are two independent AdS₃ Killing spinors defined in appendix A. Plugging these spinors into (B.2) we find the 10d 1-forms

$$K = \frac{1}{32} \left(\frac{e^A}{2} (|\chi_-^1|^2 + |\chi_-^2|^2) v^+ + \frac{e^A}{2} (|\chi_+^1|^2 + |\chi_+^2|^2) v^- - e^A (\chi_-^{1\dagger} \chi_+^1 + \chi_-^{2\dagger} \chi_+^2) u - f \xi \right),$$

$$\tilde{K} = \frac{1}{32} \left(\frac{e^A}{2} (|\chi_-^1|^2 - |\chi_-^2|^2) v^+ + \frac{e^A}{2} (|\chi_+^1|^2 - |\chi_+^2|^2) v^- - e^A (\chi_-^{1\dagger} \chi_+^1 - \chi_-^{2\dagger} \chi_+^2) u - f \tilde{\xi} \right) \quad (\text{C.2})$$

where

$$\xi_a = -i(\chi_+^{1\dagger} \gamma_a \chi_-^1 \mp \chi_+^{2\dagger} \gamma_a \chi_-^2), \quad \tilde{\xi}_a = -i(\chi_+^{1\dagger} \gamma_a \chi_-^1 \pm \chi_+^{2\dagger} \gamma_a \chi_-^2). \quad (\text{C.3})$$

The fact that $K^M \partial_M$ should be a 10d Killing vector imposes the following 7d conditions

$$\nabla_{(a} \xi_{b)} = 0, \quad (\text{C.4a})$$

$$\mathcal{L}_{\xi} A - m e^{-A} (\chi_-^{1\dagger} \chi_+^1 + \chi_-^{2\dagger} \chi_+^2) = 0, \quad (\text{C.4b})$$

$$d(e^{-A} (\chi_-^{1\dagger} \chi_+^1 + \chi_-^{2\dagger} \chi_+^2)) - m e^{-2A} \xi = 0, \quad (\text{C.4c})$$

$$d(e^{-A} (|\chi_+^1|^2 + |\chi_+^2|^2)) = d(e^{-A} (|\chi_-^1|^2 + |\chi_-^2|^2)) = 0, \quad (\text{C.4d})$$

from which it is clear that for a generic solution $\xi^a \partial_a$ is a Killing vector of the internal space but not the warp factor — unless we have $\xi = 0$. Another necessary condition is that the NS flux obeys (B.7) which leads to

$$\begin{aligned}
 d\tilde{\xi} &= \xi \lrcorner H_3, \\
 h_0(\chi_-^{1\dagger} \chi_+^1 + \chi_-^{2\dagger} \chi_+^2) &= 0, \\
 d(e^A(\chi_-^{1\dagger} \chi_+^1 - \chi_-^{2\dagger} \chi_+^2)) + m\tilde{\xi} &= 0, \quad (\text{C.5}) \\
 d(e^A(|\chi_+^1|^2 - |\chi_+^2|^2)) &= d(e^A(|\chi_-^1|^2 - |\chi_-^2|^2)) = 0, \\
 2m(|\chi_-^1|^2 - |\chi_-^2|^2) - e^A h_0(|\chi_-^1|^2 + |\chi_-^2|^2) &= 2m(|\chi_+^1|^2 - |\chi_+^2|^2) + e^A h(|\chi_+^1|^2 + |\chi_+^2|^2) = 0.
 \end{aligned}$$

So we must fix either

$$h_0 = 0, \quad \text{or} \quad \chi_-^{1\dagger} \chi_+^1 + \chi_-^{2\dagger} \chi_+^2 = 0, \quad (\text{C.6})$$

or both. When $\xi \neq 0$, (C.4c) implies we should fix $h_0 = 0$. In fact it is fairly easy to show that in this case the AdS₃ factor we are assuming gets enhanced to AdS₄ in both the metric and the fluxes (at least locally). To see this one can solve (C.4c) locally as

$$e^{-A}(\chi_-^{1\dagger} \chi_+^1 + \chi_-^{2\dagger} \chi_+^2) = \rho(r), \quad m\xi = e^{2A} \rho' dr. \quad (\text{C.7})$$

r now plays the role of a local coordinate and ρ' parameterises diffeomorphism invariance in this direction. We can implicitly fix this invariance by taking

$$\xi^a \partial_a = \partial_r \quad \Rightarrow \quad \|\xi\|^2 = \frac{1}{m} e^{2A} \rho'. \quad (\text{C.8})$$

ξ and the vectors orthogonal to it define two distributions, which are integrable by (C.4c); in other words ξ is hypersurface orthogonal. There thus exist coordinates such

$$ds^2(\text{M}_7) = (e^\xi)^2 + ds^2(\text{M}_6), \quad e^\xi = \frac{\xi}{\|\xi\|} = e^A \sqrt{\frac{\rho'}{m}} dr. \quad (\text{C.9})$$

Now ξ being a Killing vector implies

$$e^A = \sqrt{\frac{m}{\rho'}} e^{A_4}, \quad \partial_r A_4 = 0. \quad (\text{C.10})$$

Plugging this into (C.4b) then leads without loss of generality to

$$\rho = \frac{1}{m} \tanh r, \quad (\text{C.11})$$

and the 10d metric becomes

$$ds^2 = e^{2A_4} \left[m^2 \cosh^2 r ds^2(\text{AdS}_3) + dr^2 \right] + ds^2(\text{M}_6) \quad (\text{C.12})$$

which is warped AdS₄, not warped AdS₃. This pattern persists with the fluxes as well, at least away from the loci of possible sources, so in regular regions of a solution

$$\xi \neq 0 \quad \Rightarrow \quad \text{AdS}_3 \rightarrow \text{AdS}_4, \quad (\text{C.13})$$

so at best solutions with $\xi \neq 0$ can generalise AdS₄ to cases with sources placed along the AdS radial direction, and indeed we have not established that even this is necessarily possible.

Thus for AdS₃ vacua we can fix $\xi = 0$ without loss of generality. This truncates the conditions derived thus far to:

$$\xi = 0, \tag{C.14a}$$

$$\chi_-^{1\dagger}\chi_+^1 + \chi_-^{2\dagger}\chi_+^2 = 0, \tag{C.14b}$$

$$d(e^A(\chi_-^{1\dagger}\chi_+^1 - \chi_-^{2\dagger}\chi_+^2)) + m\tilde{\xi} = 0, \tag{C.14c}$$

$$|\chi_+^1|^2 \pm |\chi_+^2|^2 = e^{\pm A}c_{\pm}^{\pm}, \quad |\chi_-^1|^2 \pm |\chi_-^2|^2 = e^{\pm A}c_{\pm}^{\mp}, \tag{C.14d}$$

$$2mc_+^{\pm} + e^{3A}h_0c_+^{\pm} = 0, \quad 2mc_-^{\pm} - e^{3A}h_0c_-^{\pm} = 0, \tag{C.14e}$$

which imply $c_-^+c_+^- + c_-^-c_+^+ = 0$. With these restrictions K becomes necessarily time-like for $\mathcal{N} = (1, 1)$. It also follows that

$$\chi_-^{1\dagger}\chi_+^1 - \chi_-^{2\dagger}\chi_+^2 \neq 0, \tag{C.15}$$

or rather more specifically that $\chi_-^{1\dagger}\chi_+^1 - \chi_-^{2\dagger}\chi_+^2$ cannot be set to zero everywhere. Indeed from (C.14a), (C.14b), (C.14c) and the definitions of $(\xi, \tilde{\xi})$ it follows that

$$\chi_-^{1\dagger}\chi_+^1 - \chi_-^{2\dagger}\chi_+^2 = 0 \quad \Rightarrow \quad \chi_-^{1\dagger}\chi_+^1 = \chi_-^{2\dagger}\chi_+^2 = \chi_-^{1\dagger}\gamma_a\chi_+^1 = \chi_-^{2\dagger}\gamma_a\chi_+^2 = 0, \tag{C.16}$$

where $\{\chi, \gamma_a\chi\}$ is a basis for the space of spinors, for any χ . Applying this to (C.16) implies that one of χ_{\pm}^1 would need to be zero, and similarly for χ_{\pm}^2 . So when $\chi_-^{1\dagger}\chi_+^1 - \chi_-^{2\dagger}\chi_+^2 = 0$ everywhere only $\mathcal{N} = 1$ supersymmetry can be realised.

We now turn our attention to the conditions (B.9): one can show the 10d bi-linear decompose as

$$\begin{aligned} 4\Psi^{(10)} = & f(\Psi_{\pm}^{+-} - \Psi_{\pm}^{-+}) \mp e^A(v^- \wedge \Psi_{\mp}^{++} + v^+ \wedge \Psi_{\mp}^{--} - u \wedge (\Psi_{\mp}^{+-} + \Psi_{\mp}^{-+})) \\ & - f^{-1}e^{2A}u \wedge (v^- \wedge \Psi_{\pm}^{++} - v^+ \wedge \Psi_{\pm}^{--}) \\ & + \frac{1}{2}f^{-1}e^{2A}v^+ \wedge v^- \wedge (\Psi_{\pm}^{+-} + \Psi_{\pm}^{-+}) \pm fe^{3A}\text{vol}(\text{AdS}_3) \wedge (\Psi_{\mp}^{+-} - \Psi_{\mp}^{-+}), \end{aligned} \tag{C.17}$$

where we defined

$$\Psi_{\pm}^{st} + i\Psi_{\mp}^{st} = \chi_s^1 \otimes \chi_t^{2\dagger}, \quad s, t = \pm. \tag{C.18}$$

Likewise one can show that

$$\begin{aligned} (K_{\lrcorner} + \tilde{K} \wedge)F_{10} = & \frac{1}{32} \left[fe^{3A}\text{vol}(\text{AdS}_3) \wedge \tilde{\xi} \wedge \star_7 \lambda(f_{\pm}) - f\tilde{\xi} \wedge f_{\pm} + \frac{1}{2}(c_-^-v^+ + c_-^+v^-) \wedge f_{\pm} \right. \\ & \left. - e^A(\chi_-^{1\dagger}\chi_+^1 - \chi_-^{2\dagger}\chi_+^2)u \wedge f_{\pm} + \frac{1}{2}f^{-1}e^{3A}(c_+^-v^+ - c_+^+v^-) \wedge u \wedge \star_7 \lambda(f_{\pm}) \right]. \end{aligned} \tag{C.19}$$

Putting this all together we find (B.9) is equivalent to the 7d conditions

$$d_{H_3}(e^{A-\Phi}\Psi_{\mp}^{++}) \pm \frac{c_{\pm}^+}{16}f_{\pm} = 0, \quad (\text{C.20a})$$

$$d_{H_3}(e^{A-\Phi}\Psi_{\mp}^{--}) \pm \frac{c_{\pm}^-}{16}f_{\pm} = 0, \quad (\text{C.20b})$$

$$d_{H_3}(e^{2A-\Phi}\Psi_{\pm}^{++}) \mp 2me^{A-\Phi}\Psi_{\mp}^{++} = \frac{c_{\pm}^+}{16}e^{3A} \star_7 \lambda(f_{\pm}), \quad (\text{C.20c})$$

$$d_{H_3}(e^{2A-\Phi}\Psi_{\pm}^{--}) \pm 2me^{A-\Phi}\Psi_{\mp}^{--} = \frac{c_{\pm}^-}{16}e^{3A} \star_7 \lambda(f_{\pm}), \quad (\text{C.20d})$$

$$d_{H_3}(e^{2A-\Phi}(\Psi_{\pm}^{+-} + \Psi_{\pm}^{-+})) = 0, \quad (\text{C.20e})$$

$$d_{H_3}(e^{-\Phi}(\Psi_{\pm}^{+-} - \Psi_{\pm}^{-+})) = \frac{1}{8}\tilde{\xi} \wedge f_{\pm}, \quad (\text{C.20f})$$

$$d_{H_3}(e^{A-\Phi}(\Psi_{\mp}^{+-} + \Psi_{\mp}^{-+})) \pm me^{-\Phi}(\Psi_{\pm}^{+-} - \Psi_{\pm}^{-+}) = \mp \frac{1}{8}e^A(\chi_{-}^{1\dagger}\chi_{+}^1 - \chi_{-}^{2\dagger}\chi_{+}^2)f_{\pm}, \quad (\text{C.20g})$$

$$d_{H_3}(e^{3A-\Phi}(\Psi_{\mp}^{+-} - \Psi_{\mp}^{-+})) \pm e^{3A-\Phi}h_0(\Psi_{\pm}^{+-} - \Psi_{\pm}^{-+}) \\ \pm 3me^{2A-\Phi}(\Psi_{\pm}^{+-} + \Psi_{\pm}^{-+}) = \pm \frac{1}{8}e^{3A}\tilde{\xi} \wedge \star_7 \lambda(f_{\pm}). \quad (\text{C.20h})$$

The first four of these are simply the distinct conditions one would get from an $\mathcal{N} = (1, 0)$ and $\mathcal{N} = (0, 1)$ 10 dimensional bi-linear respectively (see the previous appendix). The Bianchi identities and equations of motion of the RR flux away from the loci of sources impose that

$$d_{H_3}f_{\pm} = 0, \quad d_{H_3}(e^{3A} \star_7 \lambda(f_{\pm})) + e^{3A}h_0f_{\pm} = 0. \quad (\text{C.21})$$

Notice that when the $dH_3 = 0$ is assumed and given (C.14e), d_{H_3} (C.20a), d_{H_3} (C.20b) and d_{H_3} (C.20f) combined with (C.20e) imply the first of these when respectively c_{\pm}^+, c_{\pm}^- and $\chi_{-}^{1\dagger}\chi_{+}^1 - \chi_{-}^{2\dagger}\chi_{+}^2$ are assumed to be non zero; the latter indeed cannot vanish for us, as we concluded in (C.15). Similarly d_{H_3} (C.20c) and d_{H_3} (C.20d) reproduce the second in (C.21); this is true in general, as both $c_{\pm}^{\pm} \neq 0$. Notice also that (C.20a)–(C.20b) imply that in IIA

$$c_{\pm}^{\pm}F_0 = 0. \quad (\text{C.22})$$

In other words a non trivial Romans mass $F_0 \neq 0$ is only possible when $c_{\pm}^{\pm} = 0$, which by (C.14e) implies that the electric NS flux $h_0 = 0$; conversely, $h_0 \neq 0$ is only possible when $F_0 = 0$. Further, in IIA the zero-form part of (C.20g), (C.20h) imply

$$e^{3A-\Phi}h_0(\Psi_0^{+-} - \Psi_0^{-+}) + 3me^{2A-\Phi}(\Psi_0^{+-} + \Psi_0^{-+}) = 0, \\ me^{-\Phi}(\Psi_0^{+-} - \Psi_0^{-+}) = -\frac{1}{8}e^A(\chi_{-}^{1\dagger}\chi_{+}^1 - \chi_{-}^{2\dagger}\chi_{+}^2)F_0. \quad (\text{C.23})$$

The second of these defines F_0 . Since one of (h_0, F_0) is zero, we always have

$$\Psi_0^{-+} + \Psi_0^{+-} = 0, \quad h_0(\Psi_0^{-+} - \Psi_0^{+-}) = 0, \quad (\text{C.24})$$

In IIB one can always use the $SL(2, \mathbb{R})$ invariance to move to a duality frame where $h_0 = 0$, so one can safely fix $h_0 = 0$ and so $c_{\pm}^{\pm} = 0$ modulo S-duality. We now note that one

cannot set $\tilde{\xi} = 0$ and preserve $\mathcal{N} = (1, 1)$ simultaneously: given the rest of the constraints this would mean that all the bi-linears $(\Psi^{\pm\pm}, \Psi^{\pm\mp})$ are proportional to each other, it then quickly becomes apparent that (C.20d)–(C.20e) cannot be made consistent with (C.20g) when $\tilde{\xi} = 0$.

It is a generic feature of this kind of reduction of 10 dimensional bi-linears on a warped product that some of the conditions (C.20a)–(C.20h) will be implied by the others. To see this one can generally exploit two identities that follow from the definition of Ψ and from supersymmetry:

$$\mathcal{L}_K \Psi^{(10)} = 0, \quad (K \lrcorner + \tilde{K} \wedge) \Psi^{(10)} = 0 \quad (\text{C.25})$$

In the case at hand, since K only has components along v^\pm , it is simple to show that the first of these holds trivially, and it is the second that yields some constraints on the 7d bi-linears. One can show that this requires

$$\tilde{\xi} \wedge (\Psi_{\pm}^{+-} - \Psi_{\pm}^{-+}) = \pm e^A (c_+^- \Psi_{\mp}^{++} + c_+^+ \Psi_{\mp}^{--}), \quad (\text{C.26a})$$

$$\tilde{\xi} \wedge (\Psi_{\mp}^{+-} + \Psi_{\mp}^{-+}) = \pm (e^A (c_+^- \Psi_{\pm}^{++} - c_+^+ \Psi_{\pm}^{--}) + (\chi_-^{1\ddagger} \chi_+^1 - \chi_-^{2\ddagger} \chi_+^2) (\Psi_{\pm}^{+-} - \Psi_{\pm}^{-+})), \quad (\text{C.26b})$$

$$2e^A \tilde{\xi} \wedge \Psi_{\mp}^{--} = \pm (c_+^- e^{2A} (\Psi_{\pm}^{+-} + \Psi_{\pm}^{-+}) + c_-^- (\Psi_{\pm}^{+-} - \Psi_{\pm}^{-+})), \quad (\text{C.26c})$$

$$2e^A \tilde{\xi} \wedge \Psi_{\mp}^{++} = \pm (-c_+^+ e^{2A} (\Psi_{\pm}^{+-} + \Psi_{\pm}^{-+}) + c_+^+ (\Psi_{\pm}^{+-} - \Psi_{\pm}^{-+})), \quad (\text{C.26d})$$

$$2\tilde{\xi} \wedge \Psi_{\pm}^{--} = \mp (c_+^- e^A (\Psi_{\mp}^{+-} - \Psi_{\mp}^{-+}) + e^{-A} c_-^- (\Psi_{\mp}^{+-} + \Psi_{\mp}^{-+}) - 2(\chi_-^{1\ddagger} \chi_+^1 - \chi_-^{2\ddagger} \chi_+^2) \Psi_{\mp}^{--}), \quad (\text{C.26e})$$

$$2\tilde{\xi} \wedge \Psi_{\pm}^{++} = \mp (c_+^+ e^A (\Psi_{\mp}^{+-} - \Psi_{\mp}^{-+}) - e^{-A} c_+^+ (\Psi_{\mp}^{+-} + \Psi_{\mp}^{-+}) + 2(\chi_-^{1\ddagger} \chi_+^1 - \chi_-^{2\ddagger} \chi_+^2) \Psi_{\mp}^{++}), \quad (\text{C.26f})$$

$$e^A \tilde{\xi} \wedge (\Psi_{\pm}^{+-} + \Psi_{\pm}^{-+}) = \mp (c_-^- \Psi_{\mp}^{++} - c_-^+ \Psi_{\mp}^{--}), \quad (\text{C.26g})$$

$$e^A \tilde{\xi} \wedge (\Psi_{\mp}^{+-} - \Psi_{\mp}^{-+}) = \mp ((c_-^- \Psi_{\pm}^{++} + c_-^+ \Psi_{\pm}^{--}) - e^A (\chi_-^{1\ddagger} \chi_+^1 - \chi_-^{2\ddagger} \chi_+^2) (\Psi_{\pm}^{+-} + \Psi_{\pm}^{-+})). \quad (\text{C.26h})$$

Using these identities one can establish that $\tilde{\xi} \wedge$ (C.20g) implies (C.20f) given (C.20c), (C.20d) and (C.14c). Similarly one can generate (C.20h) by wedging $\tilde{\xi}$ with either of (C.20c) and (C.20d), finally (C.20a) and (C.20b) give rise to (C.20e). Thus many of these conditions can easily be shown to be implied by others.

To have a sufficient system of conditions that imply $\mathcal{N} = (1, 1)$ supersymmetry, strictly speaking, we should also solve (3.1c) and (3.1d) of [66] as we did to derive the $\mathcal{N} = (1, 0)$ pairing constraint in the previous section. These are the hardest to deal with generically, though in certain cases they are implied such as for $\text{Mink}_d/\text{AdS}_d$ vacua in $d > 3$. Here we will work smart rather than hard. In the previous appendix we give necessary conditions for $\mathcal{N} = 1$ super symmetry, here now we have sufficient conditions if we are solving two independent copies of these, an $\mathcal{N} = (1, 0)$ set and an $\mathcal{N} = (0, 1)$ set — eqs. (C.20a)–(C.20b) already give all the conditions from these independent $\mathcal{N} = 1$ sub-sectors, except the pairing constraints that follow from (3.1c) and (3.1d) of [66]. Specifically these conditions take the

form

$$\begin{aligned}(\Psi_{\mp}^{++}, f_{\pm})_7 &= \mp \frac{c_+^+ m + \frac{1}{2} e^{-A} c_-^+ h_0}{4} e^{-\Phi} \text{vol}(M_7), \\(\Psi_{\mp}^{--}, f_{\pm})_7 &= \pm \frac{c_+^- m - \frac{1}{2} e^{-A} c_-^- h_0}{4} e^{-\Phi} \text{vol}(M_7),\end{aligned}\tag{C.27}$$

We have necessary conditions for supersymmetry if we also solve these — any additional conditions that one can extract by plugging the $\mathcal{N} = (1, 1)$ ansatz into (3.1c) and (3.1d) of [66] are necessarily implied by these. However, it is possible to show that the two conditions in (C.27) are dependent by (C.14) and (C.20) and in particular that the linear combination

$$(c_+^- \Psi_{\mp}^{++} + c_+^+ \Psi_{\mp}^{--}, f_{\pm})_7 = 0,\tag{C.28}$$

is implied in general. To this end one needs some identities: first off

$$(c_+^- \Psi_{\mp}^{++} + c_+^+ \Psi_{\mp}^{--}, d_{H_3}(\Psi_{\mp}^{+-} + \Psi_{\mp}^{-+}))_7 = -(d_{H_3}(c_+^- \Psi_{\mp}^{++} + c_+^+ \Psi_{\mp}^{--}), \Psi_{\mp}^{+-} + \Psi_{\mp}^{-+})_7.\tag{C.29}$$

This follows from $(c_+^- \Psi_{\mp}^{++} + c_+^+ \Psi_{\mp}^{--}, \Psi_{\mp}^{+-} + \Psi_{\mp}^{-+})_6 = 0$ and the definition of λ . The former can be easily proved with a concrete parametrisation of the 7d bi-linears such as (3.10)–(3.11) (these hold for $c_+^+ = c_+^- = 2$ and $c_-^+ = c_-^- = 0$ but generalising to generic (c_+^{\pm}, c_-^{\pm}) is straightforward). Similarly one can show that

$$(c_+^- \Psi_{\mp}^{++} + c_+^+ \Psi_{\mp}^{--}, \Psi_{\mp}^{+-} - \Psi_{\mp}^{-+})_7 = 0,\tag{C.30}$$

$$(c_+^- \Psi_{\mp}^{++} + c_+^+ \Psi_{\mp}^{--}, X_1 \wedge (\Psi_{\mp}^{+-} + \Psi_{\mp}^{-+}))_7 = (X_1 \wedge (c_+^- \Psi_{\mp}^{++} + c_+^+ \Psi_{\mp}^{--}), \Psi_{\mp}^{+-} + \Psi_{\mp}^{-+})_7 = 0,$$

for X_1 any one-form on M_7 . Using these identities one finds that $(c_+^- \Psi_{\mp}^{++} + c_+^+ \Psi_{\mp}^{--}, f_{\pm})_7 = 0$ is indeed implied given (C.20a), (C.20b), (C.20g). Thus (C.27) is implied by just

$$(c_+^- \Psi_{\mp}^{++} - c_+^+ \Psi_{\mp}^{--}, f_{\pm})_7 = \mp \left(\frac{m c_+^+ c_+^-}{2} + \frac{1}{8} e^{-A} (c_-^+ c_+^- - c_-^- c_+^+) h_0 \right) e^{-\Phi} \text{vol}(M_7).\tag{C.31}$$

Establishing whether this too is redundant is more challenging as $(c_+^- \Psi_{\mp}^{++} - c_+^+ \Psi_{\mp}^{--}, \Psi_{\mp}^{+-} + \Psi_{\mp}^{-+})_6 \neq 0$ in general. Thus one needs to exploit the torsion classes of the $SU(3) \times SU(3)$ structure that the internal manifold supports to establish whether it is also implied. In the main text we focus on the case where M_7 supports an $SU(3)$ -structure; for such manifolds we find that (C.31) is indeed implied, but this remains an open question for $SU(3) \times SU(3)$ -structure more broadly.

This concludes our analysis of $\mathcal{N} = (1, 1)$ supersymmetric AdS_3 ; a summary of these results is given in the main text in section 2 where we have fixed

$$c_+^+ = c_+^- = 2 \quad \Rightarrow \quad c_-^+ = -c_-^- = c, \quad \Rightarrow \quad e^{3A} h_0 = -mc.\tag{C.32}$$

It is not hard to see that this can be achieved without loss of generality: (C.14d) ensures that Ψ^{st} for $s, t = \pm$ has an overall $\sqrt{c_+^s c_+^t}$ factor in its definition. Since c_{\pm}^{\pm} must also have a c_{\pm}^{\pm} factor in their definition and $c_{\pm}^{\pm} > 0$, they may be factored out of (C.20a)–(C.20d). Likewise $(\chi_-^{1\ddagger} \chi_+^1 - \chi_-^{2\ddagger} \chi_+^2)$ and $\tilde{\xi}$ contain a $\sqrt{c_+^+ c_+^-}$ factor, so c_{\pm}^{\pm} can also be factored out of (C.20e)–(C.20h), making the precise values of $c_{\pm}^{\pm} > 0$ immaterial. Choosing $c_+^+ = c_+^- = 2$ then leads to the other conditions in (C.32) by (C.14e).

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